An Analytical Method of Determining Transient Stability Margin Considering Unbalanced Fault Factors

Jiuliang Liu, Tong Wang and Zengping Wang

Abstract Maintaining the transient stability of the power system is very important for the stable and safe operating. This paper presents an analytical method of determining transient stability margin quantitatively using protection information for multi-generator system. First, the additional impedance of unbalanced fault is calculated by positive sequence equivalent rule. Based on the complementary cluster center of inertia-relative motion, the multi-machine system is equivalent to a one-machine infinity-bus system. Then, through the piecewise models of generator angles, the analytical expression of the equivalent fault clearing angle and the transient stability margin are obtained. Furthermore, the analytical stability margin is put forward to quantize the transient stability after fault occurred with the acquisition of the protection information, and the effects of fault location and fault clearing time on the stability margin under unbalanced faults are analyzed. Finally, four-machine two-area system is used as test system to investigate the effectiveness and accuracy of the proposed method.

Keywords Unbalanced fault · Transient stability margin · Analytical method · Fault location

1 Introduction

Transient stability assessment is one of the main problems of stable operating of power system [1]. With the structure of power system more and more complex, the probability of a devastating accident is significantly increased, decision tables cannot accurately be used for emergency control online, causing the mismatching of the control strategies of decision tables and the current operating conditions [2]. The out-of-step separation [3] and cascading failures [4], even wide spread blackouts [5]

J. Liu · T. Wang (✉) · Z. Wang
State Key Laboratory for Alternate Electrical Power System with Renewable Energy Sources, North China Electric Power University, Beijing, China
e-mail: hdwangtong@126.com

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might occur. A rapid and effective method of transient stability assessment can prevent potential instability factors and help the system operators to implement the effective control measures.

At present, transient stability analysis is mainly based on techniques of time domain simulation [6], direct method [7] and artificial intelligence method [8]. The method of time domain simulation is based on numerical iteration, and it can be used to analyze the power system stability with complex models. However, the time domain simulation will result in the calculation burden. Also, the assessment results mainly rely on manual experience, which makes it difficult to meet the requirement of quantitative assessment and online control simultaneously. Data mining and artificial intelligence have obvious advantages in terms of computational efficiency, but it is very difficult to research the instability mechanism of power system and establish the sensitivity analysis theory. When the offline data is different from the online operation mode, it will lead to assessment errors. The direct method could quantify the transient stability of the system through the stability margin, which lays a foundation for the emergency control. The direct method mainly includes transient energy function (TEF) and extended equal area criterion (EEAC). The EEAC breaks through the limitations of traditional transient energy function and becomes applicable to any detailed models and complex scenarios, while keeping a manageable computational burden [9]. Therefore, it has received extensive attention.

Scholars have considered various influencing factors including fault location [8], fault clearing time [10] and random disturbance [11] when analyzing the transient stability of power system. However, most researchers mostly focuses on the transient stability as affected by three phase fault instead of unbalanced fault. Actually, the fault type is an important factor affecting transient stability margin [12]. According to the real operating data of the power system, the three phase fault accounts for only less than 10% of all types of faults. If all cases are considered as three phase fault, the transient stability assessment will bring the conservative results [13]. Emergency control cannot be implemented accurately, resulting in unnecessary economic losses. Therefore, it is very important to consider the unbalanced fault factors when analyzing the stability margin of power system.

This paper is organized as follows. In Sect. 2, the additional impedance is calculated when the unbalanced faults occurred in the power system, and the equation of rotor motion is equivalent to the form of one-machine infinity-bus system with the help of CCCOI-RM [9]. In Sect. 3, the piecewise models of generator angles is put forward and the analytical expression of the stability margin with respect to fault location and fault clearing time is obtained. The simulation analysis is carried out in Sect. 4, which verifies the correctness and rapidity of the analytical method proposed in this paper.

2 The Model of Unbalanced Fault

2.1 Unbalanced Fault Additional Impedance

Faults on power lines are the most common cause for the loss of power system stability. When a short-circuit fault occurs on the power line r-j, the node impedance matrix considering the point of fault f can be expressed as follow:

$$\mathbf{Z}' = \begin{bmatrix} \mathbf{Z} & Z_{qf} \\ Z_{fq} & Z_{ff} \end{bmatrix}, \ (q = 1, 2, \ldots N) \tag{1}$$

where, $\mathbf{Z}$ is the node impedance matrix of the system before the fault, N represents the dimension of $\mathbf{Z}$. Z_{qf} and Z_{fq} are the new non-diagonal elements of the node impedance matrix after fault. The values of $Z_{qf}, (q = 1, 2\ldots N)$ are as follow:

$$Z_{qf} = (1-\alpha)Z_{qr} + \alpha Z_{qj}, (q = 1, 2\ldots N) \tag{2}$$

where, $\alpha \in (0, 1)$ is the fault location, and the values of Z_{qf} and Z_{fq} are equal to each other. The new diagonal element can be obtained as follow:

$$\mathrm{Z}_{ff} = (1-\alpha)^2 \mathrm{Z}_{\mathrm{rr}} + \alpha^2 \mathrm{Z}_{\mathrm{jj}} + 2\alpha(1-\alpha)Z_{rj} + \alpha(1-\alpha)z_L \tag{3}$$

where, z_L is the impedance of power line r-j.

Both of the negative-sequence impedance matrix and zero-sequence impedance matrix can be calculated by (1), (2) and (3). In order to find the negative sequence and zero sequence equivalent impedance, it is assumed that the fault point f is injected into the unit current, and the injection currents of the other buses are all zero. Then the expression of node voltage can be obtained by:

$$\mathbf{U}' = \mathbf{Z}'\mathbf{I}' \tag{4}$$

The node voltage at fault point is numerically equal to the equivalent impedance of the negative sequence z_2 and the zero sequence z_0, respectively. By applying the positive rule, the additional impedance z_Δ can be obtained as follow:

$$\begin{array}{llll} z_\Delta = z_2 + z_0, & f^{(1)}; & z_\Delta = \frac{z_2 z_0}{z_2 + z_0}, & f^{(1,1)}; \\ z_\Delta = z_2, & f^{(2)}; & z_\Delta = 0, & f^{(3)}; \end{array} \tag{5}$$

where $f^{(1)}$,$f^{(1,1)}$,$f^{(2)}$,$f^{(3)}$ represent the single phase grounding fault, two phase grounding fault, interphase fault and three phase fault, respectively.

2.2 The Equivalent Method

The problem of direct transient stability of power systems is usually assessed by using a simplified synchronous generator model. In this paper, the rotor movement equations of the generators are as follows:

$$\begin{cases} \dot{\delta}_i = \omega_i - \omega_0 \\ M_i \dot{\omega}_i = P_{mi} - P_{ei} \end{cases}, (i = 1, 2\ldots, N_G) \tag{6}$$

where δ_i, ω_i, M_i, P_{mi} and P_{ei} are the rotor angle, angular speed, inertia constant, input mechanical power, and output electrical power of the ith generator, respectively. According to the relevant theory of CCCOI-RM [9], the equivalent rotor equation of the two generators system can be obtained as follows:

$$\begin{cases} M_K \ddot{\delta}_K = P_{\mathrm{m},K} - P_{e,K} \\ M_{T-K} \ddot{\delta}_{T-K} = P_{m,T-K} - P_{e,T-K} \end{cases} \tag{7}$$

where the corner marker 'K' and 'T−K' represent the parameters of 'K' and 'T−K' cluster, respectively. In this paper, the incoherencies within the cluster is ignored [14].

$$\begin{aligned} & M_K = \sum_{i \in K} M_i; \ddot{\delta}_K = \sum_{i \in K} M_i \ddot{\delta}_i / M_K; P_{m,K} = \sum_{i \in K} P_{mi}; P_{e,K} = \sum_{i \in K} P_{ei}; \\ & M_{T-K} = \sum_{j \in T-K} M_j; \ddot{\delta}_{T-K} = \sum_{j \in T-K} M_j \ddot{\delta}_j / M_{T-K}; P_{m,T-K} = \sum_{j \in T-K} P_{mj}; P_{e,T-K} = \sum_{j \in T-K} P_{ej} \end{aligned} \tag{8}$$

In order to perform quantitative analysis, in this paper, the equivalent system as shown in (7) is further equalized to a one-machine infinite-bus system (OMIB):

$$M_{\mathrm{eq}} \ddot{\delta} = P_{m,eq} - P_{e,eq} = P_m - [P_c + P_{\max} \sin(\delta - \gamma)] \tag{9}$$

where, $\delta = \delta_K - \delta_{T-K}$ is the equivalent rotor angle, M_{eq} is the equivalent inertia constant, $P_{m,eq} = P_m - P_c$ the equivalent input mechanical power:

$$\begin{aligned} & M_{eq} = \frac{M_K M_{T-K}}{M_T}; M_T = \sum_{i=1}^{N_G} M_i; P_m = \left(M_{T-K} \sum_{i \in K} P_{mi} - M_K \sum_{j \in T-K} P_{mj} \right) / M_T \\ & P_c = \frac{M_{T-K}}{M_T} \sum_{i \in K, \mathrm{h} \in K} E_i E_h G_{ih} - \frac{M_K}{M_T} \sum_{j \in T-K, l \in T-K} E_j E_l G_{jl} \end{aligned} \tag{10}$$

and, $P_{e,eq} = P_{\max} \sin(\delta - \gamma)$ is the equivalent output electrical power:

$$\begin{aligned} &P_{\max} = \sqrt{C^2 + D^2}; \gamma = \arctan\left(\frac{-D}{C}\right) \\ &C = \sum_{i \in K, j \in T-K} E_i E_j B_{ij}; D = \frac{M_{T-K} - M_K}{M_T}\left(\sum_{i \in K, j \in T-K} E_i E_j G_{ij}\right) \end{aligned} \tag{11}$$

3 Transient Stability Margin

3.1 Analytical Expression of Transient Stability Margin

The nature of transient stability under large disturbance is conversion of energy. When the energy accumulated in the system during the disturbance process can be converted and consumed in the network after fault, the system is stable. Otherwise, the system is unstable. For the OMIB system, transient energy function (TEF) and Equal Area Criterion (EAC) are equivalent to each other. Therefore, when the multi-machine system is equivalent to OMIB system, the stability of the system can be directly analyzed by EAC. The difference between the deceleration area and the acceleration area is defined as the stability margin indicator:

$$\begin{aligned} S_{MAR} &= S_{DEC} - S_{INC} \\ &= P_m\left(\delta_p^s + \delta_a^s - \pi - 2\gamma_a\right) + P_{c,a}\left(\pi - \delta_c - \delta_a^s + 2\gamma_a\right) + P_{c,d}\left(\delta_c - \delta_p^s\right) \\ &\quad + P_{\max,a}\left[\cos(\delta_c - \gamma_a) + \cos\left(\delta_a^s - \gamma_a\right)\right] - P_{\max,d}\left[\cos(\delta_c - \gamma_d) - \cos\left(\delta_p^s - \gamma_d\right)\right] \end{aligned} \tag{12}$$

where the corner marker ‘p’, ‘d’ and ‘a’ represent the pre-fault, during the fault and after fault, respectively. It can be seen from (10), (11) and (12) that the stability margin is mainly affected by the network parameters and fault-clearing angle. In order to analyze the influencing factors of the stability margin, it is necessary to deduce the relationship between the fault-clearing angle and fault factors.

3.2 Piecewise Model of Generator Angle

It is difficult to deduce the expression of generator angle comprising fault factors from Eq. (9) directly, because of the nonlinearity. Therefore, the piecewise models of $\sin(\delta - \gamma)$ is proposed in this paper to reduce the nonlinearity by the linear function $sin(\delta') \approx k_n\delta' + b_n, (n = 1, 2, \ldots)$, with the $\delta' = \delta - \gamma$, γ is the rotor angle deviation. The coefficient k_n and b_n can be found by the least square fitting method. Then Eq. (11) could be written to a second order linear differential equation as:

$$M_{eq}\ddot{\delta}' + P_{\max,d}k_n\delta' = P_m - P_{c,d} - P_{\max,d}b_n \tag{13}$$

The steps of the solution of the expression of $\delta^{'}$ are as follows:

Step I: Calculate the initial value of the generator angle, and the corresponding linear equation is:

$$sin(\delta') \approx k_1\delta' + b_1 \tag{14}$$

The generator angle expression can be solved as follow:

$$\delta' = c_1^1 \cos\left(\sqrt{\frac{P_{\max,d}k_1}{M_{eq}}}t\right) + \frac{P_m - P_{c,d} - P_{\max,d}b_1}{P_{\max,d}k_1} \tag{15}$$

Further, the ending time of first interval and the change rate of generator angle can be obtained:

$$\frac{d\delta'}{dt}_{(t=t_{1end})} = -c_1^1\sqrt{\frac{P_{\max,d}k_1}{M_{eq}}}\sin\left(\sqrt{\frac{P_{\max,d}k_1}{M_{eq}}}t_{1\mathrm{end}}\right) \tag{16}$$

Step II: According to $d\delta'/dt_{(t=t_{1end})}$ and $\delta'_{(t=t_{1end})}$, the generator angle expression is calculated as follow:

$$\delta' = c_2^1 \cos\left(\sqrt{\frac{P_{\max,d}k_2}{M_{eq}}}t\right) + c_2^2 \sin\left(\sqrt{\frac{P_{\max,d}k_2}{M_{eq}}}t\right) + \frac{P_m - P_{c,d} - P_{\max,d}b_2}{P_{\max,d}k_2} \tag{17}$$

where the constant value c_2^1 and c_2^2 can be solved by:

$$\begin{cases} \delta'_{(t=t_{1end})} = c_2^1 \cos\left(\sqrt{\frac{P_{\max,d}k_2}{M_{eq}}}t_{1end}\right) + c_2^2 \sin\left(\sqrt{\frac{P_{\max,d}k_2}{M_{eq}}}t_{1end}\right) + \frac{P_m - P_{c,d} - P_{\max,d}b_2}{P_{\max,d}k_2} \\ \frac{d\delta'}{dt}_{(t=t_{1end})} = \sqrt{\frac{P_{\max,d}k_2}{M_{eq}}}c_2^2 \cos\left(\sqrt{\frac{P_{\max,d}k_2}{M_{eq}}}t_{1end}\right) - \sqrt{\frac{P_{\max,d}k_2}{M_{eq}}}c_2^1 \sin\left(\sqrt{\frac{P_{\max,d}k_2}{M_{eq}}}t_{1end}\right) \end{cases} \tag{18}$$

Step III: Repeat the Step II to get the expression of generator angle during the fault persistence.

Step IV: After the fault is cleared, the formula parameters are changed with the system model, then repeat Step I, Step II and Step III, the power angle expression can be obtained:

$$\delta = c_n^1 \cos\left(\sqrt{\frac{P_{\max,d}k_n}{M_{eq}}}t\right) + c_n^2 \sin\left(\sqrt{\frac{P_{\max,d}k_n}{M_{eq}}}t\right) + \frac{P_m - P_{c,d} - P_{\max,d}b_n}{P_{\max,d}k_n} + \gamma_p \tag{19}$$

where the subscript n represents the parameter value of δ' corresponding to each interval. We can take the equivalent generator angle into Eq. (12) to calculate the system stability margin, and analyze the effect of variables on the stability margin by partial derivatives. The change rate of transient stability with fault clearing time as follow:

$$SEN_t = \frac{\partial S_M}{\partial t} = \left[P_{c,d} - P_{c,a} - P_{\max,a}\sin(\delta_c - \gamma_a) + P_{\max,d}\sin(\delta_c - \gamma_d)\right]\frac{\partial \delta_c}{\partial t} \tag{20}$$

The change rate of transient stability with fault location is:

$$SEN_\alpha = \frac{\partial S_M}{\partial \alpha} = H_1\frac{\partial \delta_c}{\partial \alpha} + H_2\frac{\partial P_{c,d}}{\partial \alpha} + H_3\frac{\partial P_{\max,d}}{\partial \alpha} - H_4\frac{\partial \gamma_d}{\partial \alpha} \tag{21}$$

where, the H_1, H_2, H_3, H_4 are constant values, which are shown as follows:

$$\begin{aligned} &H_1 = P_{c,d} - P_{c,a} + P_{\max,d}\sin(\delta_c - \gamma_d) - P_{\max,a}\sin(\delta_c - \gamma_a); H_2 {=} \delta_\mathrm{c} - \delta_p^s \\ &H_3 = \cos\left(\delta_p^s - \gamma_d\right) - \cos(\delta_c - \gamma_d); H_4 = P_{\max,d}\left[\sin(\delta_c - \gamma_d) - \sin\left(\delta_p^s - \gamma_d\right)\right] \end{aligned} \tag{22}$$

and, the $\delta_c/\partial\alpha$, $P_{c,d}/\partial\alpha$, $P_{\max,d}/\partial\alpha$, $\gamma_d/\partial\alpha$ can be solved by (10), (11) and (19).

4 Results and Analysis

The structure of the Kundur's 4-machine 2-area power system, which is shown in Fig. 1, is adopted as the test system. And software platform MATLAB R2016a is used to verify the efficiency of the proposed method. The generator ignoring the governor and the excitation system. The loads adopt the constant impedance model.

The proposed method in this paper is adopted for solving the transient stability of test system. Two phase grounding fault is occurred on the branch 7–8 at 0.2 s. The structure and parameters of the grid remain unchanged before and post the fault. When $t_c = 0.42\,\mathrm{s}$, The generator angle curves of time-domain simulation are

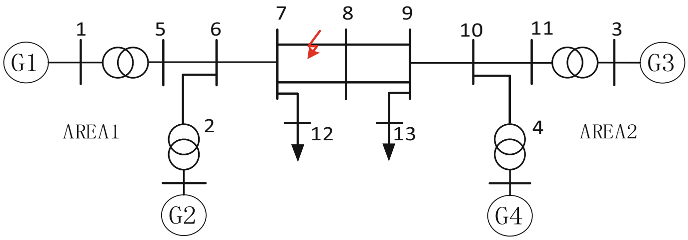

Fig. 1 Kundur's 4-machine 2-area power system

shown in Fig. 2a, and the angle curves of the equivalent OMIB system are shown in Fig. 2b. It can been shown that the analytical method can calculate the generator angle of OMIB system accurately. Transient stability margin under $\alpha \in (0.1, 0.8)$ and $t_c \in [0.21, 0.35]$ is shown in Fig. 3. It can be seen from the graph that the transient stability margin is least when $\alpha \rightarrow 0.1$ and $t_c \rightarrow 0.35$. The smaller the fault point α, that is, the closer the fault point is to the generator, which will results in the bigger fault current during the fault. The larger the fault clearing time t_c, the more acceleration energy the power system accumulates, it is easier to cause the loss of stability of power system. For simplicity, let the $t_c = 0.35\,\text{s}$ to analyze the relationship between the stability margin and α.

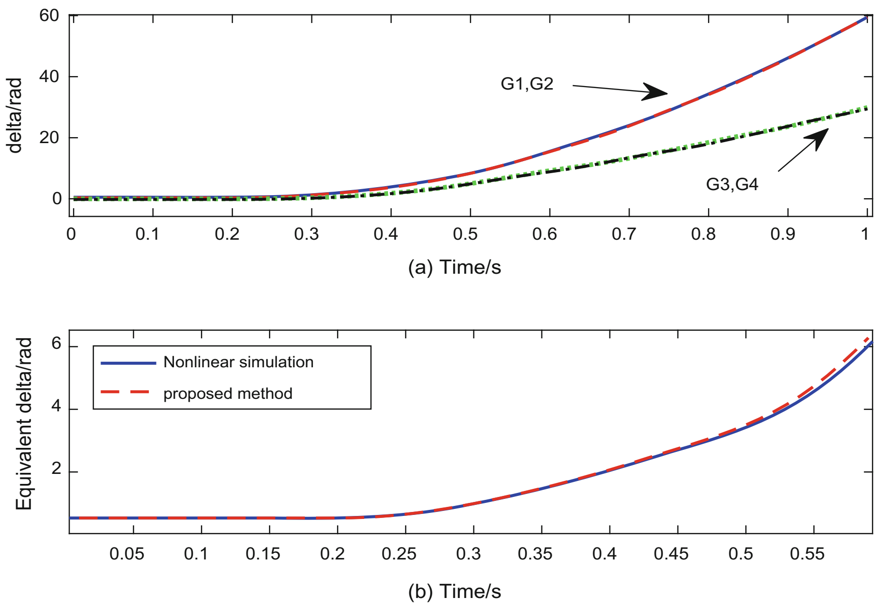

Fig. 2 Effect analysis of proposed method for the unstable systems after fault

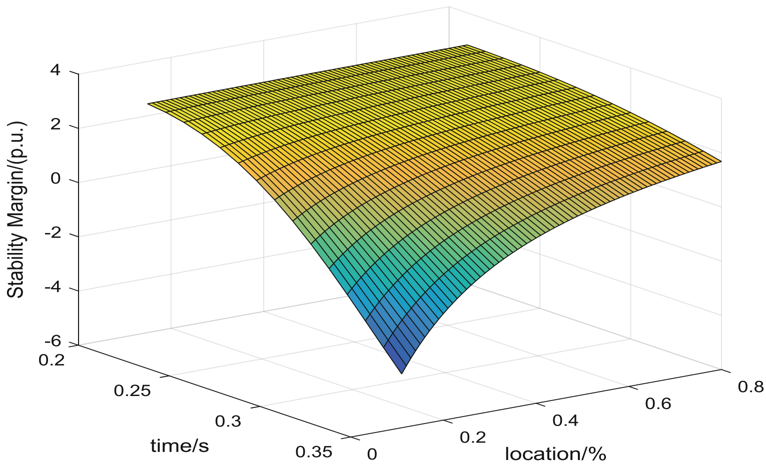

Fig. 3 Relationship between transient stability margin and fault factors

The transient stability S_{MAR} and sensitivity of fault location SEN_{α} are shown in Fig. 4. By changing the α from 10 to 80% of the line 7–8, the sensitivity of fault location SEN_{α} continues to decrease and always keeps positive value. The transient stability S_{MAR} is convexity and increases monotonically which is conforming to the variation law of SEN_{α}. According to analytical method, the power system will operate at the stability boundary when the α is 38.5%. In order to verify the accuracy of analytical method, the nonlinear simulations are carried out in the case of different fault location near the critical stability case. The simulation results are

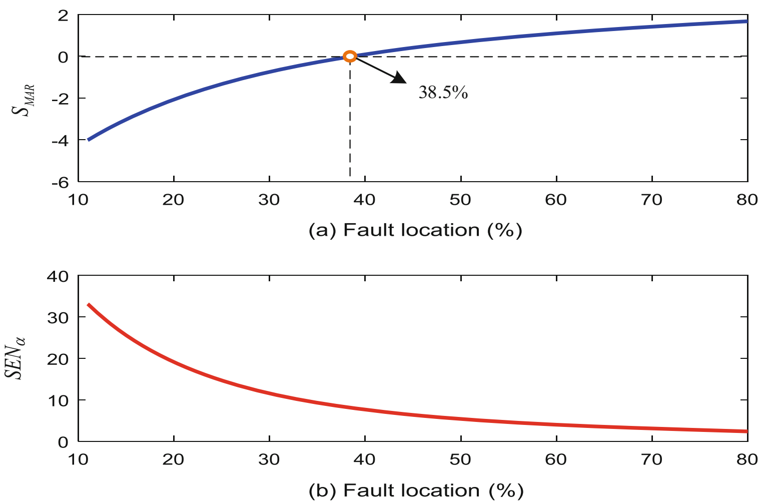

Fig. 4 The stability margin and sensitivity under different fault locations

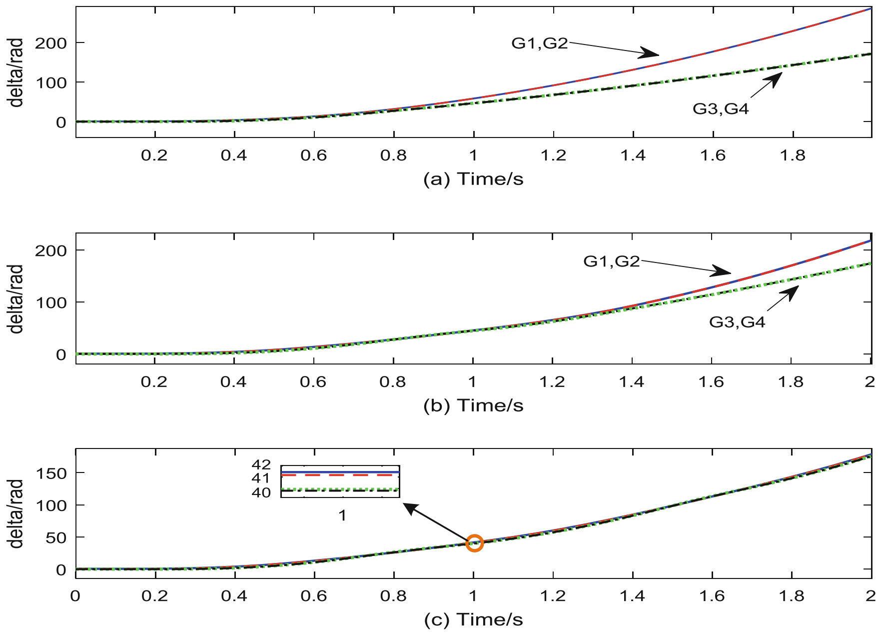

Fig. 5 The generator angle curves under different fault locations

shown in Fig. 5, in which the subplots are the generators angles with α is 38, 39, 40%, respectively. By comparing Fig. 5b, c, it can be seen that the critical fault location is between 39 and 40%. The critical fault location obtained by analytical method is 38.5%. The difference between the results of the two methods is less than 1%. The analysis process of relationship between stability margin and fault clearing time is the same as this.

In this paper, when the generator angle deviation exceeds 4π, the system is judged to be unstable. In the simulation results as shown in Fig. 5a, when $t = 1.03$ s, the generator angle deviation exceeds 4π. At this time, the actual time cost by time domain simulation is 0.460 s. The analytical method proposed in this paper costs 0.437 s. As the scale of the system increases, the computational cost in the iteration process of time domain simulation method will be further increased, and the advantages of the proposed method in this paper will be exhibited.

It can be seen that the protection information will affect the system stability margin from the above analysis. The transient stability margins under different faults obtained by analytical method and the transient stability of nonlinear simulation are as shown in Table 1. The calculation result of the formula is the same as the simulation conclusion. In the case of known protection information, the proposed method can quickly determine the stability margin of the system. It can be seen from the Table 1 that mode 6 is the most stable mode of all test cases, while mode 3 is the most unstable one.

Table 1 Stability margin corresponding to different protection information in test system

Mode	Fault type	Fault location (%)	t_c (s)	Stability margin	Nonlinear simulation
1	$f^{(1,1)}$	45	0.32	1.2762	Stable
2	$f^{(3)}$	70	0.35	−0.4684	Unstable
3	$f^{(1,1)}$	45	0.4	−1.0811	Unstable
4	$f^{(2)}$	75	0.32	2.3906	Stable
5	$f^{(3)}$	30	0.30	−0.6642	Unstable
6	$f^{(1)}$	30	0.30	2.4257	Stable

5 Conclusion

The proposed analytical method proposed in this paper considering fault type, fault location and fault clearing time can provide valuable quantitative information with respect to transient stability assessment and emergency control. The transient stability as affected by the unbalanced fault factors have been investigated and the instability mechanism of the power system is exhibited. The simulation results of test system demonstrated the accuracy, efficiency and visibility of the proposed analytical method. Based on the protection information after the fault, the transient stability of the power system can be assessed based on the analytical expression. This method could reduce the calculation burden of the numerical iterative process, thus the effective quantitative information for the guidance of subsequent emergency control.

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