Modal Analysis of Nutation Drive with Double Circular Arc Spiral Bevel Gear

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Abstract. Nutation gear transmission combines nutation drive mode with double arc tooth profile, which has the advantage of large deceleration ratio and solves various problems in the traditional involute gear profile transmission. In his pa-per, the numerical solution and MATLAB software are used to solve the free vibration equation of the nutation system. And the undamped free vibration natural characteristics of the nutation transmission system of bevel gears are obtained. Then it reveals the influence of different physical parameters on the natural frequency of the system. The study and analysis mode shape of double circular spiral bevel gears under the natural frequency are carried out by using finite element software ANSYS. The natural frequencies of the system and each double circular arc bevel gear are compared and analyzed. After it the natural frequencies of the nutation drive system with 150%, 200% and 250% average meshing stiffness and 150%, 200% and 250% nutation gear support stiffness are calculated and compared. At last chapter, we get the orders vibration pattern and know the 7th order is the most influential one by finite element analysis in the ANASY. The natural frequencies of the system and each double arc bevel gear are compared and analyzed.

Keywords: Nutation drive · Double circular arc · Spiral bevel gear · Modal analysis

1 Introduction

Nutation drive of double circular spiral bevel gears is a new type of coaxial transmission. It has the advantages of large transmission ratio, simple and compact structure, small volume, small noise, large bearing capacity, stable transmission and high efficiency [1, 2]. It can be widely used in machine tools, instruments and petrochemical industry. Modal is the inherent vibration characteristic of mechanical structure [3–6]. Every modal has a specific natural frequency, damping ratio and modal mode. Therefore, modal analysis can be used to describe the motion process of gear transmissive system.

At present, there are many researches on modal analysis of gear transmission system, and great achievements have been made. Zhang established the elastic kinematics equation of the planetary gear train with small tooth difference and carried out

D. Wang et al. (Eds.): MeTrApp 2019, MMS 79, pp. 99–109, 2020.
https://doi.org/10.1007/978-981-15-0142-5_11

modal analysis on it, which can obtain more accurate inherent characteristics [7]. Zhang studied and analyzed the influence of spiral angle of double circular arc gear on modal analysis [8]. Zhang analyzed the influence of the modulus of hyperbolic normal circular arc gear, tooth number and transmission ratio on modal analysis [9]. After that, Deng and Zhang analyzed the effects with modulus and machining tool diameter for circular tooth line of cylindrical gears on modal analysis [10]. These provide a theoretical basis for the optimization design of gear transmission.

However, the research on the modal analysis of nutation gear transmission is still blank after literature review and retrieval. Due to the influence of time-varying meshing stiffness excitation and meshing impact, it is difficult to distribute the load equally and reasonably in the nutation transmission system of bevel gears, and the uneven distribution of load will lead to the increase of vibration and noise. Furthermore, the reliability, stability, transmission precision and life of the nutation drive will be seriously affected. So, modal analysis of nutation gear transmission system has important significance for vibration reduction and noise reduction.

2 Theoretical Principle of Modal Analysis

Modal analysis is a modern method which applies the system discrimination method to the engineering vibration field to research the structural dynamic characteristics. The research is mainly on the inherent vibration characteristics of mechanical structures. Generally, modal parameters can be obtained by calculation or experimental analysis, that is, modal parameters can be obtained by finite element method or the system input and output signals collected by experiments [11, 12].

The vibration differential equation of nutation reducer system can be expressed as follows:

$$M\ddot{x} + C\ddot{x} + Kx = f(t) \tag{1}$$

In Eq. (1), M, C and K are equivalent mass matrix, damping coefficient matrix and stiffness matrix of nutation drive system. The variable parameters x, $\dot{x}$ and $\ddot{x}$ are vibration displacement, vibration velocity and vibration acceleration. Function f (t) is external excitation load vector.

The theoretical modal analysis is based on the static finite element model. In view of the inherent characteristics (natural frequency, modal mode) of the system, the influence of external load can be ignored, and the damping term can be ignored because the damping effect is small. Thus, the equations of undamped free vibration can be established as follows:

$$M\ddot{x} + K\ddot{x} = 0 \tag{2}$$

The non-diagonal factor of stiffness matrix is not 0, so it is a non-diagonal matrix. Therefore, if there is elastic coupling in the system, then the dynamic equation of the system is a nonlinear coupled differential equation system. It is assumed that the mass points in the gear system are simply harmonic motion, the vibration i of harmonic motion is ω_i. Initial phase is ω_{i}. The corresponding mode shape is X_i. Take the main vibration $x = X_i \sin(\omega_i t + \varphi_i)$ and $\ddot{x} = \omega_i^2 X_i \sin(\omega_2 + \varphi_2)$ into Eq. (2). The corresponding characteristic equation can be obtained as follows.

$$\left(K - \omega_i^2 M\right) X_i = 0 \tag{3}$$

Set

$$S = M^{-1} K \tag{4}$$

Regular matrix S is called dynamic matrix of the system, which is also called the system matrix.

For such a n-order linear and homogeneous system of Eq. (3), it is necessary to get a non-zero solution that the determinant of the coefficient matrix is zero.

$$det\left(K - \omega_i^2 M\right) = 0 \tag{5}$$

This is a real coefficient equation of degree n about ω_i^2, with n pairs of roots. The n pairs of roots are named feature pairs, feature vector X_i and feature value ω_i^2. The ω_i^2 and X_i, which meet Eq. (5), are called modal parameter. It can be used to characterize the vibration characteristics of the system. Feature vector can reflect the spatial form of gear system vibration according to natural frequency ω_i^2, so it can also be called mode vector or mode.

For the actual system structure, n vibration frequencies (from small to large) can be obtained by formula (5): $\omega_1 < \omega_2 < \omega_3 < \omega_4 < \ldots < \omega_{\mathrm{n}}$.

For each natural frequency, the amplitude of a set of nodes can be determined according to Eq. (3), X_i (i = 1, 2, 3, …, n). The ratio of each component is kept constant. The numerical value is arbitrary, which is called eigenvector. In vibration theory, it is often referred to as the vibration mode of free vibration of structure.

Based on the above theory, this paper uses modal analysis to solve the natural frequency of the nutation gear transmission system.

The nutation gear transmission system is shown in Fig. 1.

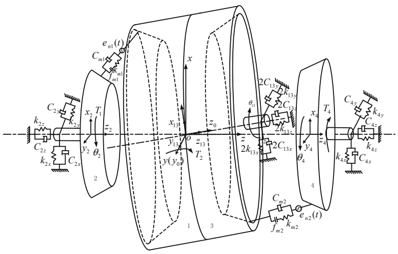

Fig. 1. Dynamic model of bevel gear nutation transmission system

Figure 1 is the dynamic model of the nutation transmission system of bevel gears [13]. It is described as follows:

Lateral bending vibration: each bevel gear is described by the translational degrees of freedom x_i, y_i (i = 13, 2, 4) in the x, y-directions. Axial vibration: each bevel gear is described translational degrees of freedom z_i in the z-direction. Torsional vibration: each bevel gear is described by the rotation of degrees of freedom θ_i around the z axis. Therefore, the outer bevel gear 1 and 3 are regarded as one whole and the system is converted into three gear parts, each gear considers 4 degrees of freedom for a total of 12 degrees of freedom. So, only the conditional parameters of three components (the numbers of them are 2, 13, 4) need to be considered.

From this dynamic model, the stiffness matrix K, Quality matrix M, and displacement vector X in the Eq. (2) can be obtained. X is [x_2, y_2, z_2, θ_2, x-$_{13}$, y_{13}, z_{13}, θ_{13}, x_4, y_4, z_4, θ_4] and M is [m_2, m_2, m_2, $J_{2,}$ m_{13}, m_{13}, m_{13}, $J_{13,}$ m_4, m_4, m_4, J_4]. The stiffness matrix K is a matrix about K_m and K_d, which is too complicate to make detailed elaboration.

3 Modal Analysis of Nutation Transmission System of Double Arc Spiral Bevel Gear

This chapter uses MATLAB to calculate the natural frequency. And the factors affecting this natural frequency are analyzed in this chapter.

3.1 Solution of Modal Analysis About Bevel Gear Nutation Transmission System

On the previous chapter, a brief introduction of theoretical principle of modal analysis and dynamic model of the nutation transmission system. Then using these equations, we can get the ω_i we need in Eq. (5).

So, take K and M of Eq. (5) into MATLAB to calculate the dynamic differential equation. The parameters we need know are shown in Table 1. In this system, Support stiffness (K_i) is no difference in each direction, and bearing damping (C_i) is the same, which means $k_{ix} = k_{iy} = k_{iz} = k_i$ and $C_{ix} = C_{iy} = C_{iz} = C_i$.

Table 1. Parameters for bearing stiffness and support damping.

Essential parameter (k_i)	Numerical value (N/m)	Essential parameter (C_i)	Numerical value (N·m/s)
Support stiffness k_2	2.2 × 108	Bearing damping C_2	787
Support stiffness k_{13}	1.7 × 108	Bearing damping C_{13}	1803
Support stiffness k_4	2.9 × 108	Bearing damping C_4	2951

Table 2. Natural frequency of bevel gear nutation transmission system.

Order	1	2	3	4	5	6
Natural frequencies/Hz	0	6600	7870	7880	7880	16160
Order	7	8	9	10	11	12
Natural frequencies/Hz	16330	296300	35970	70210	107120	285490

From Table 1, the stiffness matrix K has been defined. And the quality matrix M can be obtained by measurement. According to the Eq. (5), the natural frequencies ω_i of the bevel gear nutation transmission system are obtained as shown in Table 2.

As can be seen, the minimum natural frequency of the nutation drive system is 0 Hz, which indicates that there is a rigid body motion at the beginning. Except the initial rigid body motion, the first 5 orders′ natural frequencies of the whole nutation drive system are all small. After 6th order, the natural frequency is much large and the natural frequency span of the whole nutation transmission system is also large. Generally, the first ten natural frequencies have main influence on the system.

3.2 Modal Analysis of Nutation Transmission System with Different Mean Meshing Stiffness

The influence of different parameters on the natural frequencies of the system is very important for modal analysis. As mentioned in Sect. 2, the stiffness matrix K is a matrix about the mean meshing stiffness K_m and bearing support stiffness K_d. And in Eq. (5), quality matrix is determined with the determination of gears. Therefore, the mean meshing stiffness K_m and bearing support stiffness K_d are chosen to do comparative analysis.

Make the mean meshing stiffness of the two pairs of gears 150%, 200% and 250% times of the original one. The natural frequencies of different meshing stiffness are calculated as presented in Fig. 2.

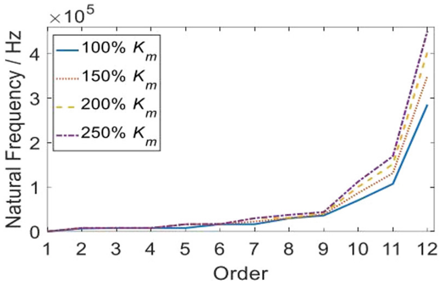

Fig. 2. Natural frequencies under different mean meshing stiffness K_m.

From Fig. 2, the trends of the natural frequency are similar. While, the natural frequency is larger when the mean meshing stiffness K_m growing. Specifically, this phenomenon is more obvious in the 5th order, 7th order and last forth orders. We can get the conclusion that the mean meshing stiffness K_m has slight effect on first 5 orders, but has more effect on the higher orders, especially, after 9th order. In order to see the law of influence more clearly, we've done a further deal with the data, which is shown in Fig. 3. The rate is relative errors compared with original meshing stiffness (100% K_m) under different meshing stiffness (150% K_m, 200% K_m and 250% K_m).

Figure 3 describes the rate of relative errors about natural frequency with growth of mean meshing stiffness K_m. The mean meshing stiffness K_m has the biggest influence on the natural frequency, but the magnitude makes little difference. After 7th order, the change of natural frequency has a positive correlation with mean meshing stiffness K_m. And the influence is unstable until the 9th order. Therefore, the meshing stiffness K_m has biggest effect on 5th order, and show more effect on the bigger order. Actually, the high order natural frequency is too high to make difference on gear transmission. So, when design a gear reducer, the meshing stiffness K_m can be ignored except on 5th order.

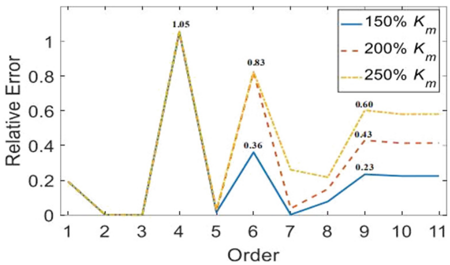

Fig. 3. The relative error of the natural frequency under different mean meshing stiffness K_m.

3.3 Modal Analysis of Nutation System Under Different Bearing Support Stiffness

In the same way, we can calculate the natural frequency of nutation transmission system when the bearing support stiffness K_d of nutation gear is 150%, 200% and 250% times of the original bearing support stiffness K_d. By comparing the natural frequency of bevel gear nutation drive system under different support stiffness K_d, the influence of bearing support stiffness K_d of nutation gear on this system is analyzed.

Through the natural frequency results of 150%, 200% and 250% of the original support stiffness K_d (Fig. 4), it can be seen that the support stiffness K_d of nutation gear has a greater effect on the low-order natural frequency, but less influence on the higher order. The natural frequencies of 10th–12th orders remain constant under different nutation gear support stiffness K_d. The bearing stiffness of nutation gear has great influence on the 6th and 7th order. The specific impact of natural frequency is shown in Fig. 5 in the same way.

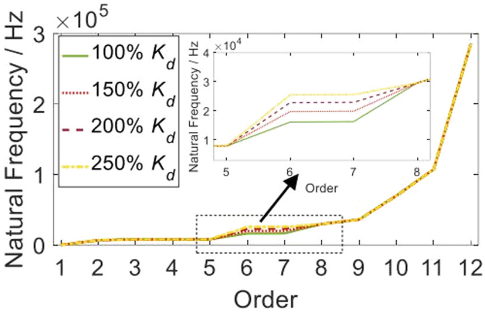

Fig. 4. Natural frequency under different bearing support stiffness K_d.

In this part, we using different support stiffness K_d to replace the mean meshing stiffness K_m as the contrast variables. Its influence is mainly on the 5th order and 6th order. And the bigger the support stiffness is, the greater the effect is. As mentioned before, influence on the lower order should be noticed more when we design such a nutation reducer, the support stiffness is necessary to check.

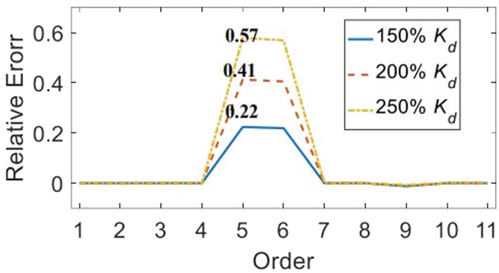

Fig. 5. Relative error of natural frequency under different support stiffness.

3.4 Modal Analysis of Nutation Transmission System of Double Arc Bevel Gear

After obtaining the natural frequency of nutation transmission system and the influence of parameters on the natural frequencies, it is necessary to complete the validation of natural frequency and vibration pattern to explore the natural frequency of the whole and the risk of resonance by using finite element method.

The finite element model is shown in Fig. 6. The material properties in this model of each double circular spiral bevel gear are material, modulus of elasticity E, Poisson's ratio μ and mass density. The material of gears is structural steel and the mass density is 7850 kg/m^3, and E = 206 GPa, μ = 0.29. The resulting natural frequency is shown in Fig. 7.

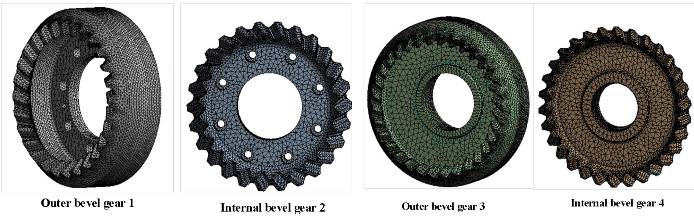

Fig. 6. Meshing diagram of gears.

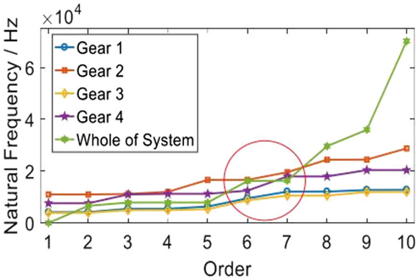

Fig. 7. Natural frequency of bevel gears and nutation transmission system

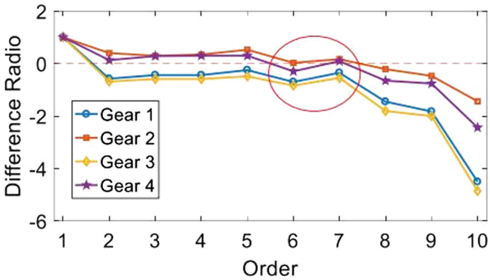

Fig. 8. Difference ratio of gears and nutation transmission system

In Figs. 7 and 8, there is no intersection between the gears and the system at the natural frequency. So, resonance does not occur under the current system. But we should notice the support stiffness to prevent resonance on 6^{th} and 7^{th} order, which each has a natural frequency of gear 2 and gear 4 that approaching the natural frequency of the system. There is a great possibility of resonance, especially when the support stiffness k_d is bigger as shown in Fig. 5.

Table 3. Vibration types of gears 1–4

Order	Type of gear 1	Type of gear 2	Type of gear 3	Type of gear 4
1	Torsional	Torsional	Torsional	Torsional
2	Torsional	Torsional	Torsional	Torsional
3	Folding	Circumferential	Folding	Folding
4	Folding	Folding	Folding	Folding
5	Umbrella	Folding	Umbrella	Bending
6	Circumferential	Bending	Circumferential	Circumferential
7	Bending	Bending	Bending	Bending
8	Bending	Circumferential	Bending	Bending
9	Radial	Bending	Radial	Radial
10	Radial	Bending	Radial	Radial

Combining with the animation and mode diagram of each order vibration process of double circular spiral bevel gear displayed by ANSYS Workbench post-processing program, the low order array of double circular spiral bevel gear can be classified into torsional vibration, diagonal vibration, umbrella vibration, circular vibration, and so on. Bending vibration and radial vibration are the types of modes of vibration (Table 3).

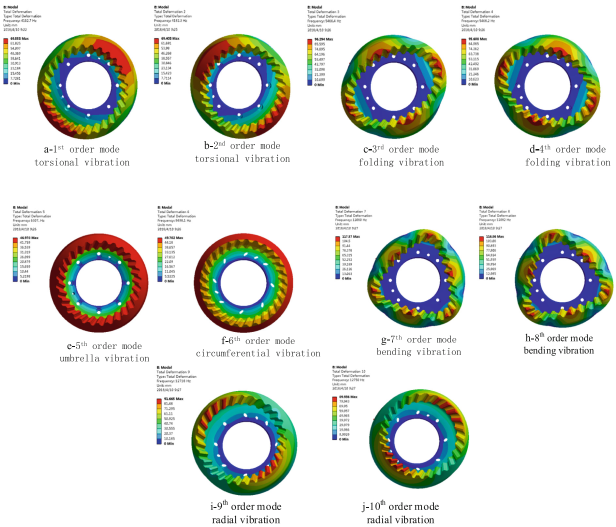

Fig. 9. First ten orders vibration patterns of gear 1.

Take the mode diagram of bevel gear 1 as an example, the shapes of each order of gear 1 are shown in the Fig. 9. The main results are as follows:

(1) The torsional vibration of bevel gear is as follows: the face of bevel gear swings forward and backward along a line of radial axis, as shown in Fig. 9(a) and (b).

(2) The main forms of counter-bending vibration are as follows: the axial direction of bevel gear is V-shaped, and it is regular polygon on the end face, as shown in Fig. 9 (c) and (d).

(3) The shape of umbrella vibration is as follows: the tooth of bevel gear retracts along the axis to the big end to form an umbrella shape, as shown in Fig. 9(e).

(4) The form of circular vibration is that there is no vibration and displacement in the axial direction of bevel gear, as shown in Fig. 9(f).

(5) Bending vibration: the main form is the regular wave array in the axial direction and the regular polygon on the end face, as shown in Fig. 9(g) and (h).

(6) Radial vibration: the main manifestation is that the bevel gear expands along the radial direction. The elliptical shape appears on the end of the bevel gear, and the axial vibration is basically non-vibrating, as shown in Fig. 9(i) and (j).

As illustrated in Fig. 9, the minimum natural frequency of outer bevel gear 1 is 4162.7 Hz and maximum natural frequency is 12750 Hz. The maximum deformation is in the 7th order. There are some natural frequencies of some gears are similar, such as 1st order and 2nd order, the 7th order and 8th order, the 9th order and 10th order. The main vibration modes of bevel gear 1 are torsional vibration, bending vibration, bevel vibration, circumferential vibration, bending vibration and radial vibration. It can be seen that the first and second order vibration modes of double circular spiral bevel gear are torsional vibration, the middle order vibration is umbrella type vibration, for the folded vibration and circular vibration, while the higher order vibration is bending vibration and radial vibration, and the frequency of adjacent order is close.

Because of the maximum deformation is in the 7th order, which is same to each gear, the resonance happened in this order is most dangerous. When design a reducer of bevel gear nutation transmission system, the resonance in 7th order needs more attention. To avoid large deformations, we need get the 7th order of nutation transmissive system and bypass it by making sure the vibration frequency at stable rotational speed away from it.

4 Conclusion

In this paper, the modal analysis of the nutation transmission system about double circular spiral bevel gear is carried out. Then it calculates the natural frequency of the nutation transmission system by using dynamic differential equation of nutation drive with double circular arc spiral bevel gear. After that, this paper studies the influence of different mean meshing stiffness and support stiffness on the natural frequencies. The comparing analysis of the natural frequencies under 150%, 200% and 250% times of meshing stiffness and support stiffness are presented. The results show that the average meshing stiffness mainly affects the higher order natural frequency of nutation transmission system, while the bearing support stiffness of the nutation gear mainly affects the low-order natural frequency of nutation drive system. So, it is better to paid more attention to the support stiffness, when design a nutation transmission system about double circular spiral bevel gear. At last, based on the ANSYS Workbench platform, the modes of each double circular spiral bevel gear in nutation drive reducer are analyzed, the first 10 modes are extracted, and the first 10 modes of double circular spiral bevel gear are judged. And find out the most influential natural frequency, which we should pay more attention on in further studies.

Acknowledgments. The authors acknowledge the support of the NSFC (Grant Nos. 51775114 and 51875105).

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