Chapter 15
A Rough Decision-Making Model for Biomaterial Selection

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1 Introduction

Biomaterials can be interpreted as the category of materials used in medical devices and their degree of sophistication has increased significantly. The benefits of engineered materials incorporate unsurprising mechanical properties and simplicity of their treatment. Wide range of scientific projects in material engineering files is shifted to biomaterials application. Those materials interact and meet with biological systems requirement for specific medical purposes are categorized as biomaterials. This section of engineering body is extracted from synthetic polymers by a variety of chemical processes utilizing metallic components, polymers, ceramics, or composite materials. It is essential to mention biomaterials are very typical in today's dental applications, surgery, and drug delivery [1, 2]. Bioengineering consists of biological, chemical, tissue engineering medicine, and material science subjects. Range of applications and utilizations like building artificial organs, rehabilitation devices, or implants to replace natural body tissues are encountered in this area. The world of

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material engineering science confronts to a brilliant concept and nowadays, many companies are trying to invest and support projects for the development of new products. An unstructured model for comparing and analyzing biomedical materials for a specific application may lead to huge failure of the product, repeated processes, considerable loss, impairment of tissue functions and overall increasing of the costs. Biomaterials have changed the demands of customers and medical services and are constantly used in human organs for treating heart diseases, coronary angioplasty, orthopaedics applications, and orthodental structures [3, 4]. The biomaterials are classified into metals, polymers, ceramics, composites, and apatite. In order to deal with the problem of human degenerative diseases, investigators effectively rely on the use of artificial or natural biomaterials for reinstating the functions of affected parts. The specification and properties each engineer seeks in a biomaterial must contain the following item: biomechanical compatibility, biocompatibility, high corrosion and wear resistance, and osseointegration [5, 6]. Biomaterials also play a vital role in fabrication of biological screening devices as well as in a large range of non-biomedical applications. Discussions on some common and familiar biomaterials are presented here. One of the vastly used biomaterials is the metallic implants which are the primary materials used for joint replacement and orthopedic applications. Exceptional thermal conductivity, excellent strength, higher fracture toughness, corrosion resistance, and hardness along with biocompatibility are the several promising properties that the metallic alloys and materials possess. Stainless steel is employed to fabricate artificial bone and becomes the predominant implant alloy due to ease in fabrication and having required mechanical properties and corrosion resistance. Cobalt-based alloys, Ti alloys, ceramic materials, zirconia ceramics, and polymeric materials are other well-known types of biomaterials [7–9]. Biomaterials are selected to simultaneously satisfy a broad range of fundamental requirements from mechanical to biological aspects. The construction of the femoral implant must meet several criteria like adequate strength, ductility, elastic modulus, wear resistance, corrosion resistance, biocompatibility, and osseointegration. For example, according to Hafezalkotob and Hafezalkotob [10], density and elastic modulus properties are also examined for compatible designs which certainly are strategically necessary implants and prostheses for material applications. Biocompatibility basically signifies the potentiality and fitness of a material for not being malignant or physiologically sensitive with living organisms. This basic requirement is considered to be the most important issue in the design and selection of implants and the material to be used for its fabrication [11]. The very required process of engineering and design decision making is how to deal with the complex system, decision-making rules, many variables, and parameters. The recognition of this approach not even build a robust decision system, it effectively carries out a quality of results and further approval.

A decision-making system comes up with situations where sort of alternatives (choices) are evaluated with respect to certain factors or criteria. It is valuable to resolve the decision problem with a well-established mechanism to reveal the optimal solution. Although, this mechanism must satisfy the policy maker's viewpoints, however, all conflicting objectives with different optimization direction cause errors and shape uncertain and incorrect conditions. This point leads academic and industrial partners to tolerate pressures which needs extraordinary endeavor. Therefore, it is argued that the key item in almost all of the engineering decisions is to draw the objectives and map the alternative options in order to overcome existing complexities. Certainly, application of conventional methods has been saturated while many engineering sectors are reforming their evaluation and measurement systems. Undoubtedly, choosing advanced methods is highly appreciated and today's material investigators in real projects understand various concepts and logics to adopt a comprehensive formula in order to take more efficient decisions. All in all, the realization of multi-criteria decision-making (MCDM) methods in such kind of situations can refine the question of what methodology fits to what decision problem and in what way. It eliminates the complexity of decision problem in a productive manner and formulates a platform to the assessment and selection of the optimal solution [12–14]. Some MCDM techniques are able to configure the decision problem containing alternatives, factors, and decision makers' (DMs) opinions, break it to hierarchical format, analyze, normalize, and finally find the solution. To name them, analytical hierarchy process (AHP) [15], technique of order preference by similarity to ideal solution (TOPSIS) [16, 17], Vlse Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [18–21], complex proportional assessment (COPRAS) [22–26], multi-objective optimization on the basis of ratio analysis (MOORA) [27], evaluation based on distance from average solution (EDAS) [28, 29] and combinative distance-based assessment (CODAS) [30, 31] are some of the examples. However, if the selection methodology is carried out randomly or disorganizedly, there will be the risk of overseeing suitable materials and criteria affecting the entire selection process, Hence, the aim of this chapter is to develop a methodology, based on rough AHP and rough CODAS methods, followed by sensitivity analysis and performance comparison to select the most suited biomaterial for a hip joint prosthesis application.

2 Literature Review on Biomaterials Selection

Despite there being many MCDM models for general material selection problems, the literature shows very less amount of works to deal with the problems on biomaterials selection. Thus, the aim of this section is to study the past researchers on biomaterial selection and figuring out their weaknesses and enable the DMs to reduce subjectivity and uncertainty to make a clearer support for a strong framework. Bahraminasab and Jahan [32] designed a comprehensive biomaterial selection model for femoral component of total knee replacement (TKR) while employing comprehensive VIKOR method. Jahan and Edwards [33] proposed a weighting technique for dependent and target-based criteria in making optimal decision for biomaterials selection and validated its appropriateness with the extended TOPSIS and comprehensive VIKOR methods. Bahraminasab et al. [34] conducted a multi-objective design optimization process for femoral component of TKR. Petković et al. [35] designed a decision support system while integrating three MCDM tools, i.e., TOPSIS, VIKOR, and weighted aggregated sum product assessment (WASPAS) methods for identifying the best biomaterial alternative for bone implants which could compensate the missing part of a long bone. Hafezalkotob and Hafezalkotob [36] explored the application of comprehensive MULTIMOORA method for hip and knee joint prosthesis materials selection. Chowdary et al. [37] proposed a strategy to prioritize some bioengineering materials under a combined MCDM model with fuzzy AHP and TOPSIS. The research recommends that Polyether ether ketone (PEEK) material is most suitable for biomedical implantations. Kabir and Lizu [38] adopted an integrated FAHP and PROMETHEE methods for selection of femoral material in TKR. Abd et al. [39] employed fuzzy TOPSIS method for hip joint prosthesis material selection. Ristić et al. [40] devised an expert system using fuzzy sets for biomaterial selection in a customized implant application. Hafezalkotob and Hafezalkotob [10] validated the application of interval MULTIMOORA method with target values of attributes based on interval distance and preference degree while utilizing two case studies on hip and knee joint prosthesis materials selection.

3 Materials and Methods

3.1 Rough Numbers and Operations

Rough numbers (RNs), consisting of the upper, lower, and boundary intervals, determine the intervals of multiple expert evaluations without requiring any additional information and relying only on the original data [41]. Hence, the obtained expert preferences objectively represent and improve the decision-making process. The definition of RNs according to Song et al. [42] is given below.

Let U be a universe containing all the objects and X be a random object from U. Then, it is assumed that there exists a set of k classes which represents a DM's preferences, $R = (J_1, J_2, \ldots, J_k)$ with the condition $J_1 < J_2 <, \ldots, < J_k$. Then for every $X \in U, J_q \in R, 1 \le q \le k$, the lower approximation $\underline{\mathrm{Apr}}(J_q)$, the upper approximation $\overline{\mathrm{Apr}}(J_q)$, and the boundary interval $\mathrm{Bnd}(J_q)$ are determined as follows:

$$\underline{\mathrm{Apr}}(J_q) = \cup\left\{X \in U/R(X) \le J_q\right\} \tag{1}$$

$$\overline{\mathrm{Apr}}(J_q) = \cup\left\{X \in U/R(X) \ge J_q\right\} \tag{2}$$

$$\begin{aligned}\mathrm{Bnd}(J_q) &= \cup\left\{X \in U/R(X) \ne J_q\right\}\\ &= \left\{X \in U/R(X) > J_q\right\} \cup \left\{X \in U/R(X) < J_q\right\}\end{aligned} \tag{3}$$

The object can be represented by a rough number with the lower limit $\underline{\mathrm{Lim}}(J_q)$ and the upper limit $\overline{\mathrm{Lim}}(J_q)$ in Eqs. (4)–(5).

$$\underline{\mathrm{Lim}}(J_q) = \frac{1}{M_\mathrm{L}} \sum R(X)|X \in \underline{\mathrm{Apr}}(J_q) \tag{4}$$

$$\overline{\mathrm{Lim}}(J_q) = \frac{1}{M_\mathrm{U}} \sum R(X)|X \in \overline{\mathrm{Apr}}(J_q) \tag{5}$$

where M_L and M_U represent the sum of objects given in the lower and upper object approximations of J_q, respectively. For object J_q, the rough boundary interval ($\mathrm{IRBnd}(J_q)$) is the interval between the lower and upper limits [43]. The rough boundary interval presents a measure of uncertainty. A bigger $\mathrm{IRBnd}(J_q)$ value shows that the variations in experts' preferences exist, while smaller values show that experts' opinions do not differ considerably. All the objects between the lower limit $\underline{\mathrm{Lim}}(J_q)$ and the upper limit $\overline{\mathrm{Lim}}(J_q)$ of the rough number $\mathrm{RN}(J_q)$ are included in $\mathrm{IRBnd}(J_q)$. Since RNs belong to a group of interval numbers, arithmetic operations applied to interval numbers are also appropriate for RNs.

3.2 R-AHP Method

As one of the most popular methods of MCDM, the AHP has widely been used for criteria weight estimation in a wide range of applications [43]. AHP is a structured technique for organizing and analyzing complex decisions. It uses the definitions of relative importance to evaluate the weights of the selection criteria. It also bestows flexibility in quantifying the consistency in DMs' preferences in a group decision-making system (GDMS). Due to the presence of uncertainty, subjectivity, and unreliability in GDM, this chapter uses a RN-based AHP method to exploit judgments and imprecision. The succeeding section provides a detail description of the adopted methodology for applying the RN-based AHP method for estimation of criteria weights.

Step 1. Pairwise comparisons of the criteria:

Assuming that there exists a group of m experts $\{e_1, e_2, \ldots, e_m\}$ and n criteria $\{c_1, c_2, \ldots c_n\}$, each expert should determine the degree of mutual influence of criteria i and $j (\forall i, j \in n)$. For this purpose, a pairwise comparative analysis of the ith and jth criteria sets for kth expert $(1 \le k \le m)$ is made and denoted by the values $\xi_{ij}^k (i, j = 1, 2, \ldots, n; k = 1, 2, \ldots, m)$, which were performed using Saaty's 9-point scale [44] and shown by the following matrix.

$$N^{(k)} = \left[\xi_{ij}^{(k)}\right]_{n\times n} = \begin{bmatrix} \xi_{11}^{(k)} & \xi_{12}^{(k)} & \cdots & \xi_{1n}^{(k)} \\ \xi_{21}^{(k)} & \xi_{22}^{(k)} & \cdots & \xi_{2n}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ \xi_{n1}^{(k)} & \xi_{n2}^{(k)} & \cdots & \xi_{nn}^{(k)} \end{bmatrix}; \quad 1 \le i, j \le n; \quad 1 \le k \le m \tag{6}$$

where $\xi_{ij}^{(k)}$ are linguistic Equations of Saaty's 9-point scale and $\xi_{ij}^{(k)} = 1$ if $i = j$.

Step 2. Estimation of weights of the Experts':

For each matrix $N^{(k)}$, consistency of the expert judgement is verified in two steps using the consistency ratio (CR) concept [45]. At first, consistency index (CI) is computed, followed by which CR is estimated using the standard relationship between CI and the random index (RI). A CR value of less than or equal to 0.10 indicates consistent judgments made by the experts [46].

The weight importance of the experts are now determined using Eqs. (7) and (8).

$$\delta_k = \frac{1}{\mathrm{CR}_k}; \quad 1 \le e \le k \tag{7}$$

where CR_k is the CR of the kth expert, and δ_k is the weight of expert k $(1 \le k \le m)$.

$$w_k = \frac{\delta_k}{\sum_{k=1}^{m} \delta_k} \tag{8}$$

where δ_k is the weight coefficient of the expert $k(1 \le k \le m)$and w_k is normalized weight coefficient of the expert k and $\sum_{k=1}^{m} w_k = 1$.

Step 3. Construction of an averaged rough comparison matrix (ARCM):

Using Eqs. (1)–(5), elements $\xi_{ij}^{(k)}$ of comparison matrix $N^{(k)}$ are now transformed into RNs as $\mathrm{RN}(\xi_{ij}^{(k)}) = \left[\underline{\mathrm{Lim}}(\xi_{ij}^{(k)}), \overline{\mathrm{Lim}}(\xi_{ij}^{(k)})\right] = \left[\xi_{ij}^{(k)-}, \xi_{ij}^{(k)+}\right]$, where $\underline{\mathrm{Lim}}(\xi_{ij}^{(k)})$ is the lower approximation of the object class $\xi_{ij}^{(k)}$, and $\overline{\mathrm{Lim}}(\xi_{ij}^{(k)})$ is the upper approximation. In this way, for each pairwise comparison matrix, rough sequences are obtained as $\mathrm{RN}(\xi_{ij}^{(k)}) = \left\{\left[\underline{\mathrm{Lim}}(\xi_{ij}^{(1)}), \overline{\mathrm{Lim}}(\xi_{ij}^{(1)})\right], \left[\underline{\mathrm{Lim}}(\xi_{ij}^{(2)}), \overline{\mathrm{Lim}}(\xi_{ij}^{(2)})\right], \ldots, \left[\underline{\mathrm{Lim}}(\xi_{ij}^{(m)}), \overline{\mathrm{Lim}}(\xi_{ij}^{(m)})\right]\right\}$.

For each matrix $N^{(k)}$, we get rough sequence $\mathrm{RN}(\xi_{ij}^{(k)}) = \left[\underline{\mathrm{Lim}}(\xi_{ij}^{(k)}), \overline{\mathrm{Lim}}(\xi_{ij}^{(k)})\right]$ on the position (i, j) and finally by applying Eq. (9), we get the averaged rough number $\mathrm{RN}(\xi_{ij}) = \left[\underline{\mathrm{Lim}}(\xi_{ij}), \overline{\mathrm{Lim}}(\xi_{ij})\right] = \left[\xi_{ij}^{-}, \xi_{ij}^{+}\right]$

$$\begin{aligned} \mathrm{RN}(\xi_{ij}) &= \mathrm{RN}\left\{\left[\xi_{ij}^{(1)-}, \xi_{ij}^{(1)+}\right], \left[\xi_{ij}^{(2)-}, \xi_{ij}^{(2)+}\right], \ldots, \left[\xi_{ij}^{(m)-}, \xi_{ij}^{(m)+}\right]\right\} \\ &= \begin{cases} \xi_{ij}^{-} = \prod_{k=1}^{m} \left(\xi_{ij}^{(k)-}\right)^{w_k} \\ \xi_{ij}^{+} = \prod_{k=1}^{m} \left(\xi_{ij}^{(k)+}\right)^{w_k} \end{cases} \end{aligned} \tag{9}$$

where $\underline{\mathrm{Lim}}(\xi_{ij}^{(k)}) = \xi_{ij}^{(k)-}$ and $\overline{\mathrm{Lim}}(\xi_{ij}^{(k)}) = \xi_{ij}^{(k)-}$, respectively, represents the lower and upper approximation of the $\mathrm{RN}(\xi_{ij}^{(k)})$.

Based on Eqs. (8) and (9), we obtain averaged rough comparison matrix as:

$$N = \begin{bmatrix} 1 & \left[\xi_{12}^{-}, \xi_{12}^{+}\right] & \cdots & \left[\xi_{1n}^{-}, \xi_{1n}^{+}\right] \\ \left[\xi_{21}^{-}, \xi_{21}^{+}\right] & 1 & \cdots & \left[\xi_{2n}^{-}, \xi_{2n}^{+}\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\xi_{n1}^{-}, \xi_{n1}^{+}\right] & \left[\xi_{n2}^{-}, \xi_{n2}^{+}\right] & \cdots & 1 \end{bmatrix}_{n \times n} \tag{10}$$

Step 4. Computation of priority vector:

Priority vector (PV) is the rough weight $\mathrm{RN}(w_j)$ which is determined for each criterion. We obtain the rough weight coefficient $\mathrm{RN}(w_j)$ by applying Eqs. (11)–(13). By applying Eq. (11), the following values are estimated.

$$\mathrm{RN}(\xi'_{ij}) = \sum\nolimits_{j=1}^{n} \mathrm{RN}(\xi_{ij}) = \left[\sum\nolimits_{j=1}^{n} \xi_{ij}^{-}, \sum\nolimits_{j=1}^{n} \xi_{ij}^{+}\right] \tag{11}$$

and dividing matrix elements of N with the values obtained from Eq. (11), the normalized matrix (W) is calculated.

$$\mathrm{RN}(\varpi_{ij}) = \left[\varpi_{ij}^{-}, \varpi_{ij}^{+}\right] = \frac{\mathrm{RN}(\xi_{ij})}{\sum_{j=1}^{n} \mathrm{RN}(\xi_{ij})} = \frac{\left[\xi_{ij}^{-}, \xi_{ij}^{+}\right]}{\left[\sum_{j=1}^{n} \xi_{ij}^{-}, \sum_{j=1}^{n} \xi_{ij}^{+}\right]} \tag{12}$$

A normalized matrix for the rough weights is obtained as follows:

$$W = \begin{bmatrix} 1 & \left[\varpi_{12}^{-}, \varpi_{12}^{+}\right] & \cdots & \left[\varpi_{1n}^{-}, \varpi_{1n}^{+}\right] \\ \left[\varpi_{21}^{-}, \varpi_{21}^{+}\right] & 1 & \cdots & \left[\varpi_{2n}^{-}, \varpi_{2n}^{+}\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\varpi_{n1}^{-}, \varpi_{n1}^{+}\right] & \left[\varpi_{n2}^{-}, \varpi_{n2}^{+}\right] & \cdots & 1 \end{bmatrix}_{n \times n} \tag{13}$$

where $\mathrm{RN}(\varpi_{ij}) = \left[\varpi_{ij}^{-}, \varpi_{ij}^{+}\right]$ represents a normalized weights coefficients of matrix (10).

The final rough criteria weights are determined using Eq. (14).

$$\mathrm{RN}(w_j) = \left[\frac{1}{n}\sum_{i=1}^{n} \varpi_{ij}^{-}, \frac{1}{n}\sum_{i=1}^{n} \varpi_{ij}^{+}\right] \tag{14}$$

The criteria weights are placed in the interval $\mathrm{RN}(w_j) = \left[w_j^{-}, w_j^{+}\right]$ where the condition is satisfied that $0 \le w_j^{-} \le w_j^{+} \le 1$ for each evaluation criteria $c_j \in C$ ($C = \{c_1, c_2, \ldots c_n\}$). Since these are rough criteria weights, using Eq. (14), the actual weight coefficients are obtained, where $\sum_{j=1}^{n} w_j^{-} \le 1$ and $\sum_{j=1}^{n} w_j^{+} \ge 1$. It satisfies the conditions $w_j \in [0, 1]$ and $j = 1, 2, \ldots, n$.

3.3 Rough CODAS Method

In this section, an extension of the original CODAS method based on RNs (R-CODAS) has been proposed to deal with the associated uncertainties. CODAS is an efficient method, introduced by Ghorabaee et al. [30]. The procedural steps of the proposed R-CODAS method are now presented below [30, 31]:

Step 1. Construct the basic rough decision matrix (Φ):

First step is evaluating b alternatives to n criteria. Evaluation of the alternatives per each criteria by $k(1 \le k \le m)$ expert is denoted as $\eta_{ij}^{(k)}$, where $i = 1,\ldots, b$; $j = 1,\ldots, n$. The judgment of k expert is presented as matrix $\Phi^{(k)} = [\eta_{ij}^{(k)}]_{b\times n}$, where $1 \le k \le m$.

$$\Phi^{(k)} = \begin{bmatrix} \eta_{11}^{(k)} & \eta_{12}^{(k)} & \cdots & \eta_{1n}^{(k)} \\ \eta_{21}^{(k)} & \eta_{22}^{(k)} & \cdots & \eta_{2n}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{b1}^{(k)} & \eta_{b2}^{(k)} & \cdots & \eta_{bn}^{(k)} \end{bmatrix}_{b\times n} ; \quad 1 \le i \le b; \quad 1 \le j \le n; \quad 1 \le k \le m \tag{15}$$

In accordance with this, $\Phi^{(1)}, \Phi^{(2)}, \ldots, \Phi^{(m)}$ matrices are the judgment matrices of each of m experts.

Using Eqs. (1)–(5), each sequence $\eta_{ij}^{(k)}(i = 1, 2, \ldots, b;\ j = 1, 2, \ldots, n)$ of $\Phi^{(k)} = [\eta_{ij}^{(k)}]_{b\times n}$ is transformed into rough sequence $\mathrm{RN}(\eta_{ij}^{(k)}) = \left[\underline{\mathrm{Lim}}(\eta_{ij}^{(k)}), \overline{\mathrm{Lim}}(\eta_{ij}^{(k)})\right] = \left[\eta_{ij}^{(k)-}, \eta_{ij}^{(k)+}\right]$, where $\underline{\mathrm{Lim}}(\eta_{ij}^{(k)}) = \eta_{ij}^{(k)-}$ and $\overline{\mathrm{Lim}}(\eta_{ij}^{(k)}) = \eta_{ij}^{(k)+}$ represent lower and upper limits of the rough sequence $\mathrm{RN}(\eta_{ij}^{(k)})$, respectively.

For each matrix $\Phi^{(k)} = [\eta_{ij}^{(k)}]_{b\times n}$, we get the rough sequence $\mathrm{RN}(\eta_{ij}^{(k)}) = \left[\underline{\mathrm{Lim}}(\eta_{ij}^{(k)}), \overline{\mathrm{Lim}}(\eta_{ij}^{(k)})\right] = \left[\eta_{ij}^{(k)-}, \eta_{ij}^{(k)+}\right]$ on the position (i, j) and finally by applying Eq. (16), we get the averaged RN $\mathrm{RN}(\eta_{ij}) = \left[\underline{\mathrm{Lim}}(\eta_{ij}), \overline{\mathrm{Lim}}(\eta_{ij})\right] = \left[\eta_{ij}^{-}, \eta_{ij}^{+}\right]$

$$\begin{aligned} \mathrm{RN}(\eta_{ij}) &= \mathrm{RN}\left\{\left[\eta_{ij}^{(1)-}, \eta_{ij}^{(1)+}\right], \left[\eta_{ij}^{(2)-}, \eta_{ij}^{(2)+}\right], \ldots, \left[\eta_{ij}^{(m)-}, \eta_{ij}^{(m)+}\right]\right\} \\ &= \begin{cases} \eta_{ij}^{-} = \frac{1}{m}\sum\limits_{k=1}^{m} \eta_{ij}^{(k)-} \\ \eta_{ij}^{+} = \frac{1}{m}\sum\limits_{k=1}^{m} \eta_{ij}^{(k)+} \end{cases} \end{aligned} \tag{16}$$

where $\underline{\mathrm{Lim}}(\eta_{ij}^{(k)}) = \eta_{ij}^{(k)-}$ and $\overline{\mathrm{Lim}}(\eta_{ij}^{(k)}) = \eta_{ij}^{(k)-}$, respectively, represent the lower and upper approximation of the $\mathrm{RN}(\eta_{ij}^{(k)})$.

Based on Eqs. (3) and (16), we obtain the following ARCM:

$$\Phi = \begin{bmatrix} [\eta_{11}^-, \eta_{11}^+] & [\eta_{12}^-, \eta_{12}^+] & \cdots & [\eta_{1n}^-, \eta_{1n}^+] \\ [\eta_{21}^-, \eta_{21}^+] & [\eta_{22}^-, \eta_{22}^+] & \cdots & [\eta_{2n}^-, \eta_{2n}^+] \\ \vdots & \vdots & \ddots & \vdots \\ [\eta_{b1}^-, \eta_{b1}^+] & [\eta_{b2}^-, \eta_{b2}^+] & \cdots & [\eta_{bn}^-, \eta_{bn}^+] \end{bmatrix}_{b\times n} \tag{17}$$

where $\mathrm{RN}(\eta_{ij}) = \left[\underline{\mathrm{Lim}}(\eta_{ij}), \overline{\mathrm{Lim}}(\eta_{ij})\right] = \left[\eta_{ij}^-, \eta_{ij}^+\right]$ denotes the value of the ith alternative for the jth criterion ($i = 1, 2, \ldots, b$; $j = 1, 2, \ldots, n$). The matrix elements $\mathrm{RN}(\eta_{ij})$ in Eq. (17) are RNs determined by the experts or by using the aggregation of the experts' decisions.

Step 2. Normalization of basic matrix element:

Determine RN normalized decision matrix $N = [\hat{\eta}_{ij}]_{b\times n}$ for beneficial and non-beneficial (cost criteria) criteria by the following expressions:

$$\hat{\eta}_{ij} = \begin{cases} \left[\frac{\eta_{ij}^- - \eta_j^-}{\eta_j^+ - \eta_j^-}, \frac{\eta_{ij}^+ - x_j^-}{\eta_j^+ - \eta_j^-}\right]; \text{ if } j \in B, \\ \left[\frac{\eta_{ij}^+ - \eta_j^+}{\eta_j^- - \eta_j^+}, \frac{\eta_{ij}^- - \eta_j^+}{\eta_j^- - \eta_j^+}\right]; \text{ if } j \in C \end{cases} \tag{18}$$

where $\eta_j^+ = \max_i(\eta_{ij})$, $\eta_j^- = \min_i(\eta_{ij})$, B and C represent the sets of beneficial and non-beneficial criteria, respectively, and $\hat{\eta}_{ij}$ denotes the normalized RN values.

Step 3. Calculate RN-weighted normalized decision matrix (R):

The RN-weighted normalized matrix $R = [r_{ij}]_{b\times n}$ is calculated as follows

$$\mathrm{RN}(r_{ij}) = \mathrm{RN}(w_j) \cdot \mathrm{RN}(\hat{\eta}_{ij}) = \left[w_j^-, w_j^+\right] \cdot \left[\hat{\eta}_{ij}^-, \hat{\eta}_{ij}^+\right] \tag{19}$$

where $\mathrm{RN}(w_j) = \left[w_j^-, w_j^+\right]$ represents the final RN criteria weight of jth criterion and $\mathrm{RN}(r_{ij}) = \left[r_{ij}^-, r_{ij}^+\right]$ indicates weighted normalized values.

Step 4. Determine FR negative-ideal solution:

We obtain the RN negative-ideal solution matrix $\mathrm{NS} = [\mathrm{RN}(ns_j)]_{1\times n}$ as follows

$$ns_j = \min_i r_{ij} = \left[\min\left\{r_{ij}^-\right\}, \min\left\{r_{ij}^+\right\}\right] \tag{20}$$

Step 5. Calculate the RN-weighted Euclidean (ED_i) *and RN-weighted Hamming* (HD_i) *distances of alternatives from the RN negative-ideal solution*

We obtain ED_i and HD_i as per [47, 48] and shown as follows:
RN-weighted Euclidean (ED_i) distances:

$$\mathrm{ED}_i = \sum_{j=1}^{n} d_E\left(r_{ij}; ns_j\right) \tag{21}$$

where $d_E\left(r_{ij}; ns_j\right)$ we obtain as follows

$$d_E\left(r_{ij}; ns_j\right) = \sqrt{\frac{\left\{r_{ij}^{-} - ns_j^{-}\right\}^2 + \left\{r_{ij}^{+} - ns_j^{+}\right\}^2}{2}} \tag{22}$$

RN-weighted Hamming (HD_i) distances

$$\mathrm{HD}_i = \sum_{j=1}^{n} d_H\left(r_{ij}; ns_j\right) \tag{23}$$

where $d_H\left(r_{ij}; ns_j\right)$ we obtain as follows

$$d_H\left(r_{ij}; ns_j\right) = \frac{\left|r_{ij}^{-} - ns_j^{-}\right| + \left|r_{ij}^{+} - ns_j^{+}\right|}{2} \tag{24}$$

Step 6. Determine the relative assessment matrix (RA):

By applying Eq. (25), we obtain elements of the relative assessment matrix $\mathrm{RA} = [p_{ie}]_{b\times b}$

$$p_{ie} = (\mathrm{ED}_i - \mathrm{ED}_e) + (g(\mathrm{ED}_i - \mathrm{ED}_e) \times (\mathrm{HD}_i - \mathrm{HD}_e)) \tag{25}$$

where $e \in \{1, 2, \ldots, b\}$ and g is a threshold function, defined as follows [49]:

$$g(x) = \begin{cases} 1 \text{ if } |x| \geq \theta \\ 0 \text{ if } |x| < \theta \end{cases} \tag{26}$$

The threshold parameter (θ) of this function can be set by the DM. In this study, we use $\theta = 0.02$ for the calculations.

Step 7. Calculate the assessment score (AS_i) of each alternative:

By applying Eq. (27), we obtain assessment score

$$\mathrm{AS}_i = \sum_{e=1}^{b} p_{ie} \tag{27}$$

The alternative with the highest assessment score is the most desirable alternative. Figure 1 illustrates the rough number-based decision-making model in a comprehensive way.

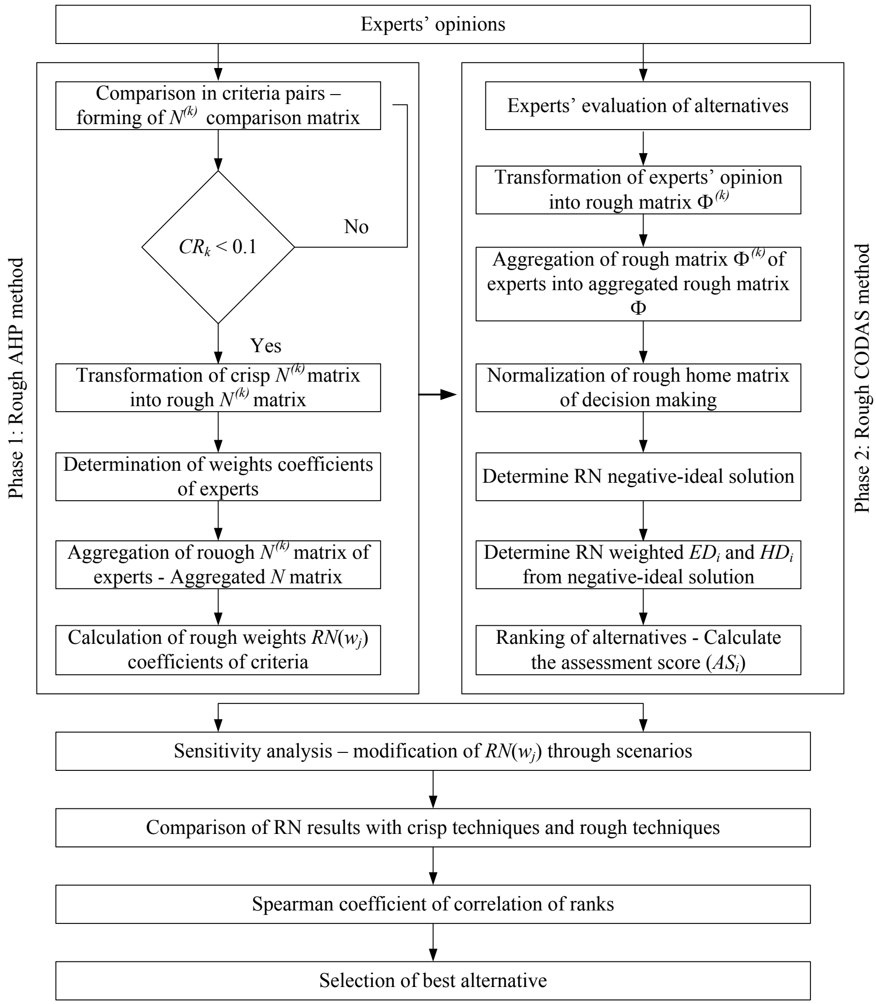

Fig. 1 Proposed rough decision-making model

4 R-AHP-CODAS Model Application

The implementation viability of the proposed R-AHP-CODAS model is now explored via a real-time illustrative example of material selection for femoral element for hip joint prosthesis. Human hip prosthesis has three major parts, namely femoral component, acetabular cup, and acetabular interface. Among these, the femoral element is a rigid metallic rod inserted into the hollow femur. The acetabular cup is normally fitted with ilium. On the other hand, the acetabular interface lies between the femoral component and the acetabular cup. It is made by various materials like metals, polymers, and ceramics to reduce the amount of frictional wear. Evaluation matrix for the considered problem consists of eleven alternative biomaterials and nine criteria which can be found in [39, 50]. Among the nine considered nine attributes, tissue tolerance (C_1), corrosion resistance (C_2), tensile strength (C_3), fatigue strength (C_4), toughness (C_5), and wear resistance (C_6) are the beneficial attributes, whereas elastic modulus (C_7), density (C_8), and cost (C_9) are the attributes regarded as non-beneficial. The concerned study included the expertise of five experts who evaluated the considered alternatives.

4.1 Implementation of R-AHP Model

Step 1. Pairwise comparisons of the criteria:

After the five experts evaluated each of the nine considered criteria, a comparison matrix between the criteria pairs has been developed, as shown in Table 1.

Step 2. Determination of weights of the experts:

After the pairwise comparison, consistency ratio is estimated. Now, using Eqs. (7) and (8), criteria weights are determined, as presented in Table 2.

$$\delta_1 = \frac{1}{\mathrm{CR}_1} = \frac{1}{0.0695} = 14.378;$$
$$\delta_2 = \frac{1}{\mathrm{CR}_2} = \frac{1}{0.0849} = 11.772;$$
$$\delta_3 = \frac{1}{\mathrm{CR}_3} = \frac{1}{0.0980} = 10.208;$$
$$\delta_4 = \frac{1}{\mathrm{CR}_4} = \frac{1}{0.0861} = 11.621;$$
$$\delta_5 = \frac{1}{\mathrm{CR}_5} = \frac{1}{0.0752} = 13.299.$$

Using Eq. (8), the experts' weights are obtained as follows:

$$w_{E1} = \frac{14.3788}{14.378 + 11.772 + 10.208 + 11.621 + 13.299} = 0.2346;$$
$$w_{E2} = \frac{11.7718}{14.378 + 11.772 + 10.208 + 11.621 + 13.299} = 0.1921;$$
$$w_{E3} = \frac{10.2080}{14.378 + 11.772 + 10.208 + 11.621 + 13.299} = 0.1666;$$
$$w_{E4} = \frac{11.6210}{14.378 + 11.772 + 10.208 + 11.621 + 13.299} = 0.1896;$$
$$w_{E5} = \frac{13.2998}{14.378 + 11.772 + 10.208 + 11.621 + 13.299} = 0.2170.$$

Table 1 Developed pairwise comparison matrix of the evaluating criteria

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	1;1;1;1;1	4;4;7;1;0.1	6;9;8;6;5	2;2;3;3;3	2;5;4;3;2	2;5;3;2;2	1;3;4;2;3	0.3;0.5;0.5;1;0.5	1;2;3;2;2
C_2	0.3;0.3;0.1;1;9	1;1;1;1;1	4;9;7;4;9	3;3;4;3;7	3;6;3;6;7	3;7;6;4;5	2;4;5;7;9	0.5;1;0.5;2;3	1;4;1;1;4
C_3	0.2;0.1;0.1;0.2;0.2	0.3;0.1;0.1;0.3;0.1	1;1;1;1;1	0.3;0.2;0.3;0.1;0.2	0.3;0.3;0.3;0.3;0.5	0.3;0.3;0.3;0.3;0.5	0.3;0.3;0.5;0.3;0.3	0.3;0.3;0.2;0.3;0.5	0.5;0.3;0.5;0.2;0.3
C_4	0.5;0.5;0.3;0.3;0.3	0.3;0.3;0.3;0.3;0.1	3;6;3;8;5	1;1;1;1;1	2;4;2;3;4	2;3;2;6;3	1;4;3;2;4	0.5;0.5;0.3;0.5;1	0.5;1;0.5;0.3;1
C_5	0.5;0.2;0.3;0.3;0.5	0.3;0.2;0.3;0.2;0.1	3;3;3;4;2	0.5;0.3;0.5;0.3;0.3	1;1;1;1;1	1;1;2;2;1	1;2;4;3;2	0.5;0.3;0.3;1;0.3	0.5;1;0.5;0.3;1
C_6	0.5;0.2;0.3;0.5;0.5	0.3;0.1;0.2;0.3;0.2	3;3;4;3;2	0.5;0.3;0.5;0.2;0.3	1;1;0.5;0.5;1	1;1;1;1;1	1;1;2;0.5;1	0.5;0.1;0.3;0.3;0.1	0.5;0.3;0.3;0.3;0.3
C_7	1;0.3;0.3;0.5;0.3	0.5;0.3;0.2;0.1;0.1	3;4;2;3;3	1;0.3;0.3;0.5;0.3	1;0.5;0.3;0.3;0.5	1;1;0.5;2;1	1;1;1;1;1	0.5;0.3;0.5;0.3;0.2	0.5;0.2;0.5;0.3;0.3
C_8	3;2;2;1;2	2;1;2;0.5;0.3	3;3;6;4;2	2;2;3;2;1	2;4;4;1;3	2;7;3;3;7	2;3;2;3;6	1;1;1;1;1	0.5;2;2;2;2
C_9	1;0.5;0.3;0.5;0.5	1;0.3;1;1;0.3	2;4;2;5;3	2;1;2;3;1	2;1;2;3;1	2;4;3;3;3	2;5;2;3;3	2;0.5;0.5;0.5;0.5	1;1;1;1;1

Table 2 CR_k matrix of comparison and weights of experts

Expert	CR_k	δ_k	w_k
E1	0.0695	14.3788	0.2346
E2	0.0849	11.7718	0.1921
E3	0.0980	10.2080	0.1666
E4	0.0861	11.6210	0.1896
E5	0.0752	13.2998	0.2170

Step 3. Calculation of ARCM:

ARCM is now estimated based on the data of Table 1 and using Eqs. (1)–(5), the components $\xi_{ij}^{(k)}$ of the CM $N^{(k)}$are transformed into rough number $RN(\xi_{ij}^{(k)}) = \left[\underline{Lim}(\xi_{ij}^{(k)}), \overline{Lim}(\xi_{ij}^{(k)})\right]$ and five rough matrices $N^{(k)}$ (k = 1, 2, … 5) are obtained. Determination of the rough elements of CM $N^{(1)}$, $N^{(2)}$, …, $N^{(5)}$, for C_1–C_3 are illustrated here. For every $N^{(k)}$ matrix, rough sequences that make up the rough number $RN(\xi_{13}^{(k)}) = \left[\underline{Lim}(\xi_{13}^{(k)}), \overline{Lim}(\xi_{13}^{(k)})\right]$ are achieved. From Table 1, for C_1–C_3 position, the object class $\xi_{13}^{(k)}$ with five elements $\xi_{13}^{(k)} = \{6; 9; 8; 6; 5\}$ is selected. Now, using Eqs. (1)–(5), we determined the rough sequences as follows:

$$\underline{Lim}(6) = \frac{1}{3}(6+6+5) = 5.67, \overline{Lim}(6) = \frac{1}{4}(6+9+8+6) = 7.25;$$
$$\underline{Lim}(9) = \frac{1}{5}(6+9+8+6+5) = 6.80, \overline{Lim}(9) = 9;$$
$$\ldots$$
$$\underline{Lim}(5) = 5, \overline{Lim}(5) = \frac{1}{5}(6+9+8+6+5) = 6.80;$$

This way, we get five rough sequences, as shown below:

$$RN(\xi_{13}^{(1)}) = [5.67, 7.25]; RN(\xi_{13}^{(2)}) = [6.8, 9.0]; RN(\xi_{13}^{(3)}) = [6.25, 8.5];$$
$$RN(\xi_{13}^{(4)}) = [5.67, 7.25]; RN(\xi_{13}^{(5)}) = [5.0, 6.8].$$

By applying Eq. (9) and the experts' weights of Table 2, we get the ARN as $RN(\xi_{13}) = \left[\underline{Lim}(\xi_{13}), \overline{Lim}(\xi_{13})\right] = \left[\xi_{13}^{-}, \xi_{13}^{+}\right]$

$$RN(\xi_{13}) = \left[\xi_{13}^{-}, \xi_{13}^{+}\right] = RN(\xi_{13}^{(1)}, \xi_{13}^{(2)}, \ldots, \xi_{13}^{(5)})$$
$$= \begin{cases} \xi_{13}^{-} = (5.67)^{0.2346} \cdot (6.80)^{0.1921} \cdots (5.00)^{0.2170} \\ \xi_{ij}^{+} = (7.25)^{0.2346} \cdot (9.0)^{0.1921} \cdots (6.80)^{0.2170} \end{cases}$$

This way, we obtained the values of Table 3.

Step 4. Computation of the PV

Table 3 Averaged matrix

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	[1, 1]	[0.96, 4.61]	[5.81, 7.65]	[2.32, 2.82]	[2.38, 3.85]	[2.18, 3.32]	[1.76, 3.18]	[0.44, 0.67]	[1.57, 2.3]
C_2	[0.37, 3.75]	[1, 1]	[5.1, 7.79]	[3.24, 4.76]	[3.91, 5.89]	[3.88, 5.9]	[3.46, 6.98]	[0.75, 1.97]	[1.38, 2.81]
C_3	[0.13, 0.17]	[0.13, 0.21]	[1, 1]	[0.17, 0.29]	[0.31, 0.4]	[0.31, 0.4]	[0.3, 0.39]	[0.25, 0.39]	[0.28, 0.44]
C_4	[0.36, 0.44]	[0.23, 0.32]	[3.69, 6.14]	[1, 1]	[2.43, 3.51]	[2.4, 3.95]	[1.85, 3.49]	[0.46, 0.68]	[0.5, 0.83]
C_5	[0.28, 0.44]	[0.18, 0.28]	[2.61, 3.32]	[0.3, 0.43]	[1, 1]	[1.13, 1.59]	[1.63, 2.97]	[0.31, 0.64]	[0.5, 0.83]
C_6	[0.33, 0.47]	[0.17, 0.27]	[2.6, 3.31]	[0.28, 0.44]	[0.68, 0.92]	[1, 1]	[0.82, 1.32]	[0.2, 0.37]	[0.31, 0.4]
C_7	[0.34, 0.65]	[0.16, 0.33]	[2.65, 3.34]	[0.31, 0.64]	[0.37, 0.69]	[0.84, 1.34]	[1, 1]	[0.28, 0.44]	[0.3, 0.44]
C_8	[1.62, 2.35]	[0.65, 1.6]	[2.69, 4.36]	[1.58, 2.3]	[1.85, 3.49]	[3.06, 5.57]	[2.41, 3.99]	[1, 1]	[1.28, 1.93]
C_9	[0.46, 0.69]	[0.46, 0.86]	[2.4, 3.9]	[1.32, 2.2]	[1.32, 2.2]	[2.6, 3.32]	[2.37, 3.59]	[0.56, 0.99]	[1, 1]

Based on the data of Table 3 and Eqs. (11) and (12), the normalized values of weights coefficients are computed, as given in Table 4.

The determination of the C_1–C_3 element of Table 4 is obtained by the following computations. Using Eq. (11), the following values are first calculated.

$$\mathrm{RN}(\xi_{13}^{'}) = \sum_{i=1}^{9} \mathrm{RN}(\xi_{i3}) = \left[\sum_{i=1}^{9} \xi_{i3}^{-}, \sum_{i=1}^{9} \xi_{i3}^{+}\right]$$
$$= \begin{cases} \sum_{i=1}^{9} \xi_{i3}^{-} = 5.81 + 5.10 + \cdots + 2.40 = 28.56 \\ \sum_{i=1}^{9} \xi_{i3}^{+} = 7.65 + 7.79 + \cdots + 3.90 = 40.81 \end{cases}$$

Then based on these values, we obtain rough number $\mathrm{RN}(\xi_{i3}^{\prime}) = \sum_{i=1}^{9} \mathrm{RN}(\xi_{i3}) = [28.56, 40.81]$. These values are further used to normalize the third column of the average interval RCM. Thus, for the C_1–C_3 position, we obtain

$$\mathrm{RN}(\varpi_{13}) = \frac{\mathrm{RN}(\xi_{13})}{\sum_{i=1}^{9} \mathrm{RN}(\xi_{i3})} = \frac{[5.81, 7.65]}{[28.56, 40.81]} = [0.14,\ 0.27]$$

Similarly, applying Eq. (11), the other values of Table 4 are achieved. Now using Eq. (14), the criteria weights are finally estimated. For example, the following values are calculated by dividing the weight of the first row of the normalized matrix by the number of criteria:

$$\mathrm{RN}(w_1) = \left[\frac{1}{9}\sum_{i=1}^{9} \varpi_{ij}^{-}, \frac{1}{9}\sum_{i=1}^{9} \varpi_{ij}^{+}\right] = \frac{1}{9} \cdot [0.962, 3.055] = [0.107,\ 0.339]$$

The same process is followed for the other weight coefficients.

$$\mathrm{RN}(w_1) = [0.107,\ 0.339]; \mathrm{RN}(w_2) = [0.130,\ 0.423]; \mathrm{RN}(w_3) = [0.018,\ 0.042];$$
$$\mathrm{RN}(w_4) = [0.066,\ 0.162]; \mathrm{RN}(w_5) = [0.041,\ 0.104]; \mathrm{RN}(w_6) = [0.032,\ 0.075];$$
$$\mathrm{RN}(w_7) = [0.032,\ 0.083]; \mathrm{RN}(w_8) = [0.106,\ 0.287]; \mathrm{RN}(w_9) = [0.073,\ 0.184].$$

4.2 Implementation of R-CODAS Model

Now, the step-by-step implementation of the R-CODAS model is explained below.

Step 1. Construct the basic RN decision matrix (Φ)

Table 4 Normalized criteria weights

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	[0.10, 0.20]	[0.1, 0.17]	[0.14, 0.27]	[0.16, 0.27]	[0.11, 0.27]	[0.08, 0.19]	[0.07, 0.20]	[0.06, 0.16]	[0.14, 0.32]
C_2	[0.04, 0.76]	[0.11, 0.25]	[0.12, 0.27]	[0.22, 0.45]	[0.18, 0.41]	[0.15, 0.34]	[0.13, 0.45]	[0.11, 0.46]	[0.13, 0.4]
C_3	[0.01, 0.04]	[0.01, 0.05]	[0.02, 0.04]	[0.01, 0.03]	[0.01, 0.03]	[0.01, 0.02]	[0.01, 0.03]	[0.03, 0.09]	[0.03, 0.06]
C_4	[0.04, 0.09]	[0.02, 0.08]	[0.09, 0.21]	[0.07, 0.09]	[0.11, 0.25]	[0.09, 0.23]	[0.07, 0.22]	[0.06, 0.16]	[0.05, 0.12]
C_5	[0.03, 0.09]	[0.02, 0.07]	[0.06, 0.12]	[0.02, 0.04]	[0.05, 0.07]	[0.04, 0.09]	[0.06, 0.19]	[0.04, 0.15]	[0.05, 0.12]
C_6	[0.03, 0.10]	[0.02, 0.07]	[0.06, 0.12]	[0.02, 0.04]	[0.03, 0.06]	[0.04, 0.06]	[0.03, 0.08]	[0.03, 0.09]	[0.03, 0.06]
C_7	[0.03, 0.13]	[0.02, 0.08]	[0.06, 0.12]	[0.02, 0.06]	[0.02, 0.05]	[0.03, 0.08]	[0.04, 0.06]	[0.04, 0.1]	[0.03, 0.06]
C_8	[0.16, 0.48]	[0.07, 0.41]	[0.07, 0.15]	[0.11, 0.22]	[0.08, 0.25]	[0.12, 0.32]	[0.09, 0.26]	[0.14, 0.24]	[0.12, 0.27]
C_9	[0.05, 0.14]	[0.05, 0.22]	[0.06, 0.14]	[0.09, 0.21]	[0.06, 0.15]	[0.10, 0.19]	[0.09, 0.23]	[0.08, 0.23]	[0.09, 0.14]

Results of the expert assessment for the considered biomaterials are shown in Table 5. By applying Eqs. (1)–(5), elements of the matrix are first translated into RNs, after which, by implementing Eq. (16), we average the RNs to obtain Table 6.

Step 2. Normalization of basic matrix element (Φ)

The normalization of the criteria is carried out using Eq. (18). The first six criteria (C_1–C_6) belongs to the max (benefit) set, and three criteria (C_7–C_9) belongs to the min (cost) set. Normalized basic RN decision matrix is shown in Table 7.

Step 3 and 4. Calculate RN-weighted normalized matrix and determine FR negative-ideal solution

By the multiplication of Table 7 and the criteria weights, we obtain the following weighted matrix.

$$R = \begin{bmatrix} [0.02, 0.23] & [0.02, 0.23] & \ldots & [0.01, 0.07] \\ [0.02, 0.22] & [0.02, 0.26] & \ldots & [0.01, 0.07] \\ \vdots & \vdots & \ddots & \vdots \\ [0.01, 0.15] & [0.02, 0.25] & \ldots & [0.01, 0.05] \end{bmatrix}_{11\times 9}$$

Now by applying Eq. (20), we obtain the RN negative-ideal solution matrix $\mathrm{NS} = [\mathrm{RN}(ns_j)]_{1\times 9}$ as given below:

$$\mathrm{NS} = \begin{bmatrix} \mathrm{RN}(ns_1) = [0.011, 0.145]; \mathrm{RN}(ns_2) = [0.017, 0.234]; \mathrm{RN}(ns_3) = [0.000, 0.002]; \\ \mathrm{RN}(ns_4) = [0.004, 0.026]; \mathrm{RN}(ns_5) = [0.002, 0.011]; \mathrm{RN}(ns_6) = [0.001, 0.008]; \\ \mathrm{RN}(ns_7) = [0.001, 0.007]; \mathrm{RN}(ns_8) = [0.011, 0.087]; \mathrm{RN}(ns_9) = [0.005, 0.035]; \end{bmatrix}_{1\times 9}$$

Step 5 and 6. Calculate the RN distances of alternatives from the RN negative-ideal solution and determine relative assessment matrix (RA)

After calculation of the RN negative-ideal solution matrix $\mathrm{NS} = [\mathrm{RN}(ns_j)]_{1\times 9}$, we obtain the ED_i and HD_i distances of the biomaterial alternatives from the RN negative-ideal solution, using Eqs. (21)–(24), as exhibited in Table 8.

In order to obtain the elements of the relative assessment matrix $\mathrm{RA} = [p_{ik}]_{11\times 11}$, we use previous obtained Euclidean distances and Hamming distances and Eqs. (26) and (27), respectively.

$$\mathrm{RA} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_{11} \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & \ldots & A_{11} \end{array} \\ \begin{bmatrix} 0.000 & -0.055 & -0.075 & \ldots & 0.090 \\ 0.055 & 0.000 & -0.010 & \ldots & 0.145 \\ 0.075 & 0.010 & 0.000 & \ldots & 0.165 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -0.090 & -0.145 & -0.165 & \ldots & 0.000 \end{bmatrix} \end{array}$$

Step 7. Calculate the assessment score (AS_i) of each biomaterial

Table 5 Experts' evaluation of biomaterials with respect to criteria

Expert 1									
Material	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
Stainless steel 316 (A_1)	10	7	517	350	8	8	200	8	1
Stainless steel 317 (A_2)	9	7	630	415	10	8.5	200	8	1.1
Stainless steel 321 (A_3)	9	7	610	410	10	8	200	7.9	1.1
Stainless steel 347 (A_4)	9	7	650	430	10	8.4	200	8	1.2
Co–Cr alloy (castable) (A_5)	10	9	655	425	2	10	238	8.3	3.7
Co–Cr alloy (wrought) (A_6)	10	9	896	600	10	10	242	9.1	4
Pure titanium (A_7)	8	10	550	315	7	8	110	4.5	1.7
Ti–6Al–4V (A_8)	8	10	985	490	7	8.3	124	4.4	1.9
Epoxy-70% glass (A_9)	7	7	680	200	3	7	22	2.1	3
Epoxy-63% carbon (A_{10})	7	7	560	170	3	7.5	56	1.6	10
Epoxy-62% aramid (A_{11})	7	7	430	130	3	7.5	29	1.4	5
…									
Expert 5									
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
A_1	10	6	516	357	8	8.7	193	8.4	1.3
A_2	10	7	633	412	10	9.3	201	8.3	1.1
A_3	9	7	610	411	10	7.0	210	7.3	1.2

(continued)

Table 5 (continued)

Expert 1									
Material	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
A_4	9	7	562	425	10	7.7	209	8.6	1.3
A_5	9	8	657	428	2	10.0	224	8.8	3.4
A_6	9	8	897	598	10	7.9	247	9.6	3.8
A_7	8	9	551	321	7	6.3	117	4.7	1.6
A_8	8	9	987	483	7	4.5	118	4.4	1.7
A_9	7	8	688	205	3	10.0	17	2.0	3.3
A_{10}	6	8	566	169	3	7.0	53	1.7	9.7
A_{11}	7	8	433	130	3	9.0	22	2.6	5.0

Table 6 Averaged rough decision matrix

Criteria/Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
A_1	[8.94, 9.83]	[6.75, 7.64]	[517.69, 522.11]	[347.46, 352.30]	[8.17, 8.69]	[8.17, 8.50]	[195.74, 199.13]	[7.78, 8.22]	[1.03, 1.18]
A_2	[8.75, 9.64]	[7.17, 8.06]	[625.95, 638.45]	[412.49, 417.43]	[10.0, 10.0]	[8.70, 9.09]	[199.89, 200.66]	[8.02, 8.23]	[1.04, 1.12]
A_3	[8.64, 8.96]	[7.47, 8.53]	[606.99, 612.36]	[405.85, 410.76]	[10.0, 10.2]	[6.65, 7.50]	[203.52, 208.56]	[7.60, 7.87]	[1.05, 1.15]
A_4	[8.65, 9.35]	[7.36, 7.84]	[610.35, 649.43]	[425.22, 428.29]	[9.24, 9.81]	[7.85, 8.22]	[201.59, 206.09]	[8.09, 8.49]	[1.08, 1.21]
A_5	[8.65, 9.35]	[8.47, 9.53]	[653.54, 658.62]	[422.98, 427.29]	[2.09, 2.37]	[9.47, 9.92]	[227.28, 234.32]	[8.33, 8.63]	[3.50, 3.66]
A_6	[9.04, 9.36]	[8.36, 9.25]	[891.71, 895.08]	[595.21, 598.25]	[9.87, 9.97]	[8.43, 9.48]	[244.76, 248.78]	[9.28, 9.69]	[3.93, 4.11]
A_7	[8.16, 8.64]	[8.36, 9.25]	[546.96, 551.21]	[312.85, 317.82]	[7.17, 7.68]	[6.74, 7.58]	[111.84, 115.40]	[4.01, 4.45]	[1.64, 1.72]
A_8	[8.04, 8.36]	[9.16, 9.64]	[978.86, 985.28]	[485.79, 491.21]	[7.12, 7.48]	[5.42, 7.34]	[118.73, 121.77]	[4.28, 4.36]	[1.75, 1.85]
A_9	[6.65, 7.35]	[6.94, 7.83]	[678.36, 688.02]	[198.07, 204.28]	[2.02, 2.76]	[7.94, 9.52]	[18.60, 21.14]	[1.99, 2.21]	[3.05, 3.20]
A_{10}	[6.36, 7.25]	[7.36, 7.84]	[557.94, 566.44]	[165.38, 169.59]	[1.95, 2.74]	[6.42, 7.14]	[52.91, 55.11]	[1.49, 1.74]	[9.70, 9.90]
A_{11}	[6.65, 7.35]	[7.36, 7.84]	[428.99, 432.29]	[127.77, 131.01]	[2.53, 2.88]	[7.90, 8.66]	[22.25, 25.90]	[1.70, 2.33]	[4.93, 5.11]

Table 7 Normalized rough decision matrix

Criteria/Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
A_1	[0.19, 0.68]	[0.13, 0.55]	[0.02, 0.05]	[0.10, 0.24]	[0.07, 0.19]	[0.05, 0.13]	[0.04, 0.10]	[0.12, 0.35]	[0.14, 0.37]
A_2	[0.18, 0.66]	[0.15, 0.61]	[0.02, 0.06]	[0.11, 0.26]	[0.08, 0.21]	[0.06, 0.14]	[0.04, 0.10]	[0.12, 0.35]	[0.15, 0.37]
A_3	[0.18, 0.59]	[0.16, 0.68]	[0.02, 0.06]	[0.11, 0.26]	[0.08, 0.21]	[0.04, 0.11]	[0.04, 0.10]	[0.13, 0.36]	[0.15, 0.37]
A_4	[0.18, 0.63]	[0.16, 0.58]	[0.02, 0.06]	[0.11, 0.26]	[0.08, 0.20]	[0.05, 0.12]	[0.04, 0.10]	[0.12, 0.34]	[0.14, 0.37]
A_5	[0.18, 0.63]	[0.21, 0.83]	[0.02, 0.06]	[0.11, 0.26]	[0.04, 0.11]	[0.06, 0.15]	[0.03, 0.09]	[0.12, 0.34]	[0.12, 0.32]
A_6	[0.19, 0.63]	[0.20, 0.79]	[0.03, 0.08]	[0.13, 0.32]	[0.08, 0.20]	[0.05, 0.14]	[0.03, 0.08]	[0.11, 0.30]	[0.12, 0.31]
A_7	[0.16, 0.56]	[0.20, 0.79]	[0.02, 0.05]	[0.09, 0.23]	[0.07, 0.18]	[0.04, 0.11]	[0.05, 0.13]	[0.17, 0.49]	[0.14, 0.36]
A_8	[0.16, 0.54]	[0.24, 0.85]	[0.04, 0.08]	[0.12, 0.29]	[0.07, 0.17]	[0.03, 0.11]	[0.05, 0.13]	[0.17, 0.48]	[0.14, 0.35]
A_9	[0.12, 0.44]	[0.14, 0.58]	[0.03, 0.06]	[0.08, 0.19]	[0.04, 0.11]	[0.05, 0.14]	[0.06, 0.17]	[0.20, 0.56]	[0.13, 0.33]
A_{10}	[0.11, 0.43]	[0.16, 0.58]	[0.02, 0.05]	[0.07, 0.18]	[0.04, 0.11]	[0.04, 0.10]	[0.06, 0.15]	[0.21, 0.57]	[0.07, 0.19]
A_{11}	[0.12, 0.44]	[0.16, 0.58]	[0.02, 0.04]	[0.07, 0.16]	[0.04, 0.12]	[0.05, 0.13]	[0.06, 0.17]	[0.20, 0.57]	[0.11, 0.29]

Table 8 Euclidean and Hamming distances

Euclidean (ED_i) distances									
Criteria/Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
A_1	0.0860	0.0000	0.0003	0.0125	0.0084	0.0016	0.0015	0.0153	0.0335
A_2	0.0797	0.0261	0.0007	0.0161	0.0101	0.0023	0.0014	0.0129	0.0335
A_3	0.0572	0.0554	0.0006	0.0157	0.0104	0.0003	0.0013	0.0171	0.0334
A_4	0.0701	0.0129	0.0007	0.0167	0.0098	0.0012	0.0013	0.0121	0.0333
A_5	0.0701	0.1175	0.0007	0.0167	0.0000	0.0033	0.0005	0.0097	0.0240
A_6	0.0706	0.1000	0.0015	0.0263	0.0101	0.0027	0.0000	0.0000	0.0223
A_7	0.0465	0.1000	0.0004	0.0105	0.0070	0.0004	0.0041	0.0536	0.0312
A_8	0.0372	0.1244	0.0018	0.0203	0.0068	0.0000	0.0038	0.0509	0.0307
A_9	0.0034	0.0118	0.0008	0.0041	0.0005	0.0028	0.0069	0.0742	0.0257
A_{10}	0.0000	0.0129	0.0004	0.0022	0.0005	0.0003	0.0058	0.0792	0.0000
A_{11}	0.0034	0.0129	0.0000	0.0000	0.0007	0.0017	0.0068	0.0771	0.0184
Hamming (HD_i) distances									
Criteria/Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
A_1	0.0941	0.0000	0.0003	0.0143	0.0095	0.0021	0.0017	0.0171	0.0383
A_2	0.0872	0.0284	0.0008	0.0186	0.0116	0.0029	0.0016	0.0147	0.0383
A_3	0.0642	0.0594	0.0007	0.0181	0.0119	0.0005	0.0014	0.0194	0.0383

(continued)

Table 8 (continued)

Euclidean (ED_i) distances									
Criteria/Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
A_4	0.0772	0.0159	0.0008	0.0193	0.0112	0.0017	0.0015	0.0136	0.0381
A_5	0.0772	0.1271	0.0009	0.0192	0.0000	0.0041	0.0006	0.0110	0.0274
A_6	0.0788	0.1090	0.0017	0.0303	0.0115	0.0033	0.0000	0.0000	0.0255
A_7	0.0520	0.1090	0.0004	0.0121	0.0080	0.0006	0.0046	0.0602	0.0357
A_8	0.0423	0.1377	0.0021	0.0233	0.0077	0.0000	0.0044	0.0577	0.0352
A_9	0.0042	0.0129	0.0010	0.0047	0.0005	0.0033	0.0078	0.0837	0.0294
A_{10}	0.0000	0.0159	0.0005	0.0025	0.0005	0.0005	0.0066	0.0893	0.0000
A_{11}	0.0042	0.0159	0.0000	0.0000	0.0008	0.0022	0.0077	0.0864	0.0211

Table 9 Materials ranks using the RN-CODAS model

Alternative	H_i	Rank
A_1	−0.650	8
A_2	−0.035	6
A_3	0.167	5
A_4	−0.644	7
A_5	1.376	3
A_6	1.195	4
A_7	1.644	2
A_8	2.202	1
A_9	−1.408	9
A_{10}	−2.214	11
A_{11}	−1.634	10

RN-CODAS criteria function for the final ranking of the alternatives $A_i(i = 1, 2, \ldots, 11)$ are calculated using Eq. (27), as shown in Table 9.

5 Sensitivity Analysis, Discussion, and Validation

Sensitivity analysis (SA) is a key component for interpreting the outcomes of any multi-criteria analysis. The main objective of SA is to analyze the robustness of the proposed R-AHP-CODAS method. It aims to determine the minimum change I criteria weights that steers some changes in the ranking preorder of the biomaterial alternatives. Figure 2 shows the SA of the proposed R-AHP-CODAS model at varying weights of different material selection criteria for the considered case study. The evaluative outcome of the case study (Table 9) is analyzed through 36 different scenarios in which one criterion has been favored over others in each scenario by increasing its weight in the following manner. In the first scenario (S_1), criterion C_1 (tissue tolerance) was favored, in the second scenario (S_2), C_2 (corrosion resistance) was favoured, and so on. Variation in the ranking preorder of the biomaterials for different circumstances is revealed in Fig. 2.

From Fig. 2, it is well understood that the ranking preorder of the biomaterials has changed slightly due to the changes in criteria weights. From a comparison among the best two alternatives (A_8) in different scenarios with the initial rank of Table 9, it is noted that the best alternative (A_8) rank has not been affected by weight variations. Analysis of the ranking through 36 scenarios of Fig. 2 shows that alternative A_8 (Ti–6Al–4V) holds its rank in 32 scenarios (88.89%), while the second-best alternative (Pure titanium) holds its rank in 23 scenarios (63.89%). However, there are few minor changes in the ranking order for some intermediate biomaterials which is also confirmed by the standard deviation (SD) of the ranks, as shown in Fig. 3.

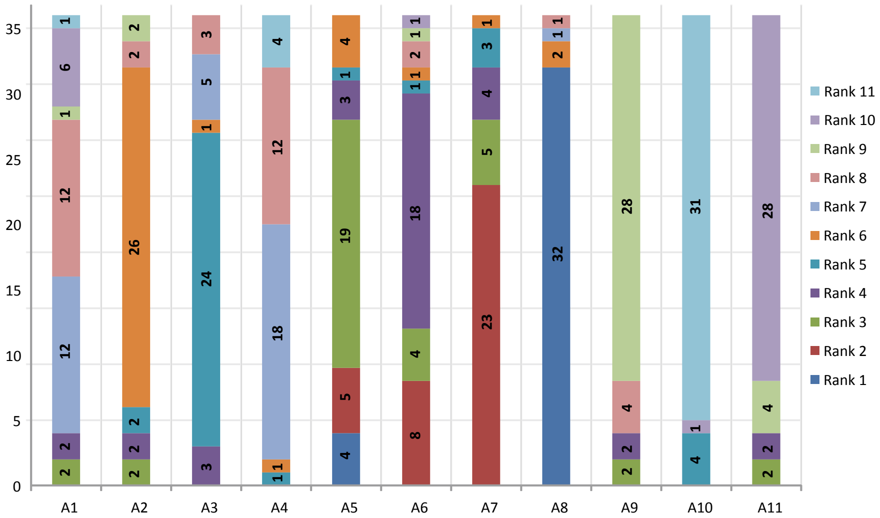

Fig. 2 Analysis of biomaterials rankings through 36 scenarios

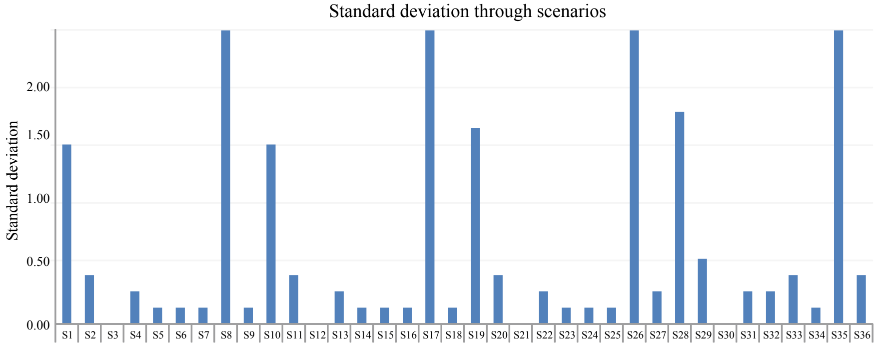

Fig. 3 SD values for different scenarios

SD values are computed based on the initial ranks of Table 9 with that of achieved through different weight changing scenarios (Fig. 2). Figure 3 shows a high ranking correlation with a SD value of less than 0.50 in 28 scenarios. The mean value of SD for the scenarios for the biomaterial selection case study is found to be 0.428, which again signifies very good ranking agreement for all the scenarios.

To arrive at a final decision for selecting the optimal biomaterial, a comparative study has now been performed between different MCDM methods, namely COPRAS [26, 52] and MABAC [51, 53], as shown in Fig. 4. Ranking of the biomaterials according to these methods shows that alternative A_8 retained its first position for all the considered MCDM methods, thus establishing its superior acceptability over other biomaterial alternatives considered in the presented case study, as shown in.

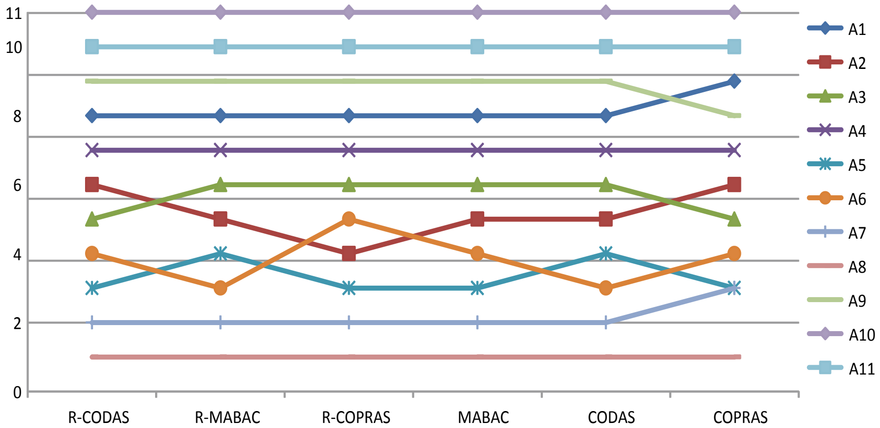

Fig. 4 Comparison of ranks of biomaterials for different MCDM methods

Table 10 Spearman's rank correlation coefficient values between R-AHP-CODAS and other MCDM methods

MCDM method	Spearman's rank correlation coefficient
R-MABAC	0.982
R-COPRAS	0.973
MABAC	0.991
CODAS	0.982
COPRAS	0.986
Average SCC	0.986

Ranking agreement and stability are validated by Spearman's rank correlation coefficient values, as exhibited in Table 10.

Table 10 shows a considerably high correlation between different MCDM methods. Spearman's rank correlation coefficient between the considered methods ranges from 0.982 to 1.00, which shows very strong correlation and ranking agreement among all these methods which ultimately establishes the reliability and credibility of the proposed model.

6 Conclusions

This paper proposes a new application of an integrated rough number-based AHP-CODAS model for selection of biomaterials. One real-life hip prosthesis joint material selection example demonstrates the potentiality and precision of the adopted model. The R-AHP method is used to determine the criteria weights, while alternative biomaterials are assessed by the R-CODAS model. In order to measure the quality of the results, weight SA and performance comparison with other well-established

MCDM methods have been carried out. Agreement between the obtained ranking orders is validated by using Spearman's rank correlation coefficients which indicates a very high rank correlation between all the considered methods, thus establishing the trustworthiness of the adopted approach. Amalgamation of fuzzy and neutrosophic theories with RNs can be the directions of future research.

References

1. Mehrali M, Thakur A, Pennisi CP, Talebian S, Arpanaei A, Nikkhah M, Dolatshahi-Pirouz A (2017) Nanoreinforced hydrogels for tissue engineering: biomaterials that are compatible with load-bearing and electroactive tissues. Adv Mater 29(8):1–26
2. Huebsch N, Mooney DJ (2009) Inspiration and application in the evolution of biomaterials. Nature 462:426–432
3. Jahan A (2012) Material selection in biomedical applications: comparing the comprehensive VIKOR and goal programming models. Int J Mater Struct Integrity 6(2–4):230–240
4. Walker J, Shadanbaz S, Woodfield TB, Staiger MP, Dias GJ (2014) Magnesium biomaterials for orthopedic application: a review from a biological perspective. J Biomed Mater Res B Appl Biomater 102(6):1316–1331
5. Sridharan R, Cameron AR, Kelly DJ, Kearney CJ, O'Brien FJ (2015) Biomaterial based modulation of macrophage polarization: a review and suggested design principles. Mater Today 18(6):313–325
6. Chiti MC, Dolmans MM, Donnez J, Amorim CA (2017) Fibrin in reproductive tissue engineering: a review on its application as a biomaterial for fertility preservation. Ann Biomed Eng 45(7):1650–1663
7. Aherwar A, Singh AK, Patnaik A (2016) Current and future biocompatibility aspects of biomaterials for hip prosthesis. AIMS Bioeng 3(1):23–43
8. Bahraminasab M, Sahari BB (2013) NiTi shape memory alloys, promising materials in orthopedic applications. In: Shape memory alloys-processing, characterization and applications, InTech, USA, pp 261–278
9. Aherwar A, Singh A, Patnaik A, Unune D (2018). Selection of molybdenum-filled hip implant material using grey relational analysis method. In: Handbook of research on emergent applications of optimization algorithms, IGI Global, USA, pp 675–692
10. Hafezalkotob A, Hafezalkotob A (2017) Interval MULTIMOORA method with target values of attributes based on interval distance and preference degree: biomaterials selection. J Ind Eng Int 13(2):181–198
11. Farag MM (2013) Materials and process selection for engineering design. CRC Press, USA
12. Mousavi-Nasab SH, Sotoudeh-Anvari A (2017) A comprehensive MCDM-based approach using TOPSIS, COPRAS and DEA as an auxiliary tool for material selection problems. Mater Des 121:237–253
13. Panchal D, Singh AK, Chatterjee P, Zavadskas EK, Ghorabaee MK (2019) A new fuzzy methodology-based structured framework for RAM and risk analysis. Appl Soft Comput 74:242–254
14. Yazdani M, Chatterjee P, Zavadskas EK, Streimikiene D (2018) A novel integrated decision-making approach for evaluation and selection of renewable energy technologies. Clean Technol Environ Policy 20(2):403–420
15. Tavana M, Yazdani M, Di Caprio D (2017) An application of an integrated ANP–QFD framework for sustainable supplier selection. Int J Logistics Res Appl 20(3):254–275
16. Chatterjee P, Chakraborty S (2017) Development of a meta-model for determination of technological value of cotton fiber using design of experiments and TOPSIS method. J Nat Fibers 15(6)

17. Chatterjee P, Chakraborty S (2017) A developed meta-model for selection of cotton fabrics using design of experiments and TOPSIS method. J Inst Eng (India): Ser E 98(2):79–90
18. Chakraborty S, Chatterjee P, Prasad K (2018) An integrated DEMATEL-VIKOR method-based approach for cotton fibre selection and evaluation. J Inst Eng (India): Ser E 99(1):63–73
19. Chatterjee P, Chakraborty S (2016) A comparative analysis of VIKOR method and its variants. Decis Sci Lett 5(4):469–486
20. Ranjan R, Chatterjee P, Chakraborty S (2016) Performance evaluation of Indian railway zones using DEMATEL and VIKOR methods. Benchmarking: an Int J 23(1):78–95
21. Chatterjee P, Athawale VM, Chakraborty S (2010) Selection of industrial robots using compromise ranking and outranking methods. Robot Comput Integr Manuf 26(5):483–489
22. Kaklauskas A, Zavadskas EK, Raslanas S, Ginevicius R, Komka A, Malinauskas P (2006) Selection of low-e windows in retrofit of public buildings by applying multiple criteria method COPRAS: a Lithuanian case. Energy Build 38(5):454–462
23. Yazdani M, Chatterjee P, Zavadskas EK, Zolfani SH (2017) Integrated QFD-MCDM framework for green supplier selection. J Clean Prod 142(4):3728–3740
24. Maity SR, Chatterjee P, Chakraborty S (2012) Cutting tool material selection using grey complex proportional assessment method. Mater Des 36:372–378
25. Chatterjee P, Chakraborty S (2012) Materials selection using COPRAS and COPRAS-G methods. Int J Mater Struct Integrity 6(2/3/4):111–133
26. Chatterjee P, Athawale VM, Chakraborty S (2011) Materials selection using complex proportional assessment and evaluation of mixed data methods. Mater Des 32(2):851–860
27. Stanujkić D, Đorđević B, Đorđević M (2013) Comparative analysis of some prominent MCDM methods: a case of ranking Serbian banks. Serb J Manage 8(2):213–241
28. Ghorabaee MK, Zavadskas EK, Olfat L, Turskis Z (2015) Multi-criteria inventory classification using a new method of evaluation based on distance from average solution (EDAS). Informatica 26(3):435–451
29. Chatterjee P, Banerjee A, Mondal S, Boral S, Chakraborty S (2018) Development of a hybrid meta-model for materials selection using design of experiments and EDAS method. Eng Trans 66(2):187–207
30. Ghorabaee MK, Zavadskas EK, Turskis Z, Antucheviciene J (2016) A new combinative distance-based assessment (CODAS) method for multi-criteria decision-making. Econ Comput Econ Cybern Stud Res 50(3):25–44
31. Panchal D, Chatterjee P, Shukla RK, Choudhury T, Tamosaitiene J (2017) Integrated fuzzy AHP-CODAS framework for maintenance decision in urea fertilizer industry. Econ Comput Econ Cybern Stud Res 51(3):179–196
32. Bahraminasab M, Jahan A (2011) Material selection for femoral component of total knee replacement using comprehensive VIKOR. Mater Des 32(8):4471–4477
33. Jahan A, Edwards KL (2013) Weighting of dependent and target-based criteria for optimal decision-making in materials selection process: biomedical applications. Mater Des 49:1000–1008
34. Bahraminasab M, Sahari BB, Edwards KL, Farahmand F, Hong TS, Arumugam M, Jahan A (2014) Multi-objective design optimization of functionally graded material for the femoral component of a total knee replacement. Mater Des 53:159–173
35. Petković D, Madić M, Radenković G, Manić M, Trajanović M (2015) Decision support system for selection of the most suitable biomedical material. In: Proceedings of 5th international conference on information society and technology, Serbia, pp 27–31
36. Hafezalkotob A, Hafezalkotob A (2015) Comprehensive MULTIMOORA method with target based attributes and integrated significant coefficients for materials selection in biomedical applications. Mater Des 87:949–959
37. Chowdary Y, Sai Ram V, Nikhil EVS, Vamsi Krishna PNS, Nagaraju D (2016) Evaluation and prioritizing of biomaterials for the application of implantation in human body using fuzzy AHP AND TOPSIS. Int J Control Theory Appl 9(40):527–533
38. Kabir G, Lizu A (2016) Material selection for femoral component of total knee replacement integrating fuzzy AHP with PROMETHEE. J Intell Fuzzy Syst 30(6):3481–3493

39. Abd K, Hussein A, Ghafil A (2016) An intelligent approach for material selection of sensitive components based on fuzzy TOPSIS and sensitivity analysis. In: Proceedings of academics world 30th international conference, Australia, pp 13–18
40. Ristić M, Manić M, Mišić D, Kosanović M, Mitković M (2017) Implant material selection using expert system. Facta Univ, Ser: Mech Eng 15(1):133–144
41. Pamucar D, Lj Gigovic, Bajic Z, Janosevic M (2017) Location selection for wind farms using GIS multi-criteria hybrid model: an approach based on fuzzy and rough numbers. Sustainability 9(8):1–24
42. Song W, Ming X, Wu Z, Zhu B (2014) A rough TOPSIS approach for failure mode and effects analysis in uncertain environments. Qual Reliab Eng Int 30(4):473–486
43. Pamucar D, Mihajlovic M, Obradovic R, Atanaskovic P (2017) Novel approach to group multi-criteria decision making based on interval rough numbers: hybrid DEMATEL-ANP-MAIRCA model. Expert Syst Appl 88:58–80
44. Saaty TL, Vargas LG (2012) Models, methods, concepts and applications of the analytic hierarchy process. Springer Science and Business Media, pp 175
45. Saaty TL (1977) A scaling method for priorities in hierarchical structures. J Math Psychol 15(3):234–281
46. Saaty TL, Tran LT (2007) On the invalidity of fuzzifying numerical judgments in the analytic Hierarchy process. Math Comput Model 46(7):962–975
47. Stevic Z, Pamucar D, Vasiljevic M, Stojic G, Korica S (2017) Novel integrated multi-criteria model for supplier selection: case study construction company. Symmetry 9(11):1–34
48. Stevic Z, Pamucar D, Zavadskas EK, Cirovic G, Prentkovskis O (2017) The selection of wagons for the internal transport of a logistics company: a novel approach based on rough BWM and rough SAW methods. Symmetry 9(11):1–25
49. Ghorabaee MK, Amiri M, Zavadskas EK, Hooshmand R, Antuchevičienė J (2017) Fuzzy extension of the CODAS method for multi-criteria market segment evaluation. J Bus Econ Manage 18(1):1–19
50. Jahan A, Mustapha F, Md YI, Sapuan SM, Bahraminasab M (2011) A comprehensive VIKOR method for material selection. Mater Des 32(3):1215–1221
51. Pamucar D, Ćirović G (2015) The selection of transport and handling resources in logistics centers using Multi-Attributive Border Approximation Area Comparison (MABAC). Expert Syst Appl 42:3016–3028
52. Badi IA, Abdulshahed AM, Shetwan AG (2018) A case study of supplier selection for a steel-making company in Libya by using the Combinative Distance-based ASsessment (CODAS) model. Decis Mak: Appl Manage Eng 1(1):1–12
53. Mukhametzyanov I, Pamucar D (2018) A sensitivity analysis in MCDM problems: a statistical approach. Decis Mak: Appl Manage Eng 1(2):51–80