Asset Analytics
Performance and Safety Management
Series Editors: Ajit Kumar Verma · P. K. Kapur · Uday Kumar

Nita H. Shah
Mandeep Mittal *Editors*

Optimization and Inventory Management

Springer

Asset Analytics

Performance and Safety Management

Series Editors

The main aim of this book series is to provide a floor for researchers, industries, asset managers, government policy makers and infrastructure operators to cooperate and collaborate among themselves to improve the performance and safety of the assets with maximum return on assets and improved utilization for the benefit of society and the environment.

Assets can be defined as any resource that will create value to the business. Assets include physical (railway, road, buildings, industrial etc.), human, and intangible assets (software, data etc.). The scope of the book series will be but not limited to:

- Optimization, modelling and analysis of assets
- Application of RAMS to the system of systems
- Interdisciplinary and multidisciplinary research to deal with sustainability issues
- Application of advanced analytics for improvement of systems
- Application of computational intelligence, IT and software systems for decisions
- Interdisciplinary approach to performance management
- Integrated approach to system efficiency and effectiveness
- Life cycle management of the assets
- Integrated risk, hazard, vulnerability analysis and assurance management
- Adaptability of the systems to the usage and environment
- Integration of data-information-knowledge for decision support
- Production rate enhancement with best practices
- Optimization of renewable and non-renewable energy resources

More information about this series at http://www.springer.com/series/15776

Nita H. Shah · Mandeep Mittal
Editors

Optimization and Inventory Management

Springer

Editors
Nita H. Shah
Department of Mathematics
Gujarat University
Ahmedabad, India

Mandeep Mittal
Department of Mathematics
Amity Institute of Applied Sciences
Amity University
Noida, India

ISSN 2522-5162 ISSN 2522-5170 (electronic)
Asset Analytics
ISBN 978-981-13-9697-7 ISBN 978-981-13-9698-4 (eBook)
https://doi.org/10.1007/978-981-13-9698-4

This Springer imprint is published by the registered company Springer Nature Singapore Pte Ltd.
The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

Contents

About the Editors

Nita H. Shah is a Professor at the Department of Mathematics, Gujarat University, Ahmedabad, India. Her primary research interest is in operations research, in particular modelling real-life problems. She has 12 books and more than 475 articles in respected international journals to her credit. In addition, she is currently serving on the editorial boards of the journals Revista Investigacion Operacional (Universidad de La Habana, Cuba), Journal of Social Science and Management, International Journal of Industrial Engineering and Computations, and Mathematics Today.

Mandeep Mittal is an Assistant Professor at the Department of Mathematics, Amity Institute of Applied Sciences, Amity University, Noida, India. After completing his Master's in Applied Mathematics at the Indian Institute of Technology (IIT) Roorkee, he obtained his Ph.D. from the University of Delhi, India. He subsequently completed his postdoctoral research at Hanyang University, South Korea. He has one book and more than 50 papers in international journals and conference proceedings to his credit. He received the Best Faculty Award from Amity School of Engineering and Technology, New Delhi, for the year 2016–2017. In addition, he is currently serving on the editorial boards of the journals Revista Investigacion Operacional, Journal of Control and Systems Engineering, and Journal of Advances in Management Sciences and Information Systems.

Chapter 1
Economic Production Quantity (EPQ) Inventory Model for a Deteriorating Item with a Two-Level Trade Credit Policy and Allowable Shortages

Ali Akbar Shaikh, Leopoldo Eduardo Cárdenas-Barrón and Sunil Tiwari

Abstract This research work derives an economic production quantity (EPQ) model, and in order to make it a bit close to reality, the stockout is allowed, and this is completely backordered. In addition to this feature, it is incorporated a two-level credit scheme when both supplier and retailer are giving a delay in payment to their respective customers with the aim of enhancing the sales. The inventory model is modeled as a constrained nonlinear optimization problem, and this is resolved by the generalized reduced gradient method (GRG). Moreover, to exemplify and certify the inventory model, five instances are given and solved. Finally, a sensitivity analysis is made for studying the influence of variations of input parameters, modifying one parameter and maintaining the others at their initial input values.

Keywords Production-inventory model · Deteriorating · Constant demand · Full backlogging · Full credit policy

1.1 Introduction

The management of inventories is one of the critical responsibilities that the managers of manufacturing firms need to do carefully. Recently, businesses are highly competitive due to globalization. So, all manufacturing firms are highly engaged in

A. A. Shaikh
Department of Mathematics, The University of Burdwan, Burdwan 713104, India
e-mail: aliashaikh@math.buruniv.ac.in

L. E. Cárdenas-Barrón (✉)
Department of Industrial and Systems Engineering, School of Engineering and Sciences, Tecnológico de Monterrey, E. Garza Sada 2501 Sur, C.P. 64849 Monterrey, Nuevo León, Mexico
e-mail: lecarden@tec.mx

S. Tiwari
The Logistics Institute - Asia Pacific, National University of Singapore, 21 Heng Mui Keng Terrace, Singapore 119613, Singapore
e-mail: sunil.tiwari047@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_1

how to promote their business in order to have a successful career with the aim of surviving in current volatile markets. For this reason, the researchers and academician are very interested in deriving inventory models that are useful in an inventory decision-making process.

In a supply chain system, there exist several stages and, in each stage, there is a customer that one needs to satisfy. A supply chain system typically involves suppliers, manufacturers, transport system, warehouses capacities, retailers, and the end consumers [8]. The core aim of a supply chain system is to deliver the right products at the right time and right location with a minimum total cost [18].

Trade credit policy is one of the essential issues that are required to promote the business in highly competitive markets. In this context, suppliers/manufacturers give a certain (fixed) time to settle the payment without any interest charge to the customers. This trade credit policy motivates to consumers to procure more goods. With this credit policy, the buyer reduces his/her inventory holding cost because this decision decreases the amount of capital investment during the permissible delay period. It is important to mention that during the interval of the permissible delay in payment, the retailer earns revenue due to the sales of the product as well as obtains interests on that revenue via a banking facility or any other investment alternative. Usually, the trade credit policy enables to increase the demand and capture more and more clientele.

In inventory management, there exist two well-known inventory models: Harris [12] and Taft [19]. The first one is identified as the economic order quantity (EOQ). The second one is called the economic production quantity (EPQ). It is relevant to remark that the EOQ inventory model is a particular case of the EPQ inventory model. Notice that the well-known EOQ inventory model is developed by putting the assumption that when a retailer buys a product, he or she must give the payment to his/her supplier when the items are delivered. For the first time, Goyal [11] introduced the concept of credit policy in an inventory model. Goyal [11] formulated a single-product inventory model with a permissible delay in payment. After that, many EOQ inventory models with trade credit policy have been appearing. In this direction, the reader can see the two comprehensive reviews related to trade credit in Chang et al. [5] and Seifert et al. [15].

The credit strategy examined in Goyal [11] consists in that supplier gives a credit period to his/her retailer. However, the retailer cannot give a credit period to his/her client. This type of problem is identified as a single-level trade credit scheme. The problem becomes more interesting if the retailer gives credit to its client too. Thus, this kind of problem is recognized as a two-level trade credit scheme for a supply chain comprised of supplier–retailer–customer. Perhaps, Huang [13] introduced an EOQ inventory model with two-level trade credit scheme by taking into consideration that the supplier gives to the retailer a delay period (M), and the latter correspondingly gives a delay period (N) to its customer. After that, this type of inventory model has also discussed by Teng [20]. Now, some dissimilarities between Teng [20] and Huang [13] inventory models are mentioned below:

(i) In Huang's [13] inventory model, if the client buys goods from the retailer at time t then he or she must pay the goods at time N. Consequently, the retailer must permit the maximum credit time period to his/her retailer up to time N.
(ii) In Teng's [20] inventory model, if the customer procures products to the retailer at time t which is within in the interval [0, T], then the customer must give payment to its retailer at time $N + t$. Here, the retailer always permits the customer a credit period N. The perspective of Teng [20] is generally applied in the business operations.

Chung and Huang [9] extended Goyal's [11] inventory model. Principally, they developed an EPQ inventory model under a single-level credit policy approach.

Generally, in the real world, there exist so many kinds of goods which deteriorate thru time. So, in inventory analysis, the deterioration cannot be ignored. Ghare and Schrader [10] formulated an EOQ inventory model by considering that the deterioration rate is known and constant. Later, a lot of research works have been done under the trade credit policy.

On the one hand, Liao [14] presented an EPQ inventory model for goods that suffer an exponential deterioration rate involving two-level trade credit scheme based on the concepts Huang's [13] inventory model. On the other hand, Chang et al. [6] discussed an EPQ inventory model with products that suffer an exponential deterioration rate considering two-level trade credit scheme by applying the concepts of Teng [20]. Table 1.1 presents some recent works related to the trade credit scheme. The acronyms IFS and SFI correspond to inventory follows shortage and shortage follows inventory, respectively.

Notice that if demand is higher than the production rate, then shortages appear. In this context, the manufacturing firm decides to cover this shortage. This research work deals with an inventory model in which it is allowed fully backlogged shortages with fully two-level trade credit scheme. The inventory model is formulated as a nonlinear constrained optimization problem. In order to exemplify and certify the inventory model, five examples are presented and solved.

This research work is designed in the following manner. Section 1.2 presents the suppositions and notation. Section 1.3 develops the inventory model. Section 1.4 solves five instances. Section 1.5 gives a sensitivity analysis. Section 1.6 exposes conclusions and research guidelines.

Table 1.1 Some related EOQ/EPQ inventory models with deterioration

Author(s)	Deterioration	Demand rate	Shortages	Level of permissible delay in payments	Inventory policies	EOQ/EPQ
Shah et al. [16]	Yes	Selling price-dependent	No	No	–	EOQ
Wu et al. [21]	Yes	Constant	No	Two levels	–	EOQ
Chen et al. [7]	Yes	Constant	No	Two levels	–	EOQ
Bhunia and Shaikh [2]	Yes	Selling price-dependent	Partial backlogging	Single level	IFS & SFI	EOQ
Bhunia et al. [3]	Yes	Linearly time-dependent	Partial backlogging	No	IFS & SFI	EOQ
Shah and Cárdenas-Barrón [17]	Yes	Constant	No	Two levels	–	EOQ
Bhunia et al. [4]	Yes	Stock-dependent	Partial backlogging	Single level	IFS	EOQ
Bhunia et al. [1]	Yes	Linearly time-dependent	Partial backlogging	Single level	IFS & SFI	EOQ
This research work	Yes	Constant demand	Full backlogging	Two-level trade credit policy	IFS	EPQ

1.2 Suppositions and Notation

1.2.1 Suppositions

The inventory model is based on the suppositions listed below:

1. Demand rate is known and constant.
2. The planning horizon is infinite.
3. Replenishment rate is instantaneous.
4. Stockout is permissible, and unsatisfied demand is fully backlogged.
5. The trade credit policy applies to both retailer and customer.

1.2.2 Notation

The following symbols are utilized during the inventory model development:

Symbol	Units	Description
c_o	\$/order	Replenishment cost
c	\$/unit	Purchasing cost
p	\$/unit	Selling price
c_h	\$/unit/unit time	Holding cost
c_b	\$/unit/unit time	Shortage cost
θ	$\theta \in (0, 1)$	Deterioration rate
P	Units/unit time	Production rate
D	Units/unit time	Demand rate
t_1	Unit time	Time when stock level attains its maximum level
t_2	Unit time	Time when the stock level touches zero
t_3	Unit time	Time when the inventory level achieves its maximum shortage level
T	Unit time	Replenishment cycle
$I(t)$	Units	Inventory level at time t; $0 \leq t \leq T$
M	Unit time	The retailer's trade credit period given by the supplier
N	Unit time	Customer's trade credit period given by the retailer
I_e	%/unit time	Interest earned by the retailer
I_p	%/unit time	Interest paid by the retailer
$TC_i(S, R)$	\$/unit time	The total cost where $i = 1, 2 \ldots, 5$
Decision variables		
S	Units	Order quantity
R	Units	Shortage level

1.3 Inventory Model Formulation

The differential equations that define the inventory level thru time t during the period $[0, T]$ are expressed below:

$$\frac{dI(t)}{dt} + \theta I(t) = P - D, \quad t \in [0, t_1] \tag{1.1}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \quad t \in (t_1, t_2] \tag{1.2}$$

$$\frac{dI(t)}{dt} = -D, \quad t \in (t_2, t_3] \tag{1.3}$$

$$\frac{dI(t)}{dt} + \theta I(t) = P - D, \quad t \in (t_3, T] \tag{1.4}$$

The following results are obtained when differential equations (1.1)–(1.4) are solved taking into account the boundary conditions $I(0) = 0, I(t_1) = S, I(t_3) = -R$, and $I(t)$ is continuous at $t = t_1, t_2$ and t_3:

$$I(t) = \frac{P - D}{\theta}\left(1 - e^{-\theta t}\right), \quad t \in [0, t_1] \tag{1.5}$$

$$I(t) = \frac{D}{\theta}\left(e^{\theta(t_2 - t)} - 1\right), \quad t \in [0, t_1] \tag{1.6}$$

$$I(t) = D(t_2 - t), \quad t \in (t_2, t_3] \tag{1.7}$$

$$I(t) = \frac{P - D}{\theta}\left(1 - e^{\theta(T - t)}\right), \quad t \in (t_3, T] \tag{1.8}$$

Using the boundary and continuity conditions, then the following results are obtained:

$$t_1 = -\frac{1}{\theta}\log\frac{P - D - S\theta}{P - D} \tag{1.9}$$

$$t_2 = t_1 + \frac{1}{\theta}\log\frac{D + S\theta}{D} \tag{1.10}$$

$$t_3 = t_2 + \frac{R}{D} \tag{1.11}$$

$$T = t_3 + \frac{1}{\theta}\log\frac{(P - D) + R\theta}{P - D} \tag{1.12}$$

The total cost of the inventory model is comprised of the terms listed below:

(a) The ordering cost (OC):

$$OC = c_o \tag{1.13}$$

(b) The inventory holding cost (*HC*):

$$HC = c_h\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right) = c_h\left[\frac{(P-D)}{\theta}\left\{t_1 + \left\{e^{-\theta t_1} - 1\right\}\right\} + \frac{D}{\theta^2}\left\{e^{\theta(t_2-t_1)} - \theta(t_2-t_1) - 1\right\}\right] \tag{1.14}$$

(c) The purchase cost (*PC*):

$$PC = c(S+R) \tag{1.15}$$

(d) The shortage cost (*SC*):

$$SC = c_b\left[\int_{t_2}^{t_3} [-I(t)]dt + \int_{t_3}^{T} [-I(t)]dt\right] = \left[\frac{D}{2}(t_3-t_2)^2 + \frac{(P-D)}{\theta^2}\left\{e^{\theta(T-t_3)} - \theta(T-t_3) - 1\right\}\right] \tag{1.16}$$

Taking into consideration the credit period given by supplier to its retailer (*M*) and credit period provided by the retailer to his/her customer (*N*), the following five cases are identified: **Case 1**: $N < M \le t_1 < t_2$, **Case 2**: $N < t_1 \le M \le t_2$, **Case 3**: $t_1 \le N < M \le t_2$, **Case 4**: $t_1 \le N < t_2 \le M$, and **Case 5**: $t_1 < t_2 \le N < M$. These are explained below.

Case 1: $N < M \le t_1 < t_2$ (Fig. 1.1)

Here, the customer's credit period (*N*) is less than the retailer credit period (*M*), where *M* is less than or equal to t_1. So, the retailer needs to pay the interest during $[M, t_2]$. Owing to customer credit period retailer can earn interest in the period $[N, M]$. Therefore, the interest paid (*IP*) is determined with $cI_c\left[\int_M^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right]$.

Hence,

$$IP = cI_c\left[\frac{(P-D)}{\theta}\left\{(t_1 - M) + \frac{1}{\theta}\left(e^{-\theta t_1} - e^{-\theta M}\right)\right\} + \frac{D}{\theta^2}\left\{e^{\theta(t_2-t_1)} - \theta(t_2-t_1) - 1\right\}\right] \tag{1.17}$$

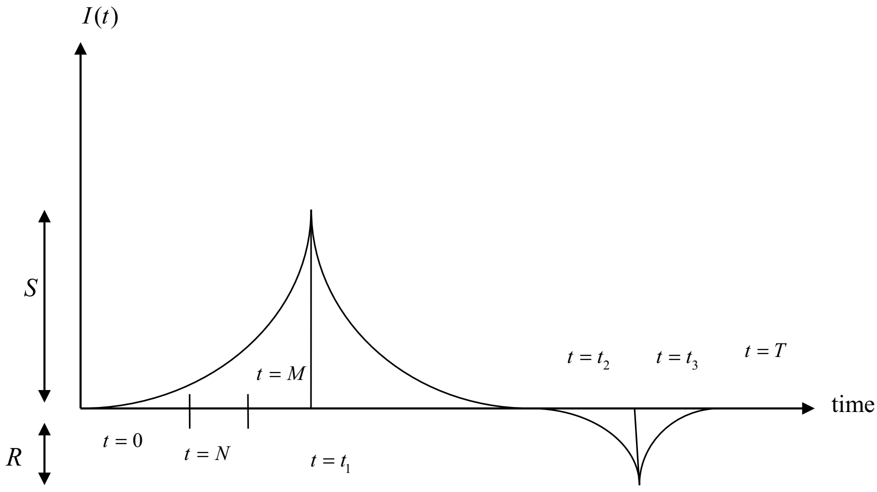


Fig. 1.1 When $N < M \leq t_1 < t_2$

Moreover, the interest earned (IE) is calculated with $pI_e \int_N^M \int_0^t Ddudt$.

Thus,

$$IE = \frac{pI_e D(M^2 - N^2)}{2} \tag{1.18}$$

Consequently, the total cost

$$TC_1(S, R) = \frac{X_1}{T} \tag{1.19}$$

where

$$X_1 = c_o + c(S + R) + HC + SC + IP - IE \tag{1.20}$$

Now, the optimization problem is formulated as follows:
Problem 1

$$\left.\begin{array}{ll} \text{Minimize} & TC_1(S, R) = \dfrac{X_1}{T} \\ \text{subject to} & N < M \leq t_1 < t_2 \end{array}\right\} \tag{1.21}$$

Case 2: $N < t_1 \leq M \leq t_2$ (Fig. 1.2)

The customer's credit period (N) is less than the retailer credit period (M) specified by the supplier where M is greater than or equal to t_1. For that reason, it is necessary

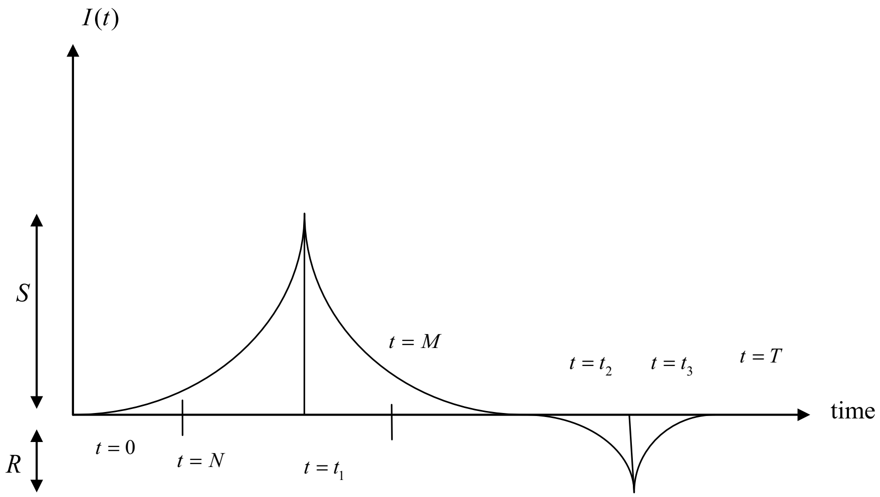


Fig. 1.2 When $N < t_1 \leq M \leq t_2$

that the retailer pays the interest for the duration of the following interval $[M, t_2]$. On the other hand, owing to customer credit period the retailer obtains interest within of the interval $[N, M]$. Then, the interest paid is obtained with $cI_c\left[\int_M^{t_2} I(t)dt\right]$.

Thus,

$$IP = cI_c\left[\frac{D}{\theta^2}\left\{e^{\theta(t_2-M)} - \theta(t_2 - M) - 1\right\}\right] \tag{1.22}$$

and the interest earned is calculated with $pI_e \int_N^M \int_0^t Ddudt$.

Therefore,

$$IE = \frac{pI_e D(M^2 - N^2)}{2} \tag{1.23}$$

Consequently, the total cost is written as

$$TC_2(S, R) = \frac{X_2}{T} \tag{1.24}$$

where

$$X_2 = c_o + c(S + R) + HC + SC + IP - IE \tag{1.25}$$

Hence, the optimization problem is

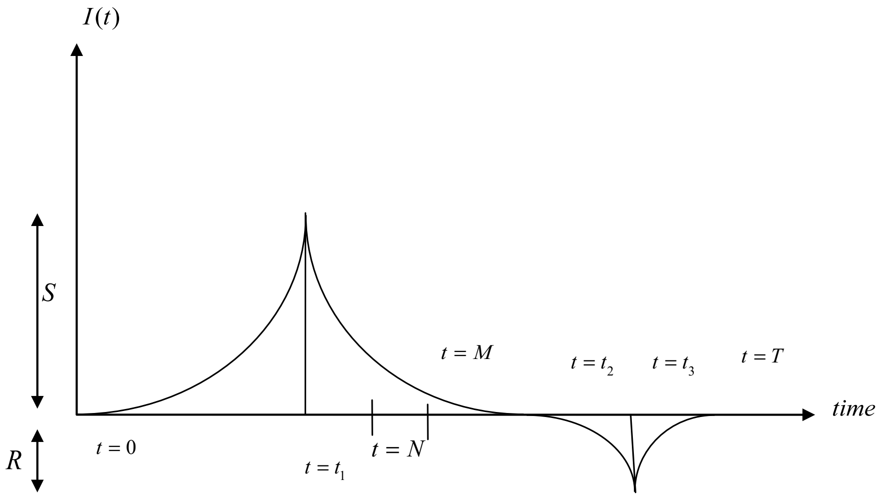


Fig. 1.3 When $t_1 \leq N < M \leq t_2$

Problem 2

$$\left.\begin{aligned} \text{Minimize} \quad & TC_2(S, R) = \frac{X_2}{T} \\ \text{subject to} \quad & N < t_1 \leq M < t_2 \end{aligned}\right\} \tag{1.26}$$

Case 3: $t_1 \leq N < M \leq t_2$ (Fig. 1.3)

Notice that the customer's credit period (N) settled by the retailer is less than the retailer's credit period (M) established by the supplier where M is less than or equal to t_2 and greater than t_1. Thus, the retailer needs to cover the interest for the time period $[M, t_2]$. In contrast, due to the customer's credit interval, the retailer gets interest through the time period $[N, M]$. So, the interest paid is computed with $cI_c\left[\int_M^{t_2} I(t)dt\right]$.

Then

$$IP = cI_c\left[\frac{D}{\theta^2}\left\{e^{\theta(t_2-M)} - \theta(t_2 - M) - 1\right\}\right] \tag{1.27}$$

and interest earned is given by $pI_e \int_N^M \int_0^t Ddudt$

$$IE = \frac{pI_e D(M^2 - N^2)}{2} \tag{1.28}$$

For that reason, the total cost is expressed as

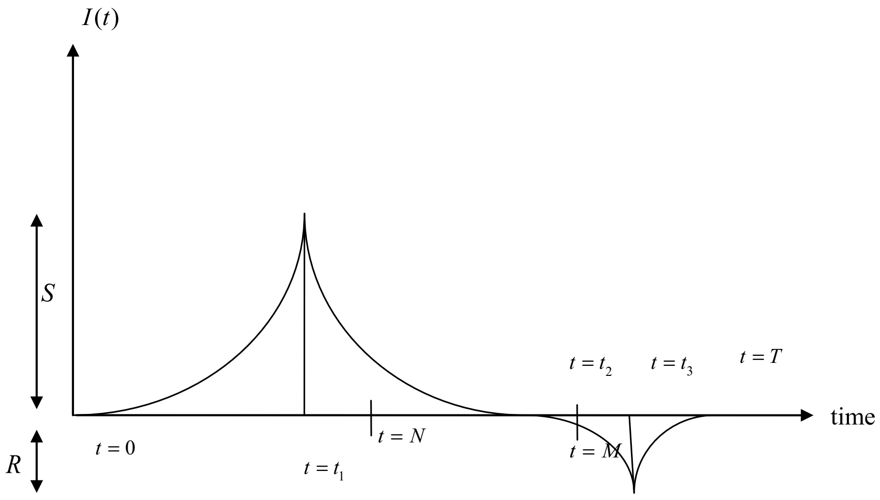


Fig. 1.4 When $t_1 \le N < t_2 \le M$

$$TC_3(S, R) = \frac{X_3}{T} \tag{1.29}$$

where

$$X_3 = c_o + c(S + R) + HC + SC + IP - IE \tag{1.30}$$

Here, the optimization problem is presented as follows:

Problem 3

$$\left.\begin{aligned} \text{Minimize} \quad & TC_3(S, R) = \frac{X_3}{T} \\ \text{subject to} \quad & t_1 \le N < M \le t_2 \end{aligned}\right\} \tag{1.31}$$

Case 4: $t_1 \le N < t_2 \le M$ (Fig. 1.4)

Here, the retailer's credit period (M) is greater than or equal to (t_2), so it is not necessary that the retailer pay the interest. Consequently, the interest paid is $IP = 0$. But retailer gains interest, and it is calculated as follows: $pI_e \int_N^M \int_0^t D du dt$.

Thus,

$$IE = \frac{pI_e D(M^2 - N^2)}{2} \tag{1.32}$$

The total cost is written as

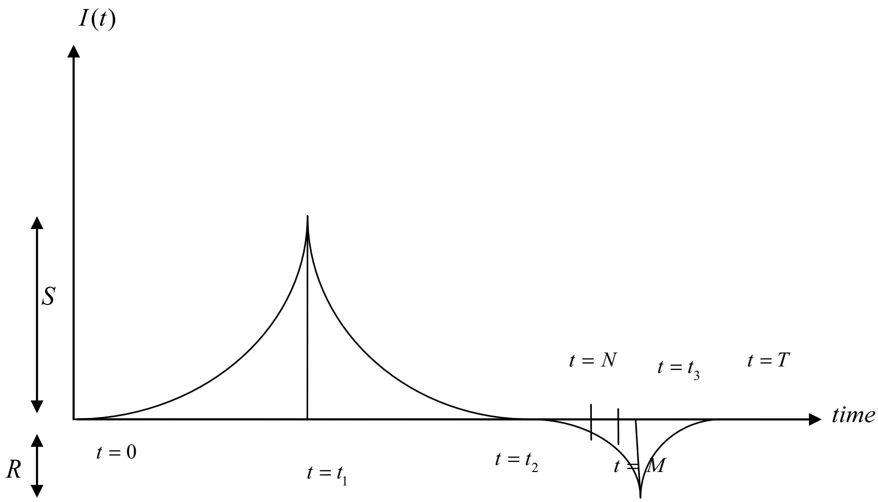


Fig. 1.5 When $t_1 < t_2 \leq N < M$

$$TC_4(S, R) = \frac{X_4}{T} \tag{1.33}$$

where

$$X_4 = c_o + c(S + R) + HC + SC + IP - IE \tag{1.34}$$

Now, the optimization problem becomes

Problem 4

$$\left.\begin{array}{ll} \text{Minimize} & TC_4(S, R) = \dfrac{X_4}{T} \\ \text{subject to} & t_1 \leq N < t_2 \leq M \end{array}\right\} \tag{1.35}$$

Case 5: When $t_1 < t_2 \leq N < M$ (Fig. 1.5)

Both the retailer credit period (M) and customer credit period (N) are greater than (t_2). So, the retailer does not need to pay interest. Therefore, the interest paid is $IP = 0$. But retailer wins interest, and it is $pI_e \int_N^M \int_0^t Ddudt$.

Thus,

$$IE = \frac{pI_e D(M^2 - N^2)}{2} \tag{1.32}$$

Here, the total cost is determined by

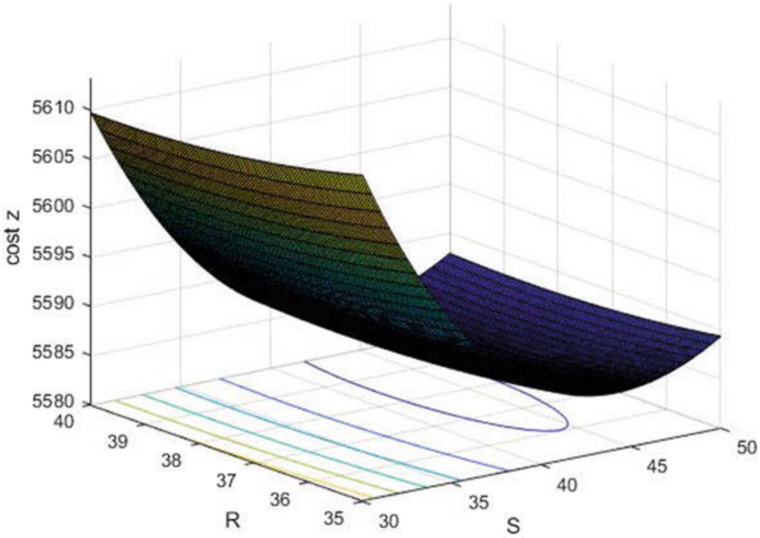


Fig. 1.6 Convexity of the objective function for Case 1

$$TC_5(S, R) = \frac{X_5}{T} \tag{1.33}$$

where

$$X_5 = c_o + c(S + R) + HC + SC + IP - IE \tag{1.34}$$

Here, the optimization problem is stated as

Problem 5

$$\left.\begin{aligned} &Minimize \quad TC_5(S, R) = \frac{X_5}{T} \\ &subject\,to \quad t_1 < t_2 \leq N < M \end{aligned}\right\} \tag{1.35}$$

The necessary conditions for optimality of objective function are $\frac{\partial TC_i()}{\partial S} = 0$ and $\frac{\partial TC_i()}{\partial R} = 0$. The sufficiency conditions are

$$\frac{\partial T^2 C_i()}{\partial S^2} \geq 0, \quad \frac{\partial T^2 C_i()}{\partial R^2} \geq 0$$

$$\text{and} \left(\frac{\partial T^2 C_i()}{\partial S^2}\right)\left(\frac{\partial T^2 C_i()}{\partial R^2}\right) - \left(\frac{\partial T^2 C_i()}{\partial S \partial R}\right)^2 \geq 0$$

The corresponding optimization problem is highly nonlinear in nature. So, it is difficult to prove the optimality analytically. For that reason, the convexity is shown graphically (see Fig. 1.6).

1.4 Numerical Examples

With the intention of exemplifying and certify the inventory model, five instances are presented and solved. Each example illustrates one case of the inventory model. The data of these instances are given in Table 1.2.

The generalized reduced gradient method (GRG) is applied in order to solve the five numerical examples. Table 1.3 presents the optimal solution for all examples.

1.5 Sensitivity Analysis

The instance 1 is utilized to analyze the influence of over/underestimation of input data on the optimal solution of the initial stock level (S), maximum shortage level (R), cycle length (T), the total cost (TC), and time periods: t_1, t_2, and t_3. The analysis is performed by modifying (decreasing/increasing) the input data by $+20\%$ to -20%. The results are computed by varying one input datum and maintaining the other data with original value. Table 1.4 displays the results of the sensitivity analysis.

From Table 1.4, the following observations are mentioned:

(i) With the increase in the value of replenishment cost c_o, the total cost (TC), highest stock (S), shortage level (R), and the replenishment cycle (T) increase, which is an obvious result.

(ii) As the holding cost c_h increases, the total cost (TC) and shortage level (R) increase whereas the highest stock (S) and the replenishment cycle (T) decrease. On the other hand, with the increase in production rate (P), the total cost (TC) and highest stock (S) increase but the replenishment cycle (T) decreases.

(iii) Higher the deterioration rate θ, higher the total cost (TC) but lesser the highest stock (S) and the replenishment cycle (T). A higher deterioration rate means more deteriorated items, which results in an increase in deterioration cost.

(iv) An increment in the shortage cost means more total cost (TC) as well as more highest stock (S). Whereas for an increase in the value of purchasing cost (c), the total cost (TC) and shortage level (R) increase but the highest stock (S) and the replenishment cycle decrease. Moreover, as the value of selling price increases (p), the total cost (TC), highest stock (S), shortage level (R), as well as the replenishment cycle (T) decrease significantly.

Table 1.2 Data for the instances

Instance	c_o $/order	D units/year	P units/year	p $/unit	c $/unit	h $/unit/year	I_p %/year	I_e %/year	M year	N year	θ	c_b $/unit/year
1	150	600	3000	45	20	15	15	9	0.1	0.05	0.05	20
2	150	500	1000	40	30	15	15	9	0.1	0.05	0.05	20
3	100	500	1000	40	25	15	12	9	0.1	0.07	0.05	20
4	200	550	900	45	30	15	12	9	0.2	0.8	0.05	25
5	170	600	800	45	35	20	12	7	0.12	0.07	0.07	30

Table 1.3 The optimal solution for all instances

Instance	Case	S	R	t_1	t_2	t_3	T	TC
1	$N < M \leq t_1 < t_2$	44.0621	38.5184	0.1105	0.1838	0.2480	0.3440	5584.208
2	$N < t_1 \leq M \leq t_2$	43.2274	39.88456	0.0866	0.1729	0.2527	0.3323	8312.618
3	$t_1 \leq N < M \leq t_2$	34.9388	31.9287	N	0.1398	0.2036	0.2674	6898.536
4	$t_1 \leq N < t_2 \leq M$	27.9441	27.9441	N	0.1307	0.1815	0.2612	7319.905
5	$t_1 < t_2 \leq N < M$	10.4871	10.4871	0.0525	N	0.0874	0.13981	6533.081

Table 1.4 Sensitivity analysis for instance 1

Parameters	% of variation in parameter	TC^*	% of the change in					
			S^*	R^*	t_1^*	t_2^*	t_3^*	T^*
c_o	−20	−1.66	−11.82	−11.82	−11.85	−11.83	−11.83	−11.82
	−10	0.80	−5.72	−5.72	−5.74	−5.73	−5.73	−5.72
	10	0.76	5.41	5.41	5.43	5.42	5.42	5.41
	20	1.48	10.56	10.56	10.60	10.57	10.57	10.56
c_h	−20	−0.70	14.25	−5.10	14.29	14.26	9.27	5.29
	−10	−0.33	6.61	−2.37	6.63	6.62	4.29	2.23
	10	0.30	−5.80	2.14	−5.81	−5.80	−3.74	−2.10
	20	0.57	−10.92	4.09	−10.95	−10.93	−7.04	−3.94
P	−20	−35.08	−18.90	−21.05	62.48	30.03	16.81	28.22
	−10	−15.49	−7.62	−8.69	23.26	10.95	5.86	10.28
	10	12.58	5.46	6.43	−15.67	−7.24	−3.70	−6.81
	20	23.00	9.52	11.35	−27.04	−12.46	−6.30	−11.72
D	−20	3.75	1.17	2.87	−22.47	−3.13	5.08	−2.16
	−10	3.27	0.85	2.19	−12.06	−2.31	1.79	−1.81
	10	−6.07	−2.75	−3.76	14.46	4.07	−0.22	3.52
	20	−14.95	−7.29	−9.24	32.57	10.52	1.49	9.33
θ	−20	−0.02	−0.25	0.33	−0.31	−0.27	−0.12	0.02
	−10	−0.01	−0.13	0.16	−0.15	−0.14	−0.06	0.01
	10	0.01	0.13	−0.16	0.15	0.14	0.06	−0.01

(continued)

Table 1.4 (continued)

Parameters	% of variation in parameter	TC^*	% of the change in					
			S^*	R^*	t_1^*	t_2^*	t_3^*	T^*
	20	0.02	0.25	−0.33	0.31	0.27	0.12	−0.02
c_b	−20	−0.74	−5.25	17.92	−5.26	−5.25	0.75	5.53
	−10	−0.34	−2.44	8.18	−2.45	−2.45	0.31	2.50
	10	0.30	2.15	−6.99	2.16	2.15	−0.21	−2.10
	20	0.57	4.06	−13.03	4.07	4.06	−0.36	−3.89
c	−20	−17.26	2.73	−0.15	2.74	2.74	1.99	1.39
	−10	−8.63	1.35	−0.07	1.35	1.35	0.98	0.69
	10	8.63	−1.32	0.06	−1.32	−1.32	−0.96	−0.68
	20	17.26	−2.60	0.11	−2.61	−2.61	−1.90	−1.34
p	−20	0.09	0.67	0.67	0.68	0.67	0.67	0.67
	−10	0.05	0.34	0.34	0.34	0.34	0.34	0.34
	10	−0.05	−0.34	−0.34	−0.34	−0.34	−0.34	−0.34
	20	−0.1	−0.68	−0.68	−0.68	−0.68	−0.68	−0.68
M	−20	0.34	2.39	2.39	2.40	2.39	2.39	2.39
	−10	0.18	1.27	1.27	1.27	1.27	1.27	1.27
	10	−0.20	−1.42	−1.42	−1.43	−1.42	−1.42	−1.42
	20	−0.42	−3.00	−3.00	−3.01	−3.01	−3.00	−3.00
N	−20	−0.06	−0.41	−0.41	−0.41	−0.41	−0.41	−0.41
	−10	−0.03	−0.21	−0.21	−0.21	−0.21	−0.21	−0.21
	10	0.03	0.24	0.24	0.24	0.24	0.24	0.24
	20	0.07	0.49	0.49	0.50	0.49	0.49	0.49

(v) As the supplier increases, the credit period (M) for its retailer, the total cost (TC), highest stock (S), shortage level (R), and the replenishment cycle (T) decrease.

(vi) As the retailer increases, the credit period (N) for its customer, the total cost (TC), highest stock (S), shortage level (R), and the replenishment cycle (T) increase.

(vii) The time periods (t_1, t_2, t_3) are highly sensitive regarding parameters c_o, c_h, P and D; moderately sensitive regarding c, c_b and M; and less sensitive regarding the rest parameters.

(viii) The cycle length (T) is very sensitive pertaining to the data of c_o, c_h and P; discreetly sensitive relating to D, c_b, c and M; and less sensitive in regard to the other data.

1.6 Conclusion

This research work develops a production-inventory model for an article that deteriorates considering fully backlogged shortages and full two-level trade credit scheme. Here, it is supposed that demand function and production rate are known and constant. A solution method for the inventory model is developed. The validation and effectiveness of the proposed inventory model are assessed through numerical examples. The findings suggest significance importance of the proposed inventory model to the retail industry for decision-making under realistic scenarios.

This inventory model can be extended by considering stock-dependent demand, price-dependent demand, stochastic demand, partial backlogging, inflation, partial trade credit policy, fuzzy-valued inventory cost, interval valued inventory costs, overtime production rate, and imperfect production processes. These are some interesting topics that researchers can explore.

References

1. Bhunia AK, Shaikh AA, Sahoo L (2016) A two-warehouse inventory model for deteriorating item under permissible delay in payment via particle swarm optimization. Int J Logist Syst Manag 24(1):45–69
2. Bhunia AK, Shaikh AA (2015) An application of PSO in a two warehouse inventory model for deteriorating item under permissible delay in payment with different inventory policies. Appl Math Comput 256(1):831–850
3. Bhunia AK, Shaikh AA, Gupta RK (2015) A study on two-warehouse partially backlogged deteriorating inventory models under inflation via particle swarm optimization. Int J Syst Sci 46(6):1036–1050
4. Bhunia AK, Shaikh AA, Pareek S, Dhaka V (2015) A memo on stock model with partial backlogging under delay in payment. Uncertain Supply Chain Manag 3(1):11–20
5. Chang CT, Teng JT, Goyal SK (2008) Inventory lot-size models under trade credits: a review. Asia-Pac J Oper Res 25:89–112
6. Chang CT, Teng JT, Chern MS (2010) Optimal manufacturer's replenishment policies for deteriorating items in a supply chain with up-stream and downstream trade credits. Int J Prod Econ 127:197–202
7. Chen SC, Cárdenas-Barrón LE, Teng JT (2014) Retailer's economic order quantity when the suppliers offers conditionally permissible delay in payments link to order quantity. Int J Prod Econ 155:284–291
8. Chopra S, Meindl P (2004) Supply chain management: strategy, planning, and operation. Prentice Hall
9. Chung KJ, Huang YF (2003) The optimal cycle time for EPQ inventory model under permissible delay in payments. Int J Prod Econ 84:307–318
10. Ghare PM, Schrader GF (1963) A model for exponentially decaying inventories. J Ind Eng 14:238–243
11. Goyal SK (1985) Economic order quantity under conditions of permissible delay in payments. J Oper Res Soc 36:335–338
12. Harris FW (1913) How many parts to make at once. Fact, Mag Manag 10(2):135–136 and 152
13. Huang YF (2003) Optimal retailers ordering policies in the EOQ model under trade credit financing. J Oper Res Soc 54:1011–1015

14. Liao JJ (2008) An EOQ model with non-instantaneous receipt and exponentially deteriorating items under two-level trade credit. Int J Prod Econ 113:852–861
15. Seifert D, Seifert RW, Protopappa-Sieke M (2013) A review of trade credit literature: opportunity for research in operations. Eur J Oper Res 231(2):245–256
16. Shah NH, Soni HN, Gupta J (2014) A note on a replenishment policy for items with price dependent demand, time-proportional deterioration and no shortages. Int J Syst Sci 45(8):1723–1727
17. Shah NH, Cárdenas-Barrón LE (2015) Retailer's decision for ordering and credit policies for deteriorating items when a supplier offers order-linked credit period or cash discount. Appl Math Comput 259:569–578
18. Simchi-Levi D, Kaminsky P, Simchi-Levi E (2008) Designing and managing the supply chain: concepts, strategies, and cases. McGraw-Hill
19. Taft EW (1918) The most economical production lot. Iron Age 101:1410–1412
20. Teng JT (2009) Optimal ordering policies for a retailer who offers distinct trade credits to its good and bad credit customers. Int J Prod Econ 119:415–423
21. Wu J, Ouyang LY, Cárdenas-Barrón LE, Goyal SK (2014) Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing. Eur J Oper Res 237(3):898–908

Chapter 2
An Economic Order Quantity (EOQ) Inventory Model for a Deteriorating Item with Interval-Valued Inventory Costs, Price-Dependent Demand, Two-Level Credit Policy, and Shortages

Ali Akbar Shaikh, Sunil Tiwari and Leopoldo Eduardo Cárdenas-Barrón

Abstract In today's competitive environment, every leading organization wishes to improve the pricing strategies in order to increase revenue, credit policy is one of the best tools of it. This research work develops an economic order quantity (EOQ) inventory model for a deteriorating item that considers interval-valued inventory costs, price dependent demand, two-level credit policy, and shortages. Due to high and uncertainty in demand, sometimes organizations have to face the situation of stock out. So, keeping this scenario in mind, this work considers the situation of partially backlogging. Here, it is developed an EOQ inventory model by considering a non-linear interval-valued constrained optimization problem. Two types of particle swarm optimization (PSO) algorithm are used to resolve it, and then the results are compared. Sensitivity analysis is done in order to investigate the impact of key parameters on decision-making. Finally, conclusions along with some managerial insights are given.

Keywords Inventory · Deterioration · Price-dependent demand · Partial shortages · Interval-valued inventory costs · Two-level credit policy

A. A. Shaikh
Department of Mathematics, The University of Burdwan, Burdwan 713104, India
e-mail: aliashaikh@math.buruniv.ac.in

S. Tiwari
The Logistics Institute-Asia Pacific, National University of Singapore, 21 Heng Mui Keng Terrace, #04-01, Singapore 119613, Singapore
e-mail: sunil.tiwari047@gmail.com

L. E. Cárdenas-Barrón (✉)
Department of Industrial and Systems Engineering, School of Engineering and Sciences, Tecnológico de Monterrey, E. Garza Sada 2501 Sur, C.P 64849 Monterrey, Nuevo León, Mexico
e-mail: lecarden@tec.mx

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_2

2.1 Introduction

In the inventory management literature, very little research work has been done in relation to considering that the inventory costs are represented in an interval-valued. Many researchers assume that the inventory costs such that the ordering cost, inventory carrying cost, and purchasing cost are expressed as a fixed value known. Nevertheless, in reality, all of the mentioned costs are imprecise numbers in nature instead of a fixed value due to the fact that generally, the inventory costs fluctuate by reason of several factors such as changes in prices. In order to explain why it needs to use an interval number rather than the fixed value number, the following reason is mentioned. Normally, the inventory carrying cost is distinct during the seasons of the year. For example, the deterioration rate is different in summer and winter. During summer time, it is necessary to use preservation technology with the intention of decreasing the deterioration percentage of some perishable products and therefore the holding cost is different from holding cost in the winter time. Another cost that also varies is the labor charges, which change over the period of time.

To overcome the problem of imprecise numbers, the researchers and academicians use the following approaches: (i) stochastic, (ii) fuzzy, and (iii) fuzzy-stochastic. In the case of the stochastic approach, the inventory data are considered as random variables with a given and known probability distribution. In the case of the fuzzy approach, the data of the inventory system and the constraints are expressed with fuzzy sets with a given and known membership function. In the case of fuzzy-stochastic approach, some inventory data are supposed to be represented by fuzzy sets and the rest of the inventory parameters are assumed random variables. But it is not an easy task to select the most suitable membership function or probability distribution.

With the aim of avoiding the complexity in the selection of the right membership function or the right probability distribution, it is suggested to use interval numbers. With this, the imprecise problem is converted to an interval-valued problem, which can be solved, by any soft computing optimization technique, such as the different versions of particle swarm optimization (PSO) or genetic algorithm (GA). In this connection, the reader can see the related works, which apply interval number into the area of inventory control. Gupta et al. [1] applied the interval concept in the field of inventory theory. They resolved an inventory problem with interval-valued inventory costs using a genetic algorithm approach. After that, Dey et al. [2] formulated an inventory model considering interval-valued lead time. Again, Gupta et al. [3] developed an inventory model using interval-valued inventory costs. Bhunia et al. [4] solved a stock-dependent inventory model with interval-valued inventory costs using particle swarm optimization (PSO). Afterward, Bhunia and Shaikh [5] built a two-warehouse inventory model with inflation, and they solved it using particle swarm optimization.

In the current competitive markets, the permissible delay in payment has a vital role in promoting the business. Normally, the suppliers give different types of facilities to retailers, and the retailers give some facilities to their direct customers. This is done with the aim of attracting more customers to acquire products. Table 2.1 presents research works related to single-level or two-level credit policy.

Table 2.1 Research works related to single- and two-level permissible delay in payment

Author(s)	Deterioration	Demand rate	Shortages	Level of permissible delay in payment	Inventory costs
Hwang and Shinn [6]	Yes	Constant	No	Single	Fixed
Chang et al. [7]	Yes	Constant	No	Single	Fixed
Abad and Jaggi [8]	No	Linearly time-dependent	No	Single	Fixed
Ouyang et al. [9]	Yes	Constant	No	Single	Fixed
Huang [10]	No	Linearly time-dependent	No	Two-level	Fixed
Huang [11]	No	Constant	No	Single	Fixed
Huang [12]	No	Constant	No	Two-level	Fixed
Sana and Chaudhuri [13]	Yes	Selling price-dependent	No	Single	Fixed
Huang and Hsu [14]	No	Constant	No	Two-level partial trade credit	Fixed
Ho et al. [15]	No	Constant	No	Two-level	Fixed
Jaggi and Khanna [16]	Yes	Inventory level dependent	Complete backlogging	Single	Fixed
Jaggi and Kausar [17]	No	Selling price-dependent	Complete backlogging	Single partial trade credit	Fixed
Jaggi and Mittal [18]	Yes	Constant	Complete backlogging	Single	Fixed
Guria et al. [19]	No	Selling price-dependent	Complete backlogging	Immediate and delay in payment	Fixed
Taleizadeh et al. [20]	No	Constant	Partial backlogging	Partial delay in payment	Fixed
Wu et al. [21]	Expiration date dependent	Constant	No	Two-level	Fixed

(continued)

Table 2.1 (continued)

Author(s)	Deterioration	Demand rate	Shortages	Level of permissible delay in payment	Inventory costs
Chen et al. [22]	Yes	Constant	No	Order quantity-dependent credit period	Fixed
Bhunia et al. [23]	Yes	Selling price-dependent	Partial backlogging	Alternative single	Fixed
Bhunia and Shaikh [24]	Yes	Selling price-dependent	Partial backlogging	Single	Fixed
Shah and Cárdenas-Barrón [25]	Yes	Constant	No	Two-level	Fixed
Bhunia et al. [26]	No	Stock-dependent	Partial backlogging	Single	Fixed
This research work	Yes	Selling price-dependent demand	Partial backlogging	Two-level credit policy	Interval-valued

It is a well-known fact that inventory cost always is not a fixed value. This means that the inventory cost lies between certain interval numbers. Therefore, the major goal of this research work is to include the interval concept in an inventory model. In this direction, this research work derives an economic order quantity (EOQ) inventory model for a deteriorating item with price-dependent demand, and interval-valued inventory costs and shortages. The shortage is partially backordered according to a rate, which is reliant on the interval of waiting time till the occurrence of next lot. The inventory model is expressed as a nonlinear interval-valued continuous optimization problem. Then, different forms of particle swarm optimization (PSO) algorithm are applied to solve it. In continuous optimization, PSO gives better results than GA. For this reason, in this research work, the latest version of PSO is used.

The remnants sections of the research work are planned as follows. Section 2.2 defines the suppositions and the notations. Section 2.3 formulates the inventory model. Section 2.4 derives the mathematical solution for three different demand functions. Section 2.5 solves some instances. Section 2.6 does a sensitivity analysis. Section 2.7 provides conclusions and lines for research.

2.2 Suppositions and Notations

The suppositions and symbols that are used to build the inventory model are listed below.

2.2.1 *Suppositions*

(i) The planning horizon is infinite.
(ii) Inventory system handles a single item.
(iii) Demand rate $D(.)$ is influenced by the selling price (p).
(iv) Inventory costs are interval-valued.
(v) The order is supplied in one delivery.
(vi) Replenishment is instantaneous.
(vii) Lead time is zero.
(viii) Stockout is partially backlogged with a backlogging rate given by $[1+\delta(T-t)]^{-1}$.
(ix) Two-level credit policy approach is assumed where the supplier gives a credit period (M) to his/her retailer, and the retailer also provides a credit facility (N) to his/her customer under certain terms and conditions. Here, it is established the following condition $N < M$.

2.2.2 *Notations*

Symbols	Description
Parameters	
$I(t)$	Inventory level at time t (units)
α	Deterioration rate ($0 < \alpha \ll 1$)
$[C_{oL}, C_{oR}]$	Interval-valued replenishment cost ($/order)
δ	Backlogging parameter
$\left[C_{pL}, C_{pR}\right]$	Interval-valued purchasing cost ($/unit)
$D(.)$	Demand rate that is dependent on price (units/unit of time)
$[C_{hL}, C_{hR}]$	Interval-valued holding cost ($/unit/unit of time)
$[C_{bL}, C_{bR}]$	Interval-valued shortage cost ($/unit/unit of time)
$[C_{lsL}, C_{lsR}]$	Interval-valued opportunity cost due to a lost sale ($/unit/unit of time)
t_1	Time in which the stock level is zero (unit of time)

(continued)

(continued)

Symbols	Description
T	Cycle length (unit of time)
M	Credit period is given to the retailer by the supplier (unit of time)
N	Credit period provided to the customer by the retailer: $N < M$ (unit of time)
I_e	Interest earned by the retailer (%/unit of time)
I_p	Interest charged by the supplier to the retailer (%/unit of time)
$\left[Z_L^{(.)}, Z_R^{(.)}\right]$	Interval-valued the total profit ($/unit of time)
Decision variables	
S	Stock level (units)
R	Shortage level (units)
B	The time period after reaching the prescribed credit time M (unit of time)

2.3 Mathematical Derivation of the Inventory Model

Initially, the retailer purchases a lot of $(S + R)$ units. After fulfilling the backordered units of the preceding cycle, the stock level is S units at $t = 0$. Then, S units start to decrease due to both consumers' demand and deterioration effect. Obviously, after a certain time period, the stock level reaches zero at the time $t = t_1$. After that, at time $t = t_1$, shortage occurs with a backlogging rate $[1 + \delta(T - t)]^{-1}$ till the time $t = T$. Then, a subsequent batch is received at T.

The behavior of the inventory $I(t)$ is modeled by the differential Eqs. (2.1) and (2.2):

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(.), \quad [0, t_1] \tag{2.1}$$

$$\frac{dI(t)}{dt} = \frac{-D(.)}{1 + \delta(T - t)}, \quad (t_1, T] \tag{2.2}$$

with the initial and boundary conditions

$$I(t) = S \text{ at } t = 0, \ I(t) = 0 \text{ at } t = t_1 \tag{2.3}$$

and

$$I(t) = -R \text{ at } t = T \tag{2.4}$$

It is significant to state that the inventory level $I(t)$ is continuous at $t = t_1$. Using the conditions (2.3) and (2.4), the solutions to the differential equations (2.1) and (2.2) are given below:

$$I(t) = -\frac{D(.)}{\theta} + \frac{D(.)}{\theta} e^{\theta(t_1 - t)}, \quad [0, t_1]$$

$$I(t) = \frac{D(.)}{\delta} \log|1 + \delta(T - t)| - R, \quad (t_1, T]$$

From condition (2.3), $I(t) = S$ at $t = 0$. Thus, the maximum inventory level is computed with

$$S = \frac{D(.)}{\theta}\left\{e^{\theta t_1} - 1\right\} \tag{2.5}$$

Using the continuity condition, hence, the shortage quantity is determined with

$$R = \frac{D(.)}{\delta} \log|1 + \delta(T - t_1)| \tag{2.6}$$

The total interval-valued inventory holding cost $C_{hol} = [C_{holL}, C_{holR}]$ of the system is expressed as follows:

$$C_{holL} = C_{hL} \int_0^{t_1} I(t)\, dt = C_{hL}\left[\frac{(S + \frac{D(.)}{\theta})}{\theta}\left(1 - e^{-\theta t_1}\right) - \frac{Dt_1}{\theta}\right] \tag{2.7}$$

and

$$C_{holR} = C_{hR} \int_0^{t_1} I(t)\, dt = C_{hR}\left[\frac{(S + \frac{D(.)}{\theta})}{\theta}\left(1 - e^{-\theta t_1}\right) - \frac{Dt_1}{\theta}\right] \tag{2.8}$$

The total interval-valued shortage cost $C_{sho} = [C_{shoL}, C_{shoR}]$ of the inventory system is given below:

$$\begin{aligned} C_{shoL} &= C_{bL} \int_{t_1}^{T} [-I(t)]\, dt \\ &= C_{bL}\left[\left(R + \frac{D(.)}{\delta}\right)(T - t_1) - \frac{D(.)}{\delta^2}\{(1 + \delta(T - t_1)) \log(1 + \delta(T - t_1))\}\right] \end{aligned} \tag{2.9}$$

$$C_{shoR} = C_{bR}\int_{t_1}^{T}[-I(t)]\,dt$$
$$= C_{bR}\left[\left(R+\frac{D(.)}{\delta}\right)(T-t_1)-\frac{D(.)}{\delta^2}\{(1+\delta(T-t_1))\log(1+\delta(T-t_1))\}\right] \quad (2.10)$$

The total interval-valued opportunity cost of lost sales $OC_{LS} = [OC_{LSL}, OC_{LSR}]$ during the entire cycle is determined by

$$OC_{LSL} = C_{lsL}\int_{t_1}^{T}\left\{1-\frac{1}{1+\delta(T-t)}\right\}D(.)\,dt$$
$$= C_{lsL}D(.)\left[(T-t_1)-\frac{\log(1+\delta(T-t_1))}{\delta}\right] \quad (2.11)$$

and

$$OC_{LSR} = C_{lsR}\int_{t_1}^{T}\left\{1-\frac{1}{1+\delta(T-t)}\right\}D(.)\,dt$$
$$= C_{lsR}D(.)\left[(T-t_1)-\frac{\log(1+\delta(T-t_1))}{\delta}\right] \quad (2.12)$$

As it was mentioned before, in two-level credit policy, the supplier provides a credit period to his/her retailer with a duration of M. Then, the retailer also gives a certain credit period to his/her client with a duration of N, where N is always less than M. Furthermore, here two cases occur: **Case 1**: $0 < N < M \le t_1$ and **Case 2**: $N < t_1 < M \le T$. Figures 2.1 and 2.2 show the behavior of the stock level over the period of time for **Case 1** and **Case 2,** respectively. Below a discussion of these two cases is given.

Case 1: $0 < N < M \le t_1$

In this case, the total amount of purchase cost of the retailer is within the following interval $\left[C_{pL}(S+R), C_{pR}(S+R)\right]$. This amount must be covered to the supplier at the time $t = M$. In this credit time period, the retailer accumulates money due to sales during [0, M] as well as the interest gained during [N, M]. Hence, the total collected amount is calculated with

$$U_1 = p\int_0^M D(.)\,dt + pI_e\int_N^M\int_N^t D(.)\,dudt + pR\{1+I_e(M-N)\}.$$

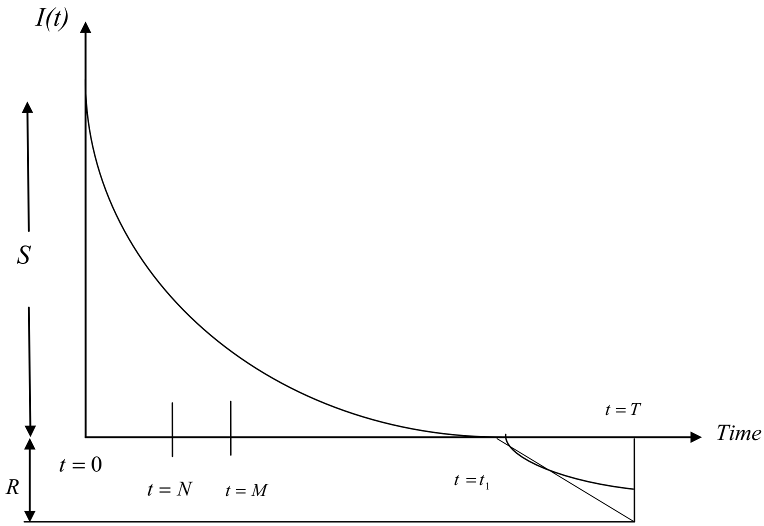


Fig. 2.1 Inventory-level behavior for Case 1

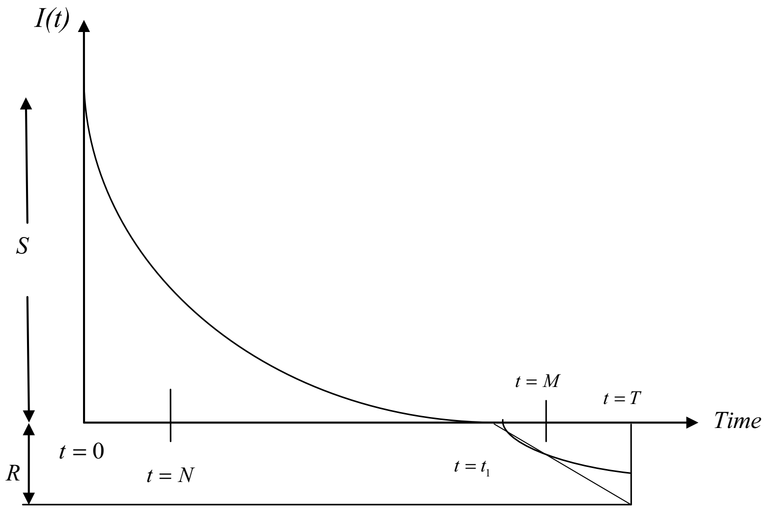


Fig. 2.2 Inventory-level behavior for Case 2

Thus,

$$U_1 = pD(.)\left\{M + I_e\frac{(M-N)^2}{2}\right\}pR\{1 + I_e(M-N)\} \tag{2.13}$$

The retailer collects U_1 and interval-valued for the purchase cost amount is $\left[C_{pL}(S+R), C_{pR}(S+R)\right]$. Here, the following two subcases occur: **Subcase 1**: $U_1 \geq \left[C_{pL}(S+R), C_{pR}(S+R)\right]$ and **Subcase 2**: $U_1 < \left[C_{pL}(S+R), C_{pR}(S+R)\right]$. These subcases are developed below:

Subcase 1: $U_1 \geq \left[C_{pL}(S+R), C_{pR}(S+R)\right]$
In this Subcase 1, the total interval-valued profit of the inventory system is written as

$$Z_{1L}(.) = \frac{X_L(.)}{T} \text{ and } Z_{1R}(.) = \frac{X_R(.)}{T} \tag{2.14}$$

where

$[X_L, X_R] =$ *<Excess amount on hand after paying the cost of purchased goods to the supplier>* + *<interest earned for excess amount in* $[M, T]$*>* + *<sales revenue in* $[M, t_1]$*>* + *<interest earned in* $[M, t_1]$*>* + *<interest earned in* $[t_1, T]$*>* − *<ordering cost>* − *<holding cost>* − *<shortage cost>* − *<cost of lost sale>*

$$X_L(.) = \left\{U_1 - C_{pR}(S+R)\right\}\{1 + I_e(T-M)\} + D(.)p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\}$$
$$\{1 + I_e(T - t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR} \tag{2.15}$$

and

$$X_R(.) = \left\{U_1 - C_{pL}(S+R)\right\}\{1 + I_e(T-M)\} + D(.)p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\}$$
$$\{1 + I_e(T - t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL} \tag{2.16}$$

Therefore, the corresponding interval-valued nonlinear optimization problem of the inventory system is written as follows.

Problem 1

$$\begin{aligned} &\text{Maximize } Z_1(.) = [Z_{1L}(.), Z_{1R}(.)] \\ &\text{subject to } 0 < N < M \leq t_1 < T \end{aligned} \tag{2.17}$$

Subcase 2: $U_1 < \left[C_{pL}(S+R), C_{pR}(S+R)\right]$
In Subcase 2, the retailer collects an amount corresponding to sales and interest earned up to $t = M$. This amount is less than the amount of the purchase cost. Taking into consideration this situation, the following two subcases happen: **Subcase**

2.1: Supplier takes a partial payment at $t = M$ of his/her retailer, and **Subcase 2.2**: Supplier does not take the partial payment at $t = M$ of his/her retailer. Now, these two subcases are discussed below.

Subcase 2.1: Supplier takes a partial payment at $t = M$ of his/her retailer.
In this subcase, it is considered that the supplier takes a partial payment and permits some time to the retailer regarding the payment of rest interval amount which is expressed as $\left[C_{pL}(S+R) - U_1, C_{pR}(S+R) - U_1\right]$. The interval relax time is $t = [B_L, B_R]$ where $[B_L, B_R] > M$. In this situation, the supplier must charge the interest of unpaid amount $\left[C_{pL}(S+R) - U_1, C_{pR}(S+R) - U_1\right]$ during the interval $[M, [B_L, B_R]]$ with interest paid rate I_p.

Thus, the total amount that must be paid to the supplier at a time $t = [B_L, B_R]$ is given by $\left[C_{pL}(S+R) - U_1, C_{pR}(S+R) - U_1\right]\left\{1 + I_p([B_L, B_R] - M)\right\}$.

On the other hand, the total available amount to the retailer is determined as < total sales revenue during the time interval $[M, [B_L, B_R]]$> + <total interest earned during the time interval $[M, [B_L, B_R]]$>. So, the total interest earned is

$$= p\int_{M}^{B_L} D(.)\,dt + pI_e\int_{M}^{B_L}\int_{M}^{t} D(.)\,dudt$$

and

$$= p\int_{M}^{B_R} D(.)\,dt + pI_e\int_{M}^{B_R}\int_{M}^{t} D(.)\,dudt$$

As a result, at the time $t = [B_L, B_R]$, the total payable amount available to the retailer is equal to the amount payable to the supplier, which is

$$\begin{aligned}&\left[C_{pL}(S+R) - U_1, C_{pR}(S+R) - U_1\right]\left\{1 + I_p(B_L - M)\right\}\\&= pD(.)(B_L - M)\left\{1 + \frac{I_e(B_L - M)}{2}\right\}\end{aligned}$$

Thus,

$$\left[C_{pL}(S+R) - U_1\right]\left\{1 + I_p(B_L - M)\right\} = pD(.)(B_L - M)\left\{1 + \frac{I_e(B_L - M)}{2}\right\}$$

and

$$\left[C_{pR}(S+R) - U_1\right]\left\{1 + I_p(B_R - M)\right\} = pD(.)(B_R - M)\left\{1 + \frac{I_e(B_R - M)}{2}\right\}$$

Consequently, the total interval-valued profit function of the inventory system is computed as

$$Z_{2L}(.) = \frac{X_L(.)}{T} \quad \text{and} \quad Z_{2R}(.) = \frac{X_R(.)}{T}$$

where

$[X_L, X_R] =$ < *sales revenue during the time interval* $[[B_L, B_R], t_1]$ > + < *interest earned thru the time interval* $[[B_L, B_R], t_1]$ > + < *interest earned through interval* $[t_1, T]$ > − < *ordering cost* > − <*holding cost* > − <*shortage cost* > − <*cost of lost sale*>

$$X_L(.) = \left\{ p \int_{B_R}^{t_1} D(.)dt + pI_e \int_{B_R}^{t_1} \int_{B_R}^{t} D(.)dudt \right\} \{1 + I_e(T - t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R(.) = \left\{ p \int_{B_L}^{t_1} D(.)dt + pI_e \int_{B_L}^{t_1} \int_{B_L}^{t} D(.)dudt \right\} \{1 + I_e(T - t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

So, in this subcase, the interval-valued constrained optimization problem is formulated as follows.

Problem 2

$$\begin{aligned} &\text{Maximize } Z_2(.) = [Z_{2L}(.), Z_{2R}(.)] \\ &\text{subject to } 0 < N < M \le t_1 < T \end{aligned} \tag{2.18}$$

Subcase 2.2: Supplier does not take the partial payment at $t = M$ of his/her retailer. In this situation, the supplier does not take a partial payment. In other words, the retailer needs to cover the credit amount to his/her supplier. This amount is calculated with $\left[C_{pL}(S + R) - U_1, C_{pR}(S + R) - U_1\right]$ after the time $t = M$. The interval time period $t = [B_L, B_R]$ when the supplier gets the full creditable amount within this time interval. Regarding this situation, supplier charges the interest for the period $[M, [B_L, B_R]]$ with interest paid rate I_p.

Therefore, the total on-hand amount available to the retailer is equal to the amount payable to the supplier at the time $t = [B_L, B_R]$. Thus,

$$C_{pL}(S + R)\left\{1 + I_p(B_L - M)\right\} = U_1[1 + I_e(B_L - N)] + pD(.)(B_L - M) \left[1 + I_e \frac{(B_L - M)}{2}\right]$$

and

$$C_{pR}(S+R)\{1+I_p(B_R-M)\} = U_1[1+I_e(B_R-N)] + pD(.)(B_R-M) \left[1+I_e\frac{(B_R-M)}{2}\right]$$

Consequently, the total interval-valued profit of the inventory system is as follows:

$$Z_{3L}(.) = \frac{X_L(.)}{T} \quad \text{and} \quad Z_{3R}(.) = \frac{X_R(.)}{T}$$

where

$[X_L, X_R] =$ < *sales revenue during the time interval* $[[B_L, B_R], t_1]$ > + < *interest earned for the duration of the time interval* $[[B_L, B_R], t_1]$ > + < *interest earned within the interval* $[t_1, T]$ > − < *ordering cost* > − <*holding cost* > − <*shortage cost* > − <*cost of lost sale*>

$$X_L(.) = \left\{ p\int_{B_R}^{t_1} D(.)dt + pI_e \int_{B_R}^{t_1}\int_{B_R}^{t} D(.)dudt \right\}\{1+I_e(T-t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R(.) = \left\{ p\int_{B_L}^{t_1} D(.)dt + pI_e \int_{B_L}^{t_1}\int_{B_L}^{t} D(.)dudt \right\}\{1+I_e(T-t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL} \tag{2.19}$$

Thus, the interval-valued constrained nonlinear optimization problem is expressed below:

Problem 3

$$\begin{aligned} &\text{Maximize } Z_3(.) = [Z_{3L}(.), Z_{3R}(.)] \\ &\text{subject to } 0 < N < M \le t_1 < T \end{aligned} \tag{2.20}$$

Case 2: $N < t_1 < M \le T$

Owing to sales revenue and interest earned, the collected amount of the retailer is computed as

$$U_2 = \left[p\int_0^{t_1} D(.)dt + pI_e \int_N^{M}\int_N^{t} D(.)dudt \right][1+I_e(M-t_1)] + pR\{1+I_e(M-N)\}$$

$$U_2 = pD(.)\left[t_1 + I_e\frac{\left(M^2 - N^2\right)}{2}\right][1 + I_e(M - t_1)] + pR\{1 + I_e(M - N)\}$$

Here, the interval-valued profit of the inventory system is formulated as

$$Z_{4L}(.) = \frac{X_L(.)}{T} \quad \text{and} \quad Z_{4R}(.) = \frac{X_R(.)}{T}$$

where

$[X_L, X_R] =$ < *Excess amount available of retailer after paying the supplier* > + < *total interest earned for that excess amount in* $[M, T]$ > − < *ordering cost* > − <*holding cost* > − <*shortage cost* > − <*cost of lost sale*>

$$X_L(.) = U_2 - (C_{oR} + C_{holR} + C_{shoR} + OC_{LSR})$$

and

$$X_R(.) = U_2 - (C_{oL} + C_{holL} + C_{shoL} + OC_{LSL})$$

For that reason, the corresponding interval-valued constrained nonlinear optimization problem is as follows.

Problem 4

$$\begin{aligned} &\text{Maximize } Z_4(.) = [Z_{4L}(.), Z_{4R}(.)] \\ &\text{subject to } N < t_1 < M \leq T \end{aligned} \tag{2.21}$$

2.4 The Solution for Three Demand Functions

$D(\cdot) = a - bp$, $a, b > 0$, $D(\cdot) = ap^{-\alpha}$ $a > 0, \alpha < 1$, **and** $D(\cdot) = ae^{(-p/k)}$, $a, k > 0$.

This section derives the mathematical expressions for three price demand functions.

4.1: When $D(\cdot) = a - bp, \quad a, b > 0$
Here,

$$U_1 = p\int_0^M (a - bp)dt + pI_e\int_N^M\int_N^t (a - bp)dudt + pR\{1 + I_e(M - N)\}$$

Case 4.1.1.

$$\text{Maximize } Z_1^{(1)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } 0 < N < M \le t_1 < T \tag{2.22}$$

where

$$X_L = \left\{U_1 - C_{pR}(S+R)\right\}\{1 + I_e(T-M)\} + (a-bp)p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\}$$
$$\{1 + I_e(T - t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{U_1 - C_{pL}(S+R)\right\}\{1 + I_e(T-M)\} + (a-bp)p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\}$$
$$\{1 + I_e(T - t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.1.2.

$$\text{Maximize } Z_2^{(1)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } 0 < N < M \le t_1 < T \tag{2.23}$$

$$\left\{C_{pL}(S+R) - U_1\right\}\left\{1 + I_p(B_L - M)\right\} = p\int_M^{B_L}(a-bp)dt + pI_e\int_M^{B_L}\int_M^{t}(a-bp)dudt$$

and

$$\left\{C_{pR}(S+R) - U_1\right\}\left\{1 + I_p(B_R - M)\right\} = p\int_M^{B_R}(a-bp)dt + pI_e\int_M^{B_R}\int_M^{t}(a-bp)dudt$$

where

$$X_L = \left\{p\int_{B_R}^{t_1}(a-bp)dt + pI_e\int_{B_R}^{t_1}\int_{B_R}^{t}(a-bp)dudt\right\}\{1 + I_e(T - t_1)\}$$
$$- C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

$$X_R = \left\{p\int_{B_L}^{t_1}(a-bp)dt + pI_e\int_{B_L}^{t_1}\int_{B_L}^{t}(a-bp)dudt\right\}\{1 + I_e(T - t_1)\}$$
$$- C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.1.3.

$$\text{Maximize } Z_3^{(1)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } 0 < N < M \le t_1 < T \tag{2.24}$$

$$C_{pL}(S+R)\{1+I_p(B_L-M)\} = U_1[1+I_e(B_L-N)] + p(a-bp)(B_L-M)\left[1+I_e\frac{(B_L-M)}{2}\right]$$

and

$$C_{pR}(S+R)\{1+I_p(B_R-M)\} = U_1[1+I_e(B_R-N)] + p(a-bp)(B_R-M)\left[1+I_e\frac{(B_R-M)}{2}\right]$$

where

$$X_L = \left\{p\int_{B_R}^{t_1}(a-bp)dt + pI_e\int_{B_R}^{t_1}\int_{B_R}^{t}(a-bp)dudt\right\}\{1+I_e(T-t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{p\int_{B_L}^{t_1}(a-bp)dt + pI_e\int_{B_L}^{t_1}\int_{B_L}^{t}(a-bp)dudt\right\}\{1+I_e(T-t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.1.4.

Here,

$$U_2 = \left[p\int_0^{t_1}(a-bp)dt + pI_e\int_N^{M}\int_N^{t}(a-bp)dudt\right][1+I_e(M-t_1)] + pR\{1+I_e(M-N)\}$$

$$\text{Maximize } Z_4^{(1)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } N < t_1 < M \le T \tag{2.25}$$

where

$$X_L = U_2 - (C_{oR} + C_{holR} + C_{shoR} + OC_{LSR})$$

and

$$X_R = U_2 - (C_{oL} + C_{holL} + C_{shoL} + OC_{LSL})$$

4.2: **When** $D(\cdot) = ap^{-\alpha} \quad a > 0, \alpha < 1$
Here,

$$U_1 = p\int_0^M ap^{-\alpha}\,dt + pI_e\int_N^M\int_N^t ap^{-\alpha}\,du\,dt + pR\{1 + I_e(M - N)\}$$

Case 4.2.1.

$$\text{Maximize } Z_1^{(2)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } 0 < N < M \le t_1 < T \qquad (2.26)$$

where

$$X_L = \{U_1 - C_{pR}(S + R)\}\{1 + I_e(T - M)\} + ap^{-\alpha}p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\}$$
$$\{1 + I_e(T - t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \{U_1 - C_{pL}(S + R)\}\{1 + I_e(T - M)\} + ap^{-\alpha}p(t_1 - M)\left\{1 + \frac{1}{2}I_e(t_1 - M)\right\}$$
$$\{1 + I_e(T - t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.2.2.

$$\text{Maximize } Z_2^{(2)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } 0 < N < M \le t_1 < T \qquad (2.27)$$

$$\{C_{pL}(S + R) - U_1\}\{1 + I_p(B_L - M)\} = p\int_M^{B_L} ap^{-\alpha}\,dt + pI_e\int_M^{B_L}\int_M^t ap^{-\alpha}\,du\,dt$$

and

$$\{C_{pR}(S + R) - U_1\}\{1 + I_p(B_R - M)\} = p\int_M^{B_R} ap^{-\alpha}\,dt + pI_e\int_M^{B_R}\int_M^t ap^{-\alpha}\,du\,dt$$

where

$$X_L = \left\{ p \int_{B_R}^{t_1} ap^{-\alpha}\, dt + pI_e \int_{B_R}^{t_1} \int_{B_R}^{t} ap^{-\alpha}\, dudt \right\} \{1 + I_e(T - t_1)\}$$
$$- C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{ p \int_{B_L}^{t_1} ap^{-\alpha}\, dt + pI_e \int_{B_L}^{t_1} \int_{B_L}^{t} ap^{-\alpha}\, dudt \right\} \{1 + I_e(T - t_1)\}$$
$$- C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.2.3.

$$\text{Maximize } Z_3^{(2)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T} \right]$$
$$\text{subject to } 0 < N < M \le t_1 < T \tag{2.28}$$

$$C_{pL}(S + R)\left\{1 + I_p(B_L - M)\right\} = U_1[1 + I_e(B_L - N)] + pap^{-\alpha}(B_L - M)$$
$$\left[1 + I_e \frac{(B_L - M)}{2}\right]$$

and

$$C_{pR}(S + R)\left\{1 + I_p(B_R - M)\right\} = U_1[1 + I_e(B_R - N)] + pap^{-\alpha}(B_R - M)$$
$$\left[1 + I_e \frac{(B_R - M)}{2}\right]$$

where

$$X_L = \left\{ p \int_{B_R}^{t_1} ap^{-\alpha}\, dt + pI_e \int_{B_R}^{t_1} \int_{B_R}^{t} ap^{-\alpha}\, dudt \right\} \{1 + I_e(T - t_1)\}$$
$$- C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{ p \int_{B_L}^{t_1} ap^{-\alpha}\, dt + pI_e \int_{B_L}^{t_1} \int_{B_L}^{t} ap^{-\alpha}\, dudt \right\} \{1 + I_e(T - t_1)\}$$
$$- C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.2.4.

Here,

$$U_2 = \left[p \int_0^{t_1} ap^{-\alpha}\, dt + pI_e \int_N^M \int_N^t ap^{-\alpha}\, dudt \right] [1 + I_e(M - t_1)] + pR\{1 + I_e(M - N)\}$$

$$\begin{aligned} &\text{Maximize } Z_4^{(2)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T} \right] \\ &\text{subject to } N < t_1 < M \le T \end{aligned} \tag{2.29}$$

where

$$X_L = U_2 - (C_{oR} + C_{holR} + C_{shoR} + OC_{LSR})$$

and

$$X_R = U_2 - (C_{oL} + C_{holL} + C_{shoL} + OC_{LSL})$$

4.3: **When** $D(\cdot) = ae^{(-p/k)}, \quad a, k > 0$
Here,

$$U_1 = p \int_0^M ae^{(-p/k)}\, dt + pI_e \int_N^M \int_N^t ae^{(-p/k)}\, dudt + pR\{1 + I_e(M - N)\}$$

Case 4.3.1.

$$\begin{aligned} &\text{Maximize } Z_1^{(3)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T} \right] \\ &\text{subject to } 0 < N < M \le t_1 < T \end{aligned} \tag{2.30}$$

where

$$X_L = \left\{U_1 - C_{pR}(S + R)\right\}\{1 + I_e(T - M)\} + ae^{(-p/k)} p(t_1 - M)\left\{1 + \frac{1}{2} I_e(t_1 - M)\right\} \{1 + I_e(T - t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{U_1 - C_{pL}(S + R)\right\}\{1 + I_e(T - M)\} + ae^{(-p/k)} p(t_1 - M)\left\{1 + \frac{1}{2} I_e(t_1 - M)\right\} \{1 + I_e(T - t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.3.2.

$$\text{Maximize } Z_2^{(3)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } 0 < N < M \leq t_1 < T \tag{2.31}$$

$$\left\{C_{pL}(S+R) - U_1\right\}\left\{1 + I_p(B_L - M)\right\} = p\int\limits_M^{B_L} ae^{(-p/k)}\,dt + pI_e\int\limits_M^{B_L}\int\limits_M^{t} ae^{(-p/k)}\,dudt$$

and

$$\left\{C_{pR}(S+R) - U_1\right\}\left\{1 + I_p(B_R - M)\right\} = p\int\limits_M^{B_R} ae^{(-p/k)}\,dt + pI_e\int\limits_M^{B_R}\int\limits_M^{t} ae^{(-p/k)}\,dudt$$

where

$$X_L = \left\{p\int\limits_{B_R}^{t_1} ae^{(-p/k)}\,dt + pI_e\int\limits_{B_R}^{t_1}\int\limits_{B_R}^{t} ae^{(-p/k)}\,dudt\right\}\{1 + I_e(T - t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{p\int\limits_{B_L}^{t_1} ae^{(-p/k)}\,dt + pI_e\int\limits_{B_L}^{t_1}\int\limits_{B_L}^{t} ae^{(-p/k)}\,dudt\right\}\{1 + I_e(T - t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.3.3.

$$\text{Maximize } Z_3^{(3)}(S, R, t_1, T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } 0 < N < M \leq t_1 < T \tag{2.32}$$

$$C_{pL}(S+R)\left\{1 + I_p(B_L - M)\right\} = U_1[1 + I_e(B_L - N)] + pae^{(-p/k)}(B_L - M)\left[1 + I_e\frac{(B_L - M)}{2}\right]$$

and

$$C_{pR}(S+R)\{1+I_p(B_R-M)\} = U_1[1+I_e(B_R-N)] + pae^{(-p/k)}(B_R-M)\left[1+I_e\frac{(B_R-M)}{2}\right]$$

where

$$X_L = \left\{p\int_{B_R}^{t_1} ae^{(-p/k)}\,dt + pI_e\int_{B_R}^{t_1}\int_{B_R}^{t} ae^{(-p/k)}\,dudt\right\}\{1+I_e(T-t_1)\} - C_{oR} - C_{holR} - C_{shoR} - OC_{LSR}$$

and

$$X_R = \left\{p\int_{B_L}^{t_1} ae^{(-p/k)}\,dt + pI_e\int_{B_L}^{t_1}\int_{B_L}^{t} ae^{(-p/k)}dudt\right\}\{1+I_e(T-t_1)\} - C_{oL} - C_{holL} - C_{shoL} - OC_{LSL}$$

Case 4.3.4.

Here,

$$U_2 = \left[p\int_{0}^{t_1} ae^{(-p/k)}\,dt + pI_e\int_{N}^{M}\int_{N}^{t} ae^{(-p/k)}\,dudt\right][1+I_e(M-t_1)] + pR\{1+I_e(M-N)\}$$

$$\text{Maximize } Z_4^{(3)}(S,R,t_1,T) = \left[\frac{X_L}{T}, \frac{X_R}{T}\right]$$
$$\text{subject to } N < t_1 < M \le T \tag{2.33}$$

where

$$X_L = U_2 - (C_{oR} + C_{holR} + C_{shoR} + OC_{LSR})$$

and

$$X_R = U_2 - (C_{oL} + C_{holL} + C_{shoL} + OC_{LSL})$$

2.5 Numerical Examples

This section provides and solves three instances with the purpose of illustrating and validating the inventory model.

The solution procedure consists of applying the theory of interval numbers and two efficient and effective soft computing techniques: Particle swarm optimization constriction (PSO-CO) and weighted quantum particle swarm optimization (WQPSO). Both soft computing algorithms are programmed in C language. The computational experiments are done on a personal computer with the following technical characteristics: Intel Core-2-Duo, 2.5 GHz Processor, and LINUX environ. It is important to remark that Kennedy and Eberhart [27], Clerc and Kennedy [28], and Clerc [29] proposed the particle swarm optimization (PSO) and particle swarm optimization constriction (PSO-CO); and Sun et al. [30, 31] introduced weighted quantum particle swarm optimization (WQPSO). Sahoo et al. [32] introduced the definitions of interval order relations between two interval numbers with the aim of solving the maximization and minimization problems.

Example 1 Consider an inventory problem in which the demand function is given by $D = a - bp$ and the following parameters: $C_{oL} = \$195$, $C_{oR} = \$200$, $\theta = 0.1$, $a = 150$, $b = 0.7$, $\delta = 1.5$, $C_{hL} = \$1$, $C_{hR} = \$1.5$, $C_{bR} = \$10$, $C_{bL} = \$8$, $C_{pL} = \$22$, $C_{pR} = \$25$, $I_e = 0.12$, $I_p = 0.15$ $N = 0.16$, $M = 0.246$, $C_{lsL} = \$18$, $C_{lsR} = \$20$, $p = \$30$.

The solution is exhibited in Tables 2.2 and 2.3, where Table 2.2 shows the solution obtained by PSO-CO and Table 2.3 displays the solution determined by WQPSO.

Example 2 Consider an inventory system in which the demand function is as follows: $D = ap^{-\alpha}$ and the following parameters: $C_{oL} = \$195$, $C_{oR} = \$200$, $\theta = 0.1$, $a = 150$, $\delta = 1.5$, $C_{hL} = \$1$, $C_{hR} = \$1.5$, $C_{bR} = \$10$, $C_{bL} = \$8$, $C_{pL} = \$22$, $C_{pR} = \$25$, $I_e = 0.12$, $I_p = 0.15$ $N = 0.16$, $M = 0.246$, $C_{lsL} = \$18$, $C_{lsR} = \$20$, $p = \$30$, $\alpha = 0.2$ (Tables 2.4 and 2.5).

Example 3 Consider that the demand function is $D = ae^{(-p/k)}$ and the following parameters: $C_{oL} = \$195$, $C_{oR} = \$200$, $\theta = 0.1$, $a = 150$, $\delta = 1.5$, $C_{hL} = \$1$, $C_{hR} = \$1.5$, $C_{bR} = \$10$, $C_{bL} = \$8$, $C_{pL} = \$22$, $C_{pR} = \$25$, $I_e = 0.12$, $I_p = 0.15$ $N = 0.16$, $M = 0.246$, $C_{lsL} = \$18$, $C_{lsR} = \$20$, $p = \$30$, $k = 40$ (Tables 2.6 and 2.7).

2.6 Sensitivity Analysis

This section provides a sensitivity analysis, which is done, based on Example 1. The sensitivity analysis is made by varying the parameters by -20 to $+20\%$. The results of the sensitivity analysis for Example 1 are shown in Table 2.8.

Table 2.2 Optimal solution for Example 1 when demand function is modeled by $D = a - bp$ and PSO-CO is applied

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
93.4681	13.5663	0.6995	0.8134	182.8536	635.1188	408.99	–	–	1.1
85.9997	13.7784	0.6454	0.7612	168.9084	606.3228	378.19	0.4630	0.5438	1.2.1
79.9735	14.0093	0.6015	0.7194	150.0555	606.3228	378.19	0.4292	0.5063	1.2.2
82.6527	14.1652	0.6210	0.7404	136.54742	581.57365	359.06	–	–	2

Table 2.3 Optimal solution for Example 1 when demand function is expressed by $D = a - bp$ and WQPSO is used

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
93.4644	13.5667	0.6995	0.8134	182.8549	635.1176	408.99	–	–	1.1
86.0001	13.7784	0.6454	0.7612	168.9082	622.6545	395.78	0.4630	0.5439	1.2.1
79.9740	14.0086	0.6015	0.7194	150.0553	606.32300	378.19	0.4292	0.5063	1.2.2
82.6534	14.1654	0.6210	0.7404	136.5472	581.5738	359.06	–	–	2

Table 2.4 Optimal solution for Example 2 when demand function is given by $D = ap^{-\alpha}$ and PSO-CO is utilized

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
70.3066	10.7814	0.8851	1.0432	13.9693	292.8948	153.43	–	–	1.1
63.6977	10.9814	0.8051	0.9665	−1.0359	279.3446	139.15	0.5841	0.6892	1.2.1
60.3069	11.0487	0.7638	0.9264	−17.0486	265.2967	124.12	0.5540	0.6557	1.2.2
61.9337	11.1585	0.7837	0.9480	−21.2679	252.0924	115.41	–	–	2

Table 2.5 Optimal solution for Example 2 when demand function is given by $D = ap^{-\alpha}$ and WQPSO is used

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
70.3070	10.7814	0.8851	1.0432	13.9692	292.8949	153.43	–	–	1.1
63.6995	10.9817	0.8051	0.9665	−1.0366	279.3454	139.15	0.5842	0.6892	1.2.1
60.3062	11.0488	0.7638	0.9264	−17.0482	265.2964	124.12	0.5540	0.6557	1.2.2
61.9330	11.1590	0.7837	0.9480	−21.2677	252.0921	115.41	–	–	2

Table 2.6 Optimal solution for Example 3 when demand function is $D = ae^{(-p/k)}$, and PSO-CO is computed

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
67.7420	10.4657	0.9131	1.0784	−0.4084	261.5048	130.55	–	–	1.1
61.2534	10.6616	0.8291	0.9980	−15.4465	247.9046	116.23	0.6027	0.7116	1.2.1
58.0977	10.7124	0.7881	0.9578	−30.9859	234.2535	101.63	0.5730	0.6785	1.2.2
59.5854	10.8172	0.8075	0.9790	−34.4253	222.0720	93.82	–	–	2

Table 2.7 Optimal solution for Example 3 when demand function is $D = ae^{(-p/k)}$ and WQPSO is applied

S	R	t_1	T	Z_L	Z_R	Average profit	B_L	B_R	Case/subcase
67.7423	10.4657	0.9131	1.0784	−0.4085	261.5049	130.55	–	–	1.1
61.2530	10.6620	0.8291	0.9980	−15.4463	247.9044	116.23	0.6027	0.7116	1.2.1
58.0976	10.7126	0.7881	0.9578	−30.9859	234.2535	101.63	0.5730	0.6785	1.2.2
59.5849	10.8174	0.8074	0.9790	−34.4251	222.0718	93.82	–	–	2

Table 2.8 Sensitivity analysis for Example 1 when $D(\cdot) = a - bp,\ a,\ b > 0$

Parameters	% change of parameters	Change						
		Z_L	Z_R	t_1	T	R	S	Average profit
$C_o = [C_{oL}, C_{oR}]$	−20	237.6248	682.45141	0.6343	0.734	11.9825	84.4757	460.04
	−10	209.5331	658.1815	0.6678	0.7748	12.7919	89.0929	433.857
	10	157.3913	613.0972	0.7296	0.8503	14.3106	97.6349	385.244
	20	132.9925	591.9853	0.7583	0.8856	15.0287	101.619	362.488
$C_h = [C_{hL}, C_{hR}]$	−20	194.2057	643.9136	0.7158	0.8281	13.3908	95.7265	419.059
	−10	188.4851	639.4885	0.7075	0.8207	13.4788	94.58	413.986
	10	177.3085	630.8032	0.6917	0.8064	13.6533	92.3891	404.055
	20	171.8473	626.5403	0.6841	0.7996	13.7397	91.3413	399.193
$C_b = [C_{bL}, C_{bR}]$	−20	184.8203	636.6107	0.6986	0.8174	14.1098	93.3359	410.715
	−10	183.8167	635.8522	0.699	0.8154	13.8327	93.4031	409.834
	10	181.9287	634.4092	0.7	0.8116	13.3099	93.5309	408.169
	20	181.0397	633.7223	0.7004	0.8098	13.0631	93.5916	407.381
$C_p = [C_{pL}, C_{pR}]$	−20	887.4738	1266.7736	0.8399	0.9522	13.3861	113.0255	1077.1237
	−10	533.2732	948.796	0.7617	0.8737	13.3512	102.0973	741.0346
	10	−164.3283	324.9279	0.648	0.7658	13.2949	86.3625	80.2998
	20	–	–	–	–	–	–	–
a	−20	83.5254	438.2659	0.7865	0.9205	12.0830	81.0094	260.90
	−10	132.1852	535.8331	0.7388	0.8616	12.8493	87.4105	334.01
	10	235.1709	735.8147	0.6665	0.7731	14.2425	99.2434	485.49
	20	288.8702	837.6921	0.6382	0.7387	14.8845	104.7820	563.28

(continued)

Table 2.8 (continued)

Parameters	% change of parameters	Change						
		Z_L	Z_R	t_1	T	R	S	Average profit
b	−20	197.3492	663.1826	0.6897	0.8014	13.7594	95.1114	430.27
	−10	190.0857	649.1373	0.6946	0.8074	13.6632	94.2924	419.61
	10	175.6537	621.1279	0.7046	0.8196	13.4685	92.6382	398.39
	20	168.4868	607.16522	0.7098	0.8260	13.3700	91.8027	387.83
θ	−20	203.7516	656.8053	0.7385	0.8498	13.2698	98.1428	430.28
	−10	193.1488	645.7845	0.7183	0.8309	13.4171	95.7234	419.47
	10	172.8476	624.7827	0.6820	0.7972	13.7166	91.3580	398.82
	20	163.1138	614.7535	0.6655	0.7822	13.8675	89.3772	388.93
δ	−20	190.2571	642.6646	0.6953	0.8292	16.0174	92.8870	416.46
	−10	186.2794	638.5865	0.6976	0.8207	14.6895	93.1987	412.43
	10	179.8720	632.1338	0.7012	0.8072	12.6036	93.7033	406.00
	20	177.2531	629.5368	0.7027	0.8019	11.7692	93.9103	403.39
$C_{ls} = [C_{lsL}, C_{lsR}]$	−20	189.3095	640.6154	0.6962	0.8275	15.4513	93.0153	414.9625
	−10	185.8718	637.7029	0.698	0.8199	14.4474	93.2545	411.7874
	10	180.1828	632.8108	0.7009	0.8078	12.7865	93.66	406.4968
	20	177.8028	630.7368	0.7022	0.8028	12.0916	93.8332	404.2698
p	−20	–	–	–	–	–	–	–
	−10	−218.2979	237.9026	0.6644	0.7786	13.4190	90.0669	9.80
	10	572.1072	1020.6209	0.7374	0.8523	13.4523	97.1063	796.36
	20	949.4445	1394.45055	0.7787	0.8958	13.4604	101.0696	1171.95

From Table 2.8, the following interpretations are mentioned:

- With the increment in the value of replenishment cost $[C_{oL}, C_{oR}]$, the cycle length (T), the time at which the inventory level reaches zero (t_1), shortage level (R), and stock level (S) increase, but average profit decreases.
- When the holding cost $[C_{hL}, C_{hR}]$ increases, then the cycle length (T), the time at which the inventory level reaches zero (t_1), stock level (S), and average profit decrease, but shortage level (R) increases.
- With the increment in the value of shortage cost $[C_{bL}, C_{bR}]$, the time at which the inventory level reaches zero (t_1) and stock level (S) increase, but the shortage level (R), the cycle length (T), and the average profit decrease.
- When the value of purchasing cost $\left[C_{pL}, C_{pR}\right]$ increases, then the shortage level (R), the cycle length (T), the time at which the inventory level reaches zero (t_1), stock level (S) and the average profit decrease.
- When the scale parameter (a) of demand increases, then the cycle length (T) and the time at which the inventory level reaches zero (t_1) decrease, whereas the shortage level (R), the stock level (S), and the average profit increase.
- When the price elasticity parameters (b) of demand increases, then the shortage level (R), the stock level (S), and the average profit decrease, whereas the cycle length (T) and the time at which the inventory level reaches zero (t_1) increase.
- With the increment in the value of the deterioration rate (θ), the cycle length (T), stock level (S), the average profit, and the time at which the inventory level reaches zero (t_1) decrease, but the shortage level (R) increases.
- When the value of the backlogging parameter δ increases, then the shortage level (R), the cycle length (T) and the average profit decrease, but the time at which the inventory level reaches zero (t_1) and stock level (S) increase.
- With the increment in the value of opportunity cost $[C_{lsL}, C_{lsR}]$, then the shortage level (R), the cycle length (T), and the average profit decrease, but the time at which the inventory level reaches zero (t_1) and stock level (S) increase.
- With the increment in the value of selling price (p), then the shortage level (R), the cycle length (T), the time at which the inventory level reaches zero (t_1), stock level (S), and the average profit increase.

2.7 Conclusion

This paper develops an inventory model for deteriorating items with interval-valued inventory costs, partial backlogging, and price-dependent demand under two-level credit policy. In order to make a more realistic scenario, the shortages are permitted, and these are partially backlogged. The proposed inventory model is very helpful for retail and manufacturing industries in developing countries where credit policy plays a significant role in decision-making. This research work aims to find the retailer's optimal replenishment policy that maximizes the total profit of the system. To solve the inventory model, two soft computing techniques are used: the PSO-CO and the WQPSO. The efficiency and effectiveness of the proposed inventory model are validated with numerical examples and a sensitivity analysis.

Finally, this research can be extended by considering: (1) stock-dependent demand, (2) inventory costs represented by a fuzzy number, (3) finite-time horizon, (4) inflation, and (5) an integrated supply chain model with two or more players with the coordination between the players, among others. These are some interesting and challenge research lines to explore by academicians and researchers.

References

1. Gupta RK, Bhunia AK, Goyal SK (2007) An application of genetic algorithm in a marketing oriented inventory model with interval valued inventory costs and three-component demand rate dependent on displayed stock level. Appl Math Comput 192(2):466–478
2. Dey JK, Mondal SK, Maiti M (2008) Two storage inventory problem with dynamic demand and interval valued lead-time over finite time horizon under inflation and time-value of money. Eur J Oper Res 185(1):170–194
3. Gupta RK, Bhunia AK, Goyal SK (2009) An application of genetic algorithm in solving an inventory model with advance payment and interval valued inventory costs. Math Comput Model 49(5–6):893–905
4. Bhunia AK, Shaikh AA, Mahato SK, Jaggi CKA (2014) A deteriorating inventory model with displayed stock-level dependent demand and partially backlogged shortages with all unit discount facilities via particle swarm optimisation. Int J Syst Sci: Oper Logist 1(3):164–180
5. Bhunia AK, Shaikh AA (2016) Investigation of two-warehouse inventory problems in interval environment under inflation via particle swarm optimization. Math Comput Model Dyn Syst 22(2):160–179
6. Hwang H, Shinn SW (1997) Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. Comput Oper Res 24(6):539–547
7. Chang CT, Ouyang LY, Teng JT (2003) An EOQ model for deteriorating items under supplier credits linked to ordering quantity. Appl Math Model 27(12):983–996
8. Abad PL, Jaggi CK (2003) A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. Int J Prod Econ 83(2):115–122
9. Ouyang LY, Wu KS, Yang CT (2006) A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Comput Ind Eng 51(4):637–651
10. Huang YF (2006) An inventory model under two levels of trade credit and limited storage space derived without derivatives. Appl Math Model 30(5):418–436
11. Huang YF (2007) Economic order quantity under conditionally permissible delay in payments. Eur J Oper Res 176(2):911–924
12. Huang YF (2007) Optimal retailer's replenishment decisions in the EPQ model under two levels of trade credit policy. Eur J Oper Res 176(3):1577–1591
13. Sana SS, Chaudhuri KS (2008) A deterministic EOQ model with delays in payments and price-discount offers. Eur J Oper Res 184(2):509–533
14. Huang YF, Hsu KH (2008) An EOQ model under retailer partial trade credit policy in supply chain. Int J Prod Econ 112(2):655–664
15. Ho CH, Ouyang LY, Su CH (2008) Optimal pricing, shipment and payment policy for an integrated supplier–buyer inventory model with two-part trade credit. Eur J Oper Res 187(2):496–510
16. Jaggi CK, Khanna A (2010) Supply chain models for deteriorating items with stock-dependent consumption rate and shortages under inflation and permissible delay in payment. Int J Math Oper Res 2(4):491–514

17. Jaggi CK, Kausar A (2011) Retailer's ordering policy in a supply chain when demand is price and credit period dependent. Int J Strat Decis Sci 2(4):61–74
18. Jaggi CK, Mittal M (2012) Retailer's ordering policy for deteriorating items with initial inspection and allowable shortages under the condition of permissible delay in payments. Int J Appl Ind Eng 1(1):64–79
19. Guria A, Das B, Mondal S, Maiti M (2013) Inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment. Appl Math Model 37(1–2):240–257
20. Taleizadeh AA, Pentico DW, Jabalameli MS, Aryanezhad M (2013) An EOQ model with partial delayed payment and partial backordering. Omega 41(2):354–368
21. Wu J, Ouyang LY, Cárdenas-Barrón LE, Goyal SK (2014) Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing. Eur J Oper Res 237(3):898–908
22. Chen SC, Cárdenas-Barrón LE, Teng JT (2014) Retailer's economic order quantity when the suppliers offers conditionally permissible delay in payments link to order quantity. Int J Prod Econ 155:284–291
23. Bhunia AK, Shaikh AA, Sahoo L (2016) A two-warehouse inventory model for deteriorating item under permissible delay in payment via particle swarm optimization. Int J Logist Syst Manag 24(1):45–69
24. Bhunia AK, Shaikh AA (2015) An Application of PSO in a two warehouse inventory model for deteriorating item under permissible delay in payment with different inventory policies. Appl Math Comput 256(1):831–850
25. Shah NH, Cárdenas-Barrón LE (2015) Retailer's decision for ordering and credit policies for deteriorating items when a supplier offers order-linked credit period or cash discount. Appl Math Comput 259:569–578
26. Bhunia AK, Shaikh AA, Pareek S, Dhaka V (2015) A memo on stock model with partial backlogging under delay in payment. Uncertain Supply Chain Manag 3(1):11–20
27. Kennedy JF, Eberhart RC (1995) Particle swarm optimization, In: Proceedings of the IEEE international conference on neural network, vol IV, Perth, Australia, pp 1942–1948
28. Clerc M (1999) The swarm and queen: towards a deterministic and adaptive particle swarm optimization. In: Proceedings of IEEE congress on evolutionary computation, Washington, DC, USA, pp 1951–1957
29. Clerc M, Kennedy JF (2002) The particle swarm: explosion, stability, and convergence in a multi-dimensional complex space. IEEE Trans Evol Comput 6(1):58–73
30. Sun J, Feng B, Xu WB (2004a) Particle swarm optimization with particles having quantum behavior. In: IEEE proceedings of congress on evolutionary computation, pp 325–331
31. Sun J, Xu WB, Feng B (2004b) A global search strategy of quantum-behaved particle swarm optimization. In: Proceedings of the 2004 IEEE conference cybernetics and intelligent systems, pp 111–116
32. Sahoo L, Bhunia AK, Kapur PK (2012) Genetic algorithm based multi-objective reliability optimization in interval environment. Comput Ind Eng 62:152–160

Chapter 3
Inventory Control Policies for Time-Dependent Deteriorating Item with Variable Demand and Two-Level Order Linked Trade Credit

Mrudul Y. Jani, Nita H. Shah and Urmila Chaudhari

Abstract In today's business world to boost the demand, vendor gives a trade credit to buyer. Moreover, most of the products lose quality over time due to environmental effects. This chapter studies an inventory policy for the item which has expiry date with two levels of trade credit depending on the quantity of order. It is considered that a supplier is ready to give a mutually agreed credit period to retailer only if the order quantity purchased by retailer is more than the predetermined quantity of order. Additionally, a retailer deals a credit limit to the end consumers. Here, time- and price-sensitive demand is debated with inflation. A retailer's main objective is to earn maximum total profit with respect to the number of replenishments throughout the finite planning horizon. Results are supported by numerical examples. Finally, a sensitivity analysis is done to develop visions for decision-makers.

Keywords Order link trade credit · Inflation · Time value of money · Maximum fixed lifetime · Price- and time-dependent demand

3.1 Introduction

In today's competitive business world, credit limit is an essential strategy for management system of supply chain. Therefore, a supplier deals with a credit limit to a retailer on purchasing cost of predetermined amount only if the stock purchased by retailer is larger than the preset order size. Trade credit attracts new retailers and

M. Y. Jani (✉)
Department of Applied Sciences and Humanities, Faculty of Engineering and Technology, PIET, Parul University, Vadodara 391760, Gujarat, India
e-mail: janimrudul07@gmail.com

N. H. Shah
Department of Mathematics, Gujarat University, Ahmedabad 380009, Gujarat, India
e-mail: nitahshah@gmail.com

U. Chaudhari
Government Polytechnic Dahod, Dahod 389151, Gujarat, India
e-mail: chaudhariurmi04@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_3

the concept of price discount occurs indirectly. Many researchers have enlightened the literature of inventory by addressing this concept. Goyal [1] perhaps was the first in introducing the term of credit limit financing in the EOQ model. Aggarwal and Jaggi [2] extended the concept of Goyal [1] for perishable items. Huang [3] considered business polices where upstream credit limit is greater than a downstream trade credit. Soni et al. [4] developed broad study of inventory policies with delay in payment options. Sana [5] derived an EOQ model of perfect and imperfect quality items that incorporates the concept of delay in payment. Ouyang [6] relaxed two assumptions of Huang's [3] work. First, the wholesaler's interest earned rate is more than the interest charged by the supplier. Second, upstream credit limit is independent of downstream credit limit. Taleizadeh et al. [7] formulated a model of half-done trade credit policy with shortages and partial backordering. Pal et al. [8] established a vendor's optimal refill size and manufacturer's manufacturing rate under the effect of trade credit. Recently, Shah et al. [9] determined optimal ordering policies for permissible delay in payment option.

Most of the articles in traditional inventory modeling are concentrated on constant demand. However, it is rare in real world. Keeping this limitation in mind, Pal et al. [10] discussed inventory models of stock-dependent demand. Wee [11] investigated inventory policies for price-sensitive demand. Jaggi et al. [12] formulated ideal replenishment strategies for permissible delay in payment-dependent demand. Shah et al. [13] examined quadratic demand-dependent EOQ model. In recent times, Shah et al. [14] derived retailer's optimum control policy for trade credit and selling price-sensitive trapezoidal demand.

In the standard traditions of inventory modeling, products are reserved for unbounded time to fulfill consumers' demand; however, this hypothesis occurs rarely. For example, drugs, blood in blood banks, and foods are deteriorated over time in warehouses. Also, some liquids like petrol and alcohol in storage evaporate with time throughout holding period. Therefore, storage of such deteriorating items is the big issue for inventory control policies. In this regard, Ghare and Schrader [15] developed inventory control policies of exponentially decaying items. Raafat [16] considered the effect of deterioration on optimal control of inventory policies. Sett et al. [17] studied inventory modeling of time-dependent deterioration. Sarkar et al. [18] established inventory control policy of product with maximum fixed life. Lately, Shah et al. [19] discussed an inventory model for three different cases, i.e., constant deterioration rate, maximum fixed life of the product, and without deterioration rate.

Nowadays, inflation is a general feature that abruptly declines the money's purchasing power. Large scale of inflation is usual in many countries. So, while giving the definition of the optimal control inventory policy, one cannot ignore the environment of inflation rate. Many researchers have recognized inventory control policies with the inflationary environment. Ray and Chaudhuri [20] and Chang et al. [21] planned an inventory policy with inflation. Sarkar [22] investigated an inventory model of finite renewal rate. Ghoreishi et al. [23] worked on an EOQ model for non-instantaneous perishable items. Lashgari et al. [24] analyzed a model for backordering and credit limit with inflation. Recently, Shah et al. [25] examined manufacturing system of imperfect item in inflationary environment.

All of the above-cited factors such as order quantity linked delay in payment, inflation, time value of money, price and time-varying demand and product's maximum fixed life have not been enlightened comprehensively in one model, but this chapter considers all the above factors together in one model. The proposed inventory model represents one supplier–one retailer for two layers of trade credit options where supplier gives credit period to retailer only if the stock purchased by retailer is more than the prescheduled order quantity. The main objective is to maximize the net current value of retailer's profit with optimum number of shipments during finite planning horizon in both the ways, analytically and numerically, under the effect of inflationary environment together with time value money.

In the remaining chapter, Sect. 3.2 represents notations together with assumptions. Section 3.3 represents a mathematical model. Numerical examples with managerial insights are discussed in Sect. 3.4. Sensitivity analysis of the proposed model is also described in Sect. 3.4. Lastly, Sect. 3.5 offers conclusions with future scopes.

3.2 Notation and Assumptions

3.2.1 Notation

A	Setup cost ($/lot)
h	Holding cost/unit/unit time (in $)
k	Net rate of constant decline in inflation
C	Purchase cost ($/unit) at $t = 0$
$C(t) = Ce^{-kt}$	Purchase cost at t
p	Selling price ($/unit) at $t = 0$; $p > C$
$p(t) = p\,e^{-kt}$	Selling price ($/unit) at t
M	Supplier deals permissible delay in payment to retailer (in years)
N	Retailer deals permissible delay in payment to customer (in years)
I_c	Interest rate paid by retailer to supplier (/$/year); $I_c > I_e$
I_e	Rate of interest earned by retailer (/$/year)
I_b	Interest rate paid by retailer to bank per dollar per year if $T > M$
n	Replenishments' number (Decision Variable)
H	Finite planning horizon
$T = \frac{H}{n}$;	Replenishment or cycle time (years)
m	Item's maximum fixed life (years)
Q	Order quantity in each replenishment (units)
Q_d	Predetermined order quantity (units)
$I(t)$	Inventory's level at t (units)
$\pi(n)$	Total profit of retailer for n^{th} replenishment (in $)

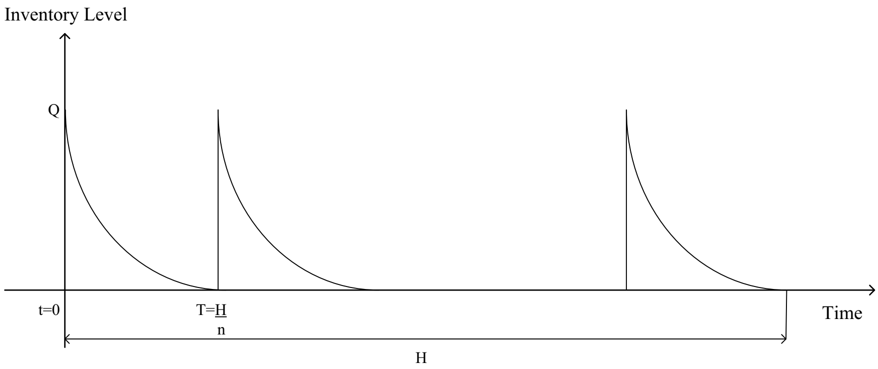


Fig. 3.1 Inventory control diagram

3.2.2 Assumptions

1. The planning horizon is finite.
2. The inventory policy deals with single product.
3. Demand rate, (say) $R(p,t) = a - bp(t)$; where $a > 0$ is scale demand and $b > 0$ is markup of selling price.
4. $\theta(t) = \frac{1}{1+m-t}$, $0 \le t \le T \le m$ is instantaneous deterioration, where $\theta(t) \le 1$ for any m.
5. Supplier is ready to give mutually agreed credit period M to retailer only if stock purchased by retailer is larger than the prearranged order quantity, i.e., $Q > Q_d$.
6. Retailer pays interest rate I_b to bank for $T > M$.
7. Constant inflation rate is considered with time value of money (Fig. 3.1).

3.3 Mathematical Model

In an interval $[0, T]$, the rate of change of inventory at any time t is given as follows:

$$\frac{dI(t)}{dt} = -R(p,t) - \theta(t)I(t), \qquad 0 \le t \le T \tag{3.1}$$

After applying a condition $I(T) = 0$, the solution of (3.1) is

$$I(t) = (1+m-t)\left(W_1 \ln\left(\frac{1+m-t}{1+m-T}\right) + W_2(t-T) - \frac{bpk^2}{4}\left(t^2 - T^2\right)\right) \tag{3.2}$$

where $W_1 = a - bp + bpk(1+m) - \frac{bpk^2(1+m)^2}{2}$ and $W_2 = bpk - \frac{bpk^2(1+m)}{2}$

Taking $I(0) = Q$ as an initial condition, Q can be derived from Eq. (3.2) as follows:

$$Q = (1+m)\left(W_1 \ln\left(\frac{1+m}{1+m-T}\right) - W_2 T + \frac{bpk^2T^2}{4}\right) \tag{3.3}$$

Relevant costs of retailer's total profit are as follows:

- Ordering/Setup Cost: $OC = \sum_{j=0}^{n} Ae^{-jkT}$
- Holding Cost: $HC = Ch\left[\sum_{j=0}^{n-1} e^{-jkT} \int_0^T I(t)dt\right]$
- Purchase Cost: $PC = CQ\left[\sum_{j=0}^{n-1} e^{-jkT}\right]$
- Sales Revenue: $SR = \left[\sum_{j=0}^{n-1} e^{-jkT} \int_0^T p(t)R(p,t)dt\right]$

Since we consider two levels order linked trade credit, the order size is more than Q_d; then, only supplier proposes credit limit M to a retailer; otherwise, at a time of receiving an order, retailer pays the total purchase cost. However, in both cases, retailer gives N to the customers. So, from the above discussion, two possible cases will occur (1) $Q < Q_d$ (2) $Q > Q_d$.

Case 1: $Q < Q_d$

In this situation, predetermined order quantity Q_d is more than retailer's order quantity Q. So, supplier does not offer M to a retailer. Nevertheless, retailer proposes N to the end customers. As an outcome, retailer takes loan from a bank at time zero and at N retailer starts to pay back. Therefore, interest payable to supplier and bank by retailer are

$$IC_1 = CI_c \sum_{j=0}^{n-1} e^{-jkT} \left(\int_0^T R(p,t)t\,dt + \int_0^N R(p,t)T\,dt\right)$$

and

$$CC_1 = CI_b \left(\sum_{j=0}^{n-1} e^{-jkT} \int_0^T I(t)dt\right)$$

respectively.

Hence, the net current total profit throughout the finite planning horizon is

$$\pi_1(n) = SR - HC - OC - PC - IC_1 - CC_1 \tag{3.4}$$

Case 2: $Q > Q_d$

In this situation, order quantity Q of retailer is more than an order quantity Q_d. Therefore, supplier gives trade credit $M(M \neq 0)$ to a retailer. Thus, possible subcases are as follows:

$$\begin{pmatrix} \text{Subcase - 1}: & M < N & & M < T \\ \text{Subcase - 2}: & M < N & & T < M \\ \text{Subcase - 3}: & M > N & M < T & \\ \text{Subcase - 4}: & M > N & T < M < T+N & \\ \text{Subcase - 5}: & M > N & T+N < M & \end{pmatrix}$$

Subcase-1: $M < N$ and $M < T$. In this subcase, due to $M < N$, interest earned to retailer is zero. On the other hand, interest payable by retailer to supplier and bank are

$$IC_2 = CI_c \sum_{j=0}^{n-1} e^{-jkT} \left(\int_0^T t\, R(p,t)\, dt + \int_0^{N-M} T\, R(p,t)\, dt \right)$$

and

$$CC_2 = CI_b \left(\sum_{j=0}^{n-1} e^{-jkT} \int_M^T I(t) dt \right)$$

respectively.

Consequently, the existing total profit for the duration of the finite planning horizon is

$$\pi_2(n) = SR - HC - OC - PC - IC_2 - CC_2 \tag{3.5}$$

Subcase-2: $M < N$ and $T < M$. Due to $T < M$, interest payable by retailer to bank is zero. Furthermore, since $M < N$, interest earned to retailer is zero. So, interest payable by retailer to supplier is $IC_3 = CI_c \sum_{j=0}^{n-1} e^{-jkT} \left(\int_0^{N-M} R(p,t) T\, dt + \int_0^T t\, R(p,t) dt \right)$.

As a result, the current total profit for the duration of the finite planning horizon is

$$\pi_3(n) = SR - HC - OC - PC - IC_3 \tag{3.6}$$

Subcase-3: $M > N$ and $M < T$. In this subcase, retailer earns interest by selling the items and retailer has to pay interest to the supplier and bank as follows:

$$IE_4 = pI_e \sum_{j=0}^{n-1} e^{-jkT} \left(\int_N^M R(p,t)t\,dt \right),$$

$$IC_4 = CI_c \sum_{j=0}^{n-1} e^{-jkT} \left(\int_0^{T+N-M} R(p,t)t\,dt \right)$$

and

$$CC_4 = CI_b \sum_{j=0}^{n-1} e^{-jkT} \left(\int_M^T I(t)dt \right)$$

Thus, the existing total profit for the duration of the finite planning horizon is

$$\pi_4(n) = SR - HC - OC - PC - IC_4 - CC_4 + IE_4 \tag{3.7}$$

Subcase-4: $M > N$ and $T < M < T + N$. Since, $T < M$, interest payable by retailer to bank is zero. Moreover, in this situation, retailer's interest earn and retailer's interest charge to supplier are

$$IE_5 = pI_e \sum_{j=0}^{n-1} e^{-jkT} \left(\int_N^M R(p,t)t\,dt \right) \text{ and}$$

$$IC_5 = CI_c \sum_{j=0}^{n-1} e^{-jkT} \left(\int_0^{T+N-M} R(p,t)t\,dt \right) \text{ respectively.}$$

So, the current total profit for the duration of the finite planning horizon is

$$\pi_5(n) = SR - HC - OC - PC - IC_5 + IE_5 \tag{3.8}$$

Subcase-5: $M > N$ and $T + N < M$. In the current subcase, since $T + N < M$:

$$IE_6 = pI_e \sum_{j=0}^{n-1} e^{-jkT} \left(\int_0^{M-N-T} R(p,t)T\,dt + \int_0^T R(p,t)t\,dt \right)$$

Consequently, the existing total profit for the duration of the finite planning horizon is

$$\pi_6(n) = SR - HC - OC - PC + IE_6 \tag{3.9}$$

Hence, total profit is given by

$$\pi(n) = \begin{cases} Q < Q_d & \pi_1(n) \\ Q > Q_d \begin{cases} M < N, & \begin{cases} \pi_2(n), & M < T \\ \pi_3(n), & T < M \end{cases} \\ M > N, & \begin{cases} \pi_4(n), & M < T \\ \pi_5(n), & T < M < T + N \\ \pi_6(n), & T + N < M \end{cases} \end{cases} \end{cases} \tag{3.10}$$

Here, model considers an algorithm for the optimum solution as follows.

Algorithm

Step 1: Calculate $\pi_i(n)$; $\forall i = 1 \ldots 6$ where $n \in \mathbb{N}$, using Eqs. (3.4)–(3.9), respectively.
Step 2: Take $n = n + 1$ where $n \in \mathbb{N}$ and once again calculate $\pi_i(n)$; $\forall i = 1 \ldots 6$ where $n \in \mathbb{N}$, using Eqs. (3.4)–(3.9), respectively.
Step 3: If $\pi_i(n + 1) < \pi_i(n)$; $\forall i = 1 \ldots 6$ where $n \in \mathbb{N}$, then the optimal number of replenishments is $n^* = n$; otherwise, go to step 2.
Step 4: Calculate optimal cycle time $T^* = \frac{H}{n^*}$
Step 5: Calculate optimum order quantity Q^* by replacing T^* in Eq. (3.3).

3.4 Numerical Examples with Sensitivity Analysis

3.4.1 Numerical Examples

3.4.1.1 Numerical Example 1 ($M > N$)

Take $a = 1000$ units, $b = 20\%$, $A = \$500$ per order, $C = \$5$ per unit, $p = \$10$ per unit, $I_e = 8\%$ $h = \$0.2$ per unit per unit time, $I_c = 12\%/\$/\text{year}$, $I_b = 8\%/\$/\text{year}$, $M = 0.15$ year, $k = 0.08$ $N = 0.06$ year, $m = 0.5$ year, $H = 5$ years, and $Q_d = 300$ units.

3.4.1.2 Numerical Example 2 ($M < N$)

Set $N = 0.2$ year in example 1.

By solving the numerical examples using the mathematical software Maple XVIII, we have Table 3.1 of optimal solutions.

From Table 3.1, we analyze that in the case ($Q > Q_d$, $M > N$, $M < T$), retailer's total profit is \$10,032.22 which is maximum, optimal number of replenishments is n* = 14, optimal cycle time is $T^* = 0.357$ year, and optimum order quantity is Q* = 407.10 units.

Table 3.1 Optimal solutions

Case	Subcase		Example number	Total profit ($)	Decision variables
$Q < Q_d$	**	**	1	Infeasible solution	Infeasible solution
$Q > Q_d$	$M < N$	$M < T$	2	9621.85	$n^* = 14$ $Q^* = 407.10$ units $T^* = 0.357$ year
		$T < M$	2	Infeasible solution	Infeasible solution
	$M > N$	$M < T$	1	**10,032.22**	$n^* = 14$ $Q^* = 407.10$ units $T^* = 0.357$ year
		$T < M < T + N$	1	Infeasible solution	Infeasible solution
		$T + N < M$	1	Infeasible solution	Infeasible solution

N.B. ** indicates not applicable case

The concavity of the retailer's total profit $\pi_4(n^*)$ for best optimal case is depicted in Fig. 3.2.

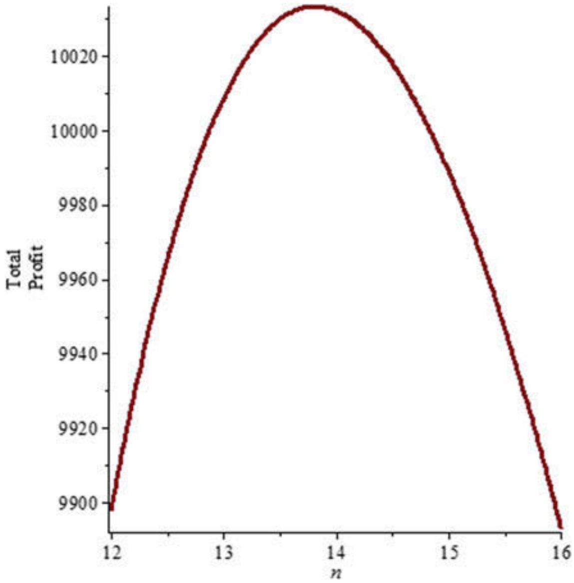


Fig. 3.2 Concavity of retailer's total profit versus number of replenishments

3.4.2 Sensitivity Analysis

Sensitivity analysis of object function for Example 1 for various inventory parameters is calculated in Table 3.2.

From Table 3.2, we analyze the following points: Increasing of finite planning horizon, scale demand, selling price, and maximum fixed life of the item increases retailer's total profit most rapidly which is fairly rational in practice. Increasing of trade credit proposed by supplier to retailer rises retailer's total profit gradually. On the other hand, purchase cost at time $t = 0$, ordering cost per unit, and constant decline rate in inflation decreases retailer's total profit promptly. Holding cost decreases retailer's total profit slowly.

Table 3.2 Sensitivity analysis

Inventory parameters	Values	Total profit ($)
A	400	11,269.69
	450	10,650.95
	500	10,032.22
	550	9413.48
	600	8794.74
C	4	15,031.72
	4.5	12,531.97
	5	10,032.27
	5.5	7532.47
	6	5032.72
a	800	6781.88
	900	8407.049
	1000	10,032.22
	1100	11,657.38
	1200	13,282.55
h	0.16	10,195.65
	0.18	10,113.94
	0.20	10,032.22
	0.22	9950.50
	0.24	9868.78
H	4	7949.86
	4.5	9074.81
	5	10,032.22
	5.5	10,824.13
	6	11,452.02

(continued)

Table 3.2 (continued)

Inventory parameters	Values	Total profit ($)
k	0.064	10,492.23
	0.072	10,259.11
	0.080	10,032.22
	0.088	9811.39
	0.096	9596.40
m	0.4	9767.65
	0.45	9905.47
	0.5	10,032.22
	0.55	10,149.18
	0.6	10,257.45
M	0.12	9901.05
	0.135	9966.99
	0.150	10,032.22
	0.165	10,096.74
	0.18	10,160.58
p	8	1791.96
	9	5912.90
	10	10,032.22
	11	14,149.90
	12	18,265.96

3.5 Conclusion

This chapter establishes an inventory policy of the item with expiry date and permissible delay in payment option. Supplier offers order quantity dependent credit limit to retailer and retailer offers unconditional credit limit to the customers. Moreover, effect of inflation together with time value of money is discussed. We have proposed the solution algorithm of the inventory model to maximize retailer's total profit by evaluating optimum number of replenishment in a finite planning horizon, optimal cycle time, and optimal order quantity for all possible cases of delay periods. In future, it is expected to spread out the offered model by considering shortages, partial backlogging, full backlogging, stochastic demand, and preservation technology investment.

References

1. Goyal SK (1985) Economic order quantity under conditions of permissible delay in payments. J Oper Res Soc 36(4):335–338
2. Aggarwal SP, Jaggi CK (1995) Ordering policies of deteriorating items under permissible delay in payments. J Oper Res Soc 46(5):658–662
3. Huang YF (2003) Optimal retailers ordering policies in EOQ model under trade credit financing. J Oper Res Soc 54(9):1011–1015
4. Soni H, Shah NH, Jaggi CK (2010) Inventory models and trade credit—a review. Control Cybern 39(3):867–882
5. Sana S (2012) An economic order quantity model for nonconforming quality products. Serv Sci 4(4):331–348
6. Ouyang LY, Yang CT, Chan YL, Cárdenas-Barrón LE (2013) A comprehensive extension of the optimal replenishment decisions under two levels of trade credit policy depending on the order quantity. Appl Math Comput 224(1):268–277
7. Taleizadeh AA, Pentico DW, Jabal-ameli MS, Aryanezhad M (2013) An EOQ model with partial delayed payment and partial backordering. Omega 41(2):354–368
8. Pal B, Sana SS, Chaudhuri KS (2014) Three stage trade credit policy in a three-layer supply chain: A production inventory model. Int J Syst Sci 45(9):1844–1868
9. Shah NH, Jani MY, Chaudhari UB (2018) Optimal ordering policy for deteriorating items under down-stream trade credit dependent quadratic demand with full up-stream trade credit and partial down-stream trade credit. Int J Math Oper Res 12(3):378–396
10. Pal S, Goswami A, Chaudhuri KS (1993) A deterministic inventory model for deteriorating items with stock-dependent demand rate. Int J Prod Econ 32(3):291–299
11. Wee HM (1997) A replenishment policy for items with a price dependent demand and a varying rate of deterioration. Prod Plan Control 8(5):494–499
12. Jaggi CK, Goyal SK, Goel SK (2008) Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments. Eur J Oper Res 190(1):130–135
13. Shah NH, Jani MY, Shah DB (2015) Economic order quantity model under trade credit and customer returns for price-sensitive quadratic demand. Rev Investig Oper 36(3):240–248
14. Shah NH, Jani MY, Chaudhari UB (2017) Retailer's optimal policies for price-credit dependent trapezoidal demand under two-level trade credit. Int J Oper Quant Manag 23(2):115–130
15. Ghare PM, Scharender GH (1963) A model for exponentially decaying inventory system. J Ind Engineering 14(5):238–243
16. Raafat F (1991) Survey of literature on continuously deteriorating inventory models. J Oper Res Society 42(1):27–37
17. Sett BK, Sarkar B, Goswami A (2012) A two-warehouse inventory model with increasing demand and time varying deterioration. Sci Iranica 19(6):1969–1977
18. Sarkar B, Saren S, Cárdenas-Barrón LE (2015) An inventory model with trade-credit policy and variable deterioration for fixed lifetime products. Ann Oper Res 229(1):677–702
19. Shah NH, Jani MY, Chaudhari UB (2017b) Optimal replenishment time for retailer under partial upstream prepayment and partial downstream overdue payment for quadratic demand. Math Comput Model Dyn Syst. https://doi.org/10.1080/13873954.2017.1324882
20. Ray J, Chaudhuri KS (1997) An EOQ model with stock-dependent demand, shortage, inflation and time discounting. Int J Prod Econ 53(2):171–180
21. Chang CT, Wu SJ, Chen LC (2009) Optimal payment time with deteriorating items under inflation and permissible delay in payment. Int J Syst Sci 40(10):985–993
22. Sarkar B (2012) An EOQ model with delay in payments and time varying deterioration rate. Math Comput Model 55(3–4):367–377
23. Ghoreishi M, Mirzazadeh A, Weber GW, Turkey A, Nakhai-Kamalabadi I (2015) Joint pricing and replenishment decisions for non-instantaneous deteriorating items with partial backlogging, inflation and selling price-dependent demand and customer returns. J Ind Manag Optim 11(3):933–949

24. Lashgari M, Taleizadeh AA, Sana SS (2016) An inventory control problem for deteriorating items with back-ordering and financial considerations under two levels of trade credit linked to order quantity. J Ind Manag Optim 12(3):1091–1119
25. Shah NH, Jani MY, Chaudhari UB (2017) Study of imperfect manufacturing system with preservation technology investment under inflationary environment for quadratic demand: A reverse logistic approach. J Adv Manuf Syst 16(1):1–18

Chapter 4
Inventory Modelling of Deteriorating Item and Preservation Technology with Advance Payment Scheme Under Quadratic Demand

Urmila Chaudhari, Nita H. Shah and Mrudul Y. Jani

Abstract This chapter comprises a single retailer and single product which deteriorates continuously. For the time-dependent deteriorating item with seasonal demand, quadratic demand is debated here which is suitable for the items whose demand with starting of the season increases initially and after end of the season, it starts to decrease. To reduce deterioration of the product, retailer needs preservation technology and due to preservation technology retailer minimizes total cost. In this chapter, the retailer has to pay a fraction of the purchase cost before the time of delivery and rest of the payment must be paid at the time of delivery. In this chapter, the optimal number of equal instalments before receiving the order quantity, replenishment time and investment of preservation technology are the decision variables that minimize the total cost. This chapter is an extension of the earlier work, as it provides the best optimal rather than the nearest minimum solution. A numerical example is delivered to demonstrate the performance of the model and to highlight certain decision-making insights.

Keywords Quadratic demand · Maximum lifetime deterioration · Preservation technology · Equal number of instalment of advance payment

---

U. Chaudhari (✉)
Government Polytechnic Dahod, Dahod 389151, Gujarat, India
e-mail: chaudhariurmi04@gmail.com

N. H. Shah
Department of Mathematics, Gujarat University, Ahmedabad 380009, Gujarat, India
e-mail: nitahshah@gmail.com

M. Y. Jani
Department of Applied Sciences and Humanities, Faculty of Engineering & Technology, PIET, Parul University, Vadodara 391760, Gujarat, India
e-mail: janimrudul07@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_4

4.1 Introduction

It is a corporate exercise by the vendor to request a retailer to advance credits for purchasing periodic and instantaneous perishable items. By doing so, a retailer may get a price reduction or an on-time distribution in return. In the current study, Taleizadeh [1] studied a model for deteriorating product for an economic order quantity (EOQ). Simultaneously, Taleizadeh [2] elaborated partial backlogging, where he obtained a near-best minimum solution using a truncated Taylor series expansion. In his circumstance, the deterioration rate reaches to zero but never extends to null. Ishii et al. [3] developed joint policies for three players, viz., vendor, wholesaler and retailer. Haq et al. [4] established the concept of joint inventory system with one vendor, several wholesalers and several retailers. Goyal and Nebebe [5] determined a model for deteriorating item under shipment policy and production for a two-layered supply chain. Woo et al. [6] studied a coordinated strategies for a manufacturer and multi-retailers. Authors measured manufacturer to be produced. Rau et al. [7] minimized an objective function of the supply chain under deteriorating items with three-echelon system. Shah et al. [8] discussed a coordinated decision when demand is quadratic. Shah et al. [9–11] developed optimal payment policies, pricing and shipment for a two-echelon system for deteriorating items under buoyant demand and up- and downstream trade credit. Shah and Shukla [12] analysed a two-layered inventory model for optimum pricing strategies and ordering under the retailer partial trade credit when demand is declining.

Due to extreme changes in the environment, most of the items loss their efficiency over time, termed as deterioration. Deterioration of goods like fruits (mango, i.e. seasonal famous fruit of India) and root vegetables. Out of many studies on deterioration product, only rare of them have considered fixed lifetime issue of deteriorating items. Ghare and Schrader [13] analysed inventory model with deterioration. Raafat [14], Shah and Shah [15], Goyal and Giri [16], and Bakker et al. [17] analysed the research articles on deteriorating items for inventory system. Chung and Cardenas-Barrón [18] developed an algorithm of stock-dependent demand and two-level trade credit in a supply chain comprising of three players for deteriorating items. Furthermore, Shah and Barrón [19] determined the retailer's verdict for credit policies and ordering for deteriorating items when a supplier offers order-linked credit period or cash discount.

On the other hand, to reduce the deterioration, Hsu et al. [20] studied an inventory model to minimize the deterioration rate of inventory for constant demand with preservation technology investment. Dye and Hsieh [21] evaluated a model for optimal cycle time with effective investment in preservation technology for deteriorating items. Hsieh and Dye [22] analysed when demand is fluctuating with time, a production inventory model incorporating the effect of protection technology investment. Recently, Shah and Shah [23] evaluated an inventory model for demand depending on price and time under inflation to optimal cycle time and preservation technology asset for deteriorating items. Later on Shah, et al. [24] developed an integrated inventory model for deteriorating item which depends on time under price and time-sensitive demand. Moreover, Shah et al. [25] established optimal policies under selling price and trade credit-dependent quadratic demand in a supply chain for time-varying deteriorating item with preservation technology.

This chapter develops an inventory model for a retailer to minimize his total cost. It is assumed that the demand rate decreases quadratically. To reduce deterioration of the product, retailer needs preservation technology and due to preservation technology retailer minimizes his total cost. In this chapter, the retailer has to pay a fraction of the purchase cost before the time of delivery and rest of the payment must pay at the time of delivery. Under above assumptions, the objective is to minimize the cost of retailer with respect to the optimal number of equal instalment before receiving the order quantity replenishment time and preservation technology investment.

The remaining of the chapter is systematized as follows. Section 4.2 contains the notations and the assumptions part that are used in model. Section 4.3 is about formulation of the proposed mathematical model. Section 4.4 validates the derived inventory model with numerical instances and its sensitivity analysis. This section also provides some managerial insights. Finally, Sect. 4.5 provides a conclusion and future research directions.

4.2 Notation and Assumptions

Following are the notation and assumption that are used in mathematical inventory model.

4.2.1 Notation

Retailer's parameters	
a	Total scale demand of the product, $a > 0$
b	Linear rate of change of demand of the product, $0 \leq b < 1$
c	Quadratic rate of change of demand of the product, $0 \leq c < 1$
A_r	Ordering cost per order incurred by the retailer ($/order)
C_r	Purchasing cost per unit item (in $)
T_t	The length of time during which the prepayments are paid, $T_t > 0$(in year)
r	The interest rate of capital cost per dollar per year, $0 \leq r \leq 1$
α	The fraction of the purchase cost to be prepaid before the time of delivery, $0 \leq \alpha \leq 1$
CC	Capital total cost
$\theta(t)$	Deterioration rate; $0 \leq \theta(t) \leq 1$
m	Fixed lifetime of the product (in years)
n	The number of equal prepayments before receiving the order quantity (decision variable)

(continued)

(continued)

Retailer's parameters	
u	Investment of preservation technology investment per unit time (in \$)(decision variable)
$f(u)$	$= 1 - \frac{1}{1+\mu u}$ amount of reduced deterioration item (in year), $\mu > 0$
$I_r(t)$	Inventory level for the retailer of item at any time t(units)
T	Cycle time (unit time) of the retailer (decision variable)
Q	Retailer's order quantity at time t
h_r	Holding cost rate for retailer per unit per annum for the product
$HC_r(t)$	Time-dependent holding cost of retailer for item (\$/unit/unit time)

Relations between parameters:

- $T \leq m$
- $0 \leq \theta(t) < 1$

Parameters of retailer:

$R(t)$	Time-dependent quadratic demand rate; $R(t) = a \cdot \left(1 + bt - ct^2\right)$, where $a > 0$ is scale demand, $0 < b, c < 1$ are rates of change of demand, respectively.
$TC_r(n, T, u)$	Total cost of the retailer per unit time (\$/unit/unit time)

Minimization of the problem for the retailer is expressed as follows:

$$\min\ TC_r(n, T, u)$$

Subject to constraints

$$T \leq m$$

4.2.2 Assumptions

1. The inventory system involves single retailer and single non-instantaneous deteriorating item.
2. The demand rate, $R(t) = a \cdot \left(1 + bt - ct^2\right)$ (say), is function of time, $a > 0$ is total scale demand, $0 \leq b < 1$ denotes the linear rate of change of demand with respect to time and $0 \leq c < 1$ denotes the quadratic rate of change of demand.
3. Time prospect is infinite.
4. Shortages are not permitted.
5. Lead time is zero or negligible.

6. The instantaneous rate of deterioration is $\theta(t) = \frac{1}{1+m-t}$, $0 \leq t \leq T \leq m$; for any finite value of m, we have $\theta(t) < 1$. If $m \to \infty$, then $\theta(t) \to 0$, i.e. the item is non-deteriorating.
7. For extremely periodic items or perishable product, the vendor frequently demands α fractions of purchasing cost PC_r to be prepaid (i.e.$\alpha \cdot PC_r$) before the time of delivery. Then the outstanding purchase cost $(1-\alpha) \cdot PC_r$ is paid at the point of delivery. Notice that if $\alpha = 0$ then the vendor does not request prepayment. Instead of if $\alpha = 1$, then the vendor requests the retailer to prepay the entire purchase cost.
8. The vendor settles to prepay $\alpha \cdot PC_r$ by n equal payments in T_t years previous to the time of delivery and pay the rest of $(1-\alpha) \cdot PC_r$ at the time of receiving.

In the next section, the proposed inventory model for the retailer is developed.

4.3 Mathematical Model

The proposed models for deteriorating items with equally multiple instalments of α fractions of the purchase cost and remaining instalment would be paid at the time of purchasing the product.

In general, the retailer pays the vendor α portions of the purchasing cost by n equal instalments in T_t years earlier to the time of purchasing. The vendor receives the remaining unpaid balance $(1-\alpha) \cdot PC_r$ instantly at the product delivery. Thereafter, the retailer's inventory level is regularly exhausted to zero by the end of the cycle T, due to the mixture of demand and deterioration. Hence, the inventory level at time t as follows:

$\frac{dI_r(t)}{dt} = -R(t) - (1 - f(u))\theta(t)I_r(t)$, $0 \leq t \leq T$ with the boundary condition $I_r(T) = 0$. Solving above differential equations, we get

$$I_r(t) = \frac{1}{\left(6\mu^2 + 7\mu u + 2\right)\mu u}(1+\mu u)\left(\begin{array}{l} (1+m-t)^{1-\frac{1}{\mu u+1}} a \left(\begin{array}{l} -2cm^2\mu^2u^2 - 2cm\mu^2tu^2 \\ -2c\mu^2t^2u^2 + 3bm\mu^2u^2 \\ +3b\mu^2tu^2 - 4cm\mu^2u^2 \\ -2c\mu^2tu^2 + 3b\mu^2u^2 \\ -4cm^2\mu u - 2cm\mu u \\ 2c\mu^2u^2 - c\mu t^2 u \\ +5bm\mu u + 2b\mu tu \\ -8cm\mu u - 2c\mu tu \\ +6\mu^2u^2 + 5b\mu u - 2cm^2 \\ -4c\mu u + 2bm - 4cm \\ +7\mu u + 2b - 2c + 2 \end{array}\right) \\ -(1+m-T)^{1-\frac{1}{\mu u+1}} a \left(\begin{array}{l} -2cm^2\mu^2u^2 - 2cm\mu^2Tu^2 \\ -2c\mu^2T^2u^2 + 3bm\mu^2u^2 \\ +3b\mu^2Tu^2 - 4cm\mu^2u^2 \\ -2c\mu^2Tu^2 + 3b\mu^2u^2 \\ -4cm^2\mu u - 2cm\mu u \\ 2c\mu^2u^2 - c\mu T^2 u \\ +5bm\mu u + 2b\mu Tu \\ -8cm\mu u - 2c\mu Tu \\ +6\mu^2u^2 + 5b\mu u - 2cm^2 \\ -4c\mu u + 2bm - 4cm \\ +7\mu u + 2b - 2c + 2 \end{array}\right) \end{array}\right)$$

Therefore, the order quantities per replenishment cycle are $Q = I_r(0)$.

Now, the total cost per unit time of retailer included the following:

- Ordering cost per unit : $OC_r = A_r$
- Purchase cost per unit : $PC_r = C_r Q$
- Inventory holding cost per unit : $HC_r = h_r \left[\int_0^T I_r(t)\ dt \right]$
- Investment for preservation technology : $PTI = u \cdot T$
- From Taleizadeh (2014), the capital cost per cycle is $CC = \frac{n+1}{2n}(\alpha\, T_t\ r\ )PC_r$

The total appropriate cost of the retailer for the product is

$$TC_r(n, T, u) = \frac{1}{T}(OC_r + PC_r + HC_r + PIT + CC)$$

The total cost function $TC_r(n, T, u)$ is a continuous function of number of instalment n, cycle time T and investment of preservation technology 'u'. We will establish endorsement of the proposed model using numerical example. The minimization of the total cost will be shown graphically for the obtained results.

4.4 Numerical Example and Sensitivity Analysis

4.4.1 Numerical Example

Example: Consider $a = 1000$ units, $b = 0.85$, $c = 0.01$, $A_r = \$100$, $C_r = \$\,40$, $h_r = \$\,10$, $\alpha = 0.4$, $T_t = 0.17$ year, $r = 0.1$, $m = 0.25$ year,$\mu = 1.7$. The values of the decision variables are total idle instalment of the prepayments that are $n = 6$, cycle time of replenishment is $T = 0.066$ years and $u = \$25.19$. This results in **retailer's minimum cost as $ 43,192.90**.

The convexity of the total cost function is obtained by the well-known Hessian matrix. Now, for a fixed value of $n = 6$, Hessian matrix for the above retailer is

$$H(6, T, u) = \begin{pmatrix} \frac{\partial^2 TC_r(n,T,u)}{\partial T^2} & \frac{\partial^2 TC_r(n,T,u)}{\partial T\, \partial u} \\ \frac{\partial^2 TC_r(n,T,u)}{\partial u\, \partial T} & \frac{\partial^2 TC_r(n,T,u)}{\partial u^2} \end{pmatrix}$$

By the above example, we get the Hessian matrix $H(6, T, u)$ at the point $(6, T, u)$

$$H(6, T, u) = \begin{pmatrix} 502336.28 & -18.36 \\ -18.36 & 6.079 \end{pmatrix}$$

As in Barrón and Sana [26], if the eigenvalues of the Hessian matrix at the solution $(6, T, u)$ are all positive, then the total cost $TC_r(6, T, u)$ is minimum at that solution. Here, eigenvalues of above Hessian matrix are $\lambda_1 = 502336.23$ and $\lambda_2 = 6.079$. So,

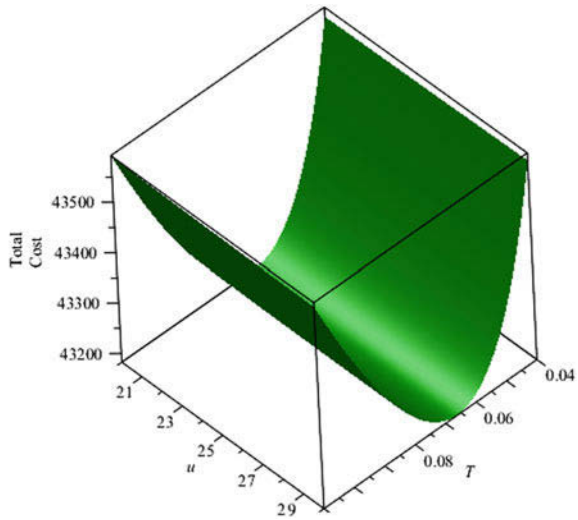


Fig. 4.1 Convexity behaviour of the cost function for $n = 6$

the cost function $TC_r(6, T, u)$ is minimum. Also, the convexity of the cost function is obtained in Fig. 4.1 with respect to cycle time and investment of preservation technology with $n = 6$.

4.4.2 Sensitivity Analysis for the Inventory Parameters

Therefore, for the changed inventory parameters, the sensitivity study of example is agreed out by changing one variable at a time as −20, −10, 10 and 20%.

In imperative to detect the sensitivity of the inventory parameters on the optimal solution, we consider the data as given in numerical example. Optimal solutions for different values of a, b, c,A_r, C,h_r,α, T_t, r, m and μ are presented in Table 4.1. The resulting statement could be made from Table 4.1.

1. In Table 4.1, holding cost rate decreases cycle time slowly. However, ordering cost per order incurred by the retailer increases cycle time rapidly, whereas scale demand, linear rate of change of demand and purchase cost decreases cycle time rapidly. In addition, change in the quadratic rate of change of demand, the fraction of the purchase cost to be prepaid before the time of delivery, a time at which prepayments are paid, the interest rate of capital cost, fixed lifetime of the product and rate of preservation technology cycle time remain constant.
2. From Table 4.1, scale demand, ordering cost per order incurred by the retailer and holding cost rate increase investment of preservation technology slowly, whereas linear rate of change of demand and fixed lifetime of the product decreases investment of preservation technology slowly. However, purchase cost increases investment of preservation technology rapidly although the rate of preservation

Table 4.1 Sensitivity analysis

Parameter	Change %	Values	T (in years)	u in $)	Total cost TC_r(in $)
a	−20	800	0.074	23.86	34,844.84
	−10	900	0.070	24.56	39,023.30
	0	1000	0.066	25.19	43,192.90
	10	1100	0.063	25.79	47,354.97
	20	1200	0.060	26.34	51,510.51
b	−20	0.680	0.072	26.25	42,954.80
	−10	0.785	0.069	25.69	43,076.34
	0	0.850	0.066	25.19	43,192.91
	10	0.935	0.064	24.74	43,305.06
	20	1.020	0.062	24.33	43,413.27
c	−20	0.008	0.066	25.19	43,193.02
	−10	0.009	0.066	25.19	43,192.96
	0	0.010	0.066	25.19	43,192.91
	10	0.011	0.066	25.19	43,192.84
	20	0.012	0.066	25.20	43,192.79
A_r	−20	80	0.059	23.72	42,873.89
	−10	90	0.063	24.49	43,037.82
	0	100	0.066	25.19	43,192.90
	10	110	0.069	25.85	43,340.44
	20	120	0.072	26.47	43,481.44
C	−20	32	0.072	23.51	34,920.69
	−10	36	0.069	24.39	39,059.30
	0	40	0.066	25.19	43,192.90
	10	44	0.064	25.93	47,322.08
	20	48	0.062	26.61	51,447.30
h_r	−20	8	0.068	25.51	43,123.37
	−10	9	0.067	25.35	43,158.35
	0	10	0.066	25.19	43,192.90
	10	11	0.065	25.04	43,227.04
	20	12	0.065	24.90	43,260.77
α	−20	0.32	0.066	25.19	43,160.26
	−10	0.36	0.066	25.19	43,176.58
	0	0.40	0.066	25.19	43,192.90
	10	0.44	0.066	25.20	43,209.23
	20	0.48	0.066	25.20	43,225.55

(continued)

Table 4.1 (continued)

Parameter	Change %	Values	T (in years)	u in \$)	Total cost TC_r(in \$)
T_t	−20	0.136	0.066	25.19	43,160.26
	−10	0.153	0.066	25.19	43,176.58
	0	0.170	0.066	25.19	43,192.90
	10	0.187	0.066	25.20	43,209.23
	20	0.204	0.066	25.20	43,225.55
r	−20	0.08	0.066	25.19	43,160.26
	−10	0.09	0.066	25.19	43,176.58
	0	0.10	0.066	25.19	43,192.90
	10	0.11	0.066	25.20	43,209.23
	20	0.12	0.066	25.20	43,225.55
m	−20	0.200	0.066	25.73	43,193.99
	−10	0.225	0.066	25.46	43,193.44
	0	0.250	0.066	25.19	43,192.91
	10	0.275	0.066	24.94	43,192.39
	20	0.300	0.066	24.69	43,191.89
μ	−20	1.360	0.066	28.08	43,198.84
	−10	1.530	0.066	26.52	43,195.63
	0	1.70	0.066	25.19	43,192.91
	10	1.87	0.066	24.05	43,190.56
	20	2.04	0.066	23.05	43,188.51

technology decreases investment of preservation technology rapidly. Furthermore, change in quadratic rate of change of demand, the fraction of the purchase cost to be prepaid before the time of delivery, a time at which prepayments are paid and the interest rate of capital cost investment of preservation technology remain constant.

3. From Table 4.1, linear rate of change of demand, holding cost rate, ordering cost per order incurred by the retailer, the fraction of the purchase cost to be prepaid before the time of delivery and t a time at which prepayments are paid and the interest rate of capital cost increases total cost slowly, whereas the rate of preservation technology decreases total cost slowly. However, scale demand and purchase cost increase total cost rapidly. In addition, change in quadratic rate of change of demand and fixed lifetime of the product total cost remain constant.

4.5 Conclusion

In this chapter, we consider retailer's model for the instantaneous deteriorating item under replenishment time, the optimal number of equal instalments before receiving the order quantity and preservation technology with quadratic demand. Due to time-dependent deteriorating item, retailer invests money on preservation technology to reduce deterioration. Moreover, the retailer has to pay a fraction of the purchase cost in an equal number of instalment before the time of delivery. The total cost of the retailer with respect to the optimal number of equal instalments before receiving the order quantity, replenishment time and investment of preservation technology is minimized. The decision policies are analysed for the decision-maker. For numerical examples, retailer reaches the minimum cost and carries out sensitivity analysis. This study will extend as the model can be further generalized by taken more items at a time. One can also analyse three-layered supply chain.

References

1. Taleizadeh AA (2014a) An economic order quantity model for deteriorating items in a purchasing system with multiple prepayments, Appl Math Model 38(23):5357–5366
2. Taleizadeh AA (2014b)An EOQ model with partial backordering and advance payments for an evaporating item. Int J Prod Econ 155(1):185–193
3. Ishii K, Takahashi K, Muramatsu R (1988) Integrate production, inventory and distribution systems. Int J Prod Res 26(3):473–482
4. Haq AN, Vrat P, Kanda A (1991) An integrated production-inventory-distribution model for manufacturer of urea: case. Int J Prod Econ 39(17):39–49
5. Goyal SK, Nebebe F (2000) Deterioration of economic production-shipment policy for a single vendor single buyer system. Eur J Oper Res 121(1):175–178
6. Woo YY, Hsu SL, Wu S (2001) An integrated inventory model for single vendor and multiple buyers with ordering cost reduction. Int J Prod Econ 73(1):203–215
7. Rau H, Wu MY, Wee HM (2003) Integrated inventory model for deteriorating items under a multi-echelon supply chain environment. Int J Prod Econ 86(1):155–168
8. Shah Nita H, Jhaveri C, Gor AS (2012) Optimal pricing and ordering policies for an integrated inventory model with quadratic demand when trade credit is linked to order quantity. J Model Manag 7(2):148–165
9. Shah Nita H, Jhaveri C, Gor AS (2009) Optimal pricing, shipment and payment policies for an integrated supplier—buyer inventory model in buoyant market with net credit. Int J Bus Insights Transform 2(2):3–12
10. Shah Nita H, Jhaveri C, Gor AS (2009) Integrated optimal solution for variable deteriorating inventory system of vendor-buyer when demand is quadratic. Can J Pure Appl Sci 3(1):713–717
11. Shah NH, Jhaveri C, Gor AS (2011) Determination of optimal pricing, shipment and payment policies for an integrated supplier-buyer deteriorating inventory model in a buoyant market with two-level trade credit. Int J Oper Res 11(2):119–135
12. Shah Nita H, Shukla KT (2010) Supply chain inventory model for optimal ordering and pricing policies under retailer's partial trade credit scenario in declining market. Dyn Contin, Discret Impuls Syst-Ser B 17(2):309–324
13. Ghare PM, Scharender GH (1963) A model for exponentially decaying inventory system. J Ind Eng 14(5):238–243

14. Raafat F (1991) Survey of literature on continuously deteriorating inventory models. J Oper Res Soc 42(1):27–37
15. Shah Nita H, Shah YK (2000) Literature survey on inventory models for deteriorating items. Econ Ann 44(145):221–237
16. Goyal SK, Giri BC (2001) Recent trends in modeling of deteriorating inventory. Eur J Oper Res 134(1):1–16
17. Bakker M, Riezebos J, Teunter R (2012) Review of inventory system with deterioration since 2001. Eur J Oper Res 221(2):275–284
18. Chung KJ, Cárdenas-Barrón LE (2013) The simplified solution procedure for deteriorating items under stock-dependent demand and two-level trade-credit in the supply chain management. Appl Math Model 37(7):4653–4660
19. Shah Nita H, Cárdenas-Barrón LE (2015) Retailer's decision for ordering and credit policies for deteriorating items when a supplier offers order-linked credit period or cash discount. Appl Math Comput 259(1):569–578
20. Hsu PH, Wee HM, Teng HM (2010) Preservation technology investment for deteriorating inventory. Int J Prod Econ 124(2):388–394
21. Dye CY, Hsieh TP (2012) An optimal replenishment policy for deteriorating items with effective investment in preservation technology. Eur J Oper Res 218(1):106–112
22. Hsieh TP, Dye CY (2013) A production inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time. J Comput Appl Math 239(1):2536
23. Shah NH, Shah AD (2014) Optimal cycle time and preservation technology investment for deteriorating items with price-sensitive stock-dependent demand under inflation. J Phys: Conf Ser 495(2014):12–17
24. Shah NH, Chaudhari UB, Jani MY (2016a) An integrated production-inventory model with preservation technology investment for time-varying deteriorating item under time and price sensitive demand. Int J Inven Res 3(1):81–98
25. Shah NH, Chaudhari UB, Jani MY (2016b) Optimal policies for time-varying deteriorating item with preservation technology under selling price and trade credit dependent quadratic demand in a supply chain. Int J Appl Comput Math. https://doi.org/10.1007/s40819-016-0141-3
26. Cárdenas-Barrón LE, Sana SS (2015) Multi-item EOQ inventory model in a two-layer supply chain while demand varies with a promotional effort. Appl Math Model 39(21):6725–6737

Chapter 5
Dynamic Pricing, Advertisement Investment and Replenishment Model for Deteriorating Items

Chetansinh R. Vaghela and Nita H. Shah

Abstract In practice, it is commonly observed that the quality and price of items are two important factors for customers to choose a product. The profit of a firm is greatly affected by these two factors, especially when their inventory has deteriorating items. Also, it is commonly observed that the product demand increases due to promotional efforts like advertisement through digital media, newspaper, etc. Thus, the spending on commercial promotion is a very crucial decision. This paper considers a replenishment model for perishable items with investment on promotion and retail price-dependent demand with a budget constraint. The deterioration rate is considered constant. An optimization problem is formulated in order to provide a pricing, promotional spending and replenishment policy, which maximize the total profit. Using Pontryagin's maximum principle, the optimal advertisement investment is obtained for a given retail price and cycle time. The closed form of the inventory level is obtained by solving the respective differential equation of inventory. The model is validated by a numerical example with hypothetical parameters in result section. The results show that the model is pretty stable and the concavity is proven graphically. The sensitivity analysis is performed in discussion section. The sensitivity analysis about key inventory parameters reveals some important managerial insights. Also, the future scope is given in conclusion section, which gives a brief idea about possible extensions of this model.

Keywords Inventory · Deteriorating items · Price-dependent demand · Advertisement investment · Budget constraint

C. R. Vaghela (✉)
Department of Mathematics, Marwadi University, Rajkot 360005, Gujarat, India
e-mail: crvaghela16@gmail.com

N. H. Shah
Department of Mathematics, Gujarat University, Ahmedabad 380009, Gujarat, India
e-mail: nitahshah@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_5

5.1 Introduction

In this modern era, technological development has connected people through electronic media, social media, etc. These media can be a good platform for the promotion of products. Promoting a product through electronic media, newspaper, etc. can help in increasing the product demand. So, the amount needs to be invested on advertisement is also one of the important managerial decision. Researchers like Chowdhury et al. [2], Kotler [7], Palanivel and Uthayakumar [15], Shah and Vaghela [29, 30] have used promotional investment in their study.

In today's competitive market, setting an optimal selling price is a very important managerial decision. The higher selling price will lead to reduced customer demand. Mishra et al. [13] used demand sensitive to retail price and stock displayed and gave a simple algorithm to optimize objective function. Lin et al. [8] considers a demand dependent on price with maximum lifetime. The model was developed for non-instantaneous deteriorating items. Shah [19] studied an inventory model with demand sensitive to trade credit and retail price. Researchers like Yang and Wee [34], Shah et al. [24, 25], Yadav et al. [33], Liu et al. [9] and Jaggi et al. [5] have worked with inventory models with price sensitive demand. Some more inspiring work in price-dependent demand includes Modak and Kelle [14], You [35], Shastri et al. [28] and Shah et al. [23].

It is very impractical to assume that the demand for an item remains constant throughout the inventory cycle. Researchers like Hariga [3], Mehta and Shah [11, 12], etc. used the time-dependent linear and exponential demand instead of constant demand. But even time trended linear and exponential demands are not much realistic because these demands lead to either uniform change or exponential rise or fall, respectively. Thus, many researchers like Sarkar et al. [17], Shah and Shah [20], Shah et al. [26, 27], Soni et al. [31] and Tripathy and Mishra [32] used a more realistic time quadratic demand in their research.

Deterioration is one of the important phenomena in the study of inventory modelling. The items in the inventory cannot stay fresh forever. Items like food, cosmetics, radioactive chemicals, etc. have significant decay rates. Thus, deterioration is a very popular parameter among researchers. In the past decades, many researchers like Jaggi and Verma [4], Maihami et al. [10], Shah et al. [22, 24, 25], etc. have incorporated different types of deterioration rate in their inventory models. Raafat [16] tried to provide literature review on deteriorating items. Bakker et al. [1] and Shah and Shah [21] also gave review articles on inventory with perishable items.

Based on the literature survey, our aim is to develop an inventory model, which uses a price and promotional investment dependent demand. The model is applicable for deteriorating items and shortages are not allowed. The managerial issues are worked out with the help of sensitivity analysis. The developed model can be useful for items like mobile phones, iron goods, garments, etc.

The paper contains five sections. Section 5.2 contains assumptions and notation used in the paper. Mathematical model is discussed in Sect. 5.3. In Sect. 5.4, Numerical example is provided. Section 5.5 gives brief conclusion of the findings.

5.2 Notation and Assumptions

5.2.1 Notation

A	Fixed cost per order (in $)
C	Cost of purchasing per unit (in $)
h	Cost of holding an item in inventory ($/unit)
θ	Deterioration rate ($0 < \theta < 1$)
Q	Initial lot size
p	The selling price per unit, where $p > C$
$E(t)$	The effort of sales team at $t \geq 0$
$R(p, E)$	Demand rate ($t \geq 0$) units
k	The coefficient of advertisement investment cost
T	Cycle time (in years)
$I(t)$	The inventory at time t (units)
$\pi(p, T)$	Seller's profit
U	The budget constraint for the promotional investment

5.2.2 Assumptions

1. The items in the inventory deteriorate at a constant rate.
2. No shortages allowed and lead time is zero.
3. The Demand rate of $R(p, t)$ is considered as $R(p, E) = (\alpha + \alpha_1 t - \alpha_2 t^2) - \beta p + \gamma E(t)$, where $\alpha > 0$ denotes the scale demand and $\alpha_1, \alpha_2 > 0$. The parameter $\beta > 0$ denotes the price elasticity.
4. The Effort level $E(t)$ at any time t is modelled as $\frac{dE(t)}{dt} + \rho E(t) = \mathrm{e(t)}, \ \mathrm{E(0)} = \mathrm{E_0}$, where $\rho > 0$ is the decay rate of effort level and denotes the scale demand and $e(t)$ is the non-negative investment rate.
5. The investment cost associated is assumed to be $IC(e(t)) = \frac{1}{2}ke^2(t), \ \mathrm{k} > 0$ [6].

5.3 Mathematical Model

In this section, we present the general formulations and solutions to the inventory model. The inventory is consumed due to demand and deterioration. Thus, the differential equation for the inventory level during the period $0 \le t \le T$ is

$$\frac{dI}{dt} + \theta I(t) = -R(p, E), \quad \text{with } I(T) = 0 \text{ and } I(0) = Q \tag{5.1}$$

The profit function is described as

$$\pi(p, T) = \frac{1}{T}\int_0^T \left(pR(p, E) - hI(t) - \frac{1}{2}ke^2(t)\right)dt - \frac{(CQ + A)}{T} \tag{5.2}$$

To optimize the objective function, we first formulate the following optimization problem:

$$\max_{p,T,e(\cdot)} \pi = \frac{1}{T}\int_0^T \left(pR(p, E) - hI(t) - \frac{1}{2}ke^2(t)\right)dt - \frac{(CQ + A)}{T}$$

Subject to

$$\begin{gathered} \frac{dI}{dt} + \theta I(t) = -R(p, E),\ I(0) = Q,\ \ I(T) = 0 \\ \frac{dE(t)}{dt} + \rho E(t) = \mathrm{e(t)},\quad e(t) \ge 0,\ \ \mathrm{E(0) = E_0} \end{gathered} \tag{5.3}$$

Here, one can observe that the replenishment quantity Q is also one of the decision variables. However, Q can be determined by setting $I(0) = Q$, once we get the inventory level $I(t)$. Thus, Q is not the explicit decision variable.

In order to solve the optimization problem (5.3), first, we solve following optimal control problem using Pontryagin's maximum principle:

$$\max_{e(\cdot)} J = \frac{1}{T}\int_0^T \left(pR(p, E) - hI(t) - \frac{1}{2}ke^2(t)\right)dt$$

Subject to

$$\begin{gathered} \frac{dI}{dt} + \theta I(t) = -R(p, E),\ I(0) = Q,\ \ I(T) = 0 \\ \frac{dE(t)}{dt} + \rho E(t) = \mathrm{e(t)},\ \ 0 \le \mathrm{e}(t) \le \sqrt{\frac{2U}{k}},\ \ E(0) = E_0, \end{gathered} \tag{5.4}$$

To find the optimal advertisement investment, we define the Hamiltonian function as [18]

$$H(e, I, E, \lambda_1, \lambda_2, t) = pR(p, E) - hI(t) - \frac{1}{2}ke^2(t) \\ + \lambda_1(-R(p, E) - \theta I(t)) + \lambda_2(-\rho E(t) + e(t)) \quad (5.5)$$

where λ_1 and λ_2 are the adjoint variables satisfying the equations

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial I} = h + \lambda_1\theta, \quad \lambda_1(0) = 0 \quad (5.6)$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial E} = -\gamma p + \gamma\lambda_1 + \rho\lambda_2, \quad \lambda_2(T) = 0 \quad (5.7)$$

By solving Eqs. (5.6) and (5.7), we have

$$\lambda_1(t) = \frac{h\left(e^{\theta t} - 1\right)}{\theta} \quad (5.8)$$

$$\lambda_2(t) = d_1 + d_2e^{\rho t} + d_3e^{\theta t}$$

where

$$d_1 = \frac{\gamma}{\rho}\left(p + \frac{h}{\theta}\right), \quad d_2 = \frac{\gamma h}{\theta(\theta - \rho)}e^{(\theta-\rho)T} - d_1e^{-\rho T} \text{ and } d_3 = \frac{\gamma h}{\theta(\theta - \rho)} \quad (5.9)$$

As the Hamiltonian function H is concave, the optimal control $e\,(t)$ maximizes the Lagrangian function at all points. Thus, the investment rate $e\,(t)$ is obtained as

$$\mathrm{e}^*(t) = \begin{cases} \sqrt{\frac{2U}{k}}, & 0 \le t \le t_1 \\ \frac{1}{k}\left(d_1 + d_2e^{\rho t} + d_3e^{\theta t}\right), & t_1 < t \le t_2 \\ 0, & t_2 < t \le T \end{cases} \quad (5.10)$$

where t_1 and t_2 satisfies the condition $\lambda_2(t_1) = \sqrt{2kU}$ and $\lambda_2(t_2) = 0$, respectively. Using the optimal investment rate in the equation $\frac{dE(t)}{dt} + \rho E(t) = \mathrm{e(t)}, \quad \mathrm{E(0) = E_0}$, we get

$$E(t) = \begin{cases} \frac{\omega}{\rho} + \mathrm{e}^{-\rho t}\left(E_0 - \frac{\omega}{\rho}\right), & 0 \le t \le t_1 \\ L_1 + L_2\,\mathrm{e}^{\rho(t_1-t)} + L_3\,\mathrm{e}^{-\rho t}\left(\mathrm{e}^{2\rho t} - \mathrm{e}^{2\rho t_1}\right) \\ \qquad + L_4\,\mathrm{e}^{-\rho t}\left(\mathrm{e}^{(\rho+\theta)t} - \mathrm{e}^{(\rho+\theta)t_1}\right), & t_1 < t \le t_2 \\ \mathrm{E(t_2)e}^{-\rho(t-t_2)}, & t_2 < t \le T \end{cases} \quad (5.11)$$

where $\omega = \sqrt{\frac{2U}{k}}$, $L_1 = \frac{d_1}{k\rho}$, $L_2 = E(t_1) - \frac{d_1}{\rho}$, $L_3 = \frac{d_2}{2k\rho}$ and $L_4 = \frac{d_3}{k(\rho+\theta)}$.

The inventory level $I(t)$ is obtained as

$$I(t) = \begin{cases} I_1(t),\ 0 \le t \le t_1 \\ I_2(t),\ t_1 < t \le t_2 \\ I_3(t),\ t_2 < t \le T \end{cases} \tag{5.12}$$

(i) For $0 \le t \le t_1$, the inventory level is

$$I_1(t) = \left(\frac{\alpha_2 t^2}{\theta} - \frac{\alpha_1 t}{\theta} - \frac{2\alpha_2 t}{\theta^2}\right) - K_1\left(e^{-t\rho} - e^{-t\theta}\right) + K_3\left(1 - e^{-t\theta}\right) + Qe^{-t\theta} \tag{5.13}$$

where $K_1 = \frac{\gamma}{\theta-\rho}\left(E_0 - \frac{\omega}{\rho}\right)$, $K_2 = \frac{2\alpha_2}{\theta^3} + \frac{\alpha_1}{\theta^2} - \frac{\alpha}{\theta} + \frac{\beta p}{\theta}$, $K_3 = K_2 - \frac{\omega\gamma}{\rho\theta}$ and $K_4 = \left(\frac{\alpha_2 t_1^2}{\theta} - \frac{\alpha_1 t_1}{\theta} - \frac{2\alpha_2 t_1}{\theta^2}\right)$

(ii) For $t_1 < t \le t_2$, the inventory level is

$$\begin{aligned} I_2(t) = {} & \left(\frac{\alpha_2 t^2}{\theta} - \frac{\alpha_1 t}{\theta} - \frac{2\alpha_2 t}{\theta^2}\right) + K_2 - \frac{\gamma L_1}{\theta} \\ & + \frac{\gamma L_4}{\theta-\rho} e^{\theta t_1 + \rho(t_1 - t)} + \frac{\gamma L_3}{\theta-\rho} e^{\rho(2t_1 - t)} - \frac{\gamma L_2}{\theta-\rho} e^{\rho(t_1 - t)} \\ & - \frac{\gamma L_3}{\theta+\rho} e^{\rho t} - \frac{\gamma L_4}{2\theta} e^{\theta t} + N_2 e^{-\theta t} \end{aligned} \tag{5.14}$$

where $N_1 = K_4 + K_2 - \frac{\gamma L_1}{\theta} - \frac{\gamma L_2}{\theta-\rho} + \frac{2\gamma\rho L_3 e^{\rho t_1}}{\theta^2-\rho^2} + \frac{\gamma L_4(\theta+\rho)e^{\theta t_1}}{2\theta(\theta-\rho)}$ and $N_2 = e^{\theta t_1}(I(t_1) - N_1)$

(iii) For $t_2 < t \le T$, the inventory level is

$$I_3(t) = \left(\frac{\alpha_2 t^2}{\theta} - \frac{\alpha_1 t}{\theta} - \frac{2\alpha_2 t}{\theta^2}\right) + K_2 - \frac{E(t_2)\gamma e^{\rho(t_2-2)}}{\theta-\rho} + N_3 e^{-\theta t} \tag{5.15}$$

where $K_5 = \left(\frac{\alpha_2 t_2^2}{\theta} - \frac{\alpha_1 t_2}{\theta} - \frac{2\alpha_2 t_2}{\theta^2}\right)$ and $N_3 = e^{\theta t_2}\left(I(t_2) + \frac{\gamma E(t_2)}{\theta-\rho} - K_5 - K_2\right)$

The replenishment quantity Q is obtained as

$$\begin{aligned} Q = I(0) = {} & -e^{\theta t_1}(M_4 + M_6) - e^{\theta t_1}(M_5 + M_7) - \frac{\gamma M_3 e^{(\theta-\rho)T}}{\theta-\rho} \\ & - e^{T\theta}\left(\frac{\alpha_2 T^2}{\theta} - \frac{2\alpha_2 T}{\theta^2} - \frac{\alpha_1 T}{\theta}\right) - K_1 + K_3 - e^{T\theta}K_4 \end{aligned} \tag{5.16}$$

where

$$M_1 = \frac{\omega}{\rho} + e^{-t_1\rho}\left(E_0 - \frac{\omega}{\rho}\right),$$

$$M_2 = e^{\rho t_1}\left(M_1 - L_1 - L_3 e^{\rho t_1} - L_4 e^{\theta t_1}\right)$$

$$M_3 = M_2 + e^{\rho t_2}\left(L_1 + L_3 e^{\rho t_2} + L_4 e^{\theta t_2}\right)$$

$$M_4 = \frac{\gamma e^{-\rho t_2}}{\theta - \rho}(M_3 - M_2) - \frac{\gamma}{\theta}(L_1 + L_4) - \frac{L_3 e^{\rho t_2}}{\theta + \rho} - L_4 \gamma t_2$$

$$M_5 = \frac{\gamma}{\theta}\left(L_1 + L_4 - \frac{\omega}{\rho}\right) + \frac{M_2 \gamma e^{-\rho t_1}}{\theta - \rho} + \frac{L_3 \gamma e^{\rho t_1}}{\theta + \rho}$$

$$M_6 = \frac{L_4 \gamma}{\theta}$$

$$M_7 = -e^{-\rho t_1} K_1 + L_4 \gamma t_1 - M_6$$

Using the classical optimization, we calculate maximum profit for the numerical example provided in the following section.

5.4 Numerical Example and Sensitivity Analysis

Example 1

$$\alpha = 240,\ \ \alpha_1 = 25,\ \ \alpha_2 = 10,\ \ \beta = 10,\ \ \gamma = 0.8,\ \ C = \$5\,,\ \ h = \$0.4,$$

$$k = 1,\ \ \rho = 0.1,\ \ A = \$50,\ \ U = \$80,\ \ \theta = 12\%.$$

Using Maple 18 software, the optimal values of decision variables are obtained as $(p^*,\ T^*) = (15.55,\ 1.32)$. The optimum replenishment quantity is obtained as $Q^* = 143.59$ units. The value of $t_1 = 0.234$ and $t_2 = 1.328 = T$. The maximum profit gained is $\pi_{\max} = \$904.69$. Figures 5.1, 5.2 and 5.3 prove the concave nature of the profit function.

Next, we perform sensitivity analysis for the data used in example 5.1. The sensitivity graphs are shown in the following Figs. 5.4, 5.5 and 5.6.

From Fig. 5.4, we can observe that

- Scale demand α increases selling price p significantly.
- A heavy decrease is observed in p when price elasticity β increases. It suggests that when the demand is highly elastic, it is not advisable to set a high selling price.
- Other parameters don't have much impact on selling price.
  From Fig. 5.5, we can observe that
- By increasing values of γ, a significant drop is observed in cycle time. This finding implies that with more efforts on advertisement, the demand gets a boost and the inventory vanishes faster.

Fig. 5.1 Concave nature of profit function with respect to p and T

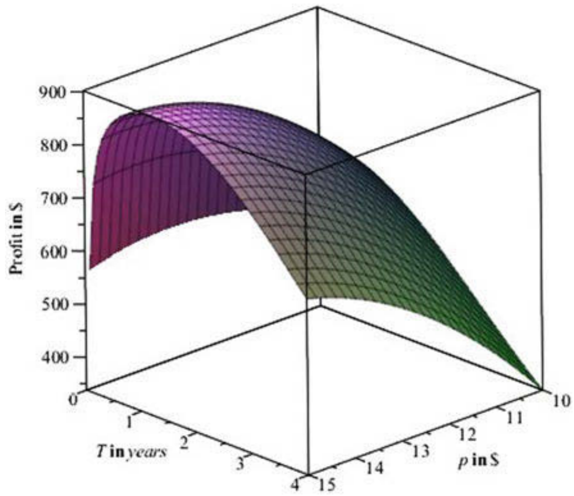
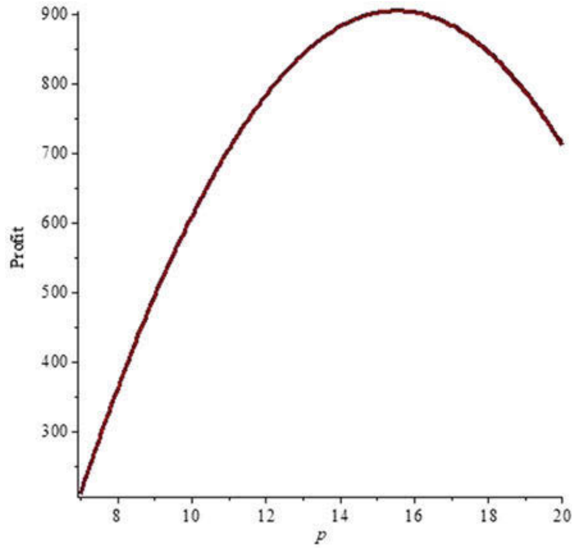


Fig. 5.2 Concavity with respect to p



- By increasing values of ρ, a significant rise is observed in cycle time. An increasing scale demand and price elasticity will lead to a significant decrease in cycle time.
- Other parameters have moderate effect on cycle time.
  From Fig. 5.6, we can observe that
- Scale demand α has big positive impact on retailer's total profit while price elasticity β and purchase cost C reduces profit heavily.
- Other parameters have a moderate effect on retailer's total profit.

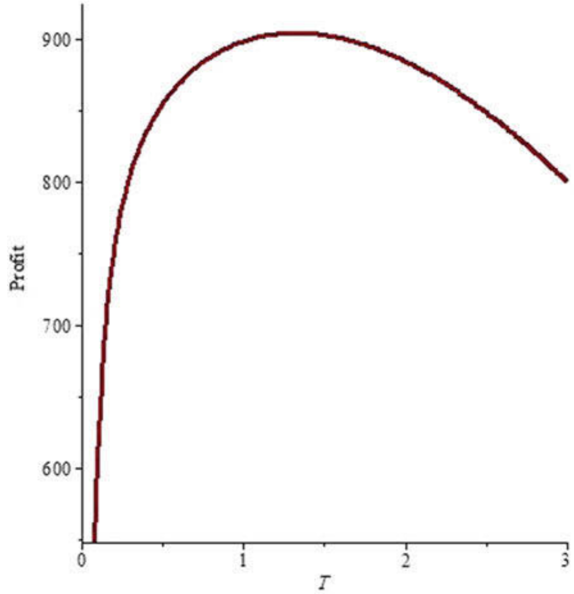


Fig. 5.3 Concavity with respect to T

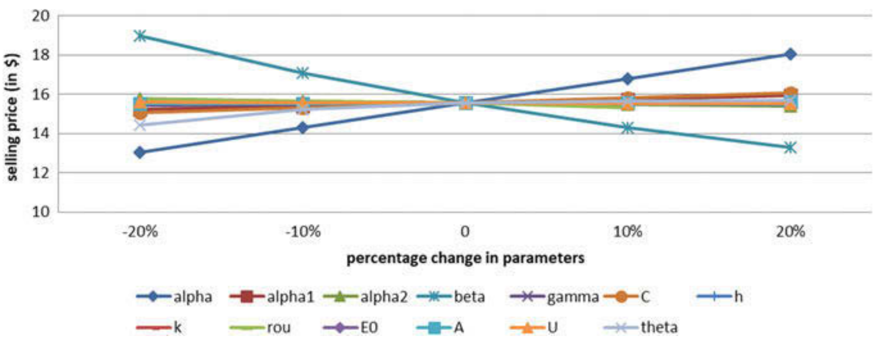


Fig. 5.4 Effects on selling price

As shown in Figs. 5.4, 5.5 and 5.6, associate increasing scale demand can increase the retail price and the total profit significantly. These findings provide insight that when the market potential is high, the player should grab the opportunity of higher investment and earn better profits by setting a higher selling price. Sensitivity analysis additionally reveals that an increasing purchase cost will decrease the total profit. A comparatively high purchase cost suggests that the merchandise is a little costly. In such situation, a player ought to scale back order amount and increase the selling price to cut back the loss. With effective promotional efforts, the demand increases and the inventory depletes faster. In such case, the player should increase order quantity and

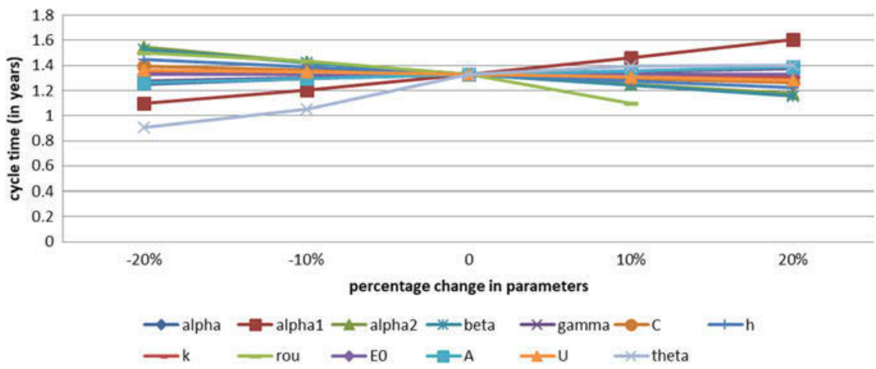


Fig. 5.5 Effects on cycle time

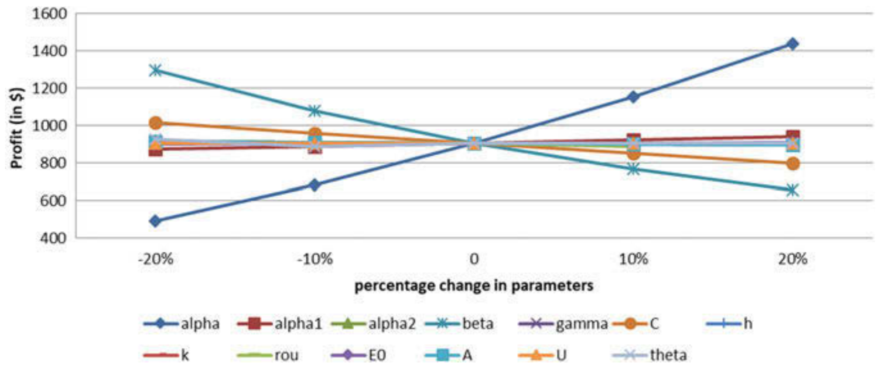


Fig. 5.6 Effects on profit

selling price to generate more profit from the sales. In case of deteriorating items, the player should invest in preservation technology at his facility to cut back the loss due to deterioration.

5.5 Conclusion

In this study, we formulated a replenishment model for a retailer when demand of the product is influenced by both price and advertisement investment. The objective function is maximized with respect to decision variables. The optimal retail price, cycle time and ordering quantity is determined using classical optimization. Finally, the model is validated by hypothetical parameter values and some useful managerial observations are derived.

For future research, the model can be extended for the study of multiple products. The competition between two firms with respect to pricing and promotional investment policy can be an interesting study. One can also use dynamic preservation technology investment for the extension.

Acknowledgements The authors thank the editor and reviewers for their constructive comments. The authors thank DST-FIST 2014, File No: MSI-097 for technical assistance for the research facilities.

References

1. Bakker M, Riezebos J, Teunter RH (2012) Review of inventory systems with deterioration since 2001. Eur J Oper Res 221(2):275–284
2. Chowdhury RR, Ghosh SK, Chaudhuri KS (2013) An inventory model for perishable items with stock and advertisement sensitive demand. Int J Manag Sci Eng Manag 9(3):169–177
3. Hariga M (1995) An EOQ model for deteriorating items with shortages and time-varying demand. J Oper Res Soc 46(4):398–404
4. Jaggi CK, Verma P (2010) An optimal replenishment policy for non-instantaneous deteriorating items with two storage facilities. Int J Serv Oper Inf 5(9):209–230
5. Jaggi CK, Tiwari S, Goel SK (2017) Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand and two storage facilities. Ann Oper Res 248(1–2):253–280
6. Karray S (2015) Cooperative promotions in the distribution channel. Omega 51:49–58
7. Kotler P (2002) Marketing management (2nd ed., The Millennium Edition). New Delhi: Prentice-Hall of India
8. Lin F, Yang ZC, Jia T (2016) Optimal pricing and ordering policies for non instantaneous deteriorating items with price dependent demand and maximum lifetime. In: Proceedings of the 6th international Asia conference on industrial engineering and management innovation, pp 411–421, Atlantis Press
9. Liu G, Zhang J, Tang W (2015) Joint dynamic pricing and investment strategy for perishable foods with price-quality dependent demand. Ann Oper Res 226(1):397–416
10. Maihami R, Karimi B, Ghomi SMTF (2016) Effect of two-echelon trade credit on pricing-inventory policy of non-instantaneous deteriorating products with probabilistic demand and deterioration functions. Ann Oper Res 257(1–2):237–273
11. Mehta NJ, Shah NH (2003) An inventory model for deteriorating items with exponentially increasing demand shortages under inflation and time discounting. Investig Oper 23(1):103–111
12. Mehta NJ, Shah NH (2004) An inventory model for deteriorating items with exponentially decreasing demand shortages under inflation and time discounting. Ind Eng J 33(4):19–23
13. Mishra U, Cárdenas-Barrón LE, Tiwari S, Shaikh AA, Treviño-Garza G (2017) An inventory model under price and stock dependent demand for controllable deterioration rate with shortages and preservation technology investment. Ann Oper Res 254(1–2):165–190
14. Modak NM, Kelle P (2019) Managing a dual-channel supply chain under price and delivery-time dependent stochastic demand. Eur J Oper Res 272(1):147–161
15. Palanivel M, Uthayakumar R (2015) Finite horizon EOQ model for non-instantaneous deteriorating items with price and advertisement dependent demand and partial backlogging under inflation. Int J Syst Sci 46(10):1762–1773
16. Raafat F (1991) Survey of literature on continuously deteriorating inventory models. J Oper Res Soc 42(1):27–37
17. Sarkar B (2012) An inventory model with reliability in an imperfect production process. Appl Math Comput 218(9):4881–4891
18. Sethi SP, Thompson GL (2000) Optimal control theory: applications to management science, 2nd edn. Kluwer Academic Publishers, Dordrecht, The Netherlands

19. Shah NH (2015) Manufacturer-retailer inventory model for deteriorating items with price-sensitive credit-linked demand under two-level trade credit financing and profit sharing contract. Cogent Eng 2(1):1012989
20. Shah NH, Shah BJ (2014) EPQ model for time-declining demand with imperfect production process under inflationary conditions and reliability. Int J Oper Res 11(3):91–99
21. Shah NH, Shah YK (2000) Literature survey on inventory models for deteriorating items. Econ Ann 44(1):221–237
22. Shah NH, Chaudhari U, Jani MY (2017) Optimal policies for time-varying deteriorating item with preservation technology under selling price and trade credit dependent quadratic demand in a supply chain. Int J Appl Comput Math 3(2):363–379
23. Shah NH, Patel DG, Shah DB (2016) EPQ model for returned/reworked inventories during imperfect production process under price-sensitive stock-dependent demand. Oper Res 1–17
24. Shah NH, Shah DB, Patel DG (2013) Optimal retail price, replenishment time and payment scenario under biddable two-part trade credit for price-sensitive trapezoidal demand. Dyn Contin, Discret Impuls Syst Ser B: Appl Algorithms 20(6b):647–673
25. Shah NH, Soni HN, Patel KA (2013) Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. Omega 41(2):421–430
26. Shah Nita H, Gor AS, Jhaveri C (2008) An integrated vendor-buyer strategy when demand is quadratic. Math Today 24(1):31–38
27. Shah Nita H, Gor AS, Jhaveri C (2009) Integrated optimal solution for variable deteriorating inventory system of vendor-buyer when demand is quadratic. Can J Pure Appl Sci 3(1):713–717
28. Shastri A, Singh SR, Yadav D, Gupta S (2014) Supply chain management for two-level trade credit financing with selling price dependent demand under the effect of preservation technology. Int J Procure Manag 7(6):695–718
29. Shah NH, Vaghela CR (2017) Economic order quantity for deteriorating items under inflation with time and advertisement dependent demand. OPSEARCH 54(1):168–180
30. Shah NH, Vaghela CR (2018) Imperfect production inventory model for time and effort dependent demand under inflation and maximum reliability. Int J Syst Sci: Oper & Logist 5(1):60–68
31. Soni H, Gor A, Patel H (2018) Vendor managed inventory model for non-instantaneous deteriorating product with quadratic demand allowing partial backlogging. Uncertain Supply Chain Manag 6(3):321–334
32. Tripathy CK, Mishra U (2010) Ordering policy for Weibull deteriorating items for quadratic demand with permissible delay in payments. Appl Math Sci 4(44):2181–2191
33. Yadav D, Singh SR, Meenu (2015) EOQ model with partial backordering for imperfect items under the effect of inflation and learning with selling price dependent demand. Int J Comput Appl 111(17):25–31
34. Yang PC, Wee HM (2006) A collaborative inventory system with permissible delay in payment for deteriorating items. Math Comput Model 43(3):209–221
35. You SP (2005) Inventory policy for products with price and time-dependent demands. J Oper Res Soc 56(7):870–873

Chapter 6
A Production Reliable Model for Imperfect Items with Random Machine Breakdown Under Learning and Forgetting

Preeti Jawla and S. R. Singh

Abstract This chapter considers the impact of preservation technology on an "economic production quantity" model in which the production process may not only shift from an "in-control" state to an "out-of-control" state but also may fail at any random point in time during production run time. Model is developed for multi-items with imperfect quality by considering the situation of random machine failure over infinite planning horizon. Demand rate is assumed to be multivariate. A reliable and flexible production inventory system is considered under learning and forgetting environment. We studied model in both crisp and fuzzy environment, and significant features of the model are illustrated by numerical experiments. So, numerical examples along with sensitivity analysis are given to show how the solution procedure works as well as the usages of research results.

Keywords Multi-item · Multivariate demand · Imperfect production · Preservation · Reliability · Volume flexibility · Learning · Forgetting · Rework · Machine breakdown · Fuzzy

6.1 Introduction

Over the past few epochs of research on Economic Production Quantity (EPQ) models, the heaps of disputes have appeared. The traditional EPQ model is often considered some unrealistic and idealistic assumptions. Thus, the development of the manufacturing inventory models needs a certain amount of relaxation from these types of assumptions to represent the actual realistic scenario to the manufacturing industries. The foremost unrealistic assumption in using the EPQ models is that a machine can work always perfectly but in reality, a production process may not always be perfect but also may face the situation of sudden machine breakdown/failure at

P. Jawla (✉)
Banasthali Vidyapith, Vanasthali, Rajasthan, India
e-mail: p.preeti.jawla@gmail.com

S. R. Singh
CCS University, Meerut, India

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_6

any random point in the duration production run. Sometimes, repair time of machine depends on the type of injury occurred in machine. Rosenblatt and Lee [25] assumed that timing from the beginning of a production run to an uncontrollable process is an exponential distribution and the defective products can be reworked at that instant moment with an extra cost. Groenevelt et al. [11] premeditated two inventory production policies in which stochastic machine breakdowns have been considered. Chung [7] discussed the bound of machine breakdown problem. Singh and Urvashi [28] discussed the effect of machine breakdown with fuzzy demand rate. Das et al. [8] had developed an economic production lot-sizing problem for imperfect items. They had considered the phenomena of random machine failure in their model under fuzzy-stochastic environment. Hsu and Hsu [12] investigated the production process in which the imperfect units are produced during the production time due to machinery fault, labor raw materials, etc. Singh and Prasher [30] developed a production inventory model for random machine breakdown with flexible manufacturing and stochastic repair time. Dey and Giri [9] developed a single vendor and single buyer integrated model with stochastic demand and imperfect production process. Pal et al. [23] developed an optimal policy for the retailer to get perfect items by conducting a screening process where slightly imperfect units are assembled and are repaired by the production firm. Jawla and Singh [17] established an imperfect production inventory model for multiple production setups. Chen [3] investigated the manufacturer-retailer's policy using the case of a two-echelon supply chain for imperfect manufacturing system. Iqbal and Sarkar [14] developed an imperfect production model for deteriorating products where the production rate is probabilistic in nature.

The reliability of the production process is also an important factor of the manufacturing system, i.e., more reliable production system means more perfection in production (fine and good quality items) and fewer imperfects. Cheng [4] established an inventory model in which they have formulated a general relation between production setup cost and reliability. This model is further extended by Cheng [5] in which they considered an EOQ model for imperfect quality items, and the production unit cost is considered as a function of quality of an item. Tripathy et al. [34] explored an inventory model in which they incorporated the effect of reliability, demand, and reliability-dependent unit production cost with excess demands then the supply. Bag et al. [2] developed a reliable economic production inventory model and they investigated the effect of investment in flexibility enhancement on the setup cost. Tripathy and Pattnaik [32] investigated an inventory policy under the effect of reliability. Sarkar et al. [26] established an inventory model for imperfect quality items in which the optimal product reliability and production rates are determined under the effect of reliability. Tripathy and Pattnaik [33] developed an inventory model under the effect of reliability consideration with optimization techniques to find the optimal solution of the problem. Paul et al. [24] established a production inventory model with the consideration of uncertainty and reliability. Mahapatra et al. [22] investigated a partial backorder inventory model in which they considered that demand is time and reliability-dependent demand and items deteriorate with time. Shah and Vaghela [27] discussed an imperfect production inventory model under inflation and maximum reliability in which demand is time- and effort-dependent. This chapter employs mathematical modeling for solving such type of manufacturing run time problem with reliability and flexibility in imperfect production system with stochastic machine breakdown.

In the production system, these two concepts (learning and forgetting) are played a vital role. Through the repetitive process of production process, managers learn to improve the quality of the product, to reduce the cost parameters, etc. These concepts are very important in improving the quality of products and many other parameters for the inventory control system. Learning is the process in which a well-arranged representation of experience is built. On the other hand, rearrangement or removal of well-arranged experience is the process of forgetting. These two processes of learning and forgetting are complementary to each other, i.e., without forgetting we cannot learn, and converse is also true. Wright [35] is one who has taken the first initiative for the concept of learning. He has formulated a relation between variables of learning in quantitative form and explored a learning curve which can elaborate performance of group as well individual, and the group includes direct or indirect labor. Towill [31] investigated the learning curve model and analyzed the level of complexity of learning models. Chiu and Chen [6] explored an optimal algorithm to solve the dynamic lot-sizing problem in which they investigated the effect of learning and forgetting in setups and production. Jaber et al. [16] presented an economic production quantity model for imperfect quality items under the effect of learning. Khan et al. [19] established an EOQ model for imperfect quality items with learning effect being considered for the process of inspection. By applying these learning models, management can enhance the performance of the organization by improving different operations, for example, improvement in utilization of capacity, better control of inventories, and managing balance among the productions and distribution functions across the chain. Konstantaras et al. [20] explored an EOQ model for imperfect quality items. In which they have applied the effect of learning to reduce some fraction of imperfect quality in each delivery over an infinite and finite planning horizon. Singh et al. [29] explored a two warehouse production inventory model for imperfect quality items. The effect of learning production cycle is well explained under the limited storage capacity. Glock and Jaber [10] investigated the effect of learning and forgetting on a multistage production inventory model with rework and scrap. Yadav et al. [36] proposed an EOQ model for imperfect quality items with price-dependent demand rate and partially backlogged shortages under the effect of learning to enhance the performance of system in an inflationary environment. Kumar and Kumar [21] developed an inventory model under the effect of learning with two level storage capacities. Jawla and Singh [18] established a reverse logistic model with the consideration of effect of learning on inventory-related costs. Agarwal et al. [1] proposed an inventory model for non-instantaneous decaying items with partial backlogging under the learning effect. Yadav et al. [37] developed a two-echelon supply chain model for imperfect production process under the effect of learning.

In this chapter, an imperfect production model with multivariate demand rate dependent on the reliability, selling price, and no. of advertisement is established in order to meet the demand under the effect of preservation technology [13] in which the manufacturing process is flexible as long as the machine is working efficiently and hence can produce as per the demand rate. But in reality, a machine cannot work properly or smoothly forever because of its technical issues like break down of its spare parts may be sooner or later. Thus, the phenomenon of machine failure is considered, and the time-to-breakdown during a production run and repair time of the machine is taken as a random variable. Model is established for multi-items under learning and forgetting environment and we have assumed that some imperfect items are produced during the production process. A reliable production system is considered, and holding cost is assumed as a function of the unit purchase cost of raw material. We develop the model in both crisp and fuzzy environment and for defuzzification of the expected total profit function, graded mean representation method is used. Finally, the imperfect economic production model for multi-items has been illustrated with the help of examples. A sensitivity analysis has been performed to study the effect of changes in some key parameters on optimal policies.

6.2 Assumptions and Notations

The assumptions and notations which are used in mathematical model formulation are given as follows.

6.2.1 Assumptions

1. Model is developed for multi-items over infinite planning horizon.
2. The demand rate of ith item is taken as function of no. of advertisement, selling price, and reliability and is given by $D_i(A_i, s_i, r_i) = A^{\gamma}{}_i(a_i - b_i s_i + c_i r_i)$, where a_i is initial demand, s_i selling price, r_i reliability, A_i no. of advertisement and $\gamma > 0, b_i > 0, c_i > 0$.
3. Model considers the situation of random machine failure/breakdown, i.e., imperfect items produces during production run time and these items are reworked.
4. Machine breakdowns occur randomly during a production period and repair time of machine is independent of machine breakdown.
5. The production cost per unit item is given by $C_{ip} = R_i + \frac{G_i}{p_i} + H_i p_i$, where R_i, G_i, H_i all are positive constants.
6. Holding cost is a function of unit purchase cost of raw material.
7. Items are deteriorated at a rate θ_i. Deteriorated items are neither repair nor replace.
8. Preservation technology is used to reduce the rate of deterioration of products.
9. Model is developed under imprecise and learning and forgetting environment.

6.2.2 Notations

The following notations are used herein:

P_i	Production rate of the given inventory system
r_i	The reliability of the production process
s_i	Selling price per cycle
A_i	No. of advertisement
θ_i	Original deterioration rate of on-hand-stock, $\theta_i > 0$
ξ_i	Preservation Technology (PT) cost of ith item for reducing deterioration rate in order to preserve the products, $\xi_i \geq 0$
k_i	Resultant deterioration rate, $k_i = \theta_i - \pi_i(\xi_i)$
S_i	Setup cost per cycle of the given inventory system for ith item which also includes advertisement cost of the products
C_{id}	Deterioration cost per cycle of the given inventory system for ith item
C_{iR}	Rework cost per cycle of the given inventory system for ith item
C_{iI}	Inspection cost per cycle of the given inventory system for ith item
$C_{ih} + \eta C$	Holding cost per cycle of the given inventory system for ith item, C is unit purchase cost of raw material and $\eta > 0$
C_{ip}	Unit production cost per cycle of the given inventory system for ith item
t	Machine repair time
$\phi_i(t)$	Pdf (probability density function) of t, $\phi_i(t) = \frac{1}{\lambda_i} e^{\frac{t}{\lambda_i}}, t, \lambda_i > 0$
$f(T_{ib})$	Probability density function of T_{ib}, $f(T_{ib}) = \alpha_i e^{-\alpha_i T_{ib}}$, $T_{ib}, \alpha_i > 0$
T_{i1}	Time when production stops
T_{ib}	Time when machine breakdown occurs (a random variable)
T_{i2}	Time when inventory of products vanishes and shortages start to accumulate which causes lost sales
$E_i(T)$	Expected duration of a production cycle for ith item
$E_i(Q)$	Expected total inventory in complete production cycle for ith item
$E_i(PC)$	Expected production cost of the given inventory system for ith item
$E_i(RC)$	Expected rework cost of the given inventory system for ith item
$E_i(HC)$	Expected holding cost of the given inventory system for ith item
$E_i(SR)$	Expected sales revenues of the given inventory system for ith item
$E_i(IC)$	Expected inspection cost of the given inventory system for ith item
$E_i(LS)$	Expected lost sales cost of the given inventory system for ith item
$E_i(TC)$	Expected total cost of the given inventory system for ith item
$E_i(TP)$	Expected total profit of the given inventory system for ith item

6.3 Mathematical Formulation of the Model

Formulation of the model with multivariate demand and exponential distribution.

6.3.1 *Crisp Model*

In the planned inventory model, we consider a multi-item imperfect production model under the effect of preservation technology, depicted in Fig. 6.1. Breakdown of the manufacturing machines is taken into account by considering its failure rate and repair time to be random (continuous). Since the machine breakdown has considered, the breakdown may occur during the production period or after the production period. The production cycle originates with nil inventories and starts at $T = T_{i0}$ and as the production process is going on, if the machine breakdown does not occur in the production period, inventory level increases and reaches its maximum level. The production run is stopped at time T_{i1} and after that due to the combined influence of demand and deterioration/damageability of the items, inventory level decreases and inventory level goes to zero level at time T_{i2} and after that machine starts to produce the items again. When the machine breakdown occurs, the production process stops at T_{ib} and the machine requires some time to repair. As we considered that repair time is also stochastic, production may not always be possible and lost sales may occur during this period.

The governing differential equation of the proposed inventory system is given by

$$I'_{i1}(t) = r_i P_i - D_i - k_i I_{i1}(t), \quad 0 \le t \le T_{i1} \tag{6.1}$$

$$I'_{i2}(t) = -D_i - k_i I_{i2}(t), \quad T_{i1} \le t \le T_{i2} \tag{6.2}$$

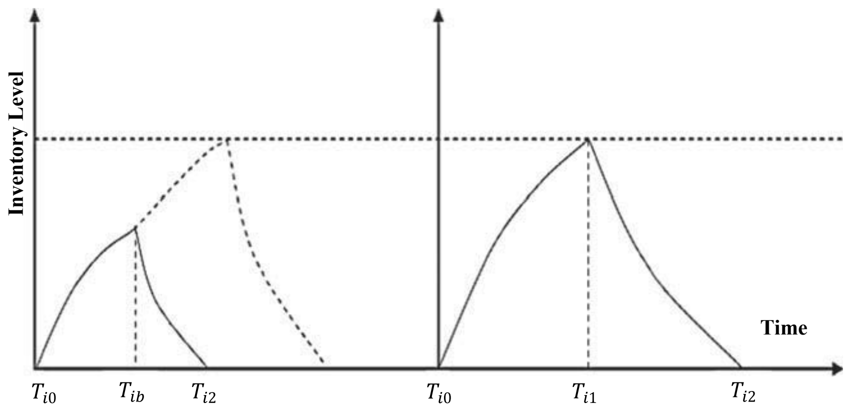


Fig. 6.1 Graphical representation of the proposed inventory system

With the initial and boundary condition $I_{i1}(0) = 0\ and\ I_{i2}(T_{i2}) = 0$.
The solutions of the Eqs. (6.1) and (6.2) are as follows:

$$I_{i1}(t) = \left(\frac{r_i P_i - D_i}{k_i}\right)(1 - e^{-k_i t}), \quad 0 \le t \le T_{i1} \tag{6.3}$$

$$I_{i2}(t) = \frac{D_i}{k_i}(e^{k_i(T_{i2}-t)} - 1), \quad T_{i1} \le t \le T_{i2} \tag{6.4}$$

Total inventory in the complete production cycle can be calculated as below:

$$Inv_i = \int_0^{T_{i1}} I_{i1}(t)dt + \int_{T_{i1}}^{T_{i2}} I_{i2}(t)dt \tag{6.5}$$

$$Inv_i = \int_0^{T_{i1}} \left(\frac{r_i P_i\ -\ D_i}{k_i}\right)(1 - e^{-k_i t})dt + \int_{T_{i1}}^{T_{i2}} \frac{D_i}{k_i}(e^{-k_i(T_{i2}-t)} - 1)dt$$

$$Inv_i = \left(\frac{r_i P_i - D_i}{2}\right)\left[T_{i1}^2 + \left(\frac{r_i P_i\ -\ D_i}{D_i}\right)\left(T_{i1} - \frac{k_i}{2}T_{i1}^2\right)^2\right] \tag{6.6}$$

If the machine breakdown occurs at $t = T_{ib}$, then (Eq. 6.6) can be formulated as

$$Inv_i = \begin{cases} \left(\frac{r_i P_i - D_i}{2}\right)\left[T_{ib}^2 + \left(\frac{r_i P_i - D_i}{D_i}\right)\left(T_{i1} - \frac{k_i}{2}T_{i1}^2\right)^2\right], \text{ for } T_{ib} < T_{i1} \\ \left(\frac{r_i P_i - D_i}{2}\right)\left[T_{i1}^2 + \left(\frac{r_i P_i - D_i}{D_i}\right)\left(T_{i1} - \frac{k_i}{2}T_{i1}^2\right)^2\right], \text{ for } T_{ib} > T_{i1} \end{cases} \tag{6.7}$$

The probability density function of machine breakdown is a function of T_{ib} and given by

$$f(T_{ib}) = \alpha_i e^{-\alpha_i T_{ib}}, \quad T_{ib} > 0 \tag{6.8}$$

Then, the expected inventory is calculated as

$$E_i(Q) = \int_{T_{ib}=0}^{T_{ib}=T_{i1}} Inv_i \alpha_i e^{-\alpha_i T_{ib}} dT_{ib} + \int_{T_{ib}=T_{i1}}^{T_{ib}=\infty} Inv_i \alpha_i e^{-\alpha_i T_{ib}} dT_{ib}$$

$$E_i(Q) = \alpha_i\left(\frac{r_i P_i - D_i}{2}\right)\left[\left(\frac{r_i P_i}{D_i}\right)\frac{T_{i1}^3}{3} - \left\{\alpha_i + (k_i + \alpha_i)\left(\frac{r_i P_i - D_i}{D_i}\right)\frac{T_{i1}^4}{4}\right\}\right]$$
$$+ e^{-\alpha_i T_{i1}}\left(\frac{r_i P_i - D_i}{D_i}\right) \tag{6.9}$$

$$\left[T_{i1}^2 + \left(\frac{r_i P_i - D_i}{D_i}\right)\left(T_{i1} - \frac{k_i}{2}T_{i1}^2\right)^2\right] \tag{6.10}$$

Now, from the equation of continuity,

$$I_{i1}(T_{i1}) = I_{i2}(T_{i2}) \tag{6.11}$$

$$\left(\frac{r_i P_i - D_i}{k_i}\right)(1 - e^{-k_i T_{i1}}) = \frac{D_i}{k_i}(e^{k_i(T_{i2}-T_{i1})} - 1)$$

To find the relation between the variables using Taylor series expansion of solution:

$$T_{i2} = T_{i1} + \left(\frac{r_i P_i - D_i}{D_i}\right)\left(T_{i1} - \frac{k_i}{2}T_{i1}^2\right) \tag{6.12}$$

Present worth sales revenue of the given inventory system: We calculate sales revenue in the complete production cycle for the given inventory system as

$$= s_i\left(\int_0^{T_{i1}} D_i dt + \int_{T_{i1}}^{T_{i2}} D_i dt\right) \tag{6.13}$$

$$= s_i D_i\left[T_{i1} + \left(\frac{r_i P_i - D_i}{D_i}\right)\left(T_{i1} - \frac{k_i}{2}T_{i1}^2\right)\right] \tag{6.14}$$

Since there is breakdown machine possibility, if machine breakdown occurs at $t = T_{ib}$, then sales revenue can be formulated as

$$Srv_i = \begin{cases} s_i D_i\left[T_{ib} + \left(\frac{r_i P_i - D_i}{D_i}\right)\left(T_{ib} - \frac{k_i}{2}T_{ib}^2\right)\right], \text{ if } T_{ib} < T_{i1} \\ s_i D_i\left[T_{i1} + \left(\frac{r_i P_i - D_i}{D_i}\right)\left(T_{i1} - \frac{k_i}{2}T_{i1}^2\right)\right], \text{ if } T_{ib} > T_{i1} \end{cases} \tag{6.15}$$

Using the probability density function of machine breakdown time T_{ib}, the expected sales revenues of complete production cycle are obtained as

$$E_i(SR) = \int_0^{T_{i1}} (Srv_i)\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} + \int_{T_{i1}}^{\infty} (Srv_i)\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} \tag{6.16}$$

$$E_i(SR) = s_i D_i\left[\alpha_i\left[\frac{T_{i1}^2}{2} + \left(\frac{r_i P_i - D_i}{D_i}\right)\left(\frac{T_{i1}^2}{2} - \frac{k_i}{2}\frac{T_{i1}^3}{3}\right) - \alpha_i\frac{T_{i1}^3}{3} - \alpha_i\left(\frac{r_i P_i - D_i}{D_i}\right)\left(\frac{T_{i1}^3}{3} - \frac{k_i}{2}\frac{T_{i1}^4}{4}\right)\right] + \left[T_{i1} + \left(\frac{r_i P_i - D_i}{D_i}\right)\left(T_{i1} - \frac{k_i}{2}T_{i1}^2\right)\right]e^{-\alpha_i T_{i1}}\right] \tag{6.17}$$

The costs which are associated with the given imperfect inventory model are defined below.

Present worth production cost of the given inventory system: Total production cost for the given inventory system is given by

$$Pdc_i = C_{ip}\int_0^{T_{i1}} r_i P_i dt = C_{ip} r_i P_i T_{i1} \tag{6.18}$$

When machine breakdown occurs at $t = T_{ib}$, then from (Eq. 6.18)

$$Pdc_i = \begin{cases} C_{ip} r_i P_i T_{ib},\ if\ T_{ib} < T_{i1} \\ C_{ip} r_i P_i T_{i1},\ if\ T_{ib} > T_{i1} \end{cases} \tag{6.19}$$

Then the total expected production cost for ith item can be formulated as

$$E_i(PC) = \int_{T_{ib}=0}^{T_{ib}=T_{i1}} (Pdc_i)\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} + \int_{T_{ib}=T_{i1}}^{T_{ib}=\infty} (Pdc_i)\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} \tag{6.20}$$

$$E_i(PC) = C_{ip} r_i P_i \left[\alpha_i\left(\frac{T_{i1}^2}{2} - \alpha_i \frac{T_{i1}^3}{3}\right) + T_{i1} e^{-\alpha_i T_{i1}}\right] \tag{6.21}$$

Present worth holding cost of the given inventory system: Expected inventory carrying cost in complete cycle:

$$E_i(HC) = (C_{ih} + \eta\, C)\, E_i(Q) \tag{6.22}$$

$$E_i(HC) = (C_{ih} + \eta\, C)\alpha_i\left(\frac{r_i P_i - D_i}{2}\right)\left[\left(\frac{r_i P_i}{D_i}\right)\frac{T_{i1}^3}{3} - \left\{\alpha_i + (k_i + \alpha_i)\left(\frac{r_i P_i - D_i}{D_i}\right)\frac{T_{i1}^4}{4}\right\}\right] + \left(\frac{r_i P_i - D_i}{D_i}\right)\left[T_{i1}^2 + \left(\frac{r_i P_i - D_i}{D_i}\right)\left(T_{i1} - \frac{k_i}{2}T_{i1}^2\right)^2\right] e^{-\alpha_i T_{i1}} \tag{6.23}$$

Present worth rework cost of the given inventory system: Total rework cost for the given inventory system is given by

$$Rwk_i = C_{iR}\int_0^{T_{i1}} r_i P_i dt = C_{iR} r_i P_i T_{i1} \tag{6.24}$$

When machine breakdown occurs at $t = T_{ib}$, then from (Eq. 6.24)

$$Rwk_i = \begin{cases} C_{iR} r_i P_i T_{ib},\ if\ T_{ib} < T_{i1} \\ C_{iR} r_i P_i T_{i1},\ if\ T_{ib} > T_{i1} \end{cases} \tag{6.25}$$

Then the total expected rework cost for ith item can be formulated as

$$E_i(RC) = \int_{T_{ib}=0}^{T_{ib}=T_{i1}} (Rwk_i)\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} + \int_{T_{ib}=T_{i1}}^{T_{ib}=\infty} (Rwk_i)\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} \quad (6.26)$$

$$E_i(RC) = C_{iR} r_i P_i \left[\alpha_i \left(\frac{T_{i1}^2}{2} - \alpha_i \frac{T_{i1}^3}{3} \right) + T_{i1} e^{-\alpha_i T_{i1}} \right] \quad (6.27)$$

Present worth deterioration cost of the given inventory system: The number of deteriorated units is as follows:

$$Det_i = \left(\int_0^{T_{i1}} r_i P_i dt - \int_{T_{i1}}^{T_{i2}} D_i dt \right) = r_i P_i T_{i1} - D_i(T_{i2} - T_{i1})$$

$$Det_i = D_i T_{i1} + (r_i P_i - D_i) \frac{k_i}{2} T_{i1}^2 \quad (6.28)$$

If the machine breakdown occurs at $t = T_{ib}$, the total deteriorating items are given by

$$Det_i = \begin{cases} D_i T_{ib} + (r_i P_i - D_i) \frac{k_i}{2} T_{ib}^2, \; if \; T_{ib} < T_{i1} \\ D_i T_{i1} + (r_i P_i - D_i) \frac{k_i}{2} T_{i1}^2, \; if \; T_{ib} > T_{i1} \end{cases} \quad (6.29)$$

By using the probability density function of machine breakdown time T_{ib}, the expected deteriorating units are given by as

$$E_i(D) = \int_{T_{ib}=0}^{T_{ib}=T_{i1}} (Det_i)\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} + \int_{T_{ib}=T_{i1}}^{T_{ib}=\infty} (Det_i)\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} \quad (6.30)$$

$$E_i(D) = \alpha_i \left[D_i \frac{T_{i1}^2}{2} + \left\{ (r_i P_i - D_i) \frac{k_i}{2} - \alpha_i D_i \right\} \frac{T_{i1}^3}{3} - \alpha_i (r_i P_i - D_i) \frac{k_i}{2} \frac{T_{i1}^4}{4} \right] + e^{-\alpha_i T_{i1}} \left[D_i T_{i1} + (r_i P_i - D_i) \frac{k_i}{2} T_{i1}^2 \right]$$

The total expected deterioration cost for ith item is given by

$$E_i(DC) = C_{id} \, E_i(D) \quad (6.31)$$

$$E_i(DC) = C_{id} \left[\alpha_i \left[D_i \frac{T_{i1}^2}{2} + \left\{ (r_i P_i - D_i) \frac{k_i}{2} - \alpha_i D_i \right\} \frac{T_{i1}^3}{3} - \alpha_i (r_i P_i - D_i) \frac{k_i}{2} \frac{T_{i1}^4}{4} \right] + e^{-\alpha_i T_{i1}} \left[D_i T_{i1} + (r_i P_i - D_i) \frac{k_i}{2} T_{i1}^2 \right] \right] \quad (6.32)$$

Present worth inspection cost of the given inventory system: Total inspection cost for the given inventory system is given by

$$Ins_i = C_{iI}\int_0^{T_{i1}} r_i P_i dt = C_{iI} r_i P_i T_{i1} \tag{6.33}$$

When machine breakdown occurs at $t = T_{ib}$, then from (Eq. 6.33)

$$Ins_i = \begin{cases} C_{iI} r_i P_i T_{ib},\ if\ T_{ib} < T_{i1} \\ C_{iI} r_i P_i T_{i1},\ if\ T_{ib} > T_{i1} \end{cases} \tag{6.34}$$

Then the total expected rework cost for *i*th item can be formulated as

$$E_i(IC) = \int_{T_{ib}=0}^{T_{ib}=T_{i1}} (Ins_i)\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} + \int_{T_{ib}=T_{i1}}^{T_{ib}=\infty} (Ins_i)\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} \tag{6.35}$$

$$E_i(IC) = C_{iI} r_i P_i \left[\alpha_i\left(\frac{T_{i1}^2}{2} - \alpha_i\frac{T_{i1}^3}{3}\right) + T_{i1} e^{-\alpha_i T_{i1}}\right] \tag{6.36}$$

Present worth lost sales cost of the given inventory system: Lost sales occurs when repair time of machine exceeds than the period in which production is not performed, i.e., non-production period T_{i2}. Here we assume that the machine repair time t is a random variable and is exponentially distributed. Exponential probability density function with mean $1/\lambda$ is given as

$$\phi_i(t) = \frac{1}{\lambda_i} e^{\frac{t}{\lambda_i}}, \quad for\ \lambda_i > 0 \tag{6.37}$$

Substituting the exponential probability density function of repair time and machine breakdown probability density function expected lost sales cost for *i*th item formulated as

$$E_i(LS) = C_{iL} D_i \int_{T_{ib}=0}^{T_{ib}=T_{i1}} \int_{t=T_{i2}}^{t=\infty} (t - T_{i2})\,\phi_i(t)\,\alpha_i e^{-\alpha_i T_{ib}} dt\, dT_{ib} \tag{6.38}$$

$$E_i(LS) = C_{iL} D_i \alpha_i \lambda_i \Bigg[T_{i1} - \alpha_i\frac{T_{i1}^2}{2} + \frac{\alpha_i}{\lambda_i}\left\{\frac{T_{i1}^3}{3} + \left(\frac{r_i P_i - D_i}{D_i}\right)\left(\frac{T_{i1}^3}{3} - \frac{k_i}{2}\frac{T_{i1}^4}{4}\right)\right\} - \frac{1}{\lambda_i}\left\{\frac{T_{i1}^2}{2}\left(\frac{r_i P_i - D_i}{D_i}\right)\left(\frac{T_{i1}^2}{2} - \frac{k_i}{2}\frac{T_{i1}^3}{3}\right)\right\}\Bigg] \tag{6.39}$$

For the present inventory model, expected total replenishment time,

$$E_i(T) = E_i(T_{i2}) + Expected\, repair\, time\, of\, the\, i\text{th item}$$

$$E_i(T) = \int_{T_{ib}=0}^{T_{ib}=T_{i1}} T_{i2}\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} + \int_{T_{ib}=T_{i1}}^{T_{ib}=\infty} T_{i2}\alpha_i e^{-\alpha_i T_{ib}} dT_{ib} + \int_{T_{ib}=0}^{T_{ib}=T_{i1}} \int_{T_{ib}=T_{i1}}^{T_{ib}=\infty} (t - T_{i2}).\phi_i(t).\alpha_i e^{-\alpha_i T_{ib}} dt.dT_{ib} \tag{6.40}$$

The expected total cost of the ith item for the given inventory model consists of setup cost, expected inventory carrying cost, expected deterioration cost, expected production cost, expected rework cost, expected lost sales cost, and expected inspection cost of ith item.

Expected total cost of ith item for the inventory system:

$$E_i(TC) = [S_i + E_i(HC) + E_i(DC) + E_i(PC) + E_i(R) + E_i(LS) + E_i(I)] \tag{6.41}$$

By using the renewal reward theorem, the expected average profit of ith item for the given inventory model can be obtained as follows:

$$E_i(AP) = \left[\frac{E_i(SR) - E_i(TC)}{E_i(T)}\right] \tag{6.42}$$

The expected total average profit for the proposed inventory model can be obtained as follows:

$$E(TAP) = \sum\nolimits_{i=1}^{m} E_i(AP) = \sum\nolimits_{i=1}^{m} \left[\frac{E_i(SR) - E_i(TC)}{E_i(T)}\right] \tag{6.43}$$

Or can be written as

$$E(TAP) = \sum\nolimits_{i=1}^{m} \frac{E_i(SR) - S_i - E_i(HC) - E_i(DC) - E_i(PC) - E_i(R) - E_i(LS) - E_i(I)}{E_i(T)} \tag{6.44}$$

From (Eq. 6.44), we get the expected total average profit for the given inventory model.

6.3.2 Model Formulation with Learning and Forgetting in Setup Cost

Full transmission of learning in setup cost: In this chapter, we consider learning effect on setup cost, i.e., setup cost follows the learning curve as described by Wright [35], given by S_{iL} which is partially constant and partially decreases due to learning effect in each cycle and is of the form $C_{iO} + \frac{C_{ios}}{n^{l_i}}$, where n is the number of cycle and $l > 0$ is the learning coefficient. Now, the expected total average profit for the proposed inventory model can be obtained as follows:

$$E(TAP) = \sum_{i=1}^{m}\left[\frac{E_i(SR) - S_{iL} - E_i(HC) - E_i(DC) - E_i(PC) - E_i(R) - E_i(LS) - E_i(I)}{E_i(T)}\right] \quad (6.45)$$

Learning and forgetting in setup cost: We developed model with the assumption that setup cost follows the learning and forgetting effect as described by the Jaber and Bonney [15]. So, setup cost for ith item is given by $S_{iLF} = S_i(y_j + 1)^{-b}$, where S_i is the first setup cost for ith item, y_j is strength of memory at beginning of setup j, and b is the learning coefficient. Now, the expected total average profit for the given inventory model can be obtained as follows:

$$E(TAP) = \sum_{i=1}^{m}\left[\frac{E_i(SR) - S_{iLF} - E_i(HC) - E_i(DC) - E_i(PC) - E_i(R) - E_i(LS) - E_i(I)}{E_i(T)}\right] \quad (6.46)$$

6.3.3 *Fuzzy Model Formulation*

In the fuzzy model, we developed model in fuzzy sense where costs are considered fuzzy in nature. In this model, we assume that the lost sale cost and deterioration cost are fuzzy triangular numbers and denoted by $\widetilde{C_{iL}}$ and $\widetilde{C_{id}}$ and given by $\widetilde{C_{iL}} = (C_{aiL}, C_{biL}, C_{ciL})$ and $\widetilde{C_{id}} = (C_{aid}, C_{bid}, C_{cid})$ where $C_{aiL}, C_{biL}, C_{ciL}, C_{aid}, C_{bid}, C_{cid}$ are nonnegative triangular fuzzy numbers.

Then, the expected average profit for the given inventory model can be expressed as

$$\begin{aligned} E_i(AP) &= \left[\frac{E_i(SR) - S_i - E_i(HC) - E_i(DC) - E_i(PC) - E_i(R) - E_i(LS) - E_i(I)}{E_i(T)}\right] \\ \widetilde{E_i}(AP) &= \left[\frac{E_i(SR) - S_i - E_i(HC) - \widetilde{E_i}(DC) - E_i(PC) - E_i(R) - \widetilde{E_i}(LS) - E_i(I)}{E_i(T)}\right] \end{aligned} \quad (6.47)$$

We defuzzify the fuzzy expected average profit by graded mean representation, where defuzzified expected total average profit is

$$\begin{aligned} \widetilde{E_i}(AP) = \frac{1}{E_i(T)}[&E_i(SR) - S_i - E_i(HC) - (E_{ai}(DC), E_{bi}(DC), E_{ci}(DC)) \\ &- E_i(PC) - E_i(R) - (E_{ai}(LS), E_{bi}(LS), E_{ci}(LS)) - E_i(I)] \end{aligned}$$

By graded mean representation, the fuzzy expected average profit is given by

$$\widetilde{E_i}(AP) = \frac{1}{6}\left\{\widetilde{E}_{ai}(AP), \widetilde{E}_{bi}(AP), \widetilde{E}_{ci}(AP)\right\}$$

where

$$\widetilde{E}_{ai}(AP) = \left[\frac{E_i(SR) - S_i - E_i(HC) - E_{ai}(DC) - E_i(PC) - E_i(R) - E_{ai}(LS) - E_i(I)}{E_i(T)}\right]$$

$$\widetilde{E}_{bi}(AP) = \left[\frac{E_i(SR) - S_i - E_i(HC) - E_{bi}(DC) - E_i(PC) - E_i(R) - E_{bi}(LS) - E_i(I)}{E_i(T)}\right]$$

$$\widetilde{E}_{ci}(AP) = \left[\frac{E_i(SR) - S_i - E_i(HC) - E_{ci}(DC) - E_i(PC) - E_i(R) - E_{ci}(LS) - E_i(I)}{E_i(T)}\right]$$

After the defuzzification of fuzzy expected average profit, using the graded mean integration formula such that:

$$\widetilde{E}_i(AP) = \frac{1}{6}\left\{\widetilde{E}_{ai}(AP) + 4\widetilde{E}_{bi}(AP) + \widetilde{E}_{ci}(AP)\right\}$$

The expected fuzzy total average profit for the given inventory model can be obtained as follows:

$$\widetilde{E}(TAP) = \sum_{i=1}^{m} \widetilde{E}_i(AP) \tag{6.48}$$

6.4 Optimal Solution Procedure

The objective of this chapter is to maximize the expected average profit per unit time $E_i(AP)$, $(i = 1, 2, .., m)$ with respect to time T_{i1} for the developed inventory model. The objective function of the chapter is highly nonlinear and continuous function of variable T_{i1}. The necessary condition for the existence of the optimal solution is

$$\frac{dE_i(AP)(T_{i1})}{dT_{i1}} = 0,\ i = 1.2.3\ldots m$$

Provided it satisfies

$$\frac{d^2E_i(AP)(T_{i1})}{dT_{i1}^2} < 0,\ i = 1.2.3\ldots m$$

To solve these highly nonlinear equations, we use software MATHEMATICA 8.0.

6.5 Numerical Examples

To demonstrate the proposed model and methodologies, we discuss numerical examples for two items. For this purpose, we have taken some initial values in appropriate units as follows.

6.5.1 Crisp Model

Then the optimal $E_1^*(AP)$ is 8963.49 at (35.6755) and $E_2^*(AP)$ is 10884 at (47.9018). Hence, the demand during the cycle is $D_1^* = 34.9951$ and $D_2^* = 35.1142$. The expected total average profit for the given inventory model is $E(TAP) = 19847.4889$ (Tables 6.1 and 6.2).

6.5.2 Effect of Learning and Forgetting on Setup Cost

From Tables 6.3 and 6.4, we find that as the number of cycle increases, setup decreases while the expected profit of the proposed inventory system increases for both the items.

Figure 6.2 shows that as we apply the effect of learning and forgetting on setup cost of the inventory system, the expected profit increases.

6.5.3 Fuzzy Model

Extensive numerical analysis has been done to gauge the impact of the level of fuzziness in the input parameters over the decision variable. To study these levels of fuzziness, we consider $\widetilde{C_{iL}}$ and $\widetilde{C_{id}}$ $(i = 1, 2, \ldots, m)$ as triangular fuzzy numbers. Rest of the input parameters are the same as previously defined.

Parameters	$i = 1$	$i = 2$
$\widetilde{C_{iL}}$	(3.5, 4, 4.5)	(4.5, 5, 5.5)
$\widetilde{C_{id}}$	(2.5, 3, 3.5)	(3.5, 4, 4.5)

by graded mean representation method, the solution of fuzzy model is

$$\widetilde{E_1}(AP) = (8971.9513, 8963.4889, 8955.0674)$$

and

$\widetilde{E_1}(AP) = (10893.0729, 10884.0001, 10874.9556)$.

The expected fuzzy total average profit $\widetilde{E}(TAP) = 19847.5005$.

Table 6.1 Initial values

Items i	a_i	b_i	c_i	r_i	α_i	A_i	s_i	R_i	G_i	H_i	δ_i	l_i
1	45	0.063	0.01	0.45	0.001	6	250	1.8	3	0.03	0.2	0.2
2	49	0.067	0.01	0.5	0.001	7	300	1.6	2.7	0.02	0.3	0.3

Items i	C_{ip}	C_{ih}	C_{iI}	S_i	C_{iR}	C_{iL}	C_{id}	P_i	η_i	λ_i
1	3.73	5	3.5	100	2.5	4	3	63.05	0.04	0.03
2	2.80	5.5	4	120	3	5	4	58.8	0.03	0.02

Table 6.2 Optimal outputs

Items i	k_i^*	D_i^*	T_{i1}^*	T_{i2}^*	$E_i^*(AP)$
1	0.125	34.9951	35.6755	43.9778	8963.4889
2	0.1	35.1142	47.9018	58.7767	10884

Table 6.3 For first item

No. of setup	Setup cost	T_{11}	T_{12}	$E_1(AP)$
1	100	35.6755	43.9778	8963.4889
2	95.19	35.6726	43.9731	8963.6023
3	88.4	35.6685	43.9662	8963.7624
4	83.25	35.6654	43.9611	8963.8839
5	79.14	35.6629	43.9570	8963.9808

Table 6.4 For second item

No. of setup	Setup cost	T_{21}	T_{22}	$E_2(AP)$
1	120	47.9018	58.7767	10884
2	114.23	47.8990	58.7726	10884.1374
3	106.08	47.8953	58.7662	10884.2830
4	99.894	47.8924	58.7615	10884.3937
5	94.96	47.8901	58.7578	10884.4818

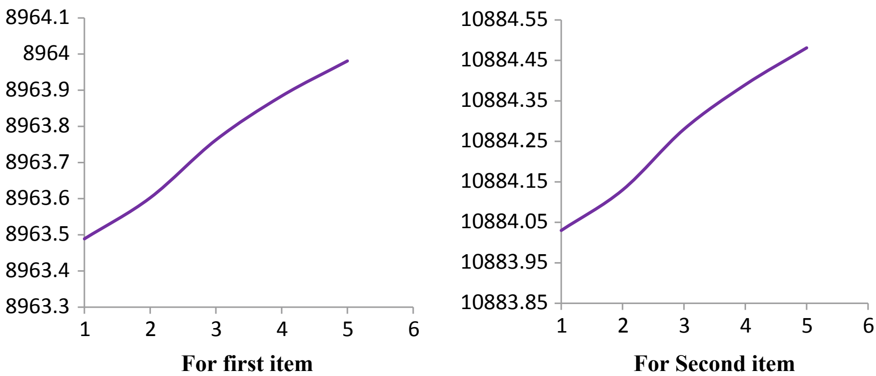


Fig. 6.2 Effects of learning and forgetting on $E_1(AP)$ and $E_2(AP)$

6.6 Concavity of the Proposed Inventory System

The concavity of the expected average profit $E_1(AP)$ and $E_2(AP)$ per unit time is shown in Fig. 6.3 w.r.t. T_{11} and T_{21}, respectively.

The 3-D plots of the expected average profit $E_1(AP)$ and $E_2(AP)$ per unit time w.r.t. two decision variables show the concavity of the function (Figs. 6.4 and 6.5).

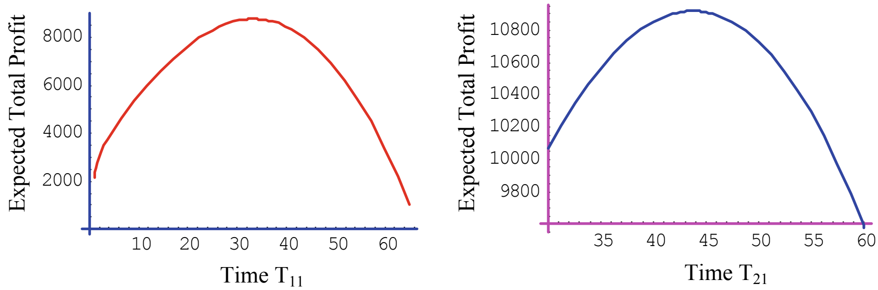


Fig. 6.3 Concavity of the $E_1(AP)$ and $E_2(AP)$ w.r.t. T_{11} and T_{21}

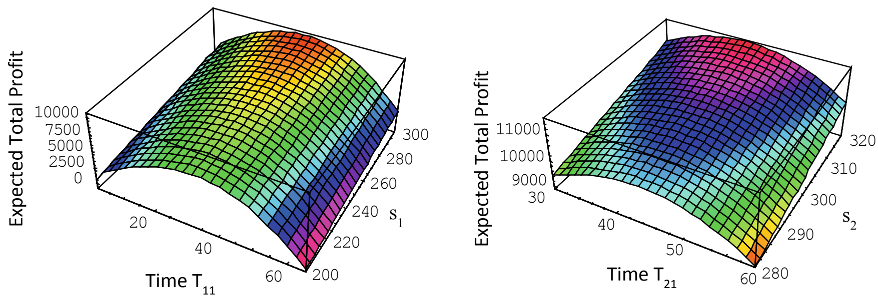


Fig. 6.4 Concavity of the $E_1(AP)$ and $E_2(AP)$

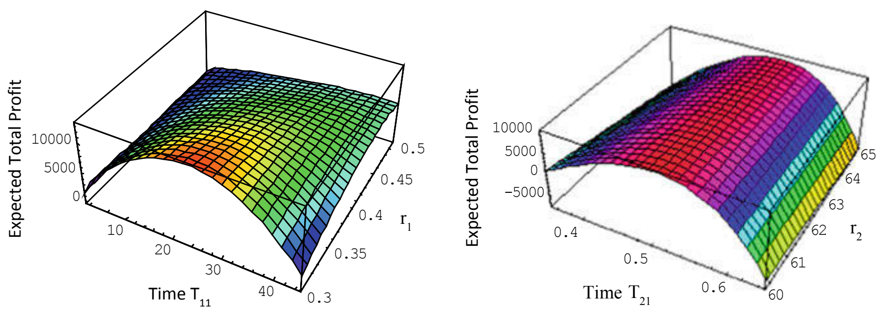


Fig. 6.5 Concavity of the $E_1(AP)$ and $E_2(AP)$

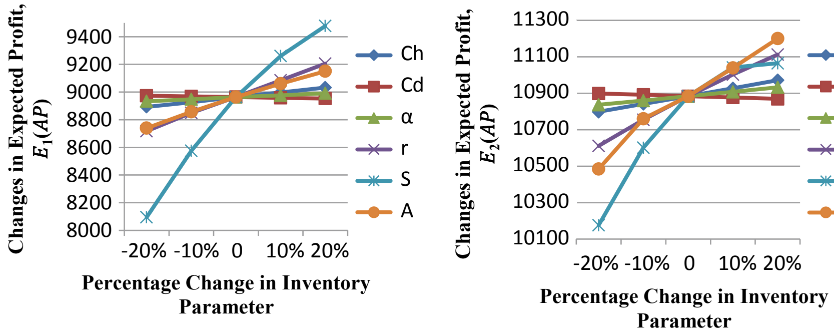


Fig. 6.6 Variation in $E_1(AP)$ and $E_2(AP)$ by changes in different parameters

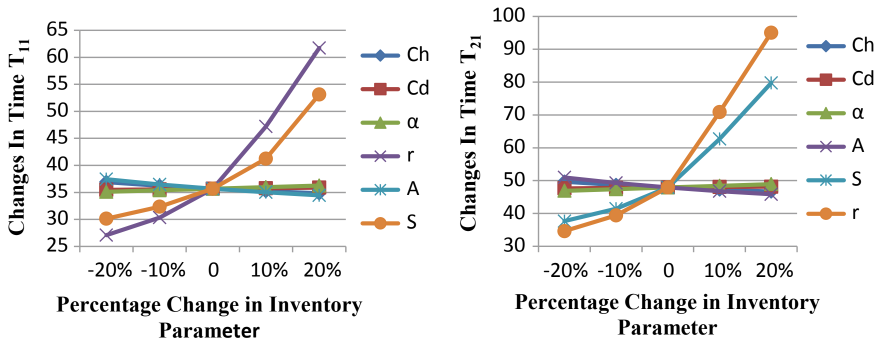


Fig. 6.7 Variation in time T_{11} and time T_{21} by changes in different parameters

6.7 Sensitivity Analysis

A sensitivity analysis has been carried out to study the effects of change in inventory parameters of the proposed model on decision variables and expected average profit function. The parameters given in examples are varied by −20, −10, 10 and 20%. The effects of variations in D_i^*, T_{i1}^*, T_{i2}^* $(i = 1, 2, \ldots, m)$ and profit $E_i^*(AP)$ $(i = 1, 2, \ldots, m)$ are shown in Figs. 6.6, 6.7, 6.8 and 6.9 (Tables 6.5 and 6.6).

6.8 Effect of Inventory Parameters on Expected Average Profit $(i = 1, 2)$

From Fig. 6.6, it is observed that the expected average profit $E_i^*(AP)$ per unit time increases with increase in holding cost C_{ih}, pdf parameter α_i, reliability r_i, no. of advertisement A_i, and selling price s_i while decreases with increase of deterioration

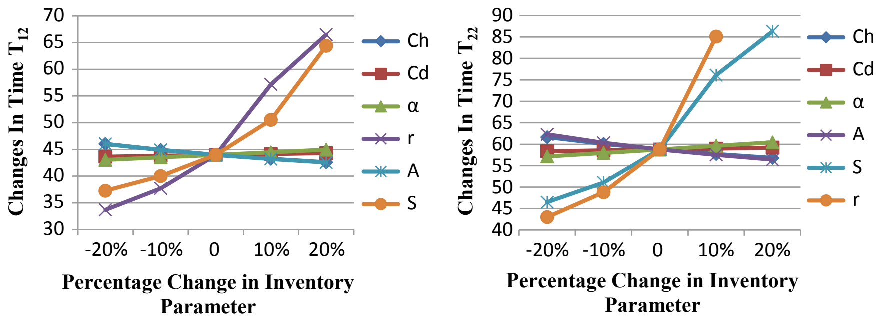


Fig. 6.8 Variation in time T_{12} and time T_{22} by changes in different parameters

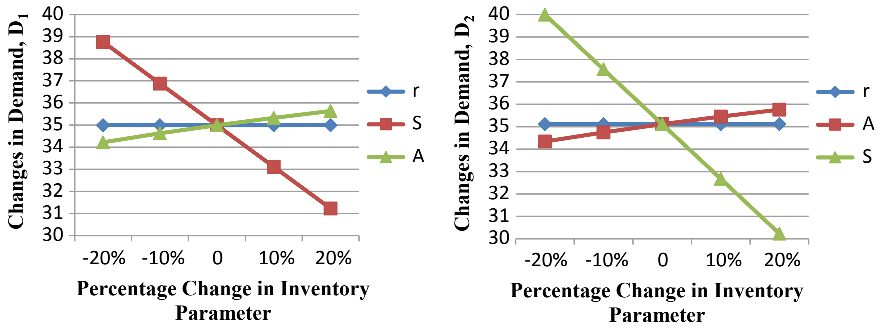


Fig. 6.9 Variation in demand D_1 and D_2 by changes in different parameters

cost C_{id} significantly. So, the manufacturer should take care of products to reduce the deterioration.

6.8.1 Effect of Inventory Parameters on Time T_{11} and Time T_{21} ($i = 1, 2$)

It is observed from Fig. 6.7 that the time T_{11}^* and T_{21}^* increase with increase in deterioration cost C_{id}, pdf parameter α_i, reliability r_i, and selling price s_i and decrease with increase of holding cost C_{ih} and no. of advertisement A_i significantly. T_{11}^* and T_{21}^* are sensitive with respect to reliability r_i and selling price s_i.

Table 6.5 Sensitivity analysis of some parameters for first item

Parameters	% change	T_{11}^*	T_{12}^*	$E_1^*(AP)$
C_{1h}	−20	36.9012	46.0181	8892.8349
	−10	36.2337	44.8996	8927.1825
	0	35.6755	43.9778	8963.4889
	+10	35.2284	43.2353	8997.1912
	+20	34.8401	42.5988	9032.6508
C_{1d}	−20	35.4721	43.6417	8973.6787
	−10	35.5741	43.8101	8968.5764
	0	35.6755	43.9778	8963.4889
	+10	35.7765	44.1450	8958.4160
	+20	35.8769	44.3115	8953.3577
α_1	−20	35.1084	43.0383	8933.4366
	−10	35.3968	43.5125	8947.6454
	0	35.6755	43.9779	8963.4889
	+10	35.9762	44.4711	8976.5253
	+20	36.2672	44.9556	8991.2007

Parameters	% change	D_1^*	T_{11}^*	T_{12}^*	$E_1^*(AP)$
r_1	−20	34.9941	27.0891	33.6861	8717.7109
	−10	34.9946	30.3396	37.6896	8849.3866
	0	34.9951	35.6755	43.9779	8963.4889
	+10	34.9957	47.1953	57.1499	9084.3051
	+20	34.9962	61.727	66.5118	9205.6101
A_1	−20	34.2229	37.4426	46.0207	8739.3414
	−10	34.6284	36.4673	44.8949	8857.0102
	0	34.9951	35.6755	43.9779	8963.4889
	+10	35.3303	35.0156	43.2111	9060.8643
	+20	35.639	34.4547	42.5575	9150.5702
s_1	−20	38.7633	30.1368	37.2745	8095.4607
	−10	36.8792	32.3695	40.0084	8575.9695
	0	34.9951	35.6755	43.9779	8963.4889
	+10	33.1111	41.2259	50.5278	9260.6476
	+20	31.2270	53.1365	64.4104	9478.0209

Table 6.6 Sensitivity analysis of some parameters for second item

Parameters	% change	T_{21}^*	T_{22}^*	$E_2^*(AP)$
C_{2h}	−20	49.6684	61.6583	10798.6016
	−10	48.6964	60.0666	10840.9437
	0	47.9018	58.7767	10884.0001
	+10	47.2396	57.7096	10927.5898
	+20	46.6789	56.8117	10971.5884
C_{2d}	−20	47.6341	58.3445	10898.5846
	−10	47.7683	58.5610	10891.2833
	0	47.9018	58.7767	10884.0001
	+10	48.0346	58.9916	10876.7349
	+20	48.1668	59.2058	10869.4875
α_2	−20	46.8864	57.1435	10836.9336
	−10	47.3931	57.9564	10860.3147
	0	47.9018	58.7767	10884.0001
	+10	48.4128	59.6051	10908.0987
	+20	48.9263	60.4417	10932.5146

Parameters	% change	D_2^*	T_{21}^*	T_{22}^*	$E_2^*(AP)$
r_2	−20	35.1129	34.6314	42.9961	10484.6881
	−10	35.1136	39.3848	48.7925	10759.9131
	0	35.1142	47.9018	58.7767	10884.0001
	+10	35.1148	70.8630	85.1035	11039.8084
	+20	35.1154	–	–	11200.0026
A_2	−20	34.3393	50.9615	62.3092	10611.2608
	−10	34.7462	49.2562	60.3425	10754.3126
	0	35.1142	47.9018	58.7767	10884.0001
	+10	35.4505	46.7927	57.4915	11002.7174
	+20	35.7603	45.8634	56.4121	11112.2164
s_2	−20	39.9978	37.6779	46.5019	10176.3307
	−10	37.5559	41.4316	51.0732	10601.0064
	0	35.1142	47.9018	58.7767	10884.0001
	+10	32.6724	62.6761	76.0711	11041.3036
	+20	30.2306	79.7629	86.3115	11064.6854

6.8.2 *Effect of Inventory Parameters on Time T_{12} and Time T_{22} $(i = 1, 2)$*

From Fig. 6.8, it is observed that T_{12}^* and T_{22}^* are sensitive with respect to reliability r_i and selling price s_i. The time T_{12}^* and T_{22}^* increase with increase in deterioration cost C_{id}, pdf parameter α_i, reliability r_i, and selling price s_i and decrease with increase of holding cost C_{ih} and no. of advertisement A_i significantly.

6.8.3 *Effect of Inventory Parameters on Demand D_1 and D_2 $(i = 1, 2)$*

It is observed from Fig. 6.9 that as we increase the value of reliability r_i and no. of advertisement A_i, demand rate parameter per unit time increases. Demand rate is sensitive with respect to selling price s_i, and they are negatively correlated to each other means by increasing values of s_i, demand rate decreases.

6.9 Conclusion

An EPQ model for multi-items with imperfect production process is developed by considering the situation of random machine failure over infinite planning horizon where machine repair time is also taken as random. To reduce the deterioration rate, use of preservation technology has taken into account. Model has been developed in both crisp and fuzzy environment. The impact of learning and forgetting has been studied and graphically shown in figures on the expected average profit function. From the graph, we observe that learning and combined effect of learning and forgetting are beneficial for the decision maker to increase the profit of the system and decrease the setup cost. In this model, a reliable and flexible production system is taken to improve the production quality and goodwill of the customer. Numerical examples and sensitivity of the inventory parameters on profit function have been studied and shown graphically.

The major contribution of the proposed model is the consideration of preservation technology and effect of learning and forgetting on setup cost in multi-item imperfect production model. From the numerical analysis, it is observed that a reliable production system is better than a non-reliable production inventory system. From the sensitive analysis, we observed that optimal solution of the proposed inventory model is highly sensitive with respect to reliability, no. of advertisement, and selling price while least sensitive with respect to holding cost and deterioration cost. So while making the inventory policy decision, the decision maker pays special attention to these factors.

A possible future research can be to extend the investigated model for stochastic deterioration rate, partially and complete backlogging, and different types of demand rates like stock-dependent demand, stochastic demand, and many more.

References

1. Agarwal A, Sangal I, Singh SR (2017) Optimal policy for non-instantaneous decaying inventory model with learning effect and partial shortages. Int J Comput Appl 161(10):13–18
2. Bag S, Chakraborty D, Roy AR (2009) A production inventory model with fuzzy random demand and with flexibility and reliability considerations. Comput Ind Eng 56:411–416
3. Chen TH (2017) Optimizing pricing, replenishment and rework decision for imperfect and deteriorating items in a manufacturing-retailer channel. Int J Prod Econ 183:539–550
4. Cheng TCE (1989) An economic production quantity model with flexibility and reliability considerations. Eur J Oper Res 39:174–179
5. Cheng TCE (1991) An economic order quantity model with demand-dependent unit production cost and imperfect production processes. IIE Trans 23(1):23–28
6. Chin HN, Chen HM (2005) An optimal algorithm for solving the dynamic lot-sizing model with learning and forgetting in setups and production. Int J Prod Econ 95(2):179–193
7. Chung KJ (1997) Bounds for production lot sizing with machine breakdowns. Comput Ind Eng 32(1):139–144
8. Das D, Roy A, Kar S (2011) A volume flexible economic production lot-sizing problem with imperfect quality and random machine failure in fuzzy-stochastic environment. Comput Math Appl 61(9):2388–2400
9. Dey O, Giri BC (2014) Optimal vendor investment for reducing defect rate in a vendor-buyer integrated system with imperfect production process. Int J Prod Econ 155:222–228
10. Glock CH, Jaber MY (2013) A multi-stage production-inventory model with learning and forgetting effects, rework and scrap. Comput Ind Eng 64(2):708–720
11. Groenevelt H, Pintelon L, Seidmann A (1992) Production lot sizing with machine breakdowns. Manag Sci 38:104–121
12. Hsu JT, Hsu LF (2013) An integrated vendor–buyer cooperative inventory model in an imperfect production process with shortage backordering. Int J Adv Manuf Technol 65(1–4):493–505
13. Hsu PH, Wee HM, Teng HM (2010) Preservation technology investment for deteriorating inventory. Int J Prod Econ 124(2):388–394
14. Iqbal MW, Sarkar B (2018) A model for imperfect production system with probabilistic rate of imperfect production for deteriorating products. DJ J Eng Appl Math 4(2):1–12
15. Jaber MY, Bonney M (2003) Lot sizing with learning forgetting in set-ups and in product quality. Int J Prod Econ 83:95–111
16. Jaber MY, Goyal SK, Imran M (2008) Economic production quantity model for items with imperfect quality subject to learning effects. Int J Prod Econ 115(1):143–150
17. Jawla Preeti, Singh SR (2016) A reverse logistic inventory model for imperfect production process with preservation technology investment under learning and inflationary environment. Uncertain Supply Chain Manag 4:107–122
18. Jawla Preeti, Singh SR (2016) Multi-item economic production quantity model for imperfect items with multiple production setups and rework under the effect of preservation technology and learning environment. Int J Ind Eng Comput 7:703–716
19. Khan M, Jaber MY, Wahab MIM (2010) Economic order quantity model for items with imperfect quality with learning in inspection. Int J Prod Econ 124(1):87–96
20. Konstantaras I, Skouri K, Jaber MY (2012) Inventory models for imperfect quality items with shortages and learning in inspection. Appl Math Model 36(11):5334–5343

21. Kumar N, Kumar S (2016) Effect of learning and salvage worth on an inventory model for deteriorating items with inventory-dependent demand rate and partial backlogging with capability constraints. Uncertain Supply Chain Manag 4(2):123–136
22. Mahapatra GS, Adak S, Mandal TK, Pal S (2017) Inventory model for deteriorating items with time and reliability dependent demand and partial backorder. Int J Oper Res 29(3):344–359
23. Pal S, Mahapatra GS, Samanta GP (2016) A three-layer supply chain EPQ model for price and stock-dependent stochastic demand with imperfect item under rework. J Uncertain Anal Appl 4(1):10
24. Paul SK, Azeem A, Sarker R, Essam D (2014) Development of a production inventory model with uncertainty and reliability considerations. Optim Eng 15(3):697–720
25. Rosenblatt MJ, Lee HL (1986) Economic production cycles with imperfect production processes. IIE Trans 18(1):48–55
26. Sarkar B, Sana SS, Chaudhuri K (2010) Optimal reliability, production lot size and safety stock in an imperfect production system. Int J Math Oper Res 2(4):467–490
27. Shah NH, Vaghela CR (2018) Imperfect production inventory model for time and effort dependent demand under inflation and maximum reliability. Int J Syst Sci Oper Logist 5(1):60–68
28. Singh SR, Urvashi (2010) A multi item inventory model with machine breakdown, flexible strategy and capacity constraint over fuzzy environment. Int J Oper Res Optim 1(1):85–94
29. Singh S, Jain S, Pareek S (2013) An imperfect quality items with learning and inflation under two limited storage capacity. Int J Ind Eng Comput 4(4):479–490
30. Singh SR, Prasher Leena (2014) A production inventory model with flexible manufacturing, random machine breakdown and stochastic repair time. Int J Ind Eng Comput 5:575–588
31. Towill DR (1982) How complex a learning curve model need we use? Radio Electron Eng 52(7):331–338
32. Tripathy PK, Pattnaik M (2009) Optimization in an inventory model with reliability consideration. Appl Math Sci 3(1):11–25
33. Tripathy PK, Pattnaik M (2011) Optimal inventory policy with reliability consideration and instantaneous receipt under imperfect production process. Int J Manag Sci Eng Manag 6(6):413–420
34. Tripathy P, Wee W, Majhi P (2003) An EOQ model with process reliability consideration. J Oper Res Soc 54:549–554
35. Wright TP (1936) Factors affecting the cost of airplanes. J Aeronaut Sci 3(4):122–128
36. Yadav D, Singh SR, Meenu (2015) EOQ model with partial backordering for imperfect items under the effect of inflation and learning with selling price dependent demand. Int J Comput Appl 111(17):25–31
37. Yadav D, Singh SR, Vandana S (2018) Integrated model with imperfect production process under the effect of learning. Malaya J Mat S(1):90–101

Chapter 7
Inventory Policies with Development Cost for Imperfect Production and Price-Stock Reliability-Dependent Demand

Nita H. Shah and Monika K. Naik

Abstract This article focuses on developing a model based on inventory dealing with the product's sell price-stock as well as reliability-dependent demand; also it undergoes a production process which is imperfect including manufacturing of perfect as well as imperfect quality products. As such, each production firm believes in the production of perfect quality goods but because of various uncontrollable barrier factors like machinery, labor, technology, and also due to the long-run process, the production, therefore, includes imperfect quality items along with perfect quality products. The products which are perfect are ready to sell out; on the other hand, the imperfect products undergo the reworking process owing a cost to become a perfect product. By inclusion of the cost of development also by modifying the raw material quality of production system, several considerations like product's reliability, the system's reliability parameter, and the reworking cost can be upgraded. The aim of this article is to calculate the firm's total profit along with the estimation of optimal values of production duration such that a manufacturer gets a maximum profit, manufacturing system's reliability parameter, and product's reliability. The classical optimization technique is utilized for calculating the optimal values. For the validation of developed models, numerical examples are demonstrated; then using the concept of eigenvalues of a Hessian matrix, we have proved the concave nature of the profit function of the system, and also the sensitivity analysis is done for each decision variables by fluctuating the inventory parameters for generating effective managerial insights.

Keywords Selling price-stock and reliability-dependent demand · Imperfect production process · Reworking process · Deteriorating items · Development cost

N. H. Shah
Department of Mathematics, Gujarat University, Ahmedabad, Gujarat, India
e-mail: nitahshah@gmail.com

M. K. Naik (✉)
Department of Applied Sciences and Humanities, PIET, Parul University, Vadodara, Gujarat, India
e-mail: monikaknaik@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_7

7.1 Introduction

In earlier studies, various EPQ models are derived; in some cases, the production analysis is done based on imperfect items. Khouja and Mehrez [17] constructed an imperfect quality inventory model and production rate as variable one with elapsed time is to be considered until the production process moves to out-of-control state to exponential one. An EPQ/EOQ concept for imperfect quality products is utilized by Salameh and Jaber [33] for deriving an inventory model. An inventory model with volume flexibility opting the production process as imperfect one was developed by Sana et al. [34]. A model dealing with the production of imperfect deteriorating items calculating the pricing policy was constructed by Chung and Wee [7] considering inspection planning and warranty period. An EPQ model for an imperfect process by computing the production lot size and the backorder size undergoing reworking process was considered by Cardenas-Barron [4].

An EPQ model with imperfect quantity items considering sales return and two-way imperfect inspection was derived by Yoo et al. [45]. An imperfect production process that computes optimal reliability, lot size for production, and safety stock was derived by Sarkar et al. [38]. An imperfect production process for a production inventory model in which the unit cost of production represents a function dealing with the reliability parameter of the product and rate of production was developed by Sana [36]. An improvement in producing an inventory model was derived by Sana [37] for imperfect quality items in a three-layer supply chain. A production process which was imperfect for time-dependent demand considering time value of money and inflation was developed by Sarkar et al. [39].

Sarkar and Sarkar [40] constructed an economic manufacturing quality model by taking the system's production in terms of probabilistic deterioration. A multi-item EPQ model over fuzzy-random planning horizon utilizing learning effect on imperfect production was developed by Manna et al. [21]. An inventory model in which the production process is imperfect dealing with the rate of production based on defective rate as well as advertisement-dependent demand was derived by Manna et al. [22]. An improved inventory model dealing with lot sizing and quality investment with quality cost analysis. An improved lot sizing quality investment with quality cost analysis for production which was imperfect and inspection processes having commercial return was proposed by Yoo et al. [46]. An EPQ model for time-dependent demand for production process which is imperfect under inflationary conditions and reliability for time-declining demand by Shah and Shah [41]. An inventory model dealing with inventory decision in a closed-loop supply chain system with learning and rework was proposed by Wakhid et al. [44].

Many researchers have contributed to the field of imperfect production like Rosenblatt and Lee [32], Ben-Daya and Hariga [3], Hayek and Salameh [14], Goyal and Cardenas-Barron [11], Chung and Hou [6], Goyal et al. [12], and Sana et al. [34]. A combined pricing and ordering policy for two echelon production inventory models which was imperfect with two cycles was developed by Pal et al. [27]. A three-layer supply chain was considered by Manna et al. [19] where an inventory model in which the production process is imperfect dealing with two storage facilities in consideration of fuzzy rough environment. Researchers like Mettas [24] and Sana [36] inserted the development cost as a function of reliability parameter only. Practically, with the rise in the time period, the development cost also increases.

Many scholars assumed the market demand a variable one depending on parameters like selling price, stock, or time. Baker and Urban [1], Mandal and Phaujdar [20] and Ray and Chaudhuri [31] derived inventory models dealing with stock-dependent demand. Datta and Pal [9], Teng and Chang [43] presented an inventory system with price sensitive and stock demand rate. Sana and Chaudhuri [35] constructed an inventory model for stock- and advertising-dependent demand. Mondal et al. [25], Panda and Maiti [28] and Chen et al. [5] presented the price-based demand rate. A fuzzy inventory model for deteriorating items with stock-dependent demand rate was presented by Indrajitsingha et al. [15].

An inventory model for deteriorating products with partial backlogging having a stock-based demand and a bound on the extreme level of inventory was represented by Min and Zhou [23]. An EOQ model was constructed by Lee and Dye [18] with partial backlogging, where stock level and deterioration rate estimated the preservation strategies and order size to rise the total profit of system to the maximum. Khara et al. [16] have considered the price reliability depending on the demand rate.

In several derived systems on inventory, the phenomena of deterioration are usually observed in the items, resulting in high harms in quality and also in quantity of items. The inventory cost and management are influenced by the deterioration of items. Ghare and Schrader [10] applied the deterioration effects in their inventory modeling. Covert and Philip [8] mainly utilized Weibull distribution as well as gamma distribution dealing with deteriorating products. A generalized model based on the concept of deterioration was developed by Philip [29]. An inventory model for deteriorating items dealing with strategies of selling price, service investment, and preservation technology with selling price service investment level rate of demand in consideration to common resource constraints was derived by Zhang et al. [47].

Many other research works considering deteriorating items were presented by Nahmias [26], Raafat [30], Shah and Shah [42], Goyal and Giri [13] and Bakker et al. [2].

This article deals with a rate of demand supposed to be a function of stock level, sale price, and reliability of the product under an imperfect production system which produces both perfect and imperfect quality products because of various types of problems such as machinery, labor, technology and the quality of the raw material, etc. A specific percentage of imperfect items are revised at a cost per unit item to become a perfect one. A development cost depending on the time-varying parameter of reliability for manufacturing system was inserted in order to reduce reworking cost and to maintain the reliability of the system dealing with machinery along with the computation of product reliability.

The practical application of this article is to be considered for the packaged products like milk products—butter, cheese; fruit juices; sliced vegetables, etc., which are deteriorating in nature, the packing may be an imperfect undergoes reworking and where demand depends on stock, sale prize, and reliability of the product.

The unit production cost is assumed to be the variable reliability parameter in the manufacturing system, the variable reliability of the item as well as time. The purpose for this article is to calculate the firm's total profit along with the calculation of optimal values for duration of production such that a manufacturer gets a maximum profit, reliability parameter of system, and product's reliability. The classical optimization technique is utilized for calculating the optimal values. For the validation of developed models, numerical examples are demonstrated; then using the concept of eigenvalues of a Hessian matrix, we have proved the concave nature of the profit function of the system, and also the sensitivity analysis is done on each decision variable by fluctuating the inventory parameters for generating operative managerial insights.

The demand rate in this paper is stock, selling price, and reliability-dependent with development cost is based on time and system reliability, and the material costs are dependent on the reliability of product for an imperfect production process for deteriorating inventory makes this article a unique one as demonstrated in Table 7.1.

In this paper, Sect. 7.2 consists of assumptions and notations of the model. In Sect. 7.3, there is the formulation of the proposed mathematical model. In Sect. 7.4,

Table 7.1 Comparative study of related literature for EPQ/EOQ models

Author(s)	EOQ/EPQ	Development cost depends on	Material cost depends on	Demand depends on	Items are deteriorating
Manna et al. [22]	EPQ	Constant (labor)	Constant	Time	No
Manna et al. [19]	EPQ	Constant (labor)	Constant	Stock-dependent	No
Sana [36]	EPQ	System reliability	Constant	Time	No
Sarkar (2012)	EPQ	System reliability	System reliability	Selling price and advertisement cost	No
Sana et al. [34]	EPQ	Labor	Constant	Constant	No
Sarkar and Sarkar [40]	EPQ	System reliability	System reliability	Time	Yes
Shah and Shah [41]	EPQ	System reliability	Constant	Time	No
Khara et al. [16]	EPQ	Time and system reliability	Reliability of product	Selling price and product's reliability	No
This article	EPQ	Time and system reliability	Reliability of product	Stock level, selling price and product's reliability	Yes

the numerical example along with sensitivity analysis has been given. In Sect. 7.5, conclusion and future research are presented.

7.2 Notations and Assumptions

7.2.1 Notations

Parameters	
I(t)	Inventory level in units at time $t \geq 0$
P	Production rate in units per year
δ_{min}	Minimum value of δ
δ_{max}	Maximum value of δ
β	Percentage of defective items reworked to become perfect
DC	Development cost for production at time t for δ (in dollars)
LE	The static cost like labor and energy costs independent of reliability parameter δ (in dollars)
TRD	The technology, resource, and design complexity cost for production when $\delta = \delta_{max}$ (in dollars)
k	The difficulties in raising the reliability of the manufacturing system
P_p	Sell price per unit perfect item (in dollars)
pim	Sell price per unit imperfect item (in dollars)
Dt	Demand depends on reliability r and sell price Pp
PC	Production cost of unit item (in dollars)
T	Fixed cycle time (in years)
Q	Total number of items produced in a production cycle (in units)
MC	Material cost depends on r (in dollars)
MC_0	Fixed material cost (in dollars)
M_1	Material cost increases the reliability of the produced product (in dollars)
h	Cost of holding per unit item per time unit (in dollars)
crew	Cost of reworking of per unit defective item to become perfect (in dollars)
m	Variation constant for tool/die costs
x	Markup for reliability
θo	Deterioration coefficient
α	Scale demand
B	Stock availability coefficient
η	Selling price parameter

(continued)

(continued)

d	The difficulties in increasing reliability of the manufacturing system
Decision variables	
δ	Reliability parameter of the manufacturing system
t_1	Duration of production (in years)
r	Reliability of the product (in years)
Functions	
Dt(I, r, Pp)	Demand rate depending on inventory level
I(δ, r)	Inventory level (in units)
ATP(t_1, δ, r)	The average total profit per period (in dollars)

7.2.2 Assumptions

1. Shortages are impermissible.
2. Inventory model undergoes a production process including perfect as well as imperfect products of a single item.
3. The production reliability is taken as $e^{-\delta t}$ (by Khara et al. [16]) where δ be the parameter for system reliability defined as

$$\delta = \frac{\text{number of defective items}}{\text{total number of produced items within a time interval}} \tag{7.1}$$

   So, δ value decreases, when the reliability of the system is increased.
4. As per assumption (7.3), it is observed that with the rise in time domain, there is a decrement in the reliability of the production system. Therefore, in order to preserve fixed reliability of the production system during the production process, a development cost increasing with time is required. Moreover, to raise the reliability of the system, the development cost must be raised. So, the development cost DC (by Khara et al. [16]) should be considered as a function of time t and δ as follows:

$$DC = LE + TRDte^{\left(\frac{d(\delta_{max}-\delta)}{\delta-\delta_{min}}\right)} \tag{7.2}$$

   Here LE is a fixed cost like labor and energy which does not depend on δ and TRD is the cost of technology, resource, and design complexity for production when $\delta = \delta_{max}$ and t = 1.
5. As, manufacturing system and quality of raw material of the product both are responsible for the reliability of the product parameterized by r. So, we consider that the material cost MC is an increasing function of the reliability of the product r given by

$$MC = MC_0 + M_1(1 - r)^{-x}\, where, \quad MC_0 > 0, M_1 > 0, x > 0 \tag{7.3}$$

6. As per the survey of the production, it can be concluded that the cost of production depends on material cost MC, development cost DC, and tool/die cost. Therefore, the cost of production PC per unit item should be of the following type:

$$PC = MC + \frac{DC}{P} + mP \tag{7.4}$$

   Here m be the proportional constant of tool or die cost, which depends on the number of produced items P.
7. Let the rate of market demand Dt represented as a function of level of inventory in stock, reliability of the product, and selling price of an item.

$$Dt\left(I, r, P_p\right) = (\alpha + IB)(1 - r)^{-b} {P_p}^{-\eta} \tag{7.5}$$

   Here, the rate of demand is an increasing function of level of stock and reliability r of the product as well as a decreasing function of selling price Pp.
8. The model is dealing with finite time horizon.
9. The inventory levels at the initial and terminal stage are zero.
10. A cost is charged for reworking/disposing of the imperfect item.
11. Let $\theta u = \theta o$, $0 \le \theta o \le 1$ be the deterioration coefficient.

7.3 Mathematical Model Formulation

In a process of long run, the imperfect production system makes a perfect and imperfect product, due to various problems related to labor, machinery, and technology. Perfect items are set for sale. In the derived model, let β percent of the items which are defective and undergone reworking process to form a perfect at a cost *pim* and left-out defective items are sold at a cheaper price per unit item. By enhancing the technology, the reworking cost may be reduced by inserting the development cost function DC which increases as the time t increases.

With P and $e^{-\delta t}$ be the constant rate of production and the reliability of the manufacturing system, respectively, the number of perfect items produced by the system is $Pe^{-\delta t}$, and the number of imperfect items is $P\left(1 - e^{-\delta t}\right)$. Let β percent of the imperfect items is reworked. In the duration $t_1 \le t \le T$, the inventory declines due to the united effects of rate of demand and level of stock. The following differential equation demonstrates the inventory level of the system:

$$\frac{dI(t)}{dt} = \begin{cases} Pe^{-\delta t} + \beta P\left(1 - e^{-\delta t}\right) - Dt \ ; & 0 \le t \le t_1 \\ -Dt - \theta u I(t) \ ; & t_1 \le t \le T \end{cases} \tag{7.6}$$

With the boundary conditions $I(0) = I(T) = 0$:

$$I(t) = \begin{cases} -\frac{P(1-r)^b Pp^\eta \left(e^{-\delta t}\beta - e^{-\delta t} - \beta + 1\right)}{-(1-r)^b Pp^\eta \delta + B} & ; \quad 0 \le t \le t_1 \\ \frac{\alpha(1-r)^{-b} Pp^{-\eta}\left(e^{\theta u T} - e^{\theta u t}\right)}{B(1-r)^{-b} Pp^{-\eta} + \theta u} & ; \quad t_1 < t \le T \end{cases} \tag{7.7}$$

The system's total profit is computed by the following components:

1. ***The Sales Revenue***:

The sales revenue of perfect items:

$$SR_p = Pp\left(\int_0^{t_1} \left(Pe^{-\delta t} + \beta P\left(1 - e^{-\delta t}\right)\right)dt\right) \tag{7.8}$$

The sales of defective items with reduced selling price:

$$SR_d = pim\left(\int_0^{t_1} (1-\beta)P\left(1 - e^{-\delta t}\right)dt\right) \tag{7.9}$$

2. ***The Total Production Cost***:

$$TPC = P\left(\int_0^{t_1} PCdt\right) \tag{7.10}$$

3. ***The Total Holding Cost***:

$$HC = h\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right) \tag{7.11}$$

4. ***The Total Reworking Cost***:

$$RWC = crew\left(\int_0^{t_1} \beta P\left(1 - e^{-\delta t}\right)\right) \tag{7.12}$$

Therefore, the average total Profit of the system is calculated by

$$ATP = \frac{1}{T}\left(SR_p + SR_d - TPC - HC - RWC\right) \tag{7.13}$$

In order to maximize the average total profit, we follow the conditions stated below.

By evaluating the partial derivatives and equating them to zero:

$$\frac{\partial ATP}{\partial \delta} = 0, \ \frac{\partial ATP}{\partial r} = 0, \ \frac{\partial ATP}{\partial t_1} = 0 \tag{7.14}$$

For testing the concave nature of the average total profit at the gained solution, the below listed algorithm is to be followed:

Step 1: First assign any particular hypothetical values to the inventory parameters.
Step 2: Solving the simultaneous equations stated in Eq. (7.14) using the mathematical software Maple XVIII to find the solution.
Step 3: Calculating all possible eigenvalues of Hessian matrix H presented below at the optimal point obtained from Eq. (7.14):

$$H = \begin{bmatrix} \frac{\partial^2 ATP}{\partial t_1^2} & \frac{\partial^2 ATP}{\partial t_1 \partial \delta} & \frac{\partial^2 ATP}{\partial t_1 \partial r} \\ \frac{\partial^2 ATP}{\partial \delta \partial t_1} & \frac{\partial^2 ATP}{\partial \delta^2} & \frac{\partial^2 ATP}{\partial \delta \partial r} \\ \frac{\partial^2 ATP}{\partial r \partial t_1} & \frac{\partial^2 ATP}{\partial r \partial \delta} & \frac{\partial^2 ATP}{\partial r^2} \end{bmatrix} \tag{7.15}$$

If each eigenvalue is positive, then the matrix is called as positive-definite matrix. Therefore, the average total profit is concave in nature and the process is stopped.

As such it is difficult to prove analytically the positive definiteness of Hessian matrix, so we prefer numerical as well as graphical way of representing the solution helping us the visualization of the concave nature of average total profit function.

7.4 Numerical Example and Sensitivity Analysis

7.4.1 Numerical Example

Example 1 Consider the following:

$P = 1000\ units, \alpha = 3, Pp = 20\ dollars/unit, b = 3, \eta = 1.001, d = 0.1,$
$LE = 10\ dollars/unit, TRD = 10\ dollars/unit, \delta_{min} = 0.01, \delta_{\boldsymbol{max}} = 0.9,$
$pim = 18\ dollars/unit, crew = 5\ dollars/unit, h = 5\ dollars/unit, \beta = 0.5,$
$T = 2\,years, B = 0.2, \theta u = 0.1.$

Solution:

The optimum values of the decision variables are as follows:

Reliability parameter of the manufacturing system $= \delta = 0.0531$.
The duration of production $= t_1 = 1.9590\,years$.

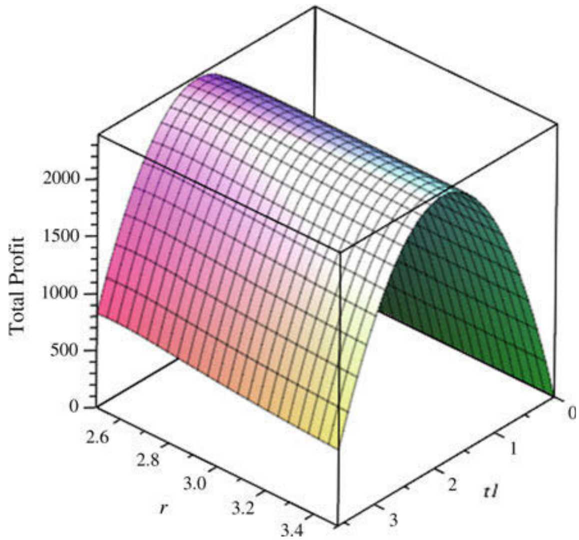


Fig. 7.1 Concavity of profit function

The Reliability of the product (in years) $r = 2.6969$.
The average total profit $ATP = 2390.4888$ dollars.

Concavity of Total Profit function:

Therefore, by applying the algorithm, the concave nature of the average total profit function is been verified which is demonstrated in Fig. 7.1. Now computing the optimum values of decision variables and also undertaking the sensitivity analysis of the values of decision variables by fluctuating the values of inventory parameters −20 to 20%. The Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 ATP}{\partial t_1^2} & \frac{\partial^2 ATP}{\partial t_1 \partial \delta} & \frac{\partial^2 ATP}{\partial t_1 \partial r} \\ \frac{\partial^2 ATP}{\partial \delta \partial t_1} & \frac{\partial^2 ATP}{\partial \delta^2} & \frac{\partial^2 ATP}{\partial \delta \partial r} \\ \frac{\partial^2 ATP}{\partial r \partial t_1} & \frac{\partial^2 ATP}{\partial r \partial \delta} & \frac{\partial^2 ATP}{\partial r^2} \end{bmatrix} = \begin{bmatrix} -1203.9912 & 830.5764 & -71.8512 \\ 830.5764 & -2.4487 \times 10^5 & 2631.111 \\ -71.8512 & 2631.111 & -620.2513 \end{bmatrix}$$

Eigenvalues of the Hessian matrix are

$$\lambda_1 = -2.4490 \times 10^5 < 0, \lambda_2 = -1207.5819 < 0, \lambda_3 = -585.4858 < 0$$

7.4.2 *Sensitivity Analysis of the Optimal Inventory Policy*

This section deals with the sensitivity analysis of the values of the decision variables with respect to various inventory parameters. Table 7.2 shows the values of decision

Table 7.2 Sensitivity analysis for decision variables with respect to various inventory parameters

Inventory parameters	Decision variables	Percentage variation of decision variables				
		−20%	−10%	0	10%	20%
P	δ	0.0621	1.4720	0.0531	0.0511	0.0496
	r	−688.833	3.9540	2.6969	2.7421	2.8066
	t_1	1.990	2.4939	1.9590	1.5503	1.1498
	ATP	2480.017	1555.0309	2390.4888	1671.5181	1017.8692
P_p	δ	0.0585	0.0548	0.0531	0.0531	0.0577
	r	3.0742	2.801	2.6969	2.6932	3.3807
	t_1	0.3856	1.1720	1.9590	1.9983	1.9875
	ATP	94.8695	863.0107	2390.4888	2486.4897	2505.8672
η	δ	0.0504	0.0504	0.0531	0.0531	0.0612
	r	2.6202	2.6203	2.6969	2.6970	−1446.7632
	t_1	1.9895	1.9895	1.9590	1.9590	1.9096
	ATP	2422.1848	2422.1810	2390.4888	2390.4642	2336.7948
d	δ	0.0447	0.0490	0.0531	0.0572	0.0684
	r	2.6566	2.6776	2.6969	2.7147	−1749.6744
	t_1	1.9814	1.9693	1.9590	1.9500	1.9017
	ATP	2420.7283	2404.6460	2390.4888	2377.7979	2322.0651
LE	δ	0.0531	0.0531	0.0531	0.0531	0.0531
	r	2.6968	2.6969	2.6969	2.6970	2.6970
	t_1	1.9598	1.9594	1.9590	1.9586	1.9581
	ATP	2392.4482	2391.4683	2390.4888	2389.5093	2388.5302
δ_{min}	δ	0.0506	0.0519	0.0531	0.0544	0.0556
	r	2.6854	2.6913	2.6969	2.7024	2.7077
	t_1	1.9633	1.9611	1.9590	1.9569	1.9550
	ATP	2396.8724	2393.6299	2390.4888	2387.4403	2384.4769
TRD	δ	0.0502	0.0542	0.0531	0.0570	0.0658
	r	2.6830	2.7013	2.6969	2.7138	−1678.4046
	t_1	1.9703	1.9602	1.9590	1.9501	1.9025
	ATP	2405.2286	2391.3240	2390.4888	2378.0879	2324.9107
δ_{max}	δ	0.0444	0.0488	0.0531	0.0574	0.0688
	r	2.6549	2.6768	2.6969	2.7155	−1748.2066
	t_1	1.9829	1.9700	1.9590	1.9494	1.9009
	ATP	2422.5322	2405.5113	2390.4888	2376.9768	2320.8915

(continued)

Table 7.2 (continued)

Inventory parameters	Decision variables	Percentage variation of decision variables				
		−20%	−10%	0	10%	20%
MC_0	δ	0.0618	0.0537	0.0531	0.0530	0.0530
	r	−1586.4048	2.7384	2.6969	2.7014	2.7060
	t_1	1.9914	1.9982	1.9590	1.9172	1.8755
	ATP	2531.8457	2488.9230	2390.4888	2293.5829	2198.7642
M1	δ	0.0530	0.0531	0.0531	0.0532	0.0532
	r	2.6868	2.6919	2.6969	2.7018	2.7067
	t_1	1.9600	1.9595	1.9590	1.9585	1.9581
	ATP	2391.5083	2390.9909	2390.4888	2390.0013	2389.5277
x	δ	0.0612	0.0612	0.0531	0.0612	0.0513
	r	−1766.3859	1875.9349	2.6969	1652.7826	2.5366
	t_1	1.9096	1.9096	1.9590	1.9096	1.9804
	ATP	2336.7947	2336.7947	2390.4888	2336.7947	2415.9664
m	δ	0.0618	0.0532	0.0531	0.0587	0.0517
	r	−1586.4048	2.6965	2.6969	−1699.7276	2.8181
	t_1	1.9914	1.9632	1.9590	1.5083	1.1432
	ATP	2531.8457	2400.2943	2390.4888	1482.7772	842.8551
pim	δ	0.0522	0.0501	0.0531	0.0531	0.0765
	r	−1546.2140	2.6883	2.6969	2.6969	−1459.4028
	t_1	1.8258	1.9184	1.9590	1.9590	1.9732
	ATP	2242.5691	2343.6451	2390.4888	2390.4888	2400.4704
crew	δ	0.0841	0.0781	0.0531	0.0698	0.0555
	r	−1434.9683	−1454.3962	2.6969	−1670.1492	2.7040
	t_1	1.9910	1.9773	1.9590	1.9517	1.9814
	ATP	2415.3901	2404.0580	2390.4888	2380.3736	2415.9136
T	δ	0.0531	0.0531	0.0531	0.0531	0.0531
	r	2.6969	2.6969	2.6969	2.6969	2.6969
	t_1	1.9590	1.9590	1.9590	1.9590	1.9589
	ATP	2988.1183	2656.1000	2390.4888	2173.1740	1992.0818
B	δ	0.0549	0.0540	0.0531	0.0523	0.0515
	r	2.7566	2.7249	2.6969	2.6720	2.6497
	t_1	1.9443	1.9514	1.9590	1.9671	1.9757
	ATP	2374.9973	2382.4910	2390.4888	2398.9743	2407.9320
θu	δ	0.0531	0.0531	0.0531	0.0531	0.0531
	r	2.6969	2.6969	2.6969	2.6969	2.6969
	t_1	1.9590	1.9590	1.9590	1.9590	1.9590
	ATP	2390.4888	2390.4887	2390.4888	2390.4887	2390.4888

variables on fluctuating the various inventory parameters in the range −20 to 20%. The following observations are extracted from Table 7.2.

Sensitivity analysis for rate of rework for imperfect quality products in units per year (P):

With the fluctuation of rework rate of imperfect quality items, the reliability parameter of the manufacturing system decreases with the declination of reliability of the product, and the duration of production decreases with an increment in the annual total profit.

Sensitivity analysis of sell price of product $\left(P_p\right)$:

The manufacturing system's reliability parameter decreases with declination in terms of reliability of the product, and the duration of production increases with an increment in the annual average total profit with regard to the fluctuation of product's selling price.

Sensitivity analysis of markup for parameter of sell price (η):

The reliability parameter of the manufacturing system increases with the uplifting of reliability of the product, and the duration of production decreases with a decrement in the annual total profit with respect to the variation of the markup for sell price parameter.

Sensitivity analysis of difficulties in increasing reliability of system (d):

With the variation in difficulties in increasing reliability of system, the system's reliability parameter increases with increment in the reliability of the product, and the duration of production decreases with a decrement in the annual total profit.

Sensitivity analysis of the static cost like in terms of labor and energy is independent of the reliability parameter $\delta(LE)$:

The reliability parameter of the manufacturing system remains constant with the uplifting of reliability of the product, the duration of production decreases with an increase in the annual total profit with respect to the variation of the static cost like in terms of labor, and energy is independent of the reliability parameter δ.

Sensitivity analysis of the minimum value of (δ_{min}):

With the variation in the minimum value of δ, the system's reliability parameter increases with increment in the reliability of the product, and the duration of production decreases with a decrement in the annual total profit.

Sensitivity analysis of the cost of technology, resource, and design complexity for production when $\delta = \delta_{\max}$ *and* $t = 1$ (TRD):

The system's reliability parameter increases along with increment in the reliability of the product, and the duration of production decreases with a decrement in the annual total profit with respect to the variation of cost of resource, technology, as well as design complexity for production of the product.

Sensitivity analysis of maximum value of $\delta(\boldsymbol{\delta_{max}})$:

With the variation in the maximum value of δ, the system's reliability parameter increases along with the increment of reliability of the product, and the duration of production decreases with a decrement in the annual total profit.

Sensitivity analysis of fixed material cost $(\boldsymbol{MC}_0)$:

The system's reliability parameter increases along with the increment of reliability of the product, and the duration of production decreases with a decrement in the annual total profit with respect to the variation of fixed material cost of the product.

Sensitivity analysis of material cost increases the reliability of the produced item $(\boldsymbol{M}1)$:

With the variation in the material cost, the system's reliability parameter increases with the increment of reliability of the product, and the duration of production decreases with a decrement in the annual total profit.

Sensitivity analysis of markup of product's reliability $(\boldsymbol{x})$:

The system's reliability parameter decreases with the decrement of reliability of the product, and the duration of production increases with an increment in the annual total profit with regard to the fluctuation of markup of reliability of the product.

Sensitivity analysis of variation constant of tool/die costs $(\boldsymbol{m})$:

With the change in the variation constant of tool/die costs, the system's reliability parameter increases with the increment of reliability of the item, and the production duration decreases with a decrement in the annual total profit.

Sensitivity analysis of selling price per unit imperfect item $(\boldsymbol{pim})$:

There is an increment in the system's reliability parameter with the increment of reliability of the item, and the duration of production also becomes lengthier with an increment in the annual total profit with regard to the fluctuation of selling price per unit imperfect item.

Sensitivity analysis of reworking cost on per unit defective item to become perfect (*crew*)

With the change in the variation constant of reworking costs, the system's reliability parameter decreases with the decrease in reliability of the product, and the duration of production shortens with a decrement in the annual total profit.

Sensitivity analysis of Replenishment cycle length (*T*):

The system's reliability parameter and the product's reliability remain constant, and the production duration decreases with a decrement in the annual total profit with regard to the variation of replenishment cycle length.

Sensitivity analysis of coefficient of Stock availability (*B*)

As the stock availability coefficient varies, the system's reliability parameter decreases with the decrement of reliability of the product, and the duration of production increases with an increment in the annual total profit.

Sensitivity analysis of coefficient of deterioration (θu):

There is no variation in the decision variable with the change in the inventory parameters. Each parameter remains constant.

7.5 Conclusion and Future Scope

This article focuses on developing a production model dealing with product's sell price-stock level as well as reliability-based demand; also it undergoes a production process which is imperfect including manufacturing of perfect and imperfect quality items. Computation of the optimal values of product's reliability parameter and reliability of product's duration for production is done. A numerical illustration yields us the following optimum solutions of decision variables like the reliability parameter of the manufacturing system be $\delta = 0.0531$, the duration of production be $t_1 = 1.9590 years$, the reliability of the product (in years) be $r = 2.6969$, and the average total profit $ATP = 2390.4888$ dollars.

By varying the various inventory parameters like the reduction in reworking cost on per unit defective item to become perfect, increment in the selling price per unit imperfect item and selling price per unit perfect item results in uplifting the total profit of the system, which is desirable. So, these managerial insights would be provided to the manufacturing firm to uplift the average total profit level of system.

Some possible future directions for extension of work are as follows:

1. For rising the demand of the firm advertising and/or service investment efforts can be utilized.
2. The concept of discounts, learning effects could be considered.

3. Shortages can be considered.
4. Preservation investment technology can be utilized to lower the deterioration effect.

Acknowledgements The authors are very much thankful to the reviewers for their deep and thorough review. The authors thank DST-FIST File# MSI-097 for the technical support to the department.

References

1. Baker RC, Urban TL (1988) A deterministic inventory system with an inventory-level dependent demand rate. J Oper Res Soc 39(9):823–831
2. Bakker M, Riezebos J, Teunter RH (2012) Review of inventory systems with deterioration since 2001. Eur J Oper Res 221:275–284. http://dx.doi.org/10.1016/j.ejor.2012.03.004
3. Ben-Daya M, Hariga M (2000) Economic lot scheduling problem with imperfect production processes. J Oper Res Soc 51(7):875–881
4. Cardenas-Barron LE (2009) Economic production quantity with rework process at a single-stage manufacturing system with planned backorders. Comput Ind Eng 57(3):1105–1113
5. Chen H, Chen Y, Chiu CH, Choi TM, Sethi S (2010) Coordination mechanism for the supply chain with lead time consideration and price-dependent demand. Eur J Oper Res 203(1):70–80
6. Chung KJ, Hou KL (2003) An optimal production run time with imperfect production processes and allowable shortages. Comput Oper Res 30(4):483–490
7. Chung CJ, Wee HM (2008) An integrated production-inventory deteriorating model for pricing policy considering imperfect production, inspection planning and warranty-period and stock-level-dependent demand. Int J Syst Sci 39(8):823–837
8. Covert RP, Philip GC (1973) An EOQ model for items with Weibull distribution deterioration. AIIE Trans 5(4):323–326
9. Datta TK, Pal K (2001) An inventory system with stock dependent, price-sensitive demand rate. Prod Plan Control 12(1):13–20
10. Ghare PM, Schrader GF (1963) A model for exponentially decaying inventory. J Ind Eng 14(2):238–243
11. Goyal SK, Cardenas-Barron LE (2002) Note on: economic production quantity model for items with imperfect quality—a practical approach. Int J Prod Econ 77(1):85–87
12. Goyal SK, Hung CK, Chen KC (2003) A simple integrated production policy of an imperfect item for vendor and buyer. Prod Plan Control 14(7):596–602
13. Goyal SK, Giri BC (2001) Recent trends in modeling of deteriorating inventory. Eur J Oper Res 134(1):1–16
14. Hayek PA, Salameh MK (2001) Production lot sizing with the reworking of imperfect quality items produced. Prod Plan Control 12(6):584–590
15. Indrajitsingha SK, Samanta PN, Misra UK (2018) A fuzzy inventory model for deteriorating items with stock dependent demand rate. Int J Logist Syst Manag 30(4):538–555
16. Khara B, Dey JK, Mondal SK (2017) An inventory model under development cost-dependent imperfect production and reliability-dependent demand. J Manag Anal 4(3):258–275
17. Khouja M, Mehrez A (1994) Economic production lot size model with variable production rate and imperfect quality. J Oper Res Soc 45(12):1405–1417
18. Lee YP, Dye CY (2012) An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. Comput Ind Eng 63(2):474–482
19. Manna AK, Dey JK, Mondal SK (2014) Three-layer supply chain in an imperfect production inventory model with two storage facilities under fuzzy rough environment. J Uncertain Anal Appl 2(1):17

20. Mandal BN, Phaujdar S (1989) An inventory model for deteriorating items and stock dependent consumption rate. J Oper Res Soc 40(5):483–488
21. Manna AK, Das B, Dey JK, Mondal SK (2017) Multi-item EPQ model with learning effect on imperfect production over fuzzy-random planning horizon. J Manag Anal 4(1):80–110
22. Manna AK, Dey JK, Mondal SK (2017) Imperfect production inventory model with production rate dependent defective rate and advertisement dependent demand. Comput Ind Eng 104:9–22
23. Min J, Zhou YW (2009) A perishable inventory model under stock-dependent selling rate and shortage-dependent partial backlogging with capacity constraint. Int J Syst Sci 40(1):33–44
24. Mettas A (2000) Reliability allocation and optimization for complex systems. In: Proceedings of the annual reliability and maintainability symposium. Institute Electrical and Electronics Engineers, Piscataway, NJ, pp 216–221
25. Mondal B, Bhunia AK, Maiti M (2003) An inventory system of ameliorating items for price dependent demand rate. Comput Ind Eng 45(3):443–456
26. Nahmias S (1982) Perishable inventory theory: a review. Oper Res 30(4):680–708
27. Pal B, Sana SS, Chaudhuri KS (2014) Joint pricing and ordering policy for two echelon imperfect production inventory model with two cycles. Int J Prod Econ 155:229–238
28. Panda D, Maiti M (2009) Multi-item inventory models with price dependent demand under flexibility and reliability consideration and imprecise space constraint: a geometric programming approach. Math Comput Model 49(9–10):1733–1749
29. Philip GC (1974) A generalized EOQ model for items with Weibull distribution. AIIE Trans 6(2):159–162
30. Raafat F (1991) Survey of literature on continuously deteriorating inventory models. J Oper Res Soc 42(1):27–37
31. Ray J, Chaudhuri KS (1997) An EOQ model with stock-dependent demand, shortage, inflation and time discounting. Int J Prod Econ 53(2):171–180
32. Rosenblatt MJ, Lee HL (1986) Economic production cycles with imperfect production processes. IIE Trans 18(1):48–55
33. Salameh MK, Jaber MY (2000) Economic production quantity model for items with imperfect quality. Int J Prod Econ 64(1):59–64
34. Sana SS, Goyal SK, Chaudhuri K (2007) An imperfect production process in a volume flexible inventory model. Int J Prod Econ 105(2):548–559
35. Sana SS, Chaudhuri KS (2008) An inventory model for stock with advertising sensitive demand. IMA J Manag Math 19(1):51–62
36. Sana SS (2010) A production inventory model in an imperfect production process. Eur J Oper Res 200(2):451–464
37. Sana SS (2011) A production-inventory model of imperfect quality products in a three-layer supply chain. Decis Support Syst 50(2):539–547
38. Sarkar B, Sana SS, Chaudhuri K (2010) Optimal reliability, production lot size and safety stock in an imperfect production system. Int J Math Oper Res 2(4):467–490
39. Sarkar B, Sana SS, Chaudhuri KS (2011) An imperfect production process for time varying demand with inflation and time value of money—an EMQ model. Expert Syst Appl 38(11):13543–13548
40. Sarkar M, Sarkar B (2013) An economic manufacturing quantity model with probabilistic deterioration in a production system. Econ Model 31:245–252
41. Shah NH, Shah BJ (2014) EPQ model for time-declining demand with imperfect production process under inflationary conditions and reliability. Int J Oper Res 11(3):091–099
42. Shah NH, Shah YK (2000) Literature survey on inventory model for deteriorating items. Econ Annals 44(145):221–237
43. Teng JT, Chang CT (2005) Economic production quantity models for deteriorating items with price- and stock-dependent demand. Comput Oper Res 32(2):297–308
44. Kurdhi N, Jauhari WA, Hendaryani O (2018) Inventory decisions in a closed-loop supply chain system with learning and rework. Int J Procure Manag 11(5):https://doi.org/10.1504/ijpm.2018.10014395

45. Yoo SH, Kim D, Park MS (2009) Economic production quantity model with imperfect-quality items, two-way imperfect inspection and sales return. Int J Prod Econ 121(1):255–265
46. Yoo SH, Kim D, Park M-S (2012) Lot sizing and quality investment with quality cost analyses for imperfect production and inspection processes with commercial return. Int J Prod Econ 140(2):922–933
47. Zhang J, Wei Q, Zhang Q, Tang W (2016) Pricing, service and preservation technology investments policy for deteriorating items under common resource constraints. Comput Ind Eng 95(1):1–9

Chapter 8
Imperfect Quality Item Inventory Models Considering Carbon Emissions

Hui-Ming Wee and Yosef Daryanto

Abstract Research on inventory models with environmental consideration has recently become a popular research stream. The amount of energy consumption and greenhouse gas emissions is influenced by inventory decisions such as delivery quantity and delivery frequencies. This chapter focuses on a supply chain system which contains a percentage of imperfect quality items in its delivered lot; we also consider carbon emission costs under a carbon tax policy. Processing the defective items, which increases carbon emission, affects supply chain decisions. We present two economic order quantity models considering carbon emission and defective items with different shortage conditions. We then study low-carbon two-echelon supply chain inventory model considering supply chain integration and imperfect quality items. Numerical examples are provided to illustrate how these models can be applied in practice. Sensitivity analysis is performed to gain more insight on changing parameters in the numerical studies.

Keywords Inventory · Supply chain · Carbon emission · Imperfect quality

8.1 Introduction and Related Literature

Existing and planned legislation penalizing high energy consumption and greenhouse gasses emission could be used to encourage many industries to develop a greener supply chain. Until 2018, at least 51 carbon pricing initiatives have been implemented or are scheduled for implementation worldwide [34]. 26 carbon tax initiatives and 25 emission trading systems have been implemented in various national and subnational jurisdictions. Besides direct energy costs, changing consumer preference is another factor driving businesses to become environmentally friendly [25].

H.-M. Wee (✉) · Y. Daryanto
Department of Industrial and Systems Engineering, Chung Yuan Christian University, Taoyuan City, Taiwan
e-mail: weehm@cycu.edu.tw

Y. Daryanto
Department of Industrial Engineering, Universitas Atma Jaya Yogyakarta, Yogyakarta, Indonesia
e-mail: yosef.daryanto@uajy.ac.id

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_8

Freight transport and material storing and handling are significant sources of CO_2 emission in logistics and supply chain activities [17]. Therefore, inventory replenishment decision on how much, when, and where to order and deliver will influence the total supply chain carbon footprint. Carbon emissions also come from the disposing or recycling of the materials [25] as well as product return from customers [18].

Direct accounting approaches based on carbon tax mechanisms can translate the environmental aspect of carbon emission into an economic parameter [10, 22]. In general, the purpose of a carbon tax is to act as a financial penalty to industries that produce emissions from their activities [2]. Bonney and Jaber [3] are among the early researchers that incorporate carbon emission cost into the EOQ model. They proposed a simple non-classical model that includes vehicle emission and waste disposal costs in addition to the ordering cost, purchase cost, holding cost, and transportation cost. The vehicle emissions cost considers the effect of the vehicle emission cost per hour, delivery distance, and vehicle's average speed. Battini et al. [1] considered the fix and variable transport cost and emission, warehousing holding cost and emission, and the emission from obsolete material collection and disposal. The warehousing emissions consider the occupied space by the inventory and the disposal emissions consider the weight of the obsolete inventory. Recently, Kazemi et al. [15] studied the impact of emission costs on the total profit of a buyer in an imperfect supply process to develop some inventory replenishment models by considering warehouse operation emission. They also developed EOQ models considering the learning effects and inspection errors of the flawed quality items.

Other researchers incorporated carbon emission cost into the supply chain inventory model. Wahab et al. [28] studied a two-echelon supply chain inventory model considering the environmental impact of transporting inventory with the aim of reducing CO_2 emission. It considered a percentage of defective items per shipment that will be transported back to the vendor. Jauhari et al. [14] considered carbon emission from transportation activities in a supply chain with unequal-sized shipment and defective products. Sarkar et al. [21] assumed a fixed and variable emission cost from transportation, and the defective items would be transported back to the vendor for rework. Focused on a cold product supply chain, Hariga et al. [11] studied the cost saving and carbon emission reduction by integrating emission cost into cold product supply chain model. Cold product supply chain consumes more fuel and electricity because it requires a special temperature-controlled truck and freezer storage unit. Tiwari et al. [24] and Daryanto and Wee [5] considered the transportation cost and emission, warehousing holding cost and emission, and emission from the disposal of deteriorated items. Recently, Daryanto et al. [6] considered variable transportation cost and carbon emission in a three-echelon supply chain.

The impact of imperfect product quality on the supply chain system has received the attention of many researchers. These problems are believed to affect the service level of the system and incur significant costs. For example, Salameh and Jaber [20] and Wee et al. [33] extended the traditional economic order quantity model considering imperfect quality items and different shortage backorders. Wee et al. [32] and Wang et al. [29] studied the EOQ model for imperfect quality items by considering the screening rate under different shortage backorders. Cárdenas-Barrón [4], Hayek and

Salameh [12], and Goyal and Cárdenas-Barrón [8] considered the impact of imperfect quality in economic production quantity models. Recently, Jaggi et al. [13] studied the impact of imperfect quality and item deterioration on two-warehouse inventory model with a permissible delay in payment. This chapter assumes that decisions dealing with the imperfect items in the supply chain will affect carbon emissions due to processing the defective items.

This introduction and literature review is followed by model developments. Section 8.2 presents the low-carbon economic order quantity (EOQ) models. The objectives of the models are to optimize the order quantity that will maximize the total profit. Section 8.3 consists of model development for low-carbon two-echelon supply chain inventory models considering supply chain integration and imperfect quality items. The objective of the model is to optimize the delivery quantity and number of deliveries per cycle that will minimize the total cost. Section 8.4 concludes this chapter with remarks about findings and further research in the future.

8.2 Low-Carbon EOQ Models for Imperfect Quality Items

This section presents the EOQ model for imperfect quality items considering a carbon emission cost. This study extends previous models by considering the fixed and variable transportation costs and emissions. The variable transportation emission depends on vehicle loads per shipment and delivery distance. The total cost also considers a warehouse emission cost by assuming average energy consumption per unit stored. This study can be used to support a company's green initiatives in reducing carbon emissions by optimizing their inventory decisions. Two EOQ models with different shortage conditions are developed.

The notation for the model is presented as follows:

Decision variables	
Q	Optimum order size (units)
$Q*$	Optimum order size without shortage
Q_B*	Optimum order size with complete backorder
B	Backorder quantity (units)
T	Cycle length (time unit)
Parameters	
D	Demand rate (units/year)
A	Purchase cost per unit ($/unit)
c	Buyer's ordering cost per order ($/order)
δ	Defective percentage of imperfect items in Q
$f(\delta)$	The probability density function of δ

(continued)

(continued)

Parameters	
x	Quality screening rate (units/minute or units/year)
Dt_s	Demand during screening time (units)
t_s	Quality screening time
u_c	Buyer's quality screening cost per unit item ($/unit)
h_b	Buyer's holding cost per unit item per year ($/unit/year)
w	Average warehouse energy consumption per unit product (kWh/unit/year)
s_p	Selling price per unit item ($/unit)
s_v	Salvage value per defective item, ($/unit; $s_v < s_p$)
d	Distance traveled from supplier to buyer (km)
t_f	Fixed transportation cost per delivery ($)
t_v	Variable transportation cost ($/liter)
c_1	Vehicle fuel consumption when empty (liter/km)
c_2	Additional vehicle fuel consumption per ton of payload (liter/km/ton)
e_1	Carbon emission cost from vehicle ($/km); $e_1 = c_1.F_e.T_x$
e_2	Additional carbon emission cost from transporting one unit item ($/unit/km); $e_2 = c_2.l.F_e.T_x$
l	Product weight (ton/unit)
F_e	Standard emission from fuel combustion ($tonCO_2$/liter)
E_e	Standard emission from electricity generation ($tonCO_2$/kWh)
T_x	Carbon price or tax ($/$tonCO_2$)
b	Backorder cost per unit item per year ($/unit/year)

This study has certain assumptions as follows:

(1) A single product is considered.
(2) Demand rate is known and constant.
(3) The replenishment is instantaneous.
(4) Customer demand and the screening process proceeds simultaneously, start from time 0, and the screening rate is greater than the demand rate, $x > D$.
(5) The defective percentage, δ, has a uniform distribution with $[\alpha, \beta]$, where $0 \leq \alpha < \beta < 1$.
(6) The δ is restricted to $E[\delta] \leq 1 - (D/x)$ to avoid a shortage during the screening period.
(7) The defective items were withdrawn as a lot at time t_s.
(8) Defective items will be sold at s_v rate immediately (no holding cost).

8.2.1 *Basic EOQ Model for Imperfect Quality Items Considering Carbon Emission*

We depict the inventory level of the sustainable EOQ for imperfect quality items without shortage in Fig. 8.1. After receiving Q units from the vendor, the buyer undertakes a quality inspection. At time t_s, the inspection is complete. As δ is the probability of the defective products, the expected defective product per delivery is δQ, and then this amount will be removed from the storage. Further, the inventory level continues to decrease over the period $[t_s, T]$ and reaches zero at time T.

The total revenue per cycle $TR(Q)$ is the sum of total sales volume of non-defective items with a sales price s_p and the sales of defective items with a sales price s_v. Hence,

$$TR(Q) = (1 - \delta)Qs_p + \delta Qs_v \tag{8.1}$$

The total cost per cycle $TC(Q)$ is presented in Eq. (8.2).

$$TC(Q) = C_O + C_P + C_I + C_H + C_T \tag{8.2}$$

in which the sum of ordering cost ($C_o = c$), purchasing cost ($C_p = AQ$), and quality screening cost ($C_I = u_c Q$).

The holding cost considers both traditional carrying cost and carbon emission cost generated by warehousing. C_H is equal to (h_b+ wE_eT_x) multiplied by the average amount of inventory per cycle, which is equivalent to the area in Salameh and Jaber [20]. Because $T = (1 - \delta)Q/D$, therefore

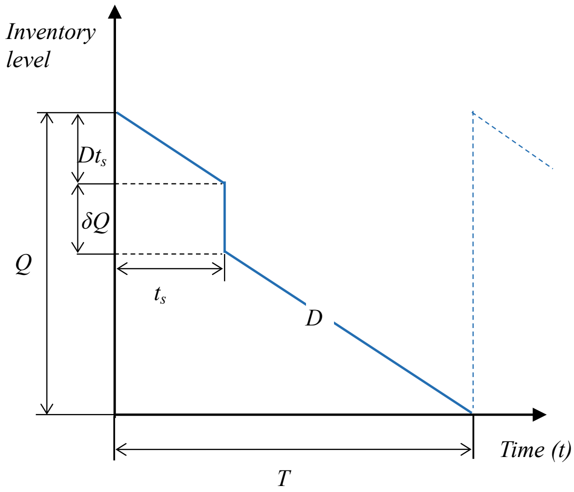


Fig. 8.1 Inventory level for imperfect quality items

$$C_H = (h_b + wE_eT_x)\left(\frac{(1-\delta)^2Q^2}{2D} + \frac{\delta Q^2}{x}\right) \tag{8.3}$$

The vendor's transportation cost consists of a fix transport cost, variable transport cost, and carbon emission cost. The variable transport cost depends on delivery distance, vehicle fuel consumption, the additional fuel consumption per ton of payload, product weight, delivery quantity, and the fuel price. The carbon emission cost depends on the delivery distance, delivery quantity, and standard vehicle emission cost for product delivery (e_1 and e_2).

$$C_{TE} = t_f + (2dc_1t_v + dc_2lQt_v) + (2de_1 + de_2Q) \tag{8.4}$$

Therefore, the total cost per cycle becomes

$$TC(Q) = c + AQ + u_cQ + (h_b + wE_eT_x)\left(\frac{(1-\delta)^2Q^2}{2D} + \frac{\delta Q^2}{x}\right) + t_f + (2dc_1t_v + dc_2lQt_v) + (2de_1 + de_2Q) \tag{8.5}$$

The objective is to maximize the total profit per unit time, *TP*, therefore

$$TP(Q) = \frac{TR(Q) - TC(Q)}{T} \tag{8.6}$$

Since the defective percentage, δ, has a uniform distribution, the expected value of *TP(Q)* is

$$ETP(Q) = D\left(s_p - s_v + \frac{h_bQ}{x} + \frac{wE_eT_xQ}{x}\right) - (h_b + wE_eT_x)\frac{Q}{2} + (h_b + wE_eT_x)\frac{Q}{2}E[\delta] + D\left(s_v - A - u_c - dc_2lt_v - de_2 - \frac{h_bQ}{x} - \frac{wE_eT_xQ}{x} - \frac{c}{Q} - \frac{t_f}{Q} - \frac{2dt_vc_1}{Q} - \frac{2de_1}{Q}\right)E\left[\frac{1}{1-\delta}\right] \tag{8.7}$$

Further, the expected total emission (*ETE*) per unit time can be derived from Eq. (8.7) as

$$ETE(Q) = \left(wE_e\left(\frac{(1-E[\delta])^2Q^2}{2D} + \frac{E[\delta]Q^2}{x}\right) + (2dc_1F_e + dc_2lF_eQ)\right)\left(\frac{D}{(1-E[\delta])Q}\right) \tag{8.8}$$

For a uniform distribution, notes that

$$E[\delta] = \int_{\alpha}^{\beta} (\delta).f(\delta)d\delta = \left(\frac{\alpha+\beta}{2}\right)$$

$$E\left[\frac{1}{1-\delta}\right] = \int_{\alpha}^{\beta} \frac{1}{1-\delta}.f(\delta)d\delta = \frac{1}{(\beta-\alpha)}\ln\left(\frac{1-\alpha}{1-\beta}\right)$$

To find Q, we must first prove the concavity of the total profit function. By taking the first and second derivatives of ETP with respect to Q yields

$$\frac{\partial ETP}{\partial Q} = D\left(\frac{h_b + wE_eT_x}{x}\right) - \left(\frac{h_b + wE_eT_x}{2}\right) + \left(\frac{h_b + wE_eT_x}{2}\right)E[\delta] + D\left(\frac{c}{Q^2} + \frac{t_f}{Q^2} + \frac{2dt_vc_1}{Q^2} + \frac{2de_1}{Q^2} - \frac{h_b}{x} - \frac{wE_eT_x}{x}\right)E\left[\frac{1}{1-\delta}\right] \tag{8.9}$$

$$\frac{\partial^2 ETP}{\partial^2 Q} = D\left(-\frac{2c}{Q^3} - \frac{2t_f}{Q^3} - \frac{4dt_vc_1}{Q^3} - \frac{4de_1}{Q^3}\right)E\left[\frac{1}{1-\delta}\right] \tag{8.10}$$

Because all the parameter values are positive, for $Q > 0$, Eq. (8.10) is always negative. Therefore, the profit function is strictly concave. By setting the first derivative equal to zero, the optimal quantity Q^* can be solved. Using Maple software, one has

$$Q^* = \sqrt{\frac{2D(c + t_f + 2dt_vc_1 + 2de_1)xE\left[\frac{1}{1-\delta}\right]}{(h_b + wE_eT_x)\left(2DE\left[\frac{1}{1-\delta}\right] - 2D + x - xE[\delta]\right)}} \tag{8.11}$$

If the transportation cost (t_f and t_r) = 0, carbon tax (T_x) = 0, and $E[\delta] = 0$ which mean all items are in perfect quality, Eq. (8.11) becomes the traditional EOQ formulae:

$$Q = \sqrt{\frac{2Dc}{h_b}}$$

8.2.2 EOQ Model with Complete Backorder Considering Carbon Emission

The inventory model for imperfect quality items with a complete backorder is depicted in Fig. 8.2 [7]. Similar to the first model, the inventory level instantaneously reduced δh unit at time t_s due to the withdrawal of the defective items. During t_2, the shortage is accumulated with D rate and will be completely backordered upon replenishment. The total revenue function $TR_B(B, Q)$ is similar to Eq. (8.11).

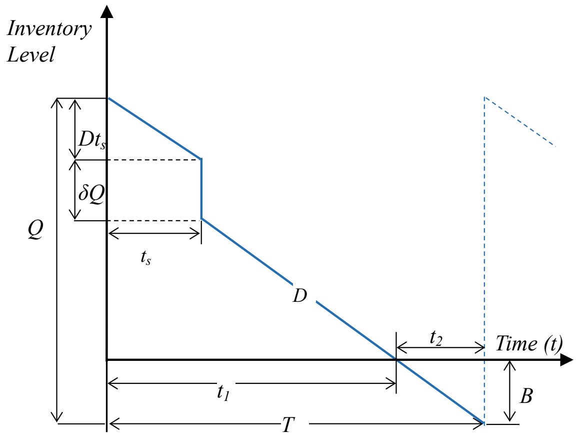


Fig. 8.2 Inventory model for imperfect quality items with a complete backorder

$TC_B\ (B,Q)$ is the total cost per cycle. Therefore, $TC_B(B,Q)$ is the sum C_O, C_P, C_I, C_H, C_T, and backorder cost (C_B) per cycle. Hence,

$$TC_B(B, Q) = C_O + C_P + C_I + C_H + C_T + C_B \tag{8.12}$$

in which C_O, C_P, C_I, and C_T are similar as in Sect. 2.1.

The holding cost considers both traditional carrying cost and carbon emission cost generated by warehousing. The average amount of inventory per cycle is equivalent to the area in Wee et al. [33].

$$C_H = (h_b + wE_eT_x)\left(\frac{1}{2}\frac{(Q - \delta Q - B)^2}{D} + \frac{\delta Q^2}{x}\right) \tag{8.13}$$

The backorder cost [33] is

$$C_B = \frac{1}{2}\frac{bB^2}{D} \tag{8.14}$$

Therefore, one has

$$TC_B(B, Q) = c + AQ + u_cQ + (h_b + wE_eT_x)\left(\frac{1}{2}\frac{(Q - \delta Q - B)^2}{D} + \frac{\delta Q^2}{x}\right) + t_f$$
$$+ (2dc_1t_v + dc_2lQt_v) + (2de_1 + de_2Q) + \frac{1}{2}\frac{bB^2}{D} \tag{8.15}$$

As $T = (1 - \delta)\ Q/D$,

$$TP_B(B, Q) = D\left(s_p - s_v + \frac{h_bQ}{x} + \frac{wE_eT_xQ}{x}\right) + (h_b + wE_eT_x)\left(B - \frac{Q}{2}\right)$$

$$+ (h_b + wE_eT_x)\frac{Q}{2}\delta + D(s_v - u_c - A - dc_2lt_v - de_2 - \frac{h_bQ}{x} - \frac{wE_eT_xQ}{x} - \frac{c}{Q} - \frac{t_f}{Q} - \frac{2dt_vc_1}{Q} - \frac{2de_1}{Q}\bigg)\bigg(\frac{1}{1-\delta}\bigg) - \frac{B^2}{2Q}(h_b + wE_eT_x + b)\bigg(\frac{1}{1-\delta}\bigg) \tag{8.16}$$

The expected total profit per unit time becomes

$$ETP_B(B, Q) = D\bigg(s_p - s_v + \frac{h_bQ}{x} + \frac{wE_eT_xQ}{x}\bigg) + (h_b + wE_eT_x)\bigg(B - \frac{Q}{2}\bigg) + (h_b + wE_eT_x)\frac{Q}{2}E[\delta] + D(s_v - u_c - A - dc_2lt_v - de_2 - \frac{h_bQ}{x} - \frac{wE_eT_xQ}{x} - \frac{c}{Q} - \frac{t_f}{Q} - \frac{2dt_vc_1}{Q} - \frac{2de_1}{Q}\bigg) E\bigg[\frac{1}{1-\delta}\bigg] - \frac{B^2}{2Q}(h_b + wE_eT_x + b)E\bigg[\frac{1}{1-\delta}\bigg] \tag{8.17}$$

To find *B* and *Q,* we must first prove the concavity of the total profit function. For the function to be concave, the following sufficient conditions must be satisfied:

$$\bigg(\frac{\partial^2 ETP}{\partial B \partial Q}\bigg)^2 - \bigg(\frac{\partial^2 ETP}{\partial B^2}\bigg)\bigg(\frac{\partial^2 ETP}{\partial Q^2}\bigg) \le 0 \tag{8.18}$$

And one or both

$$\frac{\partial^2 ETP}{\partial B^2} \le 0, \frac{\partial^2 ETP}{\partial Q^2} \le 0 \tag{8.19}$$

By taking the first derivative of *ETP* with respect to *B* and *Q* yields

$$\frac{\partial ETP}{\partial B} = h_b + wE_eT_x - \bigg(\frac{B}{Q}(h_b + wE_eT_x + b)\bigg)E\bigg[\frac{1}{1-\partial}\bigg] \tag{8.20}$$

$$\frac{\partial ETP}{\partial Q} = D\bigg(\frac{h_b + wE_eT_x}{x}\bigg) - \bigg(\frac{h_b + wE_eT_x}{2}\bigg) + \bigg(\frac{h_b + wE_eT_x}{2}\bigg)E[\delta] + \bigg(D\bigg(\frac{c}{Q^2} + \frac{t_f}{Q^2} + \frac{2dt_vc_1}{Q^2} + \frac{2de_1}{Q^2} - \frac{h_b}{x} - \frac{wE_eT_x}{x}\bigg) + \frac{B^2}{2Q^2}(h_b + wE_eT_x + b)\bigg)E\bigg[\frac{1}{1-\delta}\bigg] \tag{8.21}$$

Taking the second derivative, we have

$$\frac{\partial^2 ETP}{\partial B^2} = -\frac{(h_b + wE_eT_x + b)}{Q}E\left[\frac{1}{1-\delta}\right] \tag{8.22}$$

$$\frac{\partial^2 ETP}{\partial Q^2} = \left(D\left(-\frac{2c}{Q^3} - \frac{2t_f}{Q^3} - \frac{4dt_vc_1}{Q^3} - \frac{4de_1}{Q^3}\right) - \frac{B^2}{Q^3}(h_b + wE_eT_x + b)\right) E\left[\frac{1}{1-\delta}\right] \tag{8.23}$$

$$\frac{\partial^2 ETP}{\partial B \partial Q} = \frac{B}{Q^2}(h_b + wE_eT_x + b)E\left[\frac{1}{1-\delta}\right] \tag{8.24}$$

Substituting Eqs. (8.23) and (8.24) into (8.18), and for the positive value of all the parameter, one has

$$\left(\frac{\partial^2 ETP}{\partial B \partial Q}\right)^2 - \left(\frac{\partial^2 ETP}{\partial B^2}\right)\left(\frac{\partial^2 ETP}{\partial Q^2}\right) = -\frac{2D\left(c + t_f + 2dc_1t_v + 2de_1\right)(h_b + wE_eT_x + b)E\left[\frac{1}{1-\delta}\right]^2}{Q^4} \leq 0 \tag{8.25}$$

And from Eqs. (8.23) and (8.24), we can see that the equations in (8.19) are satisfied. Therefore, the profit function $ETP_B(B,Q)$ is strictly concave. By setting the first derivative equal to zero, Eqs. (8.20) and (8.21) can be solved simultaneously for $B*$ and Q_B*, resulting in

$$B^* = \frac{(h_b + wE_eT_x)Q}{(h_b + wE_eT_x + b)E\left[\frac{1}{1-\delta}\right]} \tag{8.26}$$

$$Q_B^* = \sqrt{\frac{\left(2Dc + 2Dt_f + 4Ddc_1t_v + 4Dde_1 + h_bB^2 + wE_eT_xB^2 + bB^2\right)xE\left[\frac{1}{1-\delta}\right]}{(h_b + wE_eT_x)\left(2DE\left[\frac{1}{1-\delta}\right] - 2D + x - xE[\delta]\right)}} \tag{8.27}$$

If the transportation cost (t_f and t_r) = 0, carbon tax (T_x) = 0, Eqs. (8.26) and (8.27) are similar to Wee et al.'s [33] as shown in Eqs. (8.28) and (8.29).

$$B^* = \frac{h_bQ}{(h_b + b)E\left[\frac{1}{1-\delta}\right]} \tag{8.28}$$

$$Q_B^* = \sqrt{\frac{\left(2Dc + h_bB^2 + bB^2\right)xE\left[\frac{1}{1-\delta}\right]}{h_b\left(2DE\left[\frac{1}{1-\delta}\right] - 2D + x - xE[\delta]\right)}} = \sqrt{\frac{\left(2Dc + B^2h_b + B^2b\right)E\left[\frac{1}{1-\delta}\right]}{h_b\left(1 - E[\delta] - \frac{2D}{x}\left(1 - E\left[\frac{1}{1-\delta}\right]\right)\right)}} \tag{8.29}$$

If the transportation cost (t_f and t_r) = 0, carbon tax (T_x) = 0, $b = 0$, and $E[\delta] = 0$ which mean shortage is not allowed and all items are in perfect quality, Eq. (8.29) becomes the traditional EOQ formulae.

Further, the expected total emission per unit time can be derived from Eq. (8.15) as

$$ETE_B(B, Q) = \left(wE_e\left(\frac{1}{2}\frac{(Q - E[\delta]Q - B)^2}{D}\right) + \frac{E[\delta]Q^2}{x} + (2dc_1F_e + dc_2lF_eQ)\right)\left(\frac{D}{(1 - E[\delta])Q}\right) \tag{8.30}$$

8.2.3 Illustrative Examples

Examples are presented to illustrate the application of the model. The value is adopted from Wee et al. [33] and Hariga et al. [11] with some modification.

Example 1 The values of the parameters are

$D = 50,000$ units/year	$A = \$25/unit$	$c = \$100$/cycle,
$h_b = \$5$/unit/year,	$u_c = \$0.5$/unit,	$x = 175,200$ unit/year,
$s_p = \$50$/unit,	$s_v = \$20$/unit,	$d = 10$ km,
$t_f = \$100$ /delivery,	$t_v = \$0.01$/unit/km,	

and the probability density function of defective items, δ, is

$$f(\delta) = \begin{cases} 25, & 0 \le \delta \le 0.04 \\ 0, & \text{otherwise} \end{cases}, \quad E[\delta] = 0.02$$

Also, several data to incorporate the carbon emissions are as follows: $c_1 = 30$ L/100 km, $w = 14.4$ kWh/unit/year, $T_x = \$75/\text{ton}CO_2$ [11], $F_e = 2.6$ kgCO_2/liter diesel fuel $= 2.6 \times 10^{-3}$ tonCO_2/liter (The US EPA, [26], and $E_e = 500$ grCO_2/kWh $= 0.5 \times 10^{-3}$ tonCO_2/kWh [16].

From Eq. (8.11), the optimal solution of Q^* is 2,167.5 units. By substituting this value into Eq. (8.7), one has *ETP* = \$1,194,683.21. Further, from (8.8), the *ETE* is 8.65 tonCO_2.

Example 2 Suppose that shortage is allowed and fully backordered. Consider the parameter in numerical Example 1 with $b = \$10$/unit/year.

By solving Eqs. (8.28) and (8.29) simultaneously, the optimal solutions are $Q^* =$ 2,636.1 units and $B = 920.9$ units. By substituting these values into Eq. (8.17), one has ETP_B= \$1,199,761.40. This expected total profit is higher than the *ETP* when the shortage is not allowed in Example 1. Further, from (8.30), the ETE_B is 4.72 tonCO_2 which means the expected total emission when backorder exists is lower.

Without considering carbon emission, from Eqs. (8.28) and (8.29), one has Q^* = 1751.8 units and B = 572.2 units. These results are similar to Wee et al. [33]. Then, applying these values into Eq. (8.17) resulting ETP_B= \$1,198,959.84. The comparison between these two ETP_B shows that the proposed model by considering emission cost results in a higher expected total profit per unit time.

8.3 Low-Carbon Supply Chain Inventory Model for Imperfect Quality Items

This section presents a single-vendor and single-buyer integrated inventory model for a single type of item containing a certain percentage of defective products per delivered lot. This study extends previous models by considering a weight-dependent transportation cost and emission, as well as the emission from the warehousing activity. It also considers a fixed inspection cost and variable inspection costs.

The buyer needs to optimize the order quantity in fulfilling the customer demand. In issuing the order, the buyer considers the probability of the defective products. The vendor performs a single-setup multiple-delivery policy (SSMD). Therefore, the vendor needs to optimize the number of deliveries per production cycle and the delivery quantity. These decisions will affect transportation and carbon emission costs. The inventory model for both the vendor and the buyer is illustrated in Fig. 8.3. The additional notation for the model is presented as follows.

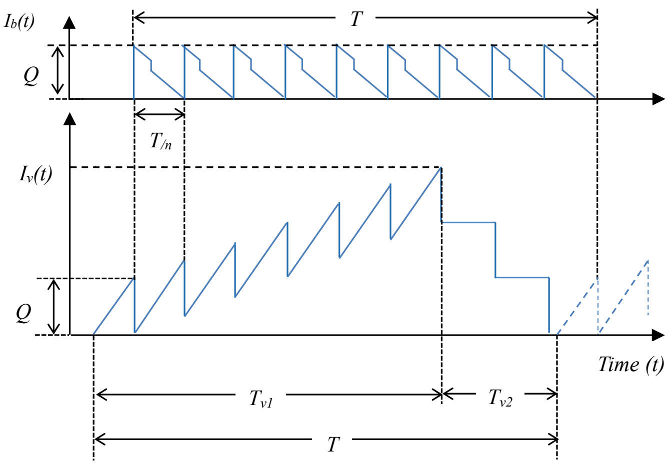


Fig. 8.3 The inventory level of the vendor and the buyer with imperfect quality

Decision variables	
n	Number of deliveries per production cycle (positive integer)
Q	Delivery quantity (unit)
Parameters	
P	Production rate (unit/year)
y	The expected good products per delivery (unit/delivery); $y = (1\text{-}\delta)Q$
T_{v1}	Production period for the vendor in each cycle
T_{v2}	Nonproduction period for the vendor in each cycle
T_b	Inventory cycle length per delivery for the buyer; $T_b = T/n$
$I_v(t)$	Vendor's inventory level at time t
$I_b(t)$	Buyer's inventory level at time t
i_c	Buyer's inspection setup cost ($/delivery)
s	Vendor's production setup cost ($/order)
h_v	Vendor's holding cost ($/unit/year)
t_f	Vendor's fixed transportation cost per delivery ($/delivery)
t_v	Vendor's variable transportation cost ($/liter)
w_e	Warehouse emission cost per unit product ($/unit/year); $w_e = wE_eT_x$
ETC_b	The buyer's total expected cost per year ($/year)
ETC_v	Vendor's total expected cost per year ($/year)
ETC	Joint total expected cost per year ($/year)
ETC_e	Joint total expected cost per year for model considering carbon emission ($/year)
ETE_b	The buyer's total expected carbon emission per year ($kgCO_2$/year)
ETE_v	Vendor's total expected carbon emission per year ($kgCO_2$/year)
ETE	Joint total expected carbon emission per year ($kgCO_2$/year)
ETE_e	Joint total expected carbon emission per year for model considering carbon emission ($kgCO_2$/year)

8.3.1 Buyer's Cost Function

In one production cycle T, there are ny good products, therefore

$$D = \frac{ny}{T} = \frac{(1-\delta)Qn}{T} \tag{8.31}$$

The buyer's total cost per year (TC_b) is given by

$$TC_b = \text{ordering cost} + \text{inspection cost} + \text{holding cost} + \text{carbon emission cost}$$

Since buyer's ordering cost per order is c, the ordering cost per year (C_o) is given by

$$C_O = c\frac{n}{T} = \frac{Dc}{(1-\delta)Q} \tag{8.32}$$

We consider a fixed inspection cost per delivery and variable inspection cost per unit product similar to Sarkar et al. [21]. Since the buyer performs a 100% inspection process with two types of inspection costs, the inspection cost per year (C_I) is given by

$$C_I = i_c\frac{n}{T} + u_c Q\frac{n}{T} = \frac{D}{(1-p)Q}i_c + \frac{D}{(1-p)}u_c \tag{8.33}$$

Figure 8.4 illustrates the buyer's inventory level at any time t. After the inspection process, the buyer only holds the good products. The buyer's inventory per delivery cycle is given by

$$\overline{I_b} = \frac{Q^2\left(2\delta D - 2x\delta + x\delta^2 + x\right)}{2xD} \tag{8.34}$$

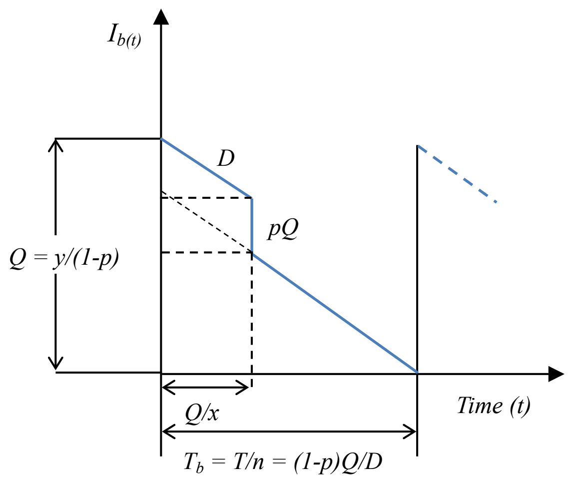


Fig. 8.4 Buyer's inventory per delivery cycle

Therefore, the inventory holding cost per year (C_{Hb}) is given by

$$C_{Hb} = h_b\left(\frac{Q^2\left(2D\delta - 2x\delta + x\delta^2 + x\right)}{2xD}\right)\left(\frac{D}{(1-\delta)Q}\right) \tag{8.35}$$

Buyer's carbon emission comes only from warehousing activity such as from the electricity it consumes. Therefore, the buyer's carbon emission cost and the expected total carbon emission per year can be calculated as follows:

$$C_{Eb} = w_e\left(\frac{Q^2\left(2D\delta - 2x\delta + x\delta^2 + x\right)}{2xD}\right)\left(\frac{D}{(1-\delta)Q}\right) \tag{8.36}$$

$$ETE_b = wE_e\left(\frac{Q^2\left(2DE[\delta] - 2xE[\delta] + xE\left[\delta^2\right] + x\right)}{2xD}\right)\left(\frac{D}{(1-\delta)Q}\right) \tag{8.37}$$

From Eqs. (8.32), (8.33), (8.35), and (8.36), the expected buyer's total cost per year (ETC_b) is

$$ETC_b = \frac{D}{E[1-\delta]}u_c + \left(\frac{D}{E[1-\delta]Q}\right)\left(c + i_c + (h_b + w_e)\left(\frac{Q^2\left(2DE[\delta] - 2xE[\delta] + xE\left[\delta^2\right] + x\right)}{2xD}\right)\right) \tag{8.38}$$

8.3.2 Vendor's Cost Function

After receiving the buyer's order, the vendor runs the production until the amount is sufficient for *n* deliveries. Figure 8.5 illustrates the vendor's inventory in one production cycle, adapted from Goyal et al. [9], Jauhari et al. [14], and Wangsa and Wee [31]. The first delivery occurs at *Q/P*. The second delivery until the *n*th delivery occurs in *(1-p)Q/D* time intervals.

The vendor's total cost per year (TC_v) is given by

TC_v = setup cost + transportation cost + holding cost + carbon emission cost

Vendor's setup cost per year (C_S) is given by

$$C_S = \frac{s}{T} = \frac{sD}{(1-p)Qn} \tag{8.39}$$

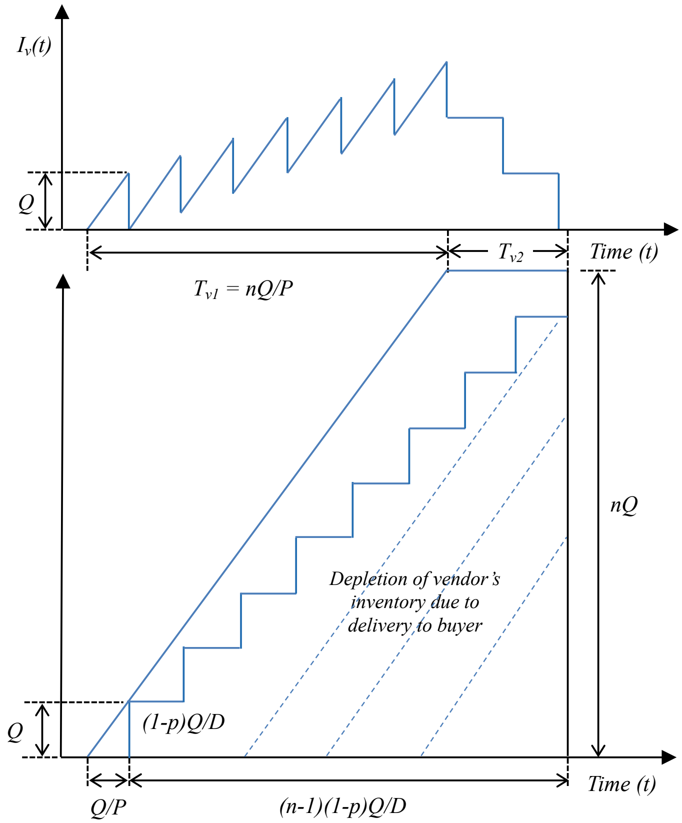


Fig. 8.5 Vendor's inventory per production cycle (T) for $n = 9$

Vendor's transportation cost (C_T) considers the number of deliveries n, fix transportation cost per delivery t_f, and variable transportation cost. Nie et al. [19], Wangsa [30], and Wangsa and Wee [31] considered the variable transportation cost which is affected by the lot size, shipping distance, and product weight. The vendor's transportation cost per delivery is given by Eq. (8.40). It considers the fix transportation cost (t_f), the transportation cost of an empty truck ($2dc_1t_v$), and then it has the cost for the truckload which depends on the delivery distance and quantity, the product weight, the fuel consumption per ton per km, and the fuel price.

$$C_T = \frac{D}{(1-\delta)Q}\left(t_f + 2dc_1t_v + dlQc_2t_v\right) \tag{8.40}$$

Wahab et al. [28] and Hariga et al. [11] emphasized the distance, fuel efficiency, actual shipment weight, and CO_2 emission per gallon of fuel in determining the emissions from transporting inventory. Therefore, considering the number of deliveries, the amount of vendor's carbon emission per year as the result of transportation activity is

$$\frac{D}{(1-\delta)Q}(2dc_1 + dQlc_2)F_e \tag{8.41}$$

Vendor's inventory is depicted in Fig. 8.5. The inventory per year can be evaluated as follows:

$$\overline{I_v} = \frac{Q}{2}\left((n-1) - \frac{(n-2)D}{(1-\delta)P}\right)$$

Hence, the vendor's holding cost per year is

$$C_{Hv} = h_v \frac{Q}{2}\left((n-1) - \frac{(n-2)D}{(1-\delta)P}\right) \tag{8.42}$$

Vendor's carbon emission from the warehousing activity is given by

$$wE_e\left(\frac{Q}{2}\left((n-1) - \frac{(n-2)D}{(1-\delta)P}\right)\right) \tag{8.43}$$

Vendor's carbon emission comes from transportation and warehousing activities. Therefore, based on Eqs. (8.41) and (8.43), the vendor's carbon emission cost and the total expected carbon emission per year can be calculated as follows:

$$C_{Ev} = \frac{D}{(1-\delta)Q}(2de_1 + dQe_2) + w_e \frac{Q}{2}\left((n-1) - \frac{(n-2)D}{(1-\delta)P}\right) \tag{8.44}$$

$$ETE_v = \frac{D}{E[1-\delta]Q}(2dc_1 + dlQc_2)F_e + wE_e\left(\frac{Q}{2}\left((n-1) - \frac{(n-2)D}{E[1-\delta]P}\right)\right) \tag{8.45}$$

From Eqs. (8.39), (8.40), (8.43), and (8.44), the expected vendor's total cost per year (ETC_v) is

$$ETC_v = \frac{sD}{E[1-\delta]Qn} + \frac{D}{E[1-\delta]Q}(t_f + 2dc_1t_v + dlQc_2t_v + (2de_1 + dQe_2)) + (h_v + w_e)\left(\frac{Q}{2}\left((n-1) - \frac{(n-2)D}{E[1-\delta]P}\right)\right) \tag{8.46}$$

8.3.3 Integrated Decision

This section provides the model when both the vendor and the buyer cooperate and make an integrated decision. The vendor and the buyer simultaneously specify Q and n that minimize the joint expected total cost ETC_e. The $ETC_e\ (Q,n)$ is the sum of ETC_v and ETC_b in Eqs. (8.38) and (8.46).

$$
\begin{aligned}
TC_e(Q,n) = {} & \left(\frac{D}{E[1-\delta]Q}\right)(c + i_c + (h_b \\
& +w_e)\left(\frac{Q^2\left(2DE[\delta] - 2xE[\delta] + xE\left[\delta^2\right] + x\right)}{2xD}\right)\Bigg) + \frac{sD}{E[1-\delta]Qn} \\
& + \frac{D}{E[1-\delta]Q}(t_f + 2dc_1t_v + dlQc_2t_v + (2de_1 + dQe_2)) + (h_v \\
& + w_e)\frac{Q}{2}\left((n-1) - \frac{(n-2)D}{E[1-\delta]P}\right) + \frac{D}{E[1-\delta]}u_c
\end{aligned}
\tag{8.47}
$$

First, we also need to prove the convexity of Eq. (8.47). Fixing n and then taking the first and second derivatives of $ETC_e(Q,n)$ with respect to Q yields

$$
\begin{aligned}
\frac{\partial ETC_e(Q,n)}{\partial Q} = {} & \frac{1}{2PQ^2xnE[1-\delta]}((PQ^2nx)(h_b + nh_v - h_v + nw_e - 2h_bE[\delta] \\
& + h_vE[\delta] + h_bE\left[\delta^2\right] - nh_vE[\delta] + w_eE\left[\delta^2\right] - w_eE[\delta] \\
& - nw_eE[\delta]) + \left(2PDQ^2nE[\delta]\right)(h_b + w_e) \\
& - (2PDnx)\left(\frac{s}{n} + c + i_c + t_f + 2dc_1t_v\right. \\
& \left.+2de_1\right) - \left(DQ^2nx\right)(nh_v - 2h_v + nw_e - 2w_e)\Big)
\end{aligned}
\tag{8.48}
$$

$$
\frac{\partial^2 ETC_e(Q,n)}{\partial Q^2} = \frac{2D\left(s + n\left(c + i_c + t_f + 2dc_1t_v + 2de_1\right)\right)}{E[1-\delta]nQ^3} \geq 0
\tag{8.49}
$$

Equation (8.49) shows that the second derivative is always positive. Therefore, for fixed n, the function $ETC_e(Q,\ n)$ is convex in Q. Setting Eq. (8.48) equal to zero gives the optimal order

$$
Q^* = \sqrt{\frac{2DPx\left(s + nC + ni_c + nt_f + 2ndc_1t_v + 2nde_1\right)}{\begin{aligned}&n((h_v + w_e)(2xD + nxP - nxD - nxPE[p]) + (h_b + w_e)\\ &\left(2DPE[p] + xPE\left[p^2\right]\right) + (h_v - 2h_b - w_e)(xPE[p]) + (h_b - h_v)(xP)\right)\end{aligned}}}
\tag{8.50}
$$

To find the optimal values of the positive integer number of deliveries n and the order quantity Q, the following algorithm can be used.

Step 1	Set $n = 1$
Step 2	From Eq. (8.50) find Q
Step 3	Substitute Q into Eq. (8.47) to calculate the corresponding $ETC_e(Q,n)$
Step 4	If $ETC_e(Q(n\text{-}1), (n\text{-}1)) \geq ETC_e(Q,n) \leq ETC_e(Q(n+1), (n+1))$, then $n = n^*$, $Q = Q^*$, and $ETC_e(Q,n) = ETC^*_e$, otherwise, set $n = n + 1$ and back to Step 2
Step 5	Substitute n^* and Q^* into Eqs. (8.37) and (8.45). Find $ETE^*_e = ETE^*_b + ETE^*_v$.

8.3.4 Illustrative Example

A numerical example is presented to illustrate the application of the model. The value is adopted from Wangsa and Wee [31], Hariga et al. [11], and Tiwari et al. [24] with some modification. The complete parameter values are presented in Table 8.1.

In an integrated decision, the vendor and the buyer cooperate and make the decision simultaneously. Applying the proposed solution procedure guide us to the optimal value of $n^* = 4$ and $Q^* = 407.50$ units with the $ETC^*_e = \$61{,}800.96$ and the

Table 8.1 Parameter values

Parameter	Value	Parameter	Value
P	40,000 units/year,	l	0.01 ton/unit
D	10,000 units/year	c_1	27 L/100 km
x	175,200 unit/year	c_2	0.57 L/100 km/ton truckload
c	\$30/order	T_x	\$75/tonCO_2
i_c	\$100/delivery	F_e	2.6 kgCO_2/l = 2.6 × 10^{-3} tonCO_2/l
u_c	\$0.5/unit	E_e	0.5 kgCO_2/kWh = 0.5 × 10^3 tonCO_2/kWh
h_b	\$45/unit/year	e_1	\$0.05265/km
s	\$3,600/setup	e_2	\$1.1115 × 10^{-5}/unit/km
h_v	\$38/unit/year	w	1.44 kWh/unit/year
t_f	\$50/delivery	w_e	\$0.054/unit/year
t_v	\$0.75/l	δ	uniformly distributed with α, β: $0 \leq \delta \leq 0.04$
d	100 km		

Notes
- Vehicle fuel consumption (c_1 & c_2) is adopted from Volvo Truck Corporation [27] for regional traffic truck
- Warehouse energy consumption (w) and carbon tax (t_x) are adopted from Hariga et al. [11]
- Fuel standard emission (F_e) is adopted from the US. EPA [26]
- Electricity standard emission (E_e) is adopted from McCarthy [16]

production quantity 1,630 units per cycle. The corresponding total costs from different values of n are given in Table 8.2. The ETE_b^*= 0.0057 tonCO_2 and the ETE_v^*= 3.6688 tonCO_2. Therefore, an integrated decision results in the total expected carbon emission per year 3.6745 tonCO_2.

When we eliminate the carbon emission costs from the model (i.e., $e_1 = e_2 = w_e = 0$), the optimal value of $n^{\#}$ remains 4. For sure the ETC is lower compared to when the emission cost is taken into account, as shown in Table 8.3. The order delivery size $Q^{\#}$ becomes 405.87 units, and therefore the optimal vendor's production quantity is 1,623.48 units per cycle. Incorporating carbon emission into the decision model will increase the delivery size and decrease the delivery frequency. By substituting $n^{\#}$ and $Q^{\#}$ into the original parameter values, the ETC_e = $61,801.41. This result is 0.0007% higher than the ETC_e of n^*. Also, the ETE_e = 3.6886 tonCO_2 which is 0.38% higher than the ETE_e of n^*. These results mean that incorporating carbon emission cost into the supply chain decision model is beneficial for the vendor and buyer who work in a country implementing carbon emission tax.

Sensitivity analysis is performed to explore the impact of the probability of defective items (δ) and the carbon tax rate (T_x), on decision variables n^*, Q^*, and the corresponding ETC_e and ETE_e. The results are shown in Fig. 8.6. It shows that as the probability of defective items increases, then both the ETC_e and ETE_e increase. When the carbon tax rate increases, then the ETC_e increases but the ETE_e decreases.

Table 8.2 Optimal solution n and Q in an integrated decision

n	Q	ETC_b ($)	ETC_v ($)	ETC_e ($)	ETE_b (tonCO_2)	ETE_v (tonCO_2)	ETE_e (tonCO_2)
1	1203.65	32,839.70	37,273.11	70,112.81	0.0502	1.3417	1.3919
2	709.62	22,674.63	40,892.94	63,567.57	0.0175	2.1708	2.1883
3	513.73	19,052.49	42,952.91	62,005.40	0.0091	2.9412	2.9504
4*	**407.50**	**17,374.83**	**44,426.13**	**61,800.96**	**0.0057**	**3.6688**	**3.6745**
5	340.55	16,533.34	45,615.56	62,148.90	0.0040	4.3604	4.3644
6	294.39	16,122.59	46,645.05	62,767.64	0.0030	5.0207	5.0237

Table 8.3 Optimal solution of n when e_1, e_2, and w_e are equal to 0

n	Q	ETC_b ($)	ETC_v ($)	ETC ($)	ETE_b (tonCO_2)	ETE_v (tonCO_2)	ETE (tonCO_2)
1	1202.74	32,788.49	37,183.48	69,971.98	0.0501	1.3426	1.3927
2	708.24	22,628.92	40,737.79	63,366.72	0.0174	2.1748	2.1922
3	512.18	19,012.37	42,734.44	61,746.81	0.0091	2.9497	2.9588
4#	**405.87**	**17,341.11**	**44,146.18**	**61,487.29**	**0.0057**	**3.6829**	**3.6886**
5	338.89	16,506.70	45,275.89	61,782.59	0.004	4.381	4.385
6	292.72	16,103.57	46,247.37	62,350.95	0.003	5.048	5.051

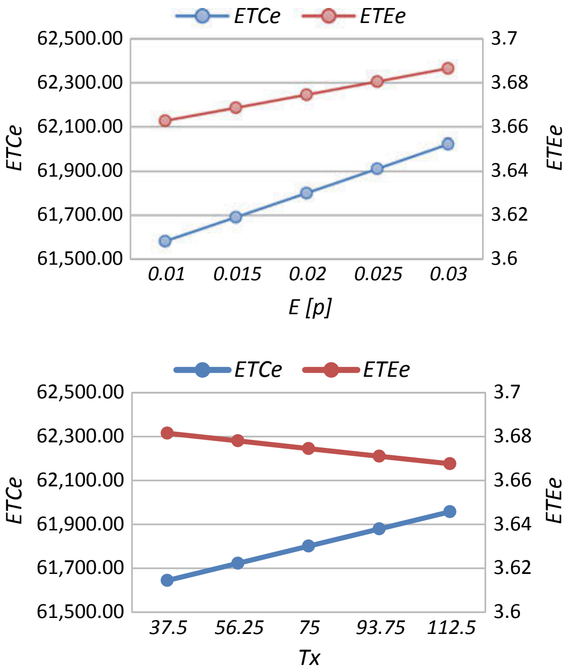


Fig. 8.6 Sensitivity analysis for different values of $E[\delta]$ and T_x

8.4 Concluding Remarks

In this chapter, low-carbon EOQ and vendor–buyer supply chain systems with defective items are discussed. The buyer performs a 100% inspection process with two types of inspection costs. Two EOQ models with different shortage situations were considered. The study incorporated carbon emissions from transportation and inventory holding activities. This study shows that incorporating an emission cost in the cost function results in a higher expected total profit per unit time. The study also shows that the expected total emission is lower when a complete backordered shortage is allowed.

The integrated vendor–buyer supply chain assumes an integrated system that tries to minimize the total cost as well as the emission cost. The decisions on the number of deliveries per production cycle and the delivery quantity affect transportation and warehousing emissions. The study shows that incorporating a carbon emission cost and implementing a carbon emission tax in the supply chain decision model is beneficial for the vendor and the buyer. The sensitivity analysis shows that the probability of defective items affects the total cost and emission. It also confirms that when the carbon taxes rate increases, the expected total emission will decrease.

This chapter considered carbon emission and imperfect quality in a supply chain system. The proposed models can be further extended by considering the possibility to return and repair the defective products. These conditions will affect the total cost and carbon emission. In addition, further study can incorporate the effect of item deterioration and partial backorder. The influence of consumers' low-carbon awareness on-demand rate such as in Tao and Xu [23] is another interesting direction for extension.

References

1. Battini D, Persona A, Sgarbossa F (2014) A sustainable EOQ model: theoretical formulation and applications. Int J Prod Econ 149:145–153
2. Benjaafar S, Li Y, Daskin M (2013) Carbon footprint and the management of supply chains: Insights from simple models. IEEE Trans Autom Sci Eng 10(1):99–116
3. Bonney M, Jaber MY (2011) Environmentally responsible inventory models: non-classical models for a non-classical era. Int J Prod Econ 133(1):43–53
4. Cárdenas-Barrón LE (2000) Observation on: Economic production quantity model for items with imperfect quality [Int J Prod Econ 64:59–64 (2000)]. Int J Prod Econ 67(2):201
5. Daryanto Y, Wee HM (2018) Single vendor-buyer integrated inventory model for deteriorating items considering carbon emission. In: Proceedings of the 8th international conference on industrial engineering and operations management (IEOM), March 2018, pp 544–555
6. Daryanto Y, Wee HM, Astanti RD (2019) Three-echelon supply chain model considering carbon emission and item deterioration. Transp Res Part E Logist Transp Rev 122:368–383
7. Daryanto Y, Wee HM, Wu KH (in press) Revisiting sustainable EOQ model considering carbon emission. Int J Manuf Technol Manag
8. Goyal SK, Cárdenas-Barrón LE (2002) Note on: economic production quantity model for items with imperfect quality-a practical approach. Int J Prod Econ 77(1):85–87
9. Goyal SK, Huang CK, Chen KC (2003) A simple integrated production policy of an imperfect item for vendor and buyer. Prod Plan Control 14(7):596–602
10. Gunasekaran A, Irani Z, Papadopoulos T (2014) Modelling and analysis of sustainable operations management: certain investigations for research and applications. J Oper Res Soc 65(6):806–823
11. Hariga M, As'ad R, Shamayleh, A (2017) Integrated economic and environmental models for a multi stage cold supply chain under carbon tax regulation. J Clean Prod 166(1):1357–1371
12. Hayek PA, Salameh MK (2001) Production lot sizing with the reworking of imperfect quality items produced. Prod Plan Control 12(6):584–590
13. Jaggi CK, Cárdenas-Barrón LE, Tiwari S, Shafi A (2017) Two-warehouse inventory model for deteriorating items with imperfect quality under the conditions of permissible delay in payments. Sci Iran Trans E 24(1):390–412
14. Jauhari WA, Pamuji AS, Rosyidi CN (2014) Cooperative inventory model for vendor-buyer system with unequal-sized shipment, defective items and carbon emission costs. Int J Logist Syst Manag 19(2):163–186
15. Kazemi N, Abdul-Rashid SH, Ghazilla RAR, Shekarian E, Zanoni S (2018) Economic order quantity models for items with imperfect quality and emission considerations. Int J Syst Sci Oper Logist 5(2):99–115
16. McCarthy JE (2013) EPA standards for greenhouse gas emissions from power plants: many questions, some answers. In: CRS Report for Congress, 7-5700. http://nationalaglawcenter.org/wp-content/uploads/assets/crs/R43127.pdf. Accessed 25 Sep 2018
17. McKinnon A (2018) Decarbonizing logistics: distributing goods in a low-carbon world. Kogan Page Ltd, London
18. Marklund J, Berling P (2017) Green inventory management. In: Bouchery Y, Corbett CJ, Fransoo JC, Tan T (eds) Sustainable supply chains, vol 4. Springer series in supply chain management. Springer, Cham

19. Nie L, Xu X, Zhan D (2006) Incorporating transportation costs into JIT lot splitting decisions for coordinated supply chains. J Adv Manuf Syst 5(1):111–121
20. Salameh MK, Jaber MY (2000) Economic production quantity model for items with imperfect quality. Int J Prod Econ 64(1):59–64
21. Sarkar B, Saren S, Sarkar M, Seo YW (2016) A Stackelberg game approach in an integrated inventory model with carbon-emission and setup cost reduction. Sustainability 8(12):1244
22. Taleizadeh AA, Soleymanfar VR, Govindan K (2018) Sustainable economic production quantity models for inventory system with shortage. J Clean Prod 174:1011–1020
23. Tao Z, Xu J (2019) Carbon-regulated EOQ models with consumers' low-carbon awareness. Sustainability 11(4):1004
24. Tiwari S, Daryanto Y, Wee HM (2018) Sustainable inventory management with deteriorating and imperfect quality items considering carbon emissions. J Clean Prod 192:281–292
25. The Carbon Trust (2006) Carbon footprint in the supply chain: the next step for business. White paper CTC 616. www.carbontrust.com/media/84932/ctc616-carbon-footprints-in-the-supply-chain.pdf. Accessed 12 Nov 2018
26. The United States Environmental Protection Agency (US EPA) (2005) Emission facts: Average carbon dioxide emissions resulting from gasoline and diesel fuel. https://nepis.epa.gov/. Accessed 12 Nov 2018
27. Volvo Truck Corporation (2018) Emissions from Volvo's truck. http://www.volvotrucks.com/content/dam/volvo/volvo-trucks/markets/global/pdf/our-trucks/Emis_eng_10110_14001.pdf, Issue 3, 09 March 2018. Accessed: 25 Sep 2018
28. Wahab MIM, Mamun SMH, Ongkunaruk P (2011) EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact. Int J Prod Econ 134(1):151–158
29. Wang WT, Wee HM, Cheng YL, Wen CL, Cárdenas-Barrón LE (2015) EOQ model for imperfect quality items with partial backorders and screening constraint. Euro J Ind Eng 9(6):744–773
30. Wangsa ID (2017) Greenhouse gas penalty and incentive policies for a joint economic lot size model with industrial and transport emissions. Int J Ind Eng Comput 8(1):453–480
31. Wangsa ID, Wee HM (2018) An integrated vendor-buyer inventory model with transportation cost and stochastic demand. Int J Syst Sci Oper Logist 5(4):295–309
32. Wee HM, Wang WT, Yang PC (2013) A production quantity model for imperfect quality items with shortage and screening constraint. Int J Prod Res 51(6):1869–1884
33. Wee HM, Yu J, Chen MC (2007) Optimal inventory model for items with imperfect quality and shortage backordering. Omega 35(1):7–11
34. World Bank & Ecofys (2018) State and trends of carbon pricing 2018. World Bank, Washington

Chapter 9
Non-instantaneous Deteriorating Model for Stock-Dependent Demand with Time-Varying Holding Cost and Random Decay Start Time

Nirmal Kumar Duari and Jobin George Varghese

Abstract In this study, we have considered an inventory model of non-instantaneous deteriorating items with stock-dependent demand. Shortages are allowed and fully backlogged. Holding cost is not always fixed; it may depend occasionally on time. That is why we have considered holding cost as constant as well as time-dependent in the model. Also, the effect of decay start time has been considered and they are random. We categorize the model into three cases. In the Cases I and II, we consider fixed decay start time with constant and time-varying holding cost, respectively. The random decay start time has been considered in the last case. Mathematical models have been derived to determine optimal-order quantity that minimizes the total cost. Optimal solution has been illustrated with numerical examples and along with that sensitivity analysis of parameters has been carried out.

Keywords Weibull distributed deterioration · Stock-dependent demand · Γ distribution · Time-dependent holding cost · Random decay start time

9.1 Introduction

Inventory structure is one of the most important aspects of operations research and is essential in commercial initiatives and engineering sectors. Little is known about the effect of investing in non-instantaneous deterioration even though the inventory system for instantaneous deteriorating items has been studied for a long time. Therefore, here the inventory model has been studied for non-instantaneous deteriorating items by bearing in mind the fact that using postponing the decay start time. As a result, the retailer can diminish the deterioration by which the retailer can reduce the financial losses, progress the customer service level, and increase business effectiveness.

N. K. Duari (✉) · J. G. Varghese
Department of Mathematics, Techno India University, EM-4, Salt Lake City, Sector V, Kolkata, West Bengal 700091, India
e-mail: abnu1985@gmail.com

J. G. Varghese
e-mail: jobingv1993@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_9

In the traditional inventory models, items conserved their physical features while they were kept in the inventory. This conjecture is obviously true for most items, but not for all cases. However, the deteriorating items are subject to continuous loss in their utility during the course of their lifetime due to degeneration, destruction, decay, and penalty of other reasons. Bearing this in mind, monitoring and preserving the inventory of deteriorating items become a challenging problem for decision-makers. Ghare and Schrader [8] recognized "a model for an exponentially decaying inventory." Dave and Patel [2] were the first to study "a deteriorating inventory with linear increasing demand when scarcities are not permissible." Recent work in this field has been done by some researchers, which include Zhou et al. [24], who gave a new "adjustable production development strategy for deteriorating items with time fluctuating demand and partial lost trade." Wu et al. [23] and Ouyang et al. [16] first combined the idea of non-instantaneous deterioration. They also established that if the retailer could efficiently decrease the declining rate of item by refining the storage facility, the entire yearly pertinent inventory cost would be dropped.

Ajanta Roy [17] developed a model when demand rate is a function of selling price, the deterioration rate is time proportional, holding cost is dependent on time, and demand rate is dependent on selling price. Lee and Hsu [11] industrialized "a manufacture model over a limited planning horizon for deteriorating items with time-dependent demand with a volume restraint." Hsu et al. [9] established a deteriorating inventory policy when the retailer capitalizes on preservation technology to decrease the degree of product decline. Chang et al. [1] gave "optimum replacement strategy for non-instantaneous deteriorating items with stock reliant on demand." Dye and Ouyang [6] calculated a "deteriorating inventory system with inconsistent demand and trade credit backing." Hung [10] gave "an inventory model with general type demand, deterioration and backorder rates." Mishra and Singh [14] dealt with "partial backlogging." Leea and Dye [12] expressed a deteriorating inventory model with "stock-dependent demand by permitting preservation technology price as a decision variable in combination with replacement policy." Maihami and Kamalabadi [13] established a joint pricing and inventory control system for "non-instantaneous deteriorating items," and assume a cost- and time-dependent demand. Sarkar [18] examined an EOQ model with "deferral in expenditures and time-varying deterioration rate." Dye and Hsieh [5] offered "an extended prototype" of Hsu et al. [9] in view of "non-instantaneous decline reliant on the length of replenishment cycle." Shah et al. [19] combined "time-varying deterioration and holding cost rates in the inventory model" where scarcities were permissible. The main idea in their model is to discover the retailer's renewal, retailing price, and advertisement policies that make the most of the retailer's profit. Mishra et al. [15] and Duari et al. [3] provided an inventory model for deteriorating matters using time-varying holding cost and time-dependent demand. Duari et al. [4] considered shortage on deteriorating items. Udayakumar and Geetha [7, 22] reflected ideal lot sizing strategy for non-instantaneous deteriorating items with value and advertisement-dependent demand under incomplete backlogging. Along with this some researchers also considered stock-dependent demand in order to consider more realistic cases. Udayakumar and Geetha [22] deliberated non-instantaneous declining items with two stages of storing under trade credit plan.

Shaikh et al. [20] considered non-instantaneous deterioration inventory model with price- and stock-dependent demand for fully backlogged shortages under inflation. Tripathi et al. [21] considered inventory models for stock-dependent demand and time-varying holding cost under different trade credits. The consideration of decay start time is important due to rapid social changes, and the fact that it can reduce the total cost significantly. This helps the retailers to reduce their economic losses. For this reason, in this article we have considered an inventory model more realistic by considering decay start time as both known and random for stock-dependent demand with time-varying holding cost for deteriorating items.

The rest of the paper is designed as follows: Sect. 9.2 defines notation and assumptions. Sections 9.3 and 9.4 derive the model development and solution procedure, respectively. Section 9.5 gives the algorithm to obtain optimal solution. Section 9.6 presents mathematical instances in order to exemplify the model and attain managerial understandings. In Sect. 9.7, we provide sensitivity analysis followed by Sect. 9.8, which provides conclusion and upcoming possibility of research.

9.2 Notations and Assumptions

Given below are the notations and assumptions that have been used to develop the projected model.

9.2.1 Notations

R Stock parameter
ϒ Stock parameter
K Setup cost
h Holding cost per unit time
S Amount of shortage of inventory during the interval $[t_1, T]$
s Shortages cost per unit time
Q Amount of on-hand inventory at the start of the cycle
Q_1 Amount of inventory at the start of the deterioration
t_0 Deterioration start time
t_1 Shortage start time
T Length of the cycle
TC Total inventory cost.

9.2.2 Assumptions

(i) Lead time is zero, i.e., the production is instantaneous.
(ii) Time horizon is infinite.
(iii) Demand is a function of on-hand inventory. The functional relationship between demand and the instantaneous inventory level q (t) is:

$$D = Rq^{\Upsilon}, R > 0, 0 < \Upsilon < 1$$

During shortage of inventory, we have considered the demand to be fixed:

$$\text{Hence } \Upsilon = 0, D = R, R > 0$$

(iv) The deterioration is non-instantaneous, i.e., in the interval $(0, t_0)$ there is no deterioration. After a certain time t_0, the deteriorations start and the rate of deterioration of the inventory is followed by a two-parameter Weibull distribution: $\Theta = \alpha\beta t^{(\beta-1)}$, where $\alpha > 0$ and $\beta > 0$ are the scale and shape parameters.
(v) Shortages are allowed and fully backlogged.
(vi) The time-dependent holding cost is given by ht^n, $n \geq 1$.
(vii) The rate of deterioration follows Weibull distribution. But the start time of the deterioration may be either known or unknown. We consider decay for both known and unknown cases. For the unknown case, the decay start time is assumed as random and follows Γ distribution. The probability density function is given by

$$f(x) = \begin{cases} \frac{1}{\Gamma(l)} l^{x-1} e^{-l} dl, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

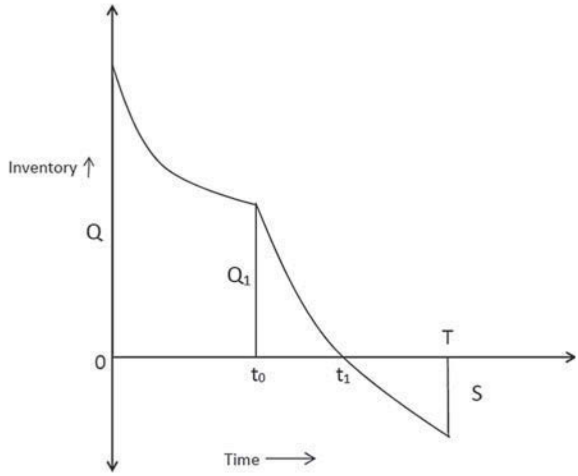
The mean of the Γ distribution is l. Hence, the random decay start time will be l.

9.3 Model Development

First, the level of inventory level reduces due to demand only. After some time, deterioration starts, the level of inventory decreases gradually due to demand of the customers and by decay of items, and then shortage arises.

At $t = 0$ when the cycle starts, the inventory is at a maximum level of Q units. During the interval $[0, t_0]$, the inventory depletes due to demand. At time $t = t_0$, the inventory level reaches Q_1 after which inventory start depleting in the

Fig. 9.1 Inventory—time representation



interval $[t_0, t_1]$ due to deterioration. At time $t = t_1$, the inventory level reaches zero after which shortage of inventory takes place in the interval $[t_1, T]$.

The changes in inventory described above at any time t are given by the following differential equations (Fig. 9.1):

$$\frac{dq}{dt} = -Rq^{\gamma}, \quad 0 \leq t \leq t_0 \tag{9.1}$$

$$\frac{dq}{dt} + \alpha\beta t^{\beta-1} q = -Rq^{\gamma}, t_0 \leq t \leq t_1 \tag{9.2}$$

And

$$\frac{dq}{dt} = -R, \quad t_1 \leq t \leq T \tag{9.3}$$

The boundary conditions are $q(0) = Q$, $q(t_0) = Q_1$, $q(t_1) = 0$, $q(T) = -S$. Then the solution of Eqs. (9.1), (9.2), and (9.3) are

$$q(t) = \left(Q^m - Rmt\right)^{\frac{1}{m}} \quad 0 \leq t \leq t_0 \tag{9.4}$$

$$q(t) = (Rm)^{\frac{1}{m}}\left[(t_1 - t) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right) + \alpha\left(t^{\beta+1} - t^{\beta} t_1\right)\right], t_0 \leq t \leq t_1 \tag{9.5}$$

where $m = 1 - \Upsilon$ (say)

$$q(t) = R(t_1 - t), t_1 \leq t \leq T \tag{9.6}$$

Now at $t = t_0$, we have $q(t) = Q_1$. From Eq. (9.4), we have

$$Q_1 = \left(Q^m - Rmt_0\right)^{\frac{1}{m}} \; and \; t_0 = \left(\frac{Q^m - Q_1^m}{Rm}\right) \tag{9.7}$$

At $t = T$, we have $q(t) = S$. From Eq. (9.6), we have

$$S = R(T - t_1) \; and \; t_1 = T - \frac{S}{R} \tag{9.8}$$

9.3.1 Different Costs for the Models

The ordering cost per cycle has been kept fixed at K per cycle.
The shortage cost per cycle has been calculated from Eqs. (9.6) and (9.8) as

$$\mathrm{SC} = \mathrm{s}\int_{t_1}^{T} q(t)dt = -s\frac{R}{2}(t_1 - T)^2 \tag{9.9}$$

The deterioration cost in the interval $[0, t_1]$ is given by

$$\mathrm{DC} = \mathrm{c}\int_{t_1}^{T} \theta(t)q(t)dt + c\int_{t_0}^{t_1} \theta(t)q(t)dt \tag{9.10}$$

Since in the interval $[0, t_0]$, the inventory depletes due to demand only, i.e., in this interval there is no deterioration; hence, the deterioration cost in the interval $[0, t_0]$ is equal to 0:

$$\text{i.e.,} \quad c\int_{t_1}^{T} \theta(t)q(t)dt = 0 \tag{9.11}$$

Hence, the deterioration cost in the interval $[t_0, t_1]$ is given by

$$\mathrm{DC} = \mathrm{c}\,\alpha\,\beta(Rm)^{\frac{1}{m}}\left\{\left(\frac{t_1^{\beta+1}}{\beta(\beta+1)} + \frac{\alpha t_1^{\beta+1}}{2\beta(\beta+1)}\right) - \left(\left(\frac{t_1 t_0^{\beta}}{\beta} - \frac{t_0^{\beta+1}}{(\beta+1)}\right) + \frac{\alpha}{\beta+1}\left(\frac{t_0^{\beta} t_1^{\beta+1}}{\beta} - \frac{t_0^{2\beta+1}}{(2\beta+1)}\right) + \alpha\left(\frac{t_0^{2\beta+1}}{(2\beta+1)} - \frac{t_1 t_0^{2\beta}}{2\beta}\right)\right)\right\} \tag{9.12}$$

In general, we considered the holding cost as fixed, i.e., a fixed amount of holding cost h is multiplied. The holding cost is given by

$$\mathrm{HC} = \mathrm{h}\int_{0}^{t_0} q(t)dt + h\int_{t_0}^{t_1} q(t)dt \tag{9.13}$$

$$= (Rm)^{\frac{1}{m}}\left\{\begin{array}{c}\frac{ht_1^2\left(\beta^2+\beta\left(2\alpha t_1^{\beta}+3\right)+2\right)}{2(\beta+1)(\beta+2)} + ht_0\left(-2\alpha\beta t_0^{\beta+1} + 2\alpha(\beta+2)t_1 t_0^{\beta}\right) \\ +ht_0\left(\left(\beta^2+3\beta+2\right)t_0 - 2(\beta+2)t_1\left(\beta+\alpha t_1^{\beta}+1\right)\right)\end{array}\right\} \\ +\frac{1}{2}hQt_0\left(2 - Q^{-m}Rt_0\right) \tag{9.14}$$

9.4 Total Cost of the Model and Solution Procedure

Total cost of the model has been calculated based on three different cases. First, we calculate total cost when the holding cost is fixed and t_0 is known. Second, we calculate total cost when the holding cost is time-dependent and t_0 is known. Third, we calculate total cost when the holding cost is time-dependent and t_0 is random.

9.4.1 Case I: Fixed Holding Cost When t_0 Is Known

The total cost of the inventory model is governed by the ordering cost, deterioration cost, holding cost, and shortage cost which has been calculated in Eqs. (9.9), (9.12), and (9.14). Therefore, the total cost per unit time is given as

$$TC = \frac{\mathrm{K} + \mathrm{DC} + \mathrm{HC} + \mathrm{SC}}{T} \tag{9.15}$$

$$\begin{aligned}&= K + \mathrm{c}\,\alpha\,\beta(Rm)^{\frac{1}{m}}\left\{\left(\frac{t_1^{\beta+1}}{\beta(\beta+1)} + \frac{\alpha t_1^{\beta+1}}{2\beta(\beta+1)}\right) - \left(\left(\frac{t_1 t_0^{\beta}}{\beta} - \frac{t_0^{\beta+1}}{(\beta+1)}\right)\right.\right. \\ &\left.\left.+ \frac{\alpha}{\beta+1}\left(\frac{t_0^{\beta} t_1^{\beta+1}}{\beta} - \frac{t_0^{2\beta+1}}{(2\beta+1)}\right) + \alpha\left(\frac{t_0^{2\beta+1}}{(2\beta+1)} - \frac{t_1 t_0^{2\beta}}{2\beta}\right)\right)\right\} \\ &+(Rm)^{\frac{1}{m}}\left\{\frac{ht_1^2\left(\beta^2+\beta\left(2\alpha t_1^{\beta}+3\right)+2\right)}{2(\beta+1)(\beta+2)} + ht_0\left(-2\alpha\beta t_0^{\beta+1} + \alpha(\beta+2)t_1 t_0^{\beta}\right)\cdot\right. \\ &\left.+ht_0\left(\left(\beta^2+3\beta+2\right)t_0 - 2(\beta+2)t_1\left(\beta+\alpha t_1^{\beta}+1\right)\right)\right\} \\ &+\frac{1}{2}hQt_0\left(2 - Q^{-m}Rt_0\right) - s\frac{R}{2}(t_1 - T)^2\end{aligned} \tag{9.16}$$

In order to minimize the total cost TC (t_0, t_1), the optimum value of t_1 can be attained by solving the equation:

$$\frac{d(TC)}{dt_0} = 0 \quad \text{And} \quad \frac{d(TC)}{dt_1} = 0. \tag{9.17}$$

$$\left\{-\frac{1}{2}hQ^{1-m}R + \frac{1}{2}hQ\left(2 - Q^{-m}Rt_0\right)\mathrm{Rm}^{\frac{1}{m}}\left(h\left(2+3\beta+\beta^2\right)t_0\right.\right.$$

$$
\begin{aligned}
&+ h\Big(\big(2+3\beta+\beta^2\big)t_0 - 2(2+\beta)t_1\Big[1+\beta+\alpha t_1^{\beta}\Big]\Big) \\
&+ h\Big(1\Big[-2\alpha\beta t_0^{1+\beta} + t_0^{\beta}t_1\alpha[2+\beta]\Big] + \Big(-2\alpha\beta(1+\beta)t_0^{\beta} \\
&+ \beta t_0^{-1+\beta}t_1\alpha[2+\beta]\Big)t_0'\Big[-2\alpha\beta t_0^{1+\beta} + t_0^{\beta}t_1\alpha[2+\beta]\Big]\Big)\Big) \\
&+ \mathrm{c}\alpha\beta[mR]^{\frac{1}{m}}\left(t_0^{\beta} - t_1(t_0t_1)^{-1+\beta} - \frac{\alpha\Big(-t_0^{2\beta} + t_0^{-1+\beta}t_1^{1+\beta}\Big)}{1+\beta}\right. \\
&\left.\left. - \ \alpha'\left[\frac{t_0^{1+2\beta}}{1+2\beta} - \frac{t_1[t_0]^{2\beta}}{2\beta}\right]\Big(t_0^{2\beta} - t_1[t_0]^{-1+2\beta}t_1'[t_0]\Big)\right)\right\} = 0
\end{aligned}
\tag{9.18}
$$

$$
\begin{aligned}
&And,\Bigg\{-Rs(-T+t_1) + \mathrm{c}a\beta[mR]^{\frac{1}{m}}\Bigg(-\frac{\alpha t_0^{\beta}t_1^{\beta}}{\beta} - t_0(t_0t_1)^{-1+\beta} \\
&+ \frac{(1+\beta)t_1^{\beta}}{\beta[1+\beta]} + \frac{\alpha(1+\beta)t_1^{\beta}}{2\beta[1+\beta]} + 1[t_0]t_1[t_0]^{-1+2\beta}\alpha'\left[\frac{t_0^{1+2\beta}}{1+2\beta} - \frac{t_1[t_0]^{2\beta}}{2\beta}\right]\Bigg) \\
&+ \mathrm{Rm}^{\frac{1}{m}}\Big(ht_0^{\beta}\alpha[2+\beta]t_0'\Big[-2\alpha\beta t_0^{1+\beta} + t_0^{\beta}t_1\alpha[2+\beta]\Big] \\
&- 2h(2+\beta)t_0\Big(1[1+\beta+\alpha] + \alpha\beta t_1^{-1+\beta}t_1'\Big[1+\beta+\alpha t_1^{\beta}\Big]\Big) \\
&+ \frac{h\Big((2t_1)\Big[2+\beta^2+\beta\Big[3+2\alpha t_1^{\beta}\Big]\Big] + 2\alpha\beta t_1^{-1+\beta}\beta'\Big[3+2\alpha t_1^{\beta}\Big]\Big(t_1^2\Big)\Big[2+\beta^2+\beta\Big[3+2\alpha t_1^{\beta}\Big]\Big)\Big)}{2(1+\beta)(2+\beta)}\Bigg\} = 0
\end{aligned}
\tag{9.19}
$$

The entire cost would be least if the determinant of the hessian matrix (H-matrix) of TC (t_0, t_1) is positive definite for $t_0 = t_0*$ and $t_1 = t_1*$ obtained from Eqs. (9.18) and (9.19), i.e.,

$$
H = \begin{vmatrix} \frac{\partial^2 TC}{\partial t_0^{*2}} & \frac{\partial^2 TC}{\partial t_1^{*}\partial t_0^{*}} \\ \frac{\partial^2 TC}{\partial t_0^{*}\partial t_1^{*}} & \frac{\partial^2 TC}{\partial t_1^{*2}} \end{vmatrix} \geq 0 \tag{9.20}
$$

9.4.2 Case II: Time-Varying Holding Cost and Known t_0

For realistic view, we consider time-dependent holding cost since inventory stored requires maintenance and calculating the cost behind that would help in obtaining accurate total cost. Therefore, we consider the holding cost as ht^n. Hence, the holding cost of the model is given by the equation:

$$
\mathrm{HC} = \int_0^{t_0} ht^n q(t)dt + \int_{t_0}^{t_1} ht^n q(t)dt \tag{9.21}
$$

$$
\begin{aligned}
&= \mathrm{hQt0}^{1+n}\left(\frac{1}{1+n} - \frac{Q^{-m}R\mathrm{t0}}{2+n}\right) + \frac{\left[\mathrm{t1}^{2+n}\left(2+n^2+\left(3+2\mathrm{t1}^{\beta}\alpha\right)\beta+\beta^2+n\left(3+\left(2+\mathrm{t1}^{\beta}\alpha\right)\beta\right)\right)\right]}{(1+n)(2+n)(1+n+\beta)(2+n+\beta)} \\
&- \frac{h\mathrm{t0}^{1+n}\left(-\frac{\mathrm{t0}}{2+n} - \frac{\mathrm{t0}\beta}{2+n} + \frac{\mathrm{t0}^{1+\beta}\alpha\beta}{2+n+\beta} + \frac{\mathrm{t1}\left((1+\beta)\left(1-\mathrm{t0}^{\beta}\alpha+\mathrm{t1}^{\beta}\alpha+\beta\right)+n\left(1+\mathrm{t1}^{\beta}\alpha+\beta-\mathrm{t0}^{\beta}\alpha(1+\beta)\right)\right)}{(1+n)(1+n+\beta)}\right)}{1+\beta}
\end{aligned}
\tag{9.22}
$$

The total cost of the inventory model is governed by the ordering cost, deterioration cost, holding cost, and shortage cost which has been calculated in Eqs. (9.9), (9.12), and (9.22):

$$
TC = \frac{\mathrm{K} + \mathrm{DC} + \mathrm{HC} + \mathrm{SC}}{T} \tag{9.23}
$$

$$
\begin{aligned}
&= hQ\mathrm{t}_0^{1+n}\left(\frac{1}{1+n} - \frac{Q^{-m}R\mathrm{t}_0}{2+n}\right) + \frac{h\mathrm{t}_1^{2+n}\left(2+n^2+\left(3+2\mathrm{t}_1^{\beta}\alpha\right)\beta+\beta^2+n\left(3+\left(2+\mathrm{t}_1^{\beta}\alpha\right)\beta\right)\right)}{(1+n)(2+n)(1+n+\beta)(2+n+\beta)} \\
&- \frac{h\mathrm{t}_0^{1+n}\left(-\frac{\mathrm{t}_0}{2+n} - \frac{\mathrm{t}_0\beta}{2+n} + \frac{\mathrm{t}_0^{1+\beta}\alpha\beta}{2+n+\beta} + \frac{\mathrm{t}_1\left((1+\beta)\left(1-\mathrm{t}_0^{\beta}\alpha+\mathrm{t}_1^{\beta}\alpha+\beta\right)+n\left(1+\mathrm{t}_1^{\beta}\alpha+\beta-\mathrm{t}_0^{\beta}\alpha(1+\beta)\right)\right)}{(1+n)(1+n+\beta)}\right)}{1+\beta} \\
&+ c\alpha\beta R m^{\frac{1}{m}}\left(\frac{\mathrm{t}_1^{1+\beta}\left(2+4\beta+\mathrm{t}_1^{\beta}\alpha(1+\beta)\right)}{2\beta(1+\beta)(1+2\beta)}\right. \\
&\left. - \frac{\mathrm{t0}^{\beta}\left(2\mathrm{t}_1^{1+\beta}\alpha(1+2\beta)+2\mathrm{t}_0\beta\left(-1+\left(-2+\mathrm{t}_0^{\beta}\alpha\right)\beta\right)-\mathrm{t1}\left(-2+\mathrm{t}_0^{\beta}\alpha\right)\left(1+3\beta+2\beta^2\right)\right)}{2\beta(1+\beta)(1+2\beta)}\right) \\
&- \frac{1}{2}sR(T-t)^2
\end{aligned}
\tag{9.24}
$$

To minimize the total cost $\mathrm{TC}(\mathrm{t}_0, \mathrm{t}_1)$ per unit time, we use the following technique and the optimal value of t_1 can be obtained by solving the following equations:

$$
\frac{d(TC)}{dt_0} = 0 \quad \text{And} \quad \frac{d(TC)}{dt_1} = 0 \tag{9.25}
$$

i.e.

$$
\begin{aligned}
&\frac{Q^{-m}\left(-cm^{\frac{1}{m}}Q^m R\mathrm{t}_0^{\beta}\alpha\beta\left(\mathrm{t}_0^{1+\beta}\alpha\beta - \mathrm{t}_0(1+\beta) - \mathrm{t}_0^{\beta}\mathrm{t}_1\alpha(1+\beta) + \mathrm{t}_1\left(1+\mathrm{t}_1^{\beta}\alpha+\beta\right)\right)\right.}{\mathrm{t0}(1+\beta)} \\
&+ \frac{h\mathrm{t}_0^{1+n}\left(Q^{1+m}(1+\beta) - QR\mathrm{t}_0(1+\beta) + Q^m\left(-\mathrm{t}_0^{1+\beta}\alpha\beta + \mathrm{t}_0(1+\beta) + \mathrm{t}_0^{\beta}\mathrm{t}_1\alpha(1+\beta) - \mathrm{t}_1\left(1+\mathrm{t}_1^{\beta}\alpha+\beta\right)\right)\right)))}{\mathrm{t}_0(1+\beta)} = 0
\end{aligned}
\tag{9.26}
$$

And

$$
\begin{aligned}
Rs(T-\mathrm{t}_1) \quad &+ \frac{h\mathrm{t}_1^{2+n}\left(2\mathrm{t}_1^{-1+\beta}\alpha\beta^2 + n\mathrm{t}_1^{-1+\beta}\alpha\beta^2\right)}{(1+n)(2+n)(1+n+\beta)(2+n+\beta)} \\
&+ \frac{h\mathrm{t}_1^{1+n}\left(2+n^2+\left(3+2\mathrm{t}_1^{\beta}\alpha\right)\beta+\beta^2+n\left(3+\left(2+\mathrm{t}_1^{\beta}\alpha\right)\beta\right)\right)}{(1+n)(1+n+\beta)(2+n+\beta)} \\
&+ cm^{\frac{1}{m}}R\alpha\beta\left(\frac{\mathrm{t}_1^{2\beta}\alpha}{2(1+2\beta)} + \frac{\mathrm{t}_1^{\beta}\left(2+4\beta+\mathrm{t}_1^{\beta}\alpha(1+\beta)\right)}{2\beta(1+2\beta)}\right. \\
&\left. - \frac{\mathrm{t}_0^{\beta}\left(2\mathrm{t}_1^{\beta}\alpha(1+\beta)(1+2\beta) - \left(-2+\mathrm{t}_0^{\beta}\alpha\right)\left(1+3\beta+2\beta^2\right)\right)}{2\beta(1+\beta)(1+2\beta)}\right)
\end{aligned}
$$

$$-\frac{ht_0^{1+n}\left(\frac{t_1\left(nt1^{-1+\beta}\alpha\beta+t_1^{-1+\beta}\alpha\beta(1+\beta)\right)}{(1+n)(1+n+\beta)}+\frac{(1+\beta)\left(1-t_0^{\beta}\alpha+t_1^{\beta}\alpha+\beta\right)+n\left(1+t_1^{\beta}\alpha+\beta-t_0^{\beta}\alpha(1+\beta)\right)}{(1+n)(1+n+\beta)}\right)}{1+\beta}=0 \tag{9.27}$$

The total cost would be minimum if the determinant of the hessian matrix (H-matrix) of TC (t_0, t_1) is positive definite for $t_0 = t_0$* and $t_1 = t_1$* obtained from Eqs. (9.26) and (9.27), i.e.,

$$H = \begin{vmatrix} \frac{\partial^2 TC}{\partial t_0^{*2}} & \frac{\partial^2 TC}{\partial t_1^* \partial t_0^*} \\ \frac{\partial^2 TC}{\partial t_0^* \partial t_1^*} & \frac{\partial^2 TC}{\partial t_1^{*2}} \end{vmatrix} \geq 0 \tag{9.28}$$

9.4.3 Case III: Time-Dependent Holding Cost When t_0 Is Random

Again, as we assume that the holding cost is time-dependent, so the holding cost is ht^n

$$\text{HC} = \int_0^{t_0} ht^n q(t)dt + \int_{t_0}^{t_1} ht^n q(t)dt \tag{9.28a}$$

The decay start time, t_0, is random and that follows Γ distribution, whose density function is

$$\text{f(x)} = \begin{cases} \frac{1}{\Gamma(l)} l^{x-1} e^{-l} dl, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases} \tag{9.29}$$

And the mean of the Γdistribution is l.

Since holding cost and deterioration cost are functions of t_0 random, the holding cost and deterioration cost should be expected holding cost and expected deterioration cost, respectively.

The expected holding cost is given as

$$\text{E(HC)} = \int_0^{l}\int_0^{t_0} ht^n q(t)dtdl + \int_0^{l}\int_{t_0}^{t_1} ht^n q(t)dtdl \tag{9.30}$$

$$= \frac{hQQ^{-m}((2+n)Q^m-(1+n)(1+l+n)R)\Gamma[1+l+n]}{(1+n)(2+n)\Gamma[l]} + \frac{\left(\begin{array}{l} \left(h\left(t_1^{2+n}(1+\beta)\left(2+n^2+\left(3+2t_1^{\beta}\alpha\right)\beta+\beta^2+n\left(3+\left(2+t_1^{\beta}\alpha\right)\beta\right)\right)\Gamma[l]+\right.\right. \\ (2+n^2+3\beta+\beta^2+n(3+2\beta))\left(1-2t_1-2t_1^{1+\beta}\alpha+\beta-2t_1\beta+n^2(1+\beta)+\right. \\ l(1+n)(1+\beta)-n\left(-2+t_1+t_1^{1+\beta}\alpha-2\beta+t_1\beta\right))\Gamma[1+l+n]- \\ (2+3n+n^2)\alpha(-t_1(1+\beta)(2+n+\beta) \\ +\beta(1+n+\beta)(1+l+n+\beta))\Gamma[1+l+n+\beta])) \end{array}\right)}{((1+n)(2+n)(1+\beta)(1+n+\beta)(2+n+\beta)\Gamma[l])} \tag{9.31}$$

The expected deterioration cost is given by

$$EDC = \frac{\left(\begin{array}{c} c\alpha\beta Rm^{\frac{1}{m}}(t_1^{1+\beta}\left(2+4\beta+t_1^{\beta}\alpha(1+\beta)\right)\Gamma[l]- \\ 2(1+2\beta)\left(t_1^{1+\beta}\alpha+t_1(1+\beta)-\beta(l+\beta)\right)\Gamma[l+\beta]+ \\ \alpha\left(-2\beta^2(l+2\beta)+t_1\left(1+3\beta+2\beta^2\right)\right)\Gamma[l+2\beta]) \end{array}\right)}{2\beta(1+\beta)(1+2\beta)\Gamma[l]} \tag{9.32}$$

The total cost of the inventory model is governed by the ordering cost, expected deterioration cost, and expected holding cost and shortage cost given by Eqs. (9.9), (9.31), and (9.32), respectively. Therefore, the expected total cost per unit is given by

$$ETC = \frac{K + EDC + EHC + SC}{T} \tag{9.33}$$

$$= \frac{hQQ^{-m}\left((2+n)Q^m-(1+n)(1+l+n)R\right)\Gamma[1+l+n]}{(1+n)(2+n)\Gamma[l]} - \frac{1}{2}sR(T-t_1)^2 + \frac{\left(\begin{array}{c} (h(t_1^{2+n}(1+\beta)(2+n^2+(3+2t_1^{\beta}\alpha)\beta+\beta^2+n(3+(2+t_1^{\beta}\alpha)\beta))\Gamma[l]+ \\ (2+n^2+3\beta+\beta^2+n(3+2\beta))(1-2t_1-2t_1^{1+\beta}\alpha+\beta-2t_1\beta+n^2(1+\beta)+ \\ l(1+n)(1+\beta)-n(-2+t_1+t_1^{1+\beta}\alpha-2\beta+t_1\beta))\Gamma[1+l+n]- \\ (2+3n+n^2)\alpha(-t_1(1+\beta)(2+n+\beta)+\beta(1+n+\beta)(1+l+n+\beta))\Gamma[1+l+n+\beta])) \end{array}\right)}{((1+n)(2+n)(1+\beta)(1+n+\beta)(2+n+\beta)\Gamma[l])} + \frac{\left(\begin{array}{c} c\alpha\beta Rm^{\frac{1}{m}}(t_1^{1+\beta}(2+4\beta+t_1^{\beta}\alpha(1+\beta))\Gamma[l]- \\ 2(1+2\beta)\left(t_1^{1+\beta}\alpha+t_1(1+\beta)-\beta(l+\beta)\right)\Gamma[l+\beta]+ \\ \alpha(-2\beta^2(l+2\beta)+t_1(1+3\beta+2\beta^2))\Gamma[l+2\beta]) \end{array}\right)}{2\beta(1+\beta)(1+2\beta)\Gamma[l]} \tag{9.34}$$

To minimize the total cost TC (t_0, t_1) per unit time, the optimal values of t_0, t_1 can be obtained by solving the following equations:

$$\frac{d(TC)}{dt_0} = 0 \quad \text{And} \quad \frac{d(TC)}{dt_1} = 0 \tag{9.35}$$

And the total cost would be minimum if the determinant of the Hessian matrix (H-matrix) of TC (t_0, t_1) is positive definite for $t_0 = t_0{*}$ and $t_1 = t_1{*}$ obtained from

Eqs. (9.35), i.e.,

$$H = \begin{vmatrix} \frac{\partial^2 TC}{\partial t_0^{*2}} & \frac{\partial^2 TC}{\partial t_1^* \partial t_0^*} \\ \frac{\partial^2 TC}{\partial t_0^* \partial t_1^*} & \frac{\partial^2 TC}{\partial t_1^{*2}} \end{vmatrix} \geq 0 \tag{9.36}$$

9.5 Algorithm to Calculate Optimum Solution

Step 1: Initialize the value of the variable α, β, ϒ, R, K, C, Q, n, and s
Step 2: Evaluate TC (t_0, t_1)
Step 3: Evaluate $\frac{d(TC)}{dt_0}$ and $\frac{d(TC)}{dt_1}$
Step 4: Solve the simultaneous equation $\frac{d(TC)}{dt_0} = 0$ and $\frac{d(TC)}{dt_1} = 0$
Step 5: Using the results obtained in step 4, evaluate det(H)
Step 6: If the value of det(H) is greater zero, then the take the solution as optimal solution and stop
Step 7: Otherwise, go to Step 1 and choose a different set of initial values of the parameters
Step 8: Stop.

9.6 Numerical Results

For numerical studies, we have considered an example to illustrate the model and examine the effectiveness of the proposed model. We set the following values of the parameters involved in the models: α = 2, β = 0.5, ϒ = 0.2, R = 20, K = 100, C = 50, n = 2, and s = 1.

The optimal solutions are shown in Table 9.1.

From the above table, we see that the total cost in Case II is minimum among the three different total costs. Therefore, the model in Case II is more profitable for known decay start time of non-instantaneous deterioration rather than the other two cases (Figs. 9.2, 9.3, and 9.4).

Table 9.1 Optimal solutions for three different cases

Case	t_0	t_1	Q_1	Total cost
I	0.384	0.95	226.82	1508.712
II	0.625	0.95	212.21	402.109
III	0.3689	0.95	227.73	1230.185

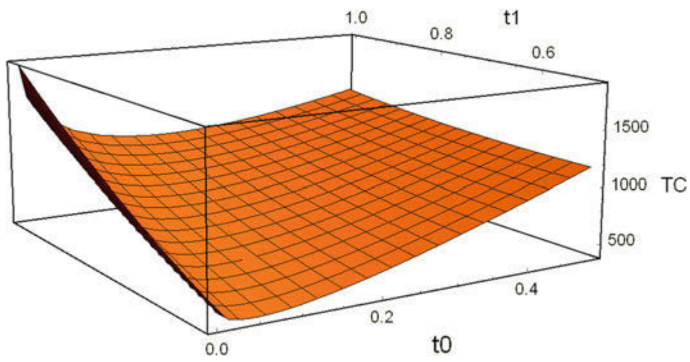


Fig. 9.2 The total cost function graphically for Case I

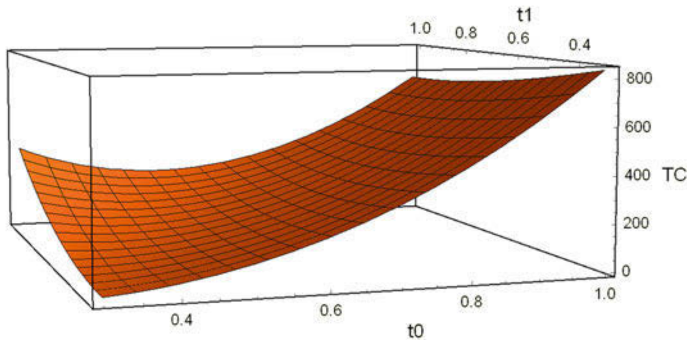


Fig. 9.3 The total cost function graphically for Case II

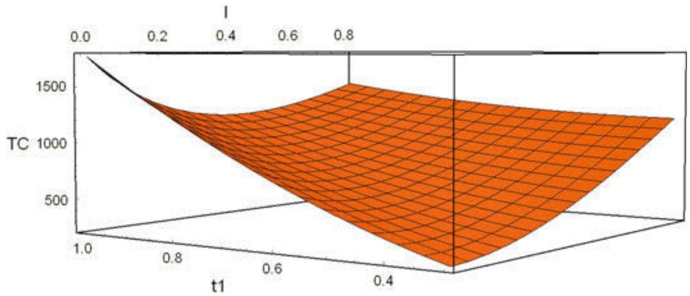


Fig. 9.4 The total cost function graphically for Case III

9.7 Sensitivity Analysis

The sensitivity of TC, Q_1, and t_0 for Case I is given in the Table 9.2.

It is seen from the above table that the total cost increases with the increase in the deterioration components. It also follows for stock parameters. The next figure

Table 9.2 Sensitivity for Case I with different parameters

Parameter	Change	Q_1	t_0	t_1	Total cost
	1.90	227.9000	0.3661	0.95	1481.42
	1.95	227.3500	0.3752	0.95	1495.32
α	2.00	226.8200	0.3840	0.95	1508.71
	2.05	226.3000	0.3926	0.95	1521.64
	2.10	225.7973	0.4010	0.95	1534.11
	0.490	227.0630	0.3801	0.95	1501.22
	0.495	226.9430	0.3821	0.95	1504.99
β	0.500	226.8200	0.3840	0.95	1508.71
	0.505	226.7050	0.3860	0.95	1512.39
	0.510	226.5891	0.3879	0.95	1516.02
	0.190	228.6284	0.3742	0.95	1497.39
	0.195	227.7452	0.3791	0.95	1503.08
ϒ	0.200	226.8239	0.3840	0.95	1508.71
	0.205	225.8626	0.3891	0.95	1514.28
	0.210	224.8595	0.3942	0.95	1519.77
	18.00	231.5912	0.3384	0.95	1436.09
	19.00	229.2564	0.3618	0.95	1474.08
R	20.00	226.8239	0.3840	0.95	1508.71
	21.00	224.3001	0.4056	0.95	1540.23
	22.00	221.6911	0.4264	0.95	1568.87
	9.00	225.1200	0.4122	0.95	1408.03
	9.50	225.9900	0.3977	0.95	1459.08
h	10.00	226.8200	0.3840	0.95	1508.71
	10.50	227.6061	0.3711	0.95	1557.99
	11.00	228.3472	0.3588	0.95	1604.01
	1.00	226.8200	0.3840	0.95	1508.71
	2.00	226.8800	0.3828	0.90	1480.47
s	3.00	229.6200	0.3400	0.85	1386.81
	4.00	231.1800	0.3110	0.80	1293.77
	5.00	233.2270	0.2770	0.75	1201.63

Fig. 9.5 Effect of stock parameters

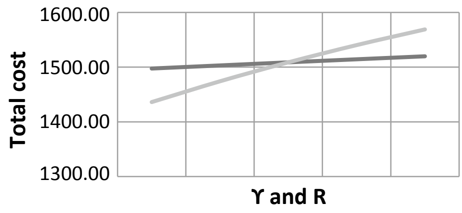
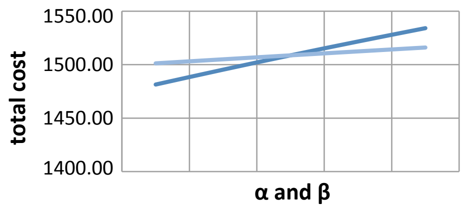


Fig. 9.6 Effect of deterioration parameters



shows graphically the effect of stock parameters and deterioration parameters on the total cost (Figs. 9.5 and 9.6).

The sensitivity of TC, Q_1, and t_0 for Case II is given in the Table 9.3.

It is seen from the above table that the total cost increases with the increase in the deterioration components. It also follows for stock parameters. The next figure shows graphically the effect of stock parameters and deterioration parameters on the total cost (Figs. 9.7 and 9.8).

The sensitivity of TC, Q_1, and t_0 for Case III is given in the Table 9.4.

The above table shows that the total cost increases with the increase in the deterioration components, while it decreases for stock parameter ϒ and increases for R. The next figure shows graphically the effect of stock parameters and deterioration parameters (Figs. 9.9 and 9.10).

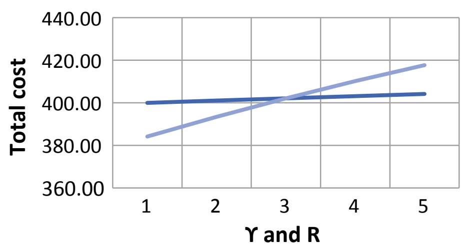
9.7.1 Observation and Managerial Insights Based on Numerical Results and Sensitivity

(1) Significant effect of deterioration components has been observed for all cases. The optimum inventory level is higher in the cases when the policy-maker has chosen the model in the Case III, and thus this makes the retailer fulfill more customers' demand and in this sense the model can maximize the retailer's profit.
(2) Stock components have had a significant phenomenon as above, and hence the retailer can draw the similar policy that follows from the earlier.
(3) Since the total cost is lesser, it is better to choose time-varying holding cost than fixed holding cost from the retailer's point of views.
(4) Consideration of shortage in inventory plays a significant role.

Table 9.3 Sensitivity for Case II with different parameters

Parameter	Change	Q_1	t_0	t_1	Total cost
	1.90	212.7400	0.6174	0.95	395.73
	1.95	212.5000	0.6214	0.95	398.95
α	2.00	212.2000	0.6250	0.95	402.11
	2.05	212.0300	0.6291	0.95	405.19
	2.10	211.8100	0.6328	0.95	408.19
	0.490	212.4105	0.6229	0.95	399.99
	0.495	212.3369	0.6241	0.95	401.06
β	0.500	212.2640	0.6250	0.95	402.11
	0.505	212.1922	0.6265	0.95	403.15
	0.510	212.1210	0.6277	0.95	404.18
	0.190	214.5860	0.6201	0.95	399.98
	0.195	213.4400	0.6227	0.95	401.06
ϒ	0.200	212.2640	0.6253	0.95	402.11
	0.205	211.0400	0.6280	0.95	403.14
	0.210	209.7800	0.6307	0.95	404.15
	18.00	217.3100	0.6018	0.944	384.17
	19.00	214.8000	0.6130	0.947	393.44
R	20.00	212.2600	0.6253	0.95	402.11
	21.00	209.6700	0.6363	0.952	410.18
	22.00	207.0500	0.6469	0.954	417.68
	9.00	211.4268	0.6392	0.95	382.47
	9.50	211.8550	0.6321	0.95	392.43
h	10.00	212.2640	0.6253	0.95	402.11
	10.50	212.6530	0.6189	0.95	411.53
	11.00	213.0250	0.6127	0.95	420.71
	1.00	212.2600	0.6250	0.95	402.11
	2.00	213.7700	0.6003	0.90	365.10
s	3.00	215.2900	0.5750	0.85	331.18
	4.00	216.8400	0.5490	0.80	300.41
	5.00	218.4200	0.5230	0.75	272.89

Fig. 9.7 Effect of stock parameters



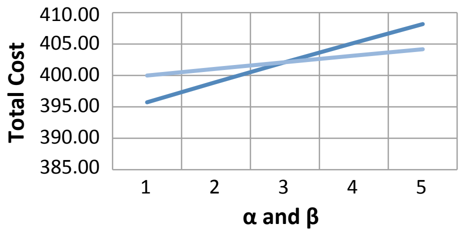


Fig. 9.8 Effect of deterioration parameters

Table 9.4 Sensitivity for Case III with different parameters

Parameter	Change	Q_1	t_0	t_1	Total cost
	1.90	229.2200	0.3440	0.95	1185.64
	1.95	228.4838	0.3565	0.95	1208.14
α	2.00	227.7350	0.3689	0.95	1230.19
	2.05	226.9830	0.3814	0.95	1251.78
	2.10	226.2260	0.3939	0.95	1272.90
	0.490	227.7634	0.3685	0.95	1230.70
	0.495	227.7497	0.3687	0.95	1230.51
β	0.500	227.7350	0.3689	0.95	1230.19
	0.505	227.7190	0.3692	0.95	1229.80
	0.510	227.7032	0.3695	0.95	1229.34
	0.190	230.1080	0.3483	0.95	1275.75
	0.195	228.9850	0.3580	0.95	1253.49
ϒ	0.200	227.7350	0.3689	0.95	1230.19
	0.205	226.0320	0.3816	0.95	1205.69
	0.210	224.7200	0.3963	0.95	1179.78
	18.00	233.8980	0.2964	0.944	1194.58
	19.00	231.0400	0.3307	0.947	1215.92
R	20.00	227.7350	0.3689	0.95	1230.19
	21.00	223.8180	0.4132	0.952	1231.29
	22.00	218.9700	0.4674	0.954	1232.46
	9.00	225.9330	0.3988	0.95	1186.90
	9.50	226.8640	0.3834	0.95	1209.06
h	10.00	227.7350	0.3689	0.95	1230.19
	10.50	228.5510	0.3551	0.95	1250.36
	11.00	229.3200	0.3427	0.95	1269.66
	1.00	227.7350	0.3689	0.95	1230.19
	2.00	229.2600	0.3430	0.90	1155.74
s	3.00	230.8100	0.3179	0.85	1082.69
	4.00	232.3700	0.2921	0.80	1011.19
	5.00	233.9540	0.2659	0.75	941.32

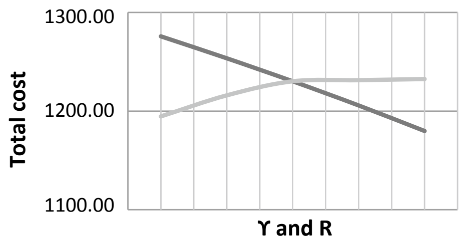


Fig. 9.9 Effect of stock parameters

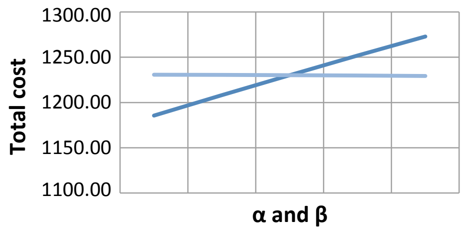


Fig. 9.10 Effect of deterioration parameters

(5) The numerical analysis shows that the solution is quite stable and the model is profitable for known decay start time rather than random start. Also, the model with random decay start time will be much profitable when the optimum inventory level is higher than others.

9.8 Concluding Remarks

We portray an inventory model for non-instantaneous deteriorating items concerning adjournment of deterioration start time to extend the traditional EOQ model. The goods with high deterioration proportion are at all times ordeal to the retailer's trade. In actual markets, the retailer can decrease the deterioration of a product and equivalent cost by making nominal capital investment in store equipment. In this study, we have developed a deteriorating stock reliant inventory model with time-varying holding cost and shortages. Results of sensitivity analysis have demonstrated many managerial insights. The non-instantaneous deterioration in inventory model is very genuine proposition for the retailers. The decay start time has been well thought out here for both known and random cases. The mathematical analysis of the model shows that the solution of the model is quite stable and is more profitable for known decay start time rather than random start on cost in one sense and on the other hand, the random decay start is much better on inventory level.

The researchers can be extended further the model by taking more substantial assumptions such as finite renewal rate, fuzzy and probabilistic demand rate and taking preservation technology, etc.

References

1. Chang CT, Teng JT, Goyal SK (2010) Optimal replenishment policies for non-instantaneous deteriorating items. Int J Prod Econ 123(1):62–68
2. Dave U, Patel LK (1981) (T, Si) policy inventory model for deteriorating items with time proportional demand. J Oper Res Soc 32:137–142
3. Duari NK, Chakrabarti T (2015) A cash flow-oriented EOQ model of deteriorating items with increasing demand rate and shortages under permissible delay in payment. Int J Oper Res 24(2):145–160
4. Duari NK, Chakrabarti T (2016) A EPQ inventory model with stochastic demand, finite production rate of deteriorating items, shortages and two-level of credit financing. IOSR J Eng (IOSRJEN) 06(08):51–59
5. Dye CY, Hsieh TP (2012) An optimal replenishment policy for deteriorating items with effective investment in preservation technology. Eur J Oper Res 218(1):106–112
6. Dye C-Y, Ouyang L-Y (2011) A particle swarm optimization for solving joint pricing and lot sizing problem with fluctuating demand and trade credit financing. Comput Ind Eng 60(1):127–137
7. Geetha KV, Udayakumar R (2016) Optimal lot sizing policy for non-instantaneous deteriorating items with price and advertisement dependent demand under partial backlogging. Int J Appl Comp Math 2:171–193
8. Ghare PM, Schrader GF (1963) A model for an exponentially decaying inventory. J Ind Eng 14:238–243
9. Hsu PH, Wee HM, Teng HM (2010) Preservation technology investment for deteriorating inventory. Int J Prod Econ 124(2):388–394
10. Hung K-C (2011) An inventory model with generalized type demand, deterioration and back order rates. Eur J Oper Res 208(3):239–242
11. Lee CC, Hsu S-L (2009) A two-warehouse production model for deteriorating inventory items with time-dependent demands. Eur J Oper Res 194(3):700–710
12. Leea Y-P, Dye C-Y (2012) An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. Comput Ind Eng 63(2):474–482
13. Maihami R, Kamalabadi IN (2012) Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand. Int J Prod Econ 136(1):116–122
14. Mishra VK, Singh LS (2011) Deteriorating inventory model for time dependent demand and holding cost with partial back logging. Int J Manag Sci Eng Manag 6(4):267–271
15. Mishra VK, Singh LS, Kumar R (2013) An inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial back logging. J Ind Eng Int 9:1–7
16. Ouyang LY, Wu KS, Yang CT (2006) A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Comput Ind Eng 51(4):637–651
17. Roy A (2008) An inventory model for deteriorating items with price dependent demand and time varying holding cost. Adv Model Optim 10:25–37
18. Sarkar B (2012) An EOQ model with delay-in-payments and time-varying deterioration rate. Math Comput Model 55:367–377
19. Shah NH, Soni HN, Patel KA (2013) Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. Omega 41(2):421–430
20. Shaikh AA, Mashud AH, Uddin MS, Khan MA (2017) Non-instantaneous deterioration inventory model with price and stock dependent demand for fully backlogged shortages under inflation. Int J Bus Forecast Mark Intel 3:152–164
21. Tripathi R, Singh D, Aneja S (2017) Inventory models for stock-dependent demand and time varying holding cost under different trade credits. Yugosl J Oper Res 28:18
22. Udayakumar R, Geetha KV (2018) An EOQ model for non-instantaneous deteriorating items with two levels of storage under trade credit policy. J Ind Eng Int 14(2):343–365

23. Wu KS, Ouyang LY, Yang CT (2006) An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial back logging. Int J Prod Econ 101(2):369–384
24. Zhou Y-W, Lau H-S, Yang S-L (2003) A new variable production scheduling strategy for deteriorating items with time-varying demand and partial lost sale. Comput Oper Res 30(12):1753–1776

Chapter 10
Stock-Dependent Inventory Model for Imperfect Items Under Permissible Delay in Payments

Vinti Dhaka, Sarla Pareek and Mandeep Mittal

Abstract In the production process, the issue of quality is always ignored which results in defective production. These defective items can be removed from the lot through the inspection process which becomes essential for the system. Demand is considered as stock dependent. It is continuously declined to meet the customer's demand which depends on the on-hand inventory up to the time t_2. After that the inventory level declines by constant demand up to time t_3. Thereafter, shortages occur and it accumulates at the rate $\psi(\tau - t)\, till\, t = \tau$ when the next batch arrives. The whole cycle repeats itself after the cycle length τ. Further, it is assumed that payment will be made to the supplier for the goods immediately after receiving the consignment. Whereas, in practice, supplier does offer a certain fixed period to the retailer for settling the account. During this period, supplier charges no interest, but beyond this period interest is being charged. On the other hand, retailer can earn interest on the revenue generated during this period. Keeping this scenario in mind, an attempt has been made to formulate an inventory policy for the retailer dealing with imperfect quality items under permissible delay in payments. Results have been analyzed with the help of a numerical example and sensitivity analysis also carried out.

Keywords Imperfect items · Permissible delay in payments · Stock-dependent demand

V. Dhaka (✉)
Department of Mathematics, Faculty of Sciences, Shree Guru Gobind Singh Tricentenary University, Gurugram 122505, India
e-mail: dhakavinti@gmail.com

S. Pareek
Department of Mathematics & Statistics, Banasthali Vidyapith, Banasthali, Rajasthan 304022, India
e-mail: psarla13@gmail.com

M. Mittal
Department of Mathematics, Amity University, Noida 201303, India
e-mail: mittal_mandeep@yahoo.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*, Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_10

10.1 Introduction

In general, demand does not depend on the factors like availability of stock, cost a product is mainly assumed in the basic inventory model in today's technology, A very common assumption of the economic order quantity is that all the units produce or purchased are of good quality. But in real situation demand rate is completely dependent on availability of stock present. For example, if there is large number of items shown on display in any store then the customers are directly influenced by the display stock and on the other hand, if the items displayed are not large then it will affect the customer as so. Levin et al. [16] concluded that goods which are displayed on the shelf of any supermarket are more opted by the customer. It encourages the consumer to buy more. Silver and Peterson [23] observed that sales at the marketing level are directly proportional to the displayed stock.

Practically, the seller provides retailer a certain time period to settle his accounts. During this time period, the retailer is free to sell all the inventories without paying any interest. After this time period, the retailer has to charge an interest on that amount to the seller. However, manager earns more profit if he delays until the final day of the extended period. Goyal [12] was the first to propose an EOQ model with trade credit. Chand and Ward [7] examined Goyal's problem with the assumptions same as the basic economic order quantity model with different results. Baron et al. [2] developed a retailer joint ordering, pricing, and preservation technology investment policies for deteriorating item under permissible delay in payments. Sarkar et al. [22] developed a model for imperfect production system with probabilistic rate of imperfect production for deteriorating products. Further, in reality, all the items produced cannot be of perfect quality. Some of the items will be imperfect also. These imperfect items were produced due to deprived production quality, insufficient and imperfect material for manufacturing. This situation was considered by Salameh and Jaber [21] and after that many more researchers are working on the same situation by considering different parameters.

Author	Year	Supply chain	Defective items	Trade credit	Stock-dependent demand
Whitin	[26]	✓		✓	
Wolfe	[27]	✓		✓	
Levin et al.	[16]	✓			
Silver and Peterson	[23]	✓			
Baker and Urban	[1]	✓			✓
Mandal and Phaujdar	[17]	✓			✓
Datta and Pal	[9]			✓	✓
Urban	[25]	✓			✓

(continued)

(continued)

Author	Year	Supply chain	Defective items	Trade credit	Stock-dependent demand
Pal et al.	[20]	✓			✓
Padmanabhan and Vrat	[18]	✓			✓
Salameh and Jaber	[21]	✓	✓		
Datta and Paul	[10]	✓			✓
Chang	[8]	✓			✓
Pal	[19]	✓		✓	✓
Hou and Lin	[13]	✓			✓
Goyal and Chang	[11]	✓			✓
Tsu-pang	[24]	✓		✓	✓
Bhunia and Shaikh	[3]	✓			✓
Bhunia et al.	[5]	✓			
Jaggi and Mittal	[15]	✓	✓	✓	
Bhunia and Shaikh	[4]	✓			
Bhunia, Shaikh, Pareek and Dhaka,	[6]	✓		✓	✓
Jaggi et al.	[14]	✓	✓	✓	
Baron et al.	[2]	✓	✓	✓	
Sarkar et al.	[22]	✓	✓		
This model		✓	✓	✓	✓

In this chapter, retailer has gone through various scenarios with imperfect quality under the permissible delay in payments. Considering these conditions with corresponding optimization cases, solved in the software MAPLE 18. This model ends with the numerical example and affectability analysis under different parameters.

Notation:

A	Cost of ordering each unit
c	Cost of purchasing each unit
λ	Screening cost
p	Selling price
p_s	Salvage value
I_p	Rate of interest paid

(continued)

(continued)

I_e	Rate of interest earned
Q	Order quantity per cycle
P	The maximum inventory level per cycle
R	Maximum shortage quantity per cycle
h	Holding cost
c_2	Backorder cost
c_3	Opportunity cost
α	Level of damaged items in Q
$f(\alpha)$	Probability density function of α
$E(\alpha)$	Expected estimation of α, which is equivalent to expected estimation of α, which is equivalent to $\int_a^b \alpha f(\alpha)d\alpha,\ 0 < a < b < 1$
t_1	Screening time
t_2	Time point at which the inventory level reaches Q_0, where Q_0 is known
t_3	Time point at which the shortages are allowed
τ	Length of the inventory cycle
M	Period of permissible delay in payments
$I_1(t)$	Inventory level at time t, where $0 \le t \le t_1$
$I_2(t)$	Inventory level at time t, where $t_1 < t \le t_2$
$I_3(t)$	Inventory level at time t, where $t_2 < t \le t_3$
$I_4(t)$	Inventory level at time t, where $t_3 < t \le \tau$
$I_{eff}(t_1)$	Effective stock level at time t_1 which does not include defective items
$TP_1(t_3, \tau)$	The total profit when $0 \le \boldsymbol{M} \le \boldsymbol{t}_1$
$TP_2(t_3, \tau)$	The total profit when $\boldsymbol{t}_1 < \boldsymbol{M} \le \boldsymbol{t}_2$
$TP_3(t_3, \tau)$	The total profit when $\boldsymbol{t}_2 < M \le \boldsymbol{t}_3$
$TP_4(t_3, \tau)$	The total profit when $\boldsymbol{t}_3 < M \le \tau$
$TP_5(t_3, \tau)$	The total profit when $\tau \le \boldsymbol{M}$

Assumptions:

- Instantaneous replenishment rate.
- Lead time is considered as negligible.
- The screening and demand proceed at the same time, but the rate of screening is greater than rate of demand.
- Defective items existing in the lot follow uniform distribution.
- Postponement in payment is offered by supplier to settle retailer's account.
- Q be the items in the system.
- $f(\alpha)$ is the known probability density function, where α is the percent of defective items.
- Screening is performed when the quantity is received by the retailer with the rate of μ units time which is more than the rate of demand in the time period $(0, t_1)$.

- The demand rate of the system depends upon the inventory available (stock) and down to a certain inventory level Q_0, where Q_0 is fixed and known, beyond that level it is presumed as constant, i.e., when the inventory(stock) level is $I(t)$ and the demand rate $I(I(t))$ of the item is considered as follows:

$$I(I(t)) = \begin{cases} \delta[I(t)]^{\beta}, 0 < t < t_1 \\ \delta[I(t)]^{\beta}, I(t) \geq Q_0, t_1 < t < t_2 \\ W, 0 \leq I(t) < Q_0, t_2 < t < t_3 \end{cases}$$

where $\delta > 0$ and $0 < \beta < 1$ are termed as scale and shape parameters, respectively, $W(> 0)$ is constant such that $W = \delta Q_0^{\beta}$.

- Shortages are allowed then it is partially backordered, that is, only a portion of shortages are backordered which is also a time function t denoted by $\psi(t)$, where t is the time till the next fulfillment with $0 \leq \theta(t) < 1$. Let the fraction is given by $\psi(t) = \frac{1}{1+\theta(t)}$, $\theta > 0$. It is to be noted that the partial backlogging reduces to a complete backlogging when $\theta \to 0$, i.e., $\psi(t) \to 1$ (Fig. 10.1).

By assuming to be expected at first, a retailer buys $Q(= P + R)$ units. It is additionally expected that each parcel may have some damaged things. "Let α be the percent of defective things with acknowledged probability density function $f(\alpha)$. Screening process has improved the situation of all the got amount at the rate λ units per unit time which is more prominent than interest rate for the period (0–t_1). In the meanwhile, screening process of the interest happens parallel to the screening procedure and is satisfied from the products which are observed to be of perfect quality through screening process, and there is a P unit of the close by stock." It ceaselessly

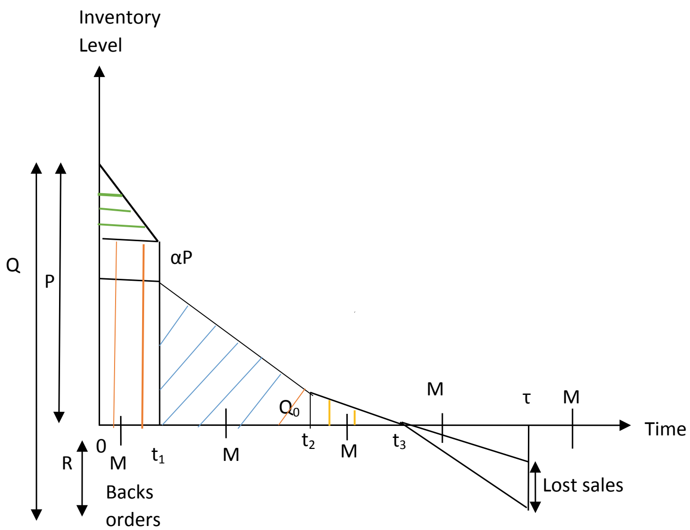


Fig. 10.1 Representation of inventory system

decays because to satisfy the customer's need which is reliant on the available stock dimension up to the time $t = t_2$ [15]. After that the stock decreases constantly up to $t = t_3$. The defective items are sold instantly after the inspection process has ended at time t_1 as a sole lot at a reduced price. After time t_1 the adequate inventory level will be $I_{eff}\ (t_1)$. From that point, shortages occur and it aggregates at the rate $\psi(\tau - t)$ till $t = \tau$ when the following cluster arrives. This entire cycle repeats itself after the cycle length τ.

The presentation of the system is shown in figure. Hence, the governing differential equation for the inventory level is as follows:

$$\frac{dI_1(t)}{dt} = -\delta[I_1(t)]^{\beta}; 0 < t < t_1. \tag{10.1}$$

Solution of differential Eq. (10.1)

$$I_1(t) = \left[-\delta t(1-\beta) + (P)^{1-\beta}\right]^{\frac{1}{1-\beta}}. \tag{10.2}$$

Now, inventory level at time t_1, including the defective items is

$$I_1(t_1) = \left[-\delta t_1(1-\beta) + (P)^{1-\beta}\right]^{\frac{1}{1-\beta}}. \tag{10.3}$$

Therefore, the numbers of defective items after screening at time t_1 is αP, the effect level of inventory at $t = t_1$ after removal of the defective items is

$$I_{eff}(t_1) = \left[-\delta t_1(1-\beta) + (P)^{1-\beta}\right]^{\frac{1}{1-\beta}} - \alpha P. \tag{10.4}$$

Now, the differential equation with the boundary condition at $t = t_1 \Rightarrow I_{eff}(t_1)$ is

$$\frac{dI_2(t)}{dt} = -\delta[I_2(t)]^{\beta}; t_1 < t < t_2. \tag{10.5}$$

Solution of differential Eq. (10.5) is

$$I_2(t) = [\delta(1-\beta)(t_1 - t)]^{\frac{1}{1-\beta}} + \left[I_{eff}(t_1)\right].$$

Put the value of $I_{eff}(t_1)$ from Eq. (10.4)

$$I_2(t) = P - [\delta(1-\beta)t]^{\frac{1}{1-\beta}} - \alpha P. \tag{10.6}$$

Now, at $t = t_2$, the demand rate becomes constant, i.e., W and the inventory level becomes zero at $t = t_3$.

In the interval (t_2, t_3) due to the effect of demand, the inventory reduces. Hence, the governing differential equation for the inventory level is

$$\frac{dI_3(t)}{dt} = -W; t_2 < t < t_3. \tag{10.7}$$

Result from (10.7) equation with the boundary condition at $t = t_3 \Rightarrow I_3(t) = 0$

$$I_3(t) = W(t_3 - t); t_2 < t < t_3. \tag{10.8}$$

Shortages are allowed and it is partially backlogged, i.e., the fraction is $\psi(t) = \frac{1}{1+\theta(t)}, \theta > 0$

$$\frac{dI_4(t)}{dt} = -\frac{W}{1 + \theta(\tau - t)}; t_3 < t < \tau. \tag{10.9}$$

Result from (10.9) equation with the condition at $t = t_3 \Rightarrow I_4(t) = 0$

$$I_4(t) = -\frac{W}{\theta}\{ln|1 + \theta(\tau - t_3)| - ln|1 + \theta(\tau - t)|\}; t_3 < t < \tau. \tag{10.10}$$

Hence, the order quantity per cycle is given as follows:

$$\begin{aligned} Q &= (P + \alpha P) + [(P)^{1-\beta}]^{\frac{1}{1-\beta}} + Wt_3 + \frac{W}{\theta}\{ln|1 + \theta(\tau - t_3)|\} \\ &= P(2 + \alpha) + Wt_3 + \frac{W}{\theta}\{ln|1 + \theta(\tau - t_3)|\}. \end{aligned}$$

Now, the other costs of the system are given below:

1. Sales revenue deals with the sum of revenue generated by the demand meet during the period $(\tau - t)$ and sale of imperfect quality items is

$$\begin{aligned} &P(1 - \alpha)P(2 + \alpha) + Wt_3 + \frac{W}{\theta}\{ln|1 + \theta(\tau - t_3)|\} \\ &+ p_s\alpha\left(P(2 + \alpha) + Wt_3 + \frac{W}{\theta}\{ln|1 + \theta(\tau - t_3)|\}\right). \end{aligned}$$

2. Ordering Cost = $\boldsymbol{A}$.
3. Purchase Cost = $\boldsymbol{cQ}$
4.

$$c\left(P(2 + \alpha) + Wt_3 + \frac{W}{\theta}\{ln|1 + \theta(\tau - t_3)|\}\right).$$

5. Screening cost = $\boldsymbol{\gamma Q}$

$$\gamma\left(P(2 + \alpha) + Wt_3 + \frac{W}{\theta}\{ln|1 + \theta(\tau - t_3)|\}\right).$$

6. Holding cost =

$$h\left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{t_2} I_2(t)dt + \int_{t_2}^{t_3} I_3(t)dt\right]$$

$$= h\left[\begin{array}{l} Pt_1 - \frac{[\delta(1-\beta)]^{\frac{1}{1-\beta}}}{2-\beta}(t_1)^{2-\beta} + P(t_2 - t_1) - \frac{[\delta(1-\beta)]^{\frac{1}{1-\beta}}}{2-\beta}\left((t_2)^{2-\beta} - (t_1)^{2-\beta}\right) \\ \quad - \alpha P(t_2 - t_1) + W\left(\frac{t_3^2}{2} + \frac{t_2^2}{2} - t_3 t_2\right) \end{array}\right].$$

7. Backorder cost:

$$c_2 \int_{t_3}^{T} \{-I_4(t)\}dt$$

$$= c_2 \int_{t_3}^{T} -\frac{W}{\theta}\{ln|1 + \theta(\tau - t_3)| - ln|1 + \theta(\tau - t)|\}dt$$

$$= c_2\left[\left(R + \frac{W}{\theta}\right)(\tau - t_3) - \frac{W}{\theta^2}\{1 + \theta(\tau - t_3)\}\log|1 + \theta(\tau - t_3)|\right].$$

8. Lost sales

$$c_3 W \int_{t_3}^{T} \left\{1 - \frac{1}{1 + \theta(\tau - t)}\right\}dt$$

$$= \frac{c_3 W}{\theta}\{\theta(\tau - t_3) - ln|1 + \theta(\tau - t_3)|\}.$$

As M is the period of permissible delay in payments offered to retailer by supplier, there arise these different cases as follows:

Case 1: $0 \le M \le t_1$,
Case 2: $t_1 < M \le t_2$,
Case 3: $t_2 < M \le t_3$,
Case 4: $t_3 < M \le \tau$, and
Case 5: $\tau \le M$.

Since the retailer's total profit consists of the following components:

$$TP_j = \textbf{Sales Revenue} - \textbf{Ordering cost} - \textbf{Holding Cost}$$
$$- \textbf{shortage cost} + \textbf{Interest Earned} - \textbf{Interest Paid}.$$

Case 1: **When** $0 < M \le t_1$

Here interest is earned from the sales up to M. Although account has to be settled at M and money is to be arranged at some stated rate of interest for financing the remaining stocks for the period M to T.

- Interest earned

$$= pI_e \int_0^M \delta[I_1(t)]^\beta t dt$$
$$= \frac{pI_e}{\delta(1-\beta)}\left[\frac{1-\beta}{2-\beta}\left(-\delta M(1-\beta)+p^{1-\beta}\right)^{\frac{2-\beta}{1-\beta}} - p^{1-\beta}(1-\beta)\left(-\delta M(1-\beta)+p^{1-\beta}\right)^{\frac{1}{2-\beta}}\right].$$

- Interest payable

$$= cI_p\left[\int_M^{t_1} I_1(t)dt + \int_{t_1}^{t_2} I_2(t)dt\right]$$
$$= cI_p\left[\begin{array}{l} -\frac{1}{2-\beta}[M(1-\beta)-t_1(1-\beta)] \\ +\left[p(t_2-t_1)-(\delta(1-\beta))^{\frac{1}{1-\beta}}\frac{1-\beta}{2-\beta}\left(t_2^{\frac{2-\beta}{1-\beta}} - t_1^{\frac{2-\beta}{1-\beta}}\right) - \alpha p(t_2-t_1)\right]\end{array}\right].$$

Case 2: **When** $t_1 < M \le t_2$

For this situation, he/she can acquire interest from sales up to M, also interest will be earned from shortages and from the sale of defective items during $(M - t_1)$.

- Interest earned

$$= pI_e \int_0^M \delta[I_1(t)]t dt + (p_s I_e \alpha Q)(M-t_1)$$
$$= \frac{pI_e}{\delta(1-\beta)}\left[\frac{1-\beta}{2-\beta}\left(-\delta M(1-\beta)+p^{1-\beta}\right)^{\frac{2-\beta}{1-\beta}} - p^{1-\beta}(1-\beta)\left(-\delta M(1-\beta)+p^{1-\beta}\right)^{\frac{1}{2-\beta}}\right]$$
$$+ p_s I_e \alpha Q(M-t_1).$$

- Interest payable

$$= cI_p \int_M^{t_2} I_2(t)dt$$
$$= cI_p\left[p(t_2-M)-(\delta(1-\beta))^{\frac{1}{1-\beta}}\frac{1-\beta}{2-\beta}\left(t_2^{\frac{2-\beta}{1-\beta}} - M^{\frac{2-\beta}{1-\beta}}\right) - \alpha p(t_2-M)\right].$$

Case 3: **When $t_2 < M \leq t_3$**

The retailer can procure interest on income created from the sales till M, and furthermore from the period (t_2, M) he acquires interest on the income produced from the sales of defective items at t_2, despite the fact that he needs to settle the account at M, for which money has to be arranged at some stated rate of interest for financing his remaining stocks for the period $(M - t_3)$.

- Interest earned

$$= pI_e \int_0^M \delta[I_2(t)]^\beta t dt + (p_s I_e \alpha Q)(M - t_2)$$
$$= pI_e\Big(P(M) + \delta(1-\beta)M^{\frac{2-\beta}{1-\beta}} + \alpha PM\Big)$$
$$+ \left(p_s I_e \alpha \left(P(2+\alpha) + Wt_3 + \frac{W}{\theta}\{ln|1 + \theta(\tau - t_3)|\}\right)\right)(M - t_2).$$

- Interest payable

$$= cI_p \int_M^{t_3} I_3(t)dt$$
$$= cI_p\left[Wt_3(t_3 - M) - \frac{(t_3 - M)^2}{2}\right].$$

Case 4: **When $t_3 < M \leq \tau$**

The retailer can earn interest on revenue generated from the sales up to M, and furthermore amid to earn interest on the revenue the period (t_3, M) interest is earned from revenue produced by the sales of defective items at t_2, though account has to be settled at M. For that, money has to be arranged at some stated rate of interest so that remaining stocks could be financed for the period $(M - \tau)$.

- Interest earned

$$= pI_e \int_0^M Wtdt + pI_e Wt_3(M - t_3) + pI_e R(M - t_1) + p_s \alpha Q I_e (M - t_1)$$
$$= pI_e WM + pI_e Wt_3(M - t_3) + pI_e R(M - t_1) + p_s \alpha Q I_e (M - t_1).$$

- Interest payable $= 0$

Case 5: **When $\tau \leq M$**

Here the time period for delay in payment M is more or equal to the total time length of the cycle τ, so interest can be earned by the retailer on cash sales during the period $(0, M)$ and does not pay interest for the items kept in stock. Hence, the interest earned is

$$= pI_e\left[\int_0^{\tau} Wt dt + (M-\tau)\int_0^{\tau} W dt + p_s I_e \alpha Q(M-t_3)\right]$$
$$= pI_e\left[\frac{W\tau^2}{2} + (M-\tau)W\tau + p_s I_e \alpha Q(M-t_3)\right].$$

Solution Procedure

The purpose of the system is to find the optimal solution and the solution is obtained from the above different cases.

The main objective is to obtain the optimal solution from the proposed inventory model. The problem is divided into five cases, on solving these cases optimal profit is to be found.

Hence, the optimal average profit of the system is given by

$$\mathbf{Z}^* = \mathbf{Maximize}\{\boldsymbol{TP}_1(\boldsymbol{t}_3,\tau), \boldsymbol{TP}_2(\boldsymbol{t}_3,\tau), \boldsymbol{TP}_3(\boldsymbol{t}_3,\tau), \boldsymbol{TP}_4(\boldsymbol{t}_3,\tau), \boldsymbol{TP}_5(\boldsymbol{t}_3,\tau)\}.$$

The main objective of the model is to maximize the average profit by taking the necessary conditions and these conditions are also equal to zero.

$$\frac{\partial TP_1(t_3,\tau)}{\partial t_3} = 0, \frac{\partial TP_1(t_3,\tau)}{\partial \tau} = 0; \frac{\partial TP_2(t_3,\tau)}{\partial t_3} = 0, \frac{\partial TP_2(t_3,\tau)}{\partial \tau} = 0; \frac{\partial TP_3(t_3,\tau)}{\partial t_3} = 0, \frac{\partial TP_3(t_3,\tau)}{\partial \tau} = 0;$$
$$\frac{\partial TP_4(t_3,\tau)}{\partial t_3} = 0, \frac{\partial TP_4(t_3,\tau)}{\partial \tau} = 0 \& \frac{\partial TP_5(t_3,\tau)}{\partial t_3} = 0, \frac{\partial TP_5(t_3,\tau)}{\partial \tau} = 0.$$

Numerical Example

For illustrating this model, consider the following values of parameters in the inventory system:

$$A = 100 perorder, c_2 = 6 perunit, c_3 = 5 perunit, \delta = 50, \beta = 0.3,$$
$$Q_0 = 15 perorder, \theta = 1.5, I_e = 0.10/12 perunittime, I_p = 0.12/12 perunittime,$$
$$M = 0.15, \alpha = 0.01 perunit, c = 25 perunit, \lambda = 0.5/unit, p_s = 10/unit,$$
$$h = 10 per/units/year, Q_0 = 15 units/year,$$
$$\mu = 17520 units/year, f(\alpha) = \begin{cases} 25, 0 \le \alpha \le 0.04 \\ 0, otherwise \end{cases} E[\alpha] = 0.02.$$

By putting these parameters in the system, the value of total profit = 46130.6, order quantity = 1000, and cycle time = 0.2440 with rate of defective items 0.02.

Affectability Analysis:

With the given numerical example, sensitivity analysis has been carried out for checking the better result.

Observation from Table 10.1

From Table 10.1, the rate of defective items is to be studied with the lot size Q. It is detected from the table that more the fraction of defective items more is the quantity to be ordered with the increment in time t_1, t_2 and τ but during time t_3 there is decrement in the rate of defective items.

Observation from Table 10.2

Table 10.2 is a relation between the rate of defective items with the inventory cost. It is observed from Table 10.2 that on increasing the rate of defective items the inventory defective cost and total profit also increase while the ordering cost and holding cost decrease.

Observation from Table 10.3
It is observed from Table 10.3 that there is an increment in the permissible delay M from 0.15 to 0.30 due to which there is also an increment in the cycle time, order quantity, and expected profit. Increment in the time period for delay in payment helps the retailer to extend the expenses to the supplier without any penalty cost, which also helps to reduce the costs sustained by the retailer and the profit also increases. In

Table 10.1 Variation of optimum quantity and cycle time with defective rate

Rate of defective	Q	t_1	t_2	t_3	τ
0.01	950.80	0.0900	0.9003	0.0270	0.2801
0.02	1000.0	0.0951	0.0951	0.0260	0.2440
0.04	1100.90	0.1107	0.1104	0.0222	0.3200
0.06	1457.24	0.1456	0.1456	0.0204	0.3456
0.08	1750.67	0.1729	0.1730	0.0140	0.3605
1.00	2900.75	0.2900	0.2940	0.0089	0.5905

Table 10.2 Variation of cost and total profit with defective rate

Rate of defective	Ordering cost	Holding cost	Defective cost	Total profit	Result obtained from
0.01	450.25	520.27	900	45646.5	Case 4
0.02	458.01	500.42	1370	46130.6	Case 4
0.03	430.23	470.62	1800	46601.7	Case 4
0.04	400.80	450.80	2300	47120.6	Case 4
0.05	380.10	400.33	2830	47608.5	Case 4
0.06	350.44	380.50	3300	48111.2	Case 4
0.07	300.97	340.67	3850	48600.7	Case 4
0.08	270.28	300.00	4300	49090.8	Case 4
0.09	230.19	250.64	4900	49550.1	Case 4
1.00	160.54	180.20	5400	50080.5	Case 4

Table 10.3 Variation in optimum quantity, cycle time and expected profit with respect to M

M	Q	t_1	τ	Expected profit	Result obtained from
0.15	5335	0.03	0.99	36,757	Case 4
0.30	5567	0.12	1.03	39,750	Case 4
0.45	5796	0.23	1.07	42,787	Case 4
0.15	5592	0.32	1.04	38,581	Case 4
0.30	5835	0.33	1.08	41,731	Case 4
0.45	6080	0.35	1.12	449,398	Case 4

order to increase his profit, the retailer should always request for long credit periods from the supplier.

10.2 Conclusion

This paper consists of a profit-maximizing inventory model with imperfect items which has been developed under stock-dependent demand under the presence of permissible delay in payments. In today's market, procedure is acquired with some opportunity reason for variety, a screening procedure is inescapable so as to guarantee the things with the sale of good quality. Here, screening rate is thought to be more than the interest rate, which empowers the retailer to satisfy the interest, out of the items which are observed to be of flawless quality, alongside the screening procedure. Moreover, exchange credit has likewise been demonstrated as a basic device for budgetary development in numerous organizations, as it fills in as a decent motivator arrangement for the purchasers. Such a circumstance is especially pervasive in huge foundations in creating nations, which bargain in electronic parts, household merchandise, and purchaser items. From the numerical examples, it has been observed that the expected profit was found in case 4 as compared with case 1, 2, 3, and 5. An affectability analysis is also conducted to indicate the analysis with respect to other parameters such as ordering cost, holding cost, defective items, etc. The model has been extended with the effect of carbon emission, learning and forgetting effect, etc.

References

1. Baker RC, Urban TL (1988) A deterministic inventory system with an inventory level dependent demand rate. J Oper Res Soc 39:823–831
2. Baron et al (2018) Retailer's joint ordering, pricing, and preservation technology investment policies for a deteriorating item under permissible delay in payments. Math Prob Eng 1–14
3. Bhunia AK, Shaikh AA (2011) A deterministic model for deteriorating items with displayed inventory level dependent demand rate incorporating marketing decision with transportation cost. Int J Ind Eng Comput 2(3):547–562

4. Bhunia AK, Shaikh AA (2014) A deterministic inventory model for deteriorating items with selling price dependent demand and three-parameter Weibull distributed deterioration. Int J Ind Eng Comput 5(3):497–510
5. Bhunia AK, Shaikh AA, Maiti AK, Maiti M (2013) A two warehouse deterministic inventory model for deteriorating items with a linear trend in time dependent demand over finite time horizon by Elitist Real-Coded Genetic Algorithm. Int J Ind Eng Comput 4(2):241–258
6. Bhunia AK, Shaikh AA, Pareek S, Dhaka V (2015) A memo on stock model with partial backlogging under delay in payments. Uncertain Supply Chain Manag 3:11–20
7. Chand S, Ward J (1987) A note on economic order quantity under conditions of permissible delay in payments. J Oper Res Soc 38:83–84
8. Chang CT (2004) Inventory models with stock dependent demand and nonlinear holding costs for deteriorating items. Asia Pac J Oper Res 21:435–446
9. Datta TK, Pal AK (1990) A note on an inventory-level-dependent demand rate. J Oper Res Soc 41:971–975
10. Datta TK, Paul K (2001) An inventory system with stock-dependent, price-sensitive demand rate. Prod Plan Control 12:13–20
11. Goyal SK, Chang CT (2009) Optimal ordering and transfer policy for an inventory with stock dependent demand. Eur J Oper Res 196:177–185
12. Goyal SK (1985) Economic order quantity under conditions of permissible delay in payments. J Oper Res Soc 36(4):335–338
13. Hou KL, Lin LC (2006) An EOQ models for deteriorating items with price & stock- dependent selling rate under inflation & time value of money. Int J Syst Sci 37:1131–1139
14. Jaggi CK, Cardenas-Barron LE, Tiwari S, Shafi AA (2017) Two warehouse inventory model for deteriorating items with imperfect quality under the conditions of permissible delay in payments Sharif University of Technology. Sci Iran E 24(1):390–412
15. Jaggi CK, Goel SK, Mittal M (2013) Credit financing in economic ordering policies for defective items with allowable shortages. Appl Math Comput 219:5268–5282
16. Levin RI, McLaughlin CP, Lamone RP, Kottas JF (1972) Production/operations management: contemporary policy for managing operating systems. McGraw-Hill, New York
17. Mandal BN, Phaujdar S (1989) An inventory model for deteriorating items and stock-dependent consumption rate. J Oper Res Soc 40:483–488
18. Padmanabhan G, Vrat P (1995) EOQ models for perishable items under stock-dependent selling rate. Eur J Oper Res 86:281–292
19. Pal P, Das CB, Panda A, Bhunia AK (2005) An application of real-coded genetic algorithm (for mixed integer non-linear programming in an optimal two-warehouse inventory policy for deteriorating items with a linear trend in demand and a fixed planning horizon. Int J Comput Math 82:163–175
20. Pal S, Goswami A, Chaudhari KS (1993) A deterministic inventory model for deteriorating items with stock dependent rate. Int J Prod Econ 32:291–299
21. Salameh MK, Jaber MY (2000) Economic production quantity model for items with imperfect quality. Int J Prod Econ 64:59–64
22. Sarkar et al (2018) A model for imperfect production system with probabilistic rate of imperfect production for deteriorating products. DJ J Eng Appl Math 4(2):1–12
23. Silver EA, Peterson R (1985) Decision systems for inventory management and production planning, 2nd edn. Wiley, Newyork
24. Tsu-Pang H, Chang-Yuan D, Liang-Yuh O (2010) Optimal lot size for an item with partial backlogging rate when demand is stimulated by inventory above a certain stock level. Math Comput Model 51:13–32
25. Urban TL (1992) An inventory model with an inventory level dependent demand rate & relaxed terminal condition. J Oper Res Soc 43:721–724
26. Whitin TM (1957) The theory of inventory management. Princeton University Press, Princeton
27. Wolfe HB (1968) A model for control of style merchandise. Ind Manag Rev 9:69–82

Chapter 11
Joint Effects of Carbon Emission, Deterioration, and Multi-stage Inspection Policy in an Integrated Inventory Model

Bijoy Kumar Shaw, Isha Sangal and Biswajit Sarkar

Abstract This paper discusses an integrated inventory model between vendor and buyer for decayed type of products. The vendor produces perfect products but may arrive some defect products in the system. To control product quality, the manufacturer inspects all the products to separate the defective products. After the first-stage inspection, the defective products are reworked at a fixed cost and again inspection takes place for the reworked products in the second-stage inspection. After completion of the second-stage inspection, the defective products are disposed at some fixed cost and delivers good products to the buyer. The delivery of good products is done by single-setup multi-delivery (SSMD) policy by consideration of fixed and variable types of transportation cost. At any stage, the good quality of products may deteriorate and the constant deterioration rate is considered for vendor and buyer, separately. Carbon is emitted from every portion of the integrated system. This issue is studied in this model and finally the joint total cost of the inventory model is minimized with the help of algebraic method. To illustrate the model numerically, some numerical examples are provided along with the sensitivity analysis and the graphical representations of those examples.

Keywords Inventory · Make-to-order policy · Inspection · Deterioration · Carbon emission

Subject Classification 90B05 · 90B06

B. K. Shaw · I. Sangal
Department of Mathematics & Statistics, Banasthali Vidyapith, Banasthali 304022, Rajasthan, India
e-mail: bkshaw.math@gmail.com

I. Sangal
e-mail: isha.sangal@gmail.com

B. Sarkar (✉)
Department of Industrial & Management Engineering, Hanyang University, Ansan, Gyeonggi-do 15588, South Korea
e-mail: bsbiswajitsarkar@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*, Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_11

11.1 Introduction

Goyal [4] introduced the integrated inventory model with unlimited production rate and this model was extended by Banerjee [1] by considering lot-for-lot production and delivery policy. Goyal [5] extended the model of Banerjee [1] by considering the delivery quantity, which was transported multiple times. How to control the lead time in an integrated system was discussed by Vijayashree and Uthayakumar [28]. Among the different ordering policies, the most used policy is make-to-order (MTO) policy where the manufacturer does not produce products until the order is received. Hadley and Whitin [6] and Silver et al. [25] developed the economic production quantity (EPQ) in MTO strategy with considering the finite demand and production quantities. The deferent policies were introduced in the delivery process and deferent delivery costs were included in the delivery process. The suitable and usable policy is single-setup-multi-delivery (SSMD) policy, which is discussed in the recent studies. The SSMD policy was introduced by Khouja [12]. During the production process, the transportation cost may be fixed or variable. These two types of transportation cost were proposed by Dey et al. [3]. To avoid shortage, the periodic ordering policy was considered by Sarker and Parija [21]. Yan et al. [30] introduced the vendor–buyer model by considering transportation cost and deterioration. Sarkar [17] improved the previous model by using advance solution methodology.

Most of the machines produce perfect products in *in-control* state which may produce defective products when the system goes to *out-of-control* state. The perfect and defective products are calculated during the whole production process and defective products are inspected and repaired. Then, the second time inspection process occurs for the repair products. Ouyang et al. [15], Yadav et al. [29], and Huang [7] discussed the imperfect products. The time-dependent demand and selling-price-dependent demand were introduced by Mittal et al. [14] in a delay in payments situations for the imperfect production system. The defective products in the system were counted and then reworked in the strategic production system under trade credit policy introduced by Khanna et al. [11]. Effort- and time-dependent demand in the imperfect production system with high reliability was discussed by Shah and Vaghela [24]. Sarkar et al. [20] extended the model of Lee and Fu [13] introduced the two-stage inspection policy through normal inspection and inspection on reworked items. Recently, Jayaswal et al. [10] discussed the imperfect products in the trade credit scenario and Taleizadeh et al. [26] introduced the same for multi-items in a single machine production system. Another important thing is that products may deteriorate in any storage. The imperfect and deterioration process in two warehouses were discussed by Jaggi et al. [9]. Shah et al. [23] focused on the deterioration process in the inventory control system for the quadratic-type demand pattern and this present study is extended from the model of Shah et al. [22]. Time-varying deterioration was introduced by Chang and Dye [2] in a partial backlogging scenario. The situation for deterioration and imperfect products was discussed by Jaggi et al. [8] where trade credit and partial backlog scenario were studied. Sarkar [16] explained about the time-varying deterioration rate in an economic order quantity model. This model introduces two-stage inspection and constant deterioration strategy with SSMD policy.

For environment issues, any production system emits CO_2 and especially carbons in the air during transportation as well as production time. Two types of cost for carbon emissions were discussed by Sarkar et al. [18] in variable setup cost. The three-echelon supply chain management model with fixed and variable carbon emission cost was studied by Sarkar et al. [19]. Tiwari et al. [27] discussed the environmental issues in an imperfect production system. This model considers these emission issues along with two-stage inspection and deterioration.

11.2 Problem Description

The problem definition, notation for this model, and assumptions are discussed in this section.

11.2.1 Problem Definition

This inventory model is integrated in nature, where the defective products are produced in the production process. Generally, the defective items are produced in *out-of-control* state for the long-run production process or machinery problems. To detect the defect items, the first-stage inspection process is adopted at the manufacturer in the manufacturing process and then reworks of those defect items. In the second-stage inspection, some defective products were found which were reworked. First-stage inspection process is completed at the starting of the production process and after finishing the reworking process, defective products are sorted out from reworked production by the second-stage inspection. Defective items after rework and inspection are disposed at some fixed cost. The perfect products were sent to the market by the make-to-order (MTO) policy, i.e., after getting the order from buyer, manufacturer produces his lot. The delivery process is done by the SSMD policy and delivers the product by *n* shipments. The delivery cost is divided into two types: constant cost per shipment and variable cost, which depends upon the per unit product transported. Also, carbon is emitted from the production and transportation process. To decrease the carbons in the air, manufacturer pays some carbon tax. This carbon emission cost is calculated for both transportation system and production system. Two types of carbon emission cost are fixed and variable costs. Some products may deteriorate in the duration storage and this research assumes the concept of deterioration and the corresponding deterioration cost. The aim of the model is to reduce the total joint cost of the integrated inventory model.

11.2.2 Notation

This subsection develops the following notation for this study:

Decision variables
n shipment number (integer number),
q delivery quantity to buyer.

Parameters
Q_0 ordering lot size (units),
p_0 production rate (units/time unit),
p rate of perfect products, i.e., $p = up_0$ (units/time unit), where $u = 1 - \alpha + \alpha\beta$,
d demand rate (units),
T cycle time (time unit),
I^v on-hand inventory for the vendor (units),
I^b on-hand inventory for the buyer (units),
A_1 vendor's setup cost ($/setup),
A_2 buyer's handling cost ($/time unit),
h_1 vendor's inventory holding cost ($/unit/time unit),
h_2 buyer's inventory holding cost ($/unit/time unit),
F constant delivery cost ($/shipment),
V variable delivery cost ($/unit),
θ constant deterioration rate,
c_θ cost for unit deteriorate item ($/unit),
c_f constant carbon emission cost ($/shipment),
c_v variable carbon emission cost ($/unit),
C_0 cost for first inspection ($/unit),
C_1 cost for reworking a defective item ($/unit),
C_2 cost for disposal item ($/unit),
α percentage of imperfect items in the manufacturing process,
β percentage of perfect items in the second time inspection for reworking items, and
I_c total inspection, rework, and disposal cost ($/time unit).

11.2.3 Assumptions

1. A single type of item is produced in this integrated inventory model. Vendor encounters with an imperfect production process whose α percentage of total produced product is defective items after first-stage inspection process. Thus, $(1 - \alpha)Q_0$ number of perfect items are left within the system.
2. Now, the imperfect products are reworked, and then by second-stage inspection process, β percentage of perfect items are detected and the rest items are disposed at some disposed cost. From this stage, $\beta\alpha Q_0$ quantities perfect items are recovered and $(1 - \beta)\alpha Q_0$ numbers of defective items are disposed.

3. The perfect items are distributed to the market by n shipment at a small quantity $q(\leq Q)$ for a fixed period $\frac{q}{d}$, where $d(d \leq p)$ is the demand rate of the buyer.
4. The perfect products flow from vendor's warehouse to buyer's warehouse. In the duration of storage, some products may deteriorate. A fixed deterioration rate θ is considered for the products in both warehouses.
5. The model follows a make-to-order (MTO) policy, i.e., there is no extra stock and products are produced on the basis of the order by buyer.
6. Constant and variable types of transportation cost are considered. Constant delivery cost is effected on a shipment and variable delivery cost is effected for handling and receiving the item.
7. Two types of carbon emission cost are considered by vendor, namely, constant per shipment and variable per unit in the production process.

11.3 Mathematical Model

This model consists of an integrated vendor–buyer business system. The model is formulated based on the make-to-order (MTO) policy, that is, vendor follows MTO policy for production. The order quantity is Q_0 for the whole system. The vendor starts the production process at a constant rate p_0. To produce the quantity Q_0, sometimes the machine may go to *out-of-control* state for long-run production system, and the system may produce some defective items at the rate of α percentage. After the first-stage inspection, αQ_0 items are defective and rest items are good. The second-stage inspection is effected after reworking and the rate of this defective item is β. Thus, the total perfect items are $Q = (1 - \alpha + \alpha\beta)Q_0 = uQ_0$ and defective items are $(1 - u)Q_0$, which are disposed at some fixed cost. The buyer does not claim anything for those disposed and deteriorated items. The production rate for perfect products is $p = up_0$.

The production and the inspection process is completed in between 0 and $\frac{Q_0}{p_0} = \frac{Q}{p}$, i.e., the time period is $[0, t_1]$ where $t_1 = \frac{Q}{p}$. In that time period, the vendor transports perfect items to the buyer at a quantity $q(q \leq Q)$ for the fixed period $\frac{q}{d}$. For the MTO production system, there is no extra stock for the sudden demand of the buyer. The production and inspection process has been stopped in the next time period t_2. Here, the replenishment cycle period is $[0, T]$, where $T = t_1 + t_2$ is the total cycle time (See the Fig. 11.1).

The total perfect products are delivered to the buyer in n shipments. Therefore, the total cycle period can be divided into n parts and the time duration of two successive deliveries is $\frac{T}{n}$. During the time period $\frac{T}{n}$, assume that x is the number of deteriorated items within the delivered items q. Then $q = x + \frac{dT}{n}$. After discussing the square and higher powers of θ, x can be expressed as $\frac{\theta q T}{2n}$. Therefore,

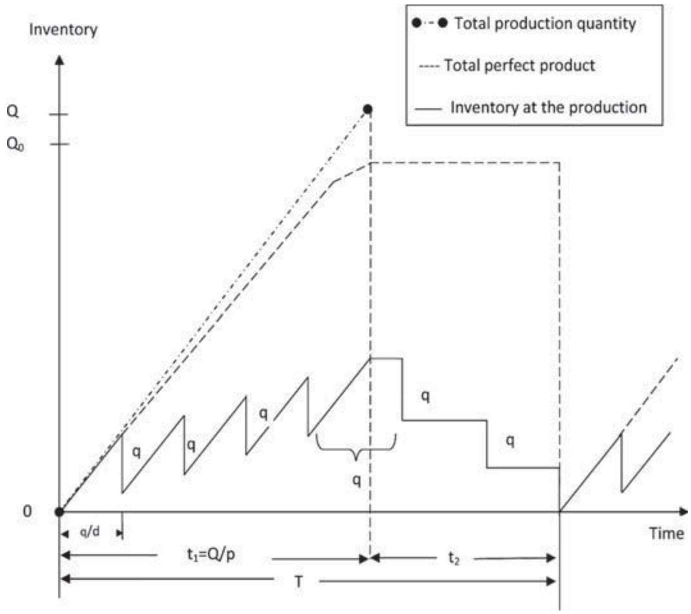


Fig. 11.1 The vendor–buyer joint inventory flow diagram

$$q = \frac{T}{n}\left(d + \frac{\theta q}{2}\right) \text{ or, } T = \frac{2nq}{2d + \theta q}.$$

11.3.1 Buyer's Model

Here, the total handling cost for the n shipments is nA_2 and the total on-hand inventory is $I^b = \frac{qT}{2}$. The holding cost for the buyer is more than the vendor and buyer only pays the extra holding cost $(h_2 - h_1)$ for the product. Again, the deterioration cost can be calculated on the average inventory. Therefore, the deterioration cost per unit time is $\frac{\theta c_\theta I^b}{T} = \frac{\theta c_\theta q}{2}$. Therefore, the total cost per cycle is calculated for the buyer as

$$\begin{aligned} TC^b &= \frac{nA_2}{T} + \frac{(h_2 - h_1)I^b}{T} + \frac{\theta c_\theta I^b}{T} \\ &= \frac{A_2(2d + \theta q)}{2q} + (h_2 - h_1 + \theta c_\theta)\frac{q}{2}. \end{aligned} \tag{11.1}$$

11.3.2 Vendor's Model

Let y be the total number of deteriorated products for the vendor in whole cycle time T. As I^v be the on-hand inventory for the vendor, then $y = \theta I^v$ or $I^v = \frac{y}{\theta}$. Therefore, it can be written that the total perfect quantity is $Q = nq + y$. The vendor has the total on-hand inventory $T\frac{(p-d)t_1}{2} + qt_1 = qT\left(\frac{n}{2} + \frac{d}{p} - \frac{dn}{2p}\right)$, where $t_1 = \frac{nq}{p} = \frac{dT}{p}$. But, due to deterioration, the vendor has I^v as on-hand inventory. Therefore (see for reference, Sarkar [17]),

$$y + \frac{\theta qT}{2} = \theta qT\left(\frac{n}{2} + \frac{d}{p} - \frac{dn}{2p}\right)$$

$$\text{or,} \quad \frac{y}{\theta} = \frac{qT}{2p}\{n(p-d) + (2d-p)\}$$

$$\text{i.e.,} \quad \frac{I^v}{T} = \frac{q}{2p}\{n(p-d) + (2d-p)\}.$$

Again, the unit inspection cost C_0 is applicable to the total production quantity Q_0 in the first-stage inspection and the total inspection cost is C_0Q_0. In the next stage, the unit rework cost C_1 is used on the imperfect product αQ_0 units and the total rework cost is $C_1\alpha Q_0$. The inspection process is applied to the reworked products αQ_0 at the same inspection cost C_0 and the total inspection cost is $C_0\alpha Q_0$. Here, total defective items are $(1-\beta)\alpha Q_0$ which are disposed at a cost C_2, and consequently the total cost for disposing is $C_2(1-\beta)\alpha Q_0$. Thus, the total inspection, rework, and disposal cost is $I_c = [C_0Q_0 + C_1\alpha Q_0 + C_0\alpha Q_0 + C_2(1-\beta)\alpha Q_0]$ and the total cost per unit time is described as

$$\frac{I_c}{T} = \frac{u_2(2d+\theta q)}{2u} + \frac{u_1\theta q}{2pu}\{n(p-d) + (2d-p)\},$$

where $u_2 = C_0(1+\alpha) + C_1\alpha + C_2\alpha(1-\beta)$.

Now, the vendor's total cost per unit time is

$$\begin{aligned} TC^{\text{v}} &= \frac{A_1}{T} + \frac{h_1 I^v}{T} + \frac{I_c}{T} + \frac{\theta c_\theta I^v}{T} + \frac{(nF + Vnq)}{T} + \frac{(nc_{\text{f}} + dc_{\text{v}})}{T} \\ &= \{A_1 + dc_{\text{v}} + n(F + c_{\text{f}})\}\frac{(2d+\theta q)}{2nq} + (h_1\text{u} + u_2\theta + u\theta c_\theta)\{n(p-d) \\ &\quad +(2d-p)\}\frac{q}{2pu} + \frac{(u_2 + uV)(2d+\theta q)}{2u}. \end{aligned} \quad (11.2)$$

11.3.3 Coordination Policy Between Vendor and Buyer

The vendor and buyer have agreed to do business in a coordinated way. Therefore, the total cost of the system can be calculated by combining both costs of the vendor and buyer. Thus, the joint inventory total cost $TC(q, n)$ for that model is the sum of buyer's cost as in Eq. (11.1) and vendor's cost as in Eq. (11.2).

$$\begin{aligned} TC(q,n) &= TC^{\mathrm{v}} + TC^{\mathrm{b}} \\ &= \{A_1 + dc_{\mathrm{v}} + n(A_2 + F + c_{\mathrm{f}})\}\frac{(2d+\theta q)}{2nq} + [\{n(p-d) \\ &\quad +(2d-p)\}(h_1 u + u_2\theta + u\theta c_\theta) + p\{u(h_2 - h_1 + \theta c_\theta + \theta V) \\ &\quad +u_2\theta\}]\frac{q}{2pu} + \frac{d(u_2+uV)}{u}. \end{aligned} \tag{11.3}$$

11.3.4 Solution Methodology

This model is solved by using algebraic method (for instance, see reference Sarkar [17]). Any algebraic function of the form $f(\mathrm{z}) = \frac{a_1}{q} + a_2 q + a_3$ may be rewritten as $f(z) = \left(\sqrt{\frac{a_1}{z}} - \sqrt{a_2 z}\right)^2 + 2\sqrt{a_1 a_2} + a_3$ and which can be solved by the algebraic method instead of the classical optimization method. The minimum value occurs for the function $f(z)$ when the square part of the function vanishes, i.e., $\sqrt{\frac{a_1}{z}} - \sqrt{a_2 z} = 0$ that implies $z = \pm\sqrt{\frac{a_1}{a_2}}$. Now, in general, the realistic value of z should be a positive number. Therefore, $z = \sqrt{\frac{a_1}{a_2}}$. For this optimum value of z, the minimum total cost is $f(z) = 2\sqrt{a_1 a_2} + a_3$.

For fixed n, Eq. (11.3) can be rewritten as

$$TC(q) = \frac{a_1}{q} + a_2 q + a_3, \tag{11.4}$$

where $a_1 = \frac{d\{A_1+dc_{\mathrm{v}}+n(A_2+F+c_{\mathrm{f}})\}}{n}$, $a_2 = [\{n(p-d) + (2d-p)\}(h_1 u + u_2\theta + u\theta c_\theta) + p\{u(h_2 - h_1 + \theta c_\theta + \theta V) + u_2\theta\}]\frac{1}{2pu}$, and $a_3 = \frac{\theta\{A_1+dc_{\mathrm{v}}+n(A_2+F+c_{\mathrm{f}})\}}{2n} + \frac{d(u_2+uV)}{u}$.

Therefore, Eq. (11.4) can be transferred into the following form:

$$TC(q) = \left(\sqrt{\frac{a_1}{q}} - \sqrt{a_2 q}\right)^2 + 2\sqrt{a_1 a_2} + a_3.$$

Thus, $TC(q)$ can attain its minimum value when $q = \sqrt{\frac{a_1}{a_2}}$ and the minimum value is $2\sqrt{a_1 a_2} + a_3$. Therefore, the optimum value of q is as follows:

$$q = \sqrt{\frac{2dpu\{A_1 + dc_{\mathrm{v}} + n(A_2 + F + c_{\mathrm{f}})\}}{n[\{n(p-d) + (2d-p)\}(h_1 u + u_2\theta + u\theta c_\theta) + p\{u(h_2 - h_1 + \theta c_\theta + \theta V) + u_2\theta\}]}}. \tag{11.5}$$

In the similar way, for fixed q, from the Eq. (11.3), we have

$$TC(n) = \frac{b_1}{n} + b_2 n + b_3, \tag{11.6}$$

where $b_1 = \frac{(A_1 + dc_{\mathrm{v}})(2d + \theta q)}{2q}$, $b_2 = (p-d)(h_1 u + u_2\theta + u\theta c_\theta)\frac{q}{2pu}$, and $b_3 = \frac{(A_2 + F + c_{\mathrm{f}})(2d + \theta q)}{2q} + [(2d-p)(h_1 u + u_2\theta + u\theta c_\theta) + p\{u(h_2 - h_1 + \theta c_\theta + \theta V) + u_2\theta\}]\frac{q}{2pu} + \frac{d(u_2 + uV)}{u}$.

With the help of algebraic method, the expression in Eq. (11.6) can be rewritten as

$$TC(n) = \left(\sqrt{\frac{b_1}{n}} - \sqrt{b_2 n}\right)^2 + 2\sqrt{b_1 b_2} + b_3.$$

Thus, $TC(n)$ can attain minimum value when $n = \sqrt{\frac{b_1}{b_2}}$ and the minimum value is $TC(n) = 2\sqrt{b_1 b_2} + b_3$. Therefore, the number of shipment is given by

$$n = \sqrt{\frac{pu(A_1 + dc_{\mathrm{v}})(2d + \theta q)}{q^2(p-d)(h_1 u + u_2\theta + u\theta c_\theta)}}. \tag{11.7}$$

Optimal interval for *q* and *n*

From Eq. (11.5), it is clear that if n increases, then q decreases. Thus, the maximum value (q_{max}) of q is obtained when $n = 1$ (since $n \geq 1$). Therefore, $q \leq q_{max}$, where

$$q_{max} = \sqrt{\frac{2dpu(A_1 + dc_{\mathrm{v}} + A_2 + F + c_{\mathrm{f}})}{(p+d)(h_1 u + u_2\theta + u\theta c_\theta) + p\{u(h_2 - h_1 + \theta c_\theta + \theta V) + u_2\theta\}}}. \tag{11.8}$$

Again, as $(h_1 u + u_2\theta + u\theta c_\theta) \leq \{u(h_2 - h_1 + \theta c_\theta + \theta V) + u_2\theta\}$, from Eq. (11.5), the minimum value of q is

$$q_{min} = \sqrt{\frac{2dpu\{A_1 + dc_{\mathrm{v}} + n(A_2 + F + c_{\mathrm{f}})\}}{n\{n(p-d) + 2d\}\{u(h_2 - h_1 + \theta c_\theta + \theta V) + u_2\theta\}}}, \tag{11.9}$$

and therefore $q \geq q_{min}$. Moreover, $n \geq 1$ implies that the maximum value (n_{max}) of n is given by

$$n_{max} = \frac{(A_1 + dc_v)\{u(h_2 - h_1 + \theta c_\theta + \theta V) + u_2\theta\}}{(h_1 u + u_2\theta + u\theta c_\theta)(A_2 + F + c_f)}\left(\frac{2d + \theta q_{max}}{2d}\right), \quad (11.10)$$

when n tends to infinity. Here, $1 \le n \le n_{max}$.

Algorithm for finding q^* and n^*

Using Eqs. (11.5) and (11.7), the optimum value of the delivery quantity q and shipment number n is calculated. Using the following algorithm, the optimum values q^* and n^* are derived, respectively. The steps of the algorithm are given below:

Step 1: As $n \geq 1$, Eq. (11.8) gives the value of q_{max} and then Eq. (11.10) finds n_{max}. Then search the interval of n, i.e., (1, n_{max}).
Step 2: Taking the sequential integer values of n from (1, n_{max}) and values of q from Eq. (11.5) is updated. Then find the total cost $TC(q, n)$ from Eq. (11.3) for the corresponding values of q and n.
Step 3: Take the minimum value of $TC(q, n)$ from step 2 for the optimum values of q^* and n^*.
Step 4: Stop.

From these steps, one can find the optimum values of n and q from Eqs. (11.5) to (11.10). Using these solutions, the minimum cost for the integrated inventory model is found.

11.4 Numerical Experiment

To validate the model numerically, the following two examples are provided:

Example 11.1 The supportive values of parameters are taken from Yan et al. [30]. $p_0 = 19,300$ units/month, $d = 4,800$ units/month, $A_1 =$ \$600/setup, $A_2 =$ \$25/month, $h_1 =$ \$6/unit/month, $h_2 =$ \$11/unit/month, $F =$ \$50/shipment, $V =$ \$1/unit, $\theta = 10\%$, $c_\theta =$ \$60/unit, $_f =$ \$0.2/shipment, $c_v =$ \$0.4/unit, $C_0 =$ \$2/units, $C_1 =$ \$2.5/units, $C_2 =$ \$2/units, $\alpha = 10\%$, and $\beta = 96\%$. Then, the optimum values are $q^* = 366.31$ units, $n^* = 5$, and the corresponding minimum cost is \$34, 392.40 (See the Fig. 11.2).

Example 11.2 Another example is provided here to describe the above model where the values of parameters are taken from Sarkar et al. [20]. $p_0 = 30$ units/month, $d = 25$ units/month, $A_1 =$ \$1, 70/setup, $A_2 =$ \$35/month, $h_1 =$ \$3/unit/month, $h_2 =$ \$5.5/unit/month, $F =$ \$20 =/shipment, $V =$ \$3/unit, $\theta = 3\%$, $c_\theta =$ \$28/unit, $c_f =$ \$9/shipment, $c_v =$ \$0.5/unit, $C_0 =$ \$2/units, $C_1 =$ \$2.5/units, $C_2 =$ \$2/units, $\alpha = 15\%$, and $\beta = 96\%$. Then, the optimum values are $q^* = 71.78$ units, $n^* = 17$, and the corresponding minimum cost is \$2, 633.04 (Fig. 11.3).

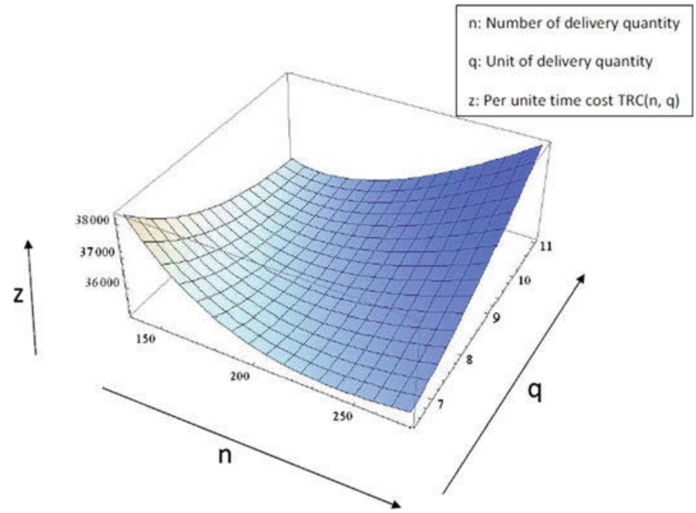


Fig. 11.2 Graphical representation of Example 11.1 of total cost $z = TC(q, n)$ with respect to the decision variables

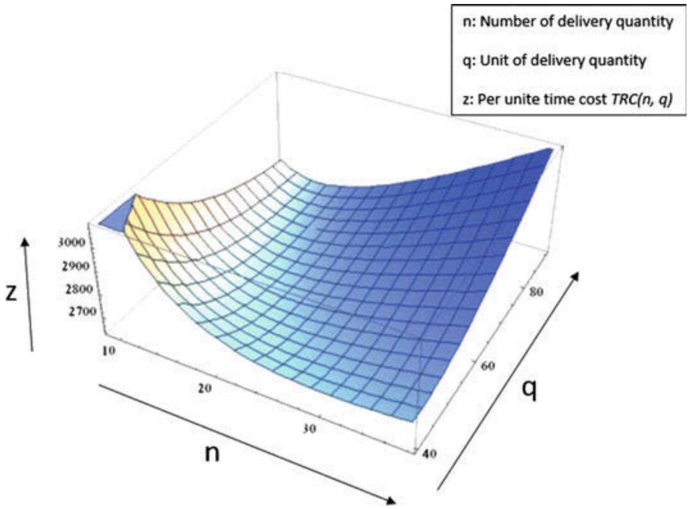


Fig. 11.3 Graphical representation of Example 11.2 of total cost $z = TC(q, n)$ with respect to the decision variables

11.5 Analysis Section

These two examples describe the validity of the model numerically. The first example generates more cost than the second example but after more investigation, anyone can see that the better example is the first example, because the total cost with respect to the delivery quantity is more which is comparatively better than the second example. If the manufacturer creates a small business, then any one of those examples is suitable for that case. However, the high value of holding cost and small delivery quantity should follow the soft products or valuable products business policy in the second example. The graphical representations show the variation of the decision variables.

11.6 Sensitivity Analysis

The major changes of the parameters are shown in the sensitivity analysis table.

The changes of parametric values for −50%, −25%, +25%, and +50% of Example 1 are shown in Table 11.1.

I. Vendor's setup cost is more sensitive than the buyer's handling cost. Vendor's setup cost is directly proportional to the total cost and negative percentage changes are more sensitive than the positive percentage changes. As setup cost involved a large amount of investment amount, the industrial manager is trying to reduce their investment for setup, that is the total cost of the system can reduce gradually. For the case of handling cost of buyer, it is more sensitive in negative sense rather than the positive changes. This implies that the total cost of buyer can be reduced if the cost of handling can be reduced.
II. The effect of changes for handling cost of buyer is hung over the total cost of the entire system. The negative percentage changes are significant than the positive changes. This implies that reduction of holding cost for the vendor can

Table 11.1 Sensitivity analysis of the total cost

Parameter	Changes (in %)	TC(q, n) (in %)	Parameter	Changes (in %)	TC(q, n) (in %)
h_1	−50	−5.14	A_1	−50	−2.61
	−25	−2.47		−25	−1.28
	+25	2.47		+25	1.25
	+50	4.48		+50	2.46
h_2	−50	−1.73	A_2	−50	−0.88
	−25	−0.83		−25	−0.43
	+25	0.76		+25	0.41
	+50	1.48		+50	0.81

be beneficial for the industry manager and it can contribute more to the cost reduction process. For buyer's core, the changes in total cost due to the changes in the holding cost are less sensitive than vendor. Even though the holding cost of buyer is more than the vendor, but as the holding quantity is less, the effect of the changes in the total cost due to the unit holding cost is not too much high relative to the vendor.

11.7 Conclusions

The integrated vendor–buyer model for single type of products was a combination of the inspection process, rework process, and the deterioration. Total cost of the entire system was minimized with using the SSMD transportation policy. As the packaged food products were needed special attention due to the preservative, inspection of the production was incorporated by vendor. Otherwise, there might be a change to deteriorate the product before its projected time period. Those packages were reworked and inspected again for safety purpose. It can help industry manager to keep their productions good enough as it is related to the health care of the society. Numerical study gave the results that based on the holding area of the buyers, either the lot size or the shipment number can be chosen. Preservation is an important issue for deteriorated products but this is not directly included in this present study. This model can be extended with the preservation technology. Even the inspection and rework are included in the present study which is a limitation of the research, which can be removed by extending the model by using backordering policy. Trade credit policy between vendor and buyer can be incorporated for the future research direction along with the uncertain demand of the buyer.

References

1. Banerjee A (1986) A joint economic lot size model for purchaser and vendor. Decis Sci 17(3):292–311
2. Chang HJ, Dye CY (1999) An EOQ model for deteriorating items with time varying demand and partial backlogging. J Oper Res Soc 50:1176–1182
3. Dey B, Sarkar B, Sarkar M, Pareek S (2019) An integrated inventory model involving discrete setup cost reduction, variable safety factor, selling-price dependent demand, and investment. Rairo-Oper Res 53(1):39–57
4. Goyal SK (1976) An integrated inventory model for a single supplier-single customer problem. Int J Prod Res 15(1):107–111
5. Goyal SK (1988) A joint economic lot size model for purchaser and vendor: a comment. Decis Sci 19(1):236–241
6. Hadley G, Whitin T (1963) Analysis of inventory systems. Prentice Hall, Englewood Cliffs, NJ
7. Huang CK (2002) An integrated vendor-buyer cooperative inventory model for items with imperfect quality. Prod Plan Control 13:355–361

8. Jaggi CK, Gautam P, Khanna A (2018) Inventory decisions for imperfect quality deteriorating items with exponential declining demand under trade credit and partially backlogged shortages. Qual IT Bus Oper, 213–229. https://doi.org/10.1007/978-981-10-5577-5-18
9. Jaggi CK, Tiwari S, Shafi AA (2015) Effect of deterioration on two-warehouse inventory model with imperfect quality. Comput Ind Eng 88:378–385
10. Jayaswal M, Sangal I, Mittal M, Malik S (2019) Effects of learning on retailer ordering policy for imperfect quality items with trade credit financing. Uncertain Supply Chain Manag 7(1):49–62
11. Khanna A, Kishore A, Jaggi CK (2017) Strategic production modeling for defective items with imperfect inspection process, rework, and sales return under two-level trade credit. Int J Ind Eng Comput 8(1):85–118
12. Khouja M (2003) Optimizing inventory decisions in a multi-stage multi-customer supply chain. Transp Res Part E Logist Transp Rev 39(3):193–208
13. Lee SD, Fu YC (2014) Join production and delivery lot sizing for a make-to-order producer-buyer supply chain with transportation cost. Trans Res Part E 66:23–35
14. Mittal M, Khanna A, Jaggi CK (2017) Retailer's ordering policy for deteriorating imperfect quality items when demand and price are time-dependent under inflationary conditions and permissible delay in payments. Int J Procure Manag 10(4):461–494
15. Ouyang LY, Chen CK, Chang HC (2002) Quality improvement, setup cost and lead-time reductions in lot size reorder point models with an imperfect production process. Comput Oper Res 29:1701–1717
16. Sarkar B (2012) An EOQ model with delay in payments and time varying deterioration rate. Math Comput Model 218:4881–4891
17. Sarkar B (2013) A production-inventory model with probabilistic deterioration in two-echelon supply chain management. Appl Math Model 37:3138–3151
18. Sarkar B, Saren S, Sinha D, Hur S (2015) Effect of unequal lot sizes, variable setup cost, and carbon emission cost in a supply chain model. Math Probl Eng 13:469–486
19. Sarkar B, Ganguly B, Sarkar M, Pareek S (2016) Effect of variable transportation and carbon emission in a three-echelon supply chain model. Trans Res Part E 91:112–128
20. Sarkar B, Shaw BK, Kim T, Sarkar M, Shin D (2017) An integrated inventory model with variable transportation cost, two-stage inspection, and defective items. J Ind Manag Optim 13(4):1975–1990
21. Sarker BR, Parija GR (1996) Optimal batch size and raw material ordering policy for a production system with a fixed-interval, lumpy demand delivery system. Eur J Oper Res 89(3):593–608
22. Shah N, Jani M, Chaudhari U (2016) Impact of future price increase on ordering policies for deteriorating items under quadratic demand. Int J Ind Eng Comput 7(3):423–436
23. Shah NH, Chaudhari U, Jani MY (2019) Inventory control policies for substitutable deteriorating items under quadratic demand. Oper Supply Chain Manag Int J 12(1):42–48
24. Shah NH, Vaghela CR (2018) Imperfect production inventory model for time and effort dependent demand under inflation and maximum reliability. Int J Syst Sci Oper Logist 5(1):60–68
25. Silver EA, Pyke DF, Peterson R (1998) Inventory management and production planning and scheduling, 3rd edn. Wiley, New York
26. Taleizadeh AA, Yadegari M, Sana SS (2019) Production models of multiple products using a single machine under quality screening and reworking policies. J Model Manag 14(1):232–259
27. Tiwari S, Daryanto Y, Wee HM (2018) Sustainable inventory management with deteriorating and imperfect quality items considering carbon emission. J Clean Prod 192:281–292
28. Vijayashree M, Uthayakumar R (2015) Integrated inventory model with controllable lead time involving investment for quality improvement in supply chain system. Int J Supply Oper Manag 2(1):2383–2525
29. Yadav R, Pareek S, Mittal M (2018) Supply chain models with imperfect quality items when end demand is sensitive to price and marketing expenditure. RAIRO-Oper Res 52(3):725–742
30. Yan C, Banerjee A, Yang L (2011) An integrated production-distribution model for a deteriorating inventory item. Int J Prod Econ 133:228–232

Chapter 12
A Note on "Inventory and Shelf-Space Optimization for Fresh Produce with Expiration Date Under Freshness-and-Stock-Dependent Demand Rate"

Hardik N. Soni and Dipali N. Suthar

Abstract In a recent paper, Chen et al. in J Oper Res Soc 67(6):884–896, [4] proposed an inventory model with freshness-expiration date and stock-dependent demand, assuming nonzero ending inventory and adopting a profit maximization function. They treated the freshness index that measures the quality of produce as linear decreasing function. However, it is evident that the degradation in quality not necessarily decreases linearly for every product. Therefore, in this work, we relax this assumption and characterize the freshness index as polynomial decreasing function to strengthen the applicability of Chen et al.'s model.

Keywords EOQ · Expiration date · Freshness-and-stock-dependent demand · Perishability

12.1 Introduction

It is frequently observed that the customers are reluctant to purchase perishable items (including most dairy products, grocery items, batteries, printer ink, etc.) whose expiration dates are approaching as it is common belief that the greater the age of an item, higher the quality degradation. As product's quality is directly correlated with customer satisfaction and reliability, it is highly important for retailers to better manage fresh-produce inventory and shelf-space allocation both. Considering these factors, Chen et al. [4] demonstrated an inventory model for perishable items wherein (1) demand rate is sensitive to freshness-expiry date and stock level, (2) the ending inventory level is nonzero because demand is positively affected by

H. N. Soni (✉)
Chimanbhai Patel Post Graduate Institute of Computer Applications, Ahmedabad 380015, Gujarat, India
e-mail: hardik_soni30@yahoo.com

D. N. Suthar
Department of Humanities and Science,
Kalol Institute of Technology & Research Centre, Kalol 382721, Gujarat, India

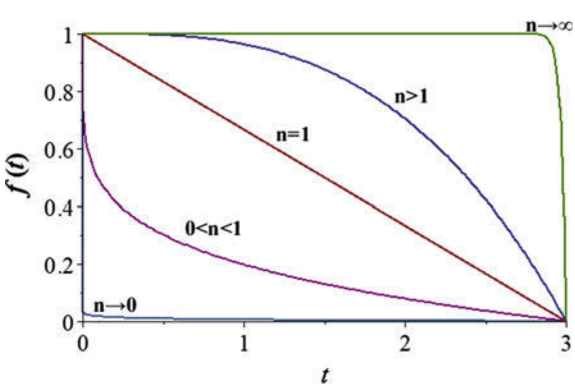
N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_12

inventory, and (3) there is a maximum inventory level because the retailer has a limited shelf space. Under these considerations, a mathematical model was formulated and an optimization procedure was developed to determine optimal shelf-space size, replenishment-cycle time, and ending inventory level that maximize the total annual profit.

Depending on the product nature, freshness indices must be followed as a function of time in order to evaluate the degradation of the product quality. Researchers such as Wu et al. [7] and Feng et al. [5] considered demand depending on freshness index which is linearly decreasing function of time declining from 1 at the beginning of the period to 0 at the expiration date. Banerjee and Agrawal [3] presented an inventory model for freshness- and price-dependent demand where linear and exponential functions were examined for freshness index. Li and Teng [6] have also considered linear function of time for freshness index to determine pricing and lot-sizing decisions for perishable goods when demand depends on selling price, reference price, product freshness, and displayed stocks. Recently, Agi and Soni [1] considered linear freshness index to determine joint pricing and inventory policy for perishable products with age, stock, and price-dependent demand rate. The aforementioned literature considers linear form of freshness decrease. However, in reality, the reduction in freshness attributed to technological factors as well as managerial factors including production services, handling and preservation practices, transportation condition, etc. Hence, reduction in freshness index cannot be well described by a linear function. Thus, in order to fill this gap, instead of linear decrease in freshness, we formulate polynomial decreasing freshness index function, whose shape is flexibly determined by changing shape parameter n [see, Eq. (12.1)]. This nonlinear degradation function over time is a generalization of many decreasing shapes depicted in Fig. 12.1: For $0 < n < 1$, convex decrease; for $n = 1$, linear decrease; for $n > 1$, concave decrease; and for $n \to \infty$, no decrease. An interested reader can refer Avinadav et al. [2] for the physical interpretation of this function.

In this study, we revisit the work of Chen et al. [4] and recast the model by considering the generalized freshness index function linked to the expiration time. Further, the model enables us to calculate the loss of profits to the retailer who overlooks the

Fig. 12.1 Freshness index for various value of n



decrease in the demand rate due to loss of freshness and uses only a stock-dependent demand function (as common in the literature). This study is organized as follows: Sect. 12.2 consists of notations and assumptions used throughout the model. A mathematical model is developed in Sect. 12.3. Numerical examples are provided for the practical implications of the model in Sect. 12.4 whereas sensitivity of the model parameters is examined in Sect. 12.5.

12.2 Notations and Assumptions

12.2.1 Notations

All notations used in this work are adopted from Chen et al. [4].

Decision variables	
E	Ending inventory level in units, with $E \geq 0$
T	Ordering cycle time in years
t_1	The length of time in years when the inventory level drops to W
Parameters	
c	Purchasing cost per unit, where $0 < c < p$
h	Holding cost per unit time
o	Ordering cost per order
m	Maximum lifetime (the time to its expiration date) per year
u	Shelf cost per unit per year
p	Selling price per unit, with $p \geq c$
s	Salvage price per unit
Q	Economic order quantity in units
W	Number of units displayed on shelf space
Variables	
$f(t)$	Freshness index at time t, which is a decreasing function during the interval [0, 1]
$D(t)$	Freshness-expiration date and stock-dependent demand rate which is close to zero at the expiration date
$I(t)$	Inventory level at time t
E^*	Optimal ending inventory level in units
Q^*	Optimal order quantity in units
T^*	Optimal ordering cycle time in years
t_1^*	Optimal length of time in years when the inventory level drops to W
$\Pi(E, T, t_1)$	Total annual profit

12.2.2 Assumptions

Basically, we adopt assumptions same as those in Chen et al. [4], which are restated below:

1. Perishable items such as food products deteriorate and reduce its freshness, nutrient value, or effectiveness with time and finally lose its usefulness. After examining all those factors, the expiration date can be approximated by experts. So, we assume product freshness index is 1 when it received and start decreasing with time and reaches to 0. To represent this problem in the mathematical model, we consider the freshness index at the time t is nonlinear degradation function from 1 initially to 0 at the maximum lifetime and given by

$$f(t) = 1 - \left(\frac{t}{m}\right)^n, \quad 0 \le t \le m. \tag{12.1}$$

2. It is widely seen that a large amount of fresh produce on the shelf increases the demand, but the display of stale items gives the contrary effect to the demand. Hence, we assume in this paper that rancid items are withdrawn from the displayed shelf space.
3. Inventory system starts with maximum Q units at zero time but only W units are displayed on the shelf and remaining units are kept in the back room. Stock in the backroom is shifted to the shelf when sales are made until the time when no more stocks remain in the backroom at the time t_1. Hence, in $[0, t_1]$ time interval, shelf space is full. This inventory system is depicted in Fig. 12.2. So,

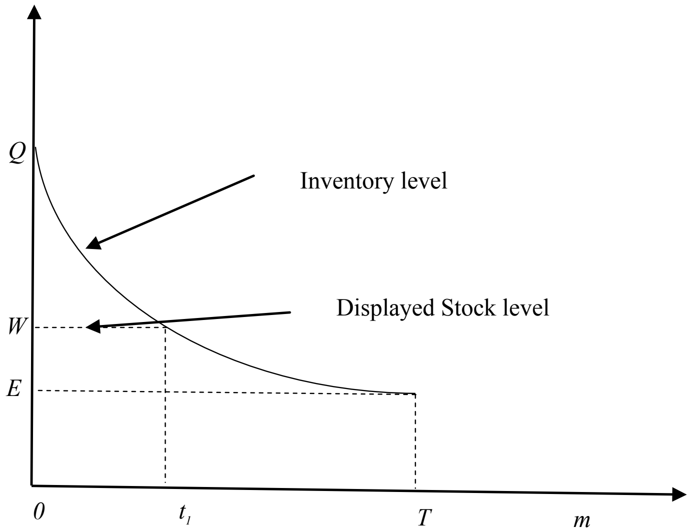


Fig. 12.2 Graphical representation of the system

demand depending on shelf space and freshness index in $[0, t_1]$ time interval can be presented by

$$D(t) = \alpha W^{\beta}\left(1 - \left(\frac{t}{m}\right)^{n}\right), \quad 0 \leq t \leq t_1 \tag{12.2}$$

where α and β ($\alpha > 0$ and $1 > \beta \geq 0$) are constant parameters. Equation (12.2) reveals that $T \leq m$. Otherwise $D(t) \leq 0$ if $t \geq m$. Further, if $\beta = 0$ and $m \to \infty$, $D(t) = \alpha$ becomes constant. Consequently, the impact of shelf space on the demand rate is decreasing return, and therefore we assume $\beta < 1$.

4. As a large display of merchandise impact positively on the demand, it is assumed that ending inventory is maintained at a nonzero level $E \geq 0$.
5. In the time span $[t_1, T]$, the shelf space is partly filled up with inventory whereas demand depends on both the displayed items and the freshness index. When the cycle ends with ending inventory E units, it is sold at the salvage price. Then new cycle commences by receiving order quantity Q units. In this time interval, demand rate can be represented as

$$D(t) = \alpha\left[I(t)^{\beta}\right]\left(1 - \left(\frac{t}{m}\right)^{n}\right), \quad t_1 \leq t \leq T. \tag{12.3}$$

6. We assume that $Q \geq W$. Otherwise, W can be reduced to Q. Consequently, $t_1 \geq 0$.
7. The holding cost is assumed to be the same for both displayed items and stored items.
8. Shortages are not permitted.
9. Replenishment rate is infinite and instantaneous.

12.3 Model Formulation

Based on the above assumptions and notation, the inventory level at different instants of time is shown in Fig. 12.2.

The inventory level $I_1(t)$ at time t in $[0, t_1]$ time interval is given by the differential equation:

$$\frac{dI_1(t)}{dt} = -\alpha W^{\beta}\left(1 - \left(\frac{t}{m}\right)^{n}\right), \quad 0 \leq t \leq t_1 \tag{12.4}$$

with the boundary condition $I_1(0) = Q$. By solving (12.4), we have

$$I_1(t) = \frac{\alpha W^{\beta} t^{n+1}}{(n+1)m^n} - \alpha W^{\beta} t + Q, \quad 0 \leq t \leq t_1 \tag{12.5}$$

Now, in $[t_1, T]$ time interval, the inventory level $I_2(t)$ at time t is formulated by the differential equation as

$$\frac{dI_2(t)}{dt} = -\alpha\big[I_2(t)^{\beta}\big]\left(1-\left(\frac{t}{m}\right)^{n}\right), t_1 \le t \le T \tag{12.6}$$

with the boundary condition $I_2(T) = E$. By solving (12.6), we obtain

$$I_2(t) = \left(\frac{\alpha(\beta-1)\big(T^{n+1}-t^{n+1}\big)}{m^n(n+1)} + E^{1-\beta} - \alpha(\beta-1)(T-t)\right)^{(1-\beta)^{-1}} \tag{12.7}$$

Now, by using $I_1(t_1) = I_2(t_1)$, we get order quantity Q as

$$Q = \left(\frac{\alpha(\beta-1)\big(T^{n+1}-t_1^{n+1}\big)}{m^n(n+1)} + E^{1-\beta} - \alpha(\beta-1)(T-t_1)\right)^{(1-\beta)^{-1}} - \frac{\alpha W^{\beta} t_1^{n+1}}{(n+1)m^n} + \alpha W^{\beta} t_1 \tag{12.8}$$

Using $W = I_2(t_1)$, we get,

$$W = \left(\frac{\alpha(\beta-1)\big(T^{n+1}-t_1^{n+1}\big)}{m^n(n+1)} + E^{1-\beta} - \alpha(\beta-1)(T-t_1)\right)^{(1-\beta)^{-1}} \tag{12.9}$$

Now, the holding cost in the time interval $[0, t_1]$ is

$$H_1 = h\int_0^{t_1} I_1(t)dt = h\int_0^{t_1}\left(\frac{\alpha W^{\beta} t^{n+1}}{(n+1)m^n} - \alpha W^{\beta} t + Q\right)dt \tag{12.10}$$

The holding cost in the time interval $[t_1, T]$ is

$$H_2 = h\int_t^{t_1} I_2(t)dt = \int_{t_1}^{T}\left(\frac{\alpha(\beta-1)\big(T^{n+1}-t^{n+1}\big)}{m^n(n+1)} + E^{1-\beta} - \alpha(\beta-1)(T-t)\right)^{(1-\beta)^{-1}}, \quad t_1 \le t \le T \tag{12.11}$$

An integration expressed in (12.11) is too complex to derive analytical solution explicitly. Additionally, H_2 contribute in less amount to overall profit. Hence, for mathematical simplicity, simpler form for H_2 can be considered as follows. In the time interval $[t_1, T]$, the average inventory level can be expressed as $(W+E)\big/2$. Therefore, the average holding cost during $[t_1, T]$ is given by

$$H_2 \approx \frac{h}{2}(W+E)(T-t_1) \tag{12.12}$$

In this paper, ending inventory level is considered nonzero which may lead to higher holding cost. Hence, the main objective of this study is to optimize ending inventory level E, ordering cycle time T, and time at which the inventory level reaches to maximum shelf-space t_1 to maximize the profit.

$$\begin{aligned}\text{The total profit} &= \text{revenue received} + \text{salvage value} - \text{purchasing cost} \\ &\quad - \text{ordering cost} - \text{holding cost} - \text{shelf space cost}\end{aligned} \tag{12.13}$$

Mathematical expression for an EOQ problem for fresh items can be presented by

$$\begin{aligned} Max\ TP(E,T,t_1) = E\Bigg[\frac{1}{T}\Big[&p(Q-E) + sE - cQ - o \\ &- h\int_0^{t_1}\left(\frac{\alpha W^{\beta} t^{n+1}}{(n+1)m^n} - \alpha W^{\beta} t + Q\right)dt \\ &- \frac{h}{2}(W+E)(T-t_1)\Big] - uW\Bigg] \end{aligned} \tag{12.14}$$

Subject to

$$\begin{aligned} Q = &\left(\frac{\alpha(\beta-1)\left(T^{n+1}-t_1^{n+1}\right)}{m^n(n+1)} + E^{1-\beta} - \alpha(\beta-1)(T-t_1)\right)^{(1-\beta)^{-1}} \\ &- \frac{\alpha W^{\beta} t_1^{n+1}}{(n+1)m^n} + \alpha W^{\beta} t_1 \geq W, \\ W = &\left(\frac{\alpha(\beta-1)\left(T^{n+1}-t_1^{n+1}\right)}{m^n(n+1)} + E^{1-\beta} - \alpha(\beta-1)(T-t_1)\right)^{(1-\beta)^{-1}} \\ &\text{and } 0 \leq E \leq W \end{aligned}$$

Likewise, as shown in Chen et al. [4], it can be established that total annual profit $TP(E,T,t_1)$ is strictly pseudo-concave in T. Also, $TP(E,T,t_1)$ is a strictly concave function in both E and t_1, for any given T. Hence, there exists a unique global optimal solution (E^*, T^*, t_1^*) that maximizes the profit.

12.4 Numerical Examples

Example 1 We have considered the same parametric values as in Chen et al. [4]. The estimated values of the model parameters are taken as $\alpha = 50$, $\beta = 0.7$, $h = \$\,4$ per unit per year, $c = \$\,20$ per unit, $m = 0.4$ years, $o = \$\,10$ per order, $u = \$\,5$ per unit, $s = \$10$ per unit, $p = 40$ per unit, $n = 1.5$. In order to maximize $TP(E, T, t_1)$, we obtain a local optimal solution (E^*, T^*, t_1^*) using Maple 18. The results obtained are as follows: the retailer's optimal cycle time is $T^* = 0.2991$ years with $t_1^* = 0.0391$ years, the optimal level of ending inventory is $E^* = 5212.16$ units, the optimal size of shelf space is $W^* = 9857.93$ units, the optimal order quantity is $Q^* = 11{,}065.85$ units, and the maximum total annual profit is $TP(E^*, T^*, t_1^*) = \$\,1, 36, 136.21$.

Example 2 If we consider this model without effect of freshness degradation, the results obtained with the same parametric values considered in Example 1 are as follows: the retailer's optimal cycle time is $T^* = 1.48$ years with $t_1^* = 0.2$ years, the optimal level of ending inventory is $E^* = 99{,}107.58$ units, the optimal size of shelf space is $W^* = 48{,}9567.67$ units, the optimal order quantity is $Q^* = 58{,}5699.88$ units, and the maximum total annual profit is $TP(E^*, T^*, t_1^*) = \$\,21, 05, 213.29$.

From above examples, it can be observed that the total profit is higher when the effect of freshness degradation is discarded. With the effect of freshness degradation, the reduction in demand is observed that resulted in less profit, which is obvious. To examine how nonlinearity of freshness index impacts on the optimal policy, we carry out the sensitivity analysis by changing the value of n.

12.5 Sensitivity Analysis

For some managerial implications, we study the effect of shape parameter, n, of freshness index function on decision variable by increasing n at the rate of 0.5 in the interval [0.5, 3]. The results are shown in Table 12.1.

The results of Table 12.1 reveal that the shape of freshness index (n) plays critical role in determining optimal shelf-space size, ending inventory level, and total annual profit. Hence, the retailer should carefully estimate the value of n to achieve maximum profit. The results suggest that the items with higher freshness degradation rate ($0 < n < 1$) should order more frequently and keep shelf-space size low that resulted in lower ending inventory level and higher profit. On the other hand, the opposite strategy should be adopted for the items whose

Table 12.1 Sensitivity analysis of shape parameters n of freshness index

n	E	T	t_1	W	TP	Q
0.5	737.26	0.2468	0.0218	1373.22	22,132.78	1517.97
1	2846.86	0.2783	0.0317	5350.38	78,285.88	5970.62
1.5	5212.16	0.2991	0.0392	9857.93	136,136.21	11,065.85
2	7382.03	0.3139	0.0448	14,027.298	186,432.90	15,809.55
2.5	9264.64	0.3248	0.0491	17,667.61	228,500.77	19,970.72
3	10,874.45	0.3333	0.0525	20,795.75	263,529.80	23,559.15

freshness degradation rate is low ($n > 1$). It is to be noted that for $n = 1$, the obtained results are same as in Chen et al. [4].

12.6 Conclusion

This paper extends the work of Chen et al. [4] by capturing the consumer's sensitivity to freshness in nonlinear manner in conjunction with shelf-space sensitive demand. Due to diverse nature of freshness degradation rate in the product, the model proposed in this paper considered generalized form of freshness index function. The general form of freshness index function allows the decision-maker to estimate consumer's sensitivity to freshness and thereby to determine proper inventory policy for perishable items in general. Results of sensitivity analysis indicate that the shape parameter, n, of freshness index function is significant factor in determination of profitable inventory policy. Besides, the model aids the retailer to cut the losses caused by freshness. Thus, the proposed model can serve as a tool for managing perishable or deteriorating inventory in general.

This paper can be extended in several ways to strengthen its applicability. For example, this work could be directly extended to allow price to be a function of freshness. Moreover, assuming price discount when the freshness index falls below the given threshold could enhance the research. It is also suggested to consider and analyze the problem presented in this paper in two or three level mode.

References

1. Agi MAN, Soni HN (2019) Joint pricing and inventory decisions for perishable products with age-, stock-, and price-dependent demand rate. J Oper Res Soc. https://doi.org/10.1080/01605682.2018.1525473
2. Avinadav T, Herbon A, Spiegel U (2013) Optimal inventory policy for a perishable item with demand function sensitive to price and time. Int J Prod Econ 144(2):497–506
3. Banerjee S, Agrawal S (2017) Inventory model for deteriorating items with freshness and price dependent demand: optimal discounting and ordering policies. Appl Math Model 52:53–64
4. Chen SC, Min J, Teng JT, Li F (2016) Inventory and shelf-space optimization for fresh produce with expiration date under freshness-and-stock-dependent demand rate. J Oper Res Soc 67(6):884–896
5. Feng L, Chan YL, Cárdenas-Barrón LE (2017) Pricing and lot-sizing polices for perishable goods when the demand depends on selling price, displayed stocks, and expiration date. Int J Prod Econ 185:11–20
6. Li R, Teng JT (2018) Pricing and lot-sizing decisions for perishable goods when demand depends on selling price, reference price, product freshness, and displayed stocks. Eur J Oper Res 270(3):1099–1108
7. Wu J, Chang CT, Cheng MC, Teng JT, Al-khateeb FB (2016) Inventory management for fresh produce when the time-varying demand depends on product freshness, stock level and expiration date. Int J Syst Sci Oper Logist 3(3):138–147

Chapter 13
EOQ Model Under Discounted Partial Advance—Partial Trade Credit Policy with Price-Dependent Demand

Swati Agrawal, Rajesh Gupta and Snigdha Banerjee

Abstract The aim of this article is to investigate an inventory model with discounted partial advance payment in a single supplier–single retailer supply chain in the presence of credit period when the demand rate is price sensitive. The lengths of the credit period, advance period, as well as rate of discount on advance payment, are specified by the supplier. Conditions for unique optimal values of the decision variables, namely, the retailer's selling price and cycle length are obtained. Optimal values of the decision variables are determined iteratively. An algorithm is developed and a numerical example is presented to demonstrate the solution algorithm. Sensitivity analysis is conducted. It is observed that optimal cycle time is affected by the two interest rates. Optimal net profit is affected by the demand rate and the discount factor. Both, the optimal cycle time, as well as the optimal net profit is affected by the supplier's selling price and the proportion of units for which the advance payment is made. Optimal retailer's selling price is significantly affected by the discount factor, supplier's selling price, price elasticity of the demand function as well as the proportion of units for which advance payment is made. We also observe that the retailer's net profit does not decrease significantly on increasing the advance period.

Keywords Inventory · Partial advance payment · Discount · Trade credit · Iso-elastic price-dependent demand

13.1 Introduction

In the competitive situation prevailing in the market, a major effort is required by suppliers to provide facilities which would, in turn, attract orders from retailers. One

S. Agrawal
Institute of Advance Computing, SAGE University, Indore, India

R. Gupta
Prestige Institute of Management, Gwalior, India

S. Banerjee (✉)
School of Statistics, Devi Ahilya University, Indore, India
e-mail: bans_1@rediffmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_13

such facility popular in a supplier–retailer contract is to offer goods on credit for some interest-free period—generally termed as the trade credit period or permissible delay in payment. The retailer may pay the entire amount or a part of it to the supplier at the end of the credit period. Once the credit period is over, interest is charged by the supplier on the remaining dues.

The benefits of trade credit policy in the context of marketing are identified as leading to increased sales and as a tool to attract new retailers who deem credit policy as a kind of price reduction. Another advantage is that due to trade credit, an established retailer may pay more promptly resulting in a reduction in the outstanding sales dues. Trade credit provides financial support to the retailer along with providing a certification of quality from the supplier.

During the last few decades, many inventory models have been developed considering the trade credit facility. Goyal [8], Teng [27], Chang et al. [3], Sarkar [22], Chen and Teng [5], Taleizadeh et al. [26], Tiwari et al. [30], Jaggi et al. [14] and many others have considered trade credit when the demand is constant.

In practice, quite often, the end customer demand at the retailer is price-sensitive. In such a situation, decisions regarding setting the retailer's selling price and order quantity are to be made by the retailer. Price-sensitive demand without trade credit has been considered by many authors, e.g., Banerjee and Sharma [2]. For an inventory model under trade credit contract with price-sensitive demand, optimal pricing policies were obtained by Hwang and Shinn [13]. Under cooperative and noncooperative structures, Abad and Jaggi [1] developed a model with price-dependent demand to obtain the retailer's optimal unit price and replenishment cycle as well as the seller's optimal selling price and credit period. Teng et al. [28] found the optimal selling price and replenishment policies considering a model with price-sensitive demand for deteriorating items. They concluded that under trade credit, the cycle time, and order quantity will decrease. Price-sensitive demands for integrated inventory models that involve trade credit have also been developed by Ouyang et al. [21], Chen and Kang [4] and Chung and Liao [6].

Ho et al. [12], Shah et al. [23] analyze the decision policy when the buyer receives a cash discount if he pays any fraction of purchase cost within a shorter allowable credit period and then clears the remaining balance in the long credit period. Such a policy is called a two-part permissible delay.

Some more realistic models have considered revenue earned through sales as well as interest earned during the credit period and even later for price-sensitive demand [15, 16, 19, 20, etc.].

Retailers are generally in search of long credit periods for the purchase of their goods, whereas this tendency may lead to financial complications for small suppliers and hence to supply crunch for the retailer. Hence, sometimes, it may be worthwhile for the supplier to demand advance payment. Zhang et al. [32] stated that advance payment is a known practice in the Chinese automobile and steel industries. Maiti et al. [18] observed that in the bricks and tiles factories in India, sometimes a price discount on advance payment is offered to the retailer if made at his own discretion.

In inventory literature, very little consideration has been given to the advance payment and its influences on inventory decisions. Maiti et al. [18] developed a stochastic inventory model with advance payment. They assumed that the retailer's procurement price depended on the fraction of the advance payment. Their model was extended by Gupta et al. [9]. However, these two papers do not consider trade credit policy. Both advance payment and trade credit were considered by Thangam [29] for constant demand. Full advance and partial advance partial credit were incorporated by Zhang et al. [32] for constant demand. They conclude that in both the payment policies, length of the period of advance payment does not affect the retailer's optimal policy.

Taleizadeh [25] studied a lot sizing model without credit period under price-dependent demand with advance payment policy when the equal installments of the advance payment of the purchase cost are specified by the supplier. For constant demand, Wu et al. [31] studied the model when the seller requires an advance-cash-credit (ACC) payment.

From the above-detailed literature review, we find that till now, very few papers have considered advance payments. Out of these few papers, some have not considered trade credit [18, 24] while others, who have considered advance payment, as well as trade credit, have regarded demand to be constant [17, 31, 32, 33] or time dependent [7]. Although Diabat et al. have considered both advance payment as well as delayed payment, the two are for different echelons in the supply chain with upstream advance payment and downstream delayed payment.

In the present paper, we consider iso-elastic price-dependent demand with partial advance payment before the supply is received when the credit period is also allowed. The aim of this article is to study an optimal inventory model that considers ordering and pricing decisions under discounted partial advance and partial credit period when the customer demand is an iso-elastic function of the retailer's selling price. We obtain the optimal price and optimal length of replenishment cycle when shortages are not allowed. We also examine how the variations in the model parameters affect the optimal solution.

The rest of this paper is organized as follows: Sect. 13.2 presents the assumptions and notations. Section 13.3 explains the working of the model, Numerical example is given in Sect. 13.4 along with algorithm, sensitivity analysis and managerial insights. In Sect. 13.5, we present the conclusions.

13.2 Notations and Assumptions

The following notations are used in this paper:

D	demand dependent on retailer's price rate per unit. $D = \alpha P_R^{-\beta}$, $\alpha, \beta > 1$.
h	unit inventory holding cost per unit time.
A	ordering cost per order.
I_1	the interest rate paid per unit time to supplier by retailer.
I_{PR}	the interest rate per unit time to be paid by retailer to financer for loan.
I_{ER}	the interest rate earned per unit time by retailer.

t_0 epoch of advance payment
M_A the retailer's advance period stipulated by the supplier.
M_R the credit period provided by the supplier to the retailer.
Net the retailer's net profit per unit time.
A_1 proportion of Q for which an advance payment is made by the retailer at epoch M_A.
A_2 proportion of Q for which payment is paid by the retailer at epoch M_R. $0 \leq A_1 + A_2 \leq 1$.
ρ discount factor for advance booking, $0 < \rho < 1$. The discount percent is $100(1-\rho)$.
T the retailer's inventory cycle length (Decision variable)
P_S supplier's unit selling price.
P_R retailer's unit selling price ($P_R > P_S$). (Decision variable)
Q the retailer's order quantity per cycle (Decision variable). $Q = DT$

* With any decision variable indicates its optimal value.

Assumptions

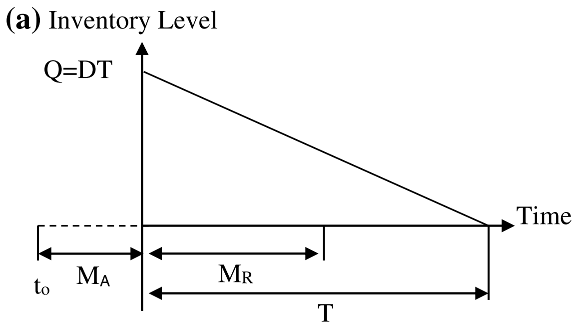
The model is developed with the following underlying assumptions:

1. The supplier provides a fixed credit period M_R to the retailer for settling the accounts.
2. The end consumer market demand rate declines with an increase in the retailer's selling price, $D(P_R) = \alpha P_R^{-\beta}$, where $\alpha > 0$ and β being, respectively, the scaling factor and the index of price elasticity. For notational simplicity, we will be interchangeably using $D(P_R)$ and D in this work.
3. The retailer starts selling the goods as soon as he receives it.
4. The earnings accumulated by the retailer is withdrawn only at epoch T, or later.
5. For the payments made to supplier at t_0 and M_R, the retailer has to take loan from the financial institution like banks—which we call financer, while for the payment made at epoch T, the retailer uses a part of the earnings accumulated till time T.
6. Shortages are not allowed.
7. Replenishment rate and time horizon are infinite.

13.3 The Model

The model is developed with the stated advance period under trade credit with a price-dependent demand so as to maximize net profit for the retailer. The retailer orders for Q units of inventory at epoch t_0, which is M_A time units before the beginning of the selling season. The ordered units arrive at the beginning of the selling season. The payments for the ordered units are made by the retailer in three parts:

Fig. 13.1 **a** Time Inventory Graph when $M_R \leq T$.



1. An advance payment at epoch t_0 for proportion A_1 of Q units is made at the discounted rate ρP_S. $0 \leq A_1 \leq 1$.
2. For the remaining quantity, payment has to be made depending on the following two cases.

Case I: $M_R \leq T$
In this case, a payment at the rate P_S for proportion A_2 of Q units is made at the epoch M_R. No interest is paid to the supplier for this delayed payment under the credit policy. $0 \leq A_1 + A_2 \leq 1$. Payment for the remaining proportion $1 - (A_1 + A_2)$ of Q units at the rate P_S along with interest charged by the supplier from M_R to T at the rate I_1. is made at epoch T.

The payments at t_0 and M_R are made by taking a loan from financer. The retailer starts selling his goods from the beginning of the selling period. The sales earnings up to Tare invested as they accumulate and interest is earned on it at the rate I_{ER}. When the selling period ends, the payment to the supplier and loan repayment and payment of interest for the loan to the financer will be made by the retailer from the sales as well as interest earnings up to T (Fig. 13.1a).
Case II: $M_R > T$
In this case, a payment for the remaining proportion $(1 - A_1)$ of Q units is made at the rate P_S at epoch M_R so as to take advantage of the credit period. No interest is to be paid for this payment, the credit period M_R being larger than the cycle length T, and no loan is to be taken by the retailer for the payment. Repayment the loan taken from the financer at t_0 and interest on it is to be repaid to the financer at epoch T, i.e., when the selling period ends (Fig. 13.1b).

13.3.1 Computation of Net

The retailer's net profit for the cycle is given by

Net = Total revenue earned – (Ordering cost + Stock holding cost + Purchase cost + Interest paid) where

Total revenue earned = Sales revenue + Interest earned

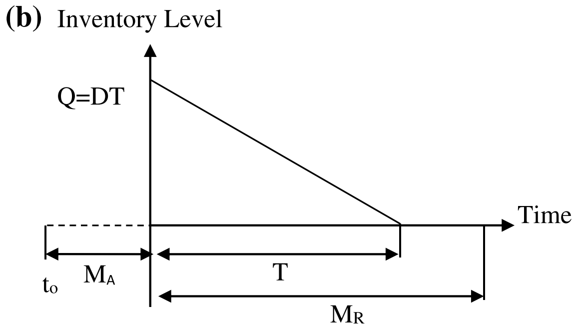


Fig. 13.1 **b** Time Inventory Graph when $M_R > T$

Ordering cost is A

Stock holding cost is $\frac{h(DT^2)}{2}$

The total revenue earned, interest earned, interest paid and net profit per unit time for Case I and Case II are as follows:

Case I: $M_R \le T$

The total purchasing cost paid at epoch M_A, M_R and T of quantity (A_1Q), (A_2Q) and $(1-(A_1+A_2))Q$, respectively, is

$$(\rho P_S)(A_1Q) + P_S(A_2Q) + P_S(1-(A_1+A_2))Q$$

The interest paid by the retailer till T, for the loan taken at the epochs t_0 and M_R, is $(T+M_A)(A_1Q)(\rho P_S)I_{PR} + (T-M_R)(A_2Q)P_SI_{PR}$

The interest paid by the retailer to the supplier for the amount paid at T is $(T - M_R)((1-(A_1+A_2))Q)P_SI_1$

Total revenue earned by the retailer is

$$P_R(DT) + \frac{P_R(DT)I_{ER}T}{2}$$

Hence, the net profit per unit time of the retailer is

$$\begin{aligned}\text{Net 1} = {} & P_RD\left(1+\frac{1}{2}I_{ER}T\right) - \frac{A}{T} - \frac{hDT}{2} - P_SD[(1-A_1)(1+I_1(T-M_R)) \\ & -A_2(I_1-I_{PR})(T-M_R) + A_1\rho(1+I_{PR}(T+M_A))]\end{aligned} \tag{13.1}$$

Case II: $M_R > T$

The total purchasing cost paid at epoch M_A and T of quantity(A_1Q) and $(1-A_1)Q$, respectively, is

$$(\rho P_S)(A_1Q) + P_S(1-A_1)Q$$

The interest paid by the retailer till T to the financer for the amount paid at the epoch t_0 is

$$(T + M_A)(A_1 Q)(\rho P_S) I_{PR}$$

Total revenue earned by the retailer is

$$P_R(DT) + \frac{P_R(DT) I_{ER} T}{2} + P_R(DT)(M_R - T) I_{ER}$$

Hence, the net profit per unit time of the retailer is

$$\mathrm{Net}\,2 = P_R D\left[1 + I_{ER}\left(M_R - \frac{T}{2}\right)\right] - \frac{A}{T} - \frac{hDT}{2} - P_S D\{A_1[1 - \rho\{1 + I_{PR}(M_A + T)\}] - 1\} \tag{13.2}$$

The overall net profit per unit time is

$$\mathrm{Net} = \begin{cases} \mathrm{Net1}; & \text{for } M_R \le T \\ \mathrm{Net2}; & \text{for } M_R > T \end{cases} \tag{13.3}$$

13.3.2 Analysis

Using assumption 3 and Q = DT, it is apparent that Net is a function of decision variables P_R and T. In order to obtain the optimal values of the decision variables analysis of the net profit function for Case I and Case II are presented:

13.3.2.1 Necessary Conditions

The first-order (necessary) conditions for maximization of Netj with respect to T and P_R are

$$\frac{\partial \mathrm{Netj}(T, P_R)}{\partial T} = 0 \qquad \frac{\partial \mathrm{Netj}(T, P_R)}{\partial P_R} = 0; \qquad j = 1, 2.$$

Differentiating (1) with respect to T and P_R, we get, respectively

$$\frac{\partial \mathrm{Net1}(T, P_R)}{\partial T} = \alpha P_R^{-\beta}\left\{\frac{I_{ER} P_R}{2} - \frac{h}{2} - P_S[(1 - A_1)I_1 - A_2(I_1 - I_{PR}) + A_1 I_{PR}\rho]\right\} + \frac{A}{T^2} \tag{13.4}$$

and

$$\frac{\partial \mathrm{Net1(T, P_R)}}{\partial \mathrm{P_R}} = \frac{1}{2}\alpha \mathrm{P_R^{-(\beta+1)}R1} \tag{13.5}$$

where

$$\begin{aligned}\mathrm{R1} = & -\mathrm{P_R(2 + I_{ER}T)(\beta - 1)} \\ & + \beta[\mathrm{hT} \\ & + 2\mathrm{P_S}\{(1 - \mathrm{A_1})(1 + \mathrm{I_1(T - M_R)}) - \mathrm{A_2(I_1 - I_{PR})(T - M_R)} \\ & + \mathrm{A_1}\rho(1 + \mathrm{I_{PR}(M_A + T)})\}]\end{aligned}$$

We note that RHS of (5) is zero iff $R_1 = 0$.
On equating (13.4) and (13.5) to zero, we get, respectively

$$\mathrm{T_1^*} = \frac{\sqrt{2\mathrm{A}}}{\sqrt{\alpha\,\mathrm{P_R^{-\beta}}[\mathrm{h - I_{ER}P_R + 2P_S((1 - A_1)I_1 - A_2(I_1 - I_{PR}) + A_1 I_{PR}\rho)}]}} \tag{13.6}$$

And on substituting for R_1, we get

$$\mathrm{P_{R1}^*} = \frac{\beta[\mathrm{hT + 2P_S\{(1 - A_1)(1 + I_1(T - M_R)) - A_2(I_1 - I_{PR})(T - M_R) + A_1\rho(1 + I_{PR}(M_A + T))\}}]}{(2 + \mathrm{I_{ER}T})(\beta - 1)} \tag{13.7}$$

Similarly, differentiating (2) with respect to T and $P_{R,}$ we get respectively

$$\frac{\partial \mathrm{Net2(T, P_R)}}{\partial \mathrm{T}} = \alpha \mathrm{P_R^{-\beta}}\left\{-\frac{\mathrm{I_{ER}P_R}}{2} - \frac{\mathrm{h}}{2} - \mathrm{A_1 I_{PR} P_S}\rho\right\} + \frac{\mathrm{A}}{\mathrm{T^2}} \tag{13.8}$$

and

$$\frac{\partial \mathrm{Net2(T, P_R)}}{\partial \mathrm{P_R}} = \frac{\alpha}{2}\mathrm{P_R^{-(\beta+1)}R2} \tag{13.9}$$

where

$$\mathrm{R2} = -\mathrm{P_R(2 + 2I_{ER}M_R - I_{ER}T)(\beta - 1)} + \beta[\mathrm{hT - 2P_S[\{A_1(1 - \rho(1 + I_{PR}(M_A + T))) - 1\}]}$$

On equating (13.8) and (13.9) to zero, we get, respectively

$$\mathrm{T_2^*} = \frac{\sqrt{2A}}{\sqrt{\alpha\,\mathrm{P_R^{-\beta}}(h + \mathrm{I_{ER}P_R + 2A_1 I_{PR} P_S}\rho)}} \tag{13.10}$$

and

$$P^*_{R2} = \frac{\beta(hT - 2P_S[A_1(1 - \rho(1 + I_{PR}(M_A + T))) - 1])}{(2 + 2I_{ER}M_R - I_{ER}T)(\beta - 1)} \quad (13.11)$$

13.3.2.2 Sufficiency Conditions

The second order (sufficiency) conditions for Netj, j = 1, 2 to be maximum with respect to T and P_R, respectively, are

(i) $\frac{\partial^2 Netj(T,P_R)}{\partial T^2} < 0$, (ii) $\frac{\partial^2 Netj(T,P_R)}{\partial P_R^2} < 0$

for which, wide sufficient conditions are derived in Appendix 1

For Net j to be jointly concave with respect to both the decision variables T and P_R, we require that Netj satisfies (i) or (ii) and

(iii) $\frac{\partial^2 Netj(T,P_R)}{\partial T^2} \frac{\partial^2 Netj(T,P_R)}{\partial P_R^2} - \left(\frac{\partial^2 Netj(T,P_R)}{\partial T \partial P_R}\right)^2 > 0$

Condition (iii) has been further discussed in Appendix 2.

13.4 Algorithm

On the basis of above theoretical results, the following solution algorithm has been developed to determine an optimal solution of the model for the given parameters α, β, A, h, I_{ER}, I_{PR}, I_1, ρ, A_1, A_2, P_S, M_A, M_R.

Step 1: Input values of all the parameters.

Step 2: We find the optimal values of T and P_R for $T \geq M_R$, i.e., T^*_1, P^*_{R1} as follows:

(i) Put j = 0. Select the initial value P^*_{R1} of P_{R1} as $P_{R10} = P_S$.
(ii) Substitute $P_R = P^*_{R1j}$in (6) and compute T^*_{1j}.
Set j = j + 1.
(iii) Substitute $T = T^*_{1j}$ in (7) to obtainP^*_{R1j+1}.
(iv) Repeat (ii) – (iii) till the values of T^*_{1j} and P^*_{R1j} stabilize, say, to T^*_1 and P^*_{R1}, respectively.
(v) Substitute T^*_1 and P^*_{R1} in (1) to obtain the optimal value of Net1*

Step 3: We find the optimal values of T and P_R for $T \geq M_R$, i.e., T^*_2, P^*_{R2} as follows:

(i) Put j = 0. Set $P_{R20} = P_S$ a guess value of P_{R2}.
(ii) Substitute $P_R = P^*_{R2j}$in (10) and compute T^*_{2j}.
Set j = j + 1.
(iii) Substitute $T = T^*_{2j}$ in (11) to obtainP^*_{R2j+1}.
(iv) Repeat (ii) – (iii) till the values of T^*_{2j} and P^*_{R2j} stabilize, say, to T^*_2 and P^*_{R2}, respectively.

(v) Substitute T_2^* and P_{R2}^* in (2) to obtain the optimal value of Net2*

Step 4: The optimal net profit is Net* = Max (Net1*, Net2*). Stop.

13.4.1 Numerical Example

In this section, we provide a numerical example to illustrate the results satisfying both the above necessary and sufficient conditions of maximization obtained in Sect. 13.3. We apply the above algorithm to obtain optimal values of the decision variables and to conduct sensitivity analysis. We consider the following values for the input parameters in proper units.

Example: Let us take the following parameter values of the inventory system as follows:$\alpha = 1{,}000{,}000$, $\beta = 2$, $h = 0.65$, $A = 50$, $I_{ER} = 0.06$, $I_{PR} = 0.09$, $I_1 = 0.1$, $\rho = 0.4$, $A_1 = 0.2$, $A_2 = 0.4$, $P_S = 5$, $M_R = 0.08$, $M_A = 0.04$.

Plots of Net1 and Net2 with respect to T and P_R for Case I ($T \geq M_R$) and Case II ($T < M_R$) are presented in Fig. 13.2a and Fig. 13.2b, respectively. From the figures, it is clear that for this set of input parameters, Net is jointly Concave function of P_R and T for both the cases.

The optimal values are as follows:

Decision variable	Case I	Case II
T*	0.0908	0.0790
P_R*	8.8525	8.8385
Net*	56084.2	56075.6

Case I provides a larger value of Net. Hence, the column under Case I provides the optimal set of values.

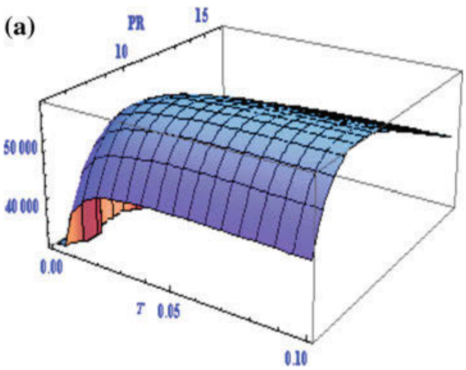


Fig. 13.2 **a** Net versus T and P_R for $M_R \leq T$,

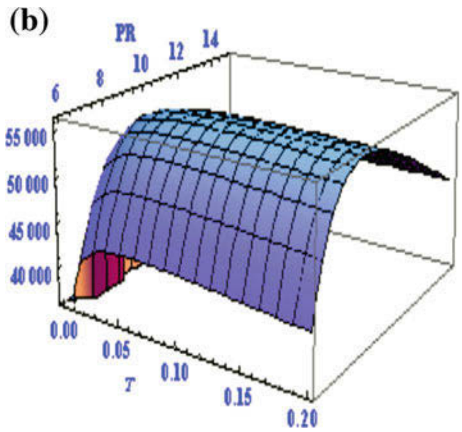


Fig. 13.2 **b** Net versus T and P_R for $M_R > T$

13.4.2 Sensitivity Analysis

We now study the effects of changes in the values of the system parameters α, β, h, A, I_{ER}, I_{PR}, I_1, ρ, A_1, A_2, P_S, M_R, M_A on the optimal values of retailer's price, cycle length and net profit.

The sensitivity analysis is performed by changing each of the parameters by + 50%, +25%, −25%, and −50% taking one parameter at a time and keeping the values of the remaining parameters unchanged.

The results for the cost parameters and other parameters of the model are presented in Table 13.1 and Table 13.2, respectively.

Table 13.1 shows the change in optimal values of the decision variables and the optimum net profit with changes in the cost parameters. We observe that increase in P_s by 50% results in increase in T* by almost 40%, increase in P_R* by almost 50% and decrease in Net* by 33%. Increase in I_{ER} by 50% results in about 18% increase in T* whereas surprisingly, this does not significantly affect the net profit. A 50% decrease in I_{PR} andI_1 results in almost 14% and 13% increase in T*, respectively. Increase in ρ by 50% results in increase in optimal cycle time by 4% and the retailers' selling price by about 5% and net profit decreases by 4.40%. Increase in A by 50% results in about 23% increases in T*. A 50% decrease in h results in about 23% increase in T*.

Table 13.2 shows the change in optimal values of the decision variables and the optimum net profit with changes in the model parameters, where the significant changes are written in bold characters. It is seen that increase in the credit period M_R by 25% results in decline in **T*** and hence, Case II becoming optimal, i.e., the inventory ordered should be such that it is sold off before the end of the credit period.

The parameters α and β are major factors that affect—the optimal values of the cycle time, retailer's price, as well as the net profit. A 25% increase/decrease in the value of α result in a proportionate increase/decrease in the value of net profit. A

Table 13.1 Sensitivity analysis of the optimal solution with change in cost parameters

Cost changing parameter	% Change in parameter value (%)	% Change in optimal values			Optimal case
		T*	P_R*	Net*	
h = 0.65	−50	**23.045**	−0.122	0.371	Case I
	−25	**9.734**	−0.061	0.176	
	25	**−5.843**	0.085	−0.142	Case II
	50	**−10.756**	0.164	−0.276	
$P_S = 5$	−50	NV	NV	NV	Case I
	−25	NV	NV	NV	
	25	**20.473**	**25.118**	**−20.061**	
	50	**39.708**	**50.276**	**−33.433**	
$I_{ER} = 0.06$	−50	**−11.614**	−0.024	−0.257	Case II
	−25	**−6.346**	−0.015	−0.133	Case I
	25	**7.845**	0.025	0.142	
	50	**17.912**	0.063	0.297	
$I_{PR} = 0.09$	−50	**13.848**	0.067	0.090	Case I
	−25	**6.261**	0.034	0.041	
	25	**−5.277**	−0.034	−0.035	
	50	**−9.805**	−0.069	−0.064	Case I = Case II
$I_1 = 0.1$	−50	**12.691**	0.108	0.037	Case I
	−25	**5.788**	0.054	0.015	
	25	**−4.942**	−0.054	−0.010	
	50	**−9.226**	−0.109	−0.015	
ρ = 0.4	−50	**−4.024**	**−4.608**	**4.826**	Case I
	−25	**−2.007**	**−2.304**	**2.356**	
	25	**1.996**	**2.305**	**−2.250**	
	50	**3.980**	**4.610**	**−4.402**	
A = 50	−50	NV	NV	NV	Case II
	−25	**−13.486**	−0.119	0.303	
	25	**12.019**	0.150	−0.232	Case I
	50	**22.924**	0.285	−0.441	

Note 'NV' indicates infeasible value

Table 13.2 Sensitivity analysis of the optimal solution with respect to model parameters

Model changing parameter	% Change in parameter value (%)	% Change in optimal values			Optimal case
		T^*	P_R^*	Net*	
$A_1 = 0.2$	−50	**5.247**	**6.786**	**−6.355**	Case I
	−25	**2.639**	**3.393**	**−3.281**	
	25	**−2.672**	**−3.392**	**3.511**	
	50	**−6.740**	**−6.784**	**7.296**	Case II
$\alpha = 1{,}000{,}000$	−50	**42.376**	0.527	**−50.405**	Case I
	−25	**15.753**	0.196	**−25.228**	
	25	**−10.711**	−0.133	**25.260**	
	50	NV	NV	NV	
$\beta = 2$	−50	NV	NV	NV	Case I
	−25	NV	NV	NV	
	25	**32.166**	**−16.333**	**−65.233**	
	50	**84.485**	**−24.213**	**−87.261**	
$M_R = 0.08$	−50	0.445	0.348	−0.345	Case I
	−25	0.222	0.174	−0.173	
	25	−0.094	−0.120	0.242	Case II
	50	−0.188	−0.241	0.485	
$M_A = 0.04$	−50	−0.021	−0.016	0.016	Case I
	−25	−0.011	−0.008	0.008	
	25	0.010	0.008	−0.008	
	50	0.021	0.017	−0.016	

Note 'NV' indicates non feasible value

25% decrease in α results in about 16% increase in the optimal cycle length, while 50% increase in β results in 84% increase in the optimal cycle length.

The results of sensitivity analysis presented above are also shown below graphically in order to enable a quicker comprehension (Fig 13.3a,b,c).

13.4.3 Managerial Insights

We find that among cost factors, increasing supplier's selling price results in a significant increase in the optimal cycle time, but a drastic decrease in the optimum profit. Other factors that result in a significant change in optimal time are the rates of interest to be paid by and earned by the retailer, ordering cost, discount factors well as the holding cost. However, other than the supplier's price, net profit is not significantly affected by cost factors. Hence supplier's price must be negotiable to attain a profitable level for the retailer. Increase in the proportion of advance

(a)

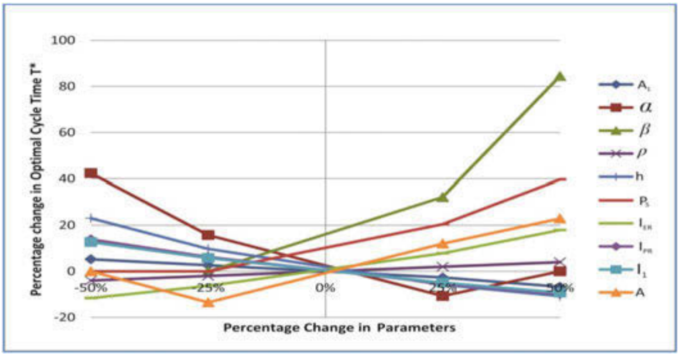


Fig. 13.3 **a** Significant change in T* with change in parameters

(b)

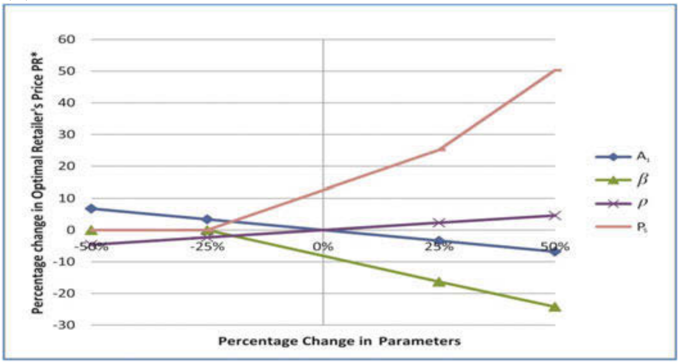


Fig. 13.3 **b** Significant change in P_R* with change in parameters

(c)

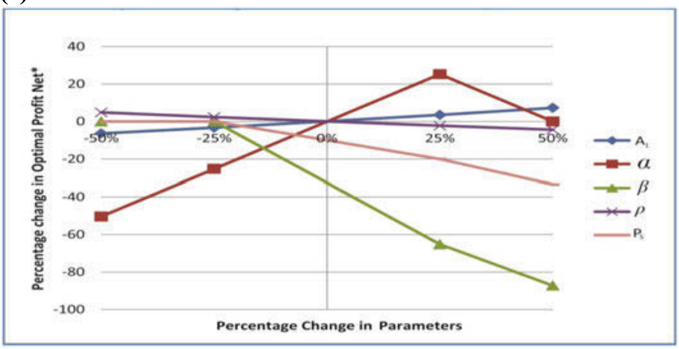


Fig. 13.3 **c** Significant change in Net* with change in parameters

payment will result in decline in optimal value of the retailer's selling price and hence increase in end customer demand. Thus, an increase in the proportion of order quantity obtained at discounted price and increase in revenue earned due to increased demand together lead to an increase in the net profit rate of the retailer. Further, an advantage of increasing A_1 is that it will contribute to increase in supplier's corpus fund. A completely opposite effect is seen when the discount factor is increased. The retailer's net profit is significantly affected by both the demand parameters. Hence, the demand rate must be estimated with care. Increase in duration of advance payment by the supplier will not result in a reduction in the retailer's net profit as in Zhang [18].

13.5 Conclusion and Future Scope

In this paper, we have discussed a payment policy for supply chains with permissible delay in payment and partial advance payment at a discounted price where the retailer's selling price is a decision variable. Iso-elastic price-dependent demand function has been considered and useful managerial insights are obtained from sensitivity analysis.

In future, other types of price-dependent demand functions may be explored for other real-life problems. Further, being an important determinant of the retailer's payment policy, discount may be optimally determined using procedure similar to Gupta et al. [10] for constant demand and Gupta et al. [11] for iso-elastic demand.

This research did not receive any specific grant from funding agencies in the public, commercial, or nonprofit sectors.

Acknowledgements The authors wish to thank the anonymous reviewers for their suggestions leading to some improvement in the presentation of this paper.

Appendix 1 (Sufficiency Conditions)

For Case I ($T \geq M_R$), the second-order derivatives with respect to T and P_R are given by differentiating (4) and (5), respectively, i.e.,

$$\frac{\partial^2 \mathrm{Net1}(T, P_R)}{\partial T^2} = -\frac{2A}{T^3} < 0$$

$$\frac{\partial^2 \mathrm{Net1}(T, P_R)}{\partial P_R^2} = -\frac{\alpha}{2} P_R^{-(\beta+1)} \left[P_R^{-1}(\beta + 1)(R1) + (\beta - 1)(2 + I_{ER} T) \right]$$

At P_R* since $\frac{\partial Net1}{\partial P_R} = 0$, we have $R_1 = 0$.

Since $R1 = 0$,

$$\frac{\partial^2 \mathrm{Net1}(T, P_R)}{\partial P_R^2} \langle 0 \text{ If } \beta - 1 \rangle 0, \text{ i.e., } \beta > 1.$$

For Case II ($T < M_R$), the second-order derivatives with respect to T and P_R are given by differentiating (8) and (9), respectively, i.e.,

$$\frac{\partial^2 \mathrm{Net2}(T, P_R)}{\partial T^2} = -\frac{2A}{T^3} < 0$$
$$\frac{\partial^2 \mathrm{Net2}(T, P_R)}{\partial P_R^2} = -\frac{\alpha}{2} P_R^{-(\beta+1)} [P_R^{-1}(\beta + 1)(R2) + (\beta - 1)(2 + 2I_{ER}M_R - I_{ER}T)$$

Since $R2 = 0$

$$\frac{\partial^2 \mathrm{Net2}(T, P_R)}{\partial P_R^2} \langle 0, \; If \; \beta - 1 \rangle 0, \text{ i.e., } \beta > 1.$$

Appendix 2 (Determinant of the Hessian Matrix)

For Case I, $T \geq M_R$, we have

$$\frac{\partial^2 \mathrm{Net1}(T, P_R)}{\partial T^2} = -\frac{2A}{T^3}$$

$$\frac{\partial^2 \mathrm{Net1}(T, P_R)}{\partial P_R^2} = -\frac{\alpha}{2} P_R^{-(\beta+2)} \left[P_R^{-1}(\beta + 1)(R1) + (2 + I_{ER}T)(\beta - 1)\right]$$

On differentiating (5), we get

$$\frac{\partial^2 \mathrm{Net1}(T, P_R)}{\partial T \partial P_R} = \frac{\alpha}{2} P_R^{-(\beta+1)} \{-P_R I_{ER}(\beta - 1)$$
$$+\beta\{h + 2P_S[(1 - A_1)I_1 - A_2(I_1 - I_{PR}) + A_1 \rho I_{PR}]\}\}$$

The determinant of this Hessian matrix for Case I is

$$\mathrm{Hessian1} = -\frac{2A}{T^3}\left\{-\frac{\alpha}{2} P_R^{-(\beta+2)} \left[P_R^{-1}(\beta + 1)(R1) + (2 + I_{ER}T)(\beta - 1)\right]\right\}$$
$$- \left[\frac{\alpha}{2} P_R^{-(\beta+1)} \{-P_R I_{ER}(\beta - 1) + \beta[h + 2P_S((1 - A_1)I_1 - A_2(I_1 - I_{PR})\right.$$
$$(+A_1 \rho I_{PR})]\}]^2$$

Since R1 = 0

$$\text{Hessian1} = \frac{A}{T^3}\alpha P_R^{-(\beta+2)}(2 + I_{ER}T)(\beta - 1)$$
$$- \left[\frac{\alpha}{2} P_R^{-(\beta+1)}\{-P_R I_{ER}(\beta - 1) + \beta[h + 2P_S((1 - A_1)I_1 - A_2(I_1 - I_{PR})$$
$$+A_1\rho I_{PR})]\}\right]^2$$

i.e.,

$$\text{Hessian1} = AA - BB$$

where

$$AA = \frac{A}{T^3}\alpha P_R^{-(\beta+2)}(2 + I_{ER}T)(\beta - 1) > 0 \text{ if } \beta > 1.$$

$$BB = \left[\frac{\alpha}{2} P_R^{-(\beta+1)}\{-P_R I_{ER}(\beta - 1) + \beta[h + 2P_S((1 - A_1)I_1 - A_2(I_1 - I_{PR}) + A_1\rho I_{PR})]\}\right]^2$$

Since $\frac{\partial^2 \text{Net1}}{\partial T^2} < 0$, the condition for joint concavity of Net1 with respect to T and P_R is AA > BB.

For Case II, for $T < M_R$, we have

$$\frac{\partial^2 \text{Net2}(T, P_R)}{\partial T^2} = \frac{-2A}{T^3}$$

$$\frac{\partial^2 \text{Net2}(T, P_R)}{\partial P_R^2} = -\frac{\alpha}{2} P_R^{-(\beta+1)}\left[P_R^{-1}(\beta + 1)(R2) + (\beta - 1)(2 + 2I_{ER}M_R - I_{ER}T)\right]$$

On differentiating (9) with respect to T, we get

$$\frac{\partial^2 \text{Net2}(T, P_R)}{\partial T \partial P_R} = \frac{\alpha}{2} P_R^{-(\beta+1)}[P_R I_{ER}(\beta - 1) + \beta(h + 2P_S A_1\rho I_{PR})]$$

The determinant of the Hessian matrix for Case II is

$$\text{Hessian2} = \frac{-2A}{T^3}\left\{-\frac{\alpha}{2} P_R^{-(\beta+1)}\left[P_R^{-1}(\beta + 1)(R2) + (\beta - 1)(2 + 2I_{ER}M_R - I_{ER}T)\right]\right\}$$
$$- \left\{\frac{\alpha}{2} P_R^{-(\beta+1)}[P_R I_{ER}(\beta - 1) + \beta(h + 2P_S A_1\rho I_{PR})]\right\}^2$$

Since at P_{R2}*, R2 = 0,

$$\text{Hessian 2} = \frac{A}{T^3}\alpha P_R^{-(\beta+1)}(\beta - 1)(2 + 2I_{ER}M_R - I_{ER}T)$$

$$- \left\{ \frac{\alpha}{2} \mathrm{P_R}^{-(\beta+1)} [\mathrm{P_R I_{ER}}(\beta - 1) + \beta(\mathrm{h} + 2\mathrm{P_S A_1} \rho \mathrm{I_{PR}})] \right\}^2$$

i.e.,

$$\text{Hessian2} = \text{CC} - \text{DD}$$

where

$$\text{CC} = \frac{\mathrm{A}}{\mathrm{T}^3} \alpha \mathrm{P_R}^{-(\beta+1)} (\beta - 1)(2 + 2\mathrm{I_{ER} M_R} - \mathrm{I_{ER} T})$$

$$\text{DD} = \left\{ \frac{\alpha}{2} \mathrm{P_R}^{-(\beta+1)} [\mathrm{P_R I_{ER}}(\beta - 1) + \beta(\mathrm{h} + 2\mathrm{P_S A_1} \rho \mathrm{I_{PR}})] \right\}^2$$

Since $\frac{\partial^2 \text{Net2}}{\partial \mathrm{T}^2} < 0$, the condition for concavity of Net2 is CC > DD.

References

1. Abad PL, Jaggi CK (2003) A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. Int J Prod Econ 83(2):115–122
2. Banerjee S, Sharma A (2010) Inventory model for seasonal demand with option to change the market. Comput Ind Eng 59(4):807–818
3. Chang CT, Ouyang LY, Teng JT, Cheng MC (2010) Optimal ordering policies for deteriorating items using a discounted cash-flow analysis when a trade credit is linked to order quantity. Comput Ind Eng 59(4):770–777
4. Chen LH, Kang FS (2010) Integrated inventory models considering the two-level trade credit policy and a price-negotiation scheme. Eur J Oper Res 205:47–58
5. Chen SC, Teng JT (2014) Retailer's optimal ordering policy for deteriorating items with maximum lifetime under supplier's trade credit financing. Appl Math Model 38(15–16):4049–4061
6. Chung K-J, Liao J-J (2011) The simplified solution algorithm for an integrated supplier–retailer inventory model with two-part trade credit in a supply chain system. Eur J Oper Res 213:156–165
7. Diabat A, Taleizadeh AA, Lashgari M (2017) A lot sizing model with partial downstream delayed payment, partial upstream advance payment, and partial backordering for deteriorating items. Journal of Manufacturing Systems 45:322–342
8. Goyal SK (1985) Economic order quantity under conditions of permissible delay in payment. J Oper Res Soc 36(4):335–338
9. Gupta R, Bhunia A, Goyal S (2009) An application of genetic algorithm in solving an inventory model with advance payment and interval valued inventory costs. Math Comput Model 49(5):893–905
10. Gupta R, Agrawal S, Banerjee S (2017) Buyer's Optimal Policy for a Deterministic Inventory Model with Discounted Advance Payment and Trade Credit in a Supply Chain System. Amity Journal of Operations Management 2(1):51–64
11. Gupta R, Banerjee S, Agrawal S (2018) Retailer's optimal payment decisions for price-dependent demand under partial advance payment and trade credit in different scenarios. Int J Sci Res Math Stat Sci 5(4):44–53
12. Ho CH, Ouyang LY, Su CH (2008) Optimal pricing, shipment and payment policy for an integrated supplier–buyer inventory model with two-part trade credit. Eur J Oper Res 187(2):496–510

13. Hwang H, Shinn SW (1997) Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. Comput Oper Res 24(6):539–547
14. Jaggi CK, Cárdenas-Barrón LE, Tiwari S, Shafi AA (2017a) Two-warehouse inventory model for deteriorating items with imperfect quality under the conditions of permissible delay in payments. Sci Iranica Trans E, Ind Eng 24(1):390–412
15. Jaggi CK, Tiwari S, Goel SK (2017) Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand and two storage facilities. Ann Oper Res 248(1–2):253–280
16. Khanna A, Gautam P, Jaggi CK (2017) Inventory modeling for deteriorating imperfect quality items with selling price dependent demand and shortage backordering under credit financing. Int JMath Eng Manag Sci 2(2):110–124
17. Lashgari M, Taleizadeh AA, Ahmadi A (2016) Partial up-stream advanced payment and partial down-stream delayed payment in a three-level supply chain. Ann Oper Res 238(1–2):329–354
18. Maiti AK, Maiti MK, Maiti M (2009) Inventory model with stochastic lead-time and price dependent demand incorporating advance payment. Appl Math Model 33(5):2433–2443
19. Mishra U, Tijerina-Aguilera J, Tiwari S, Cárdenas-Barrón LE (2018) Retailer's joint ordering, pricing, and preservation technology investment policies for a deteriorating item under permissible delay in payments. Math Probl Eng
20. Molamohamadi Z, Arshizadeh R, Ismail N (2014) An EPQ inventory model with allowable shortages for deteriorating items under trade credit policy. Discret Dyn Nat Soc
21. Ouyang L, Ho C, Su C (2009) An optimization approach for joint pricing and ordering problem in an integrated inventory system with order-size dependent trade credit. Comput Ind Eng 57(3):920–930
22. Sarkar B (2012) An EOQ model with delay in payments and time varying deterioration rate. Math Comput Model 55(3–4):367–377
23. Shah NH, Shah DB, Patel DG (2015) Optimal transfer, ordering and payment policies for joint supplier–buyer inventory model with price-sensitive trapezoidal demand and net credit. Int J Syst Sci 46(10):1752–1761
24. Taleizadeh AA (2014) An EOQ model with partial backordering and advance payments for an evaporating item. Int J Prod Econ 155:185–193
25. Taleizadeh AA (2017) Lot-sizing model with advance payment pricing and disruption in supply under planned partial backordering. Int Trans Oper Res 24(4):783–800
26. Taleizadeh AA, Lashgari M, Akram R, Heydari J (2016) Imperfect economic production quantity model with upstream trade credit periods linked to raw material order quantity and downstream trade credit periods. Appl Math Model 40(19–20):8777–8793
27. Teng JT (2002) On the economic order quantity under conditions of permissible delay in payments. J Oper Res Soc 53:915–918
28. Teng JT, Chang CT, Goyal SK (2005) Optimal pricing and ordering policy under permissible delay in payments. Int J Prod Econ 97(2):121–129
29. Thangam A (2012) Optimal price discounting and lot-sizing policies for perishable items in a supply chain under advance payment scheme and two-echelon trade credits. Int. J. Production Economics 139:459–472
30. Tiwari S, Cárdenas-Barrón LE, Khanna A, Jaggi CK (2016) Impact of trade credit and inflation on retailer's ordering policies for non-instantaneous deteriorating items in a two-warehouse environment. Int J Prod Econ 176:154–169
31. Wu J, Teng JT, Chan YL (2017) Inventory policies for perishable products with expiration dates and advance-cash-credit payment schemes. Int J Syst Sci: Oper Logist 1–17
32. Zhang Q, Tsao YC, Chen TH (2014) Economic order quantity under advance payment. Appl Math Model 38(24):5910–5921
33. Zia NP, Taleizadeh AA (2015) A lot-sizing model with backordering under hybrid linked-to-order multiple advance payments and delayed payment. Transp Res Part E: Logist Transp Rev 82:19–37

Chapter 14
Effects of Pre- and Post-Deterioration Price Discounts on Selling Price in Formulation of an Ordering Policy for an Inventory System: A Study

Mihir Suthar and Kunal T. Shukla

Abstract In present times, one of the promotional tools is offering a rebate on retail amount for raising the market needs of a product. Also, different discount rates are offered depending upon quality/originality/expediency. A non-deteriorating product maintains its quality/original conditions throughout the planning horizon. A deteriorating product may be affected by deterioration at the time of replenishment (instantaneous deterioration) or may be after some time (non-instantaneous). Retailer may offer different price discounts in each case. In this chapter, optimal ordering policies are discussed when retailer offers different price discounts to his customers, before and after deterioration starts. Moreover, the demand for a product is considered price sensitive. Pre-deterioration discount is considered to be smaller than the post-deterioration discount as per the trend. Four different situations are formulated and illustrated with support of numerical examples. Sensitivity analysis is performed to present bureaucratic insights.

14.1 Introduction

A process that prevents original usage of an item or degrades its quality is known as deteriorations. Deterioration may be observed as decay, dryness, evaporation, degradation, spoilage, etc. Ghare and Schrader [5] were the first to establish an inventory system with exponentially decayed products and deterministic demand. Ardalan [1] gave a provisionary amount rebate to design ordering policy. Wee and Yu [20] evolved provisional amount rebate replica regarding items deteriorating exponentially. Chandra [3] developed an inventory model with a price discount on backorders with ramp type demand and time varying holding cost. Rigorous survey was presented by Nahmias [9], Rafaat [12], Shah and Shah [15], Goyal and Giri [6],

M. Suthar
PDPIAS, CHARUSAT, Changa, Anand, Ahmedabad, Gujarat, India

K. T. Shukla (✉)
Vishwakarma Government Engineering College, Ahmedabad, Gujarat, India
e-mail: drkunalshukla.maths@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*, Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_14

Bakker et al. [2] regarding inventory systems for deteriorating products. However, most of the researches assumed that deterioration takes place as the replenishment of an item in inventory system. In reality, most commodities maintain their quality or original conditions over a span. That is, during this span deterioration does not occur. Normally, it is noticed that freshness of eatables, vegetables and fruits remain for less duration without any staleness. Whereas in items like volatile liquids, radioactive chemicals, trendy goods and electronic goods have more span of marinating their quality or freshness. Wu et al. [22] and Ouyang et al. [10] were developed to incorporate this occurrence in to the reserved stocks; they entitled the occurrence "non-instantaneous deterioration". Also, a suggestion was made by them to lessen the frequency of deterioration of a product by enhancing facilities at the storehouse; the entire year-end stock expenditure will decrease. Many researchers like Ouyang et al. [11], Wu et al. [23], Jaggi and Verma [7], Soni and Patel [18], Shah et al. [16], Dye [4], Wang et al. [21], Shah and Vaghela [17] and Suthar and Shukla [19] have discussed non-instantaneous deterioration in their study. Also, Mukherjee et al. [8], Saha et al. [14] and Roy et al. [13] have discussed price discount on back order in their study.

Here, optimal ordering policies are discussed when retailer offers before and after depreciating rebate in market cost. It's considered that retailer deals with an item having price-sensitive demand; shortages are not allowed; deterioration is non-instantaneous. By examining the inventory system, an algorithm is proposed to define optimal ordering policy with the aforesaid hypothesis. The chapter is outlined section wise; Sect. 14.2 deals with assumptions and notations under consideration; Mathematical formulation of an inventory system is derived in Sect. 14.3. In support of this mathematical formulation, numerical examples are presented in Sect. 14.4 along with special cases. Sensitivity analysis is presented and managerial insights are discussed in Sect. 14.5. The learning is concluded in Sect. 14.6.

14.2 Assumptions and Notations

The following assumptions and notations are used in the formulation of mathematical form of the proposed model.

1. The inventory system under consideration is for only one item. The rate of restoration is infinite. The length of planning horizon is not finite. The system does not possess shortages.
2. During an ordering cycle $0 \le t \le T$, inventory level at any instant of time is a function say $I(t)$, where T is cycle time.
3. The demand for a product is assumed to be selling price sensitive, say $D(S)$; where S is the selling price/ unit.
4. To make better demand for a product, the retailer offers a rebate on selling price to his customer. Here, we plot a general trend to offer different price discounts

before and after the effect of deterioration. Let, d_1 be pre-deterioration and d_2 be post-deterioration discount rates over selling price S.

5. $\alpha_1 = (1 - d_1)^{-\eta}, \eta \in R$ is the effect of rebate available before the start of deterioration (i.e., pre-deterioration) of a product over demand; and $\alpha_2 = (1 - d_2)^{-\eta}, \eta \in R$ is the effect of rebate offered after the start of deterioration (i.e., post-deterioration) of a product over demand.
6. During the ordering cycle, t_1 is the time up to which the product does not possess deterioration. Thereafter, it deteriorates with a rate say, $\theta(t)$. Again, it is assumed that the deteriorated product is neither repaired nor replenished during the ordering cycle.
7. EOQ Q (a decision variable), is an initial level of stock in to the inventory system.
8. Consider P_C is the purchase cost/ unit; H_C is the holding cost/ unit/ year; O_C is an ordering cost per order; $\Pi(T)$ is an average profit of an inventory system per time unit.

14.3 Mathematical Formulation

An inventory system is formulated with the assumption that retailer offers different price discounts to his customers for fresh item and deteriorated item. To design general framework, deterioration is assumed to be non-instantaneous. The depletion of stock in inventory system is showed off in Fig. 14.1.

The level of stock at any instant time can be expressed in the form of (14.1),

$$\frac{dI(t)}{dt} = \begin{cases} -D(S) & ;\ 0 \le t \le t_1 \\ -D(S) - \theta(t)I(t) & ;\ t_1 \le t \le T \end{cases} \tag{14.1}$$

where

$$I(T) = 0 \tag{14.2}$$

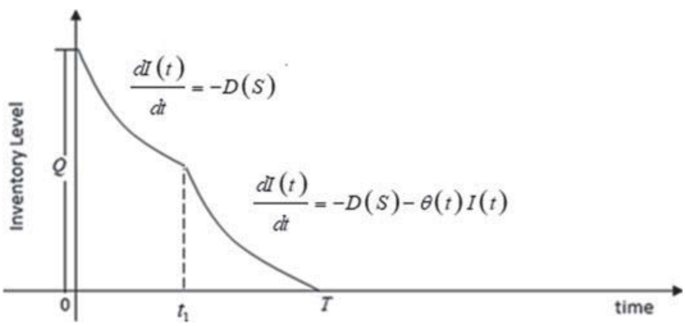


Fig. 14.1 Stock depletion during ordering cycle

Here, retailer offers $d_1\%$ rebate on selling price before deterioration starts during $[0, t_1]$ and offers $d_2\%$ rebate on selling price after deterioration starts during $[t_1, T]$. Three cases are formulated here depending upon the value of t_1.

Model 3.1: $t_1 \geq T$ (Inventory model for items without deterioration), Model 3.2: $t_1 = 0$ (Inventory model for items with instantaneous deterioration), Model 3.3: $0 < t_1 < T$ (Inventory model for items with non-instantaneous deterioration).

And Model 3.4: Generalization of Model 3.3 for various deterioration rates (i.e., except this model, the rate of deterioration is assumed to be constant).

Model 3.1: $t_1 \geq T$ (Inventory model for items without deterioration)

In this case, an item is not being deteriorated and so, level of inventory depletes due to demand only. From (14.1), the differential equation that governs the inventory system in this case is

$$\frac{dI(t)}{dt} = -D(S); \qquad 0 \leq t \leq T \tag{14.3}$$

With condition (14.2), the solution of (14.3) is

$$I(t) = D(S)(T - t); \qquad 0 \leq t \leq T \tag{14.4}$$

Using (14.4),

$$Q = I(0) = D(S) \cdot T \tag{14.5}$$

The cost components involved in the computation of total profit per ordering cycle are as follows:

1. Ordering cost:$OC = O_C$
2. Holding cost:$HC = H_C \int_0^T I(t)\,dt = \frac{H_C}{2} \cdot D(S) \cdot T^2$
3. Purchase cost: $PC = P_C \cdot Q = P_C \cdot D(S) \cdot T$
4. Sales Revenue:$SR = S \cdot (1 - d_1) \cdot \alpha_1 \cdot D(S) \cdot T = S \cdot (1 - d_1)^{1-\eta} \cdot D(S) \cdot T$

Therefore, the profit function is defined by

$$\Pi(T) = \frac{SR - OC - PC - HC}{T} \tag{14.6}$$

To find maximum value of $\Pi(T)$, the concepts of calculus are used. To solve,

$$\frac{d\Pi(T)}{dT} = 0$$

$$\Rightarrow -\frac{1}{2}H_C \cdot D(S) + \frac{O_C}{T^2} = 0$$

$$\Rightarrow T = \sqrt{\frac{2O_C}{H_C \cdot D(S)}} \tag{14.7}$$

Moreover,

$$\frac{d^2\Pi(T)}{dT^2} = \frac{-2O_C}{T^3} < 0 \tag{14.8}$$

Here, Eq. (14.8) assures that the profit function is maximized with the optimal value of T obtained from (14.7).

Model 3.2: $t_1 = 0$ (Inventory model for items with instantaneous deterioration)

In this case, an item deteriorates throughout the ordering cycle and so, the level of inventory depletes due to demand and deterioration both. From (14.1), the differential equation that governs inventory system in this case is

$$\frac{dI(t)}{dt} = -\theta I(t) - D(S)\,; \quad 0 \le t \le T \tag{14.9}$$

With condition (14.2), solution of (14.9) is

$$I(t) = \frac{D(S)}{\theta}\left(e^{\theta(T-t)} - 1\right); \quad 0 \le t \le T \tag{14.10}$$

Here, as θ and T are very small, it is assumed that

$$e^{\theta(T-t)} \approx 1 + \theta(T-t) + \frac{\theta^2(T-t)^2}{2} \approx 1 + \theta(T-t) + \frac{\theta^2\left(T^2 - t^2\right)}{2} \tag{14.11}$$

So, from (14.10) and (14.11), the solution of (14.9) is

$$I(t) = D(S)\left((T-t) + \frac{\theta\left(T^2 - t^2\right)}{2}\right); \quad 0 \le t \le T \tag{14.12}$$

Using (14.12),

$$Q = I(0) = D(S)\cdot\left(T + \frac{1}{2}\theta\cdot T^2\right) \tag{14.13}$$

The cost components involved in the computation of total profit per ordering cycle are as follows:

1. Ordering cost:$OC = O_C$
2. Holding cost:$HC = H_C\int_0^T I(t)dt = H_C\cdot D(S)\cdot\left(\frac{1}{3}\theta\cdot T^3 + \frac{1}{2}T^2\right)$
3. Purchase cost:$PC = P_C\cdot Q = P_C\cdot D(S)\cdot\left(T + \frac{1}{2}\theta\cdot T^2\right)$
4. Sales Revenue: $SR = S\cdot(1-d_2)\cdot\alpha_2\cdot D(S)\cdot T = S\cdot(1-d_2)^{1-\eta}\cdot D(S)\cdot T$

Therefore, the profit function is defined by

$$\Pi(T) = \frac{SR - OC - PC - HC}{T} \tag{14.14}$$

Again, to optimize $\Pi(T)$, the concepts of calculus are used. To solve,

$$\begin{aligned}
&\frac{d\Pi(T)}{dT} = 0 \\
&\Rightarrow -\frac{2}{3}T \cdot H_C \cdot \theta \cdot D(S) - \frac{1}{2}H_C \cdot D(S) \\
&\quad -\frac{1}{2}P_C \cdot \theta \cdot D(S) + \frac{O_C}{T^2} + \frac{1}{2}S \cdot \theta \cdot D(S)(1 - d_2)^{1-\eta} = 0 \\
&\Rightarrow T = \left[\begin{array}{l} \frac{1}{4}\frac{K}{H_C \cdot \theta \cdot D(S)} + \frac{1}{4}\frac{\theta \cdot P_C^2 \cdot D(S)}{H_C \cdot K} \\ + \frac{1}{2}\frac{P_C \cdot D(S)}{K} + \frac{1}{4}\frac{H_C \cdot D(S)}{\theta \cdot K} - \frac{1}{4}\frac{P_C}{H_C} - \frac{1}{4}\theta \end{array} \right]
\end{aligned} \tag{14.15}$$

where

$$K = \left(\begin{array}{l} -D(S)^3 \cdot (P_C \cdot \theta)^2 (P_C \cdot \theta + 3 \cdot H_C) \\ +3 \cdot (16 \cdot O_C \cdot \theta \cdot D(S) - P_C) \cdot H_C^2 \cdot \theta \cdot D(S) \\ +4 \cdot D(S)^2 \cdot H_C \cdot \theta \cdot \sqrt{6} \cdot \sqrt{24 \cdot (O_C \cdot H_C \cdot \theta)^2 - O_C \cdot D(S)(P_C \cdot \theta + H_C)^3} \\ -H_C^3 \cdot D(S)^3 \end{array} \right)$$

Moreover,
$\frac{d^2\Pi(T)}{dT^2} = \frac{-2O_C}{T^3} - \frac{2}{3}H_C \cdot \theta \cdot D(S) < 0$ assures that the profit function is maximized with the optimal value of T obtained from (14.15).

Model 3.3: $0 < t_1 < T$ ***(Inventory model for items with non-instantaneous deterioration)***

Here, an item remains unaffected over a period of time and then deteriorates for the remaining ordering cycle. This demonstrates the case that an item is non-instantaneous. From Eq. (14.1), the differential equation that governs inventory system in this case is

$$\frac{dI(t)}{dt} = \begin{cases} -D(S) & ;\ 0 < t \le t_1 \\ -D(S) - \theta I(t) & ;\ t_1 \le t < T \end{cases} \tag{14.16}$$

With condition (14.2), continuity at $t = t_1$ and approximation (14.11), the solution of (14.16) is

$$I(t) = \begin{cases} D(S)\left(T - t + \frac{\theta}{2}\left(T^2 - t_1^2\right)\right) ;\ 0 < t \le t_1 \\ D(S)\left(T - t + \frac{\theta}{2}\left(T^2 - t^2\right)\right) ;\ t_1 \le t < T \end{cases} \tag{14.17}$$

Using (14.17),

$$Q = I(0) = D(S)\left(T + \frac{\theta}{2}\left(T^2 - t_1^2\right)\right) \tag{14.18}$$

The cost components involved in the computation of total profit per ordering cycle are as follows:

1. Ordering cost: $OC = O_C$
2. Holding cost:$HC = H_C \int_0^T I(t)\,dt = H_C \cdot D(S) \cdot \left(T^3 \cdot \theta - T \cdot t_1^2 \cdot \theta + T^2\right)$
3. Purchase cost:$PC = P_C \cdot Q = P_C \cdot D(S)\left(T + \frac{\theta}{2}\left(T^2 - t_1^2\right)\right)$
4. Sales Revenue:

$$\begin{aligned} SR &= S \cdot (1 - d_1) \cdot \alpha_1 \cdot D(S) \cdot t_1 + S \cdot (1 - d_2) \cdot \alpha_2 \cdot D(S) \cdot (T - t_1) \\ &= S \cdot (1 - d_1)^{1-\eta} \cdot D(S) \cdot t_1 + S \cdot (1 - d_2)^{1-\eta} \cdot D(S) \cdot (T - t_1) \end{aligned}$$

Therefore, the profit function is defined by

$$\Pi(T) = \frac{SR - OC - PC - HC}{T} \tag{14.19}$$

Again, to optimize $\Pi(T)$, the concepts of calculus are used. To solve,

$$\frac{d\Pi(T)}{dT} = 0$$
$$\Rightarrow \frac{2O_C - P_C \cdot \theta \cdot t_1^2}{T^2} - D(S)\left(\begin{array}{c} 2 \cdot H_C \cdot \theta \cdot T + H_C + \frac{1}{2}P_C \cdot \theta \\ -\frac{S \cdot t_1(1 - d_1)}{T^2\alpha_1} + \frac{S \cdot t_1(1 - d_2)}{T^2\alpha_2} \end{array}\right) = 0 \tag{14.20}$$

One may take a help of mathematical software like Maple, Mathematica, or MATLAB to evaluate the closed form of optimal cycle time T.

Moreover, $\frac{d^2\Pi(T)}{dT^2} = \frac{1}{T^3}\left[\begin{array}{l} -2O_C - 2 \cdot H_C \cdot \theta \cdot D(S) \cdot T^3 + P_C \cdot \theta \cdot t_1^2 \cdot D(S) \\ -2 \cdot S \cdot t_1 \cdot D(S)\left((1 - d_2)^{1-\eta} - (1 - d_1)^{1-\eta}\right) \end{array}\right] < 0$ assures that the profit function is maximized with the optimal value of T obtained by (14.20).

Model 3.4: Generalization of Model 3.3 for various deterioration rates

In this case, the rate of deterioration is assumed to be time dependent and all other assumptions are as similar to Model 3.3. From Eq. (14.1), the differential equation that governs inventory system in this case is

$$\frac{dI(t)}{dt} = \begin{cases} -D(S) & ;\ 0 < t \le t_1 \\ -D(S) - \theta(t)I(t) & ;\ t_1 \le t < T \end{cases} \tag{14.21}$$

With condition (14.2), continuity at $t = t_1$, the solution of (14.21) is

$$I(t) = \begin{cases} D(S)(t_1 - t) + e^{\int_T^{t_1} -\theta(x)dx}\left(\int_T^{t_1}\left(-D(S)e^{\int_T^x \theta(x)dx}\right)dx\right) & ;\ 0 < t \le t_1 \\ e^{\int_T^{t} -\theta(x)dx}\left(\int_T^{t}\left(-D(S)e^{\int_T^x \theta(x)dx}\right)dx\right) & ;\ t_1 \le t < T \end{cases} \tag{14.22}$$

Using (14.22),

$$Q = I(0) = D(S)t_1 + e^{\int_T^{t_1} -\theta(x)dx}\left(\int_T^{t_1}\left(-D(S)e^{\int_T^x \theta(x)dx}\right)dx\right) \tag{14.23}$$

The cost components involved in the computation of total profit per ordering cycle are as follows:

1. Ordering cost: $OC = O_C$
2. Holding cost: $HC = H_C \int_0^T I(t)\,dt = H_C \cdot \left(\int_0^{t_1} I(t)dt + \int_{t_1}^T I(t)dt\right)$

$$= H_C \left(\begin{array}{l} \frac{1}{2}D(S)t_1^2 + e^{\int_T^{t_1} -\theta(x)dx}\left(\int_T^{t_1}\left(-D(S)e^{\int_T^x \theta(x)dx}\right)dx\right)t_1 \\ + \int_{t_1}^{T}\left(e^{\int_T^{t} -\theta(x)dx}\left(\int_T^{t}\left(-D(S)e^{\int_T^x \theta(x)dx}\right)dx\right)\right)dt \end{array} \right)$$

3. Purchase cost:

$$PC = P_C \cdot Q = P_C \cdot \left(D(S)t_1 + e^{\int_T^{t_1} -\theta(x)dx}\left(\int_T^{t_1}\left(-D(S)e^{\int_T^x \theta(x)dx}\right)dx\right)\right)$$

4. Sales Revenue:

$$\begin{aligned} SR &= S \cdot (1 - d_1) \cdot \alpha_1 \cdot D(S) \cdot t_1 + S \cdot (1 - d_2) \cdot \alpha_2 \cdot D(S) \cdot (T - t_1) \\ &= S \cdot (1 - d_1)^{1-\eta} \cdot D(S) \cdot t_1 + S \cdot (1 - d_2)^{1-\eta} \cdot D(S) \cdot (T - t_1) \end{aligned}$$

Therefore, the profit function is defined by

$$\Pi(T) = \frac{SR - OC - PC - HC}{T} \tag{14.24}$$

Again, to optimize $\Pi(T)$ with an available choice of deterioration rate, the concepts of calculus are used. To solve, $\frac{d\Pi(T)}{dT} = 0$ find the optimal value of cycle time T such that $\frac{d^2\Pi(T)}{dT^2} < 0$. This assures that the profit function is maximized with the optimal value of T during ordering cycle.

Remark Here, the retailer has three choices. (1) No price discount offered throughout the planning horizon, i.e., $d_1 = d_2 = 0$, (2) Only post-deterioration discounts over unit price is offered, i.e., $d_1 = 0, d_2 \neq 0$, and (3) both pre- and post-deterioration price discounts are offered, i.e., $d_1 \neq 0, d_2 \neq 0$. To compare the closed form of optimal T is difficult, so comparison is done numerically and illustrated in Example 14.6 and Fig. 14.7.

To demonstrate above cases, the examples for formulation are discussed below.

14.4 Numerical Examples

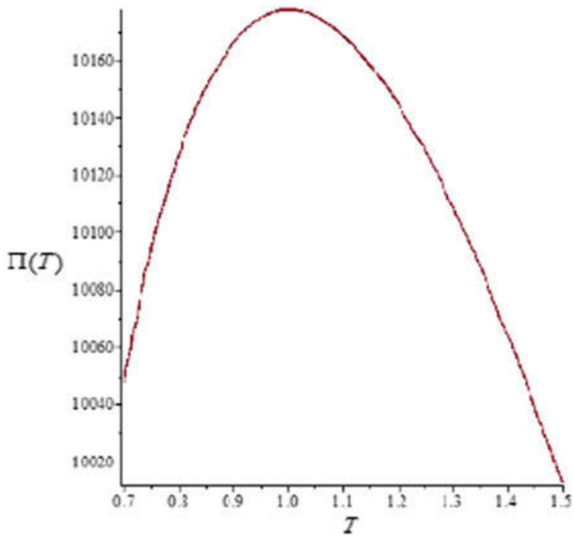
Example 14.1 (Model 3.1) Consider demand rate $D(S) = 1000 - 0.2S$, $S = \$20$, $d_1 = 0.1$, $\eta = 2$, $H_C = \$2$, $P_C = \$10$, $O_C = \$1000$. By solving (14.7), the optimal value of cycle time is $T^* = 1.0020$ years. Using (14.5) and (14.6), the optimal values of EOQ Q is 997.99 and total profit $\Pi(T)$ is \$10177.34. Concavity as shown in Fig. 14.2 validates that $\Pi(T)$ is maximum

Example 14.2 (Model 3.2) Consider demand rate $D(S) = 1000 - 0.2S$, $S = \$20$, $\eta = 2$, $H_C = \$2$, $P_C = \$10$, $\theta = 0.2$, $d_2 = 0.2$, $O_C = \$1000$. By solving (14.15), the optimal value of cycle time is $T^* = 0.6785$ years. Using (14.13) and (14.14), the optimal values of EOQ Q is 721.64 and total profit $\Pi(T)$ is \$12053.45. Concavity as shown in Fig. 14.3 validates that $\Pi(T)$ is maximum.

Example 14.3 (Model 3.3) Consider demand rate $D(S) = 1000 - 0.2S$, $S = \$20$, $d_1 = 0.1$, $d_2 = 0.2$, $t_1 = 0.2$, $\eta = 2$, $\theta = 0.2$, $H_C = \$2$, $P_C = \$10$, $O_C = \$1000$. By solving (14.20), the optimal value of cycle time is $T^* = 0.6565$ years. Using (14.18) and (14.19), the optimal values of EOQ Q is 692.87 and total profit $\Pi(T)$ is \$10517.21. Concavity as shown in Fig. 14.4 validates that $\Pi(T)$ is maximum.

To demonstrate the Model 3.3 for other price-dependent functions, the following examples are considered.

Fig. 14.2 Total profit $\Pi(T)$ with respect to T



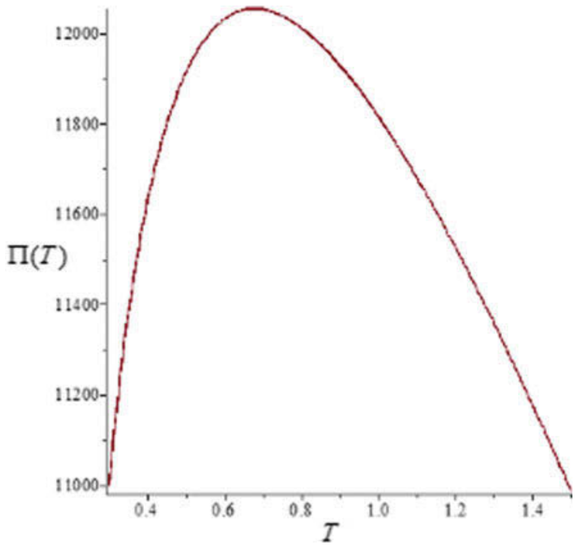


Fig. 14.3 Total profit $\Pi(T)$ with respect to T

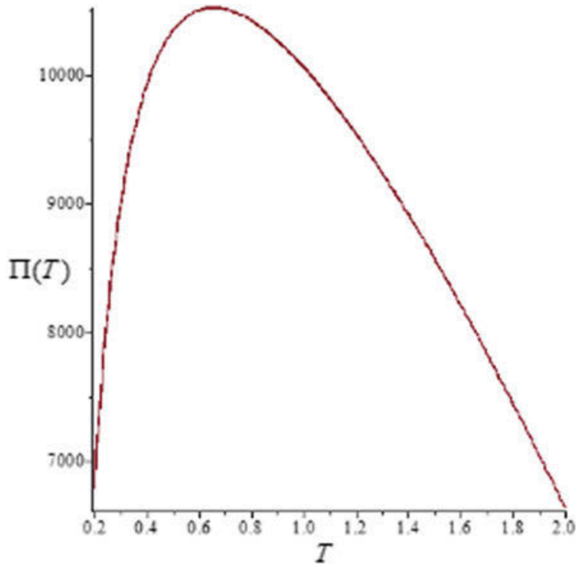
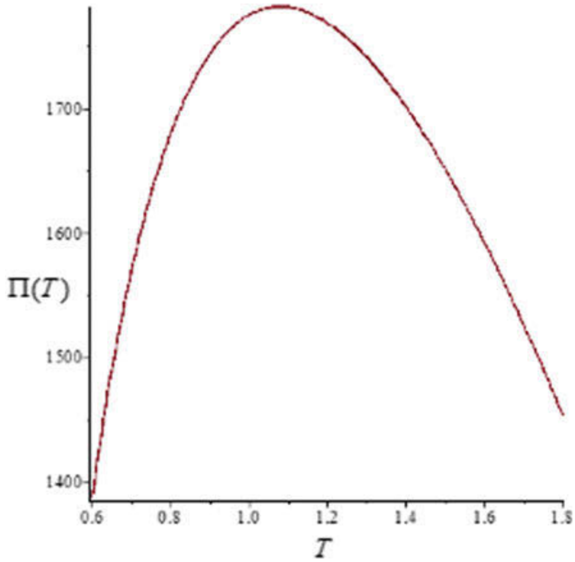


Fig. 14.4 Total profit $\Pi(T)$ with respect to T

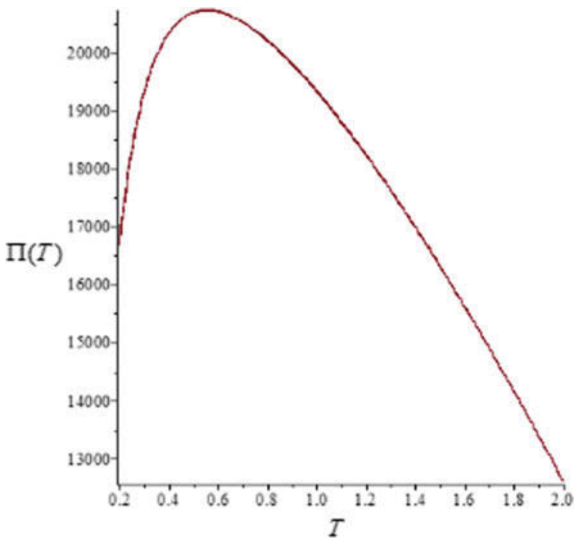
Example 14.4 Consider demand rate $D(S) = 100000 \cdot S^{-\eta}$ and all other parametric values similar to Example 14.3. Again, by solving (14.20), the optimal value of ordering cycle is $T^* = 1.0809$ years. Using (14.18) and (14.19), the optimal values of EOQ Q is 298.44 and total profit $\Pi(T)$ is \$1782.09. Concavity as shown in Fig. 14.5 validates that $\Pi(T)$ is maximum.

Fig. 14.5 Total profit $\Pi(T)$ with respect to T



Example 14.5 Consider demand rate $D(S) = 100000 \cdot e^{-0.2S}$ and all other parametric values similar to Example 14.3. Again, by solving (14.20), the optimal value of ordering cycle is $T^* = 0.5552$ years. Using (14.18) and (14.19), the optimal values of EOQ Q is 1065.96 and total profit $\Pi(T)$ is \$20724.35. Concavity as shown in Fig. 14.6 validates that $\Pi(T)$ is maximum.

Fig. 14.6 Total profit $\Pi(T)$ with respect to T



Example 14.6 Consider demand rate $D(S) = 1000 - 0.2S$, $S = \$20$, $t_1 = 0.2, \eta = 2, \theta = 0.2, H_C = \$2, P_C = \$10, O_C = \$1000$. By solving (14.20), the optimal value of cycle time T is obtained and using (14.18) and (14.19), the optimal values of Q and $\Pi(T)$.

Case	T	Q	$\Pi(T)$
$d_1 = d_2 = 0$	0.530574199	552.50	6468.76
$d_1 = 0, d_2 \neq 0$	0.739484831	787.01	9883.19
$d_1 \neq 0, d_2 \neq 0$	0.656547800	692.87	10517.21

Form Fig. 14.7, one may observe that offering the pre- and post-deterioration price discount is a better option to opt for the retailer.

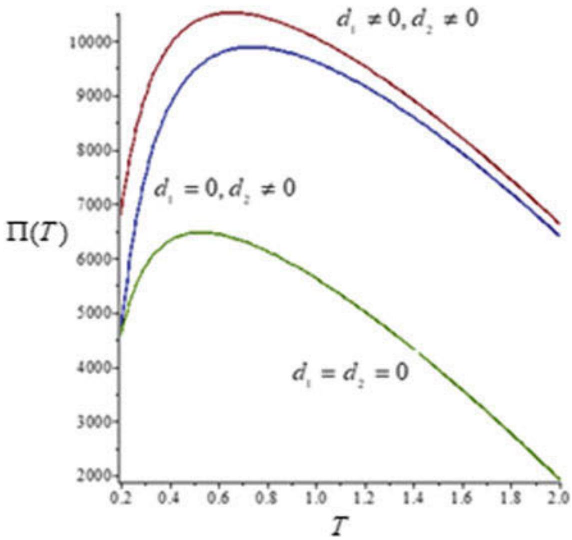


Fig. 14.7 Variation of total profit $\Pi(T)$ with respect to T

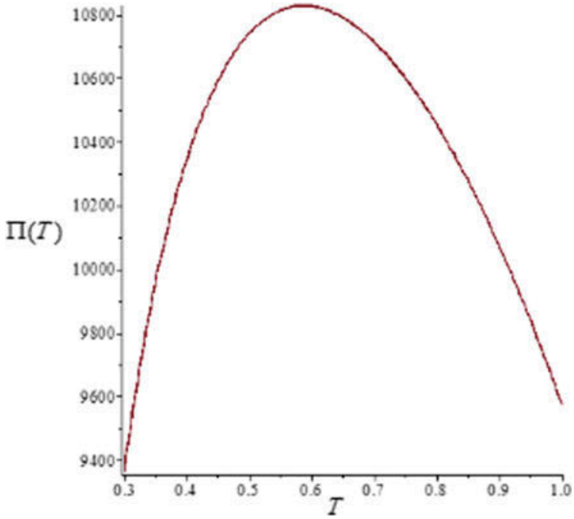


Fig. 14.8 Total profit $\Pi(T)$ with respect to T

Example 14.7 (Model 3.4) Consider the rate of deterioration depending upon maximum life m say $\theta(t) = \frac{1}{1+m-t}$, demand rate $D(S) = 1000 - 0.2S$, $S = \$20$, $d_1 = 0.1$, $d_2 = 0.2$, $t_1 = 0.2$, $\eta = 2$, $m = 1$, $H_C = \$2$, $P_C = \$10$, $O_C = \$1000$. By solving $\frac{d\Pi(T)}{dT} = 0$, the optimal value of ordering cycle is $T^* = 0.5870$ years. And the optimal values of EOQ Q is 633.23 and total profit $\Pi(T)$ is \$10827.47. Concavity as shown in Fig. 14.8 validates that $\Pi(T)$ is maximum.

On a similar path, one may try Model 3.4 for other price-sensitive demand rates and deterioration rates. To derive managerial insights, the sensitivity with respect to different parameters is outlined in the next section.

14.5 Sensitivity Analysis

Sensitivity with respect to various parameters is exhibited in Figs. 14.9, 14.10, and 14.11 as below, where standard values for parameters are as similar to Example 14.3.

From Fig. 14.9, one may observe that T is in direct proportion of selling price S, post-deterioration discount rate d_2, delay time of deterioration t_1, η, and ordering cost O_C. Also, it is in inverse proportion to demand rate $D(S)$, pre-deterioration discount rate d_1, deterioration rate θ, unit purchase cost P_C, and unit holding cost per time unit H_C.

From Fig. 14.10, one may observe that Q is in direct proportion of $D(S)$, S, d_2, t_1, and η. Also, it is in inverse proportion to d_1, θ, P_C, H_C and O_C.

From Fig. 14.11, one may observe that $\Pi(T)$ is in direct proportion of $D(S)$, S, d_1, d_2, and η. Also, it is in inverse proportion to t_1, θ, P_C, H_C and O_C.

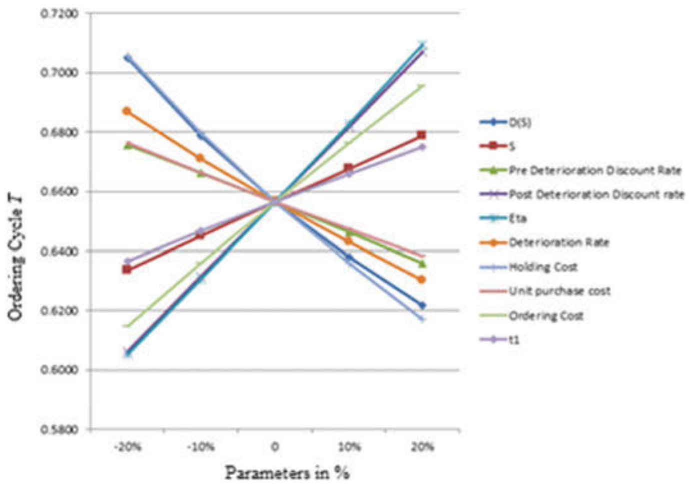


Fig. 14.9 Variation in length of ordering cycle *T*

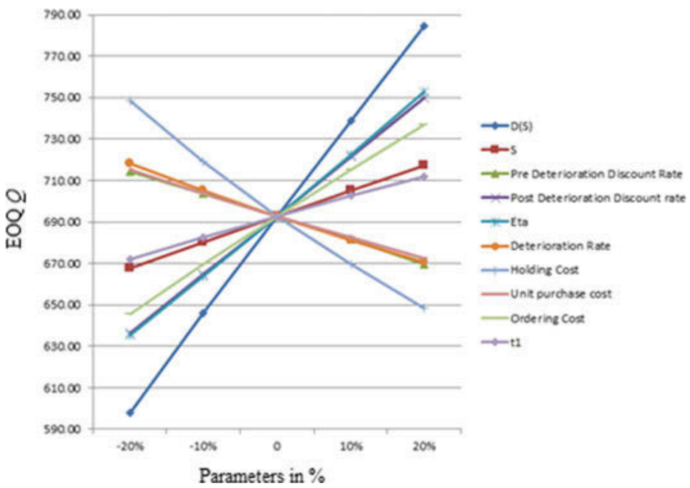


Fig. 14.10 Variation in EOQ *Q*

14.6 Conclusion

In this chapter, mathematical formulation to derive an ordering policy is discussed from retailer's point of view, when retailer offers different price discounts before and after deterioration starts. The demand of an item is price dependent and shortages are not allowed. Here, four models are discussed; (3.1) An item does not deteriorate, (3.2) Deterioration is instantaneous, (3.3) Deterioration is non-instantaneous, and (3.4) Generalization of formulation with nonconstant deterioration rates. For Model

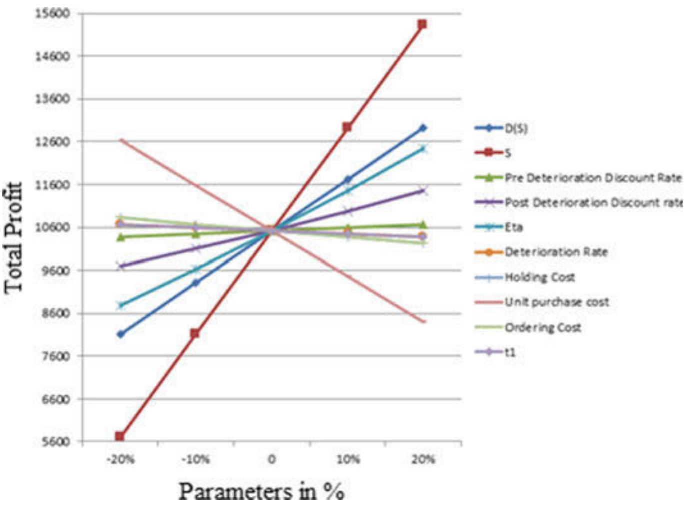


Fig. 14.11 Variation in total profit $\Pi(T)$

(3.3), it is assumed that retailer offers different price discounts on selling price, before and after deterioration. Using the formulation of Model 3.3, retailer may opt for one of the three choices. (1) No price discount offered throughout the planning horizon, (2) Just after deterioration discounts over unit price is offered, and (3) both before and after deterioration price rebate is offered. By comparing the optimal value of T numerically, retailer may opt for the best decision approach. One may extend this concept for other price-sensitive demands different demand rate and may study the effect of pre- and post-deterioration discount rates.

Acknowledgements The authors are thankful to reviewers and editors for their constructive comments for improvement of this chapter.

References

1. Ardalan A (1994) Optimal prices and order quantities when temporary price discounts result in increase in demand. Eur J Oper Res 72(1):52–61
2. Bakker M, Riezebos J, Teunter RH (2012) Review of inventory system with deterioration since 2001. Eur J Oper Res 221(2):275–284
3. Chandra S (2017) An inventory model with ramp type demand, time varying holding cost and price discount on backorders. Uncertain Supply Chain Manag 5(1):51–58
4. Dye CY (2013) The effect of preservation technology investment on a non-instantaneous deteriorating inventory model. Omega 41(5):872–880
5. Ghare PM, Schrader GF (1963) A model for exponentially decaying inventory. J Ind Eng Chem 14(2):238–243
6. Goyal SK, Giri BC (2001) Recent trend in modelling of deteriorating inventory. Eur J Oper Res 134(1):1–16

7. Jaggi CK, Verma P (2010) An optimal replenishment policy for non-instantaneous deteriorating items with two storage facilities. Int J Serv Oper Inform 5(3):209–230
8. Mukherjee A, Dey O, Giri BC (2018) An integrated imperfect production-inventory model with optimal vendor investment and backorder price discount. Inf Technol Appl Math 699:187–203
9. Nahmias S (1982) Perishable inventory theory: a review. Oper Res 23:735–749
10. Ouyang LY, Wu KS, Yang CT (2006) A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Comput Ind Eng 51(4):637–651
11. Ouyang LY, Wu KS, Yang CT (2008) Retailer's ordering policy for non-instantaneous deteriorating items with quantity discount, stock-dependent demand and stochastic backorder rate. J Chin Inst Ind Eng 25(1):62–72
12. Raafat F (1991) Survey of literature on continuously deteriorating inventory model. J Oper Res Soc 42:27–37
13. Roy SK, Pervin M, Weber GW (2018) A two-warehouse probabilistic model with price discount on backorders under two levels of trade- credit policy. J Ind Manag Optim. https://doi.org/10.3934/jimo.2018167
14. Saha S, Sen N, Nath BK (2018) Inventory model with ramp-type demand and price discount on back order for deteriorating items under partial backlogging. Appl Appl Math: Int J 13(1):472–483
15. Shah NH, Shah YK (2000) Literature survey on inventory models for deteriorating items. Econ Ann 44(1):221–237
16. Shah NH, Soni HN, Patel KA (2013) Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. Omega 41(2):421–430
17. Shah NH, Vaghela CR (2017) Retailer's replenishment and pricing decisions for non-instantaneous deterioration and price-dependent demand. Int J Ind Eng Prod Res 28(2):101–111
18. Soni HN, Patel KA (2012) Optimal pricing and inventory policies for non-instantaneous deteriorating items with permissible delay in payment: fuzzy expected value model. Int J Ind Eng Comput 3(3):281–300
19. Suthar MS, Shukla KT (2018) Retailer's optimal credit period and ordering policy for non-instantaneous deteriorating items with trapezoidal type demand. J Supply Chain Manag Syst 7(2):26–36
20. Wee HM, Yu J (1997) A deteriorating inventory model with a temporary price discount. Int J Prod Econ 53(1):51–90
21. Wang Y, Zhang J, Tang W (2015) Dynamic pricing for non-instantaneous deteriorating items. J Intell Manuf 26(4):629–640
22. Wu KS, Ouyang LY, Yang CT (2006) An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Int J Prod Econ 101(2):369–384
23. Wu KS, Ouyang LY, Yang CT (2009) Coordinating replenishment and pricing policies for non-instantaneous deteriorating items with price-sensitive demand. Int J Syst Sci 40(12):1273–1281

Chapter 15
Efficient Supplier Selection: A Way to Better Inventory Control

Shuya Zhong, Sujeet Kumar Singh and Mark Goh

Abstract Effective supplier evaluation during the purchasing process is important to business, as supplier selection and the success of inventory management depend on how and which suppliers are selected. Given the popularity of supplier selection with inventory control, this chapter presents an actual, complex supplier selection problem involving multiple products where conflicting inventory related attributes such as response time, delivery reliability, stock quantity, service level and the track record of the suppliers, are involved. The challenge for this case firm is to intertwine supplier selection with inventory management so as to yield the best space utilization, lower inventory carrying cost and increase end customer satisfaction. Our main contribution is to apply fuzzy AHP and fuzzy TOPSIS to rank and choose efficient suppliers through the linguistic ratings of a set of potential suppliers. Under the environment of global competition, accurate demand fulfilment has become more significant than ever before in supply chain management. As a result, we consider stochastic demand parameters and model the problem with the help of triangular fuzzy numbers. A multi-objective mixed integer linear optimization model is formed to assign the order volume to the selected supplier(s), with a view to executing inventory control visually on a user interface.

15.1 Introduction

Robust supplier selection is essential to establishing a sustainable and efficient supply chain partnership. To enhance supply chain performance, several criteria

S. Zhong · S. K. Singh · M. Goh (✉)
The Logistics Institute-Asia Pacific, National University of Singapore, Singapore 119613, Singapore
e-mail: mark_goh@nus.edu.sg

M. Goh
NUS Business School, National University of Singapore, Singapore 119245, Singapore

S. Zhong
Department of Engineering, Institute for Manufacturing, University of Cambridge, Cambridge CB3 0FS, UK

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*, Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_15

encompassing the ethical, social and economic dimensions need to be considered when selecting the most suitable supplier. The supplier selection process invariably involves supplier evaluation in the context of the supply chain [10].

Today, firms are more supplier dependent due to their outsourcing initiatives. Typically, when a supply chain actor seeks to design a method to assess and choose suppliers, specific technical requirements such as product quality, buffer stock levels, service levels and shelf life are introduced and measured accordingly. This calls for a range of selection methods, and draws on supplier flexibility to cater to the user requirements [10]. This makes the supplier evaluation process more critical for enhancing the overall performance of the firm. This process demands the decision makers of the firm to be cognizant of and embrace the numerous conflicting objectives and criteria during evaluation [2]. Multiple-Criteria Decision-Making (MCDM), as a technique, affords research and practice to offer and consider a suite of conflicting operational criteria, in addition to simplifying the buying firm's needs and arriving at a satisficing outcome [4, 22]. From the extant literature [1, 10, 11, 24], this area of study has grown in maturity, with recent MCDM models combining the adoption of the evaluation and implementation approaches using a myriad of quantitative as well as qualitative practices. Consequently, many MCDM systems, tools, and techniques have been developed to model and implement the supplier management practices, albeit not much of a focus yet on inventory control.

The supplier evaluation process includes a strategic focus on a win-win relationship among the buyer(s) and supplier(s), whereby close collaboration is critical and almost necessary. This needs the requisite skills and capabilities from both parties [22]. As the buying firms require a thorough identification of the skills and capabilities, this makes the process of selecting the strategic supply partners more challenging as some suppliers may be limited by their capacity or flexibility [14]. Thus, the vendor selection process necessitates the need to help the buying firms make better, holistic decisions to deal with the actual complexities in the field. In short, simplifying the complex managerial decision-making process is primary [23], but not at the expense of weakened inventory control. Inventory control is identified as one of the major drivers for any supply chain, and hence maintaining the actual required inventory level is an essential task for a firm [20]. High inventory levels enhance the service capacity to the customers. At the same time, it increases the cost of holding, whereas lowering the inventory levels may cause shortages, which consequently questions the firm's reputation [9]. Therefore, a correlated practice of supplier selection and order/inventory management is important for the competitive performance of firms. Cárdenas-Barrón et al. [3] proposed an efficient heuristics for solving the multi-product, multi-period, inventory lot sizing together with the supplier selection problem using the technique of reducing the feasible region and then optimizing it. They compared the solution with the existing methods with the help of several benchmarking instances and found their approach to be promising. Recently, Duan and Ventura [9] presented a mixed integer linear programming model addressing a coordinated inventory planning model for a serial supply chain that minimizes the overall incurred cost including procurement, inventory holding, production and transportation, incorporating the supplier's price break scheme and flexible time periods.

Several interesting works integrating the supplier's selection and inventory management are also found in Parsa et al. [17], Choudhary and Shankar [6, 7], Mazdeh et al. [15], and Purohit et al. [19], and references therein.

Given the popularity of supplier selection with inventory management, this research aims to integrate the supplier selection problem, in the presence of multiple capacitated suppliers, and inventory policy under a variety of supply contracts and service options by applying fuzzy AHP and fuzzy TOPSIS; and then the order allocation problem to the selected suppliers by developing a multi-objective mathematical model. Further, the multi-objective model is solved using the non-dominated sorting genetic algorithm II (NSGA-II).

In summary, this chapter presents an actual case of supplier selection for a Singapore-based logistics provider, who is keen to lift the operational efficiency of the firm's supply network and improve inventory management.

The rest of this chapter is set as such. Section 15.2 details the case study. The set of criteria used to select the suppliers are detailed, and an easy to apply operational framework using fuzzy AHP for supplier selection and their order management are provided. Next, we show how fuzzy TOPSIS is used to rank the suppliers. Section 15.3 dives into the equivalent chance-constrained program and highlights the sensitivity analysis performed on the results. Section 15.4 concludes.

15.2 Supplier Selection: A Case Study

This study is conducted on a logistics player involved in procuring goods from the suppliers, managing warehousing facilities and stock on behalf of the downstream client, and providing last mile delivery to healthcare corporate customers. The firm wants to efficiently select suppliers to maintain the quality of the materials procured as the final products are very time-sensitive and patient care centric. Specifically, the firm wants to reshape the healthcare supply chain ecosystem by choosing suppliers who pave the way to better inventory control overall. MCDM approaches such as AHP and TOPSIS are used to assess and select the suppliers. Then, a multi-objective optimization program is formulated to determine the optimal order allocation for the selected supplier. The genetic algorithm, NSGA-II, is used to solve the multi-objective program and a user-friendly interface is developed through MATLAB for the case firm.

One key task is to construct a set of criteria to choose the suppliers, based on an overarching consideration of the lead time, service reliability, stock quantity and supplier reputation, so as to obtain the best volumetric space usage, inventory cost, and customer satisfaction.

Criteria Set Formulation for Supplier Selection

Figure 15.1 shows the criteria used by the firm to choose the suppliers of medical examination gloves. From the compliance requirements set on the sample products, as well as the related literature review, a comprehensive decision hierarchy structure

Type	Medical examination gloves	Weight	Vendor A	B	C	D	E
Critical compliance	1. **Fully comply with tender instructions and contract conditions. The vendor cannot alter the text of the tender invitation.**		Y/N	Y/N	Y/N	Y/N	Y/N
	2. Must not be suspended or barred from participating in public sector tender invitations.		Y/N	Y/N	Y/N	Y/N	Y/N
	3. Comply with all existing health product regulations including licensing and legislative.		Y/N/DNA	Y/N/DNA	Y/N/DNA	Y/N/DNA	Y/N/DNA
	4. Mandatory requirements of the specifications		Y/N	Y/N	Y/N	Y/N	Y/N
Non-critical compliance	1. Price: tender price plus storage cost	X %					
	2. Non price (30%)						
	2a. User evaluation scores on comfort, fit, and product quality						
	2b. Product shelf life at point of delivery	Y_1 %					
	At least 4.75 years – 3%; At least 4.5 years – 2%; At least 4 years – 1%						
	Others – 0%						
	2c. Supplier track record	Y_2 %					
	At least 3 years – 2%, At least 2 years – 1%, Others – 0%						
Total							

Fig. 15.1 Evaluation criteria of suppliers. *Note* Y = comply; N = not comply; DNA = Does not apply

for supplier selection is constructed (see Fig. 15.2). The first hierarchical level is the goal, followed by the 3 main- and 11 sub-criteria. Table 15.1 shows the criteria set, comprising 3 main- and 11 sub-criteria. The suppliers who will be assessed using the criteria are determined at level 3. The firm can choose a supplier(s) through the evaluation outcomes. All the criteria are measured with the help of the data received from the requests to the suppliers in the invitation to tender.

Operational Framework for Order Management

Figure 15.3 shows a 4-stage operational framework; with Phase I: criteria set formulation, Phase II: criteria weight calculation by fuzzy AHP, Phase III: supplier ranking by fuzzy TOPSIS and Phase IV: order allocation by multi-objective programming.

Phase I. Criteria set formulation

An expert panel is formed, and five suppliers Sp_1, …, Sp_5 are considered for evaluation. The criteria set is identified, and the decision hierarchy structure is determined as shown in Fig. 15.2. This structure forms the output of Phase I, and is the input to Phase II.

Phase II. Criteria weight calculation by fuzzy AHP

Next, the criteria weights are found using fuzzy AHP, and they form the input for Phase III. AHP is a technique that structures a multi-criteria, multi-person, multi-period problem hierarchically for an easy solution [21]. Fuzzy AHP, which captures the imprecise human judgment using linguistic variables, combines AHP with fuzzy

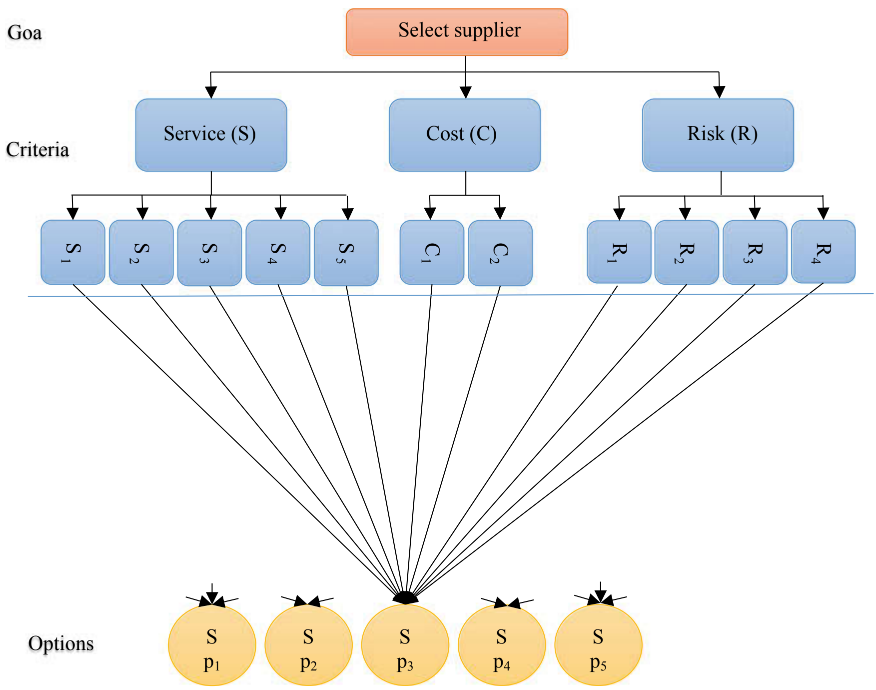


Fig. 15.2 Supplier evaluation decision hierarchy structure

Table 15.1 Supplier selection criteria set

Criteria	Sub-criteria	Description
Service (S)	Compliance with tender (S_1)	Level of compliance to the terms and conditions in the tender invitation.
	Product quality (S_2)	Portion of products that meet the firm's expectations on quality, as measured by the amount of damaged and deteriorated goods, and end user experience of the comfort and fit of the products.
	Product shelf life (S_3)	Length of the product shelf life at delivery.
	Past performance (S_4)	Track record of the supplier, measured by customer volume or transacted volumes, and the evidence of undamaged, on time, and full truckload deliveries.
	Responsiveness (S_5)	The time taken by the supplier to process an order request, arrange production, fulfil shipment, and provide after-sales service.
Cost (C)	Pricing (C_1)	Holding cost and purchase cost. Suppliers who can consolidate stock for disposal, monitor the market cost of the product, and proactively streamline the cost on the firm's behalf are preferred.
	Investment in R&D (C_2)	Portion of supplier's investment devoted to research and development activities such as new product design and technologies, and prototype development.
Risk (R)	Output flexibility (R_1)	Production and delivery flexibility level of supplier in response to critical demand surges.
	Buffer capacity (R_2)	Portion of the vendor's safety stock to cope with surge orders.
	Political & economic stability (R_3)	Political climate of the supplier's host country and government policies may affect the long term vendor relationship.
	Geographical location (R_4)	Supplier location, physical and social status. The origin country of the supplier, plant location, likelihood of the occurrence of natural calamities should be verified.

Fig. 15.3 4-phase framework for order management

Table 15.2 5-level linguistic scale to evaluate criteria weights in the pairwise comparison matrix

Linguistic term on importance	Triangular fuzzy number
Equally important	(1, 1, 3)
Moderately more	(1, 3, 5)
Strongly more	(3, 5, 7)
Very strongly more	(5, 7, 9)
Extremely more	(7, 9, 9)

set theory to solve hierarchical fuzzy problems. The steps for fuzzy AHP are as follows: [5, 13].

Step 1. Firm specifies the relative importance scale for pairwise comparison matrices.

From Table 15.2, a 5-level linguistic scale is formed, where the triangular fuzzy number $(l_{ij}, m_{ij}, u_{ij}) = (1, 1, 3)$ ranks the lowest, and (7, 9, 9) ranks the highest.

Step 2. Construct fuzzy pairwise comparison matrices.

Using the linguistic scale in Table 15.2, the firm's decision makers construct pairwise comparisons for the main- and sub-criteria as shown in Tables 15.3, 15.4, 15.5, and 15.6. For example, in Table 15.3, the cost criterion outweighs the risk

Table 15.3 Pairwise comparison matrix of main criteria (S, C, R)

Main criteria	S	C	R
S	(1,1,1)	(1/7,1/5,1/3)	(1/3,1,1)
C	(3,5,7)	(1,1,1)	(1,3,5)
R	(1,1,3)	(1/5,1/3,1)	(1,1,1)
			$\lambda_{max} = 3.1828$, $\boldsymbol{CR} = 0.1576$

Table 15.4 Pairwise comparison matrix for S

Sub-criteria	S_1	S_2	S_3	S_4	S_5
S_1	(1,1,1)	(5,7,9)	(5,7,9)	(5,7,9)	(5,7,9)
S_2	(1/9,1/7,1/5)	(1,1,1)	(3,5,7)	(5,7,9)	(1,3,5)
S_3	(1/9,1/7,1/5)	(1/7,1/5,1/3)	(1,1,1)	(1,3,5)	(1,3,5)
S_4	(1/9,1/7,1/5)	(1/9,1/7,1/5)	(1/5,1/3,1)	(1,1,1)	(1/5,1/3,1)
S_5	(1/9,1/7,1/5)	(1/5,1/3,1)	(1/5,1/3,1)	(1,3,5)	(1,1,1)
					$\lambda_{max} = 5.8539$, $\boldsymbol{CR} = 0.1906$

Table 15.5 Pairwise comparison matrix for C

Sub-criteria	C_1	C_2
C_1	(1,1,1)	(5,7,9)
C_2	(1/9,1/7,1/5)	(1,1,1)

Table 15.6 Pairwise comparison matrix for R

Sub-criteria	R_1	R_2	R_3	R_4
R_1	(1,1,1)	(1,1,3)	(3,5,7)	(1,3,5)
R_2	(1/3,1,1)	(1,1,1)	(5,7,9)	(1,3,5)
R_3	(1/7,1/5,1/3)	(1/9,1/7,1/5)	(1,1,1)	(1,1,3)
R_4	(1/5,1/3,1)	(1/5,1/3,1)	(1/3,1,1)	(1,1,1)
				$\lambda_{max} = 4.3963$, $\boldsymbol{CR} = 0.1468$

criterion and far outweighs the service criterion, while the risk and service criteria are on a par. The consistency ratio (CR) for each matrix is found (see Table 15.7). An upper bound of 0.2 is set as the threshold for the CR to determine acceptability of the comparison matrix. The decision-making group updates the initial comparison values in the pairwise comparison matrix until the CR threshold condition is satisfied.

Step 3. For each pairwise comparison matrix, compute the fuzzy synthetic extent,

$$\tilde{S}_i = \left(\sum_{j=1}^{n} l_{ij}, \sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} u_{ij}\right) \otimes \left(\frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n} u_{ij}}, \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}}, \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n} l_{ij}}\right) \tag{15.2.1}$$

Step 4. Compute the degree of possibility of $\tilde{S}_i \geq \tilde{S}_j$ between two fuzzy synthetic extents,

$$\mathrm{Pos}(\tilde{S}_i \geq \tilde{S}_j) = \begin{cases} 1, & \text{if } m_i \geq m_j \\ \frac{u_i - l_j}{(u_i - m_i) + (m_j - l_j)}, & \text{if } u_i \geq l_j,\ i, j = 1, \ldots, n;\ j \neq i. \\ 0, & \text{else} \end{cases} \tag{15.2.2}$$

Step 5. Compute the degree of possibility of $\tilde{S}_i$ over all the other fuzzy synthetic extents,

Table 15.7 Random consistency index (RI)

Size (n)	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

$$\text{Pos}\left(\tilde{S}_i \geq \tilde{S}_j | j = 1, \ldots, n; \, j \neq i\right) = \min_{j \in \{1, \cdots, n\}, j \neq i} \text{Pos}\left(\tilde{S}_i \geq \tilde{S}_j\right), i = 1, \ldots, n. \tag{15.2.3}$$

Step 6. Calculate the weight vector $W = (w_1, \cdots, w_n)^T$ of the fuzzy comparison matrices, where

$$w_i = \frac{\text{Pos}\left(\tilde{S}_i \geq \tilde{S}_j | j = 1, \ldots, n; \, j \neq i\right)}{\sum_{k=1}^{n} \text{Pos}\left(\tilde{S}_k \geq \tilde{S}_j | j = 1, \ldots, n; \, j \neq k\right)}. \tag{15.2.4}$$

After normalizing the local weights (LW) of the sub-criteria, the global weight GW can be obtained as shown in Table 15.8.

Phase III. Supplier rank determination by fuzzy TOPSIS

Once the weights of all of the 11 criteria (Table 15.8, col 5) are found, fuzzy TOPSIS is used to rank the suppliers. The corresponding output and the preference degree of the suppliers are applied in Phase IV for order allocation. TOPSIS, or the technique for order performance by similarity to the ideal solution, as highlighted by Hwang and Yoon [12], is an MCDM approach premised on the notion that the best alternative is closest to the positive ideal solution (PIS) but is farthest from the negative ideal solution (NIS). Most TOPSIS approaches view human judgment as deterministic; however, it is not always possible to measure by crisp values in practice. A better way is to use linguistic variables rather than deterministic values as fuzzy set theory performs well with linguistic values. As such, fuzzy TOPSIS is amenable to solving practical problems in fuzzy environments. The steps for fuzzy TOPSIS are set as follows [13, 18]:

Step 1. Choose the linguistic rating values for the potential suppliers with respect to the criteria.

Table 15.8 Criteria weights for supplier evaluation (criterion importance)

Main criteria	Weight of main criteria	Sub-criteria	LW of sub-criteria	GW of sub-criteria	Rank
Service (S)	0.030956	Compliance with contract (S_1)	0.585155	0.018114	6
		Product quality (S_2)	0.349624	0.010823	7
		Product shelf life (S_3)	0.064051	0.001983	8
		Track record (S_4)	0.000585	0.000018	10
		Responsiveness (S_5)	0.000585	0.000018	10
Cost (C)	0.691166	Pricing (C_1)	0.999001	0.690475	1
		Investment in R&D (C_2)	0.000999	0.00069	9
Risk (R)	0.277878	Output flexibility (R_1)	0.345761	0.096079	2
		Buffer capacity (R_2)	0.307825	0.085538	3
		Political & economic stability (R_3)	0.196295	0.054546	4
		Geographical location (R_4)	0.150119	0.041715	5

Table 15.9 5-linguistic scale to evaluate ratings of suppliers

Linguistic term	Triangular fuzzy number
Bad (B)	(1, 1, 3)
Fair (F)	(1, 3, 5)
Average (A)	(3, 5, 7)
Good (G)	(5, 7, 9)
Excellent (E)	(7, 9, 9)

Four decision makers DM_1, …, DM_4 are invited to grade all the 5 potential suppliers on the 11 criteria. Table 15.10 shows the performance ratings by linguistic terms (Table 15.9) of each decision maker on a potential supplier.

Step 2. Calculate aggregated fuzzy ratings $\tilde{X}_{ij}\left(a_{ij}, b_{ij}, c_{ij}\right)$ for the potential suppliers, where

$$a_{ij} = \min_k\left\{a_{ijk}\right\}, b_{ij} = \frac{1}{k}\sum_{k=1}^{K} b_{ijk}, c_{ij} = \max_k\left\{c_{ijk}\right\}, \tag{15.2.5}$$

and form the fuzzy decision matrix $\tilde{D}$ using $\tilde{X}_{ij}$.

Step 3. Form the normalized fuzzy decision matrix $\tilde{R}$ as

$$\tilde{R} = \left[\tilde{r}_{ij}\right]_{m\times n}, i = 1, \ldots, m;\ j = 1, \ldots, n, \tag{15.2.6}$$

where

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right)\text{and } c_j^* = \max_i c_{ij}(\text{benefit criteria}) \tag{15.2.7}$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{a_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{c_{ij}}\right)\text{and } a_j^- = \min_i a_{ij}(\text{cost criteria}). \tag{15.2.8}$$

S_5 and C_1 are the only cost criteria. The rest is the benefit criteria.

Step 4. Construct the weighted normalized matrix $\tilde{V}$ as

$$\tilde{V} = \left[\tilde{v}_{ij}\right]_{m\times n}, i = 1, \ldots, m;\ j = 1, \ldots, n, \text{where } \tilde{v}_{ij} = \tilde{r}_{ij}(\cdot)gw_j. \tag{15.2.9}$$

Step 5. Determine the fuzzy PIS (FPIS) and fuzzy NIS (FNIS) as

$$A^* = \left(\tilde{v}_1^*, \ldots, \tilde{v}_n^*\right), \text{where } \tilde{v}_j^* = (1, 1, 1)\text{for benefit criteria, and } \tilde{v}_j^* = (0, 0, 0)\text{for cost criteria}, \text{j} = 1, \ldots, \text{n} \tag{15.2.10}$$

$$A^- = \left(\tilde{v}_1^-, \ldots, \tilde{v}_n^-\right), \text{where} \tilde{v}_j^- = (0, 0, 0)\text{for benefit criteria, and } \tilde{v}_j^-$$

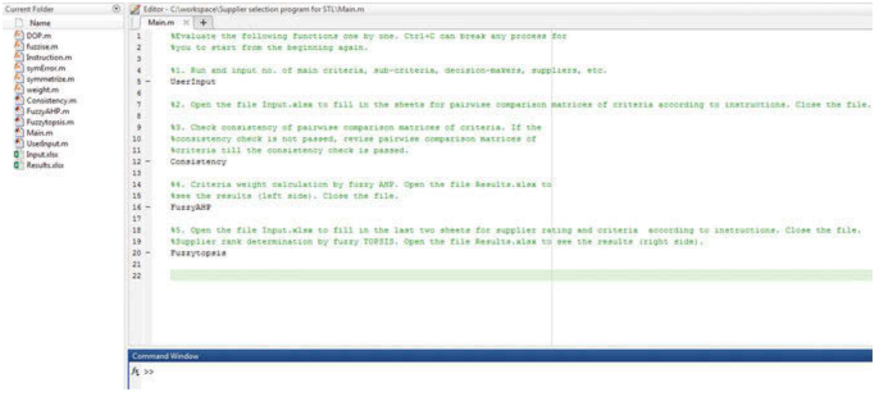


Fig. 15.4 Main interface of the program of supplier selection approach (Phases II and III)

$$=(1, 1, 1)\text{for cost criteria}, \text{j} = 1, \ldots, \text{n}. \tag{15.2.11}$$

Step 6. Find the distance of each potential supplier from FPIS and FNIS as

$$d_i^+ = \sum_{j=1}^{n} d_v\left(\tilde{v}_{ij}, \tilde{v}_j^*\right), i = 1, \ldots, m \tag{15.2.12}$$

$$d_i^- = \sum_{j=1}^{n} d_v\left(\tilde{v}_{ij}, \tilde{v}_j^-\right), i = 1, \ldots, m, \tag{15.2.13}$$

where d_v measures the distance between two fuzzy variables, i.e.

$$d_v(\tilde{x}, \tilde{y}) = \sqrt{\frac{1}{3}\left[\left(a_x - a_y\right)^2 + \left(b_x - b_y\right)^2 + \left(c_x - c_y\right)^2\right]}. \tag{15.2.14}$$

Step 7. Compute the closeness coefficient (CC_i) of each potential supplier as

$$CC_i = \frac{d_i^-}{d_i^- + d_i^+}. \tag{15.2.15}$$

Step 8. Rank the potential suppliers from best to worst, according to CC_i.

Table 15.11 shows how the potential suppliers are ranked from best to worst using CC_i. The preference degree of the suppliers is also found according to CC_i. The decision makers can then decide on the number of suppliers to choose based on the rankings. The top three suppliers (Sp_1, Sp_2 and Sp_5) are marked in red. To diversify the risk, the firm could use several vendors for one product instead of the prevailing one-vendor practice.

Figure 15.4 shows the MATLAB 17.0 program used to implement the supplier selection approach (Phases II and III).

We developed a program for the company to implement the supplier selection approach (Phases II and III) in MATLAB. Since the program is packaged as a black box, all the input operations are required and triggered in the command window of the 'Main.m' interface (see Fig. 15.4), by following the instructions in green carefully and evaluating the four functions 'UserInput', 'Consistency', 'FuzzyAHP' and 'Fuzzytopsis' successively. The first three functions pertain to Phase II, and the last function is for Phase III. The data provided by the decision makers (see Tables 15.3, 15.4, 15.5, 15.6, and 15.10) are inputs and can be found in 'Input.xlsx'. The results of the supplier selection, i.e. the contents of Tables 15.8 and 15.11 can be found in 'Results.xlsx'.

Phase IV. Order allocation by multi-objective optimization—A multi-objective program is used to determine the amount allocated to the selected suppliers Sp_1 and Sp_2. Table 15.12 shows the rest of the supplier data that are also the input data to the modelling system.

Our bi-objective program, with profit and service performance maximization, trade-off, the proportion (x: Sp_1's proportion, $1 - x$: Sp_2's proportion) from each supplier in every purchase. The profit maximization objective is the profit from holding the product. Here, only the relevant cost components are discussed. The service performance maximization objective is measured by the supplier preference degree as weights. Thus, the model is presented as

$$\max \text{PF} = (c_{S1} - c_{B1})x + (c_{S2} - c_{B2})(1 - x) \tag{15.2.16}$$

$$\max \text{SP} = w_1 x + w_2(1 - x) \tag{15.2.17}$$

$$\text{s.t.} \quad 0 < x < 1 \tag{15.2.18}$$

From Table 15.12, clearly, the two objectives conflict with each other. The results of the order proportions for suppliers Sp_1 and Sp_2 are found in Table 15.13. As the order allocation problem is formulated as profit and performance objectives with conflicting measures, only a sacrifice on one objective can bring the other closer to the optimal goal. Thus, satisficing solutions with different priorities (profit, service performance, and compromise solutions) are found for the decision makers to choose by combining practical situations. We provide three satisficing solutions for each of the three strategies.

In Table 15.13, among all the solutions, the solution with Sp_1 proportion 1% has the highest profit of 0.1098 and lowest performance level at 0.9582, while the solution with Sp_1 proportion 99% has the lowest profit of 0.0902 and highest performance level at 0.9996. It means that when the decision makers adopt a profit priority strategy, Sp_1 acts as the support of Sp_2. When the decision makers adopt a service performance priority strategy, Sp_2 acts as the support of Sp_1. Also, with more suppliers selected, the disruption risk is reduced especially when faced with major health emergencies,

Table 15.10 Linguistic ratings of suppliers by decision makers

	S_1	S_2	S_3	S_4	S_5	C_1	C_2	R_1	R_2	R_3	R_4
DM_1											
Sp_1	G	G	G	E	G	E	A	G	G	A	A
Sp_2	G	E	G	G	A	G	A	A	A	A	A
Sp_3	G	A	G	A	A	A	A	A	A	A	A
Sp_4	G	F	G	A	G	F	A	G	G	A	A
Sp_5	G	B	G	A	G	A	A	G	G	A	A
DM_2											
Sp_1	G	A	A	G	A	E	A	A	G	A	A
Sp_2	G	A	A	G	A	G	A	G	G	A	A
Sp_3	G	A	A	A	A	A	A	A	A	A	A
Sp_4	G	A	A	A	A	A	A	G	G	A	A
Sp_5	G	A	A	A	A	A	A	G	G	A	A
DM_3											
Sp_1	G	E	A	E	G	A	A	A	A	A	A
Sp_2	G	E	A	A	A	A	A	A	A	A	A
Sp_3	G	G	A	A	A	A	A	A	A	A	A
Sp_4	G	A	A	G	G	A	A	A	A	A	A
Sp_5	G	A	A	G	G	A	A	A	A	A	A
DM_4											
Sp_1	G	E	A	E	G	A	A	A	A	A	A
Sp_2	G	E	A	A	A	A	A	A	A	A	A

(continued)

Table 15.10 (continued)

	S_1	S_2	S_3	S_4	S_5	C_1	C_2	R_1	R_2	R_3	R_4
DM_1											
Sp_3	G	G	A	A	A	A	A	A	A	A	A
Sp_4	G	A	A	G	G	A	A	A	A	A	A
Sp_5	G	A	A	G	G	A	A	A	A	A	A

Table 15.11 Suppliers ranked by fuzzy TOPSIS

Alternative	CC_i	Rank	Preference degree
Sp_1	0.189756	1	1
Sp_2	0.189162	2	0.957790
Sp_3	0.185891	4	0.725322
Sp_4	0.175684	5	0
Sp_5	0.187844	3	0.864158

Table 15.12 Data related to suppliers

Supplier	Preference degree w_i	Product	Buying cost c_{Bi} ($)	Selling cost c_{Si} ($)	Profit $c_{Si} - c_{Bi}$($)
Sp_1	1	P6	0.88	0.97	0.09
		P7			
		P8			
		P9			
Sp_2	0.9578	P10	1.06	1.17	0.11
		P11			
		P12			
		P13			

Table 15.13 Order proportions for suppliers Sp_1 and Sp_2

Strategy	Order proportion for Sp_1 (%)	Order proportion for Sp_2 (%)	PF	SP
Profit priority solutions	1	99	0.1098	0.9582
	3	97	0.1094	0.9591
	4	96	0.1092	0.9595
Service performance priority solutions	99	1	0.0902	0.9996
	98	2	0.0904	0.9992
	96	4	0.0908	0.9983
Compromise solutions	50	50	0.1	0.9789
	47	53	0.1006	0.9776
	46	54	0.1008	0.9772

and reduces the potential negative impacts from the supply failures or poor decision-making in selecting unknown or untested suppliers.

15.3 Sensitivity Analysis and Managerial Insights

A sensitivity analysis is undertaken on the order allocation, so as to offer a generalized approach for handling the order allocation problem. Following Mendoza and Ventura [16], we propose a fuzzy multi-objective program for the order allocation problem that (1) determines the number of orders given to the chosen vendors (J_i: number of orders to supplier i in one order cycle. An order cycle contains orders from different suppliers, and the next order cycle only starts after the former one finishes); and (2) determine the size of each order (Q_i: quantity placed with supplier i in an order). For the purpose of contrast, we form two groups containing two suppliers (Sp_2 and Sp_1) and three suppliers (Sp_5, Sp_2 and Sp_1), respectively, for illustrating the purchase of a product called NH (disguised for data privacy) (25G * 25 mm).

Next, we assume the following. The firm can order from a supplier many times within an order cycle. Once all the orders in an order cycle are firm, the cycle is refreshed. The inventory holding cost rate is $r = 0.2$/unit/year. A triangular fuzzy annual customer demand of the NH product is $\tilde{d} \sim \mathcal{T}(200000, 291200, 700000)$, the expected annual demand with $E\left[\tilde{d}\right] = 1/4 * (200000 + 2 * 291200 + 700000)$ and the annual purchase amount from the suppliers is 370,000 ($vss_i = [(370000 - 291200)/291200]Q_i$). The firm sets the safety stock level at 90% ($ss_i = 0.9Q_i$), and the amount in each order of the firm is at least 1,200. The order quantities are capped at $M = 3, 5, 10, 15, 20, 25, 30$. Table 15.14 shows the rest of the supplier data that are also an input for the modelling system.

We build a fuzzy multi-objective programming model which has an annual cost minimization objective and a service performance maximization objective, constrained by supplier capacity and certain order quantities per order cycle. The first cost minimization objective contains the holding and purchase costs. The service performance maximization objective is measured by the supplier preference degree as weights. Thus, the model is written as

Table 15.14 Data related to suppliers

Supplier	Preference degree w_i	Unit price p_i ($)	Annual capacity (units)
Sp_1	1	62	640,000
Sp_2	0.7659	51	680,000
Sp_4	0.5861	40	650,000

$$\min\ \mathrm{AC} = r\tilde{d} \cdot \frac{\sum_{i=1}^{n} p_i J_i (0.5Q_i + ss_i)}{\sum_{i=1}^{n} J_i Q_i} + \tilde{d} \cdot \frac{\sum_{i=1}^{n} p_i J_i (Q_i + vss_i)}{\sum_{i=1}^{n} J_i Q_i} \tag{15.3.1}$$

$$\max\ \mathrm{SP} = \frac{\sum_{i=1}^{n} w_i J_i Q_i}{\sum_{i=1}^{n} J_i Q_i} \tag{15.3.2}$$

$$\text{s.t.} \quad \tilde{d} \cdot \frac{J_j Q_j}{\sum_{i=1}^{n} J_i Q_i} \le c_j,\ j = 1, 2, \ldots, n \tag{15.3.3}$$

$$\sum_{i=1}^{n} J_i = M, \tag{15.3.4}$$

$$Q_i \ge 1200, \quad i = 1, 2, \ldots, n \tag{15.3.5}$$

$$J_i, Q_i, M \in Z^{+}. \tag{15.3.6}$$

In Models (15.3.1)–(15.3.6), the first objective function and the first set of constraints are fuzzy quantities. In this model, the expected values of the fuzzy objectives are used to obtain the decisions with optimal expected returns, as well as to provide confidence levels α_j within which the fuzzy constraints hold.

Therefore, the model can be rewritten as a fuzzy chance-constrained programming by replacing the first objective function and the first set of constraints as

$$\min\ E[\mathrm{AC}] = \left(r \cdot \frac{\sum_{i=1}^{n} p_i J_i (0.5Q_i + ss_i)}{\sum_{i=1}^{n} J_i Q_i} + \frac{\sum_{i=1}^{n} p_i J_i (Q_i + vss_i)}{\sum_{i=1}^{n} J_i Q_i} \right) \cdot E\left[\tilde{d}\right] \tag{15.3.7}$$

$$\max\ \mathrm{SP} = \frac{\sum_{i=1}^{n} w_i J_i Q_i}{\sum_{i=1}^{n} J_i Q_i} \tag{15.3.8}$$

$$\text{s.t.} \quad \mathrm{Cr}\left\{ J_i Q_i - c_j \sum_{i=1}^{n} J_i Q_i \le 0 \right\} \ge \alpha_j, \quad j = 1, 2, \ldots, n, \tag{15.3.9}$$

$$\sum_{i=1}^{n} J_i = M, \tag{15.3.10}$$

$$Q_i \ge 1200, \quad i = 1, 2, \ldots, n, \tag{15.3.11}$$

$$J_i, Q_i, M \in Z^{+}. \tag{15.3.12}$$

Models (15.3.7)–(15.3.12) can be converted to the equivalent deterministic programming by transforming the chance constraint (15.3.9) as

$$\Phi^{-1}(\alpha_j) J_j Q_j - c_j \sum_{i=1}^{n} J_i Q_i \le 0, \quad j = 1, 2, \ldots, n, \tag{15.3.13}$$

where Φ^{-1} is the inverse credibility distribution of $\widetilde{d}$. A triangular fuzzy number $\xi \sim \mathcal{T}(a_1, a_2, a_3)$ would have its $\Phi^{-1}(\alpha)$ as

$$\Phi^{-1}(\alpha) = \begin{cases} (2a_2 - 2a_1)\alpha + a_1, & \text{if}\, \alpha < 0.5 \\ (2a_3 - 2a_2)\alpha + 2a_2 - a_3, & \text{if}\, \alpha \geq 0.5. \end{cases} \tag{15.3.14}$$

At the confidence level $\alpha_j = 0.85$, (15.3.13) can be rewritten as

$$\left[(2a_3 - 2a_2)\alpha_j + 2a_2 - a_3\right] J_j Q_j - c_j \sum_{i=1}^{n} J_i Q_i \leq 0, \quad j = 1, 2, \ldots, n. \tag{15.3.15}$$

We apply the non-dominated sorting genetic algorithm - II (NSGA-II) developed by Deb et al. [8] to solve the deterministic Models (15.3.7)–(15.3.8), (15.3.15), (15.3.10)–(15.3.12). NSGA-II offers a more superior solution set and yields better convergence near the Pareto frontier compared to the other multi-objective evolutionary algorithms used in most problems.

We then obtain Pareto-optimal solutions from the algorithm, and the example results of the annual procurement plans and specific purchase strategies for (a) 2-supplier case and (b) 3-supplier case are displayed in Tables 15.15, 15.16, and Tables 15.17, 15.18, respectively. The annual procurement plan, Tables 15.15 and 15.17, give an overview of the total annual amount ordered from the suppliers with corresponding cost and performance values, while the specific purchase strategy, Tables 15.16 and 15.18, are the detailed ways for the firm to place orders in each order cycle. Since our order allocation problem is formulated as cost and performance objectives with conflicting measures, only by sacrificing one objective can we bring the other closer to the optimal goal. Thus, satisficing solutions with different priorities are acquired for the decision maker to choose, by combining the practical options. Tables 15.15, 15.16, 15.17, and 15.18 provide a suite of decision choices, for example, solutions in the cases that have a different number of suppliers selected (2 and 3), different order quantity limits (3, 5, 10, 15, 20, 25, 30), as well as different priorities (cost priority, performance priority, and compromise solutions) to the two objectives.

1. Light blue lines refer to Pareto-optimal solutions on performance priority.
2. Dark blue lines refer to compromise solutions among Pareto-optimal solutions.
3. For the annual procurement plan with a cost priority solution of 30 orders/order cycle, it can only complete 0.035 order cycle in a year, so it will not involve orders from Sp_1 in the annual plan according to the purchase strategy of the same item in Table 15.16.
4. If the firm signs 1-year contracts with the suppliers, Table 15.15 shows that it is better not to deploy 30 orders in 1 order cycle because it takes too long to complete an order cycle and some suppliers cannot be assigned to any order in that year.

Table 15.15 Annual procurement plans for two suppliers Sp_2 and Sp_1

Total #orders per order cycle	Annual quantity from Sp_2	Annual quantity from Sp_1	AC ($)	SP (%)	Time per order cycle	#Annual order cycles
	367,900	2,700	60,030,000	76.78	47.2 weeks	1.1
3	12,600	358,000	72,455,000	99.28	6.4 weeks	8.1
	178,200	192,400	66,647,000	88.76	4.9 days	74.1
	361,000	9,600	60,276,000	77.22	12.5 weeks	4.2
5	6,400	364,200	72,667,000	99.67	31.5 weeks	1.7
	183,600	187,000	66,603,000	88.68	5.9 weeks	8.8
	366,700	3,900	60,165,000	77.02	29.6 weeks	1.8
10	3,400	367,200	72,788,000	99.89	48.4 weeks	1.1
	199,800	170,800	66,320,000	88.17	9.2 weeks	5.6
	367,800	2,800	60,077,000	76.86	33.6 weeks	1.5
15	6,000	364,600	72,663,000	99.66	11.6 weeks	4.5
	194,600	176,000	66,281,000	88.10	10 weeks	5.2
	365,800	4,800	60,171,000	77.03	17.8 weeks	2.9
20	2,800	367,800	72,801,000	99.91	49.8 weeks	1
	188,000	182,600	66,653,000	88.77	6.9 weeks	7.6
	368,200	2,400	60,014,000	76.75	49.7 weeks	1
25	3,400	367,200	72,787,000	99.88	48 weeks	1.1
	234,100	136,500	65,397,000	86.50	15.1 weeks	3.4
	370,600	0	59,971,000	76.67	28 years	0.035
30	1,200	369,400	72,850,000	99.997	29 years	0.034
	354,900	15,700	66,102,000	87.78	1.8 years	0.55

Note 1 White lines refer to Pareto-optimal solutions on cost priority

Table 15.16 Order allocation schedules for two suppliers—Sp_2 and Sp_1

Total #orders per order cycle	J_1 (Sp_2)	J_2 (Sp_1)	Q_1 (Sp_2)	Q_2 (Sp_1)
	2	1	167,000	2,700
3	1	2	1,400	22,100
	1	2	2,400	1,300
	3	2	28,800	1,200
5	1	4	3,200	55,300
	3	2	6,800	10,900
	9	1	23,000	3,900
10	1	9	1,700	38,100
	9	1	3,700	32,600
	14	1	16,900	2,800
15	1	14	1,200	5,800
	7	8	5,200	4,400
	18	2	6,900	1,200
20	1	19	1,400	18,600
	5	15	4,700	1,700
	23	2	15,300	1,200
25	1	24	1,700	14,200
	20	5	3,100	9,100
	29	1	359,600	36,200
30	1	29	1,200	370,600
	13	17	27,300	19,100

Managerial Insights: From Table 15.15, among the cost priority solutions (white lines), the solution with order no. 30 has the lowest cost of $59,971,000 and lowest performance level at 76.67%, while the solution with order no. 5 has the highest cost of $60,276,000 and best performance level at 77.22%. Further, for the cost priority solutions, the cost is proportional to the order quantity from Sp_2 who has the lower unit price at $51 (see Table 15.14).

Similarly, among the performance priority solutions (light blue lines), the solution with order no. 30 has the highest performance level of 99.997% and the highest cost at $72,850,000, while the solution with order no. 3 has the lowest performance level of 99.28% and the lowest cost at $72,455,000. Also, for the performance priority solutions, the performance level corresponds to the order quantity from Sp_1 who has a higher preference degree (Table 15.17).

In addition, comparing the 2-supplier case (Table 15.15) with the 3-supplier case (Table 15.17), better results on both the cost and performance criteria can be obtained from the 3-supplier case. This suggests that if more suppliers were selected by the firm for the order allocation, the supply side risk can be better diversified.

Table 15.17 Annual procurement plans for suppliers Sp_5, Sp_2, Sp_1

Total #orders per order cycle	Annual quantity from Sp_4	Annual quantity from Sp_2	Annual quantity from Sp_1	AC ($)	SP (%)	Time for one order cycle	#Annual order cycles
3	363,100	3,900	3,600	47,475,000	59.34	14.1 weeks	3.7
	7,800	2,800	360,000	72,516,000	99.45	49.7 weeks	1
	194,200	6,000	170,400	60,016,000	79.40	12.1 weeks	4.3
5	363,300	3,600	3,700	47,395,000	59.21	50.6 weeks	1
	4,000	2,400	364,200	72,661,000	99.69	49.4 weeks	1.1
	187,200	10,800	172,600	60,046,000	79.42	6.1 weeks	8.6
10	368,200	1,200	1,200	47,127,000	58.80	1 year	1
	2,400	2,400	365,800	72,723,000	99.79	50.7 weeks	1
	162,800	15,400	192,400	60,962,000	80.85	4.7 weeks	11
15	370,600	0	0	47,059,000	58.70	12.7 years	0.079
	3,800	2,700	364,100	72,823,000	99.95	12.7 years	0.079
	187,300	6,300	177,000	59,991,000	79.36	16.8 weeks	3.1
20	370,600	0	0	47,013,000	58.63	17.2 years	0.058
	6,600	9,200	354,800	72,802,000	99.92	15.9 years	0.063
	259,200	1,300	110,100	60,874,000	80.82	1.5 years	0.66
25	370,600	0	0	47,010,000	58.62	21.8 years	0.046
	10,600	25,300	334,700	72,781,000	99.88	23.1 years	0.043
	200,200	2,800	167,600	58,857,000	77.57	26.1 weeks	2
30	370,600	0	0	47,005,000	58.62	28 years	0.036
	2,400	1,200	367,000	72,844,000	99.99	27 years	0.037
	208,000	2,400	160,200	59,926,000	79.29	29.4 weeks	1.8

Table 15.18 Order allocation schedules for three suppliers Sp_5, Sp_2, Sp_1

Total #orders per order cycle	J_1 **(Sp_5)**	J_2 **(Sp_2)**	J_3 **(Sp_1)**	Q_1 **(Sp_5)**	Q_2 **(Sp_2)**	Q_3 **(Sp_1)**
	1	1	1	98,300	1,300	1,200
3	1	1	1	3,900	1,400	349,000
	1	1	1	42,000	1,500	42,600
	1	3	1	353,300	1,200	3,700
5	1	1	3	2,000	1,200	116,300
	2	1	2	10,400	1,200	10,600
	8	1	1	46,000	1,200	1,200
10	1	1	8	1,200	1,200	44,900
	2	1	7	7,400	1,400	2,500
	13	1	1	361,800	1,200	10,000
15	1	1	13	3,800	2,700	361,900
	4	1	10	14,600	2,100	5,900
	18	1	1	354,200	1,200	2,400
20	1	1	18	6,600	9,200	326,100
	12	1	7	21,600	1,300	42,900
	23	1	1	350,700	1,200	2,100
25	1	1	23	10,600	25,300	370,600
	13	1	11	7,700	1,400	7,700
	28	1	1	370,600	1,200	1,200
30	2	1	27	1,200	1,200	370,600
	16	1	13	6,500	1,200	8,000

15.4 Conclusion

This study offers some practical and implementable improvements to the current procurement methodologies by establishing a criteria set for suppliers and product selection through MCDM and multi-objective modelling. To model the supplier's ability to serve in terms of the response time, reliability, stock quantity, the necessary criteria are translated into seeking to maximize space usage, lower inventory carrying cost and increase end customer satisfaction. An operational framework has been developed to process and manage order quantities under the conditions of local sourcing, supplier selection, order allocation, contract storage, nearshore re-distribution, assured finished goods shipment and emergency manufacturing/ordering. An extant study has been undertaken and the sensitivity is analysed. Some managerial insights are provided for completeness. This study finds that if more suppliers are selected for order allocation, there could be better attainment on the cost and performance goals, as doing so mitigates the risk present in public health crisis situations, and lessens the impact of a supply disruption, or poor decision-making when choosing untested suppliers.

There are several future research directions that may be pursued after this work. Immediate extensions are incorporating some more important issues like allowing for shortages, supply availability for emergency management, and a flexible budget capacity. Another future direction could be to apply the proposed algorithm to large scale problems such as having a number of suppliers and products dispersed geographically, and the resulting problem can be integrated with the vehicle routing problem (VRP) or the inventory routing problem (IRP) with several other objectives for better trade-off performances.

References

1. Alizadeh HM, Handfield R (2019) Sustainable supplier selection and order allocation: A novel multi-objective programming model with a hybrid solution approach. Comput Ind Eng. https://doi.org/10.1016/j.cie.2019.01.011
2. Bhutta KS, Huq F (2002) Supplier selection problem: a comparison of total cost of ownership and analytic hierarchy process approaches. Supply Chain Manag Int J 7(3):126–135
3. Cárdenas-Barrón LE, González-Velarde JL, Treviño-Garza G (2015) A new approach to solve the multi-product multi-period inventory lot sizing with supplier selection problem. Comput Oper Res 64:225–232
4. Chai J, Liu JN, Ngai EW (2013) Application of decision-making techniques in supplier selection: a systematic review of literature. Expert Syst Appl 40(10):3872–3885
5. Chang DY (1996) Applications of the extent analysis method on fuzzy AHP. Eur J Oper Res 95(3):649–655
6. Choudhary D, Shankar R (2013) Joint decision of procurement lot-size, supplier selection, and carrier selection. J Purch Supply Manag 19(1):16–26
7. Choudhary D, Shankar R (2014) A goal programming model for joint decision making of inventory lot-size, supplier selection and carrier selection. Comput Ind Eng 71:1–9
8. Deb K, Pratap A, Agarwal S, Meyarivan T (2002) A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans Evol Comput 6(2):182–197
9. Duan L, Ventura JA (2019) A dynamic supplier selection and inventory management model for a serial supply chain with a novel supplier price break scheme and flexible time periods. Eur J Oper Res 272:979–998
10. Govindan K, Rajendran S, Sarkis J, Murugesan P (2015) Multi criteria decision making approaches for green supplier evaluation and selection: A literature review. J Clean Prod 98:66–83
11. Hamdan S, Cheaitou A (2017) Supplier selection and order allocation with green criteria: An MCDM and multi-objective optimization approach. Comput Oper Res 81:282–304
12. Hwang CL, Yoon K (1981) Multiple attributes decision making methods and applications. Springer, Berlin
13. Junior FRL, Osiro L, Carpinetti LCR (2014) A comparison between Fuzzy AHP and Fuzzy TOPSIS methods to supplier selection. Appl Soft Comput 21:194–209
14. Kannan D, Khodaverdi R, Olfat L, Jafarian A, Diabat A (2013) Integrated fuzzy multi criteria decision making method and multi-objective programming approach for supplier selection and order allocation in a green supply chain. J Clean Prod 47:355–367
15. Mazdeh MM, Emadikhiav M, Parsa I (2015) A heuristic to solve the dynamic lot sizing problem with supplier and quantity discounts. Comput Ind Eng 85:33–43

16. Mendoza A, Ventura JA (2012) Analytical models for supplier selection and order quantity allocation. Appl Math Model 36(8):3826–3835
17. Parsa I, Khiav M, Mazdeh M, Mehrani S (2013) A multi supplier lot sizing strategy using dynamic programming. Int J Ind Eng Comput 4(1):61–70
18. Patil SK, Kant R (2014) A fuzzy AHP-TOPSIS framework for ranking the solutions of knowledge management adoption in supply chain to overcome its barriers. Expert Syst Appl 41(2):679–693
19. Purohit AK, Choudhary D, Shankar R (2016) Inventory lot-sizing with supplier selection under non-stationary stochastic demand. Int J Prod Res 54(8):2459–2469
20. Ravindran A, Warsing DP (2013) Supply chain engineering: models and applications. CRC Press, New York
21. Saaty TL (1980) Analytic hierarchy process. McGraw Hill, New York
22. Sarkis J, Talluri S (2002) A model for strategic supplier selection. J Supply Chain Manag 38(1):18–28
23. Williamson OE (2008) Outsourcing: Transaction cost economics and supply chain management. J Supply Chain Manag 44(2):5–16
24. Yazdani M, Chatterjee P, Zavadskas EK, Zolfani SH (2017) Integrated QFD-MCDM framework for green supplier selection. J Clean Prod 142:3728–3740

Chapter 16
Supply Chain Network Optimization Through Player Selection Using Multi-objective Genetic Algorithm

Poonam Mishra, Isha Talati and Azharuddin Shaikh

Abstract This paper is an effort to study an integrated supply chain network comprising of suppliers, manufacturers, distributors, and retailers with the key objective of minimizing the overall cost of supply chain. Players of supply chain network are selected on the basis of multi criteria. Multi-objective GA has been used to select business players under constraints. Further, the output of GA is visualized through 3D-radVis techniques with respect to location, shape, range, and distribution of non-dominated Pareto front. The paper also proposes an algorithm to analyze other integrated supply chain problems belong to this class. The model is also validated through a numerical example. This model is useful to manufacturers and distributors who involved with the industries like automobile, textile, food and electronic gadgets, etc. for the sustainable supply chain management.

Keywords Supply partner selection · Multi-echelon integrated model · Multi-objective GA · 3D-RadVis visualization technique

16.1 Introduction

Due to globalization and bottleneck competition, organizations are increasingly focusing on increasing efficiency by identifying their core competencies. At the same time, organizations are required to choose supply chain partners strategically to achieve business goals. Organizations can choose different criteria for different levels of supply chain partners as per the requirement. Many of the times, these criteria are conflicting in nature that gives rise to a multi-objective problem with a set of constraints. Many traditional methods are available for optimization but they either unable to give or stuck with local minimum, while genetic algorithm works well in

P. Mishra (✉) · I. Talati
Department of Mathematics & Computer Science, School of Technology, Pandit Deendayal Petroleum University, Raisan, Gandhinagar 382007, India
e-mail: poonam.mishra@sot.pdpu.ac.in

A. Shaikh
Institute of Infrastructure Technology Research and Management, Ahmedabad 380026, Gujarat, India

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*, Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_16

this type of scenario too. After optimizing multi-objective problems, we get non-dominated solutions, which are difficult to visualize. The three-dimensional radial coordinate visualization (3D-RadVis) is capable of mapping M-dimensional objective space to three-dimensional objective space radial coordinate plot. It preserves the relative location of solutions, shape of the Pareto front, distribution of solutions, trade-off of Pareto-optimal front, and convergence trend of an optimization process.

In this article, we consider an integrated supply chain network. The network consists of suppliers, manufacturer with finite production plant, distributors, and retailers. Selection of players is done on the basis of multi criteria. We present an approach for selecting business players under constraint using multi-objective GA. We have used 3D-RadVis technique for visualization of location, shape, range, and distribution of non-dominated Pareto front. To check the effectiveness of model, numerical example is carried out. The selection has done at each stage of the supply network. We have optimized the total cost of the entire supply chain by choosing appropriate supply players.

The flow of the article is as follows, Sect. 16.2 gives literature review, Sect. 16.3 gives problem description of this supply chain model. Section 16.4 comprise of notations and assumptions. Section 16.5 demonstrates mathematical model of the problem. Section 16.6 proposes algorithm to solve the problem. Validity of algorithm and observations are shown in Sect. 16.7. Conclusion is mentioned in Sect. 16.8.

16.2 Literature Review

There is a good number of papers in literatures that have worked on supplier and vendor selection. All previous work on supplier selection can be classified into two categories (i) qualitative models and (ii) quantitative models. In qualitative models, supplier is selected on the basis of multiple criterion as price, delivery time, quality etc. Multi-objective optimization can be used to optimize these problems as they are multi-objective with conflicting goals. First, Weber and Current [18] applied a multi-objective approach for selection of supplier. Other researchers also solved multi-objective problems by different techniques like data envelopment analysis[19], multi-objective Genetic Algorithm [2], fuzzy set theory [1], fuzzy AHP and fuzzy multi-objective linear programming [14], meta heuristic algorithm [13], and NSGA-II [3].

Genetic algorithm (GA) works on Darwin's theory of "Survival of fittest". Traditional methods is normally stuck with local optimum in the complex optimization problem. For such problems, heuristic algorithm like genetic algorithm, ant colony, etc. gives global optimum. First, Schaffer [12] developeda software system VEGA for vector evaluated GA. Srinivas and Dab [15] presented a new algorithm non-dominated sorting genetic algorithm (NSGA), which eliminates the limitation of VEGA. Multi-objective GA was applied by Murata et al. [8] for flow shop scheduling problem and Parks and Miller [10] for selection of breeding. Srinivas et al. [16] analyzed sensitivity to determine parameters of GA.

In multi-objective optimization, visualization of true Pareto front or obtained non-dominated solution is very difficult. Obayashi and Sasaki [9] used self-organizing map to visualize trade-off of Pareto solution. But in self-organizing map method, the parameter space itself is not visualized and this limitation was addressed by Pryke et al. [11] applying Heat map method. To produce range, shape, and distribution in 4D, prospection method was used by Tusar and Filipic [17]. In four or more dimensions, different techniques such as 3D-RadVis, Sammon mapping, Parallel Coordinate plot, and 3D-RadVis Antenna were used by Ibrahim et al. [5], He and Yen [4], Li et al. [7], Ibrahim et al. [6], respectively.

This paper is a sincere effort to choose the best possible supply chain players on the basis of well-defined criteria, so that efficient and sustainable supply chain can be achieved for the items that need a long network-based supply chain. The total cost of supply chain is minimized using multi-objective GA. Optimization at each stage is visualized by Pareto-optimal front representation.

16.3 Problem Description

Due to globalization and competitive market optimizing resources in order to minimize the total cost of supply chain is very crucial. The selection of supply players objectively at different stages plays a vital role in total cost minimization. Thus, in the proposed model, the entire supply chain cost is minimized by choosing appropriate supply players. The problem description of the present model is given below in Fig. 16.1.

Manufacturer selects the best supplier on the base of price, transportation cost, quality, delivery tim,e and supplier supply capacity for each item. The manufacturer has finite(n) plants. He selects plant on the base of production cost, transportation cost, quality, production time, and production capacity of each item. Evaluation of best distributor is being done on the basis of purchase cost, transportation cost, distribution coverage area, delivery time, and storage capacity. While purchase cost, transportation cost, customer selection, delivery tim, and storage capacity are taken into consideration to select best retailer.

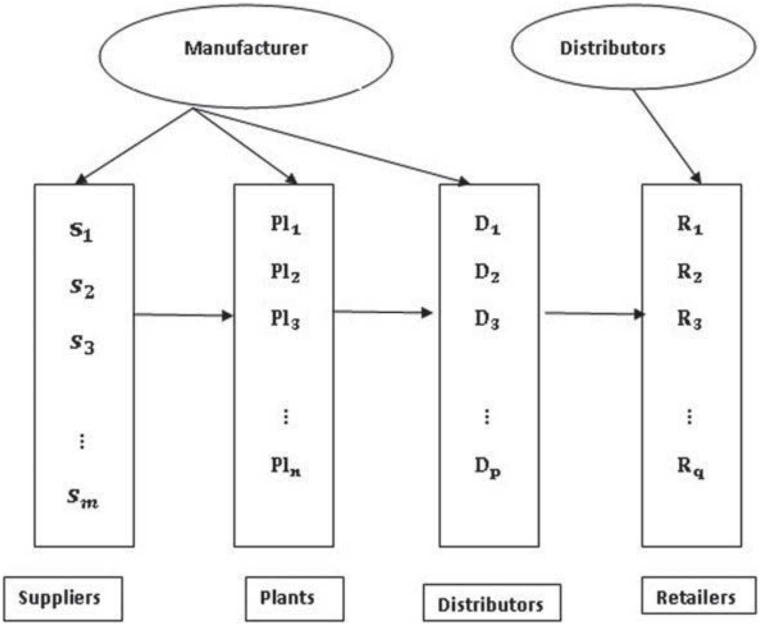


Fig. 16.1 Present model

16.4 Notations and Assumptions

16.4.1 Notations

$i = 1, 2, \ldots, t$	Index of items
$j = 1, 2, \ldots, m$	Index of candidate suppliers
$k = 1, 2, \ldots, n$	Index of plants
$\lambda = 1, 2, \ldots, p$	Index of candidate distributors
$\nu = 1, 2, \ldots, q$	Index of candidate retailers
D_i	Demand of item i
P_{ij}	Price from supplier j to manufacturer to supply item i/unit ($)
P_{ik}	Processing cost of plant k for item i/unit ($)
$P_{i\lambda}$	Purchase cost of distributor λ from manufacturer to receive item i/unit ($)
$P_{i\nu}$	Purchase cost of retailer ν from distributor to receive item i/unit ($)
T_{ijk}	Transportation cost form supplier j to plant k for item i/unit ($)
$T_{ik\lambda}$	Transportation cost form plant k to distributor λ for item i/unit ($)
$T_{i\lambda\nu}$	Transportation cost form distributor λ to retailer ν for item i/unit ($)
C_{ij}	Supply capacity of supplier j to supply item i

(continued)

(continued)

$i = 1, 2, \ldots, t$	Index of items
C_{ik}	Production capacity of plant k to produce item i
$C_{i\lambda}$	Storage capacity of distributor λ to store item i
C_{iv}	Storage capacity of retailer v to store item i
q_{ij}	Defective quality of supplier j when supplying item i
q_{ik}	Defective quality of plant k when supplying item i
$q_{i\lambda}$	Outside distribution area of distributor λ when distribute item i
u_{iv}	Unsatisfied customer from retailer v for item i
Q_{ij}	Acceptable defective quality from supplier j for item i
Q_{ik}	Acceptable defective quality from plant k for item i
$A_{i\lambda}$	Acceptable outside distribution area form distributor λ for item i
U_{iv}	Acceptable Unsatisfied customer from retailer v for item i
l_{ij}	Late delivery of supplier j when supply item i
l_{ik}	Late delivery of plant k when supply item i
$l_{i\lambda}$	Late delivery of distributor λ when supply item i
l_{iv}	Late delivery of retailer v when supply item i
L_{ij}	Acceptable late delivery form supplier j for item i
L_{ik}	Acceptable late delivery form plant k for item i
$L_{i\lambda}$	Acceptable late delivery form distributor λ for item i
L_{iv}	Acceptable late delivery form retailer _ for item i
TRC_{ijk}	Total transportation cost form supplier j to plant k for item i/unit ($)
$TRC_{ik\lambda}$	Total transportation cost form plant k to distributor λ for item i/unit ($)
$TRC_{i\lambda v}$	Total transportation cost form distributor λ to retailer v for item i/unit ($)
TC	Total cost for item i
PUC	Total purchasing cost for item i
PRC	Total processing cost for item i
TRC	Total transportation cost
MIC	Total manufacturer inventory carrying cost
DIC	Total distributor inventory carrying cost
RIC	Total retailer inventory carrying cost
INC	Total inventory carrying cost for system

16.4.2 Assumptions

- Demand of customer is deterministic.
- Supplier's supply capacity of each item is limited.
- Supplier selection is done on the base of quality and delivery performance.
- Manufacturer contains finite number of plants.

- Plants production capacity of each item is limited.
- Plant's selection is done on the base of quality and production time.
- Distributor selection is done on the base of distributor coverage area and delivery performance.
- Distributor's storage capacity of each item is limited.
- Retailer's storage capacity of each item is limited.
- Retailer selection is done on the base of customer satisfaction and delivery performance.
- Inventory carrying cost for any player of supply chain remains fixed.

16.5 Multi-echelon Inventory Model

The total cost for entire supply chain is derived as below. We want to minimize by selecting the best business players at different stages

$$\text{TC} = \text{PUC} + \text{PRC} + \text{TRC} + \text{INC} \tag{16.1}$$

The basic costs involved as below

Purchasing cost:

Purchasing cost is defined as follows:

$$PUC = \sum_i \sum_j x_{ij} P_{ij} \tag{16.2}$$

where x_{ij} = order quantity of ith item from jth supplier

Processing cost:

Total processing cost for different plants is defined as follows:

$$PRC = \sum_i \sum_k y_{ik} P_{ik} \tag{16.3}$$

where y_{ik} = Order quantity of ith item produce by kth plant

Transportation cost:

Transportation from distributor k to retailer is given below

$$TRC = \sum\nolimits_i \left(TRC_{ijk} + TRC_{ik\lambda} + TRC_{i\lambda\nu} \right) \tag{16.4}$$

where $TRC_{ijk} = \sum_k y_{ik} T_{ijk}$ and y_{ik} = Order quantity of ith item produce by kth plant

$TRC_{ik\lambda} = \sum\nolimits_\lambda z_{i\lambda} T_{ik\lambda}$ and $z_{i\lambda}$

= order quantity of ith item supply form manufacturer to distributor λ

$TRC_{i\lambda\nu} = \sum_\nu w_{i\nu} T_{i\lambda\nu}$ and $w_{i\nu}$ = order quantity of ith item from distributor to retailer k

Inventory carrying cost:
Here, we take fix carrying cost per item for any player of supply chain

$$INC = yMIC + zDIC + wRIC \tag{16.5}$$

where $y_i = \sum_i y_{ik}\ z_i = \sum_i z_{i\lambda} w_i = \sum_i w_{iv}$

The constraints are involved in present model are the following:

$$\sum_j x_{ij} \geq D_i \tag{16.6}$$

All the items customer demand must be fulfilled by the supplier.

Quality supply by the supplier to manufacturer is less than or equal to supply capacity of supplier.

$$x_{ij} \leq C_{ij} \tag{16.7}$$

Aggregate quality supply by the supplier to manufacturer must be acceptable

$$\sum_j x_{ij} q_{ij} \leq Q_i D_i \tag{16.8}$$

Aggregate delivery time taken by supplier to manufacturer must be acceptable

$$\sum_j x_{ij} l_{ij} \leq L_{ij} D_i \tag{16.9}$$

All the items customer demand must be fulfilled by the manufacturer.

$$\sum_k y_{ik} \geq D_i \tag{16.10}$$

Product produced by each plant is less than or equal to the production capacity of plants

$$y_{ik} \leq C_{ik} \tag{16.11}$$

Aggregate quality produce plant must be acceptable

$$\sum_k y_{ik} q_{ik} \leq Q_{ik} D_i \tag{16.12}$$

Aggregate delivery time taken by the plant to distributor must be acceptable

$$\sum_{k} y_{ik} l_{ik} \leq L_{ik} D_i \tag{16.13}$$

All the items customer demand must be fulfilled by the distributor.

$$\sum_{\lambda} z_{i\lambda} \geq D_i \tag{16.14}$$

Quality supply by the manufacturer to distributor is less than or equal to the storage capacity of distributor.

$$z_{i\lambda} \leq C_{i\lambda} \tag{16.15}$$

Aggregate distribution area covered by the distributor must be acceptable

$$\sum_{\lambda} z_{i\lambda} q_{i\lambda} \leq A_{i\lambda} D_i \tag{16.16}$$

Aggregate delivery time taken distributor to retailer must be acceptable

$$\sum_{\lambda} z_{i\lambda} l_{i\lambda} \leq L_{i\lambda} D_i \tag{16.17}$$

All the items customer demand must be fulfilled by retailer.

$$\sum_{\nu} w_{i\nu} \geq D_i \tag{16.18}$$

Quality supply by distributor to retailer is less than or equal to storage capacity of retailer.

$$w_{i\nu} \leq C_{i\nu} \tag{16.19}$$

Aggregate customer satisfaction must be acceptable

$$\sum_{\nu} w_{i\nu} u_{i\nu} \leq U_{i\nu} D_i \tag{16.20}$$

Aggregate delivery time taken by the retailer to customer must be acceptable

$$\sum_{\nu} w_{i\nu} l_{i\nu} \leq L_{i\nu} D_i \tag{16.21}$$

So, for the best supplier selection, we have the following objective function and constraints:

$$\min \mathrm{f}_1 = \sum_i \sum_j x_{ij} P_{ij}$$
$$\text{subject to} \sum_j x_{ij} \geq D_i;\ x_{ij} \leq C_{ij};\ \sum_j x_{ij} q_{ij} \leq Q_i D_i;\ \sum_j x_{ij} l_{ij} \leq L_{ij} D_i \tag{16.22}$$

For the best plant selection, we have the following objective function and constraints:

$$\min \mathrm{f}_2 = \sum_i \sum_k y_{ik} P_{ik}$$
$$\text{subject to} \sum_k y_{ik} \geq D_i;\ y_{ik} \leq C_{ik};\ \sum_k y_{ik} q_{ik} \leq Q_{ik} D_i;\ \sum_k y_{ik} l_{ik} \leq L_{ik} D_i \tag{16.23}$$

For the best distributor selection, we have the following objective function and constraints:

$$\min \mathrm{f}_3 = \sum_i \sum_\lambda z_{i\lambda} P_{i\lambda}$$
$$\text{subject to} \sum_\lambda z_{i\lambda} \geq D_i;\ z_{i\lambda} \leq C_{i\lambda};\ \sum_\lambda z_{i\lambda} q_{i\lambda} \leq A_{i\lambda} D_i;\ \sum_\lambda z_{i\lambda} l_{i\lambda} \leq L_{i\lambda} D_i \tag{16.24}$$

For the best retailer selection, we have the following objective function and constraints:

$$\min \mathrm{f}_4 = \sum_i \sum_\nu w_{i\nu} P_{i\nu}$$
$$\text{subject to} \sum_\nu w_{i\nu} \geq D_i;\ w_{i\nu} \leq C_{i\nu};\ \sum_\nu w_{i\nu} u_{i\nu} \leq U_{i\nu} D_i;\ \sum_\nu w_{i\nu} l_{i\nu} \leq L_{i\nu} D_i \tag{16.25}$$

16.6 Computational Algorithm

16.6.1 Multi-objective GA

he total cost function for all items and individual cost function are considered to be fitness function and minimized using the below mentioned algorithm. MATLAB 14a is used for running iterations.

1. Set numerical values for different parameters except for decision variables $x_{ij,}\mathrm{y}_{ik,}z_{i\lambda}$ and $\mathrm{w}_{i\nu}$ in the fitness function.
2. Start with an initial population of 20 chromosomes.
3. Rank the chromosomes on the basis of their fitness score.
4. Chromosomes with good fitness score will get an entry in mating pool.

5. Perform stochastic uniform crossover for reproduction. Crossover fraction is considered 0.8 and two elites are considered at each generation.
6. Again rank members of new generation on the basis of their fitness function and select members, which can create next generation.
7. Perform Step 3 and Step 4 till absolute difference between two successive members is negligible, i.e., $|x_{i+1} - x_i| < \varepsilon\left(\varepsilon = 10^{-5}\right)$.

16.6.2 3D-RadVis Visualization Technique

For N Pareto front, the solution of M objectives are as follows:

1. Compute
$$x = \frac{\sum_{i=1}^{M} f_{i,j}^{Norm} \cos(\theta_j)}{\sum_{i=1}^{M} f_{i,j}^{Norm}}; \; y = \frac{\sum_{i=1}^{M} f_{i,j}^{Norm} sin(\theta_j)}{\sum_{i=1}^{M} f_{i,j}^{Norm}};$$
where $f_i^{Norm} = \frac{f_i(x) - \min(f_i(x))}{\max(f_i(x)) - \min(f_i(x))}$
2. $px = x + 1$
3. $py = y + 1$
4. Find normal vector perpendicular to the extreme point $n = norm(z)$; where z is hyperplane.
5. Calculate $c = n \cdot z_1$
6. For $i = 1$ to N find D $= \frac{abs(f_i \cdot n - c)}{\|n\|}$
7. Finally, we convert $R = [x, y, D]$

16.7 Numerical Example and Results

Consider supply chain with three suppliers, one manufacturer with three plants, three distributors, and three retailers

$$D_i = 50;\; \text{Q}_{ij} = Q_{ik} = Q_{i\nu} = A_{i\lambda} = L_{ij} = L_{ik} = L_{i\lambda} = L_{i\nu} = 3\%(/unit);$$
$$\text{MIC} = 2(\$/\text{unit});\ \text{DIC} = 4(\$/\text{unit});\ \text{RIC} = 1 : 5(\$/\text{unit})$$

Suppliers, plants, distributors, and other retailers-related information are given in Tables 16.1, 16.2, 16.3, and 16.4, respectively.

Supplier selection:
The results obtained using multi-objective GA is shown in Table 16.6 (Appendix). Visualization of the same by 3D-RadV in MATLAB14a is shown in Fig. 16.2. At z = 0, we get the minimum total cost of all three items.

Table 16.1 Supplier Information

Supplier	Supplier 1			Supplier 2			Supplier 3		
Items	1	2	3	1	2	3	1	2	3
Price ($)	6	4	5	6	5	4.5	7	5	5
Supplier capacity	80	90	90	50	70	60	60	80	70
Quality (%)	0.1	0.2	0.2	0.2	0.1	0.1	0.1	0.2	0.3
Late delivery (month)	0.1	0.3	0.3	0.2	0.05	0.05	0.05	0.3	0.1

Table 16.2 Plants information

Plants	Plant 1			Plant 2			Plant 3		
Items	1	2	3	1	2	3	1	2	3
Production cost ($)	2.4	3.6	4.6	2.8	4.2	4.5	3.2	4.1	5
Transportation cost ($)	0.4	0.4	0.2	0.3	0.4	0.1	0.1	0.2	0.1
Production	60	90	70	80	70	60	70	80	70
Capacity	70	60	40	40	50	50	40	50	60
Quality (%)	0.25	0.1	0.2	0.2	0.1	0.1	0.1	0.2	0.15
Production time (months)	0.1	0.2	0.2	0.2	0.15	0.15	0.05	0.25	0.1

Table 16.3 Distributor information

Distributors	Distributor 1			Distributor 2			Distributor 3		
Items	1	2	3	1	2	3	1	2	3
Purchase price ($)	10.4	9.6	11.1	10.8	9.8	11.3	11.2	9.2	11.2
Transportation cost ($)	0.3	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.15
Storage capacity	70	80	70	90	80	60	80	70	70
Coverage area (%)	0.1	0.2	0.15	0.2	0.1	0.1	0.05	0.2	0.15
Late delivery (month)	0.1	0.2	0.25	0.15	0.15	0.15	0.05	0.25	0.1

Table 16.4 Retailer information

Retailer	Retailer 1			Retailer 2			Retailer 3		
Items	1	2	3	1	2	3	1	2	3
Purchase price ($)	12.4	11.2	14.1	11.8	11.4	13.5	12.2	11.1	13.8
Transportation cost ($)	0.1	0.2	0.2	0.2	0.1	0.1	0.1	0.2	0.15
Storage capacity	60	70	70	80	90	70	60	80	70
Customer no satisfaction (%)	0.01	0.05	0.15	0.12	0.1	0.1	0.05	0.2	0.2
Late delivery (month)	0.01	0.2	0.25	0.15	0.15	0.15	0.05	0.25	0.3

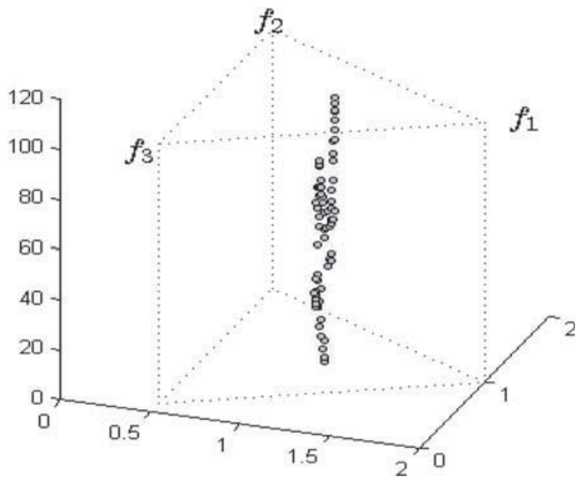


Fig. 16.2 Visualization of Pareto-optimal front representation to select supplier

From Table 16.6 (Appendix) and Fig. 16.2, it is clear that second solution is the best because it minimizes the cost and fulfill the required demand. So, the manufacturer order quantity from different supplier's are given in Table 16.5.

Plant selection:
The results obtained using multi-objective GA is shown in Table 16.7 (Appendix). Visualization of the same by 3D-RadV in MATLAB14a is shown in Fig. 16.3. At z = 0, we get the minimum total cost of all three items.

From Table 16.7 (Appendix) and Fig. 16.3, it is clear that 18th solution is the best because it minimizes the cost and fulfill the required demand. The optimal quantity sending from manufacturer to plants is shown in Table 16.5.

Distributor selection:
The results obtained using multi-objective GA is shown in Table 16.8 (Appendix). Visualization of the same by 3D-RadV in MATLAB14a is shown in Fig. 16.4. At z = 0, we get the minimum total cost of all three items.

From Table 16.8 (Appendix) and Fig. 16.4, it is clear that 57th solution is the best because it minimizes the cost and fulfills the required demand. So, the manufacturer supply from different quantities to the distributor is given in Table 16.5.

Retailer Selection:
The results obtained using multi-objective GA is shown in Table 16.9 (Appendix). Visualization of the same by 3D-RadV in MATLAB14a is shown in Fig. 16.5. At z = 0, we get the minimum total cost of all three items.

Table 16.5 Quantity order on the base of supply player selection

	S1	S2	S3	P1	P2	P3	D1	D2	D3	R1	R2	R3
Item 1	1	18	16	17	14	21	20	19	15	18	17	17
Item 2	34	0	16	29	12	12	13	25	17	17	12	22
Item 3	16	17	17	21	24	9	22	18	15	21	16	16

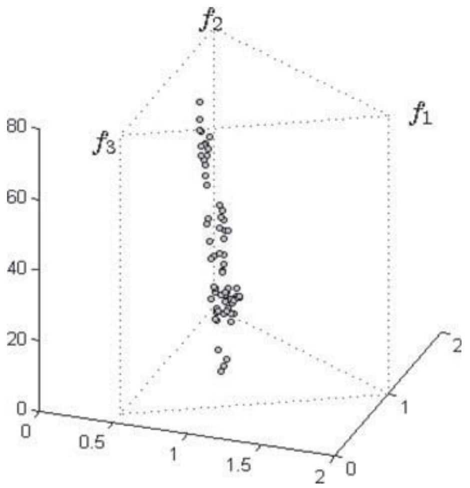


Fig. 16.3 Visualization of Pareto-optimal front representation to select plant

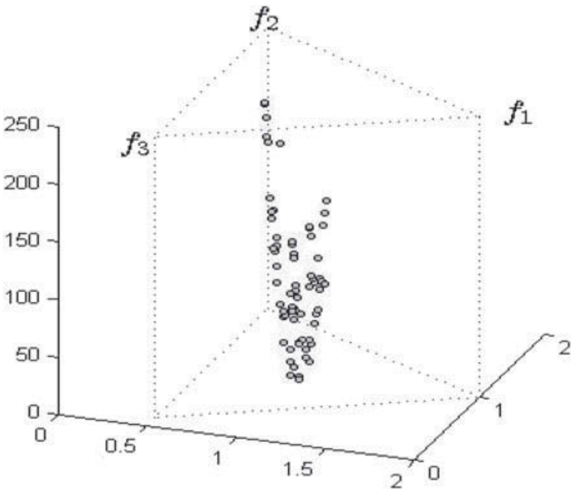


Fig. 16.4 Visualization of Pareto-optimal front representation to select distributor

From Table 16.9 (Appendix) and Fig. 16.3, it is clear that 13th solution is the best because it minimizes the cost and fulfills the required demand. So, the distributor supply quantity from different retailers is given in Table 16.5.

So, the total cost for supply chain is

$$TC = PUC + PRC + TRC + INC = 812.19 + 255 + 88.85 + 1208 = 2364.4(\$)$$

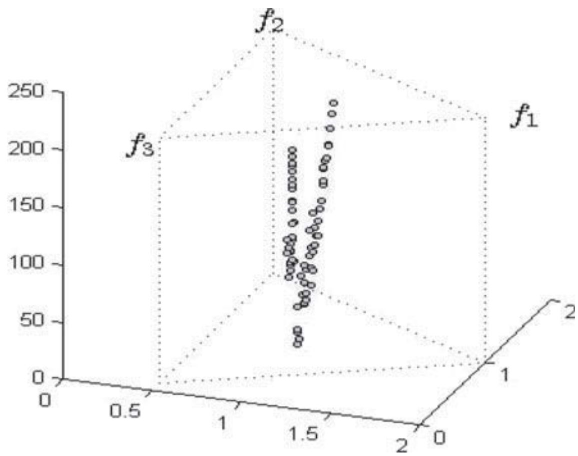


Fig. 16.5 Visualization of Pareto-optimal front representation to select retailer

16.8 Conclusions

This paper proposes total supply chain cost minimization through selecting the best supplier, plant, distributor, and retailer simultaneously. These selections criteria are production capacity, acceptable quality of the products delivery time, and distribution region and customer satisfaction. This all leads to a multi-objective constrained problem with conflicting constraints. Problem is optimized with multi-objective genetic algorithm and further, optimal case is visualized by 3D-RadVis method using MATLAB14a. Numerical example and results show that it is not only fulfill customer's demand under constraints but also minimize total cost of supply chain by selecting appropriate business players. This model may be extended with the selection of supply partners under more criteria depending on different industries, supply partner evaluation through data gathering process, etc. The multiple trade-off between these objectives is worthy of further investigation using 3D-RadVis Antenna technique.

Appendix

See Tables 16.6, 16.7, 16.8, and 16.9.

Table 16.6 Pareto-optimal front solution to select supplier

Sr. no	Multi-objective GA												3D-RadVis		
	x_1	x_2	x_3	f_{11}	x_4	x_5	x_6	f_{12}	x_7	x_8	x_9	f_{13}	X	y	d
1	14	28	8	321.24	20	23	10	261.83	19	19	18	273.81	1.06	0.99	25.80
2	18	16	16	330.75	34	0	16	233.12	16	17	17	248.33	1.11	0.98	0.00
3	20	19	13	337.34	27	33	12	360.72	12	19	18	246.61	1.04	1.10	76.49
4	23	20	16	389.24	21	28	10	293.23	12	20	19	246.24	1.13	1.04	67.27
5	26	21	18	425.36	19	34	10	319.99	11	20	19	246.18	1.14	1.06	103.54
6	20	19	13	339.14	27	32	12	356.20	12	19	18	246.59	1.04	1.10	74.90
7	22	20	16	383.63	21	25	10	279.21	12	20	19	246.27	1.13	1.03	55.96
8	21	20	14	363.87	21	22	11	266.70	12	20	18	246.31	1.12	1.02	37.35
9	16	25	8	322.48	21	19	11	253.28	20	19	18	283.71	1.06	0.97	27.30
10	26	21	18	420.15	20	33	10	316.02	11	20	19	246.19	1.14	1.06	98.25
11	17	21	12	326.54	27	10	14	243.52	18	18	17	266.59	1.09	0.98	14.12
12	21	20	14	354.94	24	25	11	298.83	12	20	18	246.40	1.09	1.05	50.80
13	20	19	12	331.08	27	31	11	348.59	12	19	18	246.65	1.04	1.10	65.89
14	26	21	18	422.12	20	33	10	315.07	11	20	19	246.19	1.14	1.06	98.84
15	15	28	8	321.80	20	23	10	264.47	18	19	18	273.21	1.06	0.99	27.30
16	25	21	18	415.06	20	32	10	308.62	11	20	19	246.20	1.14	1.06	91.04
17	26	21	18	425.40	19	34	10	319.82	11	20	19	246.18	1.14	1.06	103.46
18	23	20	17	396.94	21	28	10	292.62	11	20	19	246.24	1.14	1.04	71.36
19	17	24	9	323.73	24	15	12	249.02	19	19	18	277.20	1.07	0.97	21.80
20	21	20	14	357.60	23	28	11	307.52	12	20	19	246.35	1.09	1.06	57.32

(continued)

Table 16.6 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	x_1	x_2	x_3	f_{11}	x_4	x_5	x_6	f_{12}	x_7	x_8	x_9	f_{13}	X	y	d
21	25	21	18	417.22	20	32	10	313.46	11	20	19	246.19	1.14	1.06	95.08
22	25	21	17	410.73	20	31	10	306.57	11	20	19	246.21	1.14	1.05	87.36
23	22	22	17	398.94	21	23	10	269.78	12	20	19	246.30	1.15	1.02	59.36
24	20	19	12	330.16	26	29	12	330.20	13	19	18	246.76	1.05	1.08	54.81
25	22	20	15	372.66	22	27	10	294.26	12	20	18	246.30	1.11	1.05	58.33
26	18	19	13	327.70	28	25	13	324.48	14	19	18	248.25	1.05	1.07	50.94
27	20	20	13	350.73	23	25	11	297.03	12	20	18	246.61	1.09	1.05	47.45
28	20	19	14	345.38	26	30	12	335.26	12	20	18	246.52	1.06	1.08	66.38
29	18	24	8	323.60	21	21	10	258.91	17	19	18	267.49	1.07	0.99	21.83
30	22	20	16	385.41	21	26	10	284.80	12	20	19	246.26	1.13	1.04	60.21
31	18	18	15	329.76	31	4	16	239.13	17	17	17	252.30	1.10	0.99	5.19
32	22	22	16	393.35	21	23	10	269.29	12	20	19	246.30	1.15	1.02	55.86
33	22	20	16	380.12	21	25	10	279.39	12	20	19	246.28	1.13	1.03	54.04
34	24	20	17	401.68	20	30	10	300.88	11	20	19	246.23	1.14	1.05	78.87
35	21	20	14	359.81	23	25	11	295.07	12	20	18	246.38	1.10	1.05	51.42
36	20	19	13	342.79	26	31	12	342.95	12	20	18	246.55	1.05	1.09	69.34
37	24	20	17	403.71	20	30	10	303.90	11	20	19	246.21	1.13	1.05	81.77
38	21	20	15	368.70	21	22	11	267.13	12	20	18	246.31	1.13	1.02	40.38
39	16	25	8	322.99	21	18	11	247.30	20	19	18	282.28	1.07	0.96	23.31

(continued)

Table 16.6 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	x_1	x_2	x_3	f_{11}	x_4	x_5	x_6	f_{12}	x_7	x_8	x_9	f_{13}	X	y	d
40	25	21	18	417.73	20	32	10	312.40	11	20	19	246.20	1.14	1.06	94.76
41	26	21	18	423.91	19	34	10	317.70	11	20	19	246.18	1.14	1.06	101.38
42	19	22	9	325.70	24	32	11	337.69	14	19	18	253.27	1.03	1.08	60.32
43	20	19	12	332.91	27	31	11	346.85	12	19	18	246.64	1.04	1.09	65.93
44	24	22	12	375.38	20	21	11	259.57	12	20	19	246.61	1.14	1.01	40.05
45	20	19	13	340.19	27	32	12	354.35	12	19	18	246.58	1.04	1.10	74.44
46	23	20	16	387.25	21	26	10	287.54	12	20	19	246.26	1.13	1.04	62.85
47	24	20	17	400.71	20	30	10	301.67	11	20	19	246.22	1.13	1.05	78.76
48	17	20	13	327.44	28	8	14	241.64	18	18	17	260.72	1.09	0.98	10.17
49	15	27	8	322.71	21	24	10	274.87	18	19	18	269.78	1.06	1.01	31.85
50	17	17	16	330.18	33	2	16	235.13	17	17	17	250.15	1.11	0.98	1.89
51	20	19	12	332.88	26	28	11	327.56	13	19	18	246.74	1.05	1.08	54.84
52	20	19	12	333.96	27	30	12	340.47	12	19	18	246.62	1.04	1.09	62.84
53	16	25	9	325.96	22	23	10	272.78	14	19	18	253.57	1.07	1.02	23.17
54	19	20	11	325.60	25	32	11	338.03	13	19	18	248.49	1.04	1.09	57.69
55	16	26	9	322.64	21	19	11	254.00	19	19	18	278.67	1.07	0.98	24.89
56	20	19	14	345.99	26	29	12	330.20	12	20	18	246.52	1.06	1.08	63.81
57	20	19	14	351.33	25	28	11	320.15	12	20	18	246.47	1.07	1.07	61.06
58	16	26	8	322.09	21	20	11	256.01	19	19	18	280.60	1.06	0.98	26.85

(continued)

Table 16.6 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	x_1	x_2	x_3	f_{11}	x_4	x_5	x_6	f_{12}	x_7	x_8	x_9	f_{13}	X	y	d
59	20	20	13	348.44	25	29	12	326.24	12	20	18	246.31	1.07	1.08	62.82
60	24	20	17	408.06	20	31	10	307.54	11	20	19	246.21	1.14	1.06	86.38
61	18	17	16	335.42	32	4	16	242.85	16	17	17	248.02	1.11	0.99	8.14
62	19	17	14	331.71	31	13	14	280.65	15	18	17	247.62	1.08	1.03	27.59
63	24	21	13	374.24	20	23	10	265.38	11	20	19	246.31	1.13	1.02	42.57
64	17	22	11	327.46	21	21	10	258.08	14	19	18	254.91	1.08	1.00	16.32
65	18	22	10	324.89	22	23	12	282.35	16	18	18	258.89	1.06	1.02	31.14
66	17	24	10	324.40	23	16	12	251.92	19	19	18	276.50	1.07	0.98	23.46
67	22	20	15	371.39	22	25	10	285.54	12	20	18	246.30	1.12	1.04	52.56
68	15	26	9	322.81	22	19	11	257.19	18	18	18	269.65	1.07	0.99	21.63
69	19	22	9	324.67	21	29	11	306.73	15	19	18	257.24	1.05	1.05	44.14
70	17	24	9	323.24	21	25	11	283.33	16	19	18	263.26	1.06	1.02	33.28

Table 16.7 Pareto-optimal front solution to select plants

Sr. no	Multi-objective GA												3D-RadVis		
	y_1	y_2	y_3	f_{21}	y_4	y_5	y_6	f_{22}	y_7	y_8	y_9	f_{23}	x	y	d
1	18	17	14	153.03	21	14	15	214.77	20	43	20	398.04	0.80	0.79	70.94
2	20	20	10	151.23	21	22	19	267.91	24	24	22	333.57	0.80	0.92	63.36
3	17	15	20	159.48	26	29	19	320.20	20	27	4	237.36	0.83	1.10	42.77
4	18	16	21	166.32	23	27	19	293.91	20	26	4	236.40	0.86	1.07	30.98
5	20	18	13	153.07	21	15	15	215.21	21	42	19	389.36	0.80	0.80	66.21
6	16	18	24	181.17	16	31	20	291.95	20	27	3	235.57	0.88	1.07	37.94
7	19	14	20	163.73	31	9	11	209.66	22	27	20	328.04	0.85	0.85	33.75
8	23	20	21	195.50	21	18	18	245.66	21	25	4	236.70	0.93	1.01	20.15
9	19	14	22	169.84	32	8	10	210.51	22	23	18	304.01	0.87	0.88	23.91
10	19	16	16	156.43	25	12	14	212.93	21	42	20	395.32	0.81	0.79	70.27
11	19	19	11	151.69	21	20	18	254.73	23	29	21	349.64	0.80	0.89	65.30
12	21	19	12	157.90	22	15	18	232.56	22	25	11	278.28	0.85	0.94	14.88
13	19	18	13	152.29	22	17	17	237.83	22	35	18	356.05	0.81	0.86	59.59
14	19	16	16	156.03	25	11	14	211.61	21	44	20	403.68	0.80	0.78	74.11
15	19	18	13	151.90	23	23	19	281.65	23	29	16	324.02	0.80	0.95	66.17
16	17	15	20	159.74	26	24	18	290.42	20	27	4	238.81	0.85	1.06	26.56
17	19	18	15	156.32	21	18	18	244.69	21	33	6	282.13	0.84	0.95	23.19

(continued)

Table 16.7 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	y_1	y_2	y_3	f_{21}	y_4	y_5	y_6	f_{22}	y_7	y_8	y_9	f_{23}	x	y	d
18	17	14	21	162.73	29	12	12	225.23	21	24	9	255.00	0.88	0.96	0.00
19	23	19	22	197.50	22	17	17	240.75	21	25	5	237.98	0.94	1.00	19.21
20	19	16	19	166.12	24	21	18	272.52	21	25	7	250.88	0.86	1.03	26.88
21	17	17	17	159.22	23	29	20	313.61	20	28	3	241.48	0.83	1.09	41.19
22	18	16	17	156.94	21	21	16	248.67	20	29	7	265.93	0.85	0.98	16.50
23	19	18	13	152.24	22	17	16	235.09	22	33	20	361.82	0.80	0.85	61.31
24	18	18	22	177.08	22	15	17	229.10	21	27	4	242.87	0.91	0.98	3.52
25	31	20	23	225.14	21	14	17	217.42	22	23	5	237.82	1.00	0.97	21.60
26	25	15	21	184.24	24	11	15	212.49	22	23	13	276.50	0.91	0.92	17.47
27	25	20	23	204.01	22	16	17	234.05	21	24	5	239.46	0.95	0.99	19.95
28	17	17	22	172.47	22	28	19	299.80	20	27	4	236.22	0.87	1.08	37.83
29	20	19	12	152.04	21	18	16	237.12	22	35	20	367.68	0.80	0.85	65.75
30	18	17	17	157.53	26	27	19	309.56	21	29	4	253.45	0.83	1.07	44.79
31	28	20	23	215.76	23	15	16	230.30	21	24	5	238.00	0.97	0.99	23.73
32	19	17	14	152.38	23	14	15	216.49	21	40	20	383.81	0.80	0.81	63.35
33	32	21	23	228.67	21	14	16	214.62	22	23	6	237.41	1.00	0.97	21.78
34	18	18	24	182.97	17	24	17	251.82	20	25	5	237.54	0.91	1.02	16.96
35	27	18	23	206.05	25	13	15	223.88	22	24	6	242.00	0.96	0.98	16.72
36	20	19	14	159.39	25	22	20	286.72	22	27	5	251.92	0.84	1.04	31.79
37	21	15	15	153.22	27	13	12	219.53	21	25	18	307.74	0.84	0.89	21.67

(continued)

Table 16.7 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	y_1	y_2	y_3	f_{21}	y_4	y_5	y_6	f_{22}	y_7	y_8	y_9	f_{23}	x	y	d
38	26	18	22	201.15	19	16	17	222.64	21	26	5	248.08	0.95	0.97	16.69
39	19	19	11	151.70	21	20	18	255.57	23	28	20	342.82	0.80	0.90	61.85
40	19	17	18	165.89	25	27	18	302.33	20	26	4	236.25	0.85	1.08	35.52
41	19	16	16	155.55	25	11	14	210.96	20	47	20	413.16	0.80	0.78	78.93
42	19	18	13	153.20	21	18	17	238.49	21	34	11	316.37	0.82	0.90	37.59
43	20	18	21	181.45	27	17	17	263.27	21	24	5	236.84	0.90	1.03	22.28
44	18	18	14	152.31	18	23	18	257.99	22	30	3	260.33	0.84	1.00	15.97
45	19	20	11	153.02	21	18	18	246.04	23	27	20	336.57	0.81	0.89	53.50
46	19	14	23	172.23	33	7	10	207.49	22	22	19	306.52	0.88	0.88	24.98
47	19	15	18	160.56	27	10	13	211.25	21	38	19	371.02	0.82	0.81	57.66
48	18	13	23	167.70	34	6	9	206.71	22	22	21	313.03	0.87	0.87	25.68
49	26	18	22	201.21	25	13	14	217.35	21	24	7	249.50	0.95	0.96	14.49
50	31	19	23	220.14	22	14	16	219.26	21	23	6	241.10	0.99	0.97	21.67
51	18	17	21	171.92	22	21	19	265.86	20	26	5	240.78	0.88	1.03	20.56
52	20	17	18	170.21	22	24	18	277.27	20	28	5	245.76	0.87	1.04	29.03
53	17	18	16	154.65	19	21	19	253.79	20	29	6	264.07	0.84	0.99	17.06
54	18	18	15	158.04	23	29	19	305.36	20	28	4	246.42	0.83	1.07	38.60
55	25	16	23	194.80	28	10	12	211.33	22	23	13	277.90	0.93	0.92	23.71
56	23	17	23	192.25	27	11	13	212.25	22	23	11	269.96	0.93	0.93	18.19
57	22	15	22	180.60	30	9	12	211.65	22	24	15	290.10	0.90	0.90	22.74

(continued)

Table 16.7 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	y_1	y_2	y_3	f_{21}	y_4	y_5	y_6	f_{22}	y_7	y_8	y_9	f_{23}	x	y	d
58	19	15	20	163.34	30	9	11	209.48	21	32	20	348.41	0.84	0.83	45.19
59	18	16	20	164.75	28	13	10	216.11	22	23	13	273.85	0.88	0.92	6.78
60	19	15	17	156.95	29	11	14	227.76	21	29	10	284.72	0.85	0.93	15.28
61	18	15	18	155.90	24	19	15	248.62	21	27	14	295.24	0.83	0.94	32.79
62	19	18	14	152.44	22	14	16	223.39	21	31	19	341.89	0.82	0.86	43.16
63	17	18	22	174.79	19	21	19	253.22	20	28	4	247.83	0.89	1.01	18.99
64	17	16	20	164.72	25	28	19	311.82	20	27	4	236.79	0.85	1.09	40.63
65	17	18	23	178.03	21	21	19	261.55	21	27	4	240.35	0.89	1.03	21.35
66	17	16	21	168.66	26	14	14	226.83	21	25	7	249.98	0.89	0.97	1.45
67	18	13	23	167.72	34	7	9	206.76	22	22	21	313.01	0.87	0.87	25.71
68	17	15	20	159.46	26	29	19	320.24	20	27	4	237.40	0.83	1.10	42.81
69	19	16	16	155.53	25	11	14	211.02	20	47	20	413.20	0.80	0.78	78.98
70	32	20	22	223.66	22	14	15	216.30	21	24	6	240.34	0.99	0.97	21.56

Table 16.8 Pareto-optimal front solution to select distributors

Sr. no	Multi-objective GA												3D-RadVis		
	z_1	z_2	z_3	f_{31}	z_4	z_5	z_6	f_{32}	z_7	z_8	z_9	f_{33}	x	y	d
1	25	22	19	716.97	21	22	20	610.65	23	6	21	566.63	0.80	1.02	79.86
2	24	22	4	536.13	17	18	20	527.53	22	33	31	971.46	0.80	0.81	161.18
3	18	18	15	542.38	17	48	17	801.16	17	19	17	595.89	0.83	1.09	105.94
4	27	19	16	662.58	10	17	23	482.76	30	19	15	713.13	0.86	0.89	59.20
5	24	22	4	534.30	20	20	20	585.59	22	36	35	1059.85	0.80	0.81	244.68
6	37	22	18	824.89	13	18	21	496.89	23	21	8	580.36	0.88	0.96	84.42
7	24	22	4	534.65	19	20	20	571.68	22	34	33	1013.43	0.85	0.82	210.05
8	24	22	4	534.70	19	19	20	565.59	22	35	33	1026.84	0.93	0.81	214.31
9	35	22	19	815.10	21	21	20	600.10	22	16	11	567.28	0.87	1.01	130.79
10	25	20	15	648.23	13	26	18	558.65	21	18	11	571.67	0.81	0.99	13.06
11	23	21	18	668.70	19	28	20	649.07	21	10	19	568.51	0.80	1.04	75.26
12	40	22	19	861.34	21	20	20	593.24	22	21	7	567.46	0.85	1.01	153.64
13	24	22	4	535.37	19	20	20	574.97	22	29	28	902.51	0.81	0.86	148.33
14	23	21	6	538.02	15	18	20	516.16	23	31	29	951.27	0.80	0.81	144.06
15	24	22	19	698.47	20	25	20	634.20	22	8	21	570.38	0.80	1.03	84.94
16	16	17	17	543.51	17	50	17	814.34	16	17	16	567.53	0.85	1.11	97.83
17	37	22	19	837.85	21	21	20	597.51	22	19	9	567.44	0.84	1.01	142.53

(continued)

Table 16.8 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	z_1	z_2	z_3	f_{31}	z_4	z_5	z_6	f_{32}	z_7	z_8	z_9	f_{33}	x	y	d
18	23	19	16	622.66	8	17	25	481.87	32	18	17	758.33	0.88	0.87	61.73
19	17	17	15	545.14	17	47	17	785.76	17	18	17	588.58	0.94	1.09	94.43
20	27	19	18	692.82	19	36	18	711.64	19	19	12	567.67	0.86	1.06	124.82
21	30	22	19	760.75	21	22	20	606.03	23	11	17	566.99	0.83	1.02	102.67
22	23	21	7	542.35	13	18	21	494.65	24	28	28	923.09	0.85	0.81	117.87
23	19	18	15	562.79	14	30	22	639.38	22	18	16	635.87	0.80	1.00	47.40
24	19	18	16	567.39	18	33	17	663.84	18	17	15	568.09	0.91	1.05	25.05
25	20	19	12	553.84	16	36	19	691.16	18	19	16	603.83	1.00	1.04	53.63
26	25	21	19	701.89	20	24	20	627.93	22	7	21	569.72	0.91	1.03	82.91
27	24	22	4	534.30	20	20	20	585.45	22	36	35	1059.92	0.95	0.81	244.65
28	24	22	4	534.39	19	20	20	578.74	22	35	34	1043.08	0.87	0.81	231.10
29	24	22	4	534.36	19	20	20	584.50	22	36	35	1058.15	0.80	0.81	243.11
30	35	21	18	797.00	15	20	20	525.61	22	20	9	581.34	0.83	0.97	85.46
31	18	17	16	557.90	16	44	17	749.41	17	17	16	572.68	0.97	1.08	71.62
32	19	18	16	569.16	6	16	27	481.10	35	18	19	814.55	0.80	0.85	62.86
33	21	21	8	538.48	17	46	17	779.71	20	24	25	781.56	1.00	1.00	198.50
34	21	19	12	570.74	11	18	21	486.54	26	20	16	706.27	0.91	0.89	4.40
35	18	18	14	544.33	17	47	17	793.84	18	19	17	607.35	0.96	1.08	109.45
36	25	20	18	684.43	14	18	22	522.84	24	12	17	601.40	0.84	0.96	30.44
37	22	20	10	556.53	11	18	21	485.13	25	21	28	833.76	0.84	0.84	68.98

(continued)

Table 16.8 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	z_1	z_2	z_3	f_{31}	z_4	z_5	z_6	f_{32}	z_7	z_8	z_9	f_{33}	x	y	d
38	35	20	18	788.18	12	17	22	495.25	27	18	8	610.34	0.95	0.95	79.58
39	19	19	12	546.31	11	19	26	540.51	27	20	20	765.25	0.80	0.89	55.50
40	24	21	5	536.22	18	23	20	588.17	23	28	24	845.16	0.85	0.89	123.33
41	31	20	18	742.00	13	21	21	521.92	24	18	9	589.81	0.80	0.97	56.47
42	22	20	8	540.76	16	18	21	530.13	24	28	24	858.59	0.82	0.85	100.19
43	25	19	18	661.55	15	20	19	519.73	20	18	16	619.86	0.90	0.95	26.10
44	20	18	16	581.03	8	18	24	484.12	29	18	18	740.25	0.84	0.88	28.56
45	32	20	15	732.98	13	17	20	485.70	25	19	11	624.02	0.81	0.93	50.10
46	18	17	16	558.19	16	42	17	732.92	17	17	17	572.93	0.88	1.07	62.42
47	28	20	18	711.34	19	32	19	682.37	20	17	13	568.00	0.82	1.05	118.80
48	19	18	16	575.79	7	17	27	482.36	34	18	19	798.83	0.87	0.85	58.34
49	20	19	15	586.66	12	19	22	506.70	26	18	16	681.92	0.95	0.91	11.17
50	17	17	16	547.06	17	44	17	755.87	17	17	16	568.90	0.99	1.09	66.91
51	24	19	17	650.37	12	20	21	516.06	24	15	15	622.08	0.88	0.95	18.81
52	20	19	17	611.22	14	29	20	605.27	21	19	12	591.12	0.87	1.01	29.84
53	21	21	8	538.11	9	18	25	497.99	34	20	23	872.40	0.84	0.83	88.09
54	24	19	15	632.74	10	17	22	484.28	28	18	15	696.31	0.83	0.90	33.14
55	33	21	18	777.13	15	22	20	556.63	22	19	9	570.90	0.93	0.99	85.87
56	24	22	4	535.23	18	20	20	552.30	22	31	29	925.24	0.93	0.84	148.28
57	20	19	15	586.04	13	25	17	543.42	22	18	15	626.48	0.90	0.96	0.00

(continued)

Table 16.8 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	z_1	z_2	z_3	f_{31}	z_4	z_5	z_6	f_{32}	z_7	z_8	z_9	f_{33}	x	y	d
58	21	19	12	572.92	11	18	21	485.24	26	21	18	728.40	0.84	0.88	17.69
59	19	17	16	563.49	8	36	25	668.81	20	17	17	615.80	0.88	1.02	53.22
60	30	21	13	686.20	14	17	21	501.86	24	20	12	628.53	0.85	0.94	35.03
61	24	21	6	540.29	16	20	22	565.90	22	27	26	853.34	0.83	0.87	117.55
62	22	20	14	603.16	12	19	22	510.73	25	18	13	642.40	0.82	0.94	0.21
63	33	20	18	753.32	15	21	18	523.73	27	16	8	581.94	0.89	0.97	59.50
64	23	21	7	547.69	10	19	21	490.36	23	28	17	776.66	0.85	0.86	33.93
65	17	18	15	542.44	15	39	18	695.48	18	19	17	613.29	0.89	1.04	55.01
66	24	22	4	535.98	17	20	19	541.23	22	29	26	876.35	0.89	0.85	114.10
67	19	21	12	559.89	18	30	19	651.21	22	19	18	665.61	0.87	0.99	69.73
68	27	19	18	692.76	19	36	18	712.36	19	19	12	567.94	0.83	1.06	125.36
69	19	18	16	569.51	6	16	27	480.76	35	18	19	814.30	0.80	0.85	62.72
70	23	21	6	538.03	15	18	20	516.19	23	31	29	951.20	0.99	0.81	144.04

Table 16.9 Pareto-optimal front solution to select retailers

Sr. no	Multi-objective GA												3D-RadVis		
	w_1	w_2	w_3	f_{41}	w_4	w_5	w_6	f_{42}	w_7	w_8	w_9	f_{43}	x	y	d
1	19	19	16	664.31	18	18	22	656.32	18	18	14	697.78	0.99	0.98	50.17
2	14	18	18	611.67	19	19	22	681.06	22	18	17	791.95	0.94	0.95	88.44
3	16	17	16	611.44	18	3	31	590.08	30	17	16	890.89	0.94	0.88	92.89
4	15	18	17	611.09	21	15	26	709.10	23	17	16	786.47	0.94	0.97	101.13
5	13	19	18	610.41	18	19	20	649.07	22	18	18	810.78	0.94	0.93	80.10
6	40	17	17	908.17	17	13	20	569.68	19	16	16	723.89	1.12	0.94	156.01
7	43	17	17	942.05	17	13	20	569.53	18	16	16	704.82	1.14	0.95	164.48
8	15	19	17	611.11	18	6	28	597.82	29	17	16	868.90	0.94	0.89	84.48
9	16	17	17	611.76	25	17	22	735.57	21	17	16	760.79	0.94	0.99	101.97
10	17	17	16	611.62	17	3	30	581.76	30	16	16	877.93	0.94	0.88	80.71
11	15	18	17	611.24	26	18	28	813.16	22	17	17	779.11	0.92	1.01	157.04
12	28	18	17	761.61	17	16	21	608.26	18	17	15	698.92	1.05	0.96	79.26
13	18	17	17	621.71	17	12	22	573.41	21	16	16	736.39	0.98	0.93	0.00
14	14	19	18	610.79	22	19	24	732.29	22	17	17	794.97	0.93	0.97	119.25
15	25	17	16	711.62	17	4	29	567.76	23	16	16	773.68	1.02	0.91	70.17
16	16	17	17	614.65	17	15	20	603.94	19	17	16	717.60	0.98	0.95	2.70
17	30	18	16	789.62	17	16	21	618.70	18	17	15	698.12	1.06	0.97	101.00
18	35	17	17	852.75	17	11	23	581.95	18	16	16	701.36	1.10	0.95	118.09
19	31	18	17	804.07	17	14	20	572.37	21	16	16	755.41	1.07	0.93	115.67
20	14	18	18	611.04	24	18	26	773.06	22	17	17	786.86	0.92	0.99	138.24

(continued)

Table 16.9 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	w_1	w_2	w_3	f_{41}	w_4	w_5	w_6	f_{42}	w_7	w_8	w_9	f_{43}	x	y	d
21	17	17	16	611.78	17	3	30	567.36	30	15	16	855.48	0.95	0.88	59.53
22	13	19	18	610.76	19	18	22	676.08	22	18	17	802.45	0.94	0.95	91.09
23	26	13	16	681.64	18	16	21	624.70	18	17	15	698.11	1.01	0.97	42.11
24	33	18	16	829.75	17	15	19	589.13	18	16	16	699.83	1.09	0.95	108.08
25	30	17	16	775.07	17	6	27	568.19	22	16	16	763.22	1.05	0.92	101.01
26	38	17	17	889.66	17	14	20	583.68	18	16	16	699.90	1.11	0.95	139.56
27	47	17	17	990.25	17	14	20	570.72	18	16	16	699.89	1.16	0.95	190.15
28	36	17	17	863.13	17	16	18	585.30	18	16	16	700.24	1.10	0.95	125.37
29	13	19	18	610.44	18	19	20	646.73	22	18	18	812.00	0.94	0.93	79.48
30	28	15	17	732.82	17	16	21	612.93	18	17	15	698.76	1.04	0.96	65.24
31	19	17	16	644.15	17	4	29	573.11	28	16	16	845.02	0.97	0.89	75.50
32	15	18	17	611.36	25	18	28	799.77	22	17	17	779.32	0.92	1.01	149.50
33	15	18	17	611.60	24	17	27	768.62	23	17	17	782.93	0.92	0.99	133.74
34	42	16	17	920.77	17	15	19	590.58	18	16	16	698.27	1.13	0.96	160.56
35	33	17	17	818.49	17	15	19	575.54	18	16	16	701.67	1.09	0.95	94.79
36	14	19	17	610.92	18	13	24	615.57	25	17	17	828.30	0.95	0.91	71.17
37	31	15	17	776.70	17	15	21	604.72	18	17	15	699.04	1.06	0.96	86.00
38	50	17	17	1029.32	17	15	19	569.61	18	16	16	701.25	1.17	0.95	212.85
39	15	18	17	611.39	25	18	27	804.13	22	17	17	778.86	0.92	1.01	151.77

(continued)

Table 16.9 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	w_1	w_2	w_3	f_{41}	w_4	w_5	w_6	f_{42}	w_7	w_8	w_9	f_{43}	x	y	d
40	13	19	18	610.73	21	19	23	718.46	22	18	17	797.60	0.93	0.97	112.75
41	26	18	16	744.33	18	17	21	631.73	18	17	15	698.09	1.04	0.97	82.35
42	14	19	18	610.93	23	18	25	753.62	22	17	17	792.76	0.92	0.98	130.37
43	15	17	17	611.57	25	17	28	793.37	22	17	17	777.66	0.92	1.01	144.97
44	18	17	16	629.21	17	4	29	567.58	30	15	16	849.62	0.96	0.88	66.34
45	13	19	18	610.47	19	19	21	661.84	22	18	18	808.34	0.94	0.94	86.11
46	27	17	17	737.69	17	15	19	582.62	18	16	15	700.47	1.05	0.95	51.54
47	29	17	16	764.58	17	8	25	573.21	18	16	16	705.98	1.06	0.94	64.81
48	40	17	17	900.66	17	16	19	597.52	18	16	15	698.21	1.12	0.96	152.93
49	44	16	17	949.66	17	15	19	586.98	18	16	16	698.60	1.14	0.96	175.36
50	36	16	17	844.10	17	16	19	592.86	18	17	16	699.82	1.09	0.96	118.51
51	48	13	17	963.72	17	16	18	571.39	18	17	15	697.90	1.15	0.95	174.07
52	19	18	17	655.12	17	4	28	568.92	27	16	16	835.83	0.98	0.89	74.11
53	22	17	17	679.79	17	12	22	581.63	20	16	16	731.59	1.01	0.93	35.51
54	47	17	17	990.25	17	14	20	570.73	18	16	16	699.90	1.16	0.95	190.16
55	22	18	16	697.01	17	12	23	589.76	18	17	16	710.68	1.02	0.95	38.07
56	20	18	16	665.98	18	16	21	619.98	18	17	16	702.33	1.00	0.96	32.78
57	38	18	17	880.80	17	15	20	598.02	18	17	16	699.13	1.11	0.96	142.28
58	30	18	16	789.62	17	16	21	618.70	18	17	15	698.12	1.06	0.97	101.00

(continued)

Table 16.9 (continued)

Sr. no	Multi-objective GA												3D-RadVis		
	w_1	w_2	w_3	f_{41}	w_4	w_5	w_6	f_{42}	w_7	w_8	w_9	f_{43}	x	y	d
59	22	19	16	697.03	18	18	21	645.27	18	17	14	697.99	1.01	0.98	62.80
60	15	19	16	611.86	18	18	21	656.05	19	18	15	714.21	0.96	0.97	29.22
61	14	19	17	611.10	18	13	24	615.70	25	17	17	824.05	0.95	0.91	68.90
62	48	17	17	612.54	17	15	18	570.00	18	17	16	702.80	1.16	0.95	202.79
63	18	15	17	617.97	17	12	22	584.53	21	16	17	749.40	0.97	0.93	11.77
64	16	17	17	614.41	17	14	21	602.92	20	17	16	730.39	0.97	0.94	9.35
65	32	17	17	809.60	17	15	19	581.69	18	17	16	704.02	1.08	0.95	94.56
66	18	19	16	639.44	21	18	22	689.46	18	17	14	697.92	0.97	1.00	55.02
67	19	17	17	656.28	18	18	21	644.00	18	16	15	701.76	0.99	0.98	40.72
68	15	18	17	611.11	23	16	27	745.83	23	17	17	783.64	0.93	0.98	120.70
69	14	18	18	611.11	26	18	28	823.99	22	17	17	777.57	0.91	1.02	162.33
70	25	19	16	726.83	18	17	21	636.44	18	17	15	698.03	1.03	0.97	74.93

References

1. Amin SH, Zhang G (2012) An integrated model for closed-loop supply chain configuration and supplier selection: Multi-objective approach. Expert Syst Appl 39(8):6782–6791
2. Basnet C, Weintraub A (2009) A genetic algorithm for a bicriteria supplier selection problem. Int Trans Oper Res 16:173–187
3. Cao Y, Luo X, Kwong CK, Tang J (2014) Supplier pre-selection for platform-based products: a multi-objective approach. Int J Prod Res 52(1):1–19
4. He Z, Yen GG (2016) Visualization and performance metric in many-objective optimization. IEEE Trans Evol Comput 20(3):386–402
5. Ibrahim A, Rahnamayan S, Martin MV, Deb K (2017) 3D-RadVis Antenna:visualization and performance measure for many-objective optimization. Swarm and Evol Comput
6. Ibrahim A, Rahnamayan S, Martin MV, Deb K (2018) 3D-RadVis antenna: visualization and performance measure for many-objective optimization. Swarm Evol Comput 39:157–176
7. Li M, Zhen L, Yao X (2017) How to read many-objective solution sets in parallel coordinates. arXiv:1705.00368
8. Murata T, Ishibuchi H, Tanaka H (1996) Multi-objective genetic algorithm and its applications to flowshop scheduling. Comput Ind Eng 30(4):957–968
9. Obayashi S, Sasaki D (2003) Visualization and data mining of Pareto solutions using self-organizing map. Evolutionary multi-criterion optimization, pp 796–809
10. Parks GT, Miller I (1998) Selective breeding in a multi-objective genetic algorithm. In: Proceedings of the parallel problem solving from nature, vol. 5. Springer, Berlin, Germany, pp. 250–259
11. Pryke A, Mostaghim S, Nazemi A (2007) Heatmap visualization of population based multi objective algorithms. Evolutionary multicriterion optimization, pp 361–375
12. Schaffer JD (1984) Some experiments in machine learning using vector evaluated genetic algorithms. Doctoral dissertation, Vanderbilt University, Nashville, Tennessee
13. Seifbarghy M, Esfandiari N (2013) Modeling and solving a multi-objective supplier quota allocation problem considering transaction costs. J Intell Manuf 24(1):201–209
14. Shaw K, Shankar R, Yadav SS, Thakur LS (2012) Supplier selection using fuzzy AHP and fuzzy multi objective linear programming for developing low carbon supply chain. Expert Syst Appl 39:8182–8192
15. Srinivas N, Deb K (1995) Multi-objective function optimization using non-dominated sorting genetic algorithms. Evol Comput 2(3):221–248
16. Srinivas C, Ramgopal Reddy B, Ramjic K, Naveend R (2014) Sensitivity analysis to determine the parameters of genetic algorithm for machine layout. In: 3rd international conference on materials processing and characterisation, vol 6. Procedia Materials Science, p. 866–876
17. Tusar T, Filipic B (2015) Visualization of Pareto front approximation sin evolutionary multi-objective optimization: A critical review and the prosection method. IEEE Trans Evol Comput 19(2):225–245
18. Weber CA, Current JR (1993) Multi objective approach to vendor selection. Eur J Oper Res 68:173–184
19. Weber CA, Current JR, Desai A (2000) An optimization approach to determining the number of vendors to employ. Supply Chain Manag Int J 5(2):90–98

Chapter 17
Allocation of Order Amongst Available Suppliers Using Multi-objective Genetic Algorithm

Azharuddin Shaikh, Poonam Mishra and Isha Talati

Abstract In a supply chain, procurement of items is done on the basis of individual performance, whereas the performance of supply chain can be improved by using scientific techniques. In this chapter, we discuss the manufacturer's problem of procuring several items from the available suppliers; where, supplies from each supplier are constrained. The manufacturer needs to determine which item is to be procured from which supplier and in what quantity. The allocation of order amongst suppliers is done on the basis of multiple criteria such as unit price, quality, supply capacity, delivery time, and unit transportation cost. To demonstrate the scenario, we formulate the mathematical model, which leads to a multi-objective optimization problem. The optimization is done using multi-objective genetic algorithm, which gives a set of Pareto-optimal solutions, then we utilize 3D-RadVis technique to get the best solution. To validate the model, numerical example is presented.

Keywords Multi-objective Genetic Algorithm · Order allocation · Supplier selection · 3D-RadVis

17.1 Introduction

In supply chain activities, organizations of all shapes and sizes are outsourcing different activities to enhance their performance. Amongst these activities, selection of supply partner plays a vital role for their efficient functioning. Several criterion should be considered for selecting the supply partners. For a manufacturing firm, if the suppliers are chosen cleverly, it could lead to an effective supply network. The

A. Shaikh (✉)
Department of Mathematics, Institute of Infrastructure Technology Research and Management, Ahmedabad 380026, Gujarat, India
e-mail: azharuddin_shaikh7077@yahoo.com

P. Mishra · I. Talati
Department of Mathematics, School of Technology, Pandit Deendayal Petroleum University, Raisan, Gandhinagar 382007, India

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*, Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_17

supplier selection can be done more effectively by employing the scientific techniques. The area of selecting supplier has gained ample attention and several researchers have given their contribution for creating a sustainable supply network. Timmerman [23] formulated linear weighting models to study vendor performance evaluation. Weber and Current [27] were the first to use multi-objective programming for selecting vendors under multiple criteria. In their model, different constraints affected the number of vendors to be employed. This problem was solved by [26] with data envelopment analysis (DEA) tool. Amin and Zhang [1] studied supplier selection model in an integrated closed-loop supply chain. Shaw et al. [21] used fuzzy AHP and fuzzy multi-objective linear programming for supplier selection in developing a low carbon supply chain. Seifbarghy and Esfandiari [20] established multi-objective supplier selection model with transportation cost. A supplier preselection model was formulated by [3] for platform-based products. Izadikhah et al. [10] used DEA approach to study supplier's sustainability in the presence of dual-role factor and volume discounts. Later, [11] extended the same work by considering volume discount and negative data.

Genetic algorithm (GA) is one of the commonly used heuristic search techniques that mimics the evolutionary process of nature [5]. It is inspired by Darwin's theory of "Survival of fittest" and is one of the emerging areas of artificial intelligence. It is a calculus-free optimization method with least possibility of getting stuck at local optima. Genetic algorithm starts with a random set of solution called population. This method runs iteratively and in every iteration, population is updated by means of three basic genetic operators, i.e., selection/reproduction, crossover, and mutation. The process is continuously repeated until the desired accuracy is attained and each of this iteration is called generation. The fundamental perception of this algorithm was anticipated by Prof. J. H. Holland of the University of Michigan [7]. Thereafter, this field evolved with the contribution given by number of researchers. One can find the details on the development of this area in the books of [4, 5, 13, 18] and others. [19] used genetic algorithm in machine learning. After that, many researchers utilized GA to solve their optimization problem. Srinivasan and Deb [22] employed nondominated GA to solve their multi-objective optimization problem. Murata et al. [14] scheduled flow shop using multi-objective GA. Parks and Miller [16] did the selection of breeding using multi-objective GA. Basnet and Weintraub [2] formulated supplier selection under bi-criteria and solved it using multi-objective GA. For an overview of evolutionary algorithms for many-objective optimization problems, one can refer to [25].

In multi-objective programming (MOP), the goal is to optimize all the objective functions simultaneously. A single solution may not be optimal for all the objectives simultaneously when objectives are complex in nature. In that case, we get a Pareto-optimal solution set which means the MOP has number of optimal solutions. From this set of solution, we select an appropriate optimal solution. To visualize this set of solution in terms of quality, shape, and distribution of solution set, different methods exists in available literature. [15] used self-organizing map to reveal visualization and data mining of Pareto solutions. For population-based multi-objective algorithms, [17] proposed heatmap visualization. In case of evolutionary multi-objective optimization, refer [24] for a review on visualization of pareto front approximations. For many-objective optimization, [6] gave the performance metric for visualization; while [12] explained how to use parallel coordinates for understanding the solution set. Finally, [8, 9] gave 3D-RadVis technique for reading and measuring the performance of solution set.

In this chapter, we propose an approach to solve manufacturer problem of selecting supplier and allocating the order. After the mathematical modeling, a Pareto-optimal solution set is obtained by employing multi-objective GA. Next, based on literature survey, it has been found that three-dimensional radial coordinate visualization (3D-RadVis) technique is not used by any researcher working on supplier selection. This method maps the multi-objective function (with M-objectives) to a three-dimensional radial coordinate plot. Therefore, we use this technique to get the best solution from the Pareto-optimal solution set. The benefit of using this method is it reserves relative location and distribution of solution set preserving the convergence trend of optimization process without affecting the shape of pareto front. Further, the paper is arranged in the subsequent manner. In Sect. 17.2, notations and assumptions are given that are used to formulate mathematical model along with the description of problem. In Sect. 17.3, the mathematical model under given assumption is formulated. Multi-objective genetic algorithm (MOGA) and 3D-RadVis visualization technique are discussed in Sect. 17.4. In Sect. 17.5, numerical example is presented which is optimized by using (MOGA) and 3D-RadVis visualization technique. Then, conclusion and references concludes the paper.

17.2 Notations and Assumptions

We use the following notations and assumptions for the proposed model:

Notations

$i = 1, 2, \ldots m$	Index of Item
$j = 1, 2, \ldots p$	Index of Supplier
D_i	Demand of ith item
P_i	Manufacturer's processing cost for ith item
MIC_i	Manufacturer's inventory carrying cost for ith item
x_{ij}	Order quantity of ith item from jth supplier (decision variable)
P_{ij}	Unit purchase cost of ith item from jth supplier
O_{ij}	Unit transportation cost of ith item from jth supplier
q_{ij}	Defective quality of ith item from jth supplier
Q_{ai}	Quality acceptable for ith item
l_{ij}	Late delivery of ith item from jth supplier
L_{ai}	Late delivery acceptable for ith item
C_{ij}	Capacity of jth supplier to supply ith item
TC	Manufacturer total cost

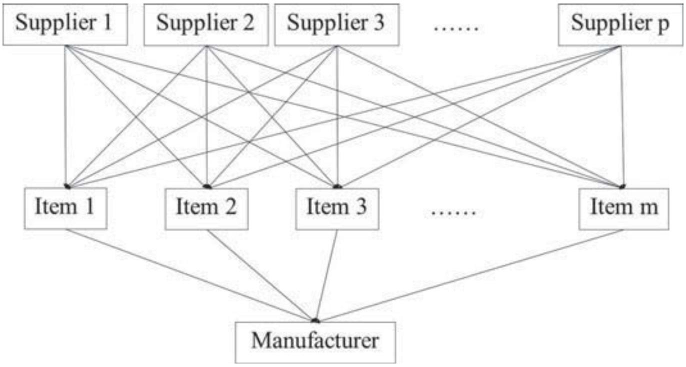


Fig. 17.1 Supply chain with single manufacturer and multiple suppliers

Assumptions

1. Demand of each item is known.
2. Supply capacity from each supplier is limited.
3. Supplier selection is done on the basis of quality, cost, delivery performance, and transportation cost.
4. Allocation of order is to be done in such a way that the demand of each item (i) is satisfied.

Problem

Here, we discuss the manufacturer's problem of procuring (m) items from the (p) available suppliers; where, supplies from each supplier is constrained. The objective is to determine which item is to be procured from which supplier and in what quantity. The allocation of order amongst suppliers is done on the basis of multiple criteria such as unit price, quality, supply capacity, delivery time, and unit transportation cost. The pictorial representation of this supply chain is shown in Fig. 17.1.

Next, we formulate the problem mathematically with an objective of minimizing four functions under given constraints. This multi-objective function is then optimized using MOGA which gives a Pareto-optimal solution set. Next, we utilize 3D-RadVis technique on this solution set to get the best solution.

17.3 Mathematical Model

In the proposed model, our aim is to meet the requirement of manufacturer at a minimal cost. Along with this objective there are certain constraints which are to be taken into consideration.

First, we consider the cost bared by manufacturer, which are as follows:
The **purchase cost** of required item i is given by the sum of product of quantity ordered from jth supplier and selling price of ith item from supplier j. Therefore, the purchase cost for all items is

$$PC = \sum_i \sum_j x_{ij} P_{ij} \tag{17.1}$$

Next, the **processing cost** for all items is given by

$$PRC = \sum_i x_i P_i \tag{17.2}$$

where, $x_i = \sum_j x_{ij}$
The **holding cost** for manufacturer is

$$HC = \sum_i x_i MIC_i \tag{17.3}$$

Finally, the **transportation cost** of item i is given by the sum of quantity ordered from supplier j times the unit transportation charge of item i from supplier j. Therefore, the transportation cost for all items is

$$TRC = \sum_i \sum_j x_{ij} O_{ij} \tag{17.4}$$

Hence, the manufacturer's total cost is given by

$$TC = PC + PRC + HC + TRC \tag{17.5}$$

The next task is to frame the multi-objective function along with the constraints involved for procuring the items.

The two objective functions of minimizing purchase cost (say f_1) and transportation cost (say f_2) have already been discussed in (17.1) and (17.4). Whereas, the other two objective of minimizing the defective quality and late delivery is given by (17.6) and (17.7).

$$f_3 = \sum_i \sum_j x_{ij} q_{ij}; \text{ Defective quality} \tag{17.6}$$

$$f_4 = \sum_i \sum_j x_{ij} l_{ij}; \text{ Late delivery} \tag{17.7}$$

Further, the constrained involved are

1. The supplied item (i) from the available suppliers should be adequate to meet the manufacturer demand. That is

$$\sum_j x_{ij} \geq D_i \tag{17.8}$$

2. The quantity of item (i) obtained from available supplier (j) should be less than or equal to supply capacity of supplier (j). That is

$$x_{ij} \leq C_{ij} \tag{17.9}$$

3. The aggregate defective quality of item (i) ordered should be less than or equal to acceptable quality of manufacturer. That is

$$\sum_j x_{ij} q_{ij} \leq Q_{ai} D_i \tag{17.10}$$

4. The aggregate late delivery time of item (i) ordered should be less than or equal to acceptable delivery time of manufacturer. That is

$$\sum_j x_{ij} l_{ij} \leq L_{ai} D_i \tag{17.11}$$

Multi-objective Function

Hence, the goal is to allocate order such that the manufacturer gets good quality of material in lesser delivery time with minimum purchase cost and transportation cost. So, the manufacturer has the following multi-objective function with certain constraints:

Minimize,

$$\begin{aligned} f_1 &= \sum_i \sum_j x_{ij} P_{ij}; && \text{Purchase cost} \\ f_2 &= \sum_i \sum_j x_{ij} O_{ij}; && \text{Transportation cost} \\ f_3 &= \sum_i \sum_j x_{ij} q_{ij}; && \text{Defective quality} \\ f_4 &= \sum_i \sum_j x_{ij} l_{ij}; && \text{Late delivery} \end{aligned} \tag{17.12}$$

Subject to,

$$\begin{aligned} &\sum_j x_{ij} \geq D_i; && \text{Demand constraint} \\ &x_{ij} \leq C_{ij}; && \text{Supplier capacity constraint} \\ &\sum_j x_{ij} q_{ij} \leq Q_{ai} D_i; && \text{Acceptable quality constraint} \\ &\sum_j x_{ij} l_{ij} \leq L_{ai} D_i; && \text{Acceptable late delivery constraint} \end{aligned}$$

17.4 Algorithm

17.4.1 Multi-objective Genetic Algorithm

Generally, a multi-objective optimization problem is represented as

$$\min\ (\bar{f}(\bar{x})) = (f_1(\bar{x}), f_2(\bar{x}), \ldots f_n(\bar{x}))$$
$$\text{subject to, } \bar{c}(\bar{x}) \leq 0$$

where $f_i : R_n \rightarrow R_m$ is the list of objective function; $\bar{c}(\bar{x})$ is the list of constraints and $\bar{x} \in S$ (feasible region).

In MOP, the aim is to optimize all the objective functions simultaneously. A single solution may not optimize all objectives simultaneously when objectives are complex in nature. In that case, we get a Pareto-optimal solution set, which means the MOP has number of optimal solutions. The solution obtained are such that one cannot further optimize any of the objective without affecting at least one of the other objective values. This Pareto-optimal solution set is termed as Pareto-optimal front.

Genetic algorithm (GA) is one of the commonly used heuristic search techniques that mimics the evolutionary process of nature. It is inspired by Darwin's theory of "Survival of fittest" and is one of the emerging area of artificial intelligence. It is a calculus free optimization method. GA starts with a random set of solution called population. This method runs iteratively and in every iteration population is updated by means of three basic genetic operators, i.e., selection/reproduction, crossover, and mutation giving successfully a better and better solution. The process is continuously repeated until the desired accuracy is attained and each of this iteration is called generation.

Here, to minimize the objective functions given in (17.12) subject to the given constraints, we use the algorithm given below:

1. For the involved parameters, allot numerical values except for the decision variables (x_{ij}).
2. Start with an initial population of 50 for five or less decision variables else with 200.
3. Evaluate the fitness value for each individual of this population and rank the individual on the basis of fitness score.
4. From the population, individuals with good fitness score will enter the mating pool.
5. Next for reproduction, perform stochastic uniform crossover with 0.8 fraction value and considering two elites at each generation.
6. For the new generation obtained repeat from Step 3.
7. Repeat the algorithm till the desired accuracy of (10^{-4}) is attained.

17.4.2 3D-RadVis Visualization Technique

In multi-objective optimization problem, imagining Pareto-optimal solution or non-dominated solutions is not an easy task. For the obtained solutions, an effective visualization technique is required to study their distribution, position, range and shape. The commonly used existing methods fails to show the shape of pareto front. Therefore, in this chapter we use three-dimensional radial coordinate visualization (3D-RadVis). This method maps the multi-objective function (with M-objectives) to a three-dimensional radial coordinate plot. Therefore, we use this technique to get the best solution from the Pareto- optimal solution set. The benefit of using this method is it reserves relative position and distribution of solution set preserving the convergence trend of optimization process without affecting the shape of pareto front. References [8, 9] for detailed explanation of this method. Next, the algorithm to obtain 3D-RadVis plot is listed below. For N Pareto-optimal, the solution of M objectives is

1. Compute

$$x = \frac{\sum_{j=1}^{M} f_{i,j}^{Norm} \cos\left(\theta_j\right)}{\sum_{j=1}^{M} f_{i,j}^{Norm}}; y = \frac{\sum_{j=1}^{M} f_{i,j}^{Norm} \sin\left(\theta_j\right)}{\sum_{j=1}^{M} f_{i,j}^{Norm}}$$

$$\text{where } f_i^{Norm} = \frac{f_i(x) - \min(f_i(x))}{\max(f_i(x)) - \min(f_i(x))}$$

2. $px = x + 1$ and $py = y + 1$
3. Find normal vector perpendicular to the extreme point $n = norm(z)$; where z is hyperplane.
4. Calculate $c = n \cdot z_1$
5. For $i = 1$ to n find $d = \frac{abs(f_i \cdot n - c)}{\|n\|}$
6. Finally, we convert 3D-RadVis $R = [x, y, d]$

17.5 Numerical Example

Example 1 To provide managerial insight, let us consider a manufacturer's problem of procuring two items from three suppliers with the available data $D_1 = 50$ units, $D_2 = 80$ units, $Q_{a1} = 5\%, Q_{a2} = 8\%, L_{a1} = 3\%, L_{a2} = 5\%, P_1 = 10\ (\$/unit), P_2 = 5\ (\$/unit), MIC_1 = 2\ (\$/unit)$, and $MIC_2 = 3\ (\$/unit)$ other details of the suppliers is given in Table 17.1.

We use the MOGA to acquire a Pareto-optimal solution set, the solution set obtained is given in Table 17.2. Next, 3D-RadVis technique in applied to this solution set and values of $R = [x, y, d]$ is also shown in Table 17.2. For the obtained solution, 3D-RadVis plot is shown in Fig. 17.2, while normalized 3D-RadVis plot is shown in

Table 17.1 Supplier Infromation

Supplier's	Supplier 1		Supplier 2		Supplier 3	
Items	1	2	1	2	1	2
Price ($)	8	20	10	15	9	18
Unit transportation cost ($)	0.8	1	1.2	1.2	0.5	1.5
Supplier capacity	40	100	30	50	50	70
Quality (%)	7	3	4	4	6	5
Late delivery per unit time	0.01	0.05	0.02	0.1	0.025	0.07

Fig. 17.2 3D-RadVis plot

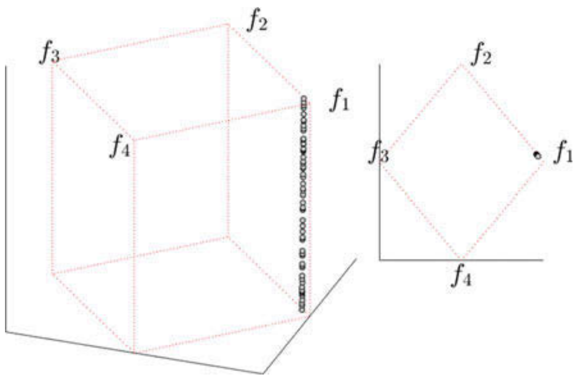


Fig. 17.3 Normalized 3D-RadVis plot

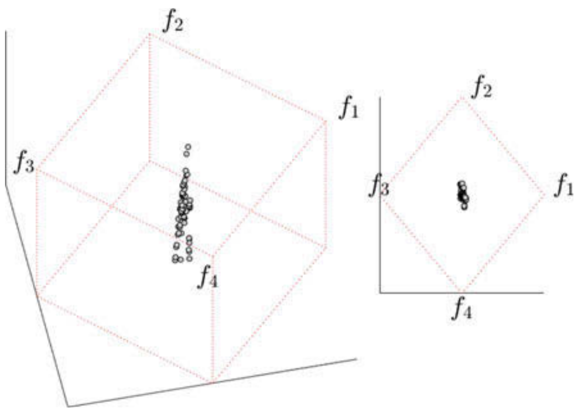


Fig. 17.3. Hence, after using this technique, the optimal order allocated among the available suppliers is highlighted in Table 17.2. Therefore, the manufacturer's total cost is $3288.32 corresponding to this solution.

Table 17.2 Pareto-optimal front for available suppliers

	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x	y	d
1	4	30	16	2	8	70	0.9114	0.0758	5.5800
2	6	28	17	72	8	0	0.9309	0.0562	57.2450
3	20	30	0	2	8	70	0.9088	0.0785	0.0000
4	7	29	15	74	6	0	0.9309	0.0564	62.8450
5	6	30	15	74	6	0	0.9308	0.0565	64.0350
6	7	28	16	73	7	1	0.9308	0.0564	61.1950
7	5	30	15	9	10	61	0.9135	0.0737	12.9250
8	10	29	12	35	8	38	0.9200	0.0672	31.1100
9	6	28	17	50	7	23	0.9250	0.0622	51.6100
10	9	28	13	55	8	17	0.9257	0.0615	43.3100
11	7	29	14	52	8	21	0.9252	0.0620	48.0450
12	7	28	16	59	8	13	0.9273	0.0598	50.3350
13	7	28	16	74	6	0	0.9311	0.0561	62.0050
14	19	30	1	6	8	66	0.9101	0.0772	4.9300
15	6	30	15	72	6	1	0.9304	0.0568	62.9000
16	3	29	18	3	8	69	0.9122	0.0750	5.4100
17	6	28	16	39	7	34	0.9218	0.0653	34.8250
18	9	28	13	48	7	26	0.9237	0.0635	42.8150
19	6	29	15	71	7	3	0.9301	0.0572	61.9250
20	5	28	17	56	8	17	0.9267	0.0605	50.1600
21	5	27	17	57	7	16	0.9270	0.0601	54.2800
22	1	25	23	3	8	69	0.9133	0.0737	4.3050
23	5	28	17	46	7	27	0.9240	0.0632	40.1500
24	7	28	16	66	8	7	0.9290	0.0581	57.5450
25	7	28	14	43	8	30	0.9226	0.0645	38.1600
26	7	28	16	59	8	13	0.9271	0.0600	48.8100
27	17	30	4	15	8	57	0.9133	0.0740	13.3850
28	0	25	25	2	8	70	0.9134	0.0736	3.0100
29	6	28	16	62	8	11	0.9281	0.0590	51.8600
30	19	30	1	4	8	68	0.9095	0.0778	1.9650
31	6	27	16	53	8	20	0.9257	0.0615	48.6000
32	5	30	15	14	10	57	0.9149	0.0723	17.0550
33	11	29	11	26	7	47	0.9175	0.0698	25.4900
34	7	29	14	64	7	9	0.9281	0.0592	55.8150
35	7	28	15	56	8	16	0.9263	0.0609	46.2850
36	6	28	16	31	8	41	0.9198	0.0674	30.3850
37	6	27	17	61	8	12	0.9281	0.0590	51.8300
38	10	29	11	28	8	44	0.9182	0.0690	27.2200

(continued)

Table 17.2 (continued)

	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x	y	d
39	17	30	3	9	8	63	0.9112	0.0761	9.3450
40	8	29	14	54	7	19	0.9256	0.0616	48.5600
41	7	28	15	36	7	37	0.9208	0.0664	32.6100
42	12	29	9	44	7	29	0.9223	0.0650	36.7200
43	11	29	11	42	8	30	0.9220	0.0652	34.8700
44	13	30	7	10	8	62	0.9123	0.0750	10.3650
45	16	30	4	12	7	61	0.9125	0.0748	16.9350
46	7	28	15	60	7	13	0.9274	0.0597	51.5850
47	5	30	15	16	10	55	0.9154	0.0718	17.9300
48	7	28	15	60	8	12	0.9274	0.0597	50.1700
49	7	29	14	20	7	52	0.9165	0.0708	21.8500
50	4	27	19	42	8	31	0.9232	0.0640	36.0800
51	6	28	16	68	8	5	0.9295	0.0577	59.3050
52	13	30	7	36	7	37	0.9196	0.0677	30.9000
53	12	29	10	24	7	49	0.9168	0.0705	23.7950
54	9	29	12	51	7	22	0.9243	0.0629	44.8900
55	6	28	17	40	8	34	0.9222	0.0649	40.2900
56	5	30	15	13	10	58	0.9145	0.0727	16.4650
57	5	30	15	2	8	70	0.9114	0.0759	5.5350
58	12	29	9	22	7	51	0.9159	0.0713	21.2800
59	13	28	9	2	8	70	0.9105	0.0767	1.3400
60	12	29	9	23	8	50	0.9162	0.0710	22.0050
61	5	30	16	11	10	60	0.9140	0.0732	13.9800
62	7	29	14	44	8	29	0.9230	0.0642	41.0100
63	6	29	15	45	8	28	0.9234	0.0638	44.3300
64	19	30	1	7	8	65	0.9104	0.0769	5.9000
65	7	29	15	51	8	22	0.9248	0.0624	50.5700
66	6	28	16	64	8	8	0.9286	0.0585	54.9050
67	6	28	16	52	8	20	0.9256	0.0616	47.9800
68	7	29	14	52	8	21	0.9252	0.0620	48.4400
69	1	25	24	8	8	64	0.9149	0.0722	8.3400
70	20	30	0	3	7	70	0.9092	0.0782	6.8650

17.6 Conclusion

This chapter discusses the manufacturer problem of allocating order among available suppliers. The selection criteria depends on quality of items delivered, delivery time taken, unit cost, and transportation cost. This scenario is modeled mathematically and it leads to multi-objective optimization problem with certain constraints. Using hypothetical data, we acquire a Pareto-optimal solution set by employing heuristic search algorithm called MOGA. Finally, we get the best solution for the manufacturer by using 3D-RadVis method.

Further, this research work can be extended by incorporating some more selection criterion. Quantity discounts is also in fashion, so discount on purchased quantity from supplier if order is above threshold quantity can also be considered. One can also have more number of manufacturing units with finite production capacity and then there could be two MOP, i.e., (1) allocating order among suppliers and (2) allocating lot size among available plants. In a similar manner, one can incorporate distributors and retailer selection.

References

1. Amin SH, Zhang G (2012) An integrated model for closed-loop supply chain configuration and supplier selection: Multi-objective approach. Expert Syst Appl 39(8):6782–6791
2. Basnet C, Weintraub A (2009) A genetic algorithm for a bicriteria supplier selection problem. Int Trans Oper Res 16(2):173–187
3. Cao Y, Luo X, Kwong C, Tang J (2014) Supplier pre-selection for platform-based products: a multi-objective approach. Int J Prod Res 52(1):1–19
4. Deb K (2012) Optimization for engineering design: Algorithms and examples. PHI Learning Pvt Ltd
5. Golberg DE (1989) Genetic algorithms in search, optimization, and machine learning. Addion Wesley, Reading
6. He Z, Yen GG (2016) Visualization and performance metric in many-objective optimization. IEEE Trans Evol Comput 20(3):386–402
7. Holland JH (1975) Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence
8. Ibrahim A, Rahnamayan S, Martin MV, Deb K (2016) 3d-radvis: Visualization of pareto front in many-objective optimization. In: 2016 IEEE congress on evolutionary computation (CEC). IEEE, pp 736–745
9. Ibrahim A, Rahnamayan S, Martin MV, Deb K (2018) 3d-radvis antenna: visualization and performance measure for many-objective optimization. Swarm Evol Comput 39:157–176
10. Izadikhah M, Saen RF, Ahmadi K (2017) How to assess sustainability of suppliers in the presence of dual-role factor and volume discounts? A data envelopment analysis approach. Asia-Pac J Oper Res 34(03):1740016
11. Izadikhah M, Saen RF, Roostaee R (2018) How to assess sustainability of suppliers in the presence of volume discount and negative data in data envelopment analysis? Ann Oper Res 1–27
12. Li M, Zhen L, Yao X (2017) How to read many-objective solution sets in parallel coordinates [educational forum]. IEEE Comput Intell Mag 12(4):88–100
13. Michalewicz Z (1996) Evolution strategies and other methods. In: Genetic Algorithms+ Data Structures = Evolution Programs. Springer, pp 159–177
14. Murata T, Ishibuchi H, Tanaka H (1996) Multi-objective genetic algorithm and its applications to flowshop scheduling. Comput Ind Eng 30(4):957–968
15. Obayashi, S. and Sasaki, D. (2003). Visualization and data mining of pareto solutions using self-organizing map. In *International Conference on Evolutionary Multi-Criterion Optimization*, pages 796–809. Springer

16. Parks GT, Miller I (1998) Selective breeding in a multiobjective genetic algorithm. In: International conference on parallel problem solving from nature. Springer, pp 250–259
17. Pryke A, Mostaghim S, Nazemi (2007) Heatmap visualization of population based multi objective algorithms. In: International conference on evolutionary multi-criterion optimization. Springer, pp 361–375
18. Sakawa M (2012) Genetic algorithms and fuzzy multiobjective optimization, vol 14. Springer Science & Business Media
19. Schaffer JD (1984) Some experiments in machine learning using vector evaluated genetic algorithms (artificial intelligence, optimization, adaptation, pattern recognition)
20. Seifbarghy M, Esfandiari N (2013) Modeling and solving a multi-objective supplier quota allocation problem considering transaction costs. J Intell Manuf 24(1):201–209
21. Shaw K, Shankar R, Yadav SS, Thakur LS (2012) Supplier selection using fuzzy ahp and fuzzy multi-objective linear programming for developing low carbon supply chain. Expert Syst Appl 39(9):8182–8192
22. Srinivasan N, Deb K (1994) Multi-objective function optimisation using non-dominated sorting genetic algorithm. Evol Comput 2(3):221–248
23. Timmerman E (1986) An approach to vendor performance evaluation. J Purch Mater Manag 22(4):2–8
24. Tušar T, Filipič B (2015) Visualization of pareto front approximations in evolutionary multi-objective optimization: a critical review and the prosection method. IEEE Trans Evol Comput 19(2):225–245
25. von Lücken C, Brizuela C, Barán B (2019) An overview on evolutionary algorithms for many-objective optimization problems. Wiley interdisciplinary reviews: data mining and knowledge discovery 9(1):e1267
26. Weber CA, Current J, Desai A (2000) An optimization approach to determining the number of vendors to employ. Supply Chain Manag Int J 5(2):90–98
27. Weber CA, Current JR (1993) A multiobjective approach to vendor selection. Eur J Oper Res 68(2):173–184

Chapter 18
Some Studies on EPQ Model of Substitutable Products Under Imprecise Environment

R. L. Das and R. K. Jana

Abstract In current scenario, big departmental stores used to work more efficiently with the items that can be substituted either with optimum order quantities or selling prices of the products. At the time of purchase, customer of one particular item transfers to relevant substitutable item because of difference in prices or the quantities that can be purchased in bulk. In this chapter, the inventory problem is determined in total profit maximization problem with crisp, random, fuzzy, fuzzy-random, rough, and fuzzy-rough constraints. The problem is solved through a gradient-based search technique—GRG (Generalized Reduced Gradient) method. The prices and optimal order quantities of substitutable items are obtained so that total profit for store owner is maximum.

Keywords Selling price · Optimal order quantities · Substitution · Inventory problems · Profit · EPQ model · Finite budget constraint

18.1 Introduction

Inventories are the essential thing needed on most of the places [9]. The function of inventory is inimitable if it passes through the channel of producer to distributor and then finally to a customer. But the absence of inventory shows a tragic moment for customers as far as future profit is concerned. When product becomes out of stock the behavior of customer changes, henceforth, the following situation arises; a customer may step aside from that shop and move to some other place or he may wait for an item or he may take similar product from the store itself. The phenomenon in the third case is known as product substitution [11]. In a nutshell, it depicts how inventory of product will not only effect the demand but also the demand of other products.

R. L. Das (✉) · R. K. Jana
Department of Applied Mathematics and Humanities, SVNIT, Surat 395007, Gujarat, India
e-mail: rakeshdas.svnit@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_18

Further, there are several researchers [1, 4, 8, 14, 15, 21, 24, 30, 33–36] who has worked with EPQ models under different segment that includes prices deterioration, price discount, trade credit, product back-logging, stock dependent, and many such criterions that gave a rise to workout with product substitution.

Later, Seyed et al. [28] showed that SQP has satisfactory performance in terms of optimum solutions, number of iterations to achieve the optimum solution, infeasibility, optimal error, and complementary. Analyzing the models having all the parameters with random variables, the problem can be expressed in various other constraints known as chance constraint, which help to convert the problem into crisp type. Charnes constraint was initially developed by Charnes et al. [5]. Further, Liu and Iwamura [17] inculcated fuzzy parameters in the problem. To reduce the problems into crisp ones, Charnes constraint is the vital source used by Panda et al. [25], Katagiri et al. [13], Liu [16]. Ultimately, Merigo et al. [19] gave an overview of fuzzy research with concerned journals, papers, institutions, and the authors with a picturesque representation that becomes easy to analyze with bibliometric indicators.

Conventional inventory models were usually developed over infinite timing and planning horizon. As any system involves cost like technological change, design, and specification change, managerial prospects change, it is reluctant to fulfill assumption of infinite planning horizon by [6, 12]. Moreover, the likeliness of some particular product also depends on seasonal business period. In order to overcome with as such knot, it is better to use finite budget horizon as random. Moon et al. [22] developed an EOQ model in random planning horizon. Astonishingly, Cárdenas–Barrón et al. [2] covered 40+ papers covering extensive scope of inventory management. Later, researchers were able to propagate with new directions. Besides, Maiti et al. [18] solved inventory models with stock dependent demand under random planning horizon. Toward this way, there are many more research work carried by Roy et al. [27] and Guria et al. [10]. Recently, in 2016, Nobil et al. [23] considered multi-product problems with nonidentical machines that consist of different production capacities with the effective use of Hybrid Genetic Algorithm that dominates the results of general algebraic modeling system. The determination of production-shipment policies for a vendor–buyer system by deriving the optimal replenishment lot size and shipment policy for an EPQ inventory model was derived by Cárdenas–Barrón et al. [3]. Taleizadeh et al. [31] showed a manufacturing process of each production cycle, where defective items are identified whilst considering scrap and the others that would convert in a perfect quality items, solving a real life case problem. In 2018, a supply chain model with integrated thermal recovery and electricity generation from industrial waste heat was analyzed by Zanoni et al. [39]. Again, Taleizadeh et al. [32] derives Vendor Managed Inventory model for two-level supply chain that optimizes the retail price along with the replenishment duration of raw materials.

Henceforth, Merigo et al. [20] portrayed average price useful for firm, countries, or regions. Moreover, while studying range of opinions and environmental uncertainties, we can calculate world average price.

- In general, the practicality of substitution between products can be beneficial to include in the needs of customer satisfaction effects on classical inventory models. Furthermore, it can determine the positions of customers changing their priorities due to optimum order quantities and selling price issues which is our main concern. Also, it is a fact that the demand of an item is influenced by the selling price of that item, i.e., whenever the selling price of an item increases, the demand of that item decreases and vice versa.
- Unit production cost is normally assumed constant in EPQ Models. Whereas, in reality, it depends on several factors such as raw materials, labors engaged, and rate of production. This involves some expenditures and hence unit production cost increases with this process.

So far, these lacunas were ignored by the researchers. The main purpose of this present study is to develop a mathematical model with and without shortages for computing the economic order quantities, where production process and substitution effect are taken into account. Further, the models are formulated in the form of a profit maximization problem with crisp, random, fuzzy, fuzzy-random, rough, and fuzzy-rough constraints and then solving it through a gradient-based search technique—GRG (Generalized Reduced Gradient) method. The optimal order quantities and qualities of substitutable products are determined so that total profit for the store owner is maximum. Also, the concavity of the models is derived with providing two illustrations for the practical usage of the proposed methods.

18.2 Mathematical Prerequisites

Following to Liu and Iwamura [17], the following Lemmas 1 and 2 are easily derived.

Lemma 1 *If* $\tilde{a} = (a_1, a_2, a_3)$ *be a TFN with* $0 < a_1$ *and b is a crisp number,* $Pos(\tilde{a} > b) \geq \alpha$ *iff* $\frac{a_3 - b}{a_3 - a_2} \geq \alpha$, *where "Pos" represents possibility measure.*

Lemma 2 *If* $\tilde{a} = (a_1, a_2, a_3)$ *be a TFN with* $0 < a_1$ *and b is a crisp number,* $Nes(\tilde{a} > b) \geq \alpha$ *iff* $\frac{b - a_1}{a_2 - a_1} \leq 1 - \alpha$, *where "Nes" represents necessity measure.*

Fuzzy-random variable [29]: Let R is the set of real numbers, $F_c(R)$ is set of all fuzzy variables, and $G_c(R)$ is all of non-empty bounded close interval. In a given probability space (Ω, F, P), a mapping $\xi : \Omega \to F_c(R)$ is called a fuzzy-random variable in (Ω, F, P), if $\forall\, \alpha \in$(0,1], the set-valued function $\xi_\alpha : \Omega \to G_c(R)$ defined by $\xi_\alpha(\omega) = (\xi(\omega))_\alpha = \{x | x \in R, \mu_{\xi(\omega)}(x) \geq \alpha\}, \forall \omega \in \Omega$, is F measurable. Different semantics of fuzzy-random variable are also presented by Xu and Zhou [37].

Theorem 1 *Let* $\tilde{\tilde{\xi}}$ *is LR fuzzy- random variable, for any* $\omega \in \Omega$, *the membership function of* $\tilde{\tilde{\xi}}(\omega)$ *is*

$$\mu_{\tilde{\bar{\xi}}(\omega)}(t) = \begin{cases} L\left(\frac{\bar{\xi}(\omega)-t}{\xi_L}\right) & for\ t \le \bar{\xi}(\omega) \\ R\left(\frac{t-\bar{\xi}(\omega)}{\xi_R}\right) & for\ t \ge \bar{\xi}(\omega) \end{cases}$$

where the random variable $\bar{\xi}(\omega)$ is normally distributed with mean m_ξ and standard deviation σ_ξ and ξ_L, ξ_R are the left and right spreads of $\tilde{\bar{\xi}}(\omega)$. The reference functions L: $[0, 1] \to [0, 1]$, R: $[0, 1] \to [0, 1]$ satisfies that $L(1) = R(1) = 0$, $L(0) = R(0) = 1$, and both are monotone function. Then

$$\begin{cases} Pr[Pos\{\tilde{\bar{\xi}}(\omega) \ge t\} \ge \delta] \ge \gamma \\ Pr[Nec\{\tilde{\bar{\xi}}(\omega) \ge t\} \ge \delta] \ge \gamma \end{cases} \text{are equivalent to}$$

$$t \le \begin{cases} m_\xi + \sigma_\xi \Phi^{-1}(1-\gamma) + \xi_R R^{-1}(\delta) \\ m_\xi + \sigma_\xi \Phi^{-1}(1-\gamma) - \xi_L L^{-1}(1-\delta) \end{cases}$$

where Φ is standard normally distributed, $\delta, \gamma \in [0, 1]$ are predetermined confidence levels.

Proof According to definition of possibility [7, 22, 38] we get, $Pos[\tilde{\bar{\xi}}(\omega) \ge t] \ge \delta \Leftrightarrow R\left[\frac{t-\bar{\xi}(\omega)}{\xi_R}\right] \le \delta \Leftrightarrow \bar{\xi}(\omega) \ge t - \xi_R R^{-1}(\delta)$. So for predetermined level $\delta, \gamma \in [0, 1]$ we have

$$\begin{aligned} & Pr[Pos\{\tilde{\bar{\xi}}(\omega) \ge t\} \ge \delta] \ge \gamma \\ \Leftrightarrow\ & Pr[\bar{\xi}(\omega) \ge t - \xi_R R^{-1}(\delta)] \ge \gamma \\ \Leftrightarrow\ & Pr[\tfrac{\bar{\xi}(\omega)-m_\xi}{\sigma_\xi} \ge \tfrac{t-\xi_R R^{-1}(\delta)-m_\xi}{\sigma_\xi}] \ge \gamma \\ \Leftrightarrow\ & \Phi\left(\frac{t-\xi_R R^{-1}(\delta)-m_\xi}{\sigma_\xi}\right) \le 1-\gamma \\ \Leftrightarrow\ & t \le m_\xi + \sigma_\xi \Phi^{-1}(1-\gamma) + \xi_R R^{-1}(\delta) \end{aligned}$$

Similarly from the measure of necessity [7, 22, 38], we have

$$Nes[\tilde{\bar{\xi}}(\omega) \ge t] \ge \delta \Leftrightarrow L\left[\frac{\bar{\xi}(\omega)-t}{\xi_L}\right] \ge 1-\delta \Leftrightarrow \bar{\xi}(\omega) \ge t + \xi_L L^{-1}(1-\delta)$$

So for predetermined level $\delta, \gamma \in [0, 1]$, we have

$$\begin{aligned} & Pr[Nes\{\tilde{\bar{\xi}}(\omega) \ge t\} \ge \delta] \ge \gamma \\ \Leftrightarrow\ & t \le m_\xi + \sigma_\xi \Phi^{-1}(1-\gamma) - \xi_L L^{-1}(1-\delta) \end{aligned}$$

The proof is complete.

Theorem 2 *Let $\hat{\xi} = ([a, b][c, d])$, $c \le a \le b \le d$ be a rough variable and a rough event is $\hat{\xi} \ge t$. Then $Tr\{\hat{\xi} \ge t\} \ge \alpha$ iff*

$$t \le \begin{cases} d - \frac{\alpha(d-c)}{\eta}, & b \le t \le d \\ \frac{\eta(b-a)+(1-\eta)b(d-c)-\alpha(d-c)(b-a)}{\eta(b-a)+(1-\eta)(d-c)}, & a \le t \le b \\ d + \frac{(1-\eta-\alpha)(d-c)}{\eta}, & c \le t \le a \\ c \end{cases}$$

Theorem 3 *Let* $\tilde{\hat{\xi}} = (\hat{\xi} - L, \hat{\xi}, \hat{\xi} + R)$ *is a fuzzy- rough variable characterized the following membership function:*

$$\mu_{\tilde{\hat{\xi}}}(t) = \begin{cases} \frac{t-\hat{\xi}+\xi_L}{\xi_L} & \textit{for } \hat{\xi} - \xi_L \le t \le \hat{\xi} \\ \frac{\hat{\xi}+\xi_R-t}{\xi_R} & \textit{for } \hat{\xi} \le t \le \hat{\xi} + \xi_R \\ 0 & \textit{otherwise.} \end{cases}$$

where ξ_L *and* ξ_R *are left and right spreads of* $\tilde{\hat{\xi}}$, *and* $\hat{\xi} = ([a, b][c, d])$ *be a rough variable, characterized by the above mentioned trust measure function, then for an event* $\tilde{\hat{\xi}} \ge t$,

$$\begin{cases} Tr[Pos(\tilde{\hat{\xi}} \ge t) \ge \beta] \ge \alpha \\ Tr[Nec(\tilde{\hat{\xi}} \ge t) \ge \beta] \ge \alpha \end{cases} \quad \textit{are equivalent to}$$

$$\begin{cases} t \le \begin{cases} d - \frac{\alpha(d-c)}{\eta} + (1-\beta)\xi_R, & b \le t - (1-\beta)\xi_R \le d \\ \frac{\eta(b-a)+(1-\eta)b(d-c)-\alpha(d-c)(b-a)}{\eta(b-a)+(1-\eta)(d-c)} + (1-\beta)\xi_R, & \\ \quad a \le t - (1-\beta)\xi_R \le b & \\ d + \frac{(1-\eta-\alpha)(d-c)}{\eta} + (1-\beta)\xi_R, & c \le t - (1-\beta)\xi_R \le a \\ c + (1-\beta)\xi_R & \end{cases} \\ t \le \begin{cases} d - \frac{\alpha(d-c)}{\eta} - \beta\xi_L, & b \le t + \beta\xi_L \le d \\ \frac{\eta(b-a)+(1-\eta)b(d-c)-\alpha(d-c)(b-a)}{\eta(b-a)+(1-\eta)(d-c)} - \beta\xi_L, & a \le t + \beta\xi_L \le b \\ d + \frac{(1-\eta-\alpha)(d-c)}{\eta} - \beta\xi_L, & c \le t + \beta\xi_L \le a \\ c - \beta\xi_L & \end{cases} \end{cases}$$

18.3 Assumptions and Notations

18.3.1 Assumptions

- Multi-product Economical Production Quantity inventory models are considered for ith items (where $i = 1, 2$). Products are substituted on the basis of their selling prices.
- Lead time is zero with single period model.
- Shortages are partially allowed in one of the model.
- The inventory system considers two substitutable items and the demand of the items are price dependent.
- The time horizon is infinite also unit production cost as well as setup cost is constant.
- The inventory in building up at a constant rate of $(P_i - D_i)$ units per unit time during $[0, t_i]$.

- During substitution, demand of a product is more or equally sensitive to the changes due to its own price than the changes due to its competitors price.
  That is, for the demand set $d_{11} \geq d_{12}$ and $d_{21} \geq d_{22}$.
- During substitution, loss of customers of product-1 is due to its own price is more or equal than the gain of customers of product-2 due to price of product-1.
  That is, for the demand set $d_{11} \geq d_{22}$ and $d_{21} \geq d_{12}$.
- For two substitutable products under price with demands(1), there is loss of sales (i.e., cust.) or no loss in the system if and only if $s_1(d_1\text{-}d_2)+s_2(d_2\text{-}d_1) \geq 0$.
  That is, for the demand set $d_{11} \geq \text{Max}(d_{12}, d_{21})$, $d_{22} \geq \text{Max}(d_{12}, d_{21})$.

18.3.2 Notations

Decision Variables:

- Q_i: Total quantity produced unit for both the items.
- s_i: Selling price for both the items.

Parameters:

- t_i: Production run time in one cycle for first and second item.
- T: Cycle time in appropriate unit.
- D_i: Effected/Resulted Demand in the market.
  where, $D_1 = d_{10} - d_{11}s_1 + d_{12}s_2$, $D_2 = d_{20} + d_{21}s_1 - d_{22}s_2$
- P_i: Production rates in unit per unit time.
- Ch_i: Holding cost per unit per unit time.
- C_{0i}: Setup cost per unit per time period.
- Pc_i: Production cost for both the item.
- d_{i0}: Market based original Demand.
- d_{i1}, d_{i2}: Measures of each products consumer demand to its own price and competitor's price, respectively.
- B: Finite Budget constraint.

18.4 Model 1 : EPQ Model Having Substitution with the Constant Demand and Same Time Period

See Fig. 18.1.

18.4.1 Model Formulation

In this EPQ model during production time, t_i inventory increases at rate P_i and decreases at rate $(P_i - D_i)$ units during a production run for the ith item. For

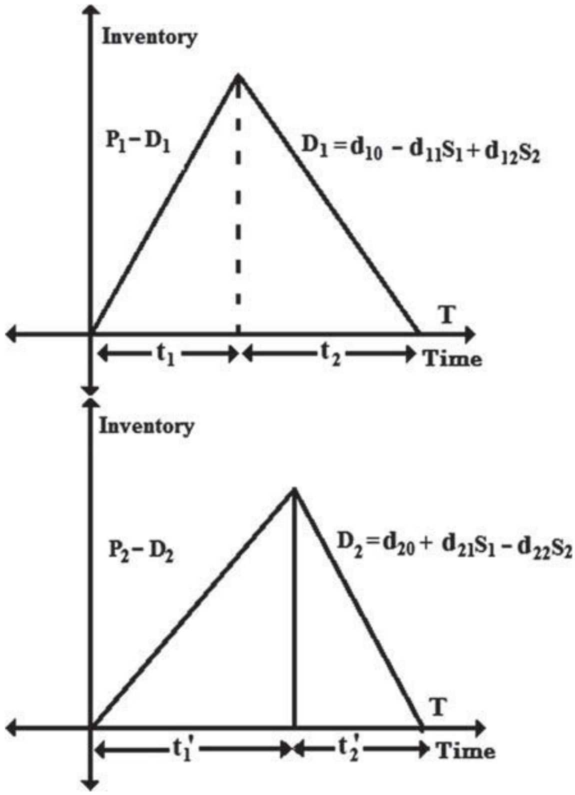


Fig. 18.1 Substitution with constant demand and same time period

multi-item production processes with different demand functions, the governing differential equations are

$$\frac{dI_1}{dt} = \begin{cases} P_1 - D_1, & 0 \le t \le t_1 \\ -D_1, & t_1 \le t \le T \end{cases}$$

$$\frac{dI_2}{dt} = \begin{cases} P_2 - D_2, & 0 \le t \le t_2 \\ -D_2, & t_2 \le t \le T \end{cases}$$

with the boundary conditions,

$$I_1(0) = 0 = I_1(T),$$
$$I_2(0) = 0 = I_2(T)$$

Also, continuity conditions hold at t_1 and t_2. Hence, solving the above equations, we have

$$I_1 = \begin{cases} (P_1 - D_1)t, & 0 \le t \le t_1 \\ D_1(T - t), & t_1 \le t \le T \end{cases}$$

$$I_2 = \begin{cases} (P_2 - D_2)t, & 0 \le t \le t_1 \\ D_2(T - t), & t_1 \le t \le T \end{cases}$$

According to our problem

$$t_1 + t_2 = t_1' + t_2' = T$$

Using continuity condition at t_1, we have

$$(P_1 - D_1)t_1 = D_1(T - t_1) \Rightarrow T = \frac{P_1 t_1}{D_1} = \frac{Q_1}{D_1}$$

Similarly, using continuity conditions at t_2 we have

$$(P_2 - D_2)t_1' = D_2(T - t_1'), \Rightarrow T = \frac{P_2 t_2}{D_2} = \frac{Q_2}{D_2} \quad \Rightarrow Q_2 = \frac{D_2 Q_1}{D_1}$$

Also,

$$D_1(t_1 + t_2) = P_1 t_1 \quad \Rightarrow (P_1 - D_1)t_1 = D_1 t_2 \Rightarrow t_2 = \frac{(P_1 - D_1)t_1}{D_1}$$

$$Similarly, \Rightarrow t_2' = \frac{(P_2 - D_2)t_1'}{D_2}$$

Again, we have

$$P_1 t_1 = Q_1 \; or \; t_1 = \frac{Q_1}{P_1} \quad Also, \; P_2 t_1' = Q_2 \; or \; t_1{}' = \frac{Q_2}{P_2}$$

Hence, we have the value of $T, \; t_1, \; t_1', \; t_2, \; t_2'$, respectively. Now, holding cost will be for complete time horizon and for multi-item would be

$$Ch_i = \frac{Ch_1}{2}(P_1 - D_1){t_1}^2 + \frac{Ch_1 D_1}{2}({t_2}^2) + \frac{Ch_2}{2}(P_2 - D_2)(t_1{}')^2 + \frac{Ch_2 D_2}{2}(t_2{}')^2$$

Substituting values of $t_2, t_2{}', t_1, t_1{}'$ in the expression of holding cost, we have

$$Ch_i = \frac{1}{2}\left[\begin{array}{l} Ch_1(P_1 - D_1){t_1}^2 + Ch_2(P_2 - D_2)(t_1{}')^2 + Ch_1 D_1 \left(\frac{(P_1-D_1)^2}{{D_1}^2}\right)(t_1)^2 \\ +Ch_2 D_2 \left(\frac{(P_2-D_2)^2}{{D_2}^2}\right)(t_1{}')^2 \end{array}\right]$$

Holding Cost for both the items will be

$$Ch_i = \frac{Ch_1}{2}(P_1 - D_1)\frac{{Q_1}^2}{P_1 D_1} + \frac{Ch_2}{2}(P_2 - D_2)\frac{D_2 {Q_1}^2}{P_2 {D_1}^2}$$

Production Cost for both the items can be determined by

$$Pc_i = Pc_1 * Q_1 + Pc_2 * Q_2 \quad or \quad Pc_1 * Q_1 + Pc_2 * \frac{D_2 Q_1}{D_1}$$

Similarly, **Setup cost** for both the item will be

$$\mathrm{C_{0i}} = C_{01} + C_{02}$$

Hence, **Total Models Cost (TMC)** can be expressed as = **SC + PC + HC** ,

$$C_{01} + C_{01} + \left\{ Pc_1 * Q_1 + Pc_2 * \frac{D_2 Q_1}{D_1} \right\} + \left\{ \frac{Ch_2 {D_2}^2 {Q_1}^2}{2{D_1}^2 P_1} + \frac{(Ch_1 D_1 + Ch_2 D_2){Q_1}^2}{2{D_1}^2} - \frac{Ch_1 {Q_1}^2}{2P_1} - \frac{Ch_2 D_2 {Q_1}^2}{{D_1}^2 P_2} \right\}$$

Also, **Total Selling Price** for both the models is given by

$$\mathrm{TSP} = Q_1 S_1 + \frac{D_2 Q_1}{D_1} S_2$$

Finally, the **Total Profit (TP) in terms of** (Q_1, s_1, s_2) will be

$$Q_1 S_1 + \frac{D_2 Q_1}{D_1} S_2 - \left\{ \begin{array}{l} C_{01} + C_{02} + \left\{ Pc_1 * Q_1 + Pc_2 * \frac{D_2 Q_1}{D_1} \right\} \\ + \left\{ \begin{array}{l} \frac{Ch_2 {D_2}^2 {Q_1}^2}{2{D_1}^2 P_1} + \frac{(Ch_1 D_1 + Ch_2 D_2){Q_1}^2}{2{D_1}^2} \\ - \frac{Ch_1 {Q_1}^2}{2P_1} - \frac{Ch_2 D_2 {Q_1}^2}{{D_1}^2 P_2} \end{array} \right\} \end{array} \right\}$$

18.5 Model 2 : EPQ Model Substitution Considering Shortage in One of the Items with Constant Demand and Same Time Period

See Fig. 18.2.

18.5.1 Model Formulation

In this EPQ model during production time, t_i inventory increases at rate P_i and decreases at rate $(P_i - D_i)$ units during a production run for the ith item. For

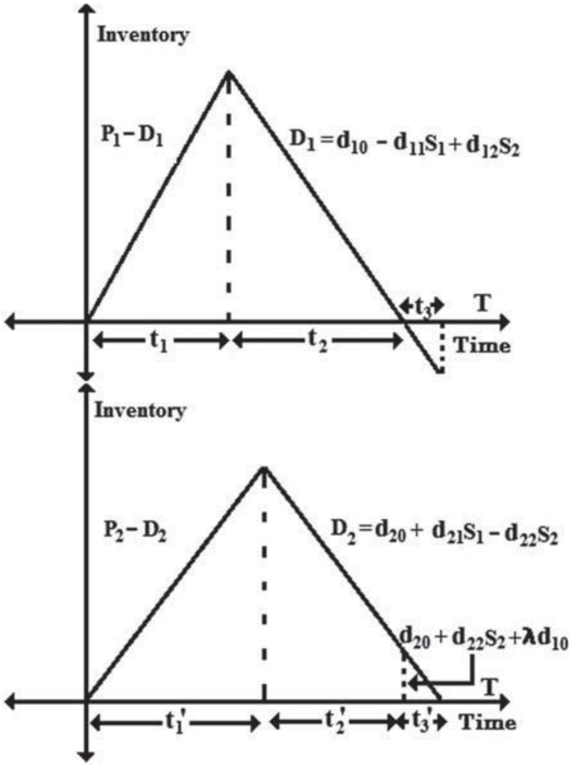


Fig. 18.2 Substitution considering shortage in one of the item with constant demand and same time period

multi-item production processes with different demand functions, the governing differential equations are

$$\frac{dI_1}{dt} = \begin{cases} P_1 - D_1, & 0 \le t \le t_1 \\ -D_1, & t_1 \le t \le t_1 + t_2 \end{cases}$$

$$\frac{dI_2}{dt} = \begin{cases} P_2 - D_2, & 0 \le t \le t_1{}' \\ -D_2 & t_1{}' \le t \le t_1{}' + t_2{}' \\ d_{20} - d_{22}S_2 + \lambda d_{10}, & t_1{}' + t_2{}' \le t \le T \end{cases}$$

Shortage cost will be $\frac{dB_1}{dt} = D_1, \quad t_1 + t_2 \le t \le T.$
with boundary conditions,

$$I_1(0) = 0 = I_1(t_1 + t_2),$$
$$I_2(0) = 0 = I_2(T)$$

Here, the continuity conditions holds at $t_1,\ t_1',\ t_1 + t_2,\ t_1' + t_2'$.

Solving the above equations we have

$$I_1 = \begin{cases} (P_1 - D_1)t, & 0 \le t \le t_1 \\ D_1(t_1 + t_2 - t), & t_1 \le t \le t_1 + t_2 \end{cases}$$
$$B_1 = D_1(t - t_1 + t_2), \qquad t_1 + t_2 \le t \le T.$$
$$I_2 = \begin{cases} (P_2 - D_2)t, & 0 \le t \le t_1{}' \\ P_2 D t_1' - D_2 t, & t_1{}' \le t \le t_1{}' + t_2{}' \\ d_{20} - d_{22}S_2 + d_{10}(T - t), & t_1{}' + t_2{}' \le t \le T \end{cases}$$

According to our problem, we have

$$\begin{aligned} & t_1 + t_2 + t_3 = t_1{}' + t_2{}' + t_3{}' = T \\ & \Rightarrow t_1 + t_2 = t_1{}' + t_2{}' \\ & \Rightarrow t_2' = t_1 + t_2 - t_1' \\ Also, \quad & t_3 = t_3' \end{aligned}$$

Again $P_1 t_1 = Q_1 \; or \; t_1 = \frac{Q_1}{P_1}$,
Similarly, $P_2 t_1' = Q_2 \; or \; t_1' = \frac{Q_2}{P_2}$
Using continuity equation at t_2 and t_2', we have

$$(P_1 - D_1)t_1 = D_1 t_2 \quad or \quad t_2 = \frac{(P_1 - D_1)t_1}{D_1}$$

Using $t_1 = \frac{Q_1}{P_1}$, Similarly, $t_2' = \frac{Q_1}{D_1} - \frac{Q_2}{P_2}$
Using continuity equation at t_3 and t_3', we have

$$\begin{aligned} & (P_2 - D_2)\, t_1{}' = D_2 t_2{}' + (d_{20} - d_{22}S_2 + \lambda d_{10})\, t_3{}' \\ & \Rightarrow t_3' = \frac{D_1 Q_2 - D_2 Q_1}{D_1(d_{20} - d_{22}S_2 + \lambda d_{10})} \end{aligned}$$

We know that $t_3 = t_3{}'$. Hence, we obtained the value of $t_1, t_2, t_3, t_1{}', t_2{}', t_3{}'$ respectively. Now, substituting them in holding cost for both the items we have.
Holding Cost (Ch_1) for **First-item** is

$$\begin{aligned} & = \frac{Ch_1}{2}\left[(P_1 - D_1)t_1{}^2 + (D_1)\, t_2{}^2\right] \\ & = \frac{Ch_1}{2}\left[(P_1 - D_1)\left(\frac{Q_1}{P_1}\right)^2 + (D_1)\left(\frac{Q_1}{D_1} - \frac{Q_2}{P_2}\right)^2\right] \end{aligned}$$

Holding Cost (Ch_2) for **Second-item** is

$$= \frac{Ch_2}{2}\left[(P_2 - D_2)\left(\frac{Q_2}{P_2}\right)^2 + (P_2 - D_2)\left(\frac{Q_2}{P_2}\right)\left(\frac{Q_1}{D_1} - \frac{Q_2}{P_2}\right)\right.$$
$$+ (d_{20} - d_{22}s_2 + \lambda d_{10})\left(\frac{Q_1}{D_1} - \frac{Q_2}{P_2}\right)\left(\frac{D_1 Q_2 - D_2 Q_1}{D_1 (d_{20} - d_{22}s_2 + \lambda d_{10})}\right)$$
$$\left. + (d_{20} - d_{22}s_2 + \lambda d_{10})\left(\frac{D_1 Q_2 - D_2 Q_1}{D_1 (d_{20} - d_{22}s_2 + \lambda d_{10})}\right)^2\right]$$

Finally, the **Holding cost** (Ch_i) for both the items will be

$$= \frac{Ch_1}{2}\left[\frac{{Q_1}^2}{P_1 D_1}(P_1 - D_1)\right] + \frac{Ch_2}{2}\left[\begin{array}{l}(P_2 - D_2)\frac{{Q_2}^2}{{P_2}^2} + 2Q_2\left(\frac{Q_1}{D_1} - \frac{Q_2}{P_2}\right)\\ -D_2\left(\frac{Q_1}{D_1} - \frac{Q_2}{P_2}\right)^2\\ + \frac{(D_1 Q_2 - D_2 Q_1)^2}{{D_1}^2 (d_{20} - d_{22}s_2 + \lambda d_{10})}\end{array}\right]$$

The **Production Cost** (Pc_i) involving in this models will be

$$\mathrm{Pc_i} = \mathrm{Pc_1} * \mathrm{Q_1} + \mathrm{Pc_2} * \mathrm{Q_2}$$

Also, **Setup cost** (SC) for both the models are,
$\mathrm{C}_{0i} = \mathrm{C}_{01} + \mathrm{C}_{02}$
Hence, **Total Model Cost (TMC)** can be expressed as
= **SC + PC + HC**,

$$C_{01} + C_{02} + Pc_1\mathrm{Q_1} + Pc_2\mathrm{Q_2} + \frac{Ch_1}{2}\left(\frac{{Q_1}^2}{P_1 D_1}(P_1 - D_1)\right)$$
$$+ \frac{Ch_2}{2}\left[\begin{array}{l}(P_2 - D_2)\frac{{Q_2}^2}{{P_2}^2} + 2Q_2\left(\frac{Q_1}{D_1} - \frac{Q_2}{P_2}\right)\\ - D_2\left(\frac{Q_1}{D_1} - \frac{Q_2}{P_2}\right)^2 + \frac{(D_1 Q_2 - D_2 Q_1)^2}{{D_1}^2 (d_{20} - d_{22}s_2 + \lambda d_{10})}\end{array}\right]$$
$$+ \frac{Sc_1}{2}\left(\frac{D_1 Q_2 - D_2 Q_1}{d_{20} - d_{22}s_2 + \lambda d_{10}}\right)^2$$

Shortage cost for both the items will be

$$\mathrm{Sc_i} = \frac{\mathrm{Sc_1}}{2}\left(\frac{\mathrm{D_1Q_2 - D_2Q_1}}{\mathrm{d_{20} - d_{22}s_2 + \lambda d_{10}}}\right)^2$$

So, **Total Selling Price** (TSP) will be

$$\mathrm{TSP} = Q_1 S_1 + Q_2 S_2$$

Henceforth, **Total Profit (TP)** in terms of Q_1,s_1,s_2 will be

$$= Q_1 S_1 + Q_2 S_2 - C_{01} + C_{02} + Pc_1 \mathrm{Q}_1 + Pc_2 \mathrm{Q}_2$$
$$+ \frac{Ch_1}{2}\left(\frac{{Q_1}^2}{P_1 D_1}(P_1 - D_1)\right)$$
$$+ \frac{Ch_2}{2}\left[\begin{array}{l} (P_2 - D_2)\frac{{Q_2}^2}{{P_2}^2} + 2Q_2\left(\frac{Q_1}{D_1} - \frac{Q_2}{P_2}\right) \\ - D_2\left(\frac{Q_1}{D_1} - \frac{Q_2}{P_2}\right)^2 + \frac{(D_1 Q_2 - D_2 Q_1)^2}{{D_1}^2(d_{20} - d_{22}s_2 + \lambda d_{10})} \end{array}\right]$$
$$+ \frac{Sc_1}{2}\left(\frac{D_1 Q_2 - D_2 Q_1}{d_{20} - d_{22}s_2 + \lambda d_{10}}\right)^2$$

18.6 Different Types of Budget Constraints

Crisp Budget Constraint

For the crisp finite budget constraint we have to consider it as $p_1 Q_1 + p_2 Q_2 \le B$.

Random Budget Constraint

In this consideration, the models remain same as developed above, except the budget constraint of the system. Here, $\bar{B}$ is random. For this type of model, we impose constraint as

$$\Pr(\bar{B} \ge p_1 Q_1 + p_2 Q_2) \ge j,$$
where $j \in (0, 1)$ is a specified permissible probability.
or, $p_1 Q_1 + p_2 Q_2 \le m_b + \sigma_b \Phi^{-1}(1 - j)$, (cf. Rao [26])

where m_b and σ_b are the expectation and standard deviation of normally distributed random variable $\bar{B}$, respectively and $\Phi^{-1}(x)$ denotes inverse function of standard normal distribution of standard normal variate $\frac{\bar{B} - m_b}{\sigma_b}$.

Fuzzy Budget Constraint

If the space horizon $\tilde{B}$ is fuzzy in nature, it can be expressed by the fuzzy constraint $\tilde{B} \ge p_1 Q_1 + p_2 Q_2$ which is interpreted in the setting of possibility and necessity theory (cf. Dubois et al. [7]). The above constraint reduces to

$$\mathrm{Pos}(\tilde{B} \ge p_1 Q_1 + p_2 Q_2) \ge \rho_1, \quad \text{and}$$
$$\mathrm{Nes}(\tilde{B} \ge p_1 Q_1 + p_2 Q_2) \ge \rho_2$$

where ρ_1 and ρ_2 represent the degree of impreciseness. Let $\tilde{B} = (B_1, B_2, B_3)$ be TFN then, using Lemmas 1 and 2, we get

$$p_1 Q_1 + p_2 Q_2 \le \begin{cases} (1 - \rho_1)B_3 + \rho_1 B_2, & \text{in possibility sense} \\ (1 - \rho_2)B_2 + \rho_2 B_1, & \text{in necessity sense.} \end{cases}$$

Fuzzy-Random Budget Constraint

In this case, the Space Constraint $\tilde{\bar{B}}$ is fuzzy-random in nature and the fuzzy-random constraint is $\tilde{\bar{B}} \geq p_1Q_1 + p_2Q_2$. It stands for the relations, which are interpreted in the setting of possibility and necessity theories (cf. Dubois et al. [7]) along with chance the constraint. The above constraint reduces to

$$\Pr\big[\mathrm{Pos}(\tilde{\bar{B}} \geq p_1Q_1 + p_2Q_2) \geq \rho_3\big] \geq j_1, \quad \text{and}$$
$$\Pr\big[\mathrm{Nes}(\tilde{\bar{B}} \geq p_1Q_1 + p_2Q_2) \geq \rho_4\big] \geq j_2$$

where (ρ_3 and ρ_4) and (j_1 and j_2) represent the degree of impreciseness and uncertainty due to randomness, respectively. Let $\tilde{\bar{B}} = (\bar{B}, B_l, B_r)$ be L-R fuzzy-random variable then, according to Theorem 1, we get

$$p_1Q_1 + p_2Q_2 \leq \begin{cases} m_b + \sigma_b\Phi^{-1}(1-j_1) + R^{-1}(\rho_3)B_r, \\ \text{in possibility sense} \\ m_b + \sigma_b\Phi^{-1}(1-j_2) - L^{-1}(1-\rho_4)B_l, \\ \text{in necessity sense.} \end{cases}$$

where m_b and σ_b are the expectation and standard deviation of normally distributed random variable $\bar{B}$, respectively, and $\Phi^{-1}(x)$ denotes inverse function of standard normal distribution of standard normal variate $\frac{\bar{B}-m_b}{\sigma_b}$.

Rough Budget Constraint

If the space constraint $\hat{B}$ is rough in nature, the rough constraint $\hat{B} \geq p_1Q_1 + p_2Q_2$ is reduced to the crisp form as $Tr(\hat{B} \geq p_1Q_1 + p_2Q_2) \geq tr_1$. ($using\ Theorem\ 2$)

$$\text{i.e. } p_1Q_1 + p_2Q_2$$
$$\leq \begin{cases} B_4 - \frac{tr_1(B_4-B_3)}{\xi_1}, & \text{if } B_2 \leq p_1Q_1 + p_2Q_2 \leq B_4 \\ \dfrac{\xi_1(B_2-B_1) + (1-\xi_1)B_2(B_4-B_3) - tr_1(B_4-B_3)(B_2-B_1)}{\xi_1(B_2-B_1) + (1-\xi_1)(B_4-B_3)}, & \text{if } B_1 \leq p_1Q_1 + p_2Q_2 \leq B_2 \\ B_4 + \frac{(1-\xi_1-tr_1)(B_4-B_3)}{\xi_1}, & \text{if } B_3 \leq p_1Q_1 + p_2Q_2 \leq B_1 \\ B_3 \end{cases}$$

where $\hat{B} = ([B_1, B_2][B_3, B_4])$, $0 \leq B_3 \leq B_1 \leq B_2 \leq B_4$, is a rough variable and $\xi_1 \in (0, 1)$ and $tr_1 \in [0, 1]$ is the confidence level.

Fuzzy-Rough Budget Constraint

If the space Constraint $\tilde{\hat{S}}$ is fuzzy-rough in nature, the fuzzy-rough constraint $\tilde{\hat{S}} \geq p_1 Q_1 + p_2 Q_2$ is reduced in the following forms which are crisp in nature.

$$\mathrm{Tr}\big[\mathrm{Pos}(\tilde{\hat{B}} \geq p_1 Q_1 + p_2 Q_2) \geq \rho_5\big] \geq tr_2, \quad \text{and}$$
$$\mathrm{Tr}\big[\mathrm{Nes}(\tilde{\hat{B}} \geq p_1 Q_1 + p_2 Q_2) \geq \rho_6\big] \geq tr_2$$

According to Theorem 3, the above constraints are finally reduced to the following forms.

$$\begin{cases} p_1 Q_1 + p_2 Q_2 \\ \leq \begin{cases} B_4 - \frac{tr_2(B_4 - B_3)}{\xi_2} + (1-\rho_5)B_R, \\ \quad \text{if} \quad B_2 \leq p_1 Q_1 + p_2 Q_2 - (1-\rho_5)B_R \leq B_4 \\ \frac{\xi_2(B_2-B_1)+(1-\xi_2)B_2(B_4-B_3)-tr_2(B_4-B_3)(B_2-B_1)}{\xi_2(B_2-B_1)+(1-\xi_2)(B_4-B_3)} \\ +(1-\rho_5)B_R, \quad \text{if} \quad B_1 \leq p_1 Q_1 + p_2 Q_2 - (1-\rho_5)B_R \leq B_2 \\ B_4 + \frac{(1-\xi_2-tr_2)(B_4-B_3)}{\xi_2} + (1-\rho_5)B_R, \\ \quad \text{if} \quad B_3 \leq p_1 Q_1 + p_2 Q_2 - (1-\rho_5)B_R \leq B_1 \\ B_3 + (1-\rho_5)B_R \end{cases} \\ and \\ p_1 Q_1 + p_2 Q_2 \\ \leq \begin{cases} B_4 - \frac{tr_2(B_4 - B_3)}{\xi_2} - \rho_6 B_L, \\ \quad \text{if} \quad B_2 \leq p_1 Q_1 + p_2 Q_2 + \rho_6 B_L \leq B_4 \\ \frac{\xi_2(B_2-B_1)+(1-\xi_2)B_2(B_4-B_3)-tr_2(B_4-B_3)(B_2-B_1)}{\xi_2(B_2-B_1)+(1-\xi_2)(B_4-B_3)} \\ -\rho_6 B_L, \quad \text{if} \quad B_1 \leq p_1 Q_1 + p_2 Q_2 + \rho_6 B_L \leq B_2 \\ B_4 + \frac{(1-\xi_2-tr_2)(B_4-B_3)}{\xi_2} + (1-\rho_6)B_R, \\ \quad \text{if} \quad B_3 \leq p_1 Q_1 + p_2 Q_2 + \rho_6 B_L \leq B_1 \\ B_3 - \rho_6 B_L \end{cases} \end{cases}$$

where $\tilde{\hat{B}} = (\hat{B} - B_L, \hat{B}, \hat{B} + B_R)$, $\hat{B} = ([B_1, B_2][B_3, B_4])$, $0 \leq B_3 \leq B_1 \leq B_2 \leq B_4$, is a fuzzy-rough variable and $\xi_2 \in (0, 1)$ and $\rho_5, \rho_6 \in [0, 1]$, $tr_2 \in [0, 1]$ are the possibility and trust confidence levels, respectively.

18.7 Solution Methodology and Numerical Solution of Both the Models

We have considered following examples:

Example 1 The demand for two items in a company is 300 and 350 units per year, respectively. Annual demand is fixed and known. The company can produce the items at rate 500 and 400 per year. Also, the holding cost of each unit is 0.50 and

0.70 paise per year. Company maintains the setup cost as 700 and 800 per production run for both items. Production cost for both the items are destined with Rs 12 and Rs 10 per year for both the items. Finally, the model has inculcated measure of each products consumer demand to its own price, i.e., $d_{11} = 10.75$ and $d_{21} = 1.70$ per unit per year along with consumer's demand to its competitor price, i.e., $d_{22} = 12.80$ and $d_{12} = 1.50$ respectively.

Finally, calculate

1: **Total Profit** (TP) with the production quantity $(Q_1,\ Q_2)$ along with the production run time $(t_1,\ t_2)$ and total production cycle time (T)? and
2: **Net effected demand** (D_1, D_2) along with selling price (s_1, s_2)?

Example 2 Considering the first example, all the parameters are same except the measure of each products consumer demand to its own price, i.e., $d_{11} = 11.75$ and $d_{21} = 1.70$ (same as Example 1) per unit per year along with consumer's demand to its competitor price, i.e., $d_{22} = 8.80$ and $d_{12} = 1.50$ (same as Example 1).

Note: The input parameters were selected for the given models based on the performance in the iteration process that has been precisely solved using GRG technique. A technique that helps us to determine the decision variable and unknowns by its own characterization. Moreover, our given input parameters enables and ensures the user for specific sensitivity values that will be perfect for any company as values considered here are from increasing to decreasing order wherein it shows the nature of the graph in 2D and 3D. Also, sensitivity analysis of the model can be considered along with the input parameters, wherein a company can decide upto what extent values can be utilized so as to get the proper optimum results along with the decision variables whilst reducing time and space complexities.

18.8 Applications and Extensions of Proposed Model

i. Different techniques are developed/presented to transform the imprecise parameters/objectives to corresponding deterministic ones. For the solution of single-objective, different optimization techniques such as GRG, GA, GA with Varying Population size (GAVP), MOGA, and Rough Age based GA (RAGA) can be developed and used.
ii. These models can be extended using the bi-fuzzy, type-2 fuzzy, random-fuzzy numbers, etc. for different model parameters.
iii. Moreover the developed optimization techniques are quite general in formulation and hence these models can be used for decision making problems in other areas such as **transportation, Supply-chain, Power-Control, Portfolio selection, etc.**

iv. Different types of optimization techniques (Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Geometric Programming (GP), and various other heuristic methods) can also be applied to the models.
v. Carbon emission due to transportation can be included in the models along with during the production.
vi. Finally, major application considering in a LED company where two types of LED—good quality and low-quality LED are produced or in the rice mills where two types of rice—fine quality and raw quality rices are produced, the products are substitutable and the customers (i.e., retailers) very often change the brand on the basis of selling price and order quantities. This analysis will be helpful for the production managers of the said LED company to fix the optimum prices, maintain the quality level, net effected demand rates, etc. for minimum cost. The responsiveness parameters (d_{10} & d_{20}) and its degree of substitution to selling price and order quantities can be obtained from the experts or may be calculated from past data. The present problem can also be applied for the managers of big departmental stores like industrial units, manufacturing units, etc., where several substitutable products are sold. In these places also, customers of one brand very often change over to other brand. Here, the replenishment may be considered as procurement/ productions with infinite rate.

18.9 Sensitivity Analysis and Discussion of Models

From Tables 18.1, 18.2, 18.3, 18.5 and 18.7.

- Table 18.1 represents the optimum results for Examples 1 and 2 for which further sensitivity analysis have been carried out in Tables 18.2 and 18.3 with the different means in the degree of substitution.
- Tables 18.2 and 18.3 especially from **sr. 1 to 5** represents the optimum results when the degree of substitution **decreases** the corresponding values of TP, Q_1^*, Q_2^*, s_1, s_2, D_1, D_2, t_1, t_2, t_3, $t_1^{'}$, $t_2^{'}$, $t_3^{'}$ and T for both the model **increases** and vice versa happens from **sr. 6 to 10**. Thus, in both the Tables 18.2 and 18.3 it can be observed that measures of each products consumer demand to its own price as well as competitor price that depends on the selling price not only effect the Quantities (Q_1^*, Q_2^*) and Total Profit (TP) but it also effects the rest of the parameters thereby achieving the accurate results.
- Tables 18.5 and 18.7 through the given input values from Tables 18.4 and 18.6 in Model 1 and Model 2 are circumscribed here. As the Budget constraints is involved in both the models in an uncertain environments such as crisp, fuzzy, random, fuzzy-random, rough, and fuzzy-rough value of B can be determined through relation $p_1Q_1 + p_2Q_2 \leq B$. Thus, value of B is known and it is maximum viz. above crisp constraint sense. **Interesting thing about Tables 18.5 and 18.7 is that as the Budget constraint(B) decreases then rest of the optimum results are decreasing except selling price (s_1, s_2) that increases.** Hence, it determines that

Table 18.1 Output results of Examples 1 and 2

Product's characteristics	Product 1	Product 2
Profit selling price	$TP = 44010.14$	
	$S_1 = 24.88$	$S_2 = 25.81$
Produced quantity	$Q_1 = 2390.87$	$Q_2 = 3808.427$
Effected demand	$D_1 = 71.25$	$D_2 = 113.50$
Production run time	$t_1 = 7.96$	$t_2 = 10.88$
Cycle time	T = 33.55	
Product's characteristics	Product 1	Product 2
Profit Selling price	$TP = 20095.91$	
	$S_1 = 23.00$	$S_2 = 35.03$
Produced quantity	$Q_1 = 1212.50$	$Q_2 = 1191.97$
Effected demand	$D_1 = 82.19$	$D_2 = 80.79$
Production run time	$t_1 = 2.42$, $t_2 = 12.32$, $t_3 = 0.10$	$t_1^{'} = 2.97$, $t_2^{'} = 11.77$, $t_3^{'} = 0.10$
Cycle time	T = 14.76	

Budget Constraint(B) should not be limited to selling price, i.e., if the quantities produced(Q_1^*, Q_2^*) decreases it doesn't mean selling price will also decrease . A shopkeeper can retain the selling price or increase whether there's a pitfall in (Q_1^* and Q_2^*) as per market condition.

From Figs. 18.3, 18.4, 18.5, 18.6, 18.7, and 18.8.

- Considering the optimal values of **Model-1**, the Total Profit(TP) is plotted in Figs. 18.3 and 18.4 against the different values of s_1, s_2 and q_1 respectively. This shows the nature of the various values corresponding from Table 18.2 and the peak level on the graph is the maximum value from same table.
- Figure 18.5 is obtained by plotting the Total Profit (TP) against the different values of Selling Prices (s_1 and s_2). This Total profit (TP) maximization is a convex function against selling price rate only.
- Considering the optimal values **Model-2**, the Total Profit (TP) is plotted in Figs. 18.6 and 18.7 against the different values of s_1, s_2, q_1, and q_2, respectively. This figure shows the nature of the various values corresponding from Table 18.3 and the peak level on the graph is the maximum value from same table.
- Figure 18.8 is obtained by plotting the Total Profit (TP) against the different values of Selling Prices (s_1 and s_2). This Total Profit (TP) maximization is a convex function against selling price rate only.

Table 18.2 Model 1 optimum values for TP, Q_1^*, Q_2^*, s_1, s_2, D_1, D_2, t_1, t_2, T

Input					Degree of substitution		Optimum result									
Sr. No	d_{11}	d_{12}	d_{21}	d_{22}	d_{11}-d_{12}	d_{22}-d_{21}	TP	Q_1^*	Q_2^*	s_1	s_2	D_1	D_2	t_1	t_2	T
1	10.75	1.50	1.70	10.80	9.25	9.10	44,010.14	2390.87	3808.42	24.88	25.81	71.25	113.50	7.96	10.88	33.55
2	10.75	2.00	2.00	10.80	8.75	8.80	51,222.31	2730.35	4072.66	25.77	26.65	76.21	113.68	9.10	11.63	35.82
3	10.75	2.50	2.30	10.80	8.25	8.50	59,854.30	3137.47	4341.81	26.72	27.62	81.76	113.15	10.45	12.40	38.37
4	10.75	3.00	2.60	10.80	7.75	8.20	70,262.15	3639.45	4602.21	27.72	28.75	88.22	111.56	12.13	13.14	41.25
5	10.75	3.50	2.90	10.80	7.25	7.90	82,932.28	4286.44	4823.70	28.76	30.11	96.17	108.22	14.28	13.78	44.57
6	12.75	1.50	1.70	12.80	11.25	11.10	20,532.14	1443.94	2487.28	21.48	22.20	59.36	102.25	4.81	7.10	24.32
7	13.75	1.50	1.70	13.80	12.25	12.10	14,085.86	1121.44	2031.41	20.19	20.82	53.55	97.00	3.73	5.80	20.94
8	14.75	1.50	1.70	14.80	13.25	13.10	9599.10	867.06	1666.62	19.09	19.62	47.83	91.94	2.89	4.76	18.12
9	15.75	1.50	1.70	15.80	14.25	14.10	6436.15	665.04	1371.63	18.13	18.59	42.19	87.03	2.21	3.91	15.75
10	16.75	1.50	1.70	16.80	15.25	15.10	4185.34	503.85	1131.10	17.30	17.68	36.63	82.25	1.67	3.23	13.75

Table 18.3 Model 2 optimum values for TP, Q_1^*, Q_2^*, s_1, s_2, D_1, D_2, t_1, t_2, $t_3 = t_3^{'}$, T

Input					Degree of substitution		Optimum result												
Sr. No	d_{11}	d_{12}	d_{21}	d_{22}	d_{11}-d_{12}	d_{22}-d_{21}	TP	Q_1^*	Q_2^*	s_1	s_2	D_1	D_2	t_1	t_2	$t_1^{'}$	$t_2^{'}$	$t_3 = t_3^{'}$	T
1	11.75	1.55	1.75	8.80	10.20	7.05	20,501.75	1234.63	1199.56	23.12	35.22	82.82	80.47	2.46	12.43	2.99	11.90	0.943	14.90
2	11.75	1.60	1.80	8.80	10.15	7.00	20,917.33	1257.39	1207.08	23.25	35.42	83.47	80.13	2.51	12.54	3.01	12.04	0.812	15.06
3	11.75	1.65	1.85	8.80	10.10	6.95	21,342.95	1280.84	1214.51	23.37	35.62	84.13	79.77	2.56	12.66	3.03	12.18	0.686	15.22
4	11.75	1.70	1.90	8.80	10.05	6.90	21,778.94	1305.00	1221.84	23.49	35.82	84.79	79.39	2.61	12.77	3.05	12.33	0.564	15.38
5	11.75	1.75	1.95	8.80	10.00	6.85	22,225.62	1329.91	1229.05	23.62	36.03	85.47	78.99	2.65	12.89	3.07	12.48	0.447	15.55
6	11.75	1.50	1.70	8.80	10.25	7.10	20,095.91	1212.50	1191.97	23.00	35.03	82.19	80.79	2.42	12.32	2.97	11.77	0.107	14.75
7	11.80	1.50	1.70	8.85	10.30	7.15	19,680.89	1196.46	1180.22	22.91	34.82	81.84	80.72	2.39	12.22	2.95	11.66	0.108	14.61
8	11.85	1.50	1.70	8.90	10.35	7.20	19,275.10	1180.66	1168.62	22.82	34.62	81.49	80.66	2.36	12.12	2.92	11.56	0.110	14.48
9	11.90	1.50	1.70	8.95	10.40	7.25	18,878.30	1165.10	1157.17	22.72	34.41	81.15	80.59	2.33	12.02	2.89	1.46	0.112	14.35
10	11.95	1.50	1.70	9.00	10.45	7.30	18,490.24	1149.77	1145.85	22.63	34.21	80.80	80.52	2.29	11.92	2.86	11.36	0.113	14.22

Table 18.4 Input values for different budget constraints of Model 1

Time horizon	Crisp	Random	Fuzzy (Pos&Nes sense)	Fuzzy-random (Pos&Nes sense)	Rough	Fuzzy-rough (Pos&Nes sense)
Related inputs for $p_1 = 0.25$& $p_2 = 0.20$	$B =$ 1359.40	$m_b = 1350$ $\sigma_b = 5$ $\Phi = 0.50$	$B_1 = 1330$ $B_2 = 1350$ $B_3 = 1340$ $\rho_1 = 0.80$ $\rho_2 = 0.30$	$m_b = 1350, \sigma_b = 25$ $B_l = 5, B_r = 7$ $\rho_3 = 0.80, \rho_4 = 0.30$ $L(x) = R(x) = 1 - x$	$B_1 = 1330, B_2 = 1350$ $B_3 = 1340, B_4 = 1355$ $\xi_1 = 0.50$ $tr_1 = 0.80$	$B_1 = 1330, B_2 = 1350$ $B_3 = 1340, B_4 = 1355$ $\xi_2 = 0.50, tr_2 = 0.80$ $B_L = 5, B_R = 7$ $\rho_5 = 0.80, \rho_6 = 0.30$

Table 18.5 Output values for different budget constraints of Model 1

Input		Optimum result										
Sr. No	Horizon	B	TP	Q_1^*	Q_2^*	s_1	s_2	D_1	D_2	t_1	t_2	T
1	Crisp	1359.40	44,010.14	2390.87	3808.42	24.88	25.81	71.25	113.50	7.96	10.88	33.55
2	Random	1352.50	44,009.41	2376.00	3792.49	24.90	25.83	70.98	113.30	7.92	10.83	33.47
3	Fuzzy-Pos. sense	1348	44,008.14	2366.31	3782.11	24.92	25.85	70.80	113.17	7.88	10.80	33.41
4	Fuzzy-Nes. sense	1344	44,006.50	2357.69	3772.87	24.94	25.86	70.65	113.05	7.85	10.77	33.37
5	Fuzzy-random Pos. sense	1331.9	44,006.06	2355.76	3770.79	24.94	25.86	70.61	113.03	7.85	10.77	33.36
6	Fuzzy-random Nes. sense	1334	44,004.94	2351.24	3765.95	24.95	25.87	70.53	112.96	7.83	10.75	33.33
7	Rough	565.42	32,338.75	724.26	1921.80	28.99	29.12	31.94	84.76	2.41	5.49	22.67
8	Fuzzy-rough Pos. sense	523.54	32,384.78	727.09	1925.27	28.98	29.11	32.03	84.83	2.42	5.50	22.69
9	Fuzzy-rough Nes. sense	520.78	32,289.30	721.23	1918.09	29.00	29.13	31.84	84.69	2.40	5.48	22.64

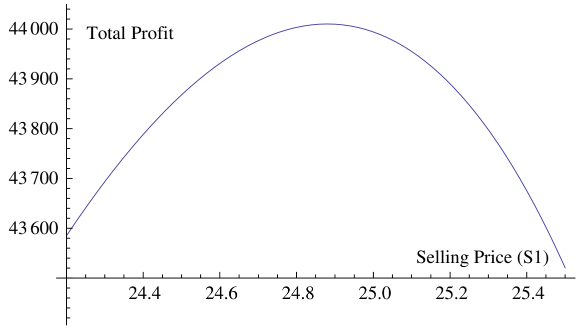
Table 18.6 Input values for different budget constraints of Model 2

Time horizon	Crisp	Random	Fuzzy (Pos&Nes sense)	Fuzzy-random (Pos&Nes sense)	Rough	Fuzzy-rough (Pos&Nes sense)
Related inputs for $p_1 = 0.25$& $p_2 = 0.20$	B=548.57	$m_b = 530$ $\sigma_b = 5$ $\Phi = 0.50$	$B_1 = 530$ $B_2 = 520$ $B_3 = 510$ $\rho_1 = 0.80$ $\rho_2 = 0.30$	$m_b = 530, \sigma_b = 5$ $B_l = 5, B_r = 7$ $\rho_3 = 0.80, \rho_4 = 0.30$ $L(x) = R(x) = 1 - x$	$B_1 = 530, B_2 = 535$ $B_3 = 500, B_4 = 540$ $\xi_1 = 0.50$ $tr_1 = 0.80$	$B_1 = 530, B_2 = 535$ $B_3 = 500, B_4 = 540$ $\xi_2 = 0.50, tr_2 = 0.80$ $B_L = 5, B_R = 7$ $\rho_5 = 0.80, \rho_6 = 0.30$

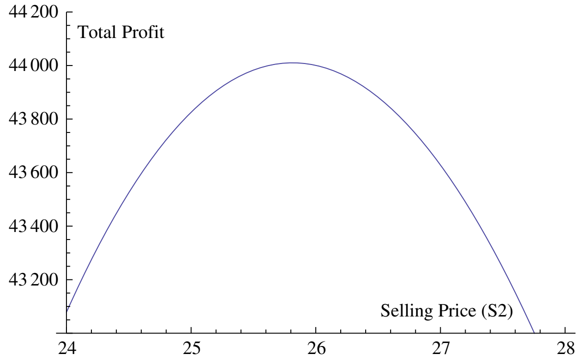
Table 18.7 Output values for different budget constraints of Model 2

Input		Optimum result													
Sr. No	Horizon	B	TP	Q_1^*	Q_2^*	s_1	s_2	D_1	D_2	t_1	t_2	t_1'	t_2'	$t_3 = t_3'$	T
1	Crisp	548.57	20,501.75	1234.63	1199.56	23.12	35.22	82.82	80.47	2.46	12.43	2.99	11.90	0.943	14.90
2	Random	532.50	20,490.32	1184.99	1181.25	23.30	35.25	80.83	80.57	2.36	12.28	2.95	11.70	0.114	14.66
3	Fuzzy-Pos. sense	523	20,472.68	1155.82	1170.22	23.40	35.26	79.63	80.62	2.31	12.20	2.92	11.58	0.126	14.51
4	Fuzzy-Nes. sense	518	20,460.11	1140.51	1164.35	23.46	35.27	79.00	80.65	2.28	12.15	2.91	11.52	0.133	14.43
5	Fuzzy-random Pos. sense	531	20,488.07	1180.38	1179.52	23.31	35.25	80.64	80.58	2.36	12.27	2.94	11.68	0.116	14.63
6	Fuzzy-random Nes. sense	533.10	20,491.16	1186.84	1181.94	23.29	35.24	80.92	80.57	2.37	12.29	2.95	11.71	0.113	14.67
7	Rough	468.55	20,209.68	991.00	1104.02	24.02	35.37	72.50	80.76	1.98	11.68	2.76	10.90	0.209	13.67
8	Fuzzy-rough Pos. sense	441.10	20,220	995.18	1105.79	24.01	35.36	72.69	80.76	1.99	11.69	2.76	10.92	0.206	13.69
9	Fuzzy-rough Nes. sense	502.02	20,198.40	986.51	1102.13	24.04	35.37	72.30	80.76	1.97	11.67	2.75	10.88	0.211	13.64

Fig. 18.3 Nature of 2D graph considering total profit against selling price (s_1) and (s_2)

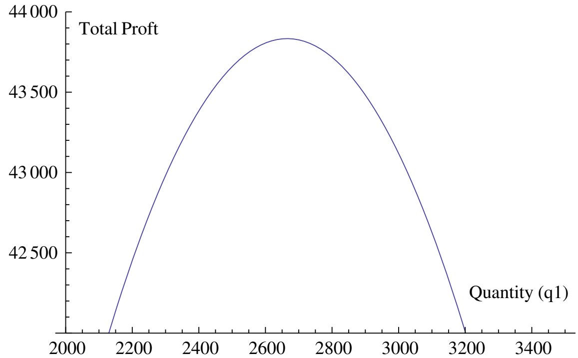


(a) Total Proft (TP) V/s Selling Price (s_1)



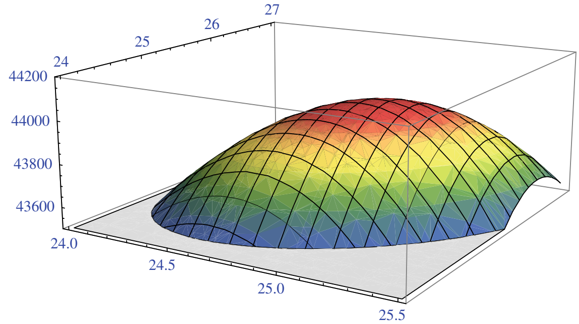
(b) Total Proft (TP) V/s Selling Price (s_2)

Fig. 18.4 Nature of 2D graph considering total profit against quantity (q_1)



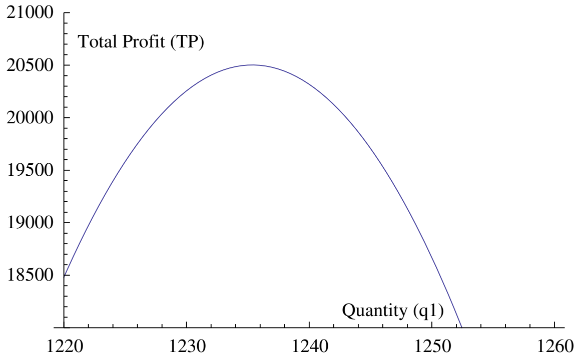
Total Proft (TP) V/s Quantity(q_1)

Fig. 18.5 Concavity property of total profit (TP) with respect to selling prices (S_1, S_2) of **Model 1**

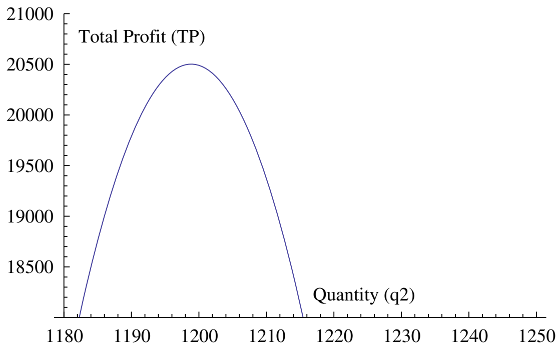


3-D Graph of Total Profit (TP) V/s Selling prices (S_1, S_2)

Fig. 18.6 Nature of 2D graph considering total profit against quantities (q_1) and (q_2)

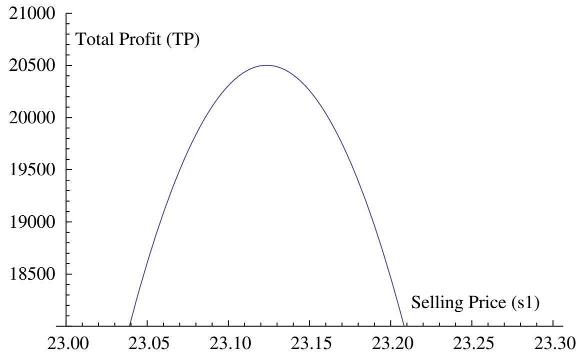


(a) Total Proft (TP) V/s Quantity (q_1)

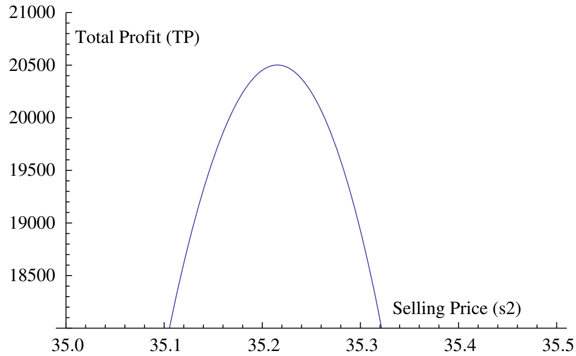


(b) Total Proft (TP) V/s Quantity (q_2)

Fig. 18.7 Nature of 2D graph considering total profit against selling prices (s_1) and (s_2)

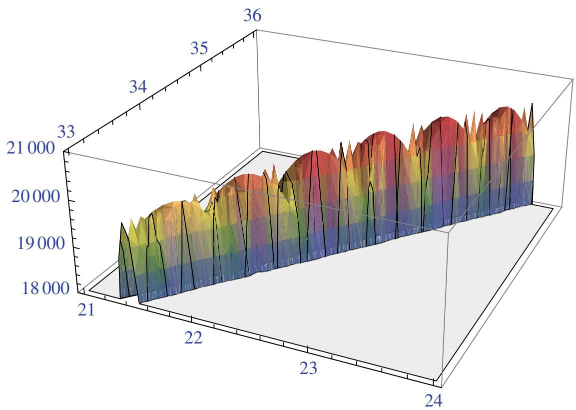


(a) Total Proft (TP) V/s Selling Price (s_1)



(b) Total Proft (TP) V/s Selling Price (s_2)

Fig. 18.8 Concavity property of total profit (TP) with respect to selling prices (S_1, S_2) of **Model 2**



3-D Graph of Total Profit (TP) V/s Selling prices (S_1, S_2)

18.10 Conclusion and Future Work

In this paper, production cost of the substitutable products on the basis of selling prices and optimum order quantities have been circumscribed over finite time horizon, optimum quantities, and production rate cycles so that the total profit is maximum. Also, sensitivity analysis have been carried out particularly from the tables based on certain inputs, which derives the total profit along with uncertain finite budget constraints. Furthermore, this paper can be extended with others types of production-inventory models such as inventory models with trade credit, two warehouses inventory system, and EPQ model with price discount whilst introducing (AUD/IQD) on the substitutable products. Moreover, considering different cases of demand units where one-way substitution, i.e., $D_2 = 0$ or $D_1 = 0$ individually can also be possible for multi-items also for those products where shortages are entertained from either of the items and vice versa. This investigation will be helpful for the managers of stores or production cum sale companies where substitutable products are produced and sold.

The virgin ideas presented in this paper are as follows:

- Reliability for the production process,
- More substitutable products,
- Supply chain system incorporating retailers and customers,
- Production cum sale of two substitutable products with can be incurred with effect of selling prices.

References

1. Bansal KK, Ahalawat N (2012) Integrated inventory models for decaying items with exponential demand under inflation. Int J Soft Comput Eng 2:578–587
2. Cárdenas-Barrón LE, Chung KJ, Treviño-Garza G (2014) Celebrating a century of the economic order quantity model in honor of Ford Whitman Harris. Int J Prod Econ 155:1–7
3. Cárdenas-Barrón LE, Treviño-Garza G, Taleizadeh AA, Treviño-Garza G, Pandian V (2015) Determining replenishment lot size and shipment policy for an EPQ inventory model with delivery and rework. Math Probl Eng 2015:1–8
4. Chang HC (2004) A note on the EPQ model with shortages and variable lead time. Inf Manag Sci 15(1):61–67
5. Charnes A, Cooper WW (1963) Deterministic equivalents for optimizing and satisficing under chance constraints. Oper Res 11:18–39
6. Chung KH, Kim YH (1989) Economic analysis of inventory systems: a rejoinder. Eng Econ 35:75–80
7. Dubois D, Prade H (1988) Possibility theory. Springer, New York
8. Garg G, Vaish B, Gupta S (2012) An economic production lot size model with price discounting for non-instantaneous deteriorating items with ramp-type production and demand rates. Int J Contemp Math Sci 7(11):531–554
9. Goyal SK (1995) A viewpoint on an EOQ model with substitutions between products. J Oper Res Soc 46:476–477

10. Guria A, Das B, Mandal S, Maiti M (2013) Inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment. Appl Math Model 37:240–257
11. Gurunani C (1983) Economic analysis of inventory systems. Int J Prod Res 21:261–277
12. Gurunani H, Drezner Z (2000) Deterministic hierarchical substitution inventory models. J Oper Res Soc 51:129–133
13. Katagiri H, Sakawa M, Kato K, Nishizaki I (2008) Interactive multi-objective fuzzy random linear programming: maximization of possibility and probability. Eur J Oper Res 188(2):530–539
14. Khedlekar UK (2012) A disruption production model with exponential demand. Int J Ind Eng Comput 3:607–616
15. Lasdon LS, Waren AD, Jain A, Ratner M (1978) Design and testing of a generalised reduced gradient code for non-linear programming. ACM Tran Math Softw 4(1):34–50
16. Liu B (2002) Theory and practice of uncertain programming. Physica-Verlag, Heidelberg
17. Liu B, Iwamura KB (1998) Chance constraint programming with fuzzy parameters. Fuzzy Sets Syst 94:227–237
18. Maiti MK, Maiti M (2006) Fuzzy inventory model with two warehouses under possibility constraints. Fuzzy Sets Syst 157:52–73
19. Merigó JM, Gil-Lafuente AM, Yager RR (2015) An overview of fuzzy research with bibliometric indicators. Appl Soft Comput 27:420–433
20. Merigó JM, Palacios-Marqués D, Benavides M (2015) Aggregation methods to calculate the average price. J Bus Res 68:1574–1580
21. Mishra SS, Singh PK (2011) A computational approach to EOQ model with power form stock dependent demand and cubic deterioration. Am J Oper Res 1:5–13
22. Moon J, Yun W (1993) An economic order quantity model with a random planning horizon. Eng Econ 39:77–86
23. Nobil AH, Sedigh AHA, Cárdenas-Barrón LE (2016) A multi-machine multi-product EPQ problem for an imperfect manufacturing system considering utilization and allocation decisions. Exp Syst Appl 56:310–319
24. Noori-daryan AAM, Cárdenas-Barrón LE (2015) Joint optimization of price, replenishment frequency, replenishment cycle and production rate in vendor managed inventory system with deteriorating items. Int J Prod Econ 159:285–295
25. Panda D, Kar S, Maity K, Maiti M (2008) A single period inventory model with imperfect production and stochastic demand under chance and imprecise constraints. Eur J Oper Res 188:121–139
26. Rao SS (2009) Engineering optimization: theory and practice, 4th edn. Wiley, Hoboken
27. Roy A, Maiti MK, Kar S, Maiti M (2009) An inventory model for a deteriorating item with displayed stock dependent demand under fuzzy inflation and time discounting over a random planning horizon. Appl Math Model 33:744–759
28. Seyed TA, Seyed HP, Gharaei A (2015) Optimization of a multiproduct economic production quantity problem with stochastic constraints using sequential quadratic programming. Knowl Based Syst 84:98–107
29. Shamsi R, Haji A, Shadrokh S, Nourbakhsh F (2009) Economic production quantity in reworkable production systems with inspection errors, scraps and backlogging. J Ind Syst Eng 3:170–188
30. Singh C, Singh SR (2012) An EPQ model with power form stock dependent demand under inflationary environment using genetic algorithm. Int J Comput Appl 48(15):25–30
31. Taleizadeh AA, Kalantari SS, Cárdenas-Barrón LE (2015) Determining optimal price, replenishment lot size and number of shipments for an EPQ model with rework and multiple shipments. J Ind Manag Optim 11(4):1059–1071
32. Taleizadeh AA, Kalantari SS, Cárdenas-Barrón LE (2016) Pricing and lot sizing for an EPQ inventory model with rework and multiple shipments. TOP 24:143–155
33. Teng JT, Ouyang LY, Chang CT (2005) Deterministic economic production quantity model with time-varying demand and cost. Appl Math Model 29:987–1003

34. Teng JT, Ouyang LY, Cheng MC (2005) An EOQ model for deteriorating items with power form stock-dependent demand. Inf Manag Sci 16(1):1–16
35. Wee HM, Wang WT, Yang PC (2013) A production quantity model for imperfect quality items with shortage and screening constraint. Int J Prod Res 51(6):1869–1884
36. Widyadana GA, Wee HM (2009) Economic Production Quantity (EPQ) deteriorating inventory model with machine breakdown and stochastic repair time. In: Proceedings of the IEEE international conference on industrial engineering and engineering management, Hong Kong, pp 548–552
37. Xu J, Zhou X (2011) Fuzzy link multiple-objective decision making. Springer, Berlin
38. Zadeh LA (1978) Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets Syst 1:3–28
39. Zanoni S, Marchi B, Pasetti M, Zavanella L (2018) A supply chain model with integrated thermal recovery and electricity generation from industrial waste heat. In: 2018 industrial efficiency conference, pp 11–13

Chapter 19
An Effective MILP Model for Food Grain Inventory Transportation in India—A Heuristic Approach

Sayan Chakraborty, Kaushik Bhattacharjee and S. P. Sarmah

Abstract In this work, we investigate a real-life inventory transportation problem faced by the Food Corporation of India (FCI). FCI is the central agency responsible for procurement, storage, and transportation of food grains over a large geographical area of India. Due to lopsided procurement and consumption of major food grains (i.e., rice & wheat) transportation of food grains across the warehouses becomes inevitable. FCI faces a significant challenge to find the optimal amount of food grains to be stored at each warehouse and transported among the warehouses to meet the demand during each period. In this study, we formulate an MILP model to determine the optimal inventory transportation decisions related to food grain transportation in India and demonstrate it via a case study. Commercial optimization packages can be used to solve the problem of this class. However, as we see, they fail to provide a solution for large size problem instances. Therefore, we propose a heuristic-based solution approach to solve the problem. It is seen that under a practical time limit, the proposed heuristic performs significantly well in terms of accuracy as compared to commercial optimizations packages. The nature of the study is generic in nature and can be also applied to various similar real-life problem scenarios.

Keywords Inventory · Food grain transportation · Heuristic · Public distribution system · India

19.1 Introduction

To meet the demand of the growing population of the country, since the green revolution back in early 1960, India has continuously improved its agronomic technology to improve crop production. Indian Public Distribution System (PDS) had its origin

S. Chakraborty (✉) · S. P. Sarmah
Department of Industrial and Systems Engineering, Indian Institute of Technology Kharagpur, Kharagpur, India
e-mail: sayanchakraborty@iitkgp.ac.in

K. Bhattacharjee
School of Management Sciences, Indian Institute of Engineering Science and Technology, Shibpur, India

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*, Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_19

in the "rationing" system that was introduced long back by the British during the time of World War II. Over this period, the system has undergone several changes and became one of the most complex PDS in the world. Food Corporation of India (FCI) is the central nodal agency responsible for procurement, storage, and transportation of food grains on behalf of Government of India (GOI). FCI procures the food grain from farmers at minimum support price (MSP) and distributes the food grains over a large geographical area of India to be sold to the consumers at a central issue price (CIP).

It is a great challenge faced by the FCI to plan the transportation of food grains to avoid shortages over the country as sufficient amount of inventory has to be maintained in each warehouse for strategic buffer level at the end of each period. FCI along with state government agencies (SGAs) procures food grains from farmers and stores them at various warehouses of the procuring states across the country. After that, food grains from warehouses of surplus states (stored food grain is more than the demand) are transported to warehouses of deficit states (Stored food grain is less than the demand). The food grain transportation flow of FCI is depicted in Fig. 19.1.

FCI has several owned and hired warehouses of different capacities across the country for storage of food grains. Due to lopsided procurement and consumption of two major food grains (i.e., rice & wheat), transportation of food grains across different warehouses is inevitable for the smooth functioning of PDS. Moreover, it may so happen that procurement of one particular food grain is very high in one state (e.g.,

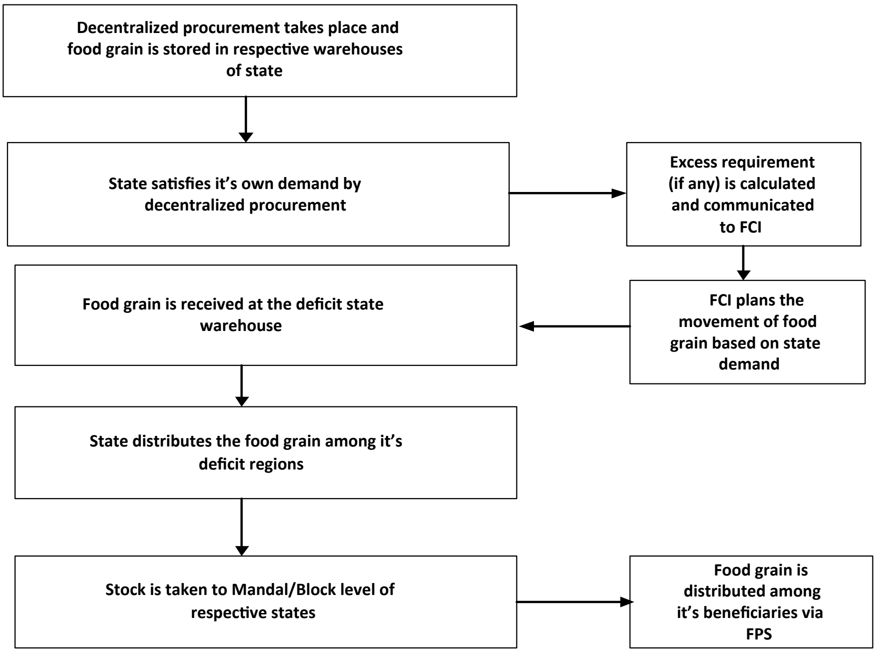


Fig. 19.1 Flow of food grain transportation under FCI

Rice procurement in Punjab), but the consumption is less due to low population. In this scenario, the food grains have to be transported from one particular warehouse (surplus) of that state to another warehouse (deficit) over the year depending upon the consumption rate and capacity of the deficit state warehouses. Due to this capacity variation, intrastate transportation also becomes necessary as well as interstate transportation.

However, due to the poor transportation plan, wastage of food grains due to improper scientific storage, underutilization of storage space, additional emergency transportation, and penalty costs and demurrage charges toward railway is observed. As per the published reports (Phillip 2016), from 2010 to 2016, a total of more than 56,000 tons of food grains were damaged due to poor transportation planning.

FCI has several owned and hired warehouses over the country for storage of food grains for future use. The procurement and storage capacity and consumption of various states are different across India. Moreover, the cultivation schedule of the two prior crops, wheat and rice varies. Wheat is harvested in Rabbi Season (December–March) and rice in Kharif season (June–September). FCI procures rice in Kharif marketing season and wheat in Rabbi marketing seasons from various farmers or agricultural cooperative societies. This procured food grains are stored at temporary storage and hence after, moved to nearest warehouses as soon as possible. Due to this, after procurement, the food grains need to transport to various other warehouses (both intrastate and interstate) according to the demand. Also, the consumption pattern and available storage capacity are different across the states, so year wise movement of food grains become inevitable between warehouses to maintain smooth operation of PDS. As per available data, there is a daily movement of 2 million gunny bags (50 kg), over an average distance of 1500 km and it accounts for an annual cost of 47.2737 billion INR [7]. Despite this, due to lack of storage capacity at the end of procuring seasons, when the food grain stock is high, FCI cannot lift the stock from the SGAs which results in carrying overcharges to SGAs. FCI has incurred 1635 crore INR from 2011 to 2012 as carryover charges to various SGAs across the country [7]. Moreover, 45% of the food grains are wasted at the distribution stage in India. Hence, in order to minimize waste, it is important to plan the transport of food grains properly. In the present scenario, food grain transportation is inevitable due to a mismatch in procurement quantity, demand, and available storage capacity with the states [5]. Hence, FCI needs to plan for optimal transportation quantity which will fulfill the demand of the beneficiaries across the country as well as minimizing the total cost incurred in the process.

By the tenth day of every month, each regional office of the FCI submits a report to the headquarter indicating the stock position, anticipated offtake, and anticipated procurement of the region. Based on that report, food grains are moved from the surplus warehouses to the deficit warehouses to meet the demand of the region. Once it is done, the food grains have to be moved downstream for distribution. PDS in India operates through a vast network of fair price shops for distribution of food grains toward beneficiaries. Each fair price shops are allocated to a distributor for stock replenishment. These distributors are affiliated to a block-level warehouse for periodic replenishment of its stock. Food grains are moved to these block-level warehouses from a central warehouse, once the central warehouse is replenished by other central warehouses of the same or neighbor states.

In this study, we formulate an MILP model to aid food grain transportation decisions among central warehouses. The remainder of the article is as follows, in Sect. 19.2, we discuss a brief overview of the related literature, and Sects. 19.3 and 19.4 defines the problem and the mathematical model respectively. In Sects. 19.5 and 19.6, solution methodology and computational results are provided. In Sect. 19.7, we present a case study on the problem and Sect. 19.8 concludes the study.

19.2 Literature Review

In the context of the inventory transportation problem, consideration and minimization of inventory storage, handling, and transportations costs are an essential aspect [4]. Jointly optimized the inventory and transportation costs in a supply chain in an integrated manner. Since that seminal work, many researchers have explored the domain of inventory transportation problem [11]. Studied decisions related to inventory and outbound transportation decisions with pre-shipping and late-shipping considerations. Later, Berman and Wang (2006) developed a nonlinear model for appropriate distribution strategy which minimizes total cost and solved it using Lagrangian relaxation and heuristics approach [2]. Formulated an MIP model to optimize allocation and transportation of customer order jointly. Kang et al. [10] studied the optimal distribution strategies minimizing the fixed vehicle cost and inventory cost. However, they only considered single capacity vehicles for their study [1]. Presented a case study of wheat transportation in Iran. They formulated the problem as an MILP to minimize the inventory and transportation cost of wheat transportation in Iran. In the agricultural industry, In the more recent context, Nguyen [14] proposed a heuristics approach for flower industry of California to minimize transportation cost. Carlson et al. (2014) investigated a case of pulp inventory distribution and planning problem of Sweden and solved it using robust optimization. [20] have taken up a MIP model to integrate multimodal transport into the cellulosic biofuel supply chain design. In their work, they have considered three different types of transport mode. They concluded that adapting various types of transport modes, the supply chain can become more cost-effective. Few other essential case studies have been undergone by Guimareaes et al. [9, 17] in the context of the food industry [18]. Investigated the transportation and storage problem of India and proposed a heuristic approach based on activity-based decision rules and [13] developed a model for perishable open-dating foods under shortages.

A real-world problem of storage and distribution of food grain faced by Food Corporation of India has been considered here in this study. The problem is characterized by heterogeneous state-bound warehouses which vary in storage capacity, demand quantity, and availability of transport. Different transport modes have been considered in this study. One can refer to [16] for a brief literature on multimodal freight transportation planning. To highlight a few, Moccia et al. (2010) studied a transportation problem in a multimodal network with shipment consolidation options [8]. Presented a dynamic programming algorithm to determine optimal intermodal freight routing. They have introduced several pruning rules to improve algorithm performance [6]. Developed a service network design model for freight consolidation carriers under resource constraints and solved it using a newly developed approach combining column generation, meta-heuristic and exact optimization techniques.

19.3 Problem Definition

The procurement of food grains is carried out every year in the producing states during specific procurement seasons each of length 3–5 months. Also, the quantity of the procured food grains varies from state to state. The food grain procured in a state during a month is stored in the warehouses of that state up to the maximum available space in the warehouses in that month itself, otherwise kept in open space. The distribution of food grains (rice and wheat) is done across various warehouses over both intrastate and interstate scenario to maintain the overall required inventory level across warehouses and to minimize the operating cost.

In case the quantity of a procured/available food grain in a warehouse of a state is not sufficient to satisfy its demand (deficit state), grain transportation is required from surplus warehouses of the same state or other states to satisfy the demand. It is seen that trucks of different capacities are the primary mode for transportation of food grains between warehouses as well as fair price shops. Third-party logistics service provider provides the trucks and availability is subject to the time period with respect to every warehouse.

The costs that are associated with this process are ordering/material handling costs, inventory holding costs, and transportation costs between warehouses. In the aspect, it is important to minimize the underutilization of hired storage capacity (truck) and warehouse storage space to minimize the total cost.

Based on this scenario, a mathematical model is developed to aid food grain transportation based on the following assumptions.

Assumptions:

i. We consider Indian PDS to have a vast network of warehouses to fulfill the demand of beneficiaries by the distribution of food grains. The warehouses are assumed to be located across the study region.
ii. Various time (transportation time, loading time, and unloading time) and cost parameters (transportation cost, loading cost at the source, and unloading cost) associated with the model are assumed to be known.
iii. The demand and consumption pattern is deterministic and known. Shortages are not allowed.
iv. The transportation cost between the two warehouses consists of a fixed cost and a variable cost incurred proportional to the quantity of food grain to be transported.

v. The quantity of food grain transported to a warehouse from any other warehouse can be delivered within a single time period.
vi. Various cost and time parameters are independent of the type of the food grain.

19.4 Mathematical Model

Graph G = {V, A} represents the distribution network model; where V is the set of vertex defined as V = {S, D}, A is the set of edges given by A = {S → D}, S is the set of source nodes and D is the set of demand nodes, respectively. This particular problem can be formulated as follows:

If cost is taken as the primary factor of decision making, then the model is given by:

Minimize:

$$\sum_{a \in A} \left(C_a^m + lC_a^m + uC_a^m\right).q_a^m + \delta_a^m.x_a^m \tag{19.1}$$

Subject to,

$$\sum_{a \in \{N \to D\}} q_a \le R_n \quad \forall n \in S \tag{19.2}$$

$$\sum_{a \in \{S \to N\}} q_a \ge Q_n \quad \forall n \in D \tag{19.3}$$

$$\max_{m \in M}\left(t_a^m + lt_a^m.q_a^m + ut_a^m.q_a^m\right) + \epsilon_a^m.x_m^a \le W_a^t \quad \forall a \in A \tag{19.4}$$

$$\sum_{a \in \{N \to D\}} q_a^m \le T_a^m \quad \forall m \in M \tag{19.5}$$

$$q_a \ge 0, x_m^a \in \{0, 1\}, \quad \forall a \in A, m \forall M \tag{19.6}$$

where,

C_a^m = Transportation cost of transport mode m at arc a
x_a^m = When transported through arc a by transport mode m, zero or one
q_a^m = Quantity of crop at arc a transported by mode m
T_a^m = Maximum available vehicle at arc a of mode m
lC_a^m = Loading cost at arc a in mode m
uc_a^m = Unloading cost at arc a in mode m
δ_a^m = Variable cost of arc a for mode m
R_n = Quantity available at node n of supply network S
Q_n = Demand quantity at node n of demand network D

W_a^t = Maximum allowable time at arc a
W_a^c = Maximum allowable cost at arc a
t_a^m = Transportation time of transport mode m at arc a
lt_a^m = Loading time at arc a for transport mode m
ut_a^m = Unloading time at arc a for transport mode m
ϵ_a^m = Variable time of arc a for mode m

The first term in the objective function (19.1) is the cost of transportation by different modes of transport from one warehouse to another, incurred if there is any transportation between the two warehouses. The second and third terms are the loading cost of food grain at the surplus warehouse and unloading cost at deficit warehouse, respectively. These costs can be called as material handling cost, if any. The last term is the variable cost, incurs if there are any other certain costs associated with the movement of food grains from one warehouse to another.

The first constraint (19.2) ensures that no surplus warehouse transports food grains beyond its ability to supply. Second constraint (19.3) enforces that the quantity supplied by all the surplus warehouses are at least as much as to satisfy the requirement of each deficit warehouses.

Third constraint (19.4) represents the consideration of allowable time constraint in the model. The first term in this constraint is the transportation time required for one mode of transport from one warehouse to another. The second and third terms are the loading time at supply point and unloading time at demand point. The last term stands for the excess variable time, if any, from one source to one destination. This constraint ensures that the maximum time required for transportation from one surplus warehouse to another deficit warehouse is within the allowable time.

Fourth constraint (19.5) ensures that the quantity transported from each surplus warehouse to each deficit warehouse by any transport mode does not exceed the available transportation capacity at that particular warehouse of that transport mode.

Fifth constraint (19.6) is the binary constraint enforces the value of 0 if there is no food grain transportation between one pair of warehouse and 1 if there is transportation between the two warehouses. Also, all the parameters and decision variables involved in this model are integers in nature. It can also be seen that the size of the problem (the number of variables and constraints) increases with the type of mode of transport involved and the number of surplus and deficit warehouses.

19.5 Solution Methodology

Commercial optimization packages can be used to solve the problem of this class. However, as we see, they fail to provide a solution for large size problem instances in a practical time limit. Therefore, we propose a heuristic-based solution approach to solve the problem. The decisions involved with this problem are (i) how much to transport from which warehouse to where? And (ii) what is the optimal combination of the mode of the vehicle to minimize the cost.

The input parameter which is necessary to know is the present inventory level of the warehouses along with the latitude and longitude of the same. The latitude and longitude of the warehouses are used for distance calculation. In order to know the monthly demand of one particular warehouse, population-based data of that region can be used. After determination of the present stock level of the all the warehouses, warehouses which are in the deficit condition can be found out. Food grains have to be moved to these deficit warehouses from nearby surplus warehouses to maintain steady inventory level over these warehouses. Figure 19.2 explains the scenario of food grain transportation in India.

After the identification of the source (surplus) and destination (deficit) warehouses, it is necessary to allocate food grains from surplus warehouses to the deficit warehouses. In order to do it, a suitable food grain transportation model has to be developed. Trucks of different capacities have been considered in this problem for food grain transportation. This makes the given problem scenario multimodal. In order to efficiently solve this problem, one approach is to transform the multimodal transportation problem to a single mode transportation problem by assigning weights for each of the modes, and then calculate the weighted cost matrix. After the weighted cost matrix is calculated, the transportation problem can be solved by the weighted cost matrix. After this solution is obtained, the exact mode and quantity from one source to another can be determined by a dynamic programming approach. We establish the following decision rule for the necessary decision-making.

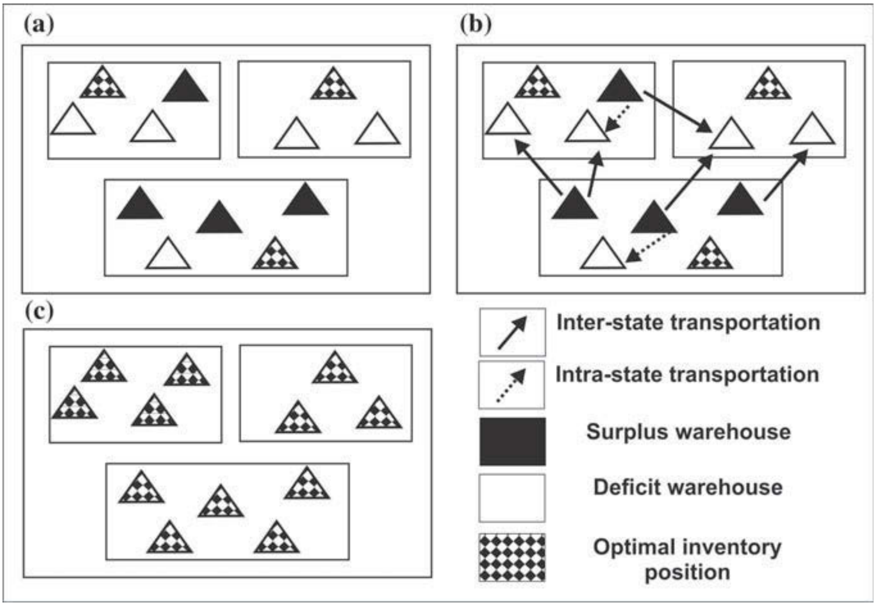


Fig. 19.2 **a** Warehouses before food grain transportation. **b** Pictorial representation of food grain transportation. **c** Warehouses after food grain transportation

Decision rule 1: The arc with higher cost and time in one node will be eliminated.
Decision rule 2: If the cost is the same in both arcs, but the time is different then the arc with the longer time is eliminated. In case the time is the same, the one with the higher cost is eliminated.

The following flowchart describes the heuristic procedure.

19.6 Results and Discussion

The quality of the proposed algorithm is tested over a large number of randomly generated problem instances. The proposed algorithm is coded on a computer with Intel Core i5-4570T CPU with 8 GB RAM. The solutions obtained using the proposed approach is compared with the exact solution obtained using IBM ILOG Cplex V12.5 optimization studio for a variety of the problem instances.

For the effective comparison of results, three scenarios have been constructed to test the performance of the algorithm. For the generation of these scenarios, a parameter Alpha (α) has been considered which is denotes the ratio between the surplus and deficit warehouse. Therefore, a higher value of α will signify more number of deficit warehouses and vice versa. After that, we consider three cases by setting up an alpha value ranging from $0.1 < \alpha < 2$.

Scenario I: $\alpha < 0.5$ (i.e., less number of deficit warehouses).
Scenario II: $0.4 < \alpha < 0.9$ (i.e., medium number deficit warehouses).
Scenario III: $\alpha > 1$ (i.e., high number of deficit warehouses).

Now, for each scenario, seven different instance sizes with five experiments for each instance have been performed, and results are compared with CPLEX solutions (See, Appendixes 1, 2, and 3). The average best solution gap obtained (1.57%) was for Scenario 1 (Alpha < 0.4, Instance size 50).The worst average solution gap obtained (11.71%) was for Scenario 2 (0.4 < Alpha < 0.8). It is also seen that the algorithm performs significantly well for Scenario 3 (Appendix 3). It is also observed that CPLEX failed to provide an optimal solution for many of the cases in practical time limit and it's convergence rate decreases significantly after ten minutes of runtime.

19.7 Case Study

The solution methodology mentioned above was implemented as a case study for a district of West Bengal named Cooch Bihar. The area of the district is 3387 km^2, and the approximate population is about 2,822,780 (Census 2011). As per the given model assumptions, nine warehouses have been identified across the district. The initial stock position has been generated randomly. The demand data for each warehouse has been generated as per the NFSA Act 2013 based on the population of the district.

Tables 19.1 and 19.2 show the related data regarding the problem area collected from various primary and secondary sources. Figure 19.3 shows a pictorial view of the study area where the green and red dots represent surplus and deficit warehouses respectively Fig. 19.4.

Based on the transportation threshold quantity and population data, the amount which has to be delivered to the deficit warehouses has been calculated. The distance matrix has been formed by the latitude and longitudinal values of the sites. Practical cost and time parameters have been considered while the problem is being solved. The decisions involved with the study problem are (i) how much to transport from which warehouse to where? And (ii) what is the optimal combination of the mode of the vehicle to minimize the cost. The obtained results are shown in Table 19.3.

Table 19.1 Data related to Cooch Behar district

Sr. No.	Name of the site	Zip code	Population	Stock position (MT)	Cut off quantity (MT)
1	CB central	735304	2303571	3240	
2	Sitalkuchi	736101	582599	816	239
3	Sitai	736135	460228	645	249
4	Haldibari	735122	93867	132	182
5	Mathabhanga	736146	383004	537	226
6	Tufangaunj	735301	133275	187	188
7	Dinhata	736167	96347	135	252
8	Cooch Behar I	736158	163708	230	701
9	Meckliganj	736159	390543	547	182

Table 19.2 Data setting of the food grain inventory transportation problem

Sr No.	Type	Vehicle 1	Vehicle 2	Vehicle 3
1	Booking Cost	1000	1500	2000
2	Capacity (Ton)	7.5	9	15
3	Cost/km (INR)	10	15	20
4	Average Speed (Km/h)	30	30	30
5	Loading Cost (INR/Ton)	30	30	30
6	Unloading Cost (INR/Ton)	30	30	30
7	Loading time (Min/Ton)	6	6	6
8	Unloading time (Min/Ton)	5	5	5

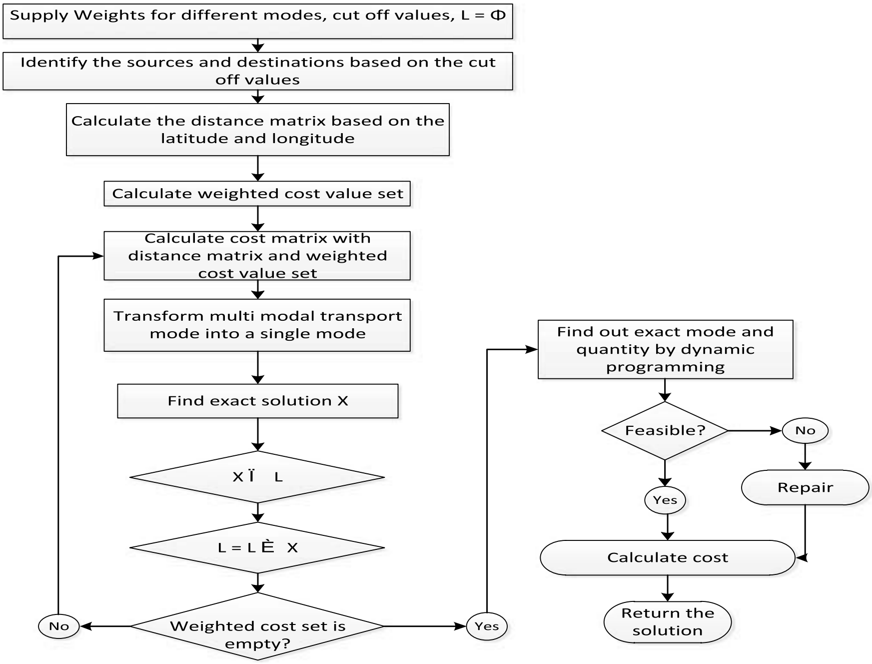


Fig. 19.3 Flowchart for the proposed solution algorithm

Fig. 19.4 Map of Cooch Behar district

Table 19.3 Results of the study

Sr. No.	Source zip code	Destination zip code	Distance (Km)	Transport quantity (Ton)	Req. no. of vehicle type 1	Req. no. of vehicle type 2	Req. no. of vehicle type 3
1	736101	735304	248	361	8	4	18
2	736135	735304	229	396	8	4	20
3	736159	735304	256	248	5	4	12
4	736146	735122	166	50	7	0	0
5	736146	735301	31	7	1	0	0
6	736159	736167	182	117	16	0	0
7	736101	736158	32	216	11	0	9
8	736146	736158	21	255	13	1	10

19.8 Conclusion

A real-life problem of food grain transportation has been considered in this study. An MILP model is formulated and solved by an efficient heuristics to determine optimal inventory transportation decisions related to food grain transportation in India. It is seen that under a practical time limit, the proposed heuristics perform significantly well regarding accuracy as compared to commercial optimization packages. For better understanding, the mentioned solution approach was demonstrated as a case study for a district of West Bengal named Cooch Bihar. The proposed solution approach is flexible and can be extended for large size problem instances to plan food grain transportation of the country. Therefore, this heuristic approach can aid as a decision support tool for the FCI. The nature of the study is generic and can be applied to various similar real-life transportation problem scenarios.

Appendix 1

Results for scenario 1

Sl. No.	Instance size	Alpha	Number of source	Number of destination	Heuristic		CPLEX		Gap (%)
					Value	Solution time	Value	Time limit (sec)	
1	50	0.2821	39	11	1651450	0.0312	1616545	3600	2.1592
2	50	0.3158	38	12	1612530	0.0312	1596100	3600	1.0294
3	50	0.3158	38	12	1479995	0.0156	1451435	3600	1.9677
4	50	0.1905	42	8	2181265	0.0156	2146540	3600	1.6177
5	50	0.2500	40	10	1530045	0.0468	1513665	3600	1.0821
6	100	0.2658	79	21	3302700	0.0468	3243850	3600	1.8142
7	100	0.1494	87	13	4535025	0.0624	4469196	3600	1.4729

(continued)

(continued)

Sl. No.	Instance size	Alpha	Number of source	Number of destination	Heuristic		CPLEX		Gap (%)
					Value	Solution time	Value	Time limit (sec)	
8	100	0.2821	78	22	3378030	0.0468	3309245	3600	2.0786
9	100	0.2658	79	21	2885435	0.0624	2846836	3600	1.3559
10	100	0.2346	81	19	2803930	0.0312	2746175	3600	2.1031
11	200	0.1628	172	28	7232030	0.1092	7106100	3600	1.7721
12	200	0.2346	162	38	7839570	0.156	7642490	3600	2.5787
13	200	0.1765	170	30	7725335	0.1248	7607955	3600	1.5429
14	200	0.2422	161	39	7247205	0.0936	7118755	3600	1.8044
15	200	0.2195	164	36	8172980	0.1248	7914580	3600	3.2649
16	300	0.2245	245	55	9210160	0.2496	9059235	3600	1.6660
17	300	0.2000	250	50	10485170	0.2028	10292360	3600	1.8733
18	300	0.2712	236	64	11873735	0.2652	11475055	3600	3.4743
19	300	0.2195	246	54	9526675	0.2184	9306545	3600	2.3653
20	300	0.1811	254	46	11082185	0.1872	10763540	3600	2.9604
21	500	0.2225	409	91	16181110	0.9516	15718415	3600	2.9436
22	500	0.2225	409	91	20159315	0.78	19193905	3600	5.0298
23	500	0.2658	395	105	16636445	0.8736	16274570	3600	2.2236
24	500	0.2107	413	87	13790340	0.8424	13522505	3600	1.9807
25	500	0.2563	398	102	18860395	0.8892	18210625	3600	3.5681
26	*800*	0.2289	651	*149*	*29411205*	*2.7456*	*28235380*	3600	4.1644
27	800	0.2214	655	145	31008890	3.1668	29714055	3600	4.3577
28	800	0.2289	651	149	25387080	3.042	24376880	3600	4.1441
29	800	0.2140	659	141	31594345	3.1668	29911875	3600	5.6248
30	800	0.2422	644	156	32154915	2.8704	30253295	3600	6.2857
31	1200	0.2371	970	230	41726305	13.96	39525370	3600	5.5684
32	1200	0.2500	960	240	51643170	11.62	48340740	3600	6.8316
33	1200	0.2513	959	241	44470050	14.6644	41633685	3600	6.8127
34	1200	0.2295	976	224	41368565	12.3548	39685200	3600	4.2418
35	1200	0.2346	972	228	37756320	10.3274	36494555	3600	3.4574

Appendix 2

Results for scenario 2

Sl. No.	Instance size	Alpha	Number of source	Number of destination	Heuristic		CPLEX		Gap (%)
					Value	Solution time	Value	Time limit (sec)	
1	50	0.5625	32	18	812325	0.3120	798040	3600	1.7900
2	50	0.5152	33	17	1042685	0.0156	1020880	3600	2.1359
3	50	0.5152	33	17	1217230	0.0468	1203455	3600	1.1446
4	50	0.3514	37	13	1475270	0.3900	1457215	3600	1.2390
5	50	0.5152	33	17	1659900	0.3744	1634145	3600	1.5761

(continued)

(continued)

Sl. No.	Instance size	Alpha	Number of source	Number of destination	Heuristic		CPLEX		Gap (%)
					Value	Solution time	Value	Time limit (sec)	
6	100	0.7857	56	44	1746685	0.2496	1734845	3600	0.6825
7	100	0.3889	72	28	3219010	0.2184	3179825	3600	1.2323
8	100	0.6949	59	41	1979970	0.5148	1924935	3600	2.8591
9	100	0.6667	60	40	1952025	0.2028	1926705	3600	1.3142
10	100	0.4706	68	32	2713960	0.1404	2641405	3600	2.7468
11	200	0.5625	128	72	4079020	0.3432	4035880	3600	1.0689
12	200	0.4697	132	62	4695785	0.3432	4646840	3600	1.0533
13	200	0.3514	148	52	6683790	0.1872	6561325	3600	1.8665
14	200	0.3986	143	57	6245100	0.2340	6127485	3600	1.9195
15	200	0.4815	135	65	4476170	0.1716	4391710	3600	1.9232
16	300	0.5306	196	104	7042970	1.0452	6821860	3600	3.2412
17	300	0.5873	189	111	6664130	0.5148	6352040	3600	4.9132
18	300	0.4634	205	95	6981260	0.5772	6783870	3600	2.9097
19	300	0.5385	195	105	7646975	0.4212	7434005	3600	2.8648
20	300	0.5789	190	110	5799355	0.4056	5686215	3600	1.9897
21	500	0.5106	331	169	11217725	2.5272	10825845	3600	3.6199
22	500	0.4620	342	158	15632570	1.6068	14790680	3600	5.6920
23	500	0.5291	327	173	12132230	1.4196	11658955	3600	4.0593
24	500	0.6393	305	195	10457775	1.5444	10070425	3600	3.8464
25	500	0.4837	337	163	11598800	1.2636	11238320	3600	3.2076
26	*800*	*0.5326*	*522*	*278*	*20317015*	*7.1136*	*18580980*	3600	*9.3431*
27	800	0.5748	508	292	23043050	7.0900	21287010	3600	8.2494
28	800	0.4981	534	266	20013165	6.8900	18659160	3600	7.2565
29	800	0.5355	521	279	18633300	6.3000	17779830	3600	4.8002
30	800	0.6097	497	303	19422815	8.0300	17808285	3600	9.0662
31	1200	0.4634	820	380	30810930	23.0200	28035140	3600	9.9011
32	1200	0.5748	762	438	28919055	24.6600	26148415	3600	10.5958
33	1200	0.5228	788	412	28866420	19.9200	26852685	3600	7.4992
34	1200	0.5464	776	424	31690120	24.7700	27829950	3600	13.8706
35	1200	0.5564	771	429	33647415	23.6100	28825775	3600	16.7268

Appendix 3

Results for scenario 3

Sl. No.	Instance size	Alpha	Number of source	Number of destination	Heuristic		CPLEX		Gap (%)
					Value	Solution Time	Value	Time limit (sec)	
1	50	6.1429	7	43	4981685	0.0398	4921825	3600	1.2162
2	50	6.1429	7	43	3223425	0.0778	3187710	3600	1.1204
3	50	7.3333	6	44	2079975	0.0381	2036070	3600	2.1564

(continued)

(continued)

Sl. No.	Instance size	Alpha	Number of source	Number of destination	Heuristic		CPLEX		Gap (%)
					Value	Solution Time	Value	Time limit (sec)	
4	50	3.5455	11	39	1494890	0.0627	1464555	3600	2.0713
5	50	2.3333	15	35	1328165	0.0498	1306285	3600	1.6750
6	100	3.3478	23	77	3299650	0.2451	3181875	3600	3.7014
7	100	3.3478	23	77	3690860	0.0727	3556420	3600	3.7802
8	100	4.2632	19	81	3642470	0.0679	3521515	3600	3.4347
9	100	3.1667	24	76	2711575	0.0668	2602320	3600	4.1984
10	100	3.1667	24	76	3641430	0.0738	3536805	3600	2.9582
11	200	5.0606	33	167	7072930	0.12	6822750	3600	3.6668
12	200	3.4444	45	155	8074245	0.1248	7733115	3600	4.4113
13	200	4.5556	36	164	7490365	0.1248	7181364	3600	4.3028
14	200	3.6512	43	157	6645620	0.1092	6394775	3600	3.9227
15	200	3.5455	44	156	7642950	0.1404	7289610	3600	4.8472
16	300	4.5556	54	246	12233600	0.234	11416165	3600	7.1603
17	300	3.9180	61	239	9274165	0.234	8949305	3600	3.6300
18	300	3.6875	64	236	8131880	0.2496	7869360	3600	3.3360
19	300	4.4545	55	245	13339635	0.2496	13402075	3600[a]	−0.4659
20	300	4.8824	51	249	11978365	0.3276	11336165	3600	5.6651
21	500	4.2083	96	404	17876425	0.78	16801105	3600	6.4003
22	500	3.5872	109	391	15447495	0.9204	14839285	3600	4.0986
23	500	4.2083	96	404	18047305	1.014	18872055	3600[a]	−4.3702
24	500	3.5872	109	391	16618755	1.2168	17630594	3600[a]	−5.7391
25	500	4.0000	100	400	18938390	0.8736	17745595	3600	6.7216
26	*800*	4.3691	149	651	*35611425*	*3.6036*	*34186968*	3600	4.1667
27	800	7.2474	97	703	35365390	3.3852	33080785	3600	6.9061
28	800	3.6784	171	629	23279945	3.744	22551970	3600	3.2280
29	800	3.5977	174	626	24362435	4.47	23197980	3600	5.0196
30	800	4.2632	152	648	31086625	4.0092	29056840	3600	6.9856
31	1200	3.7431	253	947	46173110	13.6498	45190590	3600	2.1742
32	1200	4.3333	225	975	51691665	11.2478	46962155	3600	10.0709
33	1200	4.5046	218	982	40734345	9.3758	37940585	3600	7.3635
34	1200	4.0633	237	963	44112455	13.2444	39902870	3600	10.5496
35	1200	3.7244	254	946	40565095	12.4648	37375475	3600	8.5340

[a]Exceeded CPLEX time limit

References

1. Asgari N, Farahani RZ, Rashidi-Bajgan H, Sajadieh MS (2013) Developing model-based software to optimise wheat storage and transportation a real-world application. Appl Soft Comput J 13(2):1074–1084
2. Aydinel M, Sowlati T, Cerda X, Cope E, Gerschman M (2008) Optimization of production allocation and transportation of customer orders for a leading forest products company. Math Comput Model 48(7–8):1158–1169. https://doi.org/10.1016/j.mcm.2007.12.025

3. Balani S (2013) Functioning of the public distribution system an analytical report. www.prsindia.org/administrator/uploads/general/1388728622~~TPDSThematicNote.pdf, Accessed 19 Octo 2015
4. Burns LD, Hall RW, Blumenfeld DE, Daganzo CF (1985) Distribution strategies that minimize transportation and inventory costs
5. Chakraborty S, Sarmah SP (2018) India 2025: the public distribution system and National Food Security Act. Dev Pract, 1–20
6. Crainic TG, Hewitt M, Toulouse M, Vu DM, Crainic TG (2016) Service network design with resource constraints
7. CAG (2013) Storage management and movement of food grains in Food Corporation of India, vol 3. http://164.100.47.132/paperlaidfiles/CONSUMERAFFAIRS,FOODANDPUBLICDISTRIBUTION/ReportNo7-English.pdf, Accessed 28 June 2015
8. Cho JH, Kim HS, Choi HR (2012) An intermodal transport network planning algorithm using dynamic programming—a case study: from Busan to Rotterdam in intermodal freight routing. Appl Intel 36:529–541
9. Guimarães L, Amorim P, Sperandio F, Moreira F, Almada-Lobo B (2014) Annual distribution budget in the beverage industry: a case study. Interfaces 44(6):605–626
10. Kang JH, Kim YD (2010) Coordination of inventory and transportation managements in a two-level supply chain. Int J Prod Econ 123(1):137–145
11. Lee C, Çetinkaya S, Jaruphongsa W, Lee C (2003) A dynamic model for inventory lot sizing and outbound shipment scheduling at a third-party warehouse a dynamic model for inventory lot sizing and outbound shipment scheduling at a third-party warehouse
12. Moccia L, Cordeau J, Ropke S, Valentini M (2011) Modeling and solving a multimodal transportation problem with flexible-time and scheduled services. Networks 57(1):53–68
13. Muriana C (2016) An EOQ model for perishable products with fixed shelf life under stochastic demand conditions. Eur J Oper Res. http://doi.org/10.1016/j.ejor.2016.04.036
14. Nguyen C, Toriello A, Dessouky M, Moore JE, Nguyen C, Toriello A, Ii JEM (2013) Evaluation of transportation practices in the california cut flower industry
15. Philip CM (15 June 2015) In 2 years, 40,000 tonnes of grain went down the drain. In: Times of India. http://timesofindia.indiatimes.com
16. Steadieseifi M, Dellaert NP, Nuijten W, Woensel T Van, Raoufi R (2014) Multimodal freight transportation planning: a literature review. Eur J Oper Res 233(1):1–15. https://doi.org/10.1016/j.ejor.2013.06.055
17. Sel Ç, Bilgen B (2014) Hybrid simulation and MIP based heuristic algorithm for the production and distribution planning in the soft drink industry. J Manuf Syst 33(3):385–399
18. Tanksale A, Jha JK (2016) INFOR : information systems and operational research an effective heuristic for multi-period multi-food grain inventory transportation problem in India, vol. 5986(September). http://doi.org/10.1080/03155986.2016.1231498
19. Wang Q (2014) Inbound logistic planning : minimizing transportation and inventory cost inbound logistic planning : minimizing transportation and inventory cost. http://doi.org/10.1287/trsc.1050.0130
20. Xie F, Huang Y, Eksioglu S (2014) Bioresource technology Integrating multimodal transport into cellulosic biofuel supply chain design under feedstock seasonality with a case study based on California. Biores Technol 152:15–23. https://doi.org/10.1016/j.biortech.2013.10.074

Chapter 20
Fuzzy Based Inventory Model with Credit Financing Under Learning Process

Mahesh Kumar Jayaswal, Isha Sangal and Mandeep Mittal

Abstract Whilst business dealings, the cost of items is a vital consideration for the buyer in order to purchase goods as well as to minimize the items' cost. For the accomplishment of the same, the buyer performs a new task after a fluent repetition over daily dealing of goods and this new task is entitled as learning. Nowadays, learning's awareness is increasing across various disciplines because learning effect has a direct impact on the calculation of profit or loss and it is a promotional deemed effective tool for inventory management. The supplier wants an appreciable coordination with the buyers and analyzes with full detail, the concerned cost and the demand parameters as to how suitable the demand and the respective costs should be for the buyer. Fuzzy analysis is a good tool for examining the performance as well as the output of imprecise parameters involved in the business procedure. In this paper, we are assuming the holding costs to be partially constant and reduced per shipment, owing to the learning effects for finding an optimal cycle time and optimal average cost using the notion of learning effect with trade credit financing for EOQ under the fuzzy environment. The selling price, demand rate, and ordering cost per unit have been assumed to be imprecise in nature. In addition, these entities are also called fuzzy triangular numbers. The total optimal cost in fuzzy environment is de-fuzzified with the help of the centroid method. Toward the conclusion of this paper, some numerical examples as well as sensitivity analysis have been illustrated to verify this model.

Keywords Learning effect · Inventory · Economic order quantity · Credit financing · Fuzzy environment

M. K. Jayaswal · I. Sangal
Department of Mathematics & Statistics, Banasthali Vidyapith, Banasthali, Jaipur, Rajasthan, India
e-mail: maheshjayaswal7@gmail.com

I. Sangal
e-mail: ishasangal@gmail.com

M. Mittal (✉)
Department of Mathematics, Amity Institute of Applied Sciences,
Amity University Noida, Noida, UP, India
e-mail: mittal_mandeep@yahoo.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_20

20.1 Introduction

Typically, in the business market, the supplier offers a trade credit financing policy to upgrade his sales and profit and pose as a focus for the new buyers. Consequently, in practical implementation, the supplier will permit a designated predetermined time period to arrange money for the buyer that the supplier owes to the buyer for the items delivered. Prior to the conclusion of the specified trade credit financing time, the items can be sold by the buyer; he could procure interest, and accumulate revenue for the same. In cases, where the cash could not be arranged within the stipulated time for trade credit financing, then a higher rate of interest is imposed and charged. Goyal [6] initially, recommended the formulation of EOQ utilizing trade credit financing. Shah [18, 19], Aggarwal and Jaggi [1], proposed the EOQ model with trade credit financing specifically for the articles which are decaying in quality. The Learning phenomenon is a mathematical tool which is implemented during business for reduction of ordering cost, holding costs, as well as screening cost. There have been many researchers who have been working constantly in evolving the mathematical model employing and implementing the learning effect.

The Learning curve, developed by Wright [22] is a mathematical tool. In his first attempt he successfully derived the mathematical formula which derives the relationship between learning variables in quantitative shape and also went on to derive the result in the proposition of the LC (learning curve). Again, different from the excess of review on LC, there is a scarcity of review on forgetting curves. Baloff [4] discussed about the mathematical behavior of the learning theory (learning slope varied widely and also explained with viable justification, the outcomes of the practical aspects to prepare the learning curve parameters by developed skill and analytical study in collection learning). Salameh et al. [16] considered a limited manufactured stock form (Production inventory model) with the outcome of human knowledge and also discussed about the variable demand rate and learning in time to optimize the cost. Jaber et al. [7] explained the theory of forgetting using manufacture breaks, learning curves and also discussed the optimal manufacturing amount in order to minimize the whole stock price. Jaber et al. [8] have been working on assuming optimal lot sizing using the conditions of bounded learning cases and focus more on EOQ and minimization of the whole stock cost keeping the learning curves in consideration.

Jaber et al. [9] discussed and explored a comparative study of the learning and forgetting theory and also analytically focused on the comparison of different types of models such as VRVF, VRIF, and LFCM. Jaber et al. [13] discussed about optimal lot sizing with shortage and back ordering, considering them under learning. Jaber et al. [11] discussed that in the EOQ model for imperfect quality articles, the defective percentage per batch decreases according to the LC (Learning curve).

Khan et al. [15] considered an EOQ formulation for articles with defective features that use learning in screening and maximizing production together with minimizing the cost of production. Jaber et al. [12] discussed on how to merge the average dispensation time process to give way with respect to the number of lot. Anzanello and Fogliatto [3] suggested a different kind of implementation of learning curves model and these authors focused on how this model could be used and implemented in distinct mathematical forms. Jaggi et al. [14] discussed about the production inventory model with financing policies of imperfect items under the acceptable backlogging case. Jaber et al. [10] took a manufactured stock model with LC and FC, that is the "learning and forgetting" theory in consideration and also discussed how much minimization of the number of order (shipments) of a batch from manufacturer to the subsequent cycle was required or implied. Mostly, researchers considered all the parameters of the inventory model such as demand rate, selling price, holding cost as ordering cost, etc., either as fixed, dependent on time or probabilistic in nature for the improvement of EOQ. Generally, an author assumes various types of parameters, in the formulation of inventory design or models either as a fixed, dependent on time or uncertain or flexible in nature for the improvement of the EOQ. Though, in practical scenarios, such type of components may have little formulations from the certain values and they do not necessarily follow any type of probability distribution as they are discontinuous in nature. Suppose, such components are treated as fuzzy components, then they will be other sensible. Teng et al. [21] proposed a task by using the optimal trade credit strategies and lot size policies in economic production quantity models with learning curve affecting the production costs. Givi et al. [5] discussed the modeling of worker reliability with learning and fatigue. In the paper, Sangal and Rani [17] discussed the working policy of a fuzzy environment inventory model with partial backlogging under the learning effect and they also discussed the optimal policy for non-instantaneous decaying inventory model with learning effect in the paper by Aggarwal [2]. The fuzzy inventory model with items subjected to the learning effect on the holding cost was developed. We assumed the inventory problem in which the demand parameter, ordering cost, as well as the purchasing cost to be fuzzy variables so as to control the uncertainty of business market. In this paper, we have improved an EOQ with permissible delay in payment in the fuzzy environment using learning effect. Some parameters in the considered inventory model like selling price per item, ordering cost per order, and demand rate may be fixed or not with a few imprecision in their values. In practical implications, the components of an inventory model are uncertain, inaccurate, and the determination of a maximum cycle length is difficult as it is a non-stochastic indistinguishable managerial process. Such types of problems have been tried to be solved by this model and the verification of the result has been illustrated by appropriate examples. Sensitivity analysis as well as conclusion has been presented in the last section.

20.2 Assumptions and Notations

The mathematical model is derived using the following notations and assumptions.

20.2.1 Assumptions

The subsequent assumptions have been incorporated to expand the present model:

- The demand rate for an item is imprecise in nature.
- Demand is fulfilled, and no shortages are allowed.
- Selling price of items is imprecise in nature.
- Ordering cost of an item is imprecise in nature.
- The rate of replenishment is immediate.
- Lead time is zero and insignificant.
- The supplier provides a predetermined credit period to clear up the accounts to the buyer which is suggested by Jaggi et al. [14].
- Holding cost is partially constant and partially decreases in each shipment (n) owing to the learning power of employees and suggested by Aggarwal et al. [2])

$$h(n) = ho + \frac{h_1}{n^{\lambda}}, ho, h_1 > 0 \, and \; 0 < \lambda < 1$$

20.2.2 Notations

D	fixed demand per annum
$\widetilde{D}$	fuzzy demand per annum
A_c	ordering cost per order in dollars
$\widetilde{A}_c$	fuzzy ordering cost per lot in dollar
P_c	selling price per unit
$\widetilde{P}_c$	fuzzy unit selling price
C_c	unit purchase cost
$h(n)$	unit inventory variable holding cost per lot excluding the interest charges
Q_o	order quantity
M_s	The offered trade credit by the supplier to the buyer to settle the account
IHC_1	holding cost per cycle
IC_1	interest paid/charged under the condition, $M_s \leq T_c$
IE_1	interest gained under the condition, $M_s \leq T_c$
IC_2	interest paid under the condition, $M_s \geq T_c$
IE_2	interest paid under the condition, $M_s \geq T_c$
I_c	interest paid per $ in stock per year by the retailer
I_e	interest gained per $ per year by the buyer
T_c	cycle time in year
$\Psi_1(T_c)$	the whole concerned cost per unit time with case $M_s \leq T_c$
$\Psi_2(T_c)$	the whole concerned cost per unit time with case $M_s \geq T_c$
$\widetilde{\Psi}_1(T_c)$	fuzzy whole concerned cost per unit time with case $M_s \leq T_c$
$\widetilde{\Psi}_2(T_c)$	fuzzy whole concerned cost per unit time with case $M_s \geq T_c$

$\widetilde{\Psi}_3(T_c)$	de-fuzzy whole concerned cost per unit time with case $M_s \leq T_c$
$\widetilde{\Psi}_4(T_c)$	de-fuzzy whole concerned cost per unit time with case $M_s \geq T_c$
n	number of shipment
λ	learning factor
T_{c1}	the whole cycle time under the condition, $M_s \leq T_c$
T_{c2}	the whole cycle time under the condition, $M_s \geq T_c$
$\widetilde{T}_{c1}$	de-fuzzy the whole cycle time under the condition, $M_s \leq T_c$
$\widetilde{T}_{c2}$	de-fuzzy whole cycle time under the condition, $M_s \geq T_c$

20.3 Crisp Formulation Model

Assume that $q(t)$ is the inventory stock at any time $t(0 \leq t \leq T_c)$. At the initial position the inventory level is Q_0. Now the cost parameters below, related to inventory stock (Fig. 20.1),

The cost of placing an order,

$$OC_1 = \frac{A_c}{T_c} \tag{20.1}$$

The inventory carrying cost per cycle,

$$IHC_1 = \frac{1}{T_c} h(n) \int_0^{T_c} Dt\, dt = \frac{h(n) D T_c}{2} \tag{20.2}$$

The interest paid and interest gained, are the two cases that arise depending on the lengths of T_c and M_s. These cases are presented in the graphed form in Figs. 20.2 and 20.3.

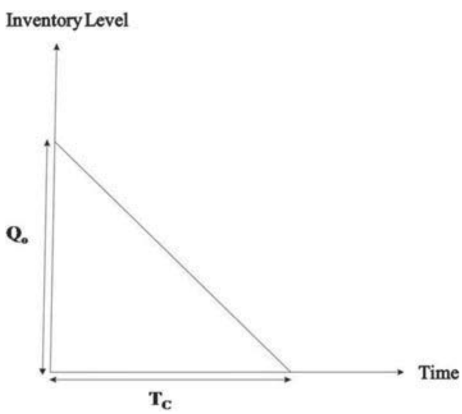


Fig. 20.1 Inventory system with order quantity and time

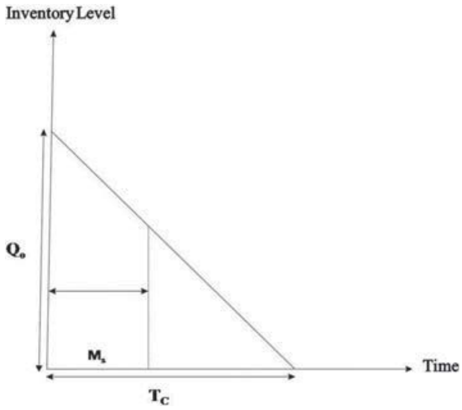


Fig. 20.2 Inventory process of trade credit financing for case-1

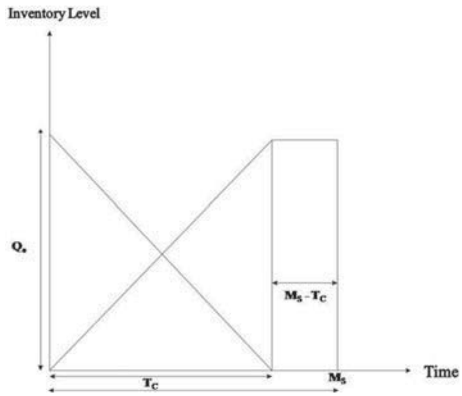


Fig. 20.3 Inventory process of trade credit financing for case-2

Case-1: $M_s \leq T_c$

The buyer gains interest at a rate of I_e on the average sales income generated for the time 0 to M_s. Further, the buyer has to settle the account at credit period M_s and must arrange for finances to pay the seller for the lasting inventory store at the precise rate of interest, I_c, from M_s to T_c.

Therefore, the buyers gain an interest for the average inventory during time period 0 to M_s, i.e., $I_e I_c D M_s^2/2T_c$ and buyers pay an interest for the unsold items after M_s which is equal to $C_c I_c D(T_c - M_s)^2/2T_c$.

Hence, the whole concerned cost per unit time is,

$$\Psi_1(T_c) = OC_1 + IHC_1 + IC_1 - IE_1$$

$$\Psi_1(T_c) = \frac{A_c}{T_c} + \frac{h(n)DT_c}{2} + \frac{C_c I_c (T_c - M_c)^2}{2T_c} - \frac{p_c I_e D M_s^2}{2T_c} \quad (20.3)$$

Case-2: $M_s \geq T_c$

In this case, no interest is payable by the buyer, who only gains an interest on the income generated from 0 to M_s and equal to $P_c I_e D\big(M_s - \frac{T_c}{2}\big)$ and $IC_2 = 0$.

Hence, the whole concerned cost per unit time is,

$$\Psi_2(T_c) = OC_1 + IHC_1 + IC_2 - IE_2$$

$$\Psi_2(T_c) = \frac{A_c}{T_c} + \frac{h(n)DT_c}{2} - P_c I_e D\left(M_s - \frac{T_c}{2}\right) \tag{20.4}$$

Hence, the whole relevant cost $\Psi(T_c)$ per time unit is,

$$\Psi(T_c) = \begin{cases} \Psi_1(T_c),\ M_s \le T_c \\ \Psi_2(T_c),\ M_s \ge T_c \end{cases} \tag{20.5}$$

where,

$$\Psi_1(T_c) = \frac{A_c}{T_c} + \frac{h(n)DT_c}{2} + \frac{C_c I_c (T_c - M_c)^2}{2T_c} - \frac{p_c I_e D M_s^2}{2T_c}$$

and

$$\Psi_2(T_c) = \frac{A_c}{T_c} + \frac{h(n)DT_c}{2} - P_c I_e D\left(M_s - \frac{T_c}{2}\right).$$

It can be easily checked or verified that $\Psi_1(M_s) = \Psi_2(M_s)$, so $\Psi(T_c)$ is a continuous function of T_c.

The necessary and sufficient conditions for $\Psi_1(T_c)$ to be optimum are as follows,

$$\frac{d\Psi_1(T_c)}{dT_c} = -\frac{A_c}{T_c^2} + \frac{h(n)D}{2} + \frac{C_c I_c D}{2} - \frac{C_c I_c D M_s^2}{2T_c^2} + \frac{P_s I_e D M_s^2}{2T_c^2}$$

and $\frac{d^2\Psi_1(T_c)}{dT_c^2} = \frac{2A_c}{T_c^3} - \frac{C_c I_c D M_s^2}{T_c^3} - \frac{P_s I_e D M_s^2}{T_c^3} > 0$ respectively. For the maximum cycle length of time T_{c1}, set $\frac{d\Psi_1(T_c)}{dT_c} = 0$ which gives

$$T_c = T_{c1} = \sqrt{\frac{2A_c + DM_s^2(C_c I_c - P_c I_e)}{D(h(n) + C_c I_c)}} \tag{20.6}$$

Now,

$$\frac{d\psi_2(T_c)}{dT_c} = -\frac{A_c}{T_c^2} + \frac{h(n)D}{2} + \frac{P_c I_e D}{2}$$

and

$$\frac{d^2\Psi_2(T_c)}{dT_c^2} = \frac{2A_c}{T_c^3} > 0$$

For the optimal cycle time T_{c2}, set $\frac{d\Psi_2(T_c)}{dT_c} = 0$ which gives

$$T_c = T_{c2}(\text{say}) = \sqrt{\frac{2A_c}{D(h(n) + P_c I_e)}} \tag{20.7}$$

20.4 Fuzzy Methodology

As per assumption, A_c, D and P_c are not known precisely and let A_c, D and P_c be defined as triangular fuzzy numbers such that $\widetilde{A}_c = [a_1, a_2, a_3]$, $\widetilde{D} = [d_1, d_2, d_3]$ and $\widetilde{P}_c = [p_1, p_2, p_3]$ where $(a_1 < a_2 < a_3)$, $(d_1 < d_2 < d_3)$, and $(p_1 < p_2 < p_3)$ are based on subjective judgments.

We apply arithmetic operators on fuzzy quantities and then de-fuzzify the same to convert them into crisp output.

The membership functions for $\widetilde{A}_c$, $\widetilde{D}$ and $\widetilde{P}_c$ are defined as follows

$$\mu_{\widetilde{A}_c}(\widetilde{A}_c) = \begin{cases} 0, & if\ A_c < a_1 \\ \frac{A_c - a_1}{a_2 - a_1}, & if\ a_1 \le A < a_2 \\ \frac{a_3 - A_c}{a_3 - a_2}, & if\ a_2 \le A_c < a_3 \\ 0, & if\ A_c \ge a_3 \end{cases} \tag{I}$$

$$\mu_{\widetilde{D}}(\widetilde{D}) = \begin{cases} 0, & if\ D < d_1 \\ \frac{D - d_1}{d_2 - d_1}, & if\ d_1 \le D < d_2 \\ \frac{d_3 - D}{d_3 - d_2}, & if\ d_2 \le D < d_3 \\ 0, & if\ D \ge d_3 \end{cases} \tag{II}$$

$$\mu_{\widetilde{P}_c}(\widetilde{P}_c) = \begin{cases} 0, & if\ P_c < p_1 \\ \frac{P_c - p_1}{p_2 - p_1}, & if\ p_1 \le P_c < p_2 \\ \frac{p_3 - P_c}{\tilde{p}_3 - \tilde{p}_2}, & if\ p_2 \le P_c < p_3 \\ 0, & if\ P_c \ge p_3 \end{cases} \tag{III}$$

The de-fuzzified $\widetilde{\Psi}_1(T_c)$ and $\widetilde{\Psi}_2(T_c)$ by centroid method are defined as illustrated below,

$$\widetilde{\Psi}_3(T_c) = \frac{\widetilde{\Psi}_{11}(T_c) + \widetilde{\Psi}_{12}(T_c) + \widetilde{\Psi}_{13}}{3T_c}. \tag{IV}$$

$$\widetilde{\Psi}_4(T_c) = \frac{\widetilde{\Psi}_{21}(T_c) + \widetilde{\Psi}_{22}(T_c) + \widetilde{\Psi}_{23}(T_c)}{3T_c}. \tag{V}$$

20.4.1 Fuzzy Inventory Model

During the formulation of the fuzzy process, we consider that the demand rate, ordering cost and selling price are all imprecise in nature and are termed as fuzzy numbers and are denoted by $\tilde{D}$, $\tilde{A}_c$ and $\tilde{P}_c$ respectively. Here, we assume that $\widetilde{D} = (d_1, d_2, d_3)$, $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{P}_c = (p_1, p_2, p_3)$ are positive triangular fuzzy numbers.

20.4.2 Derivation of $\widetilde{\Psi}_1(T_c)$ and $\widetilde{\Psi}_2(T_c)$

The fuzzy annual whole concerned cost can be formulated as,

$$\widetilde{\Psi}(T_c) = \begin{cases} \widetilde{\Psi}_1(T_c), \ M_s \le T_c \\ \widetilde{\Psi}_2(T_c), \ M_s \ge T_c \end{cases} \tag{20.8}$$

Where,

$$\widetilde{\Psi}_1(T_c) = Y_{11}\widetilde{A}_c + Y_{12}\widetilde{D} + Y_{13}\widetilde{P}_c\widetilde{D}$$

$$\widetilde{\Psi}_{11}(T_c) = Y_{11}a_1 + Y_{12}d_1 + Y_{13}p_1d_1 \tag{20.9}$$

$$\widetilde{\Psi}_{12}(T_c) = Y_{11}a_2 + Y_{12}d_2 + Y_{13}p_2d_2 \tag{20.10}$$

$$\widetilde{\Psi}_{13}(T_c) = Y_{11}a_3 + Y_{12}d_3 + Y_{13}p_3d_3 \tag{20.11}$$

and

$$\widetilde{\Psi}_2(T_c) = Y_{21}\widetilde{A}_c + Y_{22}\widetilde{D} + Y_{23}\widetilde{P}_c\widetilde{D}$$

$$\widetilde{\Psi}_{21}(T_c) = Y_{21}a_1 + Y_{22}d_1 + Y_{23}p_1d_1 \tag{20.12}$$

$$\widetilde{\Psi}_{22}(T_c) = Y_{21}a_2 + Y_{22}d_2 + Y_{23}p_2d_2 \quad (20.13)$$

$$\widetilde{\Psi}_{23}(T_c) = Y_{21}a_3 + Y_{22}d_3 + Y_{23}p_3d_3 \quad (20.14)$$

where, $Y_{11} = Y_{21} = \frac{1}{T_c}$, $Y_{12} = \frac{h(n).T_c}{2} + \frac{C_c I_c}{2}\left[T_c + \frac{M_s^2}{T_c} - 2M_s\right]$, $Y_{13} = -\frac{I_e M_s^2}{2T_c}$

$$Y_{22} = \frac{h(n).T_c}{2}, Y_{23} = I_e\left[\frac{T_c}{2} - M_s\right].$$

From Eqs. (20.9), (20.10), and (20.11) we have de-fuzzified $\widetilde{\Psi}_1(T_c)$ by centroid method which is equal to

$$\widetilde{\Psi}_3(T_c) = \frac{\widetilde{\Psi}_{11}(T_c) + \widetilde{\Psi}_{12}(T_c) + \widetilde{\Psi}_{13}(T_c)}{3T_c}.$$

$$\widetilde{\Psi}_3(T_c) = \frac{Y_{11}(a_1 + a_2 + a_3) + Y_{12}(d_1 + d_2 + d_2) + Y_{13}(p_1d_1 + p_2d_2 + p_3d_3)}{3T_c}. \quad (20.15)$$

From Eqs. (20.12), (20.13), and (20.14) we have de-fuzzified $\widetilde{\Psi}_1(T_c)$ by centroid method which is equal

$$\widetilde{\Psi}_4(T_c) = \frac{\widetilde{\Psi}_{21}(T_c) + \widetilde{\Psi}_{22}(T_c) + \widetilde{\Psi}_{23}(T_c)}{3T_c}.$$

$$\widetilde{\Psi}_4(T_c) = \frac{Y_{21}(a_1 + a_2 + a_3) + Y_{22}(d_1 + d_2 + d_2) + Y_{23}(p_1d_1 + p_2d_2 + p_3d_3)}{3T_c}. \quad (20.16)$$

20.4.3 Solution Procedure

The necessary and sufficient conditions for $\widetilde{\Psi}_3(T_c)$ to be optimum and for the optimal cycle time $\widetilde{T}_{c1}$, set $\frac{d\widetilde{\Psi}_3(T_c)}{dT_c} = 0$, from Eq. (20.15) which gives

$$\widetilde{T}_{c1} = \sqrt{\frac{2(a_1 + a_2 + a_3) + (d_1 + d_2 + d_3)M_s^2(C_c I_c - (p_1 + p_2 + p_3)I_e)}{(d_1 + d_2 + d_3)(h(n) + C_c I_c)}} \quad (20.17)$$

and for the optimal cycle time T_{c2}, set $\frac{d\widetilde{\Psi}_4(T_c)}{dT_c} = 0$, from Eq. (20.16)which gives

$$T_c = \widetilde{T}_{c2}(\text{say}) = \sqrt{\frac{2(a_1 + a_2 + a_3)}{(d_1 + d_2 + d_3)\,(h(n) + (p_1 + p_2 + p_3)I_e)}} \tag{20.18}$$

For the total cost function of this system to be convex, the sufficient condition must hold $\frac{d\widetilde{\Psi}_4(\widetilde{T}_{c2})}{dT_c} > 0$.

20.4.4 Algorithm Procedure

To find out the maximum cycle length of time (between $\widetilde{T}_{C_1}$ and $\widetilde{T}_{C_2}$) and whole maximum average cost (between $\widetilde{\Psi}_3(\widetilde{T}_{c1})$ and $\widetilde{\Psi}_4(\widetilde{T}_{c2})$) for various values of M_s, the following algorithms employed in the form different sequential steps and followed [14].

Step-1: Find out $\widetilde{T}_{c1}$ and $\widetilde{T}_{c2}$ by solving Eqs. (20.17) and (20.18).
Step-2: If $\widetilde{T}_{c1} \le M_s$, then calculate $\widetilde{T}c_2$ and $\widetilde{\Psi}_4(\widetilde{T}_{c2})$, otherwise go to step-3.
Step-3: If $\widetilde{T}_{c1} > M_s$, then calculate $\widetilde{\Psi}_3(\widetilde{T}_{c1})$.
Step-4: Find out the concerning cycle length of time and whole maximum cost.

20.5 Model Illustrated Examples

The almost inventory parameters have taken from Shah et al. [20];

$$\widetilde{A}_c = (48, 50, 52),\ \widetilde{P}_c = (118, 120, 122),\ \widetilde{D} = (480, 500, 520),\ I_e = 0.12/year,\ I_c = 0.15/year,$$
$$M_S = 0.068\ \ year, n = 1,\ \ \lambda = 0.10,\ h_1 = \$5/unit/year,\ h_0 = \$4/unit/year, C_c = \$50/unit$$

After using algorithm we got fuzzy optimal cycle time, $\widetilde{T}_{c2} = 0.061$ *year* and fuzzy total cost corresponding fuzzy optimal cycle time is $\widetilde{\Psi}_4(\widetilde{T}_{c2}) = 136.097\$$. After simplification of Eqs. (20.16) with the help of mathematica software 0.8, we got, $\frac{d^2\Psi_4(\widetilde{T}_{c2})}{dT_{c2}^2} = 29 > 0$, where $\widetilde{T}_{c2} = 0.061$ *year* which represents the convexity of the cost function for the retailer's and shown in Fig. 20.4.

20.6 Sensitivity Analysis

20.6.1 Observations

From Table 20.1, we observed that, when the values of M_s increased, the maximum cycle length of time and the optimal average cost for the retailer did not increase owing

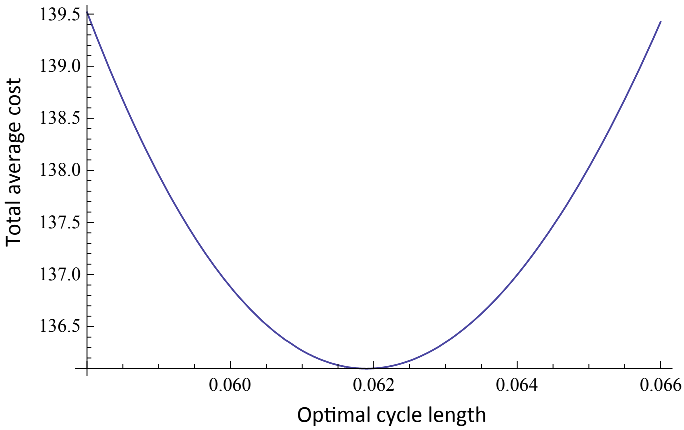


Fig. 20.4 Convexity of total average cost

Table 20.1 Impact of trade credit on cycle length and average cost

Fuzzy unit selling price $\widetilde{P}_c$	Trade credit period (in year) M_s	Optimal cycle (in year) $\widetilde{T}_{c2}$ In year	Average cost $\widetilde{\Psi}_4(\widetilde{T}_{c2})$
(118, 120, 122)	0.063	0.061	254
(118, 120, 122)	0.065	0.061	195
(118, 120, 122)	0.068	0.061	136

to the learning phenomenon. We analyzed that the retailer would not order additional quantity to receive the gain of delayed cash payment more often. From Table 20.2, we analyzed that whenever the number of shipments increased, the optimal cycle time increased but the average cost decreased in each shipment due to the learning effect. From Table 20.3, we observed that when the values of learning rate increased the maximum cycle length of time increased and the average cost decreased in each shipment owing to the process of learning. From Fig. 20.5, we were able to analyze that whenever the number of shipments increased, the average cost decreased in each shipment due to the phenomenon of learning.

Table 20.2 Variation of fuzzy average cost and optimal cycle length with respect to number of shipments under learning effect

Number of shipment n	Optimal cycle $\widetilde{T}_{c2}$	Average cost for the case-2 $\widetilde{\Psi}_4(\widetilde{T}_{c2})$
1	0.061	136
2	0.062	131
3	0.062	129
4	0.062	128

Table 20.3 Impact of learning factor with variable shipment on cycle length and average cost

Learning factor γ	Optimal cycle $\widetilde{T}_{c2}$	Average cost for the case-2 $\widetilde{\Psi}_4(\widetilde{T}_{c2})$
0.10	0.061	136
0.20	0.062	128
0.30	0.062	118
0.40	0.062	109

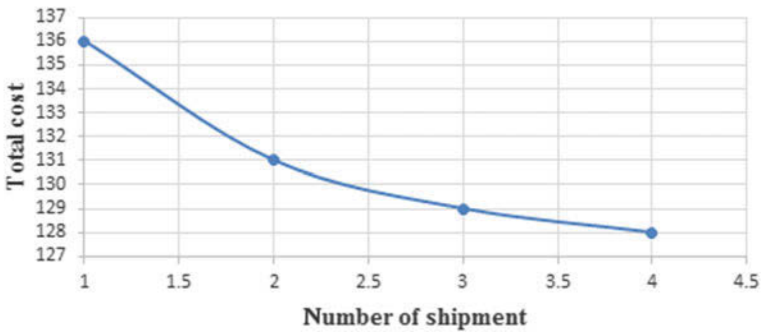


Fig. 20.5 Variation of cost with the number of shipments

20.7 Conclusion

This paper covered as well as reported both the mathematical and management areas and also went on to design an intelligent automation policy and a preventive inventory cost maintenance which employed and implemented the learning effect. This combination of work has been done in this article for contributing to the industrial sector. In general, some industries have some constraints to apply machinery system instead of human labor. This area requires extensive research to find out the optimal solution with profit maximization or cost minimization. In this contemporary research paper, we have attempted to find a solution to such a type of problem by implementing the learning effect. In this paper, an EOQ model has been modified under the fuzzy environment where permissible setback in cash is allowed from the seller to his buyer under the learning effect. The fuzzy whole variable cost and the fuzzy cycle length of time have been derived and de-fuzzified. The fuzzy optimal total cost and the fuzzy optimal cycle length of time have been obtained by employing the centroid method. Sensitivity analysis as well as observations exposed that a longer value of the allowable setback time period reduced the whole cycle length of time and the whole cost of the retailer under the learning effect.

References

1. Aggarwal SP, Jaggi CK (1995) Ordering policies of deteriorating items under permissible delay in payments. J Oper Res Soc 46:658–662
2. Aggarwal A, Sangal I, Singh SR (2017) Optimal policy for non-instantaneous decaying inventory model with learning effect with partial shortages. Int J Comput Appl 161:13–18
3. Anzanello MJ, Fogliatto FS (2011) Learning curve model and applications: Literature review and research directions. Int J Ind Ergon 41:573–583
4. Baloff N (1966) Startups in machine-intensive production system. Int J Eng 17:132–141
5. Givi ZS, Jaber MY, Neumann WP (2015) Modeling worker reliability with learning and fatigue. Appl Math Model 39:5186–5199
6. Goyal SK (1985) Economic order quantity under condition of permissible delay in payment. J Oper Res Soc 36:335–338
7. Jaber MY, Bonney M (1996) Optimal lot sizing under learning considerations: the bounded learning case. Appl Math Model 20:750–755
8. Jaber MY, Bonney M (1996) Production breaks and learning curve: the forgetting phenomenon. Appl Math Model 2:157–169
9. Jaber MY, Bonney M (1997) A comparative study of learning curves with forgetting. Appl Math Model 21:523–531
10. Jaber MY, Bonney M (2003) Lot sizing with learning and forgetting in set-ups and in product quality. Int J Prod Econ 83:95–111
11. Jaber MY, Goyal SK, Imran M (2008) Economic production quantity model for items with imperfect quality subjected to learning effects. Int J Prod Econ 115:143–150
12. Jaber MY, Khan M (2010) Managing yield by lot splitting in a serial production line with learning rework and scrap. Int J Prod Econ 124:32–39
13. Jaber MY, Salameh MK (1995) Optimal lot sizing under learning considerations: Shortages allowed and backordered. Appl Math Model 19:307–310
14. Jaggi CK, Goyal SK, Mittal M (2013) Credit financing in economic order policies for defective items with allowable shortages. Appl Math Comput 219:5268–5282
15. Khan M, Jaber MY, Wahab MIM (2010) Economic order quantity model for items with imperfect quality with learning in inspection. Int J Prod Econ 124:87–96
16. Salameh MK, Abdul- Malak MAU, Jaber MY (1993) Mathematical modeling of the effect of human learning in the finite production inventory model. Appl Math Model 17:613–615
17. Sangal I, Aggarwal A, Rani S (2016) A fuzzy environment inventory model with partial backlogging under learning effect. Int J Comput Appl 137:25–38
18. Shah NH (1993) A lot-size model for exponentially decaying inventory when delay in payments is permissible. Cahiers du CERO 35:115–123
19. Shah NH (1993) A probabilistic order level system when delay in payments is permissible. J Korea 18:175–182
20. Shah N, Pareek S, Sangal I (2012) EOQ in fuzzy environment and trade credit. Int J Ind Eng Comput 3(2):133–144
21. Teng JT, Lou KR, Wang L (2014) Optimal trade credit and lot size policies in economic production quantity models with learning curve production costs. Int J Prod Econ 155:318–323
22. Wright TP (1936) Factors affecting the cost of airplanes. J Aeronaut Sci 3:122–128

Chapter 21
A Fuzzy Two-Echelon Supply Chain Model for Deteriorating Items with Time Varying Holding Cost Involving Lead Time as a Decision Variable

Srabani Shee and Tripti Chakrabarti

Abstract In this paper, we have developed a two-stage supply chain production-inventory model for deteriorating product with time-dependent demand under fuzzy environment. Here we describe an EOQ model with changeable lead time and time-dependent holding cost. This situation is very common in the market, once an enterprise has some key technology or product that others have not, as a supplier, it can decide the prices and lead time of the technology or product to the buyers or retailers according to its need. Then the retailer determines his optimal order strategy, i.e., decides on the quantity of products to order from the suppliers. Under this circumstance, the problem that lead time, as a controllable variable of the supplier, and how it affects the cost to the supplier, retailer and whole supply chain is very important to the supplier and retailer because double-win benefits is a base of existence for the supply chain. In reality it is seen that we cannot define all parameters precisely due to imprecision or uncertainty in the environment. So we have defined the inventory parameters, such as set up cost, stock-out cost, and deterioration cost as triangular fuzzy numbers. The signed distance method and graded mean integration method have been used for defuzzification. To illustrate the proposed model a numerical example and sensitivity analysis with respect to different associated parameters has been presented.

Keywords EOQ · Time-dependent holding cost · Lead time · Deterioration · Supply chain · Fuzzy · Triangular fuzzy numbers · Defuzzification · Signed distance method · Graded mean integration method

---

S. Shee (✉) · T. Chakrabarti
Department of Applied Mathematics, University of Calcutta, 92 A.P.C. Road, 700009 Kolkata, India
e-mail: srabanishee68@gmail.com

T. Chakrabarti
Basic Science, Techno India University, Kolkata, India
e-mail: triptichakrabarti@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*, Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_21

21.1 Introduction

The Economic Order Quantity (EOQ) model, where cumulative holding cost is a convex function of time, is in contrast with the classic EOQ model where holding cost is a linear function of time. Ferguson et al. [1] considered an EOQ model with nonlinear holding cost to the inventory management of perishables. Misra [2] developed an inventory model for time-dependent holding cost and deterioration with salvage value and shortages. Mahata and Goswami [3] developed a fuzzy EOQ model with stock-dependent demand rate and nonlinear holding cost by taking fuzzy deterioration rate. In recent years, EOQ research has many new directions, such as economic order quantity with random supplier capacity, quantity discounts for the vendor's benefit and the buyer–vendor coordination of inventory, ordering, pricing, etc. Viswanathan [4] studied the optimal strategy for the integrated vendor–buyer inventory model. Goyal and Gupta [5] reviewed integrated inventory models: the buyer–vendor coordination. Piplani and Viswannathan [6] coordinated supply chain inventories through common replenishment epochs. Liao and Shyu [7] developed a model that can be used to determine the length of lead time that minimizes the expected total cost. The lead time is the only decision variable in many researchers' model. Hariga and Ben-Dayu [8] studied a continuous review inventory model where the lead time, the recorder point, and the ordering quantity are decision variables. Hsiao and Lin [9] proposed a buyer–vendor EOQ model with changeable lead time in supply chain. Ben-Daya and Raouf [10] studied a model where both lead time and order quantity are considered as decision variables. Ouyang et al. [11] extended Ben-Dayu and Raouf's model considering shortages where the total amount of stock-outs is considered as a mixture of backorders and lost sales.

The control and maintenance of any inventory of deteriorating items plays an important role in any supply chain management system as most physical goods such as food products and beverages, pharmaceuticals, radioactive substance, gasoline, etc. deteriorate over time. Various researchers have investigated these issues over time. Misra [12] first studied optimum production lot size model for a system with deteriorating inventory. Goyal and Giri [13] developed a production-inventory model of a product with time varying demand, production, and deterioration rates. Yang and Wee [14], proposed a multi-lot-size production-inventory system for deteriorating items where production and demand rates are constant. Sana et al. [15] considered a production-inventory model for deteriorating items with trended demand and shortages. Manna and Chiang [16] proposed an economic production quantity model for deteriorating items where demand rate is ramp type. Al-Khamis et al. [17] developed an optimal policy for a finite horizon batching inventory model. Pal et al. [18] considered a production-inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness.

In the crisp environment, all parameters associated with the model such as production rate, demand rate, deterioration rate, set up cost, holding cost, shortage cost, etc. are known and have definite value but in reality, most of the variables are highly uncertain. In such situations, these variables are described as fuzzy parameters.

Several researchers like Jaggi et al. [19], Yao and Chiang [20], Wang et al. [21], Yao and Lee [22], Kao and Hsu [23], Dutta et al. [24] and Saha [25] have developed inventory models under fuzzy environment. In this area, a lot of research papers have been published by several researchers viz., Bera et al. [26], He et al. [27], Dutta and Kumar [28], Mishra et al. [29] etc. Priyan and Manivannan [30] represented an Optimal inventory modeling of supply chain system involving quality inspection errors and fuzzy effective rate. Sonia et al. [31] proposed a two-warehouse inventory model under conditionally permissible delay in payment where the deterioration rate and demand rate are fuzzy in nature.

In this paper, we consider an EOQ model involving lead time as a decision variable with time varying holding cost on an Integrated System in Supply Chain. Here we considered various costs, such as setup cost, holding cost, cost of deteriorating items taken as triangular fuzzy numbers and demand rate is time dependent. Later on, the fuzzy total cost is defuzzified by using signed distance method and graded mead integration method. The problem is then solved by using LINGO 17.0 software.

21.2 Assumptions and Notations

The proposed model is developed under the following notations and assumptions.

21.2.1 Notations

1. $D(t) = ae^{bt}, where\ 0\langle b\langle 1, t\rangle 0, a\rangle 0$, the market's demand rate is exponentially increasing in nature
2. Q is the Retailer's initial inventory level (quantity)
3. θ is the constant deterioration rate, $0 < \theta < 1$
4. $h_r(t)$ is the Retailer's holding cost per unit increases with the time t, $h_r(t) = c_1 t^n, where\, c_1\, and\, n \geq 1\, are\, constant$
5. $\widetilde{h_r}$ is the Retailer's fuzzy holding cost per unit
6. $h_s(t)$ is the Supplier's holding cost per unit which is a linear function of time t, $h_s(t) = h_1 + h_2 t, where\, h_1, h_2 > 0\, are\, constant$
7. $\widetilde{h_s}$ is the Supplier's fuzzy holding cost per unit
8. $I_r(t)$ is the Retailer's inventory level at any time t
9. $I_s(t)$ is the Supplier's inventory level at any time t
10. c_2 is the Retailer's stock-out cost per unit
11. $\widetilde{c_2}$ is the Retailer's fuzzy stock-out cost per unit
12. c_3 is the Retailer's set up cost for each order
13. $\widetilde{c_3}$ is the Retailer's fuzzy set up cost for each order
14. c_4 is the Supplier's set up cost for each order
15. $\widetilde{c_4}$ is the Supplier's fuzzy set up cost for each order
16. c_5 is the Retailer's deterioration cost per unit

17. $\widetilde{c_5}$ is the Retailer's fuzzy deterioration cost per unit
18. c_6 is the Supplier's deterioration cost per unit
19. $\widetilde{c_6}$ is the Supplier's fuzzy deterioration cost per unit
20. T is the Retailer's order cycle time, a decision variable
21. L is the Supplier's lead time, a decision variable
22. TC is the total cost per unit time
23. $\widetilde{TC}$ is the fuzzified value of TC
24. TC_s is the defuzzified value of $\widetilde{TC}$ when signed distance method of defuzzification is used
25. TC_G is the defuzzified value of $\widetilde{TC}$ when graded mean integration method of defuzzification is used.

21.2.2 Assumptions

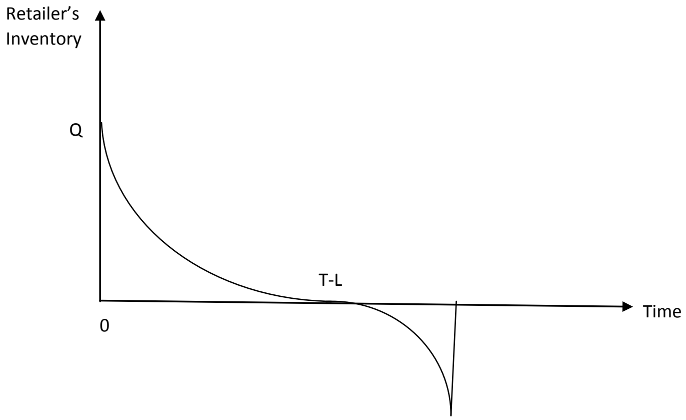
1. The inventory system involves production of single item.
2. The set up cost and deterioration cost are fuzzy.
3. The deterioration rate is constant fraction of on hand inventory.
4. The demand for product is time dependent.
5. The model is developed for finite time horizon.

21.3 Mathematical Model

In the distribution channel system with one retailer and one supplier, the supplier holds monopolistic status and the retailer is the follower. When retailer's inventory is zero, he issues orders to the supplier immediately. The supplier always delivers the product to him after a span of L in order to get the biggest profits or for another reason that causes the retailer to be out of stock.

21.3.1 Retailer's Model

The retailer's initial inventory level is Q at time $t = 0$. The inventory level gradually depletes to zero at time $t = T - L$ due to demand and deterioration. The changes in inventory level can be described by the following differential equations:



Retailer's Inventory model

$$\frac{dI_r(t)}{dt} + \theta I_r(t) = -ae^{bt}, 0 \le t \le T - L \tag{21.1}$$

$$\frac{dI_r(t)}{dt} = -ae^{bt}, T - L \le t \le T \tag{21.2}$$

With the boundary conditions

$$I_r(0) = Q, I_r(T - L) = 0 \tag{21.3}$$

Solving (21.1) and (21.2) we get,

$$I_r(t) = Qe^{-\theta t} + \frac{a}{b+\theta}\left(e^{-\theta t} - e^{bt}\right), 0 \le t \le T - L \tag{21.4}$$

$$I_r(t) = \frac{a}{b}\left(e^{b(T-L)} - e^{bt}\right), T - L \le t \le T \tag{21.5}$$

From the condition $I_r(T - L) = 0$ and the Eq. (21.4) we have,

$$Q = \frac{a}{b+\theta}\left(e^{(b+\theta)(T-L)} - 1\right) \tag{21.6}$$

Now, the holding cost of retailer

$$= c_1 \int_0^{T-L} t^n \left\{ Qe^{-\theta t} + \frac{a}{b+\theta}\left(e^{-\theta t} - e^{bt}\right) \right\} dt$$

$$= c_1 \left\{ \frac{Q(T-L)^{n+1}}{n+1} - \frac{(Q\theta + a)(T-L)^{n+2}}{n+2} \right\}$$

Stock-out cost of retailer

$$= -c_2 \int_{T-L}^{T} \left\{ \frac{a}{b}\left(e^{b(T-L)} - e^{bt}\right) \right\} dt$$

$$= \frac{c_2 a}{b^2}\left\{e^{bT} - (bL+1)e^{b(T-L)}\right\}$$

Deterioration cost of retailer

$$= c_5 \int_{0}^{T-L} \theta \left\{ Qe^{-\theta t} + \frac{a}{b+\theta}\left(e^{-\theta t} - e^{bt}\right) \right\} dt$$

$$= c_5\theta \left\{ Q(T-L) - \frac{(Q\theta + a)(T-L)^2}{2} \right\}$$

Set up cost of retailer

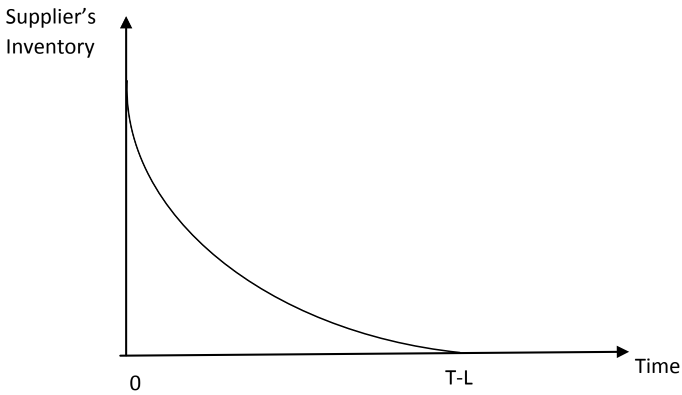
$$= c_3$$

Therefore, the retailer's average total cost in cycle T is

$$\begin{aligned} C_r(T, L) = \frac{1}{T}\Bigg[& c_1 \left\{ \frac{Q(T-L)^{n+1}}{n+1} - \frac{(Q\theta + a)(T-L)^{n+2}}{n+2} \right\} \\ & + \frac{c_2 a}{b^2}\left\{e^{bT} - (bL+1)e^{b(T-L)}\right\} \\ & + c_5\theta \left\{ Q(T-L) - \frac{(Q\theta + a)(T-L)^2}{2} \right\} + c_3 \Bigg] \end{aligned} \tag{21.7}$$

21.3.2 Supplier's Model

The inventory cycle starts at $t = 0$. The initial inventory level is Q. The inventory level gradually depletes to zero at time $t = T - L$ due to demand and deterioration. The change in inventory level can be described by the following differential equations:



Supplier's Inventory model

$$\frac{dI_s(t)}{dt} + \theta I_s(t) = -ae^{bt}, 0 \le t \le T - L \tag{21.8}$$

With the boundary conditions

$$I_s(0) = Q, I_s(T - L) = 0 \tag{21.9}$$

Solving (21.8) we get,

$$I_s(t) = Qe^{-\theta t} + \frac{a}{b+\theta}\left(e^{-\theta t} - e^{bt}\right), 0 \le t \le T - L \tag{21.10}$$

Now, the Supplier's holding cost-

$$= \int_0^{T-L} (h_1 + h_2 t)\left\{Qe^{-\theta t} + \frac{a}{b+\theta}\left(e^{-\theta t} - e^{bt}\right)\right\}dt$$

$$= \left[Qh_1(T-L) + \{Qh_2 - h_1(Q\theta + a)\}\frac{(T-L)^2}{2} - h_2(Q\theta + a)\frac{(T-L)^3}{3}\right]$$

The deterioration cost of Supplier-

$$= c_6 \int_0^{T-L} \theta\left\{Qe^{-\theta t} + \frac{a}{b+\theta}\left(e^{-\theta t} - e^{bt}\right)\right\}dt$$

$$= c_6\theta\left\{Q(T-L) - \frac{(Q\theta + a)(T-L)^2}{2}\right\}$$

Set up cost of retailer

$$= c_4$$

Therefore, the supplier's average total cost in cycle T is

$$C_s(T, L) = \frac{1}{T}\Bigg[Qh_1(T - L) + \{Qh_2 - h_1(Q\theta + a)\}\frac{(T - L)^2}{2}$$
$$4 - h_2(Q\theta + a)\frac{(T - L)^3}{3}$$
$$+c_6\theta\left\{Q(T - L) - \frac{(Q\theta + a)(T - L)^2}{2}\right\} + c_4\Bigg] \tag{21.11}$$

Therefore, the total average cost in cycle T is

$$TC(T, L) = C_r(T, L) + C_s(T, L)$$
$$= \frac{1}{T}\Bigg[c_1\left\{\frac{Q(T - L)^{n+1}}{n + 1} - \frac{(Q\theta + a)(T - L)^{n+2}}{n + 2}\right\}$$
$$+ \frac{c_2 a}{b^2}\left\{e^{bT} - (bL + 1)e^{b(T-L)}\right\} + Qh_1(T - L)$$
$$+ \{Qh_2 - h_1(Q\theta + a)\}\frac{(T - L)^2}{2} - h_2(Q\theta + a)\frac{(T - L)^3}{3}$$
$$+(c_5 + c_6)\theta\left\{Q(T - L) - \frac{(Q\theta + a)(T - L)^2}{2}\right\} + c_3 + c_4\Bigg] \tag{21.12}$$

The objective in the following is to find the solutions for the optimal values of T and L (say T^* and L^*) that minimize the total average cost $TC(T, L)$.

The necessary condition for minimization of $TC(T, L)$ are

$$\frac{\partial TC(T, L)}{\partial T} = 0$$

and

$$\frac{\partial TC(T, L)}{\partial L} = 0 \tag{21.13}$$

The sufficient condition for minimization of $TC(T, L)$ requires that it must be a convex function for $T > 0, L > 0$.

Now the function $TC(T, L)$ will be convex if

$$\begin{vmatrix} \frac{\partial^2 TC(T,L)}{\partial T^2} & \frac{\partial^2 TC(T,L)}{\partial T \partial L} \\ \frac{\partial^2 TC(T,L)}{\partial L \partial T} & \frac{\partial^2 TC(T,L)}{\partial L^2} \end{vmatrix} > 0 \tag{21.14}$$

Equation (21.13) can be solved simultaneously by some computer-oriented numerical technique to obtain retailer's optimal order cycle time T^* and supplier's optimal lead time L^*.

21.3.3 Fuzzy Model

Next we fuzzify the parameters $c_1, c_2, c_3, c_4, c_5, c_6, h_1, h_2$

Let, $\widetilde{c_1} = (x_1, y_1, z_1)$, $\widetilde{c_2} = (x_2, y_2, z_2)$, $\widetilde{c_3} = (x_3, y_3, z_3)$, $\widetilde{c_4} = (x_4, y_4, z_4)$, $\widetilde{c_5} = (x_5, y_5, z_5)$, $\widetilde{c_6} = (x_6, y_6, z_6)$, $\widetilde{h_1} = (\alpha_1, \alpha_2, \alpha_3)$, $\widetilde{h_2} = (\beta_1, \beta_2, \beta_3)$.

Then, $\widetilde{TC} = \frac{1}{T}\left[\widetilde{c_1}U + \widetilde{c_2}V + (\widetilde{c_3} + \widetilde{c_4}) + (\widetilde{c_5} + \widetilde{c_6})W + \widetilde{h_1}X + \widetilde{h_2}Y\right]$

where,

$$U = \left\{\frac{Q(T-L)^{n+1}}{n+1} - \frac{(Q\theta + a)(T-L)^{n+2}}{n+2}\right\}$$

$$V = \frac{a}{b^2}\left\{e^{bT} - (bL+1)e^{b(T-L)}\right\}$$

$$W = \theta\left\{Q(T-L) - \frac{(Q\theta + a)(T-L)^2}{2}\right\}$$

$$X = \left\{Q(T-L) - \frac{(Q\theta + a)(T-L)^2}{2}\right\}$$

$$Y = \left\{Q\frac{(T-L)^2}{2} - (Q\theta + a)\frac{(T-L)^3}{3}\right\}$$

Now, $\widetilde{TC} = (TC_1, TC_2, TC_3)$ (say)

where,

$$TC_1 = \frac{1}{T}[x_1U + x_2V + (x_3 + x_4) + (x_5 + x_6)W + \alpha_1X + \beta_1Y]$$

$$TC_2 = \frac{1}{T}[y_1U + y_2V + (y_3 + y_4) + (y_5 + y_6)W + \alpha_2X + \beta_2Y]$$

$$TC_3 = \frac{1}{T}[z_1U + z_2V + (z_3 + z_4) + (z_5 + z_6)W + \alpha_3X + \beta_3Y]$$

(i) **Signed Distance Method**

$$\begin{aligned} TC_s &= \frac{1}{4}(TC_1 + 2TC_2 + TC_3) \\ &= \frac{1}{4T}[(x_1 + 2y_1 + z_1)U + (x_2 + 2y_2 + z_2)V \\ &\quad + \{(x_3 + x_4) + 2(y_3 + y_4) + (z_3 + z_4)\} \end{aligned}$$

$$+\{(x_5+x_6)+2(y_5+y_6)+(z_5+z_6)\}W$$
$$+(\alpha_1+2\alpha_2+\alpha_3)X+(\beta_1+2\beta_2+\beta_3)Y]$$

The objective in the following is to find the solutions for the optimal values of T_s and L_s that minimize the total average cost $TC_s(T, L)$.

The necessary condition for minimization of $TC_s(T, L)$ are
$\frac{\partial TC_s(T,L)}{\partial T}=0$ and $\frac{\partial TC_s(T,L)}{\partial L}=0$

The sufficient condition for minimization of $TC_s(T, L)$ requires that it must be a convex function for $T>0, L>0$.

Now the function $TC(T, L)$ will be convex if

$$\begin{vmatrix} \frac{\partial^2 TC_s(T,L)}{\partial T^2} & \frac{\partial^2 TC_s(T,L)}{\partial T\partial L} \\ \frac{\partial^2 TC_s(T,L)}{\partial L\partial T} & \frac{\partial^2 TC_s(T,L)}{\partial L^2} \end{vmatrix} > 0$$

(ii) **Graded Mean Integration method**

$$TC_G=\frac{1}{6}(TC_1+4TC_2+TC_3)=\frac{1}{6T}[(x_1+4y_1+z_1)U+(x_2+4y_2+z_2)V$$
$$+\{(x_3+x_4)+4(y_3+y_4)+(z_3+z_4)\}$$
$$+\{(x_5+x_6)+4(y_5+y_6)+(z_5+z_6)\}W$$
$$+(\alpha_1+4\alpha_2+\alpha_3)X+(\beta_1+4\beta_2+\beta_3)Y]$$

The objective in the following is to find the solutions for the optimal values of T_G and L_G that minimize the total average cost $TC_G(T, L)$.

The necessary condition for minimization of $TC_G(T, L)$ are
$\frac{\partial TC_G(T,L)}{\partial T}=0$ and $\frac{\partial TC_G(T,L)}{\partial L}=0$

The sufficient condition for minimization of $TC_G(T, L)$ requires that it must be a convex function for $T>0, L>0$.

Now the function $TC(T, L)$ will be convex if

$$\begin{vmatrix} \frac{\partial^2 TC_G(T,L)}{\partial T^2} & \frac{\partial^2 TC_G(T,L)}{\partial T\partial L} \\ \frac{\partial^2 TC_G(T,L)}{\partial L\partial T} & \frac{\partial^2 TC_G(T,L)}{\partial L^2} \end{vmatrix} > 0$$

.

21.4 Numerical Examples

We consider the following numerical values of the parameters in appropriate units to analyze the model:

$c_1=2, c_2=8, c_3=100, c_4=200, c_5=5, c_6=7, h_1=0.2, \theta=0.01, h_2=0.1, a=40, b=0.5, n=2$ in appropriate units.

Table 21.1 Optimal solutions for various values of "n"

n	Retailer's optimal order cycle time T^*	Supplier's optimal lead time L^*	Retailer's optimal cost C_r^*	Supplier's optimal cost C_s^*	Total optimal cost TC^*
2	2.2109	1.0634	102.4426	96.8688	199.3114
3	2.2025	1.0505	99.3425	97.3046	196.6471
4	2.1937	1.0469	97.6576	97.6171	195.2747
5	2.1857	1.0470	96.6479	97.8549	194.5028
6	2.1787	1.0486	96.0011	98.0431	194.0442
7	2.1726	1.0508	95.5658	98.1965	193.7623
8	2.1674	1.0531	95.2620	98.3241	193.5861

Equations (21.12) and (21.13) are now solved simultaneously for the above parameter values using a gradient-based nonlinear optimization technique (LINGO 17.0) and get the results shown in Table 21.1. It is verified that all the solutions in Table 21.1 for different values of n, satisfy the convexity condition for $TC(T, L)$.

For Fuzzy Model, we consider the following numerical values of the parameters in appropriate units to analyze the Fuzzy model:

$$\tilde{c}_1 = (1, 2, 3), c_2 = (7, 8, 9), \tilde{c}_3 = (95, 100, 105),$$
$$\tilde{c}_4 = (195, 200, 205), \tilde{c}_5 = (4, 5, 6), \tilde{c}_6 = (6, 7, 8),$$

$$\widetilde{h_1} = (0.1, 0.2, 0.3), \widetilde{h_2} = (0.05, 0.1, 0.15), \theta = 0.01, a = 40, b = 0.5, n = 2$$

We obtain, $TC_s = 198.1792$ and Retailer's optimal order cycle time $T_s = 2.2053$ and Supplier's optimal lead time $L_s = 1.0594$ for Signed Distance Method.

$TC_G = 198.5569$ and Retailer's optimal order cycle time $T_G = 2.2072$ and Supplier's optimal lead time $L_G = 1.0607$ for Graded Mean Integration Method.

21.5 Sensitivity Analysis

The sensitivity analysis is performed by changing the value of each of the parameters a, b, θ, n, taking one parameter at each time and keeping the remaining parameters unchanged. We now study the sensitivity of the optimal solution to changes in the values of different parameters associated with the model (Figs. 21.1, 21.2, 21.3, 21.4, 21.5, 21.6, 21.7 and 21.8).

Fig. 21.1 Impact of $\boldsymbol{a}$ on TC_S

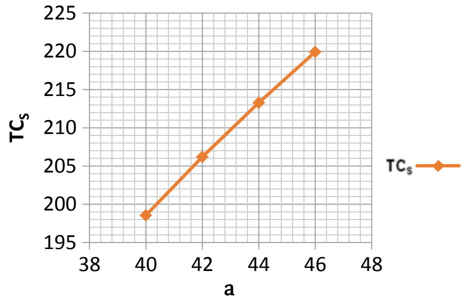


Fig. 21.2 Impact of $\boldsymbol{a}$ on TC_G

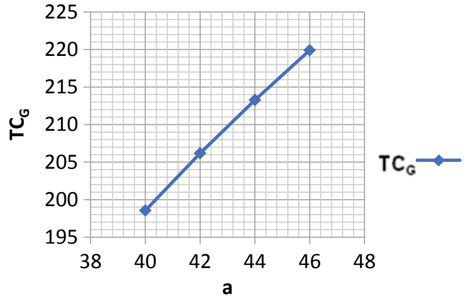


Fig. 21.3 Impact of $\boldsymbol{b}$ on TC_S

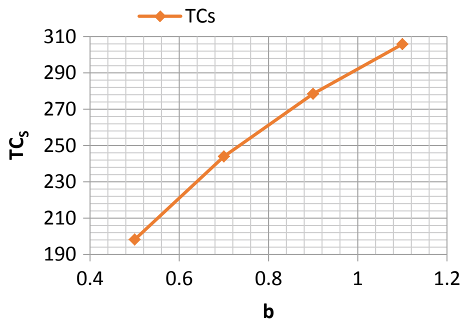
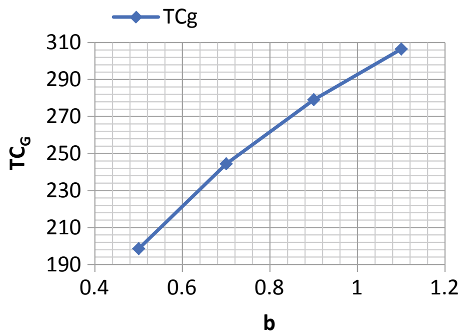


Fig. 21.4 Impact of $\boldsymbol{b}$ on TC_G



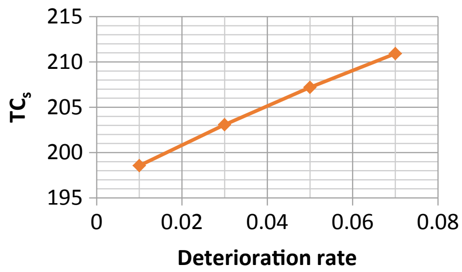


Fig. 21.5 Impact of deterioration rate on TC_S

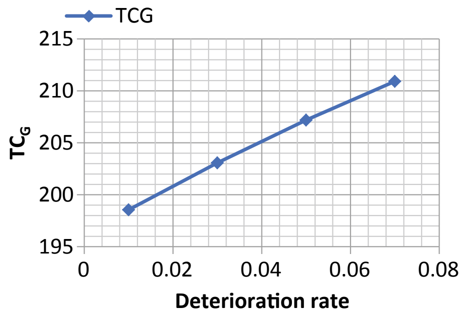


Fig. 21.6 Impact of deterioration rate on TC_G

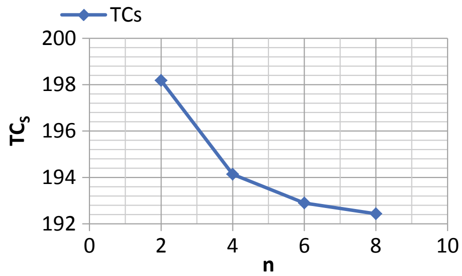


Fig. 21.7 Impact of n on TC_S

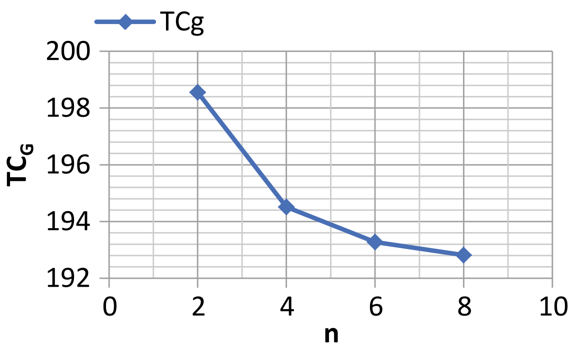


Fig. 21.8 Impact of n on TC_G

Table 21.2 Sensitivity on "a"

Change value	Signed distance method			Graded mean integration method		
a	T_s	L_s	TC_s	T_G	L_G	TC_G
40	2.2053	1.0594	198.1792	2.2072	1.0607	198.5569
42	2.1274	1.0038	205.7881	2.1292	1.0051	206.1796
44	2.0598	0.9561	212.8665	2.0616	0.9574	213.2709
46	2.0003	0.9145	219.4997	2.0020	0.9157	219.9161

Table 21.3 Sensitivity on "b"

Change value	Signed distance method			Graded mean integration method		
b	T_s	L_s	TC_s	T_G	L_G	TC_G
0.5	2.2053	1.0594	198.1792	2.2072	1.0607	198.5569
0.7	1.7098	0.5700	243.9281	1.7111	0.5708	244.4153
0.9	1.4506	0.3377	278.4718	1.4516	0.3381	279.0460
1.1	1.2926	0.2138	305.7972	1.2934	0.2141	306.4417

Table 21.4 Sensitivity on "θ"

Change value	Signed distance method			Graded mean integration method		
θ	T_s	L_s	TC_s	T_G	L_G	TC_G
0.01	2.2053	1.0594	198.1792	2.2072	1.0607	198.5569
0.03	2.1973	1.0973	202.6988	2.1992	1.0987	203.0779
0.05	2.1885	1.1338	206.8084	2.1904	1.1352	207.1890
0.07	2.1793	1.1688	210.5376	2.1813	1.1702	210.9199

21.5.1 Observation

The following are noted on the basis of the sensitivity analysis-

1. From Tables 21.2 and 21.3 it is observed that, an increase in parameter $'a'$ and $'b'$ causes increment in total cost for both the models (TC_s and TC_G). In contrast, the rise in these two parameters results in decrease in cycle time and lead time for both the developed models. Also we observed that TC_s and TC_G are moderately sensitive to change in $'a'$. On the other side TC_s and TC_G are highly sensitive due to the changes in the value of $'b'$.
2. As the deterioration rate increases hence the total cost TC_s and TC_G and the lead time increases but the cycle time decreases for both models (from Table 21.4). Here also TC_s and TC_G are moderately sensitive due to change in $'\theta'$.
3. The total cost, cycle time, and lead time (for both models) decreases as the parameter "n" increases (from Table 21.5).

Table 21.5 Sensitivity on "*n*"

Change value	Signed distance method			Graded mean integration method		
n	T_s	L_s	TC_s	T_G	L_G	TC_G
2	2.2053	1.0594	198.1792	2.2070	1.0607	198.5569
4	2.1883	1.0426	194.1337	2.1901	0.0441	194.5143
6	2.1733	1.0441	192.8953	2.1751	1.0456	193.2785
8	2.1621	1.0485	192.4312	2.1638	1.0501	192.8165

21.6 Conclusion

In this paper, we have developed the optimal order strategy of a supplier retailer's inventory model for deteriorating items under fuzzy environment. If we take $h_2 = 0$ in Supplier's holding cost and n = 0 in retailer's holding cost, then both holding costs become constant. In that case the supplier will gain more profit compared to the retailer. Taking holding cost and demand rate constant, some researchers have developed a model where it is observed that supplier's average total cost decreases and the retailer's average total cost increases. We observed that the total cost is minimum corresponding to the value of the cycle time T when Signed distance method of defuzzification is used. Also, the cycle time T and the lead time L are minimum with corresponding total cost when the signed distance method is used. In future the obtained optimal solutions can be improved by using different algebraic procedure or Geometric Programming Problem approach for the case of Posynomial functions which arise in Engineering problems.

References

1. Ferguson M, Jayaraman V, Souza GC, Note: an application of the EOQ model with nonlinear holding cost to inventory management of perishables
2. Mishra VK (2012) Inventory model for time dependent holding cost and deterioration with salvage value and shortages. J Math Comput Sci 4(1): 37–47
3. Mahata GC, Goswami A (2009) Fuzzy EOQ models foe deteriorating items with stock dependent demand & nonlinear holding costs. Int J Math Comput Sci 5(2):94–98
4. Viswanathan S (1998) Optimal strategy for the integrated vendor-buyer inventory model. Eur J Oper Res 105:38–42
5. Goyal SK, Gupta YP (1989) Integrated inventory models: the buyer-vendor coordination (review). Eur J Oper Res 41:261–269
6. Piplani P, Viswannathan S (2001) Coordinating supply chain inventories through common replenishment epochs. Eur J Oper Res 129:277–286
7. Liao CJ, Shyu CH (1991) An analytical determination of lead time with normal demand. Int J Oper Prod Manag 11:72–79
8. Hariga M, Ben-Daya M (1999) Some stochastic inventory models with deterministic variable lead time. Eur J Oper Res 113:42–51
9. Hsiao JM, Lin C (2005) A buyer-vendor EOQ model with changeable lead-time in supply chain. Int J Adv Manuf Technol 26:917–921

10. Ben-Daya M, Raouf A (1994) Inventory models involving lead time as decision variable. J Oper Res Soc 45:579–582
11. Ouyang LY, Yeh NC, Wu KS (1996) Mixture inventory models with backorders and lost sales for variable lead-time. J Oper Res Soc 47:829–832
12. Misra RB (1975) Optimum production lot size model for a system with deteriorating inventory. Int J Prod Res 13(5): 495–505
13. Goyal SK, Giri BC (2003) The production–inventory problem of a product with time varying demand, production and deterioration rates. Eur J Oper Res 147(3):549–557
14. Yang PC, Wee HM (2003) An integrated multi-lot-size production inventory model for deteriorating item. Comput Oper Res 30(5):671–682
15. Sana S, Goyal SK, Chaudhuri KS (2004) A production–inventory model for a deteriorating item with trended demand and shortages. Eur J Oper Res 157(2):357–371
16. Manna SK, Chiang C (2010) Economic production quantity models for deteriorating items with ramp type demand. Int J Oper Res 7(4):429
17. Al-Khamis TM, Benkherouf L, Omar M (2014) Optimal policies for a finite horizon batching inventory model. Int J Syst Sci 45:2196–2202
18. Pal S, Mahapatra GS, Samanta GP (2015) A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness. Econ Model 46:334–345
19. Jaggi K et al (2013) Fuzzy inventory model for deteriorating items with time-varying demand and shortages. Am J Oper Res 2(6):81–92
20. Yao JS, Chiang J (2003) Inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. Eur J Oper Res 148(2):401–409
21. Wang X, Tang W, Zhao R (2007) Fuzzy economic order quantity inventory models without backordering. Tsinghua Sci Technol 12(1):91–96
22. Yao JS, Lee HM (1999) Fuzzy inventory with or without backorder for fuzzy order quantity with trapezoid fuzzy number. Fuzzy Sets Syst 105(3): 311–337
23. Kao C, Hsu WK (2002) A single-period inventory model with fuzzy demand. Comput Math Appl 43(6–7):841–848
24. Dutta P, Chakraborty D, Roy AR (2005) A single-period inventory model with fuzzy random variable demand. Math Comput Model 41(8–9):915–922
25. Saha S (2017) Fuzzy inventory model for deteriorating items in a supply chain system with price dependent demand and without backorder. Am J Eng Res 6(6):183–187
26. Bera UK, Bhunia AK, Maiti M (2013) Optimal partial backordering two-storage inventory model for deteriorating items with variable demand. Int J Oper Res 16(1):96
27. He W, Wei HE, Fuyuan XU (2013) Inventory model for deteriorating items with time-dependent partial backlogging rate and inventory level-dependent demand rate. J Comput Appl 33(8):2390–2393
28. Dutta D, Kumar P (2015) A partial backlogging inventory model for deteriorating items with time-varying demand and holding cost. Int J Math Oper Res 7(3):281
29. Mishra N, Mishra SP, Mishra S, Panda J, Misra UK (2015) Inventory model of deteriorating items for linear holding cost with time dependent demand. Math J Interdiscip Sci 4(1):29–36
30. Priyan S, Manivannan P (2017) Optimal inventory modelling of supply chain system involving quality inspection errors and fuzzy effective rate. Opsearch 54:21–43
31. Sonia S, Abolfazl M, Ehsan S (2016) A two-warehouse inventory model with fuzzy deterioration rate and fuzzy demand rate under conditionally permissible delay in payment. J. Ind. Prod. Eng. 33:134–142

Chapter 22
Transportation-Inventory Model for Electronic Markets Under Time Varying Demand, Retailer's Incentives and Product Exchange Scheme

Arindum Mukhopadhyay

Abstract Exchange offers are popular in many businesses to attract new customers. Availability of novel varieties, stiff competition, and regulatory restriction of discarding old products enhances to facilitate such offers in the market. Although, the reduction of out-of-pocket expenses to the customers helps to increase the sale of products; it also elevates the decision problem for managing the inventories of the exchanged products for the retailers. In view of this, the present article addresses inventory decision modeling in a system where a customer can buy products from the electronic-market retailer either by paying the full price or getting some price-discount for exchanging old products. Retailer bears transportation costs for the new and exchanged products. Four models are formulated and numerical examples are presented. Sensitivity analysis is also performed to understand the effect of various parameters in the models.

Keywords Exchange offer · Inventory · Time varying demand · Transportation · Optimization

22.1 Introduction

The inventory systems are indispensable in any business. Be it manufacturing, agribusiness, healthcare, logistics or any other; without storing an inventory one cannot continue smooth operations. In particular, retail business is completely based on storage of inventories and sometimes it also generates demands for the products. Nowadays, there have been trends of various kinds of exchange offers in the electronic marketplace of products such as utensils, smartphones, computers, batteries, automobiles, televisions, etc. The commonality among these products is that, all of them are more or less consumer durables. They are driven by trends, fashion, or

A. Mukhopadhyay (✉)
Operations and Quantitative Techniques Area, Indian Institute of Management Shillong, Shillong, India
e-mail: arindum.iims@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*, Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_22

technology which advances with time. On one side, the long life of the product prevents customers from buying a new one; keeping average demand of such products low for the customers. From the perspectives of the retailers, it is difficult to sell products to the consumers who are already using some older version or older model of that product. This motivates retailers to incentivize their customers to purchase the newer version of the product by giving a discount in lieu of their old functioning products. Also the retailer can use it to increase the selling of products during low-demand period or to liquidate excess inventories. The retailer may also sell the product without the exchange offer and the exchange offer works as discount to a segment of customers who want to upgrade from their older models to the latest ones. The products bought by the company can be used for various purposes—it can be sold to some secondary markets to price sensitive low-budget customers either in the same form or after some modifications. Sometimes, few parts of the products may be used to assemble to produce new products, which reduces the cost of manufacturing new products.

Exchange offers are often driven by external factors. There may exist sometimes mandatory regulations by authorities which prevents the user of the product to continue using it. In many countries, Automobiles cannot be used after a certain number of years because it may create much more pollution than the latest models. Besides this, the trends of the consumers; who are inclined toward sustainability also motivates such kind of offers because it prevents waste disposals (which may be hazardous sometimes), and reduces resource consumptions for manufacturing (such resources may be rare or costly raw materials or components). It has been observed that, in automobiles there has been usage of $25-30\%$ of used high quality materials which not only reduces manufacturing cost (Belvedre [3]), but also addresses the greater dimensions of environment consciousness. Due to these advantages more and more consumer durables retailers are moving toward the trend of selling products using exchange offers. On one hand, this reduces the out-of-pocket cost to customers; on the other hand it creates problem for the retail manager for managing inventories of both types of products viz. new and exchanged. In particular, following new challenges a manager of a company faces in company's product exchange offers—1. Exactly specifying criteria for selecting eligible product for exchange in terms of quality and specifications 2. Managing transportation of the exchanged product from customer to company 3. Managing inventory of exchanged products 4. Refurbishment and 5. Selling it to secondary market. From the retailers' perspectives, the exchanged old product may be of variety of uses for its company. Firstly, there is a secondary market with abundant customers who may consider functioning older products due to low cost. Sometimes, retailer may also consider to send the old exchanged products to the manufacturer, who can use it for remanufacturing, component reuse and remanufacturing, or raw material recovery.

In essence, this article presents some inventory models with product exchange on the basis of two way transportation for decision on optimal policy. Although, the models deal with two different types of storage for new and exchanged items it is assumed that single transportation mode is used for forward and reverse logistics. This assumption will reduce the complexity of the model. Furthermore, the objective of our model is to maximize the average total profit. The model proposed in this article will enhance the future research on inventory systems that deal with exchange offers and reverse logistics. The rest of this manuscript is organized as follows. The literature review is presented in Sect. 22.2. In Sect. 22.3 the notation and assumptions used in this paper are provided. The mathematical models are formulated in Sect. 22.4. Sections 22.5 and 22.6 presents sensitivity analysis and managerial insights respectively. Finally, the conclusions and future research are provided in Sect. 22.7.

22.2 Literature Review

Because this article constitutes of two types of inventory models, namely inventory model with time varying demand and inventory model for exchange offer; It seems fair that the review of literature needs to incorporate both types separately. Many authors have worked in the area of deterministic inventory models with time varying demand. The article by Silver and Meal [37] was the first to consider the modeling of EOQ for the general case of a deterministic time varying demand pattern. It developed a computable method to solve the problem, but it was computationally tedious for obtaining the reorder time and thus unimplementable. To overcome this issue, Silver and Meal [37] approached the same inventory model with a novel heuristics that generates an approximate operating schedule incurring a negligible additional cost. With a different approach to obtain analytic solution for the optimal policy for an inventory system with linear time varying demand over a finite planning horizon, Donaldson [11] investigated and discussed some characteristics of the model. Ritchie [34] investigated an inventory model with linear time varying demand and obtained a practical solution method for the optimal policy. Various other researchers, notably Phelps [32] and Mitra et al. [28] made remarkable contributions in developing heuristic solution procedures. Goswami and Chaudhuri [16] investigated an EOQ model for deteriorating items with shortages and a linear trend in demand. Goswami and Chaudhuri [17] considered two warehouse facilities when the demand was varying linearly over time. Datta and Pal [9] developed alternative method for improvement in solution procedure in the previous models of linearly time varying demand by assuming that successive replenishment cycles are in Arithmetic Progression. The method provides optimal number of replenishments and optimal replenishment time. [21] incorporated some improvement in the model of Goswami and Chaudhuri [16] and provided implementable examples. Goyal et al. [18] approached linearly time varying demand with shortage using novel replenishment policy where replenishment cycle starts with shortage and after a period of shortage replenishment is made. They have developed four heuristics procedures to follow the new replenishment procedure. Bose et al. [4] incorporated the concept of Inflation and time-discounting in an inventory model with linearly time varying demand and shown that some of the previous models can be developed as a special case of their model. Chakrabarti and Chaudhuri [5] developed an inventory model with linear time varying demand, where shortage in every replenishment cycle was considered within finite time horizon. Chang et al. [6] incorporated permissible delay of payment in an inventory model with time varying demand. Goyal and Giri [19] investigated on an inventory model of deteriorating products with linear time varying demand, time varying production rate, and time varying deterioration rate. Alamri and Balkhi [1] investigated an inventory model for a perishable product with time varying demand, in which effect of learning and forgetting effects were also considered. Lee an Hsu [25] applied heuristics method in an inventory system with deteriorating products under time varying demand when inventory was stored in two warehouses. Sarkar [35] investigated an inventory model with time varying demand under delay in payment, when deterioration rate is considered as dependent on time. Mishra et al. [27] investigated an inventory model with deteriorating products under time varying demand rate, deterioration rate and holding cost functions. Mukhopadhyay and Goswami [30] investigated an inventory system for the deteriorating product where demand and holding cost were linear functions of time. They have obtained optimal policy and provided some implications of their model. Singh et al. [39] studied an inventory system with linear time varying demand and time varying deterioration rate where they applied a customized Newton–Raphson Based solution approach. Uthayakumar and Karuppasamy [41] considered linear time varying demand and two other demand formulations for healthcare industries applications in their article where they have obtained optimal policy in terms of optimal replenishment time, economic order quantity, and total cost. Recently, Mukhopadhyay [31] developed an inventory system with time varying demand under uncertainty for imperfect items with product deterioration, variable selling price, partial backlogging, and selling price dependent time varying demand. It was shown that his model was a generalization of previous other models.

Although there is not much literature on product exchange offers in inventory systems, there are various studies on recycling and reverse logistics which address similar aspects, as in exchange offers. Mitra [29] studied remanufacturing resulted from product exchange offer in terms of revenue management perspective and developed a pricing model to maximize the expected revenue from the recovered products. Seitz [36] explored through in-depth case studies within the remanufacturing facilities of a major European Vehicle Manufacturer to identify the motive for the exchange offer. Pourmohammadi et al. [33] applied case study approach to discuss the positive impact of product exchange. Geyer and Doctori Blass [14] observed that product exchange offer has positive effect on the behavior of cell-phone customers who usually simply stored the old phones instead of using reverse logistics channel. Das and Dutta [8]applied system dynamics framework to investigate inclusion of Product Exchange and Three Way Recovery policy in closed-loop supply chain system. Daaboul [7] observed that cost is the major motivator for a manufacturer to go for reverse logistics and developed a method to integrate reverse logistics design with environmental impacts by closed-loop product lifecycle management. Belvedere and Grando [3] explored reverse logistics in Renault car company and its closed-loop supply chains. Islam and Huda [22] reviewed the literature on Reverse logistics and closed-loop supply chain of Waste Electrical and Electronic Equipment in which some concepts of product exchange is also covered.

22.3 Mathematical Model

Notations and Assumptions

For convenience, the following notation is used throughout the entire paper:
D_1: demand rate of the product without exchange offer
D_2: demand rate of the product with exchange offer
T: cycle time
I: inventory size
y: order quantity of new product
y_1: quantity of new product sold without exchange
y_2: quantity of new product sold with exchange
c_o: order cost
c_{h1}: inventory holding cost per unit per unit-time for new product
c_{h1}: inventory holding cost per unit per unit-time for exchanged product
c_t: transportation cost
Δ: discount per product, a random variable
$E(\bullet)$: Expected value of random variable
p: selling price of the new product
R_0: selling price of the refurbished exchanged product
r_1, r_2: lower and upper limit of the exchange rebate respectively
HC_1: total holding cost for new products
HC_2: total holding cost for exchanged products
OC: total ordering cost for new products
PC: total purchase cost for new products
TrC: total transportation cost
a, b, c, d: positive constants as demand-rate parameters.

Next, the inventory models are based on the following assumptions:

1. Single category of product is considered.
2. Demand is deterministic.
3. Assuming product is abundant, shortage will be prohibited.
4. Exchange products are similar in size to the new product. This assumption will help in calculation of transportation costs.
5. The product is sold with or without exchange and the exchange is applicable to same category of product. The demand for without exchange D_1 is independent of D_2.
6. The exchanged goods have lesser value than new products.
7. The valuation of exchanged product follows uniform distribution.
8. The inventory level at the end of the planning horizon will be zero.
9. The cost parameters are deterministic.
10. There are different holding cost for new and exchanged items.
11. The product is transported through single transportation mode.
12. For the demand parameters to be meaningful, we must have $a \geq b \cdot T, c \geq d \cdot T$.

22.4 Model Formulation

In all of our models we assume that demand starts when inventory lot size is y. There are two types of demand: one is without product exchange (D_1) and another is along with product exchange (D_2). We assume that both types of demands occur together during selling season and one order−cycle ends as the inventory becomes zero. Based on the aforementioned assumptions and notations, total cost will be formulated for the retailer which will help to determine optimal lot size and optimal total cost for various models. In these models, the transportation and holding cost have been given more attention, because of the electronic retailing and two types of products (new and old) respectively.

In order to capture various modeling environment, we consider four situations:
(i) All the cost components and demand are constant
(ii) Transportation cost depends on quantity
(iii) Transportation cost depends on quantity and holding costs depend on time
(iv) Transportation cost depends on quantity, Demand is declining function w.r.t. time, and holding costs depend on time (Figs. 22.1, 22.2, 22.3 and 22.4).

Constant Demand and constant Cost components

Before calculating cost, let us calculate the inventory level at any time $t \in [0, T]$. During this interval the inventory of new item is governed by equation:

$$\frac{dI}{dt} = -(D_1 + D_2), \quad I(0) = y \tag{22.1}$$

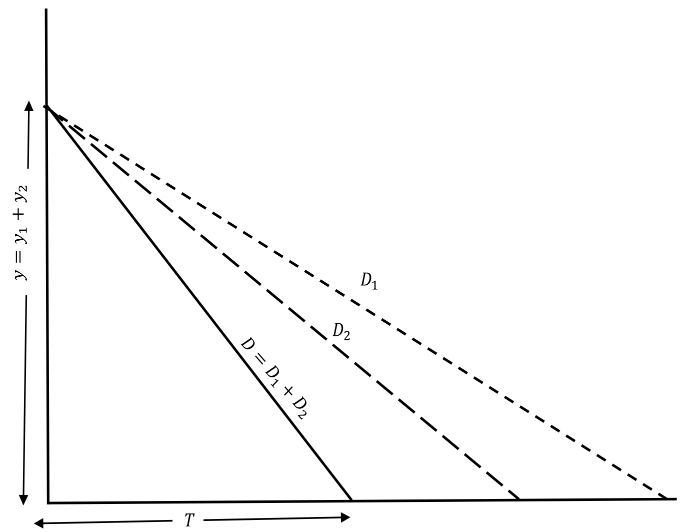


Fig. 22.1 Inventory-Time diagram under constant demand when $D_1 < D_2$

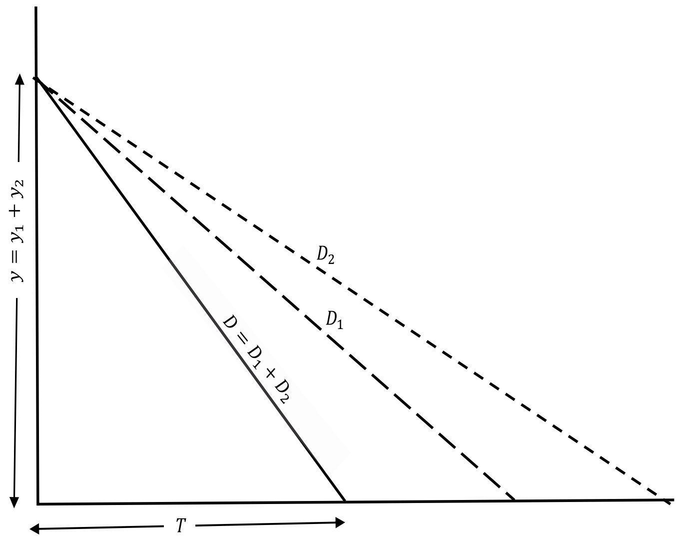


Fig. 22.2 Inventory-Time diagram under constant demand when $D_1 > D_2$

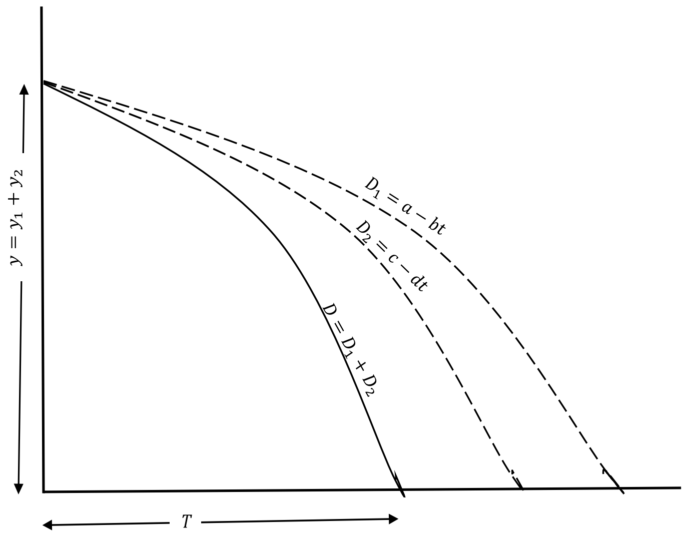


Fig. 22.3 Inventory-Time diagram under variable demand when $a > c, b \leq d$

This gives

$$I(t) = y - (D_1 + D_2)t \tag{22.2}$$

From (22.2), at $t = T$, $y = 0$ giving $y = (D_1 + D_2)T$

The inventory of used item during $t \in [0, T]$ is governed by equation:

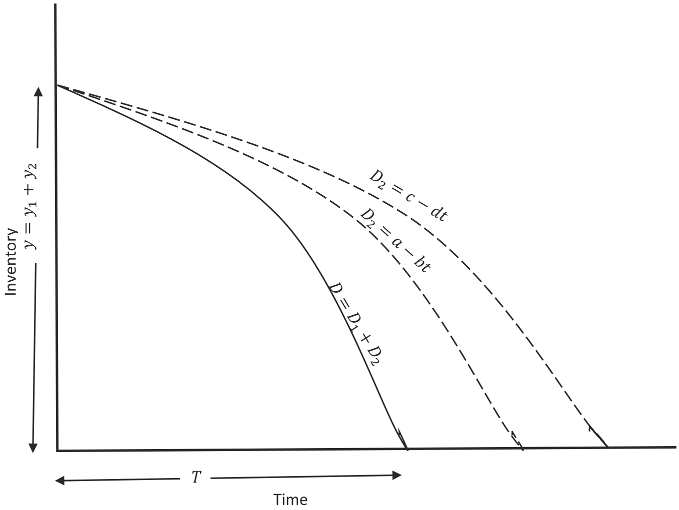


Fig. 22.4 Inventory-Time diagram under variable demand when $a < c, b \geq d$

$$\frac{dJ}{dt} = D_2, \quad J(0) = 0 \tag{22.3}$$

This gives

$$J(t) = D_2 t \tag{22.4}$$

We calculate various costs as follows:
Purchase cost $(PC) = c_p \cdot y = c_p(D_1 + D_2)T$
Ordering cost$(OC) = c_o$
Inventory holding cost for new items: $(HC_1) = c_{h1} \int_0^T I(t)dt = c_{h1}\left[yT - (D_1 + D_2)\frac{T^2}{2}\right]$
Putting the value of y, we get
$HC_1 = \frac{c_{h1}(D_1+D_2)T^2}{2}$
Inventory holding cost for exchanged items:
$(HC_2) = c_{h2} \int_0^T J(t)dt = c_{h2}\left[D_2 \frac{T^2}{2}\right]$
Since for the exchanged items, onward and return transportation are used; so transportation cost will be incurred twice. This gives,
Transportation Cost $(TrC) = c_t y_1 + 2c_t y_2 = c_t(D_1 \cdot T + 2 \cdot D_2.T)$
Total cost(TC) $= OC + PC + HC_1 + HC_2 + TrC$
$TC = c_o + c_p(D_1 + D_2)T + \frac{c_{h1}(D_1+D_2)T^2}{2} + c_{h2}\left[D_2 \frac{T^2}{2}\right] + c_t(D_1 \cdot T + 2 \cdot D_2.T$
Because under random phenomenon, it can be possible that the exchange value of old product returned by the customer can be any number between minimum possible exchange value r_1 and maximum possible exchange value r_2. That means any exchange price between r_1 and r_2 are equally likely. This is the reason for selecting uniform distribution.

Now average exchange value= Mean of the distribution $U(r_1, r_2) = \frac{r_1+r_2}{2}$
Revenue from selling new products:
= (Revenue from selling without exchange) + (Expected revenue from selling with exchange)
$= D_1Tp + D_2T(p - \frac{r_1+r_2}{2}) = pT(D_1 + D_2) - D_2T\frac{r_1+r_2}{2}$
Revenue from reselling old products after some refurbishment:
$= D_2TR_0$
Now we can calculate, "Total Revenue":
$TR(T) = pT(D_1 + D_2) + D_2T(R_0 - \frac{r_1+r_2}{2})$
Total Average Profit(TAP) is calculated as follows:
$TAP(T) = \frac{TR(T)-TC(T)}{T}$

$$\begin{aligned} TAP(T) &= p(D_1 + D_2) + D_2\left(R_0 - \frac{r_1 + r_2}{2}\right) - \frac{c_o}{T} - c_p(D_1 + D_2) \\ &\quad - \frac{c_{h1}(D_1 + D_2)T}{2} - c_{h2}D_2\frac{T}{2} - c_t(D_1 + 2D_2) \end{aligned} \tag{22.5}$$

In order to optimize $TAP(T)$, we need to differentiate w.r.t. T $TAP'(T) = 0$, gives

$$T = \sqrt{\frac{2c_o}{c_{h1}(D_1 + D_2) + c_{h2}D_2}} \tag{22.6}$$

It can be observed that from above expression for T, that optimal cycle time independent of transportation cost.

Constant Demand and variable Transportation cost

Sometimes it is observed that transportation cost is variable rather than being a constant (for example, it may depend on time, distance, or quantity). We shall consider only quantity-dependent transportation cost because for a local retailer in a town or city the distance of picking-up is constant on average. Similarly, time is also less important factor for such kind of transportation. Therefore it is assumed that transportation cost is dependent on quantity transported. Including the variable transportation cost gives

$$c_t(q) = \lambda_0 + \lambda_1 q$$

This give the expression for total transportation cost as follows:

$$\begin{aligned} TrC &= (\lambda_0 + \lambda_1 D_1 T)D_1T + 2(\lambda_0 + \lambda_1 D_2 T)D_2T \\ &= \lambda_0(D_1 + 2D_2) + \lambda_1(D_1^2 + 2D_2^2)T^2 \end{aligned} \tag{22.7}$$

As per the aforementioned assumption, the other cost components are remaining unchanged; modified expression for total cost changes.

Now we can calculate, Total cost(TC) $= OC + PC + HC_1 + HC_2 + LC$

$$TC = c_o + c_p(D_1 + D_2)T + \frac{c_{h1}(D_1 + D_2)T^2}{2} + c_{h2}\left[D_2\frac{T^2}{2}\right] + \lambda_0(D_1 + 2D_2) + \lambda_1(D_1^2 + 2D_2^2)T^2 \tag{22.8}$$

This gives the modified expression for total average profit for this situation,

$TAP(T) = \frac{TR(T)-TC(T)}{T}$

$$TAP(T) = p(D_1 + D_2) + D_2\left(R_0 - \frac{r_1 + r_2}{2}\right) - \frac{c_o}{T} + c_p(D_1 + D_2) - \frac{c_{h1}(D_1 + D_2)T}{2} - c_{h2}D_2\frac{T}{2} - \lambda_0(D_1 + 2D_2) - \lambda_1(D_1^2 + 2D_2^2)T \tag{22.9}$$

Differentiating with respect to variable T;
$TAP'(T) = 0$ gives the expression of optimal cycle time as follows:

$$T = \sqrt{\frac{2c_o}{c_{h1}D_1 + c_{h2}(D_1 + D_2) + 2\lambda_1(D_1^2 + 2D_2^2)}} \tag{22.10}$$

As it is obvious from above expression for T, that optimal cycle time depend on the transportation cost.

Quantity-dependent transportation cost and time varying holding costs

It may be possible that holding cost may be dependent on the duration in which products are stored and storing longer may cost more to the retailer. This is related to opportunity cost in some manner.
$c_{h1}(t) = \mu_0 + \mu_1 t$
$c_{h2}(t) = \nu_0 + \nu_1 t$
This gives new expressions for both types of holding costs:

$$HC_1 = \int_0^T c_{h2}(t)I(t)dt = \int_0^T (\mu_0 + \mu_1 t)\{y - (D_1 + D_2)t\}dt$$

$$HC_1 = \mu_0(D_1 + D_2)T^2 + (\mu_1 - \mu_0(D_1 + D_2))\frac{T^2}{2} - \mu_1(D_1 + D_2)\frac{T^3}{3} \tag{22.11}$$

$$HC_2 = \int_0^T c_{h2}(t)J(t)dt = \int_0^T (\nu_0 + \nu_1 t)D_2 t dt = \nu_0\frac{T^2}{2} + \nu_1 D_2\frac{T^3}{3} \tag{22.12}$$

Due to above-mentioned changes, we can calculate new expression of Total cost(TC) as follows:

$TC = OC + PC + HC_1 + HC_2 + LC$

$$TC = c_o + c_p(D_1 + D_2)T + \mu_0(D_1 + D_2)T^2 + (\mu_1 - \mu_0(D_1 + D_2))\frac{T^2}{2} - \mu_1(D_1 + D_2)\frac{T^3}{3} + \nu_0\frac{T^2}{2} + \nu_1 D_2\frac{T^3}{3} + \lambda_0(D_1 + 2D_2) + \lambda_1(D_1^2 + 2D_2^2)T^2 \tag{22.13}$$

Total Average Profit(TAP) is calculated as follows:

$TAP(T) = \frac{TR(T) - TC(T)}{T}$

$$\begin{aligned} TAP(T) = {} & p(D_1 + D_2) + D_2\left(R_0 - \frac{r_1 + r_2}{2}\right) - \frac{c_o}{T} - c_p(D_1 + D_2) \\ & -\mu_0(D_1 + D_2)T - (\mu_1 - \mu_0(D_1 + D_2))\frac{T}{2} - \mu_1(D_1 + D_2)\frac{T^2}{3} - \nu_0\frac{T}{2} \\ & -\nu_1 D_2\frac{T^2}{3} - \lambda_0(D_1 + 2D_2) - \lambda_1(D_1^2 + 2D_2^2)T \end{aligned} \tag{22.14}$$

Given above formulation of $TAP(T)$, we now calculate first and second order derivatives respectively.

$$\begin{aligned} TAP'(T) = {} & \frac{c_o}{T^2} - \mu_0(D_1 + D_2) - \left(\frac{\mu_1 - \mu_0(D_1 + D_2))}{2} - \frac{2T\mu_1(D_1 + D_2)}{3}\right) \\ & -\frac{\nu_0}{2} - \frac{2\nu_1 D_2 T}{3} - \lambda_1(D_1^2 + 2D_2^2) \end{aligned} \tag{22.15}$$

The formulation of $TAP'(T)$ shows that $TAP'(T) = 0$ is a cubic equation in T.

$$TAP''(T) = -\frac{2}{3}\left[\frac{3c_o}{T^3} + \mu_1 D_1 + \nu_1 D_2\right] \tag{22.16}$$

Because all the terms inside [] is positive, we can say that $TAP''(T) < 0$. Thus, $TAP(T)$ is concave function of T and hence atleast one of the positive root of $TAP'(T) = 0$ gives the optimal solution.

Quantity-dependent transportation cost with time varying demand and holding costs

In addition to the above, let us assume that demand and holding costs are functions of time. There is a valid reason for assuming such criteria as the demand for the durable goods decline with time due to either availability of multiple options for purchase from competitors or better options due to technological innovation or incremental improvement (in terms of capacity, quality, efficiency, or cost).

$D_1(t) = a - bt$

$D_2(t) = c - dt$

We now find the new expressions for y, I, J, HC_1, and HC_2
The expression of total inventory becomes,
$y = \int_0^T (D_1(t) + D_2(t))dt = (a+c)T - \frac{(b+d)T^2}{2}$
The inventory of new products are governed by differential as follows

$$\frac{dI}{dt} = -(D_1 + D_2), \quad I(0) = y \tag{22.17}$$

$$I(t) = y - (a+c)t + (b+d)\frac{t^2}{2} \tag{22.18}$$

The inventory of exchanged products are governed by expression as follows

$$\frac{dJ}{dt} = D_2, \quad J(0) = 0 \tag{22.19}$$

$$J(t) = ct - \frac{dt^2}{2} \tag{22.20}$$

Based on above $I(t)$ and $J(t)$, expressions of both the holding costs are obtained.
$HC_1 = \int_0^T (\mu_0 + \mu_1 t)(y - (a+c)t + (b+d)\frac{t^2}{2})dt$

$$HC_1 = \mu_0 \left[yT - (a+c)\frac{T^2}{2} + (b+d)\frac{T^3}{6} \right] + \mu_1 \left[y\frac{T^2}{2} - (a+c)\frac{T^3}{3} + (b+d)\frac{T^4}{8} \right] \tag{22.21}$$

$HC_2 = \int_0^T (\nu_0 + \nu_1 t)(ct - \frac{dt^2}{2})dt$

$$HC_2 = \nu_0 \left[\frac{cT^2}{2} - \frac{dT^3}{6} \right] + \nu_1 \left[\frac{cT^3}{3} - \frac{dT^4}{8} \right] \tag{22.22}$$

Expressions of transportation cost are obtained as follows:
$TrC = (\lambda_0 + \lambda_1 y_1)y_1 + (\lambda_0 + \lambda_1 y_2)y_2$
where $y_1 = \int_0^T (D_1(t))dt = aT - \frac{bT^2}{2}$
and, $y_2 = cT - \frac{dT^2}{2}$
This gives, transportation cost:
$TrC = \lambda_0\big[(a+c)T - \frac{(b+d)T^2}{2}\big] + \lambda_1\big[\big(aT - \frac{bT^2}{2}\big)^2 + \big(cT - \frac{dT^2}{2}\big)^2\big]$
Based on the above, we calculate total cost:

$$\begin{aligned} TC = c_o + c_p \left[(a+c)T - (b+d)\frac{T^2}{2} \right] + \mu_0 \left[yT - (a+c)\frac{T^2}{2} + (b+d)\frac{T^3}{6} \right] \\ +\mu_1 \left[y\frac{T^2}{2} - (a+c)\frac{T^3}{3} - (b+d)\frac{T^4}{8} \right] \\ +\nu_0 \left[\frac{cT^2}{2} - \frac{dT^3}{6} \right] + \nu_1 \left[\frac{cT^3}{3} - \frac{dT^4}{8} \right] \end{aligned} \tag{22.23}$$

The expression of total revenue is given by:
$TR = py_1 + y_2(p - \frac{r_1+r_2}{2})$
$= p(aT - \frac{bT^2}{2}) + (p - \frac{r_1+r_2}{2}(cT - \frac{dT^2}{2})$
Finally, the expression of total average profit is calculated as follows:

$TAP(T) = \frac{TR-TC}{T}$

$$TAP(T) = p\left(a - \frac{bT}{2}\right) + \left(R_0 - \frac{r_1+r_2}{2}\right)\left(c - \frac{dT}{2}\right) - \frac{c_o}{T} - c_p\left[a + c - (b+d)\frac{T}{2}\right]$$
$$-\mu_0\left[y - (a+c)\frac{T}{2} + (b+d)\frac{T^2}{6}\right] - \mu_1\left[y\frac{T}{2} - (a+c)\frac{T^2}{3} + (b+d)\frac{T^3}{8}\right]$$
$$-\nu_0\left[\frac{cT}{2} - \frac{dT^2}{6}\right] - \nu_1\left[\frac{cT^2}{3} - \frac{dT^3}{8}\right] \quad (22.24)$$

We have, $y = y_1 + y_2 = aT - \frac{bT^2}{2} + cT - \frac{dT^2}{2}$; putting the value of y in above, we get

$$TAP'(T) = -\frac{pb}{2} - \frac{d}{2}\left(R_0 - \frac{r_1+r_2}{2}\right) + \frac{c_o}{T^2} + c_p\frac{b+d}{2} - \mu_0\left[\frac{a+c}{2} - \frac{2(b+d)T}{3}\right]$$
$$-\mu_1\left[(a+c)T - \frac{3(b+d)T^2}{4}\right] - \nu_0\left[\frac{c}{2} - \frac{dT^2}{2}\right] - \nu_1\left[\frac{2cT}{3} - \frac{3dT^2}{8}\right]$$
$$(22.25)$$

The structure of the above expression clearly shows that equation $TAP'(T) = 0$ is a polynomial equation of degree 4 in T. So we can obtain the solution using numerical methods by MS-Excel software. Convexity for particular parameter value can be shown graphically.

Theorem 22.1 *The model in Sect. 3.2.1 can be derived from model in Sect. 3.2.2.*

Proof Putting $\lambda_1 = 0$ in the model 3.2.2., we get $c_t = \lambda_0$. Finally, putting in (22.9), we get the expression of T identical as (22.6).
Under that special condition, total average profit can be seen as identical in both the models. □

22.5 Numerical Examples and Sensitivity Analysis

Using the aforementioned procedure for obtaining optimal solution, the optimum values of decision variables T and BB and total optimal profit have been calculated for some initial values of the parameters of our model. Afterwards, in the sensitivity analysis, by changing each parameter of the model from −30, −10% to 10, 30%, effect on the change in optimal policy will be observed.

22.5.1 Numerical Problem

The models developed above is illustrated by the following numerical data:
For 3.2.1 $D_1 = 30{,}000$ products per year, $D_2 = 20{,}000$ products per year, $c_p = 100\$$ per product, $c_o = 3000\$$, $c_{h1} = 20\$$ per product per year, $c_{h2} = 5\$$ per product per year, $p = 200\$$, $R_0 = 50\$$, $r_1 = 20\$$, $r_2 = 60\$$ $c_t = 0.5\$$ per product\$
We obtain optimal values as $T^* = 0.739$ Years, $y^* = 3692.75$, $TAP^* = 5{,}083{,}759.62\$$
For 3.2.2, We incorporate transportation cost parameters $\lambda_0 = 0.5$, $\lambda_1 = 0.01$; other parameters remain equal to the previous, we obtain:
$T^* = 0.0149$, $y^* = 743.979$, $TAP^* = 4{,}761{,}763.097$
For 3.2.3, We use $\mu_0 = 20$, $\mu_1 = 0.4$ instead of c_{h1} Also, $\nu_0 = 5$, $\nu_1 = 0.1$ instead of c_{h2} other parameters remain equal to the previous, we obtain:
We obtain $T^* = 0.013$, $y^* = 707.979$, $TAP^* = 4{,}537{,}613.51$
The reason behind decrements in the optimal values is due to the effect of increased transportation cost, which depends on the quantity. This motivates lower order quantity and cycle time and profit reduces correspondingly. For 3.2.4, We use $a = 30{,}000$ and $b = 2$ instead of D_2; $c = 20{,}000$, $d = 3$, instead of D_2. Other parameters are identical as earlier.
We obtain $T^* = 0.0137$, $y^* = 699.302$, $TAP^* = 4{,}495{,}237.51$
In this scenario, the reason for decrement of profit is due to reduction in demand rate. This also motivates to order less than non-decreasing demand scenario; hence the optimal order quantity reduces in this case (Figs. 22.5 and 22.6).

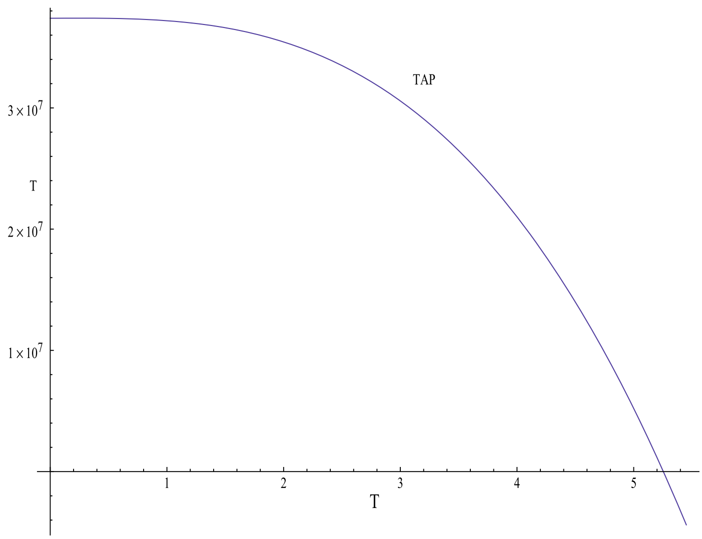


Fig. 22.5 *TAP* versus *T* for case 3.2.3

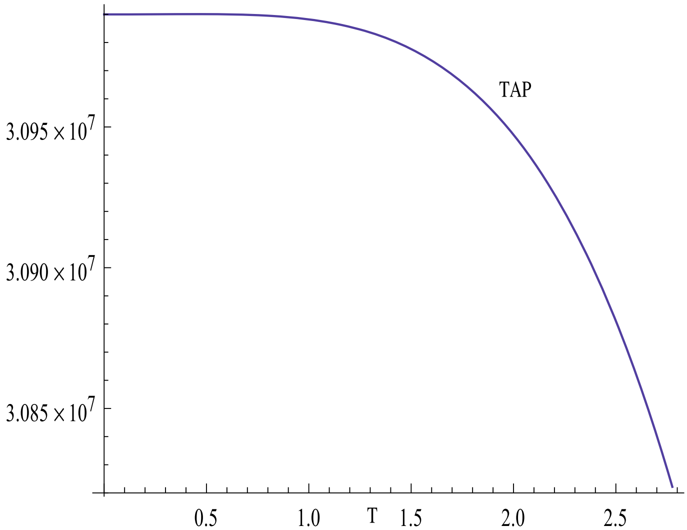


Fig. 22.6 *TAP* versus *T* for case 3.2.4

Table 22.1 Sensitivity analysis for model 3.2.1

Parameter	% change	Value	T	y	*TAP*
c_t	−40	0.3	0.07385	3692.745	5,097,759.62
	−20	0.4	0.07385	3692.745	5,090,759.62
	20	0.6	0.07385	3692.745	5,076,759.62
	40	0.7	0.07385	3692.745	5,069,759.62
c_{h1}	−40	12	0.0926	4629.1	5,100,192.59
	−20	16	0.0817	4082.483	5,091,515.31
	20	24	0.0679	3396.831	5,076,682.39
	40	28	0.0632	3162.278	5,070,131.67
c_{h12}	−40	3	0.0752	3761.77	5,085,250.39
	−20	4	0.0745	3726.78	5,084,501.55
	20	6	0.0731	3659.62	5,083,024.39
	40	7	0.0725	3627.38	5,082,295.71

22.5.2 Sensitivity Analysis

It can be observed in Table 22.1, that effect of change on the vital cost parameters which are the holding costs and transportation cost is considered. We can observe that due to constant nature of the transportation cost, its effect can not be observed in the order quantity; although it has an effect on total average profit. Increase of the transportation cost reduces total average profit proportionately.

Table 22.2 Sensitivity analysis for model 3.2.2

Parameter	% change	Value	T	y	TAP
λ_0	−40	0.3	0.01488	743.97	4,775,763.10
	−20	0.4	0.01488	743.97	4,768,763.1
	20	0.6	0.01488	743.97	4,754,763.10
	40	0.7	0.01488	743.97	4,747,763.10
λ_1	−40	0.006	0.00618	308.999	4,194,124.11
	−20	0.008	0.005357	267.835	4,044,910.72
	20	0.012	0.004378	218.88	3,794,379.70
	40	0.014	0.004054	202.693	3,684,932.43
c_{h1}	−40	12	0.004797	239.87	3,914,320.18
	−20	16	0.004795	239.78	3,913,840.54
	20	24	0.004727	239.59	3,912,881.79
	40	28	0.004790	239.50	3,912,402.69
c_{h2}	−40	3	0.004794	239.70	3,913,456.95
	−20	4	0.004794	239.69	3,913,409.01
	20	6	0.004793	239.67	3,913,313.14
	40	7	0.004793	239.66	3,913,265.20

In Table 22.2, the effect of change on the transportation and holding cost parameters are shown. One can observe that there is inverse effect of variable transportation cost component on the order quantity; which means higher transportation cost implies lower order quantity. On the other hand, the fixed component of the transportation cost has no effect on the order quantity but it has an effect on total average profit. Increase in the fixed transportation cost reduces total average profit proportionately. Also for the holding cost of new items c_{h1}, we can observe that there is marginal effect on the order quantity but there is moderate effect on the total average profit. Lastly, the holding cost of exchanged items c_{h1}, we can observe that there is further marginal effect on the order quantity but there is moderate effect on the total average profit.

22.6 Managerial Implications

In the era of cut-throat competition in the electronic retailing markets it is difficult to remain competitive. To have more market share, various retailers offer assortment of schemes to sell their products. One of the most popular scheme is product exchange offer where customers can exchange their old products with new ones by paying marginal difference in price. Based on those situations, the models formulated in this article suggest various insights for managers. The basic model shows that if all costs are fixed then cycle time and order quantity are independent of transportation cost.

But, when the transportation cost becomes dependent upon the quantity transported, the cycle time and optimal order quantity both depend on the transportation cost in an inverse manner. In other words, higher transportation cost motivates the retailer to order in lower quantities. This predicts that in electronic retailing, transportation can be one of the vital cost which managers should take into account. Besides this, the models also show that declining demand has drastic effect on the profit of the retailer.

22.7 Conclusion

The model formulated in this article contributes to the inventory system literature in several unique manners. First, unlike previous inventory models, this model addresses the situation when product is sold in the electronic-market with optional exchange offer. This implies that the customer can buy the product either by paying full price or paying discounted price after availing exchange offer. The optimal policy of the model shows that there is a significant role of transportation cost when transportation cost depends on the quantity transported. This can be vital. Secondly, the model formulated in this article partially addresses the environmental aspects of inventory modeling, where reverse logistics have been explored. This makes our model more applicable for recent business trends. One can further explore to study quantitatively the effect of product exchange offers in sales of such products which further affect the inventory behavior. Besides this, multiple warehouses, imperfect products, and supply network can also be included in the formulation of the some advanced models.

References

1. Alamri AA, Balkhi ZT (2007) The effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand and deterioration rates. Int J Prod Econ 107(1):125–138
2. Alqahtani AY, Gupta SM (2018) Money-back guarantee warranty policy with preventive maintenance strategy for sensor-embedded remanufactured products. J Ind Eng Int 1–16
3. Belvedere V, Grando A (2017) Reverse logistics and closed-loop supply chain. Sustainable Operations and Supply Chain Management. Wiley, New York
4. Bose S, Goswami A, Chaudhuri KS (1995) An EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting. J Oper Res Soc 46(6):771–782
5. Chakrabarti T, Chaudhuri KS (1997) An EOQ model for deteriorating items with a linear trend in demand and shortages in all cycles. Int J Prod Econ 49(3):205–213
6. Chang HJ, Hung CH, Dye CY (2001) An inventory model for deteriorating items with linear trend demand under the condition of permissible delay in payments. Prod Plan Control 12(3):274–282
7. Daaboul J, Le Duigou J, Penciuc D, Eynard B (2016) An integrated closed-loop product lifecycle management approach for reverse logistics design. Prod Plan Control 27(13):1062–1077

8. Das D, Dutta P (2013) A system dynamics framework for integrated reverse supply chain with three way recovery and product exchange policy. Comput Ind Eng 66(4):720–733
9. Datta TK, Pal AK (1992) A note on a replenishment policy for an inventory model with linear trend in demand and shortages. J Oper Res Soc 24(5):993–1001
10. De Kumar S, Kundu PK, Goswami A (2003) An economic production quantity inventory model involving fuzzy demand rate and fuzzy deterioration rate. J Appl Math Comput 12(1–2):251
11. Donaldson W (1977) Inventory replenishment policy for a linear trend in demand an analytical solution. Oper Res Q 28(3):663–670
12. Diewert WE (1974) Intertemporal consumer theory and the demand for durables. Econom J Econ Soc 42:497–516
13. Dye CY, Chang HJ, Teng JT (2006) A deteriorating inventory model with time-varying demand and shortage-dependent partial backlogging. Eur J Oper Res 172(2):417–429
14. Geyer R, Blass VD (2010) The economics of cell phone reuse and recycling. Int J Adv Manuf Technol 47(5–8):515–525
15. Giri BC, Goswami A, Chaudhuri KS (1996) An EOQ model for deteriorating items with time varying demand and costs. J Oper Res Soc 47(11):1398–1405
16. Goswami A, Chaudhuri KS (1991) An EOQ model for deteriorating items with shortages and a linear trend in demand. J Oper Res Soc 42(12):1105–1110
17. Goswami A, Chaudhuri KS (1992) An economic order quantity model for items with two levels of storage for a linear trend in demand. J Oper Res Soc 43(2):157–167
18. Goyal S, Hariga M, Alyan A (1996) The trended inventory lot sizing problem with shortages under a new replenishment policy. J Oper Res Soc 43(2):1286–1295
19. Goyal SK, Giri BC (2003) The productioninventory problem of a product with time varying demand, production and deterioration rates. Eur J Oper Res 147(3):549–557
20. Guide VDR Jr (2000) Production planning and control for remanufacturing: industry practice and research needs. J Oper Manag 18(4):467–483
21. Hariga M (1995) An EOQ model for deteriorating items with shortages and time-varying demand. J Oper Res Soc 46(3):398–404
22. Islam MT, Huda N (2018) Reverse logistics and closed-loop supply chain of Waste Electrical and Electronic Equipment (WEEE)/E-waste: a comprehensive literature review. Resour Conserv Recycl 137:48–75
23. Kostecki M (ed) (2013) The Durable Use of Consumer Products: New Options for Business and Consumption. Springer Science Business Media, Dordrecht
24. Lau AHL, Lau H-S (2003) Effects of a demand-curves shape on the optimal solutions of a multi-echelon inventory/pricing model. Eur J Oper Res 147(3):530–548
25. Lee CC, Hsu SL (2009) A two-warehouse production model for deteriorating inventory items with time-dependent demands. Eur J Oper Res 194(3):700–710
26. Menon P, Vijayaraghavan P (2016) Exchange offer as a sales promotion tool for consumer durables: a content analysis of Indian print advertisement. Thriving in a New World Economy. Springer, Cham, pp 256–257
27. Mishra VK, Singh LS, Kumar R (2013) An inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging. J Ind Eng Int 9(1):4
28. Mitra A, Cox J, Jesse R Jr (1984) A note on determining order quantities with a linear trend in demand. J Oper Res Soc 35(2):141–144
29. Mitra S (2007) Revenue management for remanufactured products. Omega 35(5):553–562
30. Mukhopadhyay A., Goswami A. (2014), Deteriorating inventory model with variable holding cost and price dependent time varying demand. In: Proceedings of ICBIM-2014 (IEEE Explore), pp 14–19
31. Mukhopadhyay A (2018) An Uncertain inventory model for deteriorating products with imperfections, variable selling price, partial backlogging and selling price dependent time varying demand. Mukhopadhyay Int J Bus Forecast Mark Intell (Article in Press.)
32. Phelps R (1980) Optimal inventory rule for a linear trend in demand with a constant replenishment period. J Oper Res Soc 31(5):439–442

33. Pourmohammadi H, Rahimi M, Dessouky M (2008) Sustainable reverse logistics for distribution of industrial waste/byproducts: a joint optimization of operation and environmental costs. Supply Chain Forum Int J 9(1):2–17
34. Ritchie E (1984) The EOQ for linear increasing demand: a simple optimal solution. J Oper Res Soc 35(10):949–952
35. Sarkar B (2012) An EOQ model with delay in payments and time varying deterioration rate. Math Comput Model 55(3–4):367–377
36. Seitz MA (2007) A critical assessment of motives for product recovery: the case of engine remanufacturing. J Clean Prod 15(11–12):1147–1157
37. Silver E, Meal H (1969) A simple modification of the EOQ for the case of a varying demand rate. Prod Invent Manag 10(4):52–65
38. Silver EA, Meal H (1973) A heuristic for selecting lot size quantities for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment. Prod Invent Manag 14(2):64–74
39. Singh T, Mishra PJ, Pattanayak H (2017) An optimal policy for deteriorating items with time-proportional deterioration rate and constant and time-dependent linear demand rate. J Ind Eng Int 13(4):455–463
40. Srivastava SK, Srivastava RK (2006) Managing product returns for reverse logistics. Int J Phys Distrib Logist Manag 36(7):524–546
41. Uthayakumar R, Karuppasamy SK (2018) An EOQ model for deteriorating items with different types of time-varying demand in healthcare industries. J Anal 1–16

Chapter 23
Electronic Components' Supply Chain Management of Electronic Industrial Development for Warehouse and Its Impact on the Environment Using Particle Swarm Optimization Algorithm

Ajay Singh Yadav, Anupam Swami, Navin Ahlawat, Dhowmya Bhatt and Geethanjali Kher

Abstract The electronic component model inventory is a method of balancing investments to achieve the service-level goal. Here you can see the electronic components warehouse and the distribution centers for the electronic components inventory. Environmental heritage policy, electronic components, and electronic component warehouses are very important issues, as the supply chain of electronic components for chemicals is directly linked to people's lives. Variable electronic component order quantity, economical electronic component order quantity, electronic component time order quantity, electronic component removal order quantity, and electronic environment component order quantity inventory rules are usually used in the modern trend for inventory management of parts of electronics. However, effectively managing the inventory of a substance in electronic components is a difficult problem due to its properties. We have proposed the PSO inventory policy for resellers of electronic component warehouses in the electronic supply chain. We also model

A. S. Yadav (✉)
Department of Mathematics, SRM Institute of Science and Technology, Delhi-NCR Campus Ghaziabad, Ghaziabad, Uttar Pradesh, India
e-mail: ajay29011984@gmail.com

A. Swami
Department of Mathematics, Government Post Graduate College, Sambhal, Uttar Pradesh, India
e-mail: swami.anupam@gmail.com

N. Ahlawat
Department of Computer Science, SRM Institute of Science and Technology, Delhi-NCR Campus Ghaziabad, Ghaziabad, Uttar Pradesh, India
e-mail: navinahlawat@gmail.com

D. Bhatt
Department of Information Technology, SRM Institute of Science and Technology, Delhi-NCR Campus Ghaziabad, Ghaziabad, Uttar Pradesh, India
e-mail: dhowmya.b@ncr.srmuniv.ac.in

G. Kher
Department of Computer Science, Kirori Mal College, Delhi, India
e-mail: geetanjalikher@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_23

a model designed to measure the effectiveness of administrative strategies. The proposed algorithm for artificial bee colony (PSO) determines the optimal product at the time of the order using the current stock. The simulation results show that the ABC algorithm for artificial bee colonies is an effective way to manage electronic component warehouses and electronic component supply chains.

Keywords Electronic parts' supply chain · Electronic components warehouses · Electronic components inventory · Particle swarm optimization algorithm (PSO) and economic order quantity of electronic components

23.1 Introduction

As explained above, the storage center for electronic components and distribution of electronic components occupies an important place in the business unit. Every entrepreneur needs it during the transaction, whether it is finished products or raw materials. In the current market scenario and the globalization of the market, the business climate is very competitive and no one wants to lose goodwill in the market and tries to satisfy customer demand. To this end, resellers and retailers always store the goods in their store. In this damn commercial environment, sellers offer discounts on bulk purchases during the festival season, and they also offer a loan program to fund resellers to attract their retailers. In order to benefit from these suppliers' instructions, retailers needed more space to store the products they had purchased as part of the offer. However, because of their small footprint in crowded markets, retailers face storage problems in their own stores and therefore require different storage spaces to store their overbought products. To solve this problem, they rent another storage space for a short rental period. This rented warehouse becomes additional storage space provided by private/public or government agencies, and these spaces are used as secondary storage space. The acquisition of leased space for storage integrated the concept of storage and distribution center for electronic components into inventory modeling. In the electronic component modeling inventory, the concept of electronic component warehouse and electronic component distribution centers was introduced for the first time in the concept of electronic components warehouse and distribution centers, with limited capacity (electronic component distribution centers) and others with unlimited capacity (Electronic Components Warehouse). In this concept, it is often assumed that the accounting costs of electronic component warehouse items are higher than those of electronic component distribution centers because of better storage facilities provided by the owner of an additional warehouse. Therefore, it is economical to first consume goods held in the stock of electronic components in order to reduce the cost of maintaining the stock of electronic components. The latter will discuss the benefits and limitations of the electronic component warehouse and electronic component modeling distribution centers versus the electronic component modeling model at a warehouse.

Today, the environment is a necessary issue, a more lively topic, but today people do not know about it. Outside of rural society, even in cosmopolitan life, this is not so popular. Therefore, environmental protection has become a simple state program. This relationship is very close to the whole society. If there is no natural connection between people, environmental protection remains an impossible dream. Direct communication with the environment is inherent. In our environment, we find all kinds of animals, plants, and other living things. They all make the environment together. In various scientific disciplines such as physics, chemistry, biology, etc., the basic principles of the subject and related experimental subjects are considered. However, the current requirement is to focus on practical environmental knowledge and associated practical knowledge. Modern society should be broader in environmental issues. In addition, information about preventive measures is needed to deal with this. In this mechanical age, we are experiencing this situation. Contamination stands before us to destroy the whole environment in the form of a curse. The whole world is in serious test. However, there is not enough reference material. In fact, it is necessary for knowledge to relate to the environment so that the public can easily understand the problem. In such a disgusting situation, society must fulfill its duties and responsibilities. This can create an understanding of the environment in society. In fact, living and nonliving creatures together form nature. Air, water, and earth are in nonliving areas, while living things form in relation to animals and plants. The important relationship between these components is that they depend on their vital relationships. Although man is the most conscious and sensitive person in the world of the Jiva, he depends on other animals, plants, air, water, and earth to meet his needs. The organisms present in the human environment form the structure of plants, air, water, and soil. Education through education is a powerful tool for the diverse development of human life. Their main goal is to give people physical, mental, social, cultural, and spiritual wisdom. Knowledge of the natural environment is very important to achieve the goals of education. The tradition of knowledge about the natural environment has been part of Indian culture from the very beginning. However, in the modern materialist era, circumstances will be different. On the one hand, new inventions are emerging in various fields of science and technology. On the other hand, the same speed affects the human environment. By acquiring knowledge of the links between the environment and education, many important functions in this sense can be performed. The environment is closely related to science. However, in their education, there are no scientific subtleties. Students should learn nature and environmental knowledge in a simple and understandable language. This knowledge should initially be superficial in introductory form. Other technical aspects should be taken into account. The price of most electronic components' supply chains will be highly correlated with the overall profitability of the supply chain, which is the difference between the income received by the customer and the total value of the supply chain. The higher the profitability of the supply chains of electronic components, the higher the supply chain. The success of the supply chain of electronic components should be measured by the profitability of the supply chain of electronic components, and not by the profit obtained at each stage. Cash flows are only a means of exchange in the supply chain if different owners are at different stages.

All information, products, or cash flows lead to costs in the supply chain. Therefore, proper control over these rivers is key to the success of the electronic component supply chain. Effective supply chain management of electronic components includes managing the properties and products of the supply chain of electronic components, information, and flow of funds to maximize the overall profitability of the supply chain of electronic components. In fact, in order for everything to be done correctly and quickly, especially for a large number of products, someone needs a system that performs a number of tasks. Prepare forecasts, calculate the appropriate safety margin, set an economical order quantity, set the optimum discount amount, correct the discrepancies, and ensure complete transparency of changes in the supply chain of electronic components so that they can immediately respond to this modification. The supply chain of electronic components is only as strong as the relationship between sellers, buyers, and other participants. It is important to consider these other companies and suppliers as partners in the success of the supply chain of electronic components, and this should be the main priority of the organization.

23.2 Literature Review and Survey of Electronic Components' Supply Chain Management

Nagurney et al. [1] presented a supply chain network operations management of a blood banking system with cost and risk minimization. Yadav and Swami [2] analyzed an integrated supply chain model for deteriorating items with linear stock-dependent demand under imprecise and inflationary environment. Yadav and Swami [3] discuss a partial backlogging production-inventory lot-size model with time-varying holding cost and Weibull deterioration. Yadav et al. [4] presented a supply chain inventory model for decaying items with two warehouses and partial ordering under inflation. Yadav et al. [5] proposed an inventory model for deteriorating items with two warehouses and variable holding cost. Yadav et al. [6] analyzed an inventory of electronic components model for deteriorating items with warehousing using genetic algorithm. Yadav et al. [7] discussed an analysis of green supply chain inventory management for warehouse with environmental collaboration and sustainability performance using genetic algorithm. Yadav and Kumar [8] presented an electronic components' supply chain management for warehouse with environmental collaboration and neural networks. Yadav et al. [9] analyzed an effect of inflation on a two-warehouse inventory model for deteriorating items with time-varying demand and shortages. Yadav et al. [10] discussed an inflationary inventory model for deteriorating items under two storage systems. Yadav et al. [11] proposed a fuzzy-based two-warehouse inventory model for non-instantaneous deteriorating items with conditionally permissible delay in payment. Yadav [12] analyzed an analysis of supply chain management in inventory optimization for warehouse with logistics using genetic algorithm. Yadav et al. [13] discussed a supply chain inventory model for two warehouses with soft computing optimization. Yadav et al. [14] presented a multi-objective optimization for electronic component inventory model and deteriorating items with two warehouses using genetic algorithm. Yadav [15] analyzed a modeling and analysis of supply chain inventory model with two warehouses and economic load dispatch problem using genetic algorithm. Yadav et al. [16] discussed a particle swarm optimization for inventory of auto-industry model for two warehouses with deteriorating items. Yadav et al. [17] presented a supply chain management of chemical industry for deteriorating items with warehouse using genetic algorithm. Yadav [18] discussed an analysis of seven stages of supply chain management in electronic component inventory optimization for warehouse with economic load dispatch using GA and PSO. Yadav et al. [19] gave a multi-objective genetic algorithm optimization in inventory model for deteriorating items with shortages using supply chain management. Yadav et al. [20] analyzed a supply chain management in inventory optimization for deteriorating items with genetic algorithm. Yadav et al. [21] discussed a modeling and analysis of supply chain management in inventory optimization for deteriorating items with genetic algorithm and particle swarm optimization. Yadav et al. [22] presented a multi-objective particle swarm optimization and genetic algorithm in inventory model for deteriorating items with shortages using supply chain management. Yadav et al. [23] proposed a soft computing optimization of two-warehouse inventory model with genetic algorithm. Yadav et al. [24] analyzed a multi-objective genetic algorithm involving green supply chain management. Yadav et al. [25] presented a multi-objective particle swarm optimization algorithm involving green supply chain inventory management. Yadav et al. [26] gave a green supply chain management for warehouse with particle swarm optimization algorithm. Yadav et al. [27] analyzed an analysis of seven stages of supply chain management in electronic component inventory optimization for warehouse with economic load dispatch using genetic algorithm. Yadav et al. [28] discussed an analysis of six stages of supply chain management in inventory optimization for warehouse with artificial bee colony algorithm using genetic algorithm. Yadav et al. [29] presented an analysis of electronic component inventory optimization in six stages of supply chain management for warehouse with abc using genetic algorithm and PSO. Yadav et al. [30] discussed a two-warehouse inventory model with ramp-type demand and partial backordering for Weibull distribution deterioration. Yadav et al. [31] proposed a two-storage model for deteriorating items with holding cost under inflation and genetic algorithms. Singh et al. [32] analyzed a two-warehouse model for deteriorating items with holding cost under particle swarm optimization. Singh et al. [33] presented a two-warehouse model for deteriorating items with holding cost under inflation and soft computing techniques. Sharma et al. [34] gave an optimal ordering policy for non-instantaneous deteriorating items with conditionally permissible delay in payment under two storage managements. Yadav et al. [35] discussed an analysis of genetic algorithm and particle swarm optimization for warehouse with supply chain management in inventory control. Swami et al. [36] analyzed inventory policies for deteriorating item with stock-dependent demand and variable holding costs under permissible delay in payment. Swami et al. [37] discussed an inventory model with price-sensitive demand, variable holding cost, and trade credit under inflation. Gupta et al. [38] proposed a binary multi-objective genetic algorithm and

PSO involving supply chain inventory optimization with shortages, inflation. Yadav et al. [39] analyzed a soft computing optimization based two-warehouse inventory model for deteriorating items with shortages using genetic algorithm. Yadav and Swami [40] presented a two-warehouse inventory model for deteriorating items with ramp-type demand rate and inflation. Yadav and Swami [41] discussed an effect of permissible delay on two-warehouse inventory model for deteriorating items with shortages. Yadav and Swami [42] analyzed a two-warehouse inventory model for decaying items with exponential demand and variable holding cost.

23.3 Related Works

23.3.1 Electronic Parts' Supply Chain

The ecological source of raw materials is almost identical to other logistics. Spare parts for electronics are exported to electronic components warehouses in the process of production and packaging at an electronic component production plant. The delivery method is determined by the characteristics of each raw material. Figure 23.1 illustrates the raw materials supply chain process. Parts of electronics are divided into prescription and general types. Eco-friendly packaging allows you to purchase electronic components in distribution centers for electronic components and electronic components. However, this requires a recipe, which was published on the recipe electronic components. Electronic component warehouses transfer the electronic com-

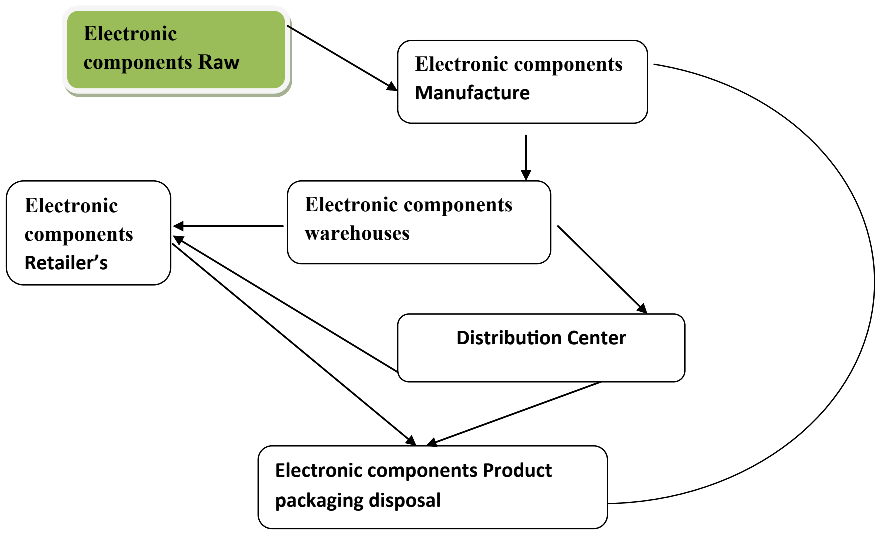


Fig. 23.1 Electronic parts' supply chain

ponents directive to electronic distribution centers or the electronic component seller in accordance with the distribution guidelines. This latter process differs according to the type of raw material used. In particular, the seller must notify the government of the delivery, purchase, and use of therapeutic products. Electronic component distribution centers provide the patient with electronic materials, such as injections or blends. Electronic component resource stores sell general recipes or electronic component resources according to the rules. This process provides electronic components for the final disposal of electronic products.

23.3.2 Electronic Components Inventory Policy

Listing policy is open to uncertainty at all stages of the supply chain. For this strategic work remain raw materials and products. The ideal number in the list is close to 0. However, stocks of electronic components absorb some reserve reserves due to uncertainty. A reduction problem may arise if the stock is set at the lowest point and vice versa. In accordance with the optimal inventory management strategy, the list policy can be divided into variable order quantity, economical order quantity of electronic components, order quantity of electronic components time, and order quantity. The number of variable commands is also called the V-system. If the list falls below the threshold, the agent automatically controls the specified number of products. It is easy to control the number of control variables, and the product is suitable that it is impossible to control and forecast demand. Due to an extensive list of variable order quantities, the cause of the warehouse can cause problems. TOQEC is also called the P system. The agent controls the deficit by regularly checking stock. However, it should ensure the safety of stocks compared to other methods, since inventory instability is considered only in the following list. Order quantity MRP is a combination of variables, order quantity and. The order of the manager is the sum of R if the list falls below the threshold. R is obtained by reducing the amount of electricity to the appropriate amount. The amount of environmental control (EQE) is a periodic guide. The manager checks the stock for the normal order duration U and adds an R in advance. This policy is useful when demand is stable. Otherwise, it is difficult to determine the quantity ordered.

23.3.3 Particle Swarm Optimization Algorithm

Optimization of a swarm of particles is initiated by a set of random solutions, and a random velocity is assigned to each potential solution. Solutions, called particles, then fall into the problem space. Each particle follows its coordinates in the space of tasks related to the best solution or suitability, provided that the value of the physical state is also preserved. This value is called pbest. The generic version of PSO is another best value for money: the best value for money that has ever been achieved by a fraction of the population. This value is called gbest. At each stage, when a particle changes direction and moves to its pbest and gbest, this is a global version of PSO, when in addition to pbest each particle is looking for a better solution, called nbest or lbest, in the local environment topological environment. Particles are achieved. The process is called a local version of PSO.

P: = 0
$\{M_x, N_x, U_x, V_x\}_{x=1}^{X}$: = initialize()
for a: = 1: U
for b: = 1: X
for r: = 1: R
$n_{xc}^{(a+1)} = yn_{xc}^{a} + c_1 d_1 \left[V_{xc} - m_{xc}^{a}\right] + c_2 d_2 \left[U_{xc} - m_{xc}^{a}\right]$
$M_x^{a+1} = M_x^a + mN_x^a + \in^a$
end
M_x: = enforce Constraints(X)
Y_x := f(M_x)
if $M_x \nleq$ e $\forall$ e $\in$ P
P: = e $\in$P/e $\nless M_x$
P: = P $\cup\, M_x$
end
end
if $M_x \leq V_x \vee (M_x \nless V_x \wedge V_x \nless M_x)$
V_x: = M_x
end
U_x: = selectGuide(X, A)
end

We are using those basic steps for finding the optimal resources for an organization in medium-range prospective using MATLAB software package.

23.4 Model Design

23.4.1 Electronic Parts' Supply Chain

Based on the analysis of process management results, we prepare simulations for a global configuration of electronic component inventory management and a pharmaceutical supply chain. To implement methods of environmental management of stocks, we need to analyze the goods and their properties in the supply chain. The data component for analyzing the list data includes the maximum and minimum stock, the order cycle, and the delivery time. And we determine sales prices, sales, delivery costs, order numbers, average stocks, stock prices, and net income based on data components. Resource management models should be modeled and emulated. The supply chain includes an electronic component plant, electronic component warehouses, electronic component distribution centers, and electronic component retailers. The plant of electronic components develops and manufactures new electronic components from raw materials. Produced electronic components are stored in packaging and labeling conditions depending on these types and sizes. The company needed a number of pieces of electronics in each warehouse of electronic components to handle ships.

The electronic component warehouse imports electronic components from an electronic component plant. Imported electronic components are stored in a container by type and characteristics. The production of electronic components requires a reduction in raw materials. Ordering the required number of electronic components in electronic component warehouses requires environmentally sound inventory management, which takes into account the costs of maintenance and administration. Due to the nature of the supply chain, the environmental distribution center and the storage of raw materials are located at a lower level. Electronic component warehouses are asked to supply electronic components from distribution centers for electronic components and pharmaceuticals and export the required quantity. As a result, it is difficult to predict demand. The environmentally friendly distribution center receives electronic components from an electronic component warehouse. Electronic parts can be used directly from electronic components distribution centers with a doctor's prescription. Unused list can be issued in the nearest resources. This delivery occurs when the electronic component vendor requests electronic components. The ecological retailer is the last step in the supply chain, providing direct electronic components for packaging electronic components. Universal raw materials can be sold directly to the electronic components of the product packaging products. On the contrary, proof is required for some parts of the electronics. Spare parts for electronics can be obtained from the electronic component warehouse or electronic distribution centers. There is an urgent need to get through the distribution centers of electronic components. Bulk purchases are part of the electronic component warehouse.

23.4.2 Electronic Parts Model

The supply chain consists of four stages: production of electronic components, warehouses of electronic components, distribution centers for electronic components, and storage of goods. Parts of electronics were classified into six types, called A and J values, and are associated with the demand for products for production, as well as classification data for specific products. There are differences in the delivery time, price, and demand for each product. Product margin is 40% of sales. We believe that, despite the product type, the list and shipping cost are the same. The parameters of the raw material models are presented in Table 23.1.

Table 23.1 Characteristic by electronic parts

Product	Rate	Demand	Lead time	Holding cost (%)	Cost of loss (%)	Environment cost (%)	Packaging disposal cost (%)
EC1	19,174	288.96	3 year	Price * 19	Price * 110	Price * 115	Price * 113
EC2	18,156	255.25	3 year	Price * 19	Price * 110	Price * 115	Price * 113
EC3	17,256	244.77	3 year	Price * 19	Price * 110	Price * 115	Price * 113
EC4	15,126	266.89	3 year	Price * 19	Price * 110	Price * 115	Price * 113
EC5	16,756	277.49	3 year	Price * 19	Price * 110	Price * 115	Price * 113
EC6	11,256	256.63	3 year	Price * 19	Price * 110	Price * 115	Price * 113
EC7	11,789	257.23	3 year	Price * 19	Price * 110	Price * 115	Price * 113
EC8	14,759	212.78	3 year	Price * 19	Price * 110	Price * 115	Price * 113
EC9	12,358	290.13	3 year	Price * 19	Price * 110	Price * 115	Price * 113
EC10	13,459	260.36	3 year	Price * 9	Price * 10	Price * 15	Price * 113

23.4.3 Electronic Components Inventory Policy

In this study, we measure the reduction in quantity and frequency for the supply chain of electronic components, the net profit and cost of stocks of electronic components, individual stocks, and the total stock of electronic components. Due to a defect, expenses are only added to the retail store. In contrast, transportation and environmental costs arise in both phases. We select the variable order quantity of electronic components, the economical quantity of the order of electronic components, the quantity of the temporary order, the quantity of the order for removing electronic components, and the quantity of the order of electronic components of the environment and electronic components. The proposed algorithm for artificial bee colonies calculates the optimal order at certain points in time. The particle swarm optimization algorithm uses current minimum and maximum stocks to calculate orders for each electronic component. Particle swarm optimization algorithm calculates the optimal order for each electronic part at the time of ordering. We have determined the local mass and the total calculation at 0.74. This is the most commonly used value for finding the optimal point. The number of iterations is set to 112. The optimal order is calculated after the 2100th repetition. All calculations are based on the fitness function given in Eq. 23.1.

$$\begin{aligned}\text{Format function} = {} & \text{minimum requirement} + \text{quantity} + \text{maximum quantity} \\ & + \text{order quantity} + \text{order quantity for disposal} \\ & + \text{order quantity in the environment}\end{aligned} \tag{23.1}$$

This formatting function calculates the current and quantity ordered. For this calculation, the ordered quantity must be greater than the minimum required capacity. GA increases net profit using a fitness function to maintain minimal potential.

Table 23.2 Detail method

Method	Note
Economic order quantity of electronic components	Order quantity = Max electronic components inventory capacity * 74% Order time = Max electronic components inventory capacity * 60%
Variables order quantity of electronic components	Order quantity = Max electronic components inventory capacity * 40% Order time = Max electronic components inventory capacity * 60%
Time order quantity of electronic components	Order quantity = Max electronic components inventory capacity—current electronic components inventory Order time = 3 Year
Disposal order quantity of electronic components	Waste order quantity = Max electronic components inventory capacity—current electronic components inventory Order time = Max electronic components inventory capacity * 60%
Environment order quantity of electronic components	(Water + Air + Waste) Quantity = Max electronic components inventory capacity * 10% Order time = 10 Day
Particle swarm optimization algorithm (PSO)	Order quantity = PSO result Order time = Max electronic components inventory capacity * 65%

Table 23.2 shows the specific configurations of individual methods. Order time is 4 days, and time is 60% of the maximum. Results are calculated separately for each electronic component.

23.5 Industrial Development and Its Impact on Environment

Outstanding technological advances have increased our ability to produce goods and improve living conditions. However, it caused less important events, such as pollution. The impact of deteriorating quality of life has been affected. In the past, improvements in the quality of life created by new technologies have adversely affected the environment. Recently, however, some doubts have arisen that advances in technology will improve the quality of life. It can be seen that the increase in productivity not only aggravates the deterioration of the quality of raw materials but also reduces the environment due to the loss of waste. On one hand, the environment is a source of energy and materials that are converted into goods and services to meet

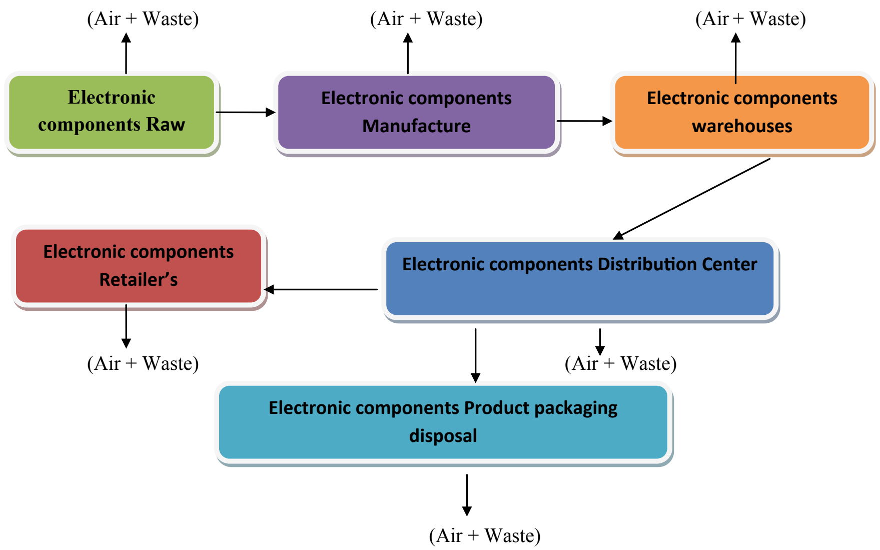


Fig. 23.2 The environmental impacts of each stage

the needs of people. On the other hand, it is synchronized with waste and emissions produced by producers and consumers (Fig. 23.2).

The number and variety of chemicals used daily are increasing rapidly. Their design sometimes uses new chemicals whose effects on health are unknown or harmful. Although these chemicals are of great benefit to society, they potentially threaten the waste generated during the production process. Millions of tons of toxic or other hazardous substances are released into the environment every year. One of the most disturbing features of the problem is that the long-term effects of exposure to chemicals are very small. We now know that some of these periods can cause cancer, that nervousness can be delayed, distortions of children in cities, and mutations. Many other chemicals are likely to have similar effects, but since they take time to identify, and their causes are difficult to determine, we still do not know who is dangerous. The situation is also problematic, since these chemicals are very complex in the environment and can spread and turn into other substances with different effects.

23.6 Simulation

The simulated raw materials for electronic components are used to compare and check the inventory management list of electronic components in the supply chain. As shown in the figures, the virtual system includes 110 manufacturers of electronic components, wholesale, 110 centers of distribution of electronic components, and

155 commodity stores. Table 23.3 presents the 155 parameters of raw materials for modeling.

Supply chain modeling was carried out with each management method for 700 virtual days. We obtained the results by calculating the sum of 5000 simulations. To evaluate each inventory management method for electronic components, we compare sales prices, sales invoices, order numbers, shipping costs, stock prices, and net income. Equation 10 describes the calculation of distribution costs, stock prices, and net income.

$$\textbf{Electronic components delivery Cost} = \int_{1}^{10} \text{Electronic components sales account}_1 * \text{Electroniccomponents account}_1 * 1.29 + \text{Environment Cost} + \text{packaging disposal Cost}$$

$$\textbf{Electronic components stock Cost} = \int_{1}^{10} \text{current Electronic components inventory quantity}_1 * 1.798 + \text{Environment Cost} + \text{packaging disposal Cost}$$

$$\textbf{Electronic components net Profit} = \int_{1}^{10} (\text{Electronic components sales cost} * 1.23)$$

Table 23.3 Product parameter

Product	Max electronic components inventory capacity	Minimum required electronic components inventory capacity	Initial electronic components inventory	Price	Demand (%)
EC-1	18,000	24,000	41,000	36,000	15.3
EC-2	17,910	22,100	41,100	32,100	15.1
EC-3	17,820	22,090	41,090	33,000	15.0
EC-4	17,620	22,080	41,080	34,000	14.9
EC-5	17,930	22,070	41,170	35,000	14.0
EC-6	17,710	22,060	41,360	36,000	11.1
EC-7	17,610	22,050	41,950	37,000	10.4
EC-8	16,560	22,010	41,810	38,000	09.7
EC-9	16,640	22,020	41,520	31,000	09.2
EC-10	16,120	22,070	41,470	32,000	08.4

$$- \text{ Electronic components delivery cost} - \text{Electronic componentsstock cost} + \text{Environment Cost} + \text{packagingdisposal Cost}$$

$$\textbf{Electronic components Transportation Costs} = \int_{10}^{100} \text{Electronic components Sales Account}_1 * \text{Electronics parts Account}_1 * 11.29 + \text{Environmental Costs} + \text{Packaging Disposal Costs}$$

$$\textbf{Electronic components inventory Costs} = \int_{1}^{10} \text{Current Electronic components inventory}_1 * \ 11,798 + \text{Environmental Costs} + \text{Packaging Disposal Costs}$$

$$\textbf{Electronic components Net Profit} = \int_{10}^{100} (\text{Electronic components Sales Costs} * 1.23) - \text{ Electronic components Transportation Costs} - \text{Electronic components Storage Costs} + \ \text{Environmental Costs} + \text{Packaging Disposal Costs}$$

.

23.6.1 Simulation Result

Simulation results are shown in Table 23.4.

Table 23.4 Simulation result electronic components supply

Product	Electronic components sale price	Electronic components sale account	Electronic components order count	Electronic components delivery cost	Electronic components stock price	Electronic components net profit
VOQEC	31,279.413	42.218	35.480	1,948.382	17,760.334	25,803.048
TOQEC	32,322.720	41.405	32.685	1,979.682	19,235.473	23,813.933
DOQEC	33,263.033	41.278	36.181	1,857.891	18,408.974	23,838.348
EOQEC	35,258.522	41.896	33.176	1,817.756	19,335.446	23,050.207
PSO	38,761.324	41.275	31.362	1,902.840	16,657.568	26,044.121
Total average	34,377.002	41.614	37.777	1,921.310	18,519.559	24,509.932

This simulation determines the optimal policy and reduces the gap between the number of orders and the sales account. The order of the MRP electronic components and the time control of the electronic components TOQ are not suitable, since they have the greatest difference (41.280) between the quantity of the order of the model and the sales invoice. Variable control loudness 83.262 and datasets 54.903 for the number of electronic components for environmental monitoring, which are relatively small differences. However, the proposed algorithm for an artificial bee colony has the slightest difference. We find that the proposed model is the optimal listing policy for the supply chain of medical services with the highest net profit. As a result of the simulation, the raw materials for electronic components of an algorithm for optimizing a swarm of particles are the most efficient method for managing the stocks of electronic components in the supply chain. The algorithm of an artificial bee family order organizes a predefined set of lists. The particle swarm optimization algorithm is usually a problem with an additional list. However, drug orders vary considerably. In addition, medication must control many elements. An artificial bee colony algorithm may be a suitable method for characterization.

23.7 Conclusion

The purpose of this work is to analyze the effective method of accounting for control of warehouses of electronic components in the supply chain of raw materials. We analyze the supply chain of raw materials and carry out modeling and simulation. List policy is an important factor in determining the time and quantity of the order. It is also important to manage optimal supply chain benefits. Therefore, in order to increase profits, it is necessary to reduce the trade-off between consumption and control. This letter suggests list strategies using a particle swarm optimization algorithm. The proposed particle optimization algorithm calculates the optimal order from the existing stock at the expected standard time. We compare order quantity variables, economical order quantity for electronic components, order duration, order quantity for removal of electronic components, amount of environmental control, and an algorithm for an artificial bee colony. The simulation results show the effectiveness of orders related to the remaining orders and the specified number of orders. The particle swarm optimization algorithm satisfies both conditions, and the raw material supply chain for electronic components is a useful way to manage electronic component storage policies. The limitations of this study are as follows. It is difficult to imagine the number of distribution centers for electronic components and electronic components. In addition, we did not take into account the characteristics of demand.

References

1. Nagurney A, Masoumi AH, Yu M (2012) Supply chain network operations management of a blood banking system with cost and risk minimization. CMS 6(2):205–231
2. Yadav AS, Swami A (2018) Integrated supply chain model for deteriorating items with linear stock dependent demand under imprecise and inflationary environment. Int J Procure Manag 11(6):684–704
3. Yadav AS, Swami A (2018) A partial backlogging production-inventory lot-size model with time-varying holding cost and weibull deterioration. Int J Procure Manag 11(5):639–649
4. Yadav AS, Swami A, Kumar S (2018) A supply chain inventory model for decaying items with two ware-house and partial ordering under inflation. Int J Pure Appl Math 120(6):3053–3088
5. Yadav AS, Swami A, Kumar S (2018) An inventory model for deteriorating items with two warehouses and variable holding cost. Int J Pure Appl Math 120(6):3069–3086
6. Yadav AS, Swami A, Kumar S (2018) Inventory of electronic components model for deteriorating items with warehousing using genetic algorithm. Int J Pure Appl Math 119(16):2955–2966
7. Yadav AS, Johri M, Singh J, Uppal S (2018) Analysis of green supply chain inventory management for warehouse with environmental collaboration and sustainability performance using genetic algorithm. Int J Pure Appl Math 118(20):155–161
8. Yadav AS, Kumar S (2017) Electronic components supply chain management for warehouse with environmental collaboration & neural networks. Int J Pure Appl Math 117(17):169–177
9. Yadav AS, Taygi B, Sharma S, Swami A (2017) Effect of inflation on a two-warehouse inventory model for deteriorating items with time varying demand and shortages. Int J Procure Manag 10(6):761–775
10. Yadav AS, Mahapatra RP, Sharma S, Swami A (2017) An inflationary inventory model for deteriorating items under two storage systems. Int J Econ Res 14(9):29–40
11. Yadav AS, Sharma S, Swami A (2017) A fuzzy based two-warehouse inventory model for non instantaneous deteriorating items with conditionally permissible delay in payment. Int J Control Theory Appl 10(11):107–123
12. Yadav AS (2017) Analysis of supply chain management in inventory optimization for warehouse with logistics using genetic algorithm. Int J Control Theory Appl 10(10):1–12
13. Yadav AS, Swami A, Kher G, Kumar S (2017) Supply chain inventory model for two warehouses with soft computing optimization. Int J Appl Bus Econ Res 15(4):41–55
14. Yadav AS, Mishra R, Kumar S, Yadav S (2016) Multi objective optimization for electronic component inventory model & deteriorating items with two-warehouse using genetic algorithm. Int J Control Theory Appl 9(2):881–892
15. Yadav AS (2017) Modeling and analysis of supply chain inventory model with two-warehouses and economic load dispatch problem using genetic algorithm. Int J Eng Technol 9(1):33–44
16. Yadav AS, Swami A, Kher G (2018) Particle Swarm optimization of inventory model with two-warehouses. Asian J Math Comput Res 23(1):17–26
17. Yadav AS, Maheshwari P, Swami A, Pandey G (2018) A supply chain management of chemical industry for deteriorating items with warehouse using genetic algorithm. Selforganizology 5(1–2):41–51
18. Yadav AS (2017) Analysis of seven stages supply chain management in electronic component inventory optimization for warehouse with economic load dispatch using GA And PSO. Asian J Math Comput Res 16(4):208–219
19. Yadav AS, Garg A, Gupta K, Swami A (2017) Multi-objective genetic algorithm optimization in Inventory model for deteriorating items with shortages using supply chain management IPASJ. Int J Comput Sci 5(6):15–35
20. Yadav AS, Garg A, Swami A, Kher G (2017) A supply chain management in inventory optimization for deteriorating items with genetic algorithm. Int J Emerg Trends Technol Comput Sci 6(3):335–352
21. Yadav AS, Maheshwari P, Garg A, Swami A, Kher G (2017) Modeling & analysis of supply chain management in inventory optimization for deteriorating items with genetic algorithm and particle swarm optimization. Int J Appl Innov Eng Manag 6(6):86–107

22. Yadav AS, Garg A, Gupta K, Swami A (2017) Multi-objective particle swarm optimization and genetic algorithm in Inventory model for deteriorating items with shortages using supply chain management. Int J Appl Innov Eng Manag 6(6):130–144
23. Yadav AS, Maheshwari P, Swami A, Kher G (2017) Soft computing optimization of two warehouse inventory model with genetic algorithm. Asian J Math Comput Res 19(4):214–223
24. Yadav AS, Swami A, Kher G (2017) Multi-objective genetic algorithm involving green supply chain management. Int J Sci Adv Res Technol 3(9):132–138
25. Yadav AS, Swami A, Kher G (2017) Multi-objective particle swarm optimization algorithm involving green supply chain inventory management. Int J Sci Adv Res Technol 3(9):240–246
26. Yadav AS, Swami A, Pandey G (2017) Green supply chain management for warehouse with particle swarm optimization algorithm. Int J Sci Adv Res Technol 3(10):769–775
27. Yadav AS, Swami A, Kher G, Garg A (2017) Analysis of seven stages supply chain management in electronic component inventory optimization for warehouse with economic load dispatch using genetic algorithm. Selforganizology 4(2):18–29
28. Yadav AS, Maheshwari P, Swami A, Garg A (2017) Analysis of six stages supply chain management in inventory optimization for warehouse with artificial bee colony algorithm using genetic algorithm. Selforganizology 4(3):41–51
29. Yadav AS, Swami A, Gupta CB, Garg A (2016) Analysis of electronic component inventory optimization in six stages supply chain management for warehouse with ABC using genetic algorithm and PSO. Selforganizology. 4(4):52–64
30. Yadav AS, Sharam S, Swami A (2016) Two warehouse inventory model with ramp type demand and partial backordering for weibull distribution deterioration. Int J Comput Appl 140(4):15–25
31. Yadav AS, Swami A, Singh RK (2016) A two-storage model for deteriorating items with holding cost under inflation and genetic algorithms international journal of advanced engineering. Manag Sci 2(4):251–258
32. Singh RK, Yadav AS, Swami A (2016) A two-warehouse model for deteriorating items with holding cost under particle swarm optimization. Int J Adv Eng Manag Sci 2(6):858–864
33. Singh RK, Yadav AS, Swami A (2016) A two-warehouse model for deteriorating items with holding cost under inflation and soft computing techniques. Int J Adv Eng Manag Sci 2(6):869–876
34. Sharma S, Yadav AS, Swami A (2016) An optimal ordering policy for non-instantaneous deteriorating items with conditionally permissible delay in payment under two storage management. Int J Comput Appl 147(1):16–25
35. Yadav AS, Maheshwari P, Swami A (2016) Analysis of Genetic Algorithm and Particle Swarm Optimization for warehouse with Supply Chain management in Inventory control. Int J Comput Appl 145(5):10–17
36. Swami A, Singh SR, Pareek S, Yadav AS (2015) Inventory policies for deteriorating item with stock dependent demand and variable holding costs under permissible delay in payment. Int J Appl Innov Eng Manag 4(2):89–99
37. Swami A, Pareek S, Singh SR, Yadav AS (2015) An inventory model with price sensitive demand, variable holding cost and trade-credit under inflation. Int J Curr Res 7(6):17312–17321
38. Gupta K, Yadav AS, Garg A, Swami A (2015) A binary multi-objective genetic algorithm & PSO involving supply chain inventory optimization with shortages, inflation. Int J Appl Innov Eng Manag 4(8):37–44
39. Yadav AS, Gupta K, Garg A, Swami A (2015) A Soft computing optimization based two warehouse inventory model for deteriorating items with shortages using genetic algorithm. Int J Comput Appl 126(13):7–16
40. Yadav AS, Swami A (2014) Two-warehouse inventory model for deteriorating items with ramp-type demand rate and inflation. Am J Math Sci 3(1):137–144
41. Yadav AS, Swami A (2013) Effect of permissible delay on two-warehouse inventory model for deteriorating items with shortages. Int J Appl Innov Eng Manag 2(3):65–71
42. Yadav AS, Swami A (2013) A two-warehouse inventory model for decaying items with exponential demand and variable holding cost. Int Inventive Eng Sci 1(5):18–22

Chapter 24
Interpretive Structural Modeling to Understand Factors Influencing Buying Behavior of Air Freshener

Deepti Aggrawal, Jagvinder Singh, Anurag Kumar and Adarsh Anand

Abstract In order to gain "loyalty", a firm has to maintain "quality" that defines acceptability of its offerings. In the era of globalization, all industries face cutting-edge competition and it becomes difficult to survive for less qualitative products. This paper is an attempt to study attributes preferred by air freshener buyers. A multi-scaling technique, interpretive structural modeling (ISM), has been applied to understand and find the contextual relationship amongst the various attributes under study. Furthermore, MICMAC classification has been done to determine the autonomous, dependent, linkage and independent nature of the factors. Out of 14 attributes, only 5 comes out to be independent which are the deriving attributes for the rest of attributes. The findings can also help company policymakers with understanding the interrelated attributes associated with trustworthiness in the context of air freshener industry and implement them in effective strategic planning. This study has been carried out in one of the metropolitan cities of India and results obtained are significant.

Keywords Air freshener product · Interpretive structural modeling (ISM) · Matriced' Impacts Croise's Multiplication Appliqúee a UN Classment

D. Aggrawal · J. Singh
University School of Management and Entrepreneurship, DTU, Delhi, India
e-mail: deepti.aggrawal@gmail.com

J. Singh
e-mail: jagvinder.singh@dtu.in

A. Kumar (✉) · A. Anand
Department of O.R, University of Delhi, Delhi, India
e-mail: anuragkumar.du.aor@gmail.com

A. Anand
e-mail: adarsh.anand86@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_24

24.1 Introduction

With the advent of globalization which refers to the Borderless world, a product or service can be accessed anywhere in the world. In the era of Internet, it becomes imperative for the company to keep on studying the consumer behavior in continuous manner. The moment consumer feels dissatisfied, so they start switching from one company to another company. In order to keep the consumer base intact, organizations tend to adopt various strategies to attract customers, for example, substitution approach, guarantee, discounts, etc., and try to hold their buyers with them for a longer period of time. There is immense accessibility of the items in the market; in this way, it winds up noticeably troublesome for the organization to distinguish the critical traits of customers for purchasing a specific item. For example, some arrangement of attributes like cost and configuration can be essential for a couple of customers; however, on the other hand, quality and accessibility can be vital for some different customers. These attributes assume a pivotal part and have a genuine effect on the basic decision-making of the customers. However, this investigation is mainly focused on the attributes for purchasing an air freshener in India, while stressing factors like quality, cost, accessibility and so forth. According to a survey conducted for this study, "Quality" can be considered as the most vital characteristic for most of the customers in India [9]. According to W.A. foster, "Quality is never an accident; it is always the result of high intention, sincere effort, intelligent direction and skillful execution. It represents the wise choice of many alternatives" [2]. Along these lines, quality assumes an imperative part in gaining consumer loyalty and it encourages the buyers to buy and repurchase the items with the goal that the organization can upgrade their business and make benefits [6].

Air fresheners are consumer products that lie in the category of cosmetics, used in homes or commercial places that typically emit fragrance. Being a standout amongst the most crowded nation on the planet, it serves a huge marketplace for the companies which deals in the air freshener industry. Customer needs today are quite dynamic and challenging due to the presence of high competition in the market. In case of air freshener, demand of the customers goes beyond from good fragrance to air purification and pleasant environment through organic, natural and environment-friendly products. Air freshener industry works on the strategic reconciliation of market requirements and the operational capabilities of the firm. Therefore, it requires consistent product developments and innovation to attract consumers through aggressive campaigns and advertisement. Organizations are urging Indian buyers to use air fresheners in their day-to-day lives to enhance air cleanliness. Indeed, Delhi Metro prepares the advertisement which has been placed inside the metro coaches "Use antiperspirant to keep the surroundings hygienic". Nowadays, it has been turned out to be basic practice to utilize surrounding smells in different shopping spots to keep the environment wonderful and it has some good effects on the spending habit and emotions as well [5, 7, 8]. Various forms of air freshener available in the market range from sprays, candles, oils and plug-ins. These air care products are available in a variety of fragrances like lavender, jasmine, rose, sandalwood, lemon, apple, etc.

Indian air care market is largely divided into three segments: room fresheners, bathroom fresheners and car fresheners. Due to huge demand of car fresheners in India, the car freshener market considered to be the most successful market in the context of air freshener. In India, the market for room fresheners is growing at a stable rate, whereas according to "**India Air Care Market Outlook, 2022**" air freshener market for cars is expected to see a Compounded Annual Growth Rate (CAGR) of more than 15% over next 5 years.

Today, the Indian market for air fresheners witnessed the growing importance of aromatherapy in homes, and consumer preference for having the fragrant environment. Aromatherapy is getting very popular among Indian consumer because it has diverse applications, including speed up the healing process, ease depression and strengthen the immune system. Manufacturers are trying each and every possible effort to make the consumers aware about the products they use through different media; however, it still has a long way to go. For the time being, only Indian metro cities and urban areas are generating most of the revenue for the air freshener industry, but the rural penetration is almost negligible because the type of population resides in rural India is comparatively rigid in accepting any product belongs to cosmetic Industry but this can present ample opportunities to the air freshener industry. Therefore, in this work, our main objective was to identify the contextual relationship among the various attributes related to air freshener industry. A combination of interpretive structural modeling (ISM) and MICMAC analysis has been applied for the successful execution of this study.

24.2 Research Methodology

ISM is the technique which is used for identification and encapsulating relationship among the distinct variables. Hence, this methodology gives a clear picture of complex real-life problems so that the managers, researchers, and individuals can take some relevant actions by interpreting the results given by ISM and MICMAC.

ISM was first developed by Warfield in 1974 though the applications of ISM have been used many times in past [15, 16, 17] used ISM in different fields of the supply chain management. Attri et al. in [3] described the merits and demerits of using ISM methodology. The application of ISM technique can be found in various diverse fields of the supply chain and operations management [4, 14], whereas Mishra et al. [13] incorporated the ISM technique along with some other MCDM techniques to identify the deriving factors in the context of international manufacturing network. Though ISM does not provide any statistical validation of the study, the results are efficient and easy to interpret for firm managers and experts. Lamba and Singh [12] applied ISM with other MCDM techniques to identify key enablers in the domain of supply chain management.

Air care industries are incorporating the notion of "free samples" offer for the promotional purpose so that the companies can get to know how the sample is behaving and diffusion is taking place into the market. Here, positive word-of-mouth of free samples can develop and create artificial demand before the launching of the actual product into the market. All these efforts put in before launching have positive effect on the diffusion and adoption of the product in highly competitive market [1]. Kannan et al. [11] discussed the steps contained in the methodology of ISM.

(a) Analyze the problem. Distinguish the different factors which are important to this particular study. These factors can be found using various MCDM techniques available. In the present study, all the 14 factors as mentioned and described in Table 24.1 are taken into consideration which is found important by the survey done in the region of Delhi NCR.
(b) Once the variables are identified, contextual relationship is examined.
(c) Development of structural self-interaction matrix (SSIM). This matrix performs pair-wise comparison and gives a relationship among various factors.
(d) Reachability matrix is formed through SSIM matrix; thereafter, transitivity step is performed thoroughly. Different levels created through previously obtained SSIM matrix.
(e) Based on relationship among factors assigned in reachability matrix the digraph is drawn.

Table 24.1 Important attributes for purchasing an air freshener

S.No	Attributes	Definition
C1	Pricing	Refers to the amount required as payment for something be it product or service
C2	Packaging	Packaging basically a marketing strategy to make product attractive by the way it looks from outside
C3	Fragrance	A pleasant smell out of perfume, flower, etc.
C4	Long lasting	Anything which is continuing for a long period of time
C5	Variety	Variety means variant of diversity the absence of uniformity
C6	Availability	Availability defined as the presence of that particular product and service in the market
C7	Brand	Brand is just an identification mark by any well-known brand or regular brand
C8	Aromatherapy	It is basically a process to insert nature into products or services in order to market them
C9	Odor control	It is defined as unpleasant smell out of anything
C10	Disinfectant	Any process to destroy the microbes
C11	Discounts	A deduction from the regular cost of something
C12	Advertisements	In order to make the product or service, popular advertisement is done through media allocation problem
C13	Recommendations	To give suggestions in the favor of something or someone
C14	Prestige	Refers to good reputation earned with the aspect of time and word-of-mouth

(f) Various factors have been clustered through MICMAC analysis to validate ISM findings.

24.3 Case Study

A survey was conducted, and the sample size of 150 respondents was selected of different age groups and gender. For this, a questionnaire was designed and the study was done through personalized interviews in the region of Delhi and NCR. It was ensured that the respondents understand the interrelationships among the attributes, and thereafter their opinions have been collected. Further, the respondent's opinions are considered as the input and found to have the practical validity.

Structural self-interaction matrix (SSIM)

The contextual relationship between the two selected factors, i.e. factors i to factor j in a pair-wise comparison is directed using the following four symbols:

(a) V denotes how factor *i* is related to factor *j* (i.e. "factor *i* influences factor *j*").
(b) A denotes how factor *j* is related to factor *i* (i.e. "factor *j* influences factor *i*").
(c) X denotes the relation from both directions (i.e. "both factors *i* and *j* influence each other").
(d) O denotes factors having no relation in between (i.e. "factors *i* and *j* are not related").

Based on above-mentioned contextual relationships, further the SSIM matrix is formed. To avoid any kind of ambiguities, SSIM should be examined by the experts from industry or academic, and then only final SSIM should be formed; otherwise, chances of flaws would be high in the SSIM matrix. Refer Fig. 24.1 to understand the SSIM matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14
C1	X	V	A	A	V	V	X	V	A	A	A	V	V	V
C2		X	A	A	A	A	A	A	A	A	A	A	A	A
C3			X	X	V	V	V	X	X	X	V	V	V	V
C4				X	V	V	V	A	X	A	V	V	V	V
C5					X	A	A	A	A	A	A	V	A	A
C6						X	A	A	A	A	V	V	V	V
C7							X	V	A	A	A	V	V	V
C8								X	X	X	V	V	V	V
C9									X	X	V	V	V	V
C10										X	V	V	V	V
C11											X	V	V	V
C12												X	A	A
C13													X	V
C14														X

Fig. 24.1 Structural self-interactive matrix (SSIM)

Step 1: Initial Reachability Matrix: In this step, an initial reachability matrix is formed through structural self-interaction matrix which was obtained in the previous step. Here, SSIM is transformed into a binary matrix called an initial reachability matrix by replacing these four symbols (i.e. V, A, X or O) of SSIM with 1 s or 0 s. The replacement procedure is given below:

(a) If the entry (i, j) in the SSIM is V, then entry (i, j) in the reachability matrix gets the value 1 and the entry (j, i) gets the value 0.
(b) If the entry (i, j) in the SSIM is A, then entry (i, j) in the matrix gets the value 0 and the (j, i) entry gets the value 1.
(c) If the entry (i, j) in the SSIM is X, then both the entries (i, j) and (j, i) in the matrix get the value 1.
(d) If the entry (i, j) in the SSIM is O, then both the entries (i, j) and (j, i) in the matrix get the value 0.

After following the above-mentioned steps, the initial reachability matrix as presented in Fig. 24.2 can be obtained. Thereafter, final reachability matrix is obtained by checking the initial reachability matrix for transitive links.

Step 2: Using these steps, SSIM can be transformed into reachability matrix.

(a) If the entry (i, j) in the SSIM is V, then entry (i, j) in the reachability matrix gets the value 1 and the entry (j, i) gets the value 0.
(b) If the entry (i, j) in the SSIM is A, then entry (i, j) in the matrix gets the value 0 and the (j, i) entry gets the value 1.
(c) If the entry (i, j) in the SSIM is X, then both the entries (i, j) and (j, i) in the matrix get the value 1.
(d) If the entry (i, j) in the SSIM is O, then both the entries (i, j) and (j, i) in the matrix get the value 0.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14
C1	1	1	0	0	1	1	1	1	0	0	0	1	1	1
C2	0	1	0	0	0	0	0	0	0	0	0	0	0	0
C3	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C4	1	1	1	1	1	1	1	0	1	0	1	1	1	1
C5	0	1	0	0	1	0	0	0	0	0	0	1	0	0
C6	0	1	0	0	1	1	0	0	0	0	1	1	1	1
C7	1	1	0	0	1	1	1	1	0	0	0	1	1	1
C8	0	1	1	1	1	1	0	1	1	1	1	1	1	1
C9	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C10	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C11	1	1	0	0	1	0	1	0	0	0	1	1	1	1
C12	0	1	0	0	0	0	0	0	0	0	0	1	0	0
C13	0	1	0	0	1	0	0	0	0	0	0	1	1	1
C14	0	1	0	0	1	0	0	0	0	0	0	1	0	1

Fig. 24.2 Initial reachability matrix

Final reachability matrix

The concept of transitivity becomes important to achieve final reachability matrix. Transitivity states that if an element A is related to B and somehow B is related to C, then it can be concluded that A is certainly related to C. Using this approach, a matrix as represented through Fig. 24.3 can be obtained.

Conical Matrix

Next, to obtain a conical form as shown in Fig. 24.4, factors are clustered at the same level across the columns and rows of the final reachability matrix. Adding up the number of ones in the rows provides the driving power. On the similar lines, summing up the number of ones in the columns provides the dependence of the factors.

Step 3: Level partitions: Based on final reachability matrix, both reachability and the antecedent sets are obtained for every parameter. Next, the intersection of these sets is obtained to determine the different levels to which the factor belongs. If both reachability and the antecedent sets found to be identical, then that specific factor secures the topmost position in the ISM progression. If topmost level parameter is obtained, it will be dropped from the list for the next iteration. A similar procedure is performed again so as to identify the factors for the following level. Following this procedure, levels for all the parameters are obtained.

In this particular study of air freshener, reachability set, antecedent set, intersection set and levels of different parameters considered are shown to be deduced in iterations (given in the form of Fig. 24.5a–g).

Step 4: Build digraph on the basis of relationship obtained in reachability matrix after removing transitive links.

Initial digraph which includes transitive links is obtained from the conical form and again digraph is formed after eliminating the indirect links. However, a digraph is basically the representation of the elements and their interdependence on each

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14
C1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C2	0	1	0	0	0	0	0	0	0	0	0	0	0	0
C3	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C4	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C5	0	1	0	0	1	0	0	0	0	0	0	1	0	0
C6	1	1	0	0	1	1	1	0	0	0	1	1	1	1
C7	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C8	1	1		1	1	1	1	1	1	1	1	1	1	1
C9	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C10	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C11	1	1	0	0	1	1	1	1	0	0	1	1	1	1
C12	0	1	0	0	0	0	0	0	0	0	0	1	0	0
C13	0	1	0	0	1	0	0	0	0	0	0	1	1	1
C14	0	1	0	0	1	0	0	0	0	0	0	1	0	1

Fig. 24.3 Final reachability matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	D.P
C1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**14**
C2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	**1**
C3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**14**
C4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**14**
C5	0	1	0	0	1	0	0	0	0	0	0	1	0	0	**3**
C6	1	1	0	0	1	1	1	0	0	0	1	1	1	1	**9**
C7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**14**
C8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**14**
C9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**14**
C10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**14**
C11	1	1	0	0	1	1	1	1	0	0	1	1	1	1	**10**
C12	0	1	0	0	0	0	0	0	0	0	0	1	0	0	**2**
C13	0	1	0	0	1	0	0	0	0	0	0	1	1	1	**5**
C14	0	1	0	0	1	0	0	0	0	0	0	1	0	1	**4**
Dep	**9**	**14**	**7**	**7**	**12**	**9**	**9**	**8**	**7**	**7**	**9**	**13**	**10**	**11**	

Fig. 24.4 Conical matrix

other. ISM model is developed which does not contain any transitive links as shown in Fig. 24.6.

Step 5: After obtaining the driving power and dependence, MICMAC analysis is conducted. MICMAC analysis is a graphical representation and validation of ISM findings and not absolute in nature; it only gives the direction to the results obtained from ISM up to some extent. It basically divides the graph into four distinct groups such as autonomous parameter (I), dependent parameters (II), linkage parameters (III) and independent parameters (IV).

The structure of the MICMAC can be designed using the driving and dependence power of all the attributes that were calculated while forming the conical form of the final reachability matrix. In this study, we have considered dependence on x-axis, whereas deriving power on y-axis and there is no set rule for making MICMAC analysis.

1. **Autonomous parameters**: This quadrant contains all those parameters which exhibit weak deriving power as well as weak dependence.
2. **Linkage parameters**: This quadrant contains all those parameters which exhibit strong driving power and strong dependence. Set of attributes which lie in this quadrant is important as they show high deriving power.
3. **Dependent parameters**: This quadrant contains all those parameters which exhibit weak drive power but strong dependence. These sets of attributes are considered to be less important.

(a)

Variable Names	Reachability Set	Antecedents Set	Intersection Set	Level
c1	c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	0
c2	c2	c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c2	1
c3	c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c4	c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c5	c2 c5 c12	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c13 c14	c5	0
c6	c1 c2 c5 c6 c7 c11 c12 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c11	0
c7	c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	0
c8	c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c7 c8 c9 c10 c11	c1 c3 c4 c7 c8 c9 c10 c11	0
c9	c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c10	c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c11	c1 c2 c5 c6 c7 c8 c11 c12 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c8 c11	0
c12	c2 c12	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c12	0
c13	c2 c5 c12 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c13	0
c14	c2 c5 c12 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13 c14	c14	0

(b)

Variable Names	Reachability Set	Antecedents Set	Intersection Set	Level
c1	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	0
c3	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c4	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c5	c5 c12	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c13 c14	c5	0
c6	c1 c5 c6 c7 c11 c12 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c11	0
c7	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	0
c8	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c7 c8 c9 c10 c11	c1 c3 c4 c7 c8 c9 c10 c11	0
c9	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c10	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c11	c1 c5 c6 c7 c8 c11 c12 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c8 c11	0
c12	c12	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14	c12	1
c13	c5 c12 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c13	0
c14	c5 c12 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13 c14	c14	0

Fig. 24.5 **a** Level partitions (Iteration 1), **b** Level partitions (Iteration 2), **c** Level partitions (Iteration 3), **d** Level partitions (Iteration 4), **e** Level partitions (Iteration 5), **f** Level partitions (Iteration 6), **g** Level partitions (Iteration 7)

(c)

Variable Names	Reachability Set	Antecedents Set	Intersection Set	Level
c1	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	0
c3	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c4	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c5	c5	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c13 c14	c5	*1*
c6	c1 c5 c6 c7 c11 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c11	0
c7	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	0
c8	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c7 c8 c9 c10 c11	c1 c3 c4 c7 c8 c9 c10 c11	0
c9	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c10	c1 c3 c4 c5 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c11	c1 c5 c6 c7 c8 c11 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c8 c11	0
c13	c5 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c13	0
c14	c5 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13 c14	c14	0

(d)

Variable Names	Reachability Set	Antecedents Set	Intersection Set	Level
c1	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	0
c3	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c4	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c6	c1 c6 c7 c11 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c11	0
c7	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	0
c8	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c7 c8 c9 c10 c11	c1 c3 c4 c7 c8 c9 c10 c11	0
c9	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c10	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13 c14	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c11	c1 c6 c7 c8 c11 c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c8 c11	0
c13	c13 c14	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c13	0
c14	c14	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13 c14	c14	*1*

(e)

Variable Names	Reachability Set	Antecedents Set	Intersection Set	Level
c1	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	0
c3	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c4	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c6	c1 c6 c7 c11 c13	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c11	0
c7	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	0
c8	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c1 c3 c4 c7 c8 c9 c10 c11	c1 c3 c4 c7 c8 c9 c10 c11	0
c9	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c10	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c11	c1 c6 c7 c8 c11 c13	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c8 c11	0
c13	c13	c1 c3 c4 c6 c7 c8 c9 c10 c11 c13	c13	*1*

Fig. 24.5 (continued)

(f)

Variable Names	Reachability Set	Antecedents Set	Intersection Set	Level
c1	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	1
c3	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c4	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c6	c1 c6 c7 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c11	1
c7	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	1
c8	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c7 c8 c9 c10 c11	c1 c3 c4 c7 c8 c9 c10 c11	0
c9	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c10	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c3 c4 c7 c8 c9 c10	c1 c3 c4 c7 c8 c9 c10	0
c11	c1 c6 c7 c8 c11	c1 c3 c4 c6 c7 c8 c9 c10 c11	c1 c6 c7 c8 c11	1

(g)

Variable Names	Reachability Set	Antecedents Set	Intersection Set	Level
c3	c3 c4 c8 c9 c10	c3 c4 c8 c9 c10	c3 c4 c8 c9 c10	1
c4	c3 c4 c8 c9 c10	c3 c4 c8 c9 c10	c3 c4 c8 c9 c10	1
c8	c3 c4 c8 c9 c10	c3 c4 c8 c9 c10	c3 c4 c8 c9 c10	1
c9	c3 c4 c8 c9 c10	c3 c4 c8 c9 c10	c3 c4 c8 c9 c10	1
c10	c3 c4 c8 c9 c10	c3 c4 c8 c9 c10	c3 c4 c8 c9 c10	1

Fig. 24.5 (continued)

4. **Independent parameters**: This quadrant contains all those parameters which show strong drive power but weak dependence power which clearly states that these sets of attributes are the most important.

The classification using MICMAC analysis as done by Anand and Bansal [2] has been utilized to have the categorization of all the attributes under consideration into aforesaid four categories. Figure 24.7 shows the result of this analysis.

24.4 Managerial Implications

This particular research work provides significant and relevant insights about the various factors which are important and company should give emphasis upon in the context of air freshener industry. The selected set of attributes is taken from various sources such as literature, expert's opinion, etc. and not all fourteen (14) attributes are equally crucial for the growth of the air freshener industry. This research work provides significant analysis and interrelationships of the driving factors which can be taken and extended in future research works also.

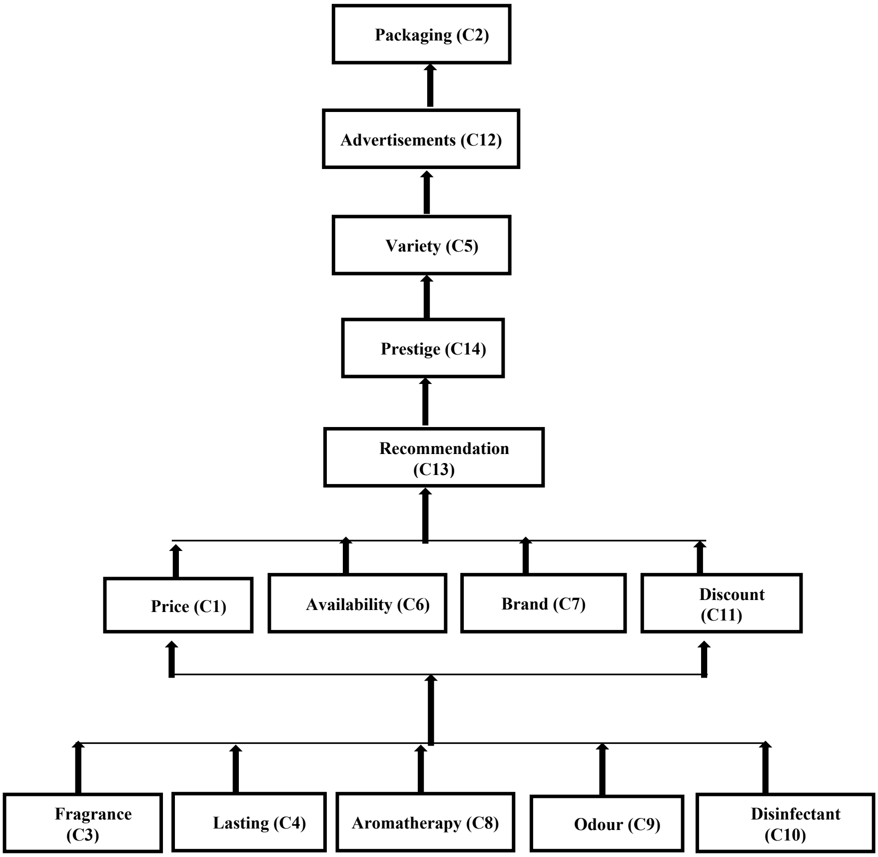


Fig. 24.6 Digraph of attributes for air freshener

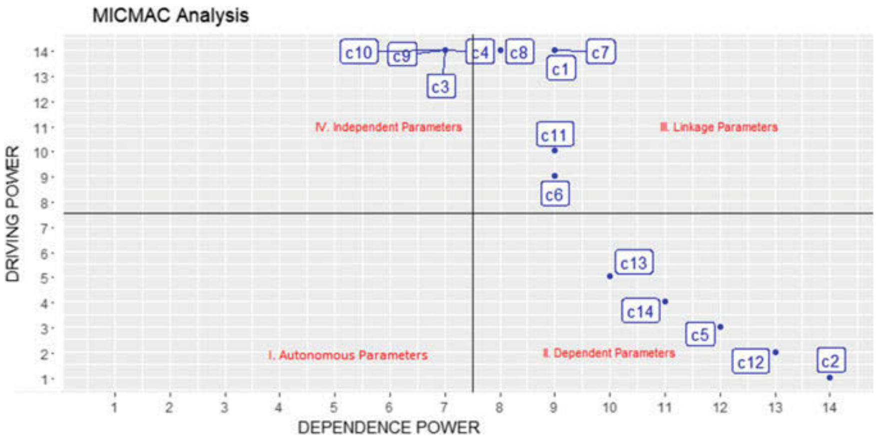


Fig. 24.7 MICMAC analysis

24.5 Conclusion and Future Research Scope

This research tries to justify the objective of the study which revolves around the attributes that drive an air freshener industry and to identify the inter-relationship among those attributes using ISM and MICMAC analysis. This entire analysis if used properly can generate more revenue and business value for any particular firm. With the help of these influenced and influential attributes, some important insights can be gained and used further in order to support decision-making. Therefore, using ISM and MICMAC analysis for qualitative attributes of an air freshener, it can be concluded that all the considered attributes of the air freshener are significant but fragrance, lasting, aromatherapy, odor, and disinfectant are the attributes which provide a qualitative advantage and increases acceptability of the product in the market.

Using MICMAC analysis, apart from the attributes C3, C4, C9 and C10, all the other attributes are clustered under the category of linkage parameters and dependent parameters. It clearly conveys that the factors considered in the study are detrimental to the success of the air freshener industry and must be significantly taken into consideration to grab the competitive advantage.

The analysis conducted provides the key enablers of an air freshener industry by creating a hierarchical representation of attributes and classifying them under four relatively diverse clusters based on their dependence as well as driving power. This study successfully builds the basis by providing the key drivers of an air freshener industry and this current research can also be further extended by incorporating various MCDM techniques like AHP, TOPSIS, and Fuzzy AHP to prioritize the identified factors. Also, this can be integrated with quantitative mathematical models to optimize the overall cost while considering the qualitative parameters in the form of weights obtained from these ranking methods. The industries like air freshener should be continuously studied since the products belonging to this industry are choice-based products and customers can quickly switchover in case they are not satisfied. These industries need to focus on these enabling factors to attract new customers and to ensure the loyalty of existing customers.

References

1. Anand A, Singh O, Agarwal M, Aggarwal R (2014) Modeling adoption process based on awareness and motivation of consumers. In: Proceedings of 3rd international conference on reliability, infocom technologies and optimization, October 2014. IEEE, pp 1–6
2. Anand A, Bansal G (2017) Interpretive structural modeling for attributes of software quality. J Adv Manag Res 14(3):256–269
3. Attri R, Dev N, Sharma V (2013) Interpretive structural modelling (ISM) approach: an overview. Res J Manag Sci 2319:1171
4. Bag S (2016) Building theory of green procurement using fuzzy TISM and fuzzy DEMATEL methods. Int J Appl Manag Sci Eng (IJAMSE) 3(2):21–49
5. Bone PF, Ellen PS (1999) Scents in the marketplace: explaining a fraction of olfaction. J Retail 75(2):243–262

6. Caruana A (2002) Service loyalty: the effects of service quality and the mediating role of customer satisfaction. Eur J Mark 36(7/8):811–828. https://doi.org/10.1108/03090560210430818
7. Chebat JC, Michon R (2003) Impact of ambient odors on mall shoppers' emotions, cognition, and spending: a test of competitive causal theories. J Bus Res 56(7):529–539
8. Craik KH (1973) Environmental psychology. Annu Rev Psychol 24:403–422
9. Garvin DA (1984) Product quality: an important strategic weapon. Bus Horiz 27(3):40–43
10. India Air Care Market Outlook, 2022. https://www.researchandmarkets.com/research/hqt3kk/india_air_care
11. Kannan G, Pokharel S, Kumar PS (2009) A hybrid approach using ISM and fuzzy TOPSIS for the selection of reverse logistics provider. Resour Conserv Recycl 54(1):28–36
12. Lamba K, Singh SP (2018) Modeling big data enablers for operations and supply chain management. Int J Logist Manag 29(2):629–658
13. Mishra S, Singh SP, Johansen J, Cheng Y, Farooq S (2018) Evaluating indicators for international manufacturing network under circular economy. Manag Decis
14. Pfohl HC, Gallus P, Thomas D (2011) Interpretive structural modeling of supply chain risks. Int J Phys Distrib Logist Manag 41(9):839–859
15. Raj T, Shankar R, Suhaib M (2008) An ISM approach for modeling the enablers of flexible manufacturing system: The case for India. Int J Prod Res 46. https://doi.org/10.1080/00207540701429926
16. Sachdeva N, Singh O, Kapur PK (2015) Modeling critical success factors for adoption of big data analytics project: an ISM-MICMAC based analysis. Commun Dependability Qual Manag 18(4):93–110
17. Thakkar J, Kanda A, Deshmukh SG (2008) Interpretive structural modeling (ISM) of IT-enablers for Indian manufacturing SMEs. Inf Manag Comput Secur 16(2):113–136

Chapter 25
Decision-Making with Temporal Association Rule Mining and Clustering in Supply Chains

Reshu Agarwal

Abstract Timely identification of recently rising patterns is required in business process. Data mining methods are most appropriate for the characterization, valuable examples extraction, and predications which are essential for business support and decision-making. Some research studies have also expanded the use of this idea in inventory management. However, not very many research analyzes have considered the utilization of the data mining approach for supply chain inventory management. In this chapter, two unique cases for supply chain inventory management dependent on cross-selling effect are presented. First, the cross-selling effect in different clusters is characterized as a basis for deciding the significance of items. Second, the cross-selling in different time periods is considered as a criterion for ranking inventory items. An example is devised to approve the outcomes. It is illustrated that by using this modified approach, the ranking of items may get affected resulting in higher profit.

Keywords Clustering · Temporal association rule mining · Cross-selling effect · Data mining · Inventory management · Supply chain management

25.1 Introduction

Supply chain management includes the development of items and services from providers to wholesalers. It includes the progression of data and items between and among supply chain stages to amplify profitability [26]. However, it is outstanding that huge data has been produced and put away on each hub of the entire supply chain, which is expanding tremendously like a snow slide. Going up against such monstrous data, it is troublesome for a venture to discover out the guidelines among providers and clients on the premise of its own business information [27]. There are occurrences where effectiveness in supply chain can be guaranteed by efficiencies in inventory. In spite of the fact that inventory is viewed as an obligation to effective

R. Agarwal (✉)
Amity Institute of Information Technology, Amity University, Noida, India
e-mail: agarwal.reshu3@gmail.com

N. H. Shah and M. Mittal (eds.), *Optimization and Inventory Management*,
Asset Analytics, https://doi.org/10.1007/978-981-13-9698-4_25

supply chain management, supply chain managers directors recognize the need for inventory.

Inventory management is the way toward guaranteeing that an organization dependably has the items it needs available and that it keeps costs as low as could reasonably be expected. Inventory control procedures are utilized by the inventory control association inside the system of one of the fundamental inventory models. Inventory control systems speak to the operational part of inventory management and help to understand the targets of inventory management and control. A few procedures of inventory control are being used and it relies upon the comfort of the firm to receive any of the strategies. What ought to be focused, in any case, is the need to cover all items of inventory and all stages, i.e., from the phase of receipt from suppliers to the phase of their utilization. Historically, ABC analysis has been founded on the foundation of dollar volume [25]. In early ABC analysis, inventory was organized in view of its dollar use, which is unit demand multiplied by unit cost. After some time, overseeing inventory in view of dollar utilization has transformed into a system of overseeing inventory in view of a huge number of criteria. There are various criteria like lead time, criticality, obsolescence, etc. that can be used for characterization of inventories [11, 12]. Flores and Whybark [14] proposed a framework-based methodology. Two criteria can be utilized to make a joint criteria network. However, the methodology is ordinarily hard to utilize when more criteria must be considered. A few distinct criteria like decision-making tools have likewise been utilized for the reason. Cohen and Ernst [11] and Ernst and Cohen [13] have used cluster analysis to gather similar items. Ramanathan [24] built up an optimization model to explain the issue. The significant idea of this model is unequivocally like the possibility of data envelopment analysis (DEA). The model first changes overall criteria estimation into a scalar which is a weighted total of measures under individual criteria. Further, the loads are scaled by DEA. Finally, the items are arranged according to the scores made.

However, a linear optimization was needed for each item. The handling time can be long when the amount of inventory items is tremendous in size of thousands of items in inventory. Zhou and Fan [33] proposed an expanded form of Ramanathan's model by consolidating some adjusting highlights. They utilized two arrangements of weights that are most positive and slightest great for everything. Ng [22] proposed a straightforward model for optimization. The model changes overall criteria proportions of an inventory item into a scalar score. With authentic change, this model procures the scores of inventory items without a linear optimization. Besides, this model presentation was versatile as it could without quite a bit of a stretch organize additional information from decision-makers for inventory characterization. Regardless, Ng model prompts a condition where the score of everything was free of the weights got from the model. That is, the weights do not have any part to decide significant score of everything and this may incite a situation where an item is inappropriately characterized. Ozan and Mustafa [23] proposed a stock gathering structure dependent on the fuzzy AHP to help a reasonable multi-criteria inventory arrangement. However, the criteria, for instance, cross-selling effect portrayed by Anand et al. [8]

ought to likewise be viewed as when requesting inventory items. In this section, two cases are examined for characterization of inventories with cross-selling effect.

Firstly, the cross-selling inside each group can be characterized as a paradigm for assessing the significance of items. It is proved that a lot of items change their positions in the ranking list of ABC classification. Also, due to cross-selling effect, many items moved to A grouping, in spite of the fact that they previously do not belong to it. Moreover, various items that belong to C grouping have been advanced into higher grouping in view of their high cross-selling effect. Furthermore, the cross-selling with time periods into association rules can be described for evaluating the importance of items. It is proved that different items that for the most part had a grouping with the B or C group were moved into the A group due to cross-selling effect. A model has appeared to endorse the results.

25.2 Background

In this section, the topics such as association rules, temporal association rules, clustering, and the classification of inventories were explained.

Association Rules

Association rule mining, a champion among the most basic and all around explored procedures of data mining, was first exhibited by Agrawal et al. [1]. It means to find interesting relationships, patterns among transaction databases or data archives.

Example: In an online book shop, there are ceaselessly several experiences after you buy two or three books, for example, when you have purchased the book Operating System, a summary of related books, for example, Let us C 50%, Data Structure 30% will be appearing to you as suggestion for in addition getting. Two association rules can be found in this example. The main communicates that when the book Operating System is obtained, 50% of the time the book Let us C is purchased together and the second one communicates that when the book Operating System is acquired, 30% of the time the book Data Structure is purchased. The found association rules can be utilized by the executives to expand the viability related to promoting, advertising, stock, and stock area on the floor.

Temporal Association Rules

Temporal association rules can be more profitable and educational than fundamental association rules [18, 19]. For instance, as opposed to the fundamental association rule {Christmas} ⇒ {cake}, mining from the temporal data we can get all the more understanding guideline that the support of {Christmas} ⇒ {cake} ricochets to 60% during winter season normally. This fact clearly suggests that retailers can make increasingly powerful progression system by using temporal association rules.

Clustering

Clustering is the way toward gathering a lot of physical or conceptual items into classes of similar objects, so that objects inside a similar group must be similar to some extent, additionally they ought to be not at all similar to those objects in different groups [15].

Most clustering applications are utilized as a part of market division. For instance, in light of the cost, deposit, and draw instances of the customers, a bank can make a group of customers on the basis of their credits to houses or with various budget designs. For this circumstance, the bank can give a superior administrator, and moreover guarantee that every one of the credits can be recovered.

Classification of Inventories

Inventory classification can empower an association to control its stock by diminishing the proportion of stock they have close eventually growing the stock turnover extent. Both of which influence an organization's appropriation to arrange more proficient and lower its general cost. Generally, ABC approach is used to classify inventory items into three groups. The ABC approach gives a technique for organizing items that will highly influence general stock expense. It moreover gives a way to deal with supply supervisors for recognizing items that require differing controls and oversight. These groupings are as follows:

Class A: These are high-income items that record for 80% of annual sales and 20% of inventory.
Class B: These items represent 15% of annual sales.
Class C: These items represent 5% of annual sales. These are low-volume sales items.

For many items, however, ABC classification is sometimes not suitable for inventory control. Therefore, it is important to consider cross-selling effect, when classifying inventory items. An association-rule-based approach was proposed by Brijs et al. [9] to solve the problem of product assortment in general stores. They developed a product assortment model by considering association rules and cross-selling effect. Further this model was extended to deal with large baskets and category management in practice by Brijs et al. [10]. The biggest disadvantage of this model was that it does not provide relative ranking of selected items. This factor is very important in classification of inventories. Hence, this model was not used to classify inventory items. The strength of relationship between items was considered in the model proposed by Kaku [16]. This model was further extended by Kaku and Xiao [17] considering the factors of cross-selling effect and ABC classification. However, they did not propose a method to classify inventory items that are correlated. Further, this drawback was removed in model proposed by Xiao et al. [31]. They proposed a method to classify correlated items utilizing the idea of loss profit together with cross-selling effect. The inventory items are ranked to find the most profitable item sets. Loss rule was used to arrange items [29, 30]. The loss profit of item/item set is described as the standard for finding the importance of items, in light of which inventory items are classified. They elucidated that to pass judgment on the importance of an item (set), it is not simply by looking advantage it obtains when it is on the rack, yet in addition the loss profit it might take away when it is truant or stock out. Further, this approach was extended for requesting arrangement utilizing association rule mining [2–7, 20, 21]. However, a very little research is done considering interrelationships between items. Hence, how to treat relationship is a test when creating inventory models.

25.3 Mathematical Model

In this section, how to arrange inventory items which are interrelated is explained by using the idea of cross-selling effect.

Case 1: The cross-selling within each cluster can be defined as a criterion for evaluating the importance of item. The large items and small items in the clustering algorithm are used to minimize the cost [28]. The algorithm as described by Wang et al. [28] is shown in Fig. 25.1.

Further, Yin et al. [32] prescribe another establishment of expected dollar usage (EDU) to rank inventory items according to their importance. To figure the EDU of an item, frequent item set is managed as an interesting item, the dollar usage of which can be processed like an ordinary item. Now, the ranking will be done by considering all frequent item sets together with individual items.

Consider an item set X in an inventory transaction database Db with support sup(X), then

$$\sup(X) = \frac{|\mathrm{X(t)}|}{\mathrm{D}}$$

where X(t) = {t in D/t contains X} in transaction t.

```
/* Allocation phase */

    (1) while not end of the file do
    (2) read the next transaction;
    (3) allocate t to an existing or a new cluster Ci ;
    (4) write  <t, Ci>;
/* Refinement phase */

    (5) repeat
    (6) not_ moved = true;
    (7) while not end of the file do
    (8) read the next transaction < t, Ci>;
    (9) move t to an existing cluster Cj to minimize Cost
        C;
    (10) if Ci ≠ Cjthen
    (11) write < t, Cj>;
    (12) not_moved=false;
    (13) eliminate any empty cluster;
    (14) until not moved;
```

Fig. 25.1 Outline of clustering algorithm

Now, EDU can be found using the following formula:

$$C_x = \sup(X) \sum\nolimits_{i \in X} p_i$$

where C_x represents EDU of an item set X, p_i is the cost of single item in X, and $\sum_{i \in X} p_i$ is the set's price.

Case 2: The cross-selling within each time period can be defined as a criterion for evaluating the importance of item. The working of algorithm can be explained in the following five steps:

Step 1: First, time periods were used to partition database.
Step 2: The frequent item sets are calculated using association rule mining algorithm.
Step 3: Rank items are beginning with biggest esteem, after calculating EDU of all frequent item sets and of all single items.
Step 4: All frequent item sets are replaced in the ranking list by their contained items, so that their interior order does not change. Further, check the positioning rundown from the earliest starting point to the end and pull back the duplicate items that have shown up for the second time to make every one of the items one of a kind in the rundown.
Step 5: On the basis of new ranking list, classify items according to ABC classification. The new "A" gathering is finished by picking items of the new ranking list from the most beginning stage as far as possible till the total EDU of picked items accomplishes 80% of total dollar usage.

25.4 Numerical Example

In this section, the ranking list of inventory items is prepared on the basis of Yin et al. [32] model parameters. Moreover, EDU has been determined for inventory items in each cluster which was not considered by Yin et al. [32].

Consider the database set D and the item set, I = {a, b, c, d, e, f, g, h, i}. The transaction set TID = {TID1, TID2, TID3, TID4, TID5, TID6} is shown in Table 25.1.

Further, inventory classification is decided by utilizing rules acquired by a priori algorithm after clustering the data. By applying clustering algorithm on transaction database of Table 25.1, clusters C_1 = {TID1, TID2, TID3, TID4} and C_2 = {TID5, TID6} were got. Then a priori algorithm is applied to the two groups. The item sets {a, b, c} and {d, g} are most frequent item sets found in group C_1 and C_2, respectively.

Table 25.1 An inventory transaction database

TID	TID1	TID2	TID3	TID4	TID5	TID6
ITEMS	a, b, c	a, b, c, d	a, b, c, e	a, b, f	d, g, h	d, g, i

Let the costs of items be a = \$4, b = \$5, c = \$3, d = \$5, e = \$1, f = \$1, g = \$8, h = \$2, and i = \$1. By utilizing customary ABC grouping, let the dollar utilization of items be a = \$16, b = \$20, c = \$9, d = \$15, e = \$1, f = \$1, g = \$16, h = \$2, i = \$1. At that point rank things in slipping request beginning with the biggest estimation of dollar utilization; the positioning rundown is (bagdchefi).

Consider cluster C_1. Let minimum support = 2. Table 25.2 shows the generation of all large frequent item sets in cluster C_1.

By positioning these individual items and frequent item sets of Table 25.2 in dropping request beginning with the biggest estimation of EDU, we can get the accompanying list of item sets as follows:

{ab}, {ac}, {abc}, {bc}, {a}, {b}, {c}

As indicated by the proposed calculation, the ranking list of items is obtained by replacing the frequent item sets with their components.

{abacabcbcabc}.

At long last, the ranking list is checked from the earliest starting point to the end. Every rehashed item is pulled back. The ranking list is given below:

{abc}.

Similarly, considering cluster C_2, the ranking list obtained is {dg}.

By applying this approach, stock out of item "a" may come about a bigger misfortune than itself on the grounds that the strong cross-selling relationship with other

Table 25.2 The progress of finding frequent item sets in cluster C_1

Transaction Id	{Items}	Length	Support	EDU	Is it frequent?
One large item set					
1	{a}	1	4	4 * 4 = 16	Y
2	{b}	1	4	4 * 5 = 20	Y
3	{c}	1	3	3 * 3 = 9	Y
4	{d}	1	1	1 * 2 = 2	N
5	{e}	1	1	1 * 1 = 1	N
6	{f}	1	1	1 * 1 = 1	N
Two large item set					
7	{ab}	2	4	4 * (4 + 5) = 36	Y
8	{ac}	2	3	3 * (4 + 3) = 21	Y
9	{bc}	2	3	3 * (5 + 3) = 24	Y
Three large item set					
10	{abc}	3	3	3 * (4 + 5+3) = 24	Y

profitable items. This approach helps stock administrator to perceive high-benefit items in each group, with the goal that he/she wins benefit and effortlessly oversees stocks.

Further, Yin et al. [32] do not calculate EDU for inventory items in each time period. Table 25.3 shows the transaction set TID = {TID1, TID2, TID3, TID4, TID5, TID6, TID7, TID8, TID9, TID10, TID11, TID12}.The item set I consists of {P, Q, R, S, T, U} and the cost of items are P = \$5, Q = \$4, R = \$3, S = \$2, T = \$3, and U = \$1. According to ABC classification, the dollar usages of items are P = \$15, Q = \$32, R = \$18, S = \$14, T = \$15, U = \$3. After ranking items in descending order of dollar usages, we get a ranking list as (QRPTSU).

Let us consider the time period from January to March. Let minimum support = 2. Table 25.4 shows the generation of all large frequent item sets in time span January to March.

The individual items and frequent item sets of Table 25.4 are ranked in descending order of their EDU, to obtain item sets as follows:

{QR}, {Q}, {QS}, {QT}, {QRT}, {QRS}, {R}, {RT}, {T}, {P}, {S},
{QU}, {RS}, {ST}, {TU}, {U}

According to the proposed algorithm, the ranking list is obtained by replacing the frequent item sets with their elements as follows:

{QRQQSQTQRTQRSRRTTPSQURSSTTUU}

At last, scanning of ranking list is done from starting to end. All repeated items are withdrawn. The ranking list is given below:

{QRSTPU}

Similarly, considering time period from February to March, the ranking list obtained is {QRTSPU}. The ranking list for time span of March only is {QUTRS}.

Results show that, if item S is not in stock, it will result in larger loss than itself because of its cross-selling effect with other items. Also, it is to be noticed that item set {QR} is key frequent item set since it has the highest EDU in time period from January to March. This methodology helps stock manager to perceive high-benefit items in each time period, with the goal that he/she gains benefit and effectively oversees stocks.

25.5 Conclusion and Future Scope

Every business has an inventory management system which tracks the physical movement of inventory. An appropriate inventory management framework setup will not

Table 25.3 An inventory transaction database

Jan-01				Feb-01				Mar-01			
TID1	TID2	TID3	TID4	TID5	TID6	TID7	TID8	TID9	TID10	TID11	TID12
Q, S	Q, R, S	Q, R	P, S	Q, R, T	S, T	P, Q, R	Q, R, S, T	R, T, U	Q, T, U	P, S	Q, S, U

Table 25.4 The progress of finding frequent item sets in time period from January to March

Transaction Id	{Items}	Length	Support	EDU	Is it frequent?
One large item set					
1	{P}	1	3	3 * 5 = 15	Y
2	{Q}	1	8	8 * 4 = 32	Y
3	{R}	1	6	6 * 3 = 18	Y
4	{S}	1	7	7 * 2 = 14	Y
5	{T}	1	5	5 * 3 = 15	Y
6	{U}	1	3	3 * 1 = 3	Y
Two large item set					
7	{PQ}	2	1		N
8	{PR}	2	1		N
9	{PS}	2	1		N
10	{PT}	2	0		N
11	{PU}	2	0		N
12	{QR}	2	5	5 * (4 + 3) = 35	Y
13	{QS}	2	4	4 * (4 + 2) = 24	Y
14	{QT}	2	3	3 * (4 + 3) = 21	Y
15	{QU}	2	2	2 * (4 + 1) = 10	Y
16	{RS}	2	2	2 * (3 + 2) = 10	Y
17	{RT}	2	3	3 * (3 + 3) = 18	Y
18	{RU}	2	1		N
19	{ST}	2	2	2 * (2 + 3) = 10	Y
20	{SU}	2	1		N
21	{TU}	2	2	2 * (3 + 1) = 8	Y
Three large item set					
22	{QRS}	3	2	2 * (4 + 3+2) = 18	Y
23	{QRT}	3	2	2 * (4 + 3+3) = 20	Y
24	{QRU}	3	0		N
25	{QSU}	3	1		N
26	{QTU}	3	1		N
27	{RST}	3	1		N
28	{RTU}	3	1		N
29	{STU}	3	0		N
Four large item set					
30	{QRST}	4	1		N

just enhance consumer loyalty and maintenance; however, it will likewise be a significant advance for your warehouse management and operations. In this chapter, stock items which are interrelated to each other are grouped by utilizing the idea of "cross-selling effect". Further, the inventories are classified considering cross-selling effect in two ways. First, the cross-selling effect in different clusters is characterized as a basis for assessing the significance of items. Second, the cross-selling in different time periods is considered as a criterion for ranking inventory items. Numerous items that customarily do not belong to A grouping have been shifted into the A group by the cross-selling effect, and furthermore numerous items that generally belong to C grouping have been advanced into higher gathering as a result of their high benefits. A numerical example was presented to illustrate the new approach. Future studies ought to create anticipating models that incorporate learning of stock strategies in the frequent item sets in light of the fact that the frequent item set has items that can be from different classes and can correlate with each other.

References

1. Agrawal R, Imielinski T, Swami (1993) Mining association rules between sets of items in large databases. In: Proceedings of the ACM SIGMOD international conference on management of data, New York, NY, USA, pp. 207–216
2. Agarwal R (2017) Optimal order quantity and inventory classification using clustering. Int J Appl Manag Sci Eng 4(2):41–52
3. Agarwal R (2017) Ordering policy and inventory classification using temporal association rule mining. Int J Prod Manag Assess Technol 6(1):37–49
4. Agarwal R (2017) Decision making with association rule mining and clustering in supply chains. Int J Data Netw Sci 1(1):11–18
5. Agarwal R (2017) Opportunity cost estimation using temporal association rule mining. Int J Serv Sci 6(3/4):261–272
6. Agarwal R, Mittal M, Pareek S (2016) Optimal inventory classification using data mining techniques. Optimal inventory control and management techniques, IGI Global Publisher, pp 236–255
7. Agarwal R, Mittal M, Pareek S (2018) Optimal ordering policy with inventory classification using data mining techniques. Promoting business process improvement through inventory control techniques. IGI Global, pp 305–326
8. Anand SS, Hughes JG, Bell DA, Patrick AR (1997) Tackling the cross-sales problem using data mining. In: Proceedings of the 2nd Pacific-Asia conference on knowledge discovery and data mining, Hong Kong, pp 331–343
9. Brijs T, Swinnen G, Vanhoof K, Wets G (1999) Using association rules for product assortment decisions: a case study. In: Proceedings of the 5th ACM SIGKDD international conference on knowledge discovery & data mining, New York, USA, pp 254–260
10. Brijs T, Swinnen G, Vanhoof K, Wets G (2000) A data mining framework for optimal product selection in retail supermarket data: The generalized PROFSET model. In: Proceedings of the 6th ACM SIGKDD international conference on knowledge discovery & data mining, New York, USA, pp 300–304
11. Cohen MA, Ernst R (1988) Multi-item classification and generic inventory stock control Policies. Prod Invent Manag J 29(3):6–8
12. Chase RB, Aquilano NJ, Jacobs FR (1998) Production and operations management. Irwin/McGraw Hill, New York

13. Ernst R, Cohen MA (1990) Operations related groups (ORGs): a clustering procedure for production/inventory systems. J Oper Manag 9(4):574–598
14. Flores BE, Whybark DC (1987) Implementing multiple criteria ABC analysis. J Oper Manag 7(1 & 2):79–85
15. Han J, Kamber M (2006) Data mining: concepts and techniques. Morgan Kaufmann Publishers, San Francisco
16. Kaku I (2004) A data mining framework for classification of inventories. In: Proceedings of the 5th Asia Pacific industrial engineering and management systems, Japan, pp 450–455
17. Kaku I, Xiao Y (2008) A new algorithm of inventory classification based on the association rules. Int J Serv Sci 1(2):148–163
18. Lee C-H, Chen M-S, Lin C-R (2003) Progressive partition miner: an efficient algorithm for mining general temporal association rules. J IEEE Trans Knowl Data Eng 15(4):1004–1017
19. Li Y, Ning P, Wang XS, Jajodia S (2001) Discovering calendar-based temporal association rules. In: Proceedings of the 8th international symposium on temporal representation and reasoning, Itlay, pp 111–118
20. Mittal M, Pareek S, Agarwal R (2015) Loss profit estimation using association rule mining with clustering. Manag Sci Lett 5(2):167–174
21. Mittal M, Pareek S, Agarwal R (2015) Ordering policy using temporal association rule mining. Int J Data Sci 1(2):157–171
22. Ng WL (2007) A simple classifier for multiple criteria ABC analysis. Eur J Oper Res 177(1):344–353
23. Ozan C, Mustafa SC (2008) A web-based decision support system for multi-criteria inventory classification using fuzzy AHP methodology. Exp Syst Appl 35(3):1367–1378
24. Ramanathan R (2006) ABC inventory classification with multiple-criteria using weighted linear optimization. Comput Oper Res 33(3):695–700
25. Silver EA, Pyke DF, Peterson R (1998) Inventory management & production planning & scheduling, 3rd edn. Wiley, New York
26. Tan KC, Kannan VR, Handfield RB (1998) Supply chain management: supplier performance and firm performance. Int J Purch Mater Manag 34(3):2–9
27. Towill DR (1996) Time compression and supply chain management – a Guided Tour. Logist Inf Manag 9(6):41–53
28. Wang K, Xu C, Liu B (1999) Clustering transactions using large items. In: Proceedings of the 8th international conference on information and knowledge management, USA, pp 483–490
29. Wong RC, Fu AW, Wang K (2003) MPIS: maximal-profit item selection with cross-selling consideration. In: Proceedings of the 3rd IEEE international conference on data mining, USA, pp 371–378
30. Wong RC, Fu AW, Wang K (2005) Data mining for inventory item selection with cross-selling consideration. Data Min Knowl Disc 11(1):81–112
31. Xiao Y, Zhang R, Kaku I (2011) A new approach of inventory classification based on loss profit. Exp Syst Appl 38(8):9382–9391
32. Yin Y, Kaku I, Tang J, Zhu JM (2011) Data mining concepts, methods and applications in management and engineering design. Springer, London
33. Zhou P, Fan L (2007) A note on multi-criteria ABC inventory classification using weighted linear optimization. Eur J Oper Res 182(3):1488–1491