Chapter 2
An Economic Order Quantity (EOQ) Inventory Model for a Deteriorating Item with Interval-Valued Inventory Costs, Price-Dependent Demand, Two-Level Credit Policy, and Shortages

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Abstract In today's competitive environment, every leading organization wishes to improve the pricing strategies in order to increase revenue, credit policy is one of the best tools of it. This research work develops an economic order quantity (EOQ) inventory model for a deteriorating item that considers interval-valued inventory costs, price dependent demand, two-level credit policy, and shortages. Due to high and uncertainty in demand, sometimes organizations have to face the situation of stock out. So, keeping this scenario in mind, this work considers the situation of partially backlogging. Here, it is developed an EOQ inventory model by considering a non-linear interval-valued constrained optimization problem. Two types of particle swarm optimization (PSO) algorithm are used to resolve it, and then the results are compared. Sensitivity analysis is done in order to investigate the impact of key parameters on decision-making. Finally, conclusions along with some managerial insights are given.

Keywords Inventory · Deterioration · Price-dependent demand · Partial shortages · Interval-valued inventory costs · Two-level credit policy

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2.1 Introduction

In the inventory management literature, very little research work has been done in relation to considering that the inventory costs are represented in an interval-valued. Many researchers assume that the inventory costs such that the ordering cost, inventory carrying cost, and purchasing cost are expressed as a fixed value known. Nevertheless, in reality, all of the mentioned costs are imprecise numbers in nature instead of a fixed value due to the fact that generally, the inventory costs fluctuate by reason of several factors such as changes in prices. In order to explain why it needs to use an interval number rather than the fixed value number, the following reason is mentioned. Normally, the inventory carrying cost is distinct during the seasons of the year. For example, the deterioration rate is different in summer and winter. During summer time, it is necessary to use preservation technology with the intention of decreasing the deterioration percentage of some perishable products and therefore the holding cost is different from holding cost in the winter time. Another cost that also varies is the labor charges, which change over the period of time.

To overcome the problem of imprecise numbers, the researchers and academicians use the following approaches: (i) stochastic, (ii) fuzzy, and (iii) fuzzy-stochastic. In the case of the stochastic approach, the inventory data are considered as random variables with a given and known probability distribution. In the case of the fuzzy approach, the data of the inventory system and the constraints are expressed with fuzzy sets with a given and known membership function. In the case of fuzzy-stochastic approach, some inventory data are supposed to be represented by fuzzy sets and the rest of the inventory parameters are assumed random variables. But it is not an easy task to select the most suitable membership function or probability distribution.

With the aim of avoiding the complexity in the selection of the right membership function or the right probability distribution, it is suggested to use interval numbers. With this, the imprecise problem is converted to an interval-valued problem, which can be solved, by any soft computing optimization technique, such as the different versions of particle swarm optimization (PSO) or genetic algorithm (GA). In this connection, the reader can see the related works, which apply interval number into the area of inventory control. Gupta et al. [1] applied the interval concept in the field of inventory theory. They resolved an inventory problem with interval-valued inventory costs using a genetic algorithm approach. After that, Dey et al. [2] formulated an inventory model considering interval-valued lead time. Again, Gupta et al. [3] developed an inventory model using interval-valued inventory costs. Bhunia et al. [4] solved a stock-dependent inventory model with interval-valued inventory costs using particle swarm optimization (PSO). Afterward, Bhunia and Shaikh [5] built a two-warehouse inventory model with inflation, and they solved it using particle swarm optimization.

In the current competitive markets, the permissible delay in payment has a vital role in promoting the business. Normally, the suppliers give different types of facilities to retailers, and the retailers give some facilities to their direct customers. This is done with the aim of attracting more customers to acquire products. Table 2.1 presents research works related to single-level or two-level credit policy.

Table 2.1 Research works related to single- and two-level permissible delay in payment

Author(s)	Deterioration	Demand rate	Shortages	Level of permissible delay in payment	Inventory costs
Hwang and Shinn [6]	Yes	Constant	No	Single	Fixed
Chang et al. [7]	Yes	Constant	No	Single	Fixed
Abad and Jaggi [8]	No	Linearly time-dependent	No	Single	Fixed
Ouyang et al. [9]	Yes	Constant	No	Single	Fixed
Huang [10]	No	Linearly time-dependent	No	Two-level	Fixed
Huang [11]	No	Constant	No	Single	Fixed
Huang [12]	No	Constant	No	Two-level	Fixed
Sana and Chaudhuri [13]	Yes	Selling price-dependent	No	Single	Fixed
Huang and Hsu [14]	No	Constant	No	Two-level partial trade credit	Fixed
Ho et al. [15]	No	Constant	No	Two-level	Fixed
Jaggi and Khanna [16]	Yes	Inventory level dependent	Complete backlogging	Single	Fixed
Jaggi and Kausar [17]	No	Selling price-dependent	Complete backlogging	Single partial trade credit	Fixed
Jaggi and Mittal [18]	Yes	Constant	Complete backlogging	Single	Fixed
Guria et al. [19]	No	Selling price-dependent	Complete backlogging	Immediate and delay in payment	Fixed
Taleizadeh et al. [20]	No	Constant	Partial backlogging	Partial delay in payment	Fixed
Wu et al. [21]	Expiration date dependent	Constant	No	Two-level	Fixed

(continued)

Table 2.1 (continued)

Author(s)	Deterioration	Demand rate	Shortages	Level of permissible delay in payment	Inventory costs
Chen et al. [22]	Yes	Constant	No	Order quantity-dependent credit period	Fixed
Bhunia et al. [23]	Yes	Selling price-dependent	Partial backlogging	Alternative single	Fixed
Bhunia and Shaikh [24]	Yes	Selling price-dependent	Partial backlogging	Single	Fixed
Shah and Cárdenas-Barrón [25]	Yes	Constant	No	Two-level	Fixed
Bhunia et al. [26]	No	Stock-dependent	Partial backlogging	Single	Fixed
This research work	Yes	Selling price-dependent demand	Partial backlogging	Two-level credit policy	Interval-valued

It is a well-known fact that inventory cost always is not a fixed value. This means that the inventory cost lies between certain interval numbers. Therefore, the major goal of this research work is to include the interval concept in an inventory model. In this direction, this research work derives an economic order quantity (EOQ) inventory model for a deteriorating item with price-dependent demand, and interval-valued inventory costs and shortages. The shortage is partially backordered according to a rate, which is reliant on the interval of waiting time till the occurrence of next lot. The inventory model is expressed as a nonlinear interval-valued continuous optimization problem. Then, different forms of particle swarm optimization (PSO) algorithm are applied to solve it. In continuous optimization, PSO gives better results than GA. For this reason, in this research work, the latest version of PSO is used.

The remnants sections of the research work are planned as follows. Section 2.2 defines the suppositions and the notations. Section 2.3 formulates the inventory model. Section 2.4 derives the mathematical solution for three different demand functions. Section 2.5 solves some instances. Section 2.6 does a sensitivity analysis. Section 2.7 provides conclusions and lines for research.

2.2 Suppositions and Notations

The suppositions and symbols that are used to build the inventory model are listed below.

2.2.1 *Suppositions*

(i) The planning horizon is infinite.
(ii) Inventory system handles a single item.
(iii) Demand rate $D(.)$ is influenced by the selling price (p).
(iv) Inventory costs are interval-valued.
(v) The order is supplied in one delivery.
(vi) Replenishment is instantaneous.
(vii) Lead time is zero.
(viii) Stockout is partially backlogged with a backlogging rate given by $[1 + \delta(T - t)]^{-1}$.
(ix) Two-level credit policy approach is assumed where the supplier gives a credit period (M) to his/her retailer, and the retailer also provides a credit facility (N) to his/her customer under certain terms and conditions. Here, it is established the following condition $N < M$.

2.2.2 *Notations*

Symbols	Description
Parameters	
$I(t)$	Inventory level at time t (units)
α	Deterioration rate ($0 < \alpha \ll 1$)
$[C_{oL}, C_{oR}]$	Interval-valued replenishment cost ($/order)
δ	Backlogging parameter
$\left[C_{pL}, C_{pR}\right]$	Interval-valued purchasing cost ($/unit)
$D(.)$	Demand rate that is dependent on price (units/unit of time)
$[C_{hL}, C_{hR}]$	Interval-valued holding cost ($/unit/unit of time)
$[C_{bL}, C_{bR}]$	Interval-valued shortage cost ($/unit/unit of time)
$[C_{lsL}, C_{lsR}]$	Interval-valued opportunity cost due to a lost sale ($/unit/unit of time)
t_1	Time in which the stock level is zero (unit of time)

(continued)

(continued)

Symbols	Description
T	Cycle length (unit of time)
M	Credit period is given to the retailer by the supplier (unit of time)
N	Credit period provided to the customer by the retailer: $N < M$ (unit of time)
I_e	Interest earned by the retailer (%/unit of time)
I_p	Interest charged by the supplier to the retailer (%/unit of time)
$\left[Z_L^{(.)}, Z_R^{(.)}\right]$	Interval-valued the total profit ($/unit of time)
Decision variables	
S	Stock level (units)
R	Shortage level (units)
B	The time period after reaching the prescribed credit time M (unit of time)

2.3 Mathematical Derivation of the Inventory Model

Initially, the retailer purchases a lot of $(S + R)$ units. After fulfilling the backordered units of the preceding cycle, the stock level is S units at $t = 0$. Then, S units start to decrease due to both consumers' demand and deterioration effect. Obviously, after a certain time period, the stock level reaches zero at the time $t = t_1$. After that, at time $t = t_1$, shortage occurs with a backlogging rate $[1 + \delta(T - t)]^{-1}$ till the time $t = T$. Then, a subsequent batch is received at T.

The behavior of the inventory $I(t)$ is modeled by the differential Eqs. (2.1) and (2.2):

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(.), \quad [0, t_1] \tag{2.1}$$

$$\frac{dI(t)}{dt} = \frac{-D(.)}{1 + \delta(T - t)}, \quad (t_1, T] \tag{2.2}$$

with the initial and boundary conditions

$$I(t) = S \text{ at } t = 0, \; I(t) = 0 \text{ at } t = t_1 \tag{2.3}$$

and

$$I(t) = -R \text{ at } t = T \tag{2.4}$$

It is significant to state that the inventory level $I(t)$ is continuous at $t = t_1$. Using the conditions (2.3) and (2.4), the solutions to the differential equations (2.1) and (2.2) are given below:

$$I(t) = -\frac{D(.)}{\theta} + \frac{D(.)}{\theta}e^{\theta(t_1-t)}, \quad [0, t_1]$$
$$I(t) = \frac{D(.)}{\delta}\log|1 + \delta(T - t)| - R, \quad (t_1, T]$$

From condition (2.3), $I(t) = S$ at $t = 0$. Thus, the maximum inventory level is computed with

$$S = \frac{D(.)}{\theta}\left\{e^{\theta t_1} - 1\right\} \tag{2.5}$$

Using the continuity condition, hence, the shortage quantity is determined with

$$R = \frac{D(.)}{\delta}\log|1 + \delta(T - t_1)| \tag{2.6}$$

The total interval-valued inventory holding cost $C_{hol} = [C_{holL}, C_{holR}]$ of the system is expressed as follows:

$$C_{holL} = C_{hL}\int_0^{t_1} I(t)\,dt = C_{hL}\left[\frac{(S + \frac{D(.)}{\theta})}{\theta}\left(1 - e^{-\theta t_1}\right) - \frac{Dt_1}{\theta}\right] \tag{2.7}$$

and

$$C_{holR} = C_{hR}\int_0^{t_1} I(t)\,dt = C_{hR}\left[\frac{(S + \frac{D(.)}{\theta})}{\theta}\left(1 - e^{-\theta t_1}\right) - \frac{Dt_1}{\theta}\right] \tag{2.8}$$

The total interval-valued shortage cost $C_{sho} = [C_{shoL}, C_{shoR}]$ of the inventory system is given below:

$$\begin{aligned} C_{shoL} &= C_{bL}\int_{t_1}^{T} [-I(t)]\,dt \\ &= C_{bL}\left[\left(R + \frac{D(.)}{\delta}\right)(T - t_1) - \frac{D(.)}{\delta^2}\{(1 + \delta(T - t_1))\log(1 + \delta(T - t_1))\}\right] \end{aligned} \tag{2.9}$$

$$C_{shoR} = C_{bR} \int_{t_1}^{T} [-I(t)]\,dt$$
$$= C_{bR}\left[\left(R + \frac{D(.)}{\delta}\right)(T - t_1) - \frac{D(.)}{\delta^2}\{(1 + \delta(T - t_1)) \log(1 + \delta(T - t_1))\}\right] \tag{2.10}$$

The total interval-valued opportunity cost of lost sales $OC_{LS} = [OC_{LSL}, OC_{LSR}]$ during the entire cycle is determined by

$$OC_{LSL} = C_{lsL} \int_{t_1}^{T} \left\{1 - \frac{1}{1 + \delta(T - t)}\right\} D(.)\,dt$$
$$= C_{lsL} D(.)\left[(T - t_1) - \frac{\log(1 + \delta(T - t_1))}{\delta}\right] \tag{2.11}$$

and

$$OC_{LSR} = C_{lsR} \int_{t_1}^{T} \left\{1 - \frac{1}{1 + \delta(T - t)}\right\} D(.)\,dt$$
$$= C_{lsR} D(.)\left[(T - t_1) - \frac{\log(1 + \delta(T - t_1))}{\delta}\right] \tag{2.12}$$

As it was mentioned before, in two-level credit policy, the supplier provides a credit period to his/her retailer with a duration of M. Then, the retailer also gives a certain credit period to his/her client with a duration of N, where N is always less than M. Furthermore, here two cases occur: **Case 1**: $0 < N < M \leq t_1$ and **Case 2**: $N < t_1 < M \leq T$. Figures 2.1 and 2.2 show the behavior of the stock level over the period of time for **Case 1** and **Case 2,** respectively. Below a discussion of these two cases is given.

Case 1: $0 < N < M \leq t_1$

In this case, the total amount of purchase cost of the retailer is within the following interval $\left[C_{pL}(S + R), C_{pR}(S + R)\right]$. This amount must be covered to the supplier at the time $t = M$. In this credit time period, the retailer accumulates money due to sales during [0, M] as well as the interest gained during [N, M]. Hence, the total collected amount is calculated with

$$U_1 = p \int_{0}^{M} D(.)\,dt + pI_e \int_{N}^{M}\int_{N}^{t} D(.)\,du dt + pR\{1 + I_e(M - N)\}.$$

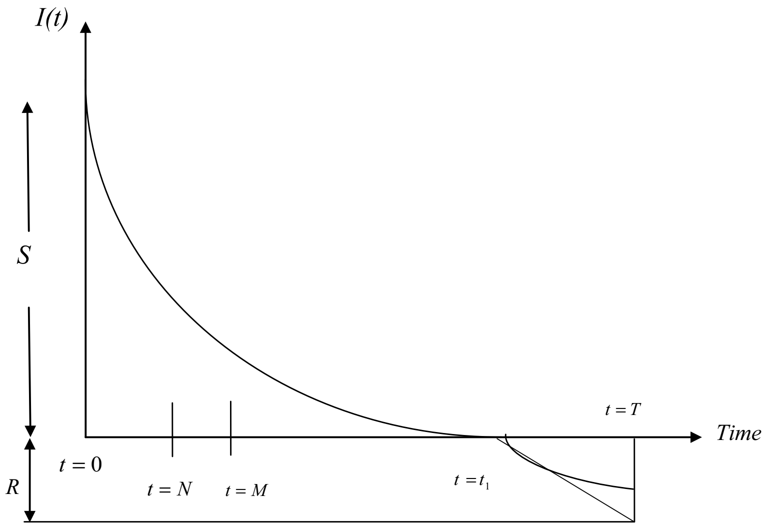

Fig. 2.1 Inventory-level behavior for Case 1

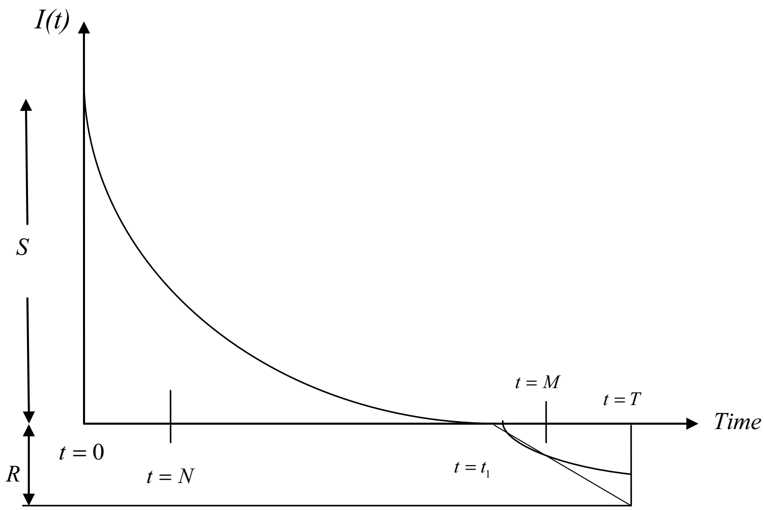

Fig. 2.2 Inventory-level behavior for Case 2

Thus,

$$U_1 = pD(.)\left\{M + I_e\frac{(M-N)^2}{2}\right\}pR\{1 + I_e(M-N)\} \tag{2.13}$$

The retailer collects U_1 and interval-valued for the purchase cost amount is $\left[C_{pL}(S+R), C_{pR}(S+R)\right]$. Here, the following two subcases occur: **Subcase 1**: $U_1 \geq \left[C_{pL}(S+R), C_{pR}(S+R)\right]$ and **Subcase 2**: $U_1 < \left[C_{pL}(S+R), C_{pR}(S+R)\right]$. These subcases are developed below:

Subcase 1: $U_1 \geq \left[C_{pL}(S+R), C_{pR}(S+R)\right]$
In this Subcase 1, the total interval-valued profit of the inventory system is written as

$$Z_{1L}(.) = \frac{X_L(.)}{T} \text{ and } Z_{1R}(.) = \frac{X_R(.)}{T} \tag{2.14}$$

where

$[X_L, X_R]$ = *<Excess amount on hand after paying the cost of purchased goods to the supplier>* + *<interest earned for excess amount in* $[M, T]$*>* + *<sales revenue in* $[M, t_1]$*>* + *<interest earned in* $[M, t_1]$*>* + *<interest earned in* $[t_1, T]$*>* − *<ordering cost>* − *<holding cost>* − *<shortage cost>* − *<cost of lost sale>*

$$\begin{aligned} X_L(.) = &\left\{U_1 - C_{pR}(S+R)\right\}\{1 + I_e(T-M)\} + D(.)p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\} \\ &\{1 + I_e(T - t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR} \end{aligned} \tag{2.15}$$

and

$$\begin{aligned} X_R(.) = &\left\{U_1 - C_{pL}(S+R)\right\}\{1 + I_e(T-M)\} + D(.)p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\} \\ &\{1 + I_e(T - t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL} \end{aligned} \tag{2.16}$$

Therefore, the corresponding interval-valued nonlinear optimization problem of the inventory system is written as follows.

Problem 1

$$\begin{aligned} &\text{Maximize } Z_1(.) = [Z_{1L}(.), Z_{1R}(.)] \\ &\text{subject to } 0 < N < M \leq t_1 < T \end{aligned} \tag{2.17}$$

Subcase 2: $U_1 < \left[C_{pL}(S+R), C_{pR}(S+R)\right]$
In Subcase 2, the retailer collects an amount corresponding to sales and interest earned up to $t = M$. This amount is less than the amount of the purchase cost. Taking into consideration this situation, the following two subcases happen: **Subcase**

2.1: Supplier takes a partial payment at $t = M$ of his/her retailer, and **Subcase 2.2**: Supplier does not take the partial payment at $t = M$ of his/her retailer. Now, these two subcases are discussed below.

Subcase 2.1: Supplier takes a partial payment at $t = M$ of his/her retailer.
In this subcase, it is considered that the supplier takes a partial payment and permits some time to the retailer regarding the payment of rest interval amount which is expressed as $\left[C_{pL}(S+R) - U_1, C_{pR}(S+R) - U_1\right]$. The interval relax time is $t = [B_L, B_R]$ where $[B_L, B_R] > M$. In this situation, the supplier must charge the interest of unpaid amount $\left[C_{pL}(S+R) - U_1, C_{pR}(S+R) - U_1\right]$ during the interval $[M, [B_L, B_R]]$ with interest paid rate I_p.

Thus, the total amount that must be paid to the supplier at a time $t = [B_L, B_R]$ is given by $\left[C_{pL}(S+R) - U_1, C_{pR}(S+R) - U_1\right]\left\{1 + I_p([B_L, B_R] - M)\right\}$.

On the other hand, the total available amount to the retailer is determined as < total sales revenue during the time interval $[M, [B_L, B_R]]$> + <total interest earned during the time interval $[M, [B_L, B_R]]$>. So, the total interest earned is

$$= p\int_M^{B_L} D(.)\,dt + pI_e\int_M^{B_L}\int_M^{t} D(.)\,dudt$$

and

$$= p\int_M^{B_R} D(.)\,dt + pI_e\int_M^{B_R}\int_M^{t} D(.)\,dudt$$

As a result, at the time $t = [B_L, B_R]$, the total payable amount available to the retailer is equal to the amount payable to the supplier, which is

$$\begin{aligned}&\left[C_{pL}(S+R) - U_1, C_{pR}(S+R) - U_1\right]\left\{1 + I_p(B_L - M)\right\}\\&= pD(.)(B_L - M)\left\{1 + \frac{I_e(B_L - M)}{2}\right\}\end{aligned}$$

Thus,

$$\left[C_{pL}(S+R) - U_1\right]\left\{1 + I_p(B_L - M)\right\} = pD(.)(B_L - M)\left\{1 + \frac{I_e(B_L - M)}{2}\right\}$$

and

$$\left[C_{pR}(S+R) - U_1\right]\left\{1 + I_p(B_R - M)\right\} = pD(.)(B_R - M)\left\{1 + \frac{I_e(B_R - M)}{2}\right\}$$

Consequently, the total interval-valued profit function of the inventory system is computed as

$$Z_{2L}(.) = \frac{X_L(.)}{T} \quad \text{and} \quad Z_{2R}(.) = \frac{X_R(.)}{T}$$

where

$[X_L, X_R]$ = < *sales revenue during the time interval* $[[B_L, B_R], t_1]$ > + < *interest earned thru the time interval* $[[B_L, B_R], t_1]$ > + < *interest earned through interval* $[t_1, T]$ > − < *ordering cost* > − <*holding cost* > − <*shortage cost* > − <*cost of lost sale*>

$$X_L(.) = \left\{ p\int_{B_R}^{t_1} D(.)dt + pI_e \int_{B_R}^{t_1}\int_{B_R}^{t} D(.)dudt \right\}\{1 + I_e(T - t_1)\}$$
$$- C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R(.) = \left\{ p\int_{B_L}^{t_1} D(.)dt + pI_e \int_{B_L}^{t_1}\int_{B_L}^{t} D(.)dudt \right\}\{1 + I_e(T - t_1)\}$$
$$- C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

So, in this subcase, the interval-valued constrained optimization problem is formulated as follows.

Problem 2

$$\begin{aligned} &\text{Maximize } Z_2(.) = [Z_{2L}(.), Z_{2R}(.)] \\ &\text{subject to } 0 < N < M \le t_1 < T \end{aligned} \tag{2.18}$$

Subcase 2.2: Supplier does not take the partial payment at $t = M$ of his/her retailer. In this situation, the supplier does not take a partial payment. In other words, the retailer needs to cover the credit amount to his/her supplier. This amount is calculated with $\left[C_{pL}(S + R) - U_1, C_{pR}(S + R) - U_1\right]$ after the time $t = M$. The interval time period $t = [B_L, B_R]$ when the supplier gets the full creditable amount within this time interval. Regarding this situation, supplier charges the interest for the period $[M, [B_L, B_R]]$ with interest paid rate I_p.

Therefore, the total on-hand amount available to the retailer is equal to the amount payable to the supplier at the time $t = [B_L, B_R]$. Thus,

$$C_{pL}(S + R)\left\{1 + I_p(B_L - M)\right\} = U_1[1 + I_e(B_L - N)] + pD(.)(B_L - M)$$
$$\left[1 + I_e\frac{(B_L - M)}{2}\right]$$

and

$$C_{pR}(S+R)\left\{1+I_p(B_R-M)\right\} = U_1[1+I_e(B_R-N)] + pD(.)(B_R-M)\left[1+I_e\frac{(B_R-M)}{2}\right]$$

Consequently, the total interval-valued profit of the inventory system is as follows:

$$Z_{3L}(.) = \frac{X_L(.)}{T} \quad \text{and} \quad Z_{3R}(.) = \frac{X_R(.)}{T}$$

where

$[X_L, X_R]$ = < *sales revenue during the time interval* $[[B_L, B_R], t_1]$ > + < *interest earned for the duration of the time interval* $[[B_L, B_R], t_1]$ > + < *interest earned within the interval* $[t_1, T]$ > − < *ordering cost* > − <*holding cost* > − <*shortage cost* > − <*cost of lost sale*>

$$X_L(.) = \left\{p\int_{B_R}^{t_1} D(.)dt + pI_e\int_{B_R}^{t_1}\int_{B_R}^{t} D(.)dudt\right\}\{1+I_e(T-t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R(.) = \left\{p\int_{B_L}^{t_1} D(.)dt + pI_e\int_{B_L}^{t_1}\int_{B_L}^{t} D(.)dudt\right\}\{1+I_e(T-t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL} \tag{2.19}$$

Thus, the interval-valued constrained nonlinear optimization problem is expressed below:

Problem 3

$$\begin{aligned} &\text{Maximize } Z_3(.) = [Z_{3L}(.), Z_{3R}(.)] \\ &\text{subject to } 0 < N < M \le t_1 < T \end{aligned} \tag{2.20}$$

Case 2: $N < t_1 < M \le T$

Owing to sales revenue and interest earned, the collected amount of the retailer is computed as

$$U_2 = \left[p\int_0^{t_1} D(.)dt + pI_e\int_N^M\int_N^t D(.)dudt\right][1+I_e(M-t_1)] + pR\{1+I_e(M-N)\}$$

$$U_2 = pD(.)\left[t_1 + I_e\frac{\left(M^2 - N^2\right)}{2}\right][1 + I_e(M - t_1)] + pR\{1 + I_e(M - N)\}$$

Here, the interval-valued profit of the inventory system is formulated as

$$Z_{4L}(.) = \frac{X_L(.)}{T} \quad \text{and} \quad Z_{4R}(.) = \frac{X_R(.)}{T}$$

where

$[X_L, X_R] =$ < *Excess amount available of retailer after paying the supplier* > + < *total interest earned for that excess amount in* $[M, T]$ > − < *ordering cost* > − <*holding cost* > − <*shortage cost* > − <*cost of lost sale*>

$$X_L(.) = U_2 - (C_{oR} + C_{holR} + C_{shoR} + OC_{LSR})$$

and

$$X_R(.) = U_2 - (C_{oL} + C_{holL} + C_{shoL} + OC_{LSL})$$

For that reason, the corresponding interval-valued constrained nonlinear optimization problem is as follows.

Problem 4

$$\begin{aligned} &\text{Maximize } Z_4(.) = [Z_{4L}(.), Z_{4R}(.)] \\ &\text{subject to } N < t_1 < M \leq T \end{aligned} \tag{2.21}$$

2.4 The Solution for Three Demand Functions

$D(\cdot) = a - bp,\ a, b > 0,\ D(\cdot) = ap^{-\alpha}\ \ a > 0, \alpha < 1,$ **and** $D(\cdot) = ae^{(-p/k)},\ a, k > 0.$

This section derives the mathematical expressions for three price demand functions.

4.1: When $D(\cdot) = a - bp, \quad a, b > 0$

Here,

$$U_1 = p\int_0^M (a - bp)dt + pI_e\int_N^M\int_N^t (a - bp)dudt + pR\{1 + I_e(M - N)\}$$

Case 4.1.1.

$$\begin{aligned} &\text{Maximize } Z_1^{(1)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right] \\ &\text{subject to } 0 < N < M \le t_1 < T \end{aligned} \tag{2.22}$$

where

$$\begin{aligned} X_L = &\left\{U_1 - C_{pR}(S+R)\right\}\{1 + I_e(T - M)\} + (a - bp)p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\} \\ &\{1 + I_e(T - t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR} \end{aligned}$$

and

$$\begin{aligned} X_R = &\left\{U_1 - C_{pL}(S+R)\right\}\{1 + I_e(T - M)\} + (a - bp)p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\} \\ &\{1 + I_e(T - t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL} \end{aligned}$$

Case 4.1.2.

$$\begin{aligned} &\text{Maximize } Z_2^{(1)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right] \\ &\text{subject to } 0 < N < M \le t_1 < T \end{aligned} \tag{2.23}$$

$$\left\{C_{pL}(S+R) - U_1\right\}\left\{1 + I_p(B_L - M)\right\} = p\int_M^{B_L}(a - bp)dt + pI_e\int_M^{B_L}\int_M^{t}(a - bp)dudt$$

and

$$\left\{C_{pR}(S+R) - U_1\right\}\left\{1 + I_p(B_R - M)\right\} = p\int_M^{B_R}(a - bp)dt + pI_e\int_M^{B_R}\int_M^{t}(a - bp)dudt$$

where

$$\begin{aligned} X_L = &\left\{p\int_{B_R}^{t_1}(a - bp)dt + pI_e\int_{B_R}^{t_1}\int_{B_R}^{t}(a - bp)dudt\right\}\{1 + I_e(T - t_1)\} \\ &- C_{oR} - C_{holR} - C_{shoR} - OC_{LSR} \end{aligned}$$

$$\begin{aligned} X_R = &\left\{p\int_{B_L}^{t_1}(a - bp)dt + pI_e\int_{B_L}^{t_1}\int_{B_L}^{t}(a - bp)dudt\right\}\{1 + I_e(T - t_1)\} \\ &- C_{oL} - C_{holL} - C_{shoL} - OC_{LSL} \end{aligned}$$

Case 4.1.3.

$$\text{Maximize } Z_3^{(1)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } 0 < N < M \le t_1 < T \tag{2.24}$$

$$C_{pL}(S+R)\{1 + I_p(B_L - M)\} = U_1[1 + I_e(B_L - N)] + p(a - bp)(B_L - M)\left[1 + I_e\frac{(B_L - M)}{2}\right]$$

and

$$C_{pR}(S+R)\{1 + I_p(B_R - M)\} = U_1[1 + I_e(B_R - N)] + p(a - bp)(B_R - M)$$
$$\left[1 + I_e\frac{(B_R - M)}{2}\right]$$

where

$$X_L = \left\{p\int_{B_R}^{t_1}(a - bp)dt + pI_e\int_{B_R}^{t_1}\int_{B_R}^{t}(a - bp)dudt\right\}\{1 + I_e(T - t_1)\}$$
$$- C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{p\int_{B_L}^{t_1}(a - bp)dt + pI_e\int_{B_L}^{t_1}\int_{B_L}^{t}(a - bp)dudt\right\}\{1 + I_e(T - t_1)\}$$
$$- C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.1.4.

Here,

$$U_2 = \left[p\int_{0}^{t_1}(a - bp)dt + pI_e\int_{N}^{M}\int_{N}^{t}(a - bp)dudt\right][1 + I_e(M - t_1)]$$
$$+ pR\{1 + I_e(M - N)\}$$

$$\text{Maximize } Z_4^{(1)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } N < t_1 < M \le T \tag{2.25}$$

where

$$X_L = U_2 - (C_{oR} + C_{holR} + C_{shoR} + OC_{LSR})$$

and

$$X_R = U_2 - (C_{oL} + C_{holL} + C_{shoL} + OC_{LSL})$$

4.2: **When** $D(\cdot) = ap^{-\alpha} \quad a > 0, \alpha < 1$
Here,

$$U_1 = p\int_0^M ap^{-\alpha}\,dt + pI_e\int_N^M\int_N^t ap^{-\alpha}\,dudt + pR\{1 + I_e(M - N)\}$$

Case 4.2.1.

$$\begin{aligned} &\text{Maximize } Z_1^{(2)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right] \\ &\text{subject to } 0 < N < M \le t_1 < T \end{aligned} \tag{2.26}$$

where

$$X_L = \left\{U_1 - C_{pR}(S + R)\right\}\{1 + I_e(T - M)\} + ap^{-\alpha}p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\}$$
$$\{1 + I_e(T - t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{U_1 - C_{pL}(S + R)\right\}\{1 + I_e(T - M)\} + ap^{-\alpha}p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\}$$
$$\{1 + I_e(T - t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.2.2.

$$\begin{aligned} &\text{Maximize } Z_2^{(2)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right] \\ &\text{subject to } 0 < N < M \le t_1 < T \end{aligned} \tag{2.27}$$

$$\left\{C_{pL}(S + R) - U_1\right\}\left\{1 + I_p(B_L - M)\right\} = p\int_M^{B_L} ap^{-\alpha}\,dt + pI_e\int_M^{B_L}\int_M^t ap^{-\alpha}\,dudt$$

and

$$\left\{C_{pR}(S + R) - U_1\right\}\left\{1 + I_p(B_R - M)\right\} = p\int_M^{B_R} ap^{-\alpha}\,dt + pI_e\int_M^{B_R}\int_M^t ap^{-\alpha}\,dudt$$

where

$$X_L = \left\{ p \int_{B_R}^{t_1} ap^{-\alpha}\, dt + pI_e \int_{B_R}^{t_1} \int_{B_R}^{t} ap^{-\alpha}\, dudt \right\} \{1 + I_e(T - t_1)\}$$
$$- C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{ p \int_{B_L}^{t_1} ap^{-\alpha}\, dt + pI_e \int_{B_L}^{t_1} \int_{B_L}^{t} ap^{-\alpha}\, dudt \right\} \{1 + I_e(T - t_1)\}$$
$$- C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.2.3.

$$\begin{aligned} &\text{Maximize } Z_3^{(2)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T} \right] \\ &\text{subject to } 0 < N < M \le t_1 < T \end{aligned} \tag{2.28}$$

$$C_{pL}(S + R)\big\{1 + I_p(B_L - M)\big\} = U_1[1 + I_e(B_L - N)] + pap^{-\alpha}(B_L - M) \left[1 + I_e \frac{(B_L - M)}{2} \right]$$

and

$$C_{pR}(S + R)\big\{1 + I_p(B_R - M)\big\} = U_1[1 + I_e(B_R - N)] + pap^{-\alpha}(B_R - M) \left[1 + I_e \frac{(B_R - M)}{2} \right]$$

where

$$X_L = \left\{ p \int_{B_R}^{t_1} ap^{-\alpha}\, dt + pI_e \int_{B_R}^{t_1} \int_{B_R}^{t} ap^{-\alpha}\, dudt \right\} \{1 + I_e(T - t_1)\}$$
$$- C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{ p \int_{B_L}^{t_1} ap^{-\alpha}\, dt + pI_e \int_{B_L}^{t_1} \int_{B_L}^{t} ap^{-\alpha}\, dudt \right\} \{1 + I_e(T - t_1)\}$$
$$- C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.2.4.

Here,

$$U_2 = \left[p \int_0^{t_1} ap^{-\alpha}\, dt + pI_e \int_N^M \int_N^t ap^{-\alpha}\, dudt \right][1 + I_e(M - t_1)] \\ + pR\{1 + I_e(M - N)\}$$

$$\text{Maximize } Z_4^{(2)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T} \right] \\ \text{subject to } N < t_1 < M \le T \tag{2.29}$$

where

$$X_L = U_2 - (C_{oR} + C_{holR} + C_{shoR} + OC_{LSR})$$

and

$$X_R = U_2 - (C_{oL} + C_{holL} + C_{shoL} + OC_{LSL})$$

4.3: **When** $D(\cdot) = ae^{(-p/k)}, \quad a, k > 0$

Here,

$$U_1 = p \int_0^M ae^{(-p/k)}\, dt + pI_e \int_N^M \int_N^t ae^{(-p/k)}\, dudt + pR\{1 + I_e(M - N)\}$$

Case 4.3.1.

$$\text{Maximize } Z_1^{(3)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T} \right] \\ \text{subject to } 0 < N < M \le t_1 < T \tag{2.30}$$

where

$$X_L = \{U_1 - C_{pR}(S + R)\}\{1 + I_e(T - M)\} + ae^{(-p/k)} p(t_1 - M)\left\{1 + \frac{1}{2} I_e(t_1 - M)\right\} \\ \{1 + I_e(T - t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \{U_1 - C_{pL}(S + R)\}\{1 + I_e(T - M)\} + ae^{(-p/k)} p(t_1 - M)\left\{1 + \frac{1}{2} I_e(t_1 - M)\right\} \\ \{1 + I_e(T - t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.3.2.

$$\text{Maximize } Z_2^{(3)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } 0 < N < M \le t_1 < T \tag{2.31}$$

$$\left\{C_{pL}(S+R) - U_1\right\}\left\{1 + I_p(B_L - M)\right\} = p\int_M^{B_L} ae^{(-p/k)}\,dt + pI_e\int_M^{B_L}\int_M^t ae^{(-p/k)}\,dudt$$

and

$$\left\{C_{pR}(S+R) - U_1\right\}\left\{1 + I_p(B_R - M)\right\} = p\int_M^{B_R} ae^{(-p/k)}\,dt + pI_e\int_M^{B_R}\int_M^t ae^{(-p/k)}\,dudt$$

where

$$X_L = \left\{p\int_{B_R}^{t_1} ae^{(-p/k)}\,dt + pI_e\int_{B_R}^{t_1}\int_{B_R}^t ae^{(-p/k)}\,dudt\right\}\{1 + I_e(T - t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{p\int_{B_L}^{t_1} ae^{(-p/k)}\,dt + pI_e\int_{B_L}^{t_1}\int_{B_L}^t ae^{(-p/k)}\,dudt\right\}\{1 + I_e(T - t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.3.3.

$$\text{Maximize } Z_3^{(3)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } 0 < N < M \le t_1 < T \tag{2.32}$$

$$C_{pL}(S+R)\left\{1 + I_p(B_L - M)\right\} = U_1[1 + I_e(B_L - N)] + pae^{(-p/k)}(B_L - M)\left[1 + I_e\frac{(B_L - M)}{2}\right]$$

and

$$C_{pR}(S+R)\left\{1+I_p(B_R-M)\right\} = U_1[1+I_e(B_R-N)] + pae^{(-p/k)}(B_R-M)\left[1+I_e\frac{(B_R-M)}{2}\right]$$

where

$$X_L = \left\{p\int_{B_R}^{t_1} ae^{(-p/k)}\,dt + pI_e\int_{B_R}^{t_1}\int_{B_R}^{t} ae^{(-p/k)}\,dudt\right\}\{1+I_e(T-t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{p\int_{B_L}^{t_1} ae^{(-p/k)}\,dt + pI_e\int_{B_L}^{t_1}\int_{B_L}^{t} ae^{(-p/k)}dudt\right\}\{1+I_e(T-t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.3.4.

Here,

$$U_2 = \left[p\int_{0}^{t_1} ae^{(-p/k)}\,dt + pI_e\int_{N}^{M}\int_{N}^{t} ae^{(-p/k)}\,dudt\right][1+I_e(M-t_1)] + pR\{1+I_e(M-N)\}$$

$$\begin{aligned}&\text{Maximize } Z_4^{(3)}(S,R,t_1,T) = \left[\frac{X_L}{T},\frac{X_R}{T}\right]\\&\text{subject to } N < t_1 < M \le T\end{aligned} \tag{2.33}$$

where

$$X_L = U_2 - (C_{oR} + C_{holR} + C_{shoR} + OC_{LSR})$$

and

$$X_R = U_2 - (C_{oL} + C_{holL} + C_{shoL} + OC_{LSL})$$

2.5 Numerical Examples

This section provides and solves three instances with the purpose of illustrating and validating the inventory model.

The solution procedure consists of applying the theory of interval numbers and two efficient and effective soft computing techniques: Particle swarm optimization constriction (PSO-CO) and weighted quantum particle swarm optimization (WQPSO). Both soft computing algorithms are programmed in C language. The computational experiments are done on a personal computer with the following technical characteristics: Intel Core-2-Duo, 2.5 GHz Processor, and LINUX environ. It is important to remark that Kennedy and Eberhart [27], Clerc and Kennedy [28], and Clerc [29] proposed the particle swarm optimization (PSO) and particle swarm optimization constriction (PSO-CO); and Sun et al. [30, 31] introduced weighted quantum particle swarm optimization (WQPSO). Sahoo et al. [32] introduced the definitions of interval order relations between two interval numbers with the aim of solving the maximization and minimization problems.

Example 1 Consider an inventory problem in which the demand function is given by $D = a - bp$ and the following parameters: $C_{oL} = \$195$, $C_{oR} = \$200$, $\theta = 0.1$, $a = 150$, $b = 0.7$, $\delta = 1.5$, $C_{hL} = \$1$, $C_{hR} = \$1.5$, $C_{bR} = \$10$, $C_{bL} = \$8$, $C_{pL} = \$22$, $C_{pR} = \$25$, $I_e = 0.12$, $I_p = 0.15$ $N = 0.16$, $M = 0.246$, $C_{lsL} = \$18$, $C_{lsR} = \$20$, $p = \$30$.

The solution is exhibited in Tables 2.2 and 2.3, where Table 2.2 shows the solution obtained by PSO-CO and Table 2.3 displays the solution determined by WQPSO.

Example 2 Consider an inventory system in which the demand function is as follows: $D = ap^{-\alpha}$ and the following parameters: $C_{oL} = \$195$, $C_{oR} = \$200$, $\theta = 0.1$, $a = 150$, $\delta = 1.5$, $C_{hL} = \$1$, $C_{hR} = \$1.5$, $C_{bR} = \$10$, $C_{bL} = \$8$, $C_{pL} = \$22$, $C_{pR} = \$25$, $I_e = 0.12$, $I_p = 0.15$ $N = 0.16$, $M = 0.246$, $C_{lsL} = \$18$, $C_{lsR} = \$20$, $p = \$30$, $\alpha = 0.2$ (Tables 2.4 and 2.5).

Example 3 Consider that the demand function is $D = ae^{(-p/k)}$ and the following parameters: $C_{oL} = \$195$, $C_{oR} = \$200$, $\theta = 0.1$, $a = 150$, $\delta = 1.5$, $C_{hL} = \$1$, $C_{hR} = \$1.5$, $C_{bR} = \$10$, $C_{bL} = \$8$, $C_{pL} = \$22$, $C_{pR} = \$25$, $I_e = 0.12$, $I_p = 0.15$ $N = 0.16$, $M = 0.246$, $C_{lsL} = \$18$, $C_{lsR} = \$20$, $p = \$30$, $k = 40$ (Tables 2.6 and 2.7).

2.6 Sensitivity Analysis

This section provides a sensitivity analysis, which is done, based on Example 1. The sensitivity analysis is made by varying the parameters by −20 to +20%. The results of the sensitivity analysis for Example 1 are shown in Table 2.8.

Table 2.2 Optimal solution for Example 1 when demand function is modeled by $D = a - bp$ and PSO-CO is applied

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
93.4681	13.5663	0.6995	0.8134	182.8536	635.1188	408.99	–	–	1.1
85.9997	13.7784	0.6454	0.7612	168.9084	606.3228	378.19	0.4630	0.5438	1.2.1
79.9735	14.0093	0.6015	0.7194	150.0555	606.3228	378.19	0.4292	0.5063	1.2.2
82.6527	14.1652	0.6210	0.7404	136.54742	581.57365	359.06	–	–	2

Table 2.3 Optimal solution for Example 1 when demand function is expressed by $D = a - bp$ and WQPSO is used

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
93.4644	13.5667	0.6995	0.8134	182.8549	635.1176	408.99	–	–	1.1
86.0001	13.7784	0.6454	0.7612	168.9082	622.6545	395.78	0.4630	0.5439	1.2.1
79.9740	14.0086	0.6015	0.7194	150.0553	606.32300	378.19	0.4292	0.5063	1.2.2
82.6534	14.1654	0.6210	0.7404	136.5472	581.5738	359.06	–	–	2

Table 2.4 Optimal solution for Example 2 when demand function is given by $D = ap^{-\alpha}$ and PSO-CO is utilized

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
70.3066	10.7814	0.8851	1.0432	13.9693	292.8948	153.43	–	–	1.1
63.6977	10.9814	0.8051	0.9665	−1.0359	279.3446	139.15	0.5841	0.6892	1.2.1
60.3069	11.0487	0.7638	0.9264	−17.0486	265.2967	124.12	0.5540	0.6557	1.2.2
61.9337	11.1585	0.7837	0.9480	−21.2679	252.0924	115.41	–	–	2

Table 2.5 Optimal solution for Example 2 when demand function is given by $D = ap^{-\alpha}$ and WQPSO is used

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
70.3070	10.7814	0.8851	1.0432	13.9692	292.8949	153.43	–	–	1.1
63.6995	10.9817	0.8051	0.9665	−1.0366	279.3454	139.15	0.5842	0.6892	1.2.1
60.3062	11.0488	0.7638	0.9264	−17.0482	265.2964	124.12	0.5540	0.6557	1.2.2
61.9330	11.1590	0.7837	0.9480	−21.2677	252.0921	115.41	–	–	2

Table 2.6 Optimal solution for Example 3 when demand function is $D = ae^{(-p/k)}$, and PSO-CO is computed

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
67.7420	10.4657	0.9131	1.0784	−0.4084	261.5048	130.55	–	–	1.1
61.2534	10.6616	0.8291	0.9980	−15.4465	247.9046	116.23	0.6027	0.7116	1.2.1
58.0977	10.7124	0.7881	0.9578	−30.9859	234.2535	101.63	0.5730	0.6785	1.2.2
59.5854	10.8172	0.8075	0.9790	−34.4253	222.0720	93.82	–	–	2

Table 2.7 Optimal solution for Example 3 when demand function is $D = ae^{(-p/k)}$ and WQPSO is applied

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
67.7423	10.4657	0.9131	1.0784	−0.4085	261.5049	130.55	–	–	1.1
61.2530	10.6620	0.8291	0.9980	−15.4463	247.9044	116.23	0.6027	0.7116	1.2.1
58.0976	10.7126	0.7881	0.9578	−30.9859	234.2535	101.63	0.5730	0.6785	1.2.2
59.5849	10.8174	0.8074	0.9790	−34.4251	222.0718	93.82	–	–	2

Table 2.8 Sensitivity analysis for Example 1 when $D(\cdot) = a - bp,\ a,\ b > 0$

Parameters	% change of parameters	Change						
		Z_L	Z_R	t_1	T	R	S	Average profit
$C_o = [C_{oL}, C_{oR}]$	−20	237.6248	682.45141	0.6343	0.734	11.9825	84.4757	460.04
	−10	209.5331	658.1815	0.6678	0.7748	12.7919	89.0929	433.857
	10	157.3913	613.0972	0.7296	0.8503	14.3106	97.6349	385.244
	20	132.9925	591.9853	0.7583	0.8856	15.0287	101.619	362.488
$C_h = [C_{hL}, C_{hR}]$	−20	194.2057	643.9136	0.7158	0.8281	13.3908	95.7265	419.059
	−10	188.4851	639.4885	0.7075	0.8207	13.4788	94.58	413.986
	10	177.3085	630.8032	0.6917	0.8064	13.6533	92.3891	404.055
	20	171.8473	626.5403	0.6841	0.7996	13.7397	91.3413	399.193
$C_b = [C_{bL}, C_{bR}]$	−20	184.8203	636.6107	0.6986	0.8174	14.1098	93.3359	410.715
	−10	183.8167	635.8522	0.699	0.8154	13.8327	93.4031	409.834
	10	181.9287	634.4092	0.7	0.8116	13.3099	93.5309	408.169
	20	181.0397	633.7223	0.7004	0.8098	13.0631	93.5916	407.381
$C_p = [C_{pL}, C_{pR}]$	−20	887.4738	1266.7736	0.8399	0.9522	13.3861	113.0255	1077.1237
	−10	533.2732	948.796	0.7617	0.8737	13.3512	102.0973	741.0346
	10	−164.3283	324.9279	0.648	0.7658	13.2949	86.3625	80.2998
	20	–	–	–	–	–	–	–
a	−20	83.5254	438.2659	0.7865	0.9205	12.0830	81.0094	260.90
	−10	132.1852	535.8331	0.7388	0.8616	12.8493	87.4105	334.01
	10	235.1709	735.8147	0.6665	0.7731	14.2425	99.2434	485.49
	20	288.8702	837.6921	0.6382	0.7387	14.8845	104.7820	563.28

(continued)

Table 2.8 (continued)

Parameters	% change of parameters	Change						
		Z_L	Z_R	t_1	T	R	S	Average profit
b	−20	197.3492	663.1826	0.6897	0.8014	13.7594	95.1114	430.27
	−10	190.0857	649.1373	0.6946	0.8074	13.6632	94.2924	419.61
	10	175.6537	621.1279	0.7046	0.8196	13.4685	92.6382	398.39
	20	168.4868	607.16522	0.7098	0.8260	13.3700	91.8027	387.83
θ	−20	203.7516	656.8053	0.7385	0.8498	13.2698	98.1428	430.28
	−10	193.1488	645.7845	0.7183	0.8309	13.4171	95.7234	419.47
	10	172.8476	624.7827	0.6820	0.7972	13.7166	91.3580	398.82
	20	163.1138	614.7535	0.6655	0.7822	13.8675	89.3772	388.93
δ	−20	190.2571	642.6646	0.6953	0.8292	16.0174	92.8870	416.46
	−10	186.2794	638.5865	0.6976	0.8207	14.6895	93.1987	412.43
	10	179.8720	632.1338	0.7012	0.8072	12.6036	93.7033	406.00
	20	177.2531	629.5368	0.7027	0.8019	11.7692	93.9103	403.39
$C_{ls} = [C_{lsL}, C_{lsR}]$	−20	189.3095	640.6154	0.6962	0.8275	15.4513	93.0153	414.9625
	−10	185.8718	637.7029	0.698	0.8199	14.4474	93.2545	411.7874
	10	180.1828	632.8108	0.7009	0.8078	12.7865	93.66	406.4968
	20	177.8028	630.7368	0.7022	0.8028	12.0916	93.8332	404.2698
p	−20	–	–	–	–	–	–	–
	−10	−218.2979	237.9026	0.6644	0.7786	13.4190	90.0669	9.80
	10	572.1072	1020.6209	0.7374	0.8523	13.4523	97.1063	796.36
	20	949.4445	1394.45055	0.7787	0.8958	13.4604	101.0696	1171.95

From Table 2.8, the following interpretations are mentioned:

- With the increment in the value of replenishment cost $[C_{oL}, C_{oR}]$, the cycle length (T), the time at which the inventory level reaches zero (t_1), shortage level (R), and stock level (S) increase, but average profit decreases.
- When the holding cost $[C_{hL}, C_{hR}]$ increases, then the cycle length (T), the time at which the inventory level reaches zero (t_1), stock level (S), and average profit decrease, but shortage level (R) increases.
- With the increment in the value of shortage cost $[C_{bL}, C_{bR}]$, the time at which the inventory level reaches zero (t_1) and stock level (S) increase, but the shortage level (R), the cycle length (T), and the average profit decrease.
- When the value of purchasing cost $\left[C_{pL}, C_{pR}\right]$ increases, then the shortage level (R), the cycle length (T), the time at which the inventory level reaches zero (t_1), stock level (S) and the average profit decrease.
- When the scale parameter (a) of demand increases, then the cycle length (T) and the time at which the inventory level reaches zero (t_1) decrease, whereas the shortage level (R), the stock level (S), and the average profit increase.
- When the price elasticity parameters (b) of demand increases, then the shortage level (R), the stock level (S), and the average profit decrease, whereas the cycle length (T) and the time at which the inventory level reaches zero (t_1) increase.
- With the increment in the value of the deterioration rate (θ), the cycle length (T), stock level (S), the average profit, and the time at which the inventory level reaches zero (t_1) decrease, but the shortage level (R) increases.
- When the value of the backlogging parameter δ increases, then the shortage level (R), the cycle length (T) and the average profit decrease, but the time at which the inventory level reaches zero (t_1) and stock level (S) increase.
- With the increment in the value of opportunity cost $[C_{lsL}, C_{lsR}]$, then the shortage level (R), the cycle length (T), and the average profit decrease, but the time at which the inventory level reaches zero (t_1) and stock level (S) increase.
- With the increment in the value of selling price (p), then the shortage level (R), the cycle length (T), the time at which the inventory level reaches zero (t_1), stock level (S), and the average profit increase.

2.7 Conclusion

This paper develops an inventory model for deteriorating items with interval-valued inventory costs, partial backlogging, and price-dependent demand under two-level credit policy. In order to make a more realistic scenario, the shortages are permitted, and these are partially backlogged. The proposed inventory model is very helpful for retail and manufacturing industries in developing countries where credit policy plays a significant role in decision-making. This research work aims to find the retailer's optimal replenishment policy that maximizes the total profit of the system. To solve the inventory model, two soft computing techniques are used: the PSO-CO and the WQPSO. The efficiency and effectiveness of the proposed inventory model are validated with numerical examples and a sensitivity analysis.

Finally, this research can be extended by considering: (1) stock-dependent demand, (2) inventory costs represented by a fuzzy number, (3) finite-time horizon, (4) inflation, and (5) an integrated supply chain model with two or more players with the coordination between the players, among others. These are some interesting and challenge research lines to explore by academicians and researchers.

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