An Integral Relation Associated with a General Class of Polynomials and the Aleph Function

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Abstract A new finite integral involving two general class of polynomials with the Aleph function has been obtained in the present paper. This integral is supposed to be one of the most universal integral evaluated until now and act as a key component from which we can obtain as its different special cases, integrals relating a large number of simpler special functions and polynomials. Some useful unique cases of the main outcome have also been discussed in the paper.

Keywords The general class of polynomials · Aleph-function

2000 Mathematics Subject Classifications 26A33 · 33C60

1 Introduction

The Aleph-function is a new generalization of the well-known H-function [1] and the I-function [2, 3].

The Aleph-function is defined and represented as follows [4, 5].

$$\aleph[z] = \aleph^{M,N}_{P_i,Q_i,\tau_i;r}[z] = \aleph^{M,N}_{P_i,Q_i,\tau_i;r}\left[z \,\middle|\, \begin{matrix} (a_j, A_j)_{1,N}, \ldots, [\tau_i(a_j, A_j)]_{N+1,P_i} \\ (b_j, B_j)_{1,M}, \ldots, [\tau_i(b_j, B_j)]_{M+1,Q_i} \end{matrix}\right]$$
$$= \frac{1}{2\pi\omega}\int_L \Phi(\xi) z^{-\xi} d\xi \tag{1.1}$$

for all $z \neq 0$, where $\omega = \sqrt{-1}$ and

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J. Singh et al. (eds.), *Mathematical Modelling, Applied Analysis and Computation*, Springer Proceedings in Mathematics & Statistics 272,
https://doi.org/10.1007/978-981-13-9608-3_8

$$\Phi(\xi) = \frac{\prod_{j=1}^{M} \Gamma(b_j + B_j\xi) \prod_{j=1}^{N} \Gamma(1 - a_j - A_j\xi)}{\sum_{i=1}^{r} \tau_i \prod_{j=N+1}^{P_i} \Gamma(a_{ji} + A_{ji}\xi) \prod_{j=M+1}^{Q_i} \Gamma(1 - b_{ji} - B_{ji}\xi)} \tag{1.2}$$

The path of integration $L = L_{i\Upsilon\infty}, \Upsilon \in R$ extends from $\Upsilon - i\infty$ to $\Upsilon + i\infty$. The poles of $\Gamma(b_j + B_j\xi)$, $j = \overline{1, M}$. which do not coincide to the poles of $\Gamma(1 - a_j - A_j\xi), j = \overline{1, N}$ are taken as simple poles. The parameters p_i, q_i are non-negative integers $0 \leq N \leq P_i$, $1 \leq M \leq Q_i$, $\tau_i > 0$ for $i = \overline{1, r}$. The parameters $A_j, B_j, A_{ji}, B_{ji} > 0$ and $a_j, b_j, a_{ji}, b_{ji} \in C$. The product in (1.2) is interpreted as unity. The existence conditions for the described integral (1.1) are given beneath:

$$\theta_\ell > 0, \ |\arg(z)| < \frac{\pi}{2}\theta_\ell, \ell = \overline{1, r}; \tag{1.3}$$

$$\theta_\ell > 0, |\arg(z)| < \frac{\pi}{2}\theta_\ell \text{ and } \mathrm{Re}\{\zeta_\ell\} + 1 < 0, \tag{1.4}$$

where

$$\theta_\ell = \sum_{j=1}^{N} A_j + \sum_{j=1}^{M} B_j - \tau_\ell \left(\sum_{j=N+1}^{P_\ell} A_{j\ell} + \sum_{j=M+1}^{Q_\ell} B_{j\ell} \right) \tag{1.5}$$

$$\zeta_\ell = \sum_{j=1}^{M} b_j - \sum_{j=1}^{N} a_j + \tau_\ell \left(\sum_{j=M+1}^{Q_\ell} b_{j\ell} - \sum_{j=N+1}^{P_\ell} a_{j\ell} \right) + \frac{1}{2}(P_\ell - Q_\ell), \ell = \overline{1, r}, \tag{1.6}$$

Note 1 The simplification of the sum in the denominator of (1.2) in terms of a polynomial in ξ, the factor of this polynomial can be uttered by a fraction of Euler's Gamma function leading to H-function, see [6], p. 325.

Note 2 It might be seen that there is no recorded name given to (1.1), compared to [5]. The Mellin transform of this function is coefficient of $z^{-\zeta}$ in the integrand of (1.1).

Note 3 Taking $\tau_i = 1, i = 1, \ldots, r$, in (1.1), the ℵ-function lessens to the notable I-function [3].

Note 4 Putting $r = 1$ and $\tau_1 = \tau_2 = \ldots = \tau_3 = 1$, then ℵ-function reduces to the known H-function [7].

Following definition of general class of polynomials is required which was introduced by Srivastava [8, Eq. (1)].

$$S_n^m[x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} x^k, \qquad n = 0, 1, 2, \ldots \tag{1.7}$$

Here the coefficients $A_{n,k}(n, k \geq 0)$ are subjective real or complex constants, whereas M_1 is an arbitrarily chosen positive integer.

On suitably specializing the coefficients $A_{n,k}$ occurring in (1.7), the general class polynomials $S_n^m[x]$ can be reduced to the known traditional orthogonal polynomials and the generalized hypergeometric polynomials as its particular cases. These incorporate, among others, the Hermite polynomials, the Jacobi polynomials, the Laguerre polynomials, the Bessel polynomials, the Gould-Hopper polynomials and a couple of others.

2 Main Result

$$\int_a^b (x-a)^{u-1}(b-x)^{v-1}(x-c)^{-u-v}$$

$$.\aleph^{M,N}_{P_i,Q_i,\tau_i;r}\left[z\left(\frac{x-a}{x-c}\right)^s\left(\frac{b-x}{x-c}\right)^t \middle| \begin{matrix}(a_j,A_j)_{1,N},\ldots,[\tau_i(a_j,A_j)]_{N+1,P_i}\\ (b_j,B_j)_{1,M},\ldots,[\tau_i(b_j,B_j)]_{M+1,Q_i}\end{matrix}\right]$$

$$.S^{m_1}_{n_1}\left[z_1\left(\frac{x-a}{x-c}\right)^{\lambda}\left(\frac{b-x}{x-c}\right)^{\mu}\right]S^{m_2}_{n_2}\left[z_2\left(\frac{x-a}{x-c}\right)^{\lambda'}\left(\frac{b-x}{x-c}\right)^{\mu'}\right]dx$$

$$=\sum_{k_1=0}^{[n_1/m_1]}\sum_{k_2=0}^{[n_2/m_2]}\frac{(-n_1)_{m_1k_1}(-n_2)_{m_2k_2}}{k_1!k_2!}A_{n_1,k_1}\ A_{n_2,k_2}z_1^{k_1}z_2^{k_2}$$

$$.(b-a)^{u+v+(\lambda+\mu)k_1+(\lambda'+\mu')k_2-1}(b-c)^{-u-\lambda k_1-\lambda' k_2}(a-c)^{-v-\mu k_1-\mu' k_2}$$

$$.\aleph^{M,N+2}_{P_i+2,Q_i+1,\tau_i;r}\left[z\left(\frac{b-a}{b-c}\right)^s\left(\frac{b-a}{a-c}\right)^t \middle| \begin{matrix}(1-u-\lambda k_1-\lambda' k_2,s),(1-v-\mu k_1-\mu' k_2,t)\\ (b_j,B_j)_{1,M},\ldots,[\tau_i(b_j,B_j)]_{M+1,Q_i}\end{matrix}\right.$$

$$\left.\begin{matrix}(a_j,\alpha_j)_{1,N},\ \tau_j(a_j,\alpha_j)_{N+1,P_i;r}\\ (1-u-v-\lambda k_1-\mu k_1-\lambda' k_2-\mu' k_2,s+t)\end{matrix}\right], \tag{2.1}$$

where $s>0$, $t>0$, $\mathrm{Re}\,(u+s\,b_j/\beta_j)>0$, $\mathrm{Re}\,(v+t\,b_j/\beta_j)>0$,

$j=1,\ldots,M$, λ, λ', μ and μ' are positive integers. A_{n_1,k_1} and A_{n_2,k_2} ($n_1,\ k_1, n_2,\ k_2 \geq 0$) are arbitrary constants, real or complex.

Proof To establish (2.1), expressing the ℵ-function by (1.2) and general class of polynomials by (1.7), then the order of summations and integration are interchanged (which is justified due to the absolute convergence of the integral in the process), we calculate the integral with the help of a result ([7], p. 287 (3.119)), and get the desired outcome.

3 Special Cases

(A) Taking $S_n^2[y] = y^{n/2}H_n\left[\frac{1}{2\sqrt{y}}\right]$ in the result obtained in (2.1) to the case of Hermite polynomials ([9], Eq. (5.5.4), p. 106 and [3], p. 158)

in which case $m_1 = 2$, $A_{n_1,k_1} = (-1)^{k_1}$ and also letting $m_2 = 2$, $A_{n_2,k_2} = (-1)^{k_2}$, we have

$$\int_a^b (x-a)^{u-1}(b-x)^{v-1}(x-c)^{-u-v}$$
$$.\aleph^{M,N}_{P_i,Q_i,\tau_i;r}\left[z\left(\frac{x-a}{x-c}\right)^s\left(\frac{b-x}{x-c}\right)^t \middle| \begin{matrix}(a_j, A_j)_{1,N}, \ldots, [\tau_i(a_j, A_j)]_{N+1,P_i} \\ (b_j, B_j)_{1,M}, \ldots, [\tau_i(b_j, B_j)]_{M+1,Q_i}\end{matrix}\right]$$
$$.\left[z_1\left(\frac{x-a}{x-c}\right)^{\lambda}\left(\frac{b-x}{x-c}\right)^{\mu}\right]^{n_1/2} H_{n_1}\left[\frac{1}{2\sqrt{z_1\left(\frac{x-a}{x-c}\right)^{\lambda}\left(\frac{b-x}{x-c}\right)^{\mu}}}\right]$$
$$.\left[z_2\left(\frac{x-a}{x-c}\right)^{\lambda'}\left(\frac{b-x}{x-c}\right)^{\mu'}\right]^{n_2/2} H_{n_2}\left[\frac{1}{2\sqrt{z_2\left(\frac{x-a}{x-c}\right)^{\lambda'}\left(\frac{b-x}{x-c}\right)^{\mu'}}}\right]dx$$
$$= \sum_{k_1=0}^{[n_1/m_1]}\sum_{k_2=0}^{n_2/m_2}\frac{(-n_1)_{2k_1}(-n_2)_{2k_2}}{k_1!k_2!}(-1)^{k_1}(-1)^{k_2}z_1^{k_1}z_2^{k_2}$$
$$.(b-a)^{u+v+(\lambda+\mu)k_1+(\lambda'+\mu')k_2-1}(b-c)^{-u-\lambda k_1-\lambda' k_2}(a-c)^{-v-\mu k_1-\mu' k_2}$$
$$.\aleph^{M,N+2}_{P_i+2,Q_i+1,\tau_i;r}\left[z\left(\frac{b-a}{b-c}\right)^s\left(\frac{b-a}{a-c}\right)^s \middle| \begin{matrix}(1-u-\lambda k_1-\lambda' k_2, s),(1-v-\mu k_1-\mu' k_2, t) \\ (b_j, B_j)_{1,M}, \ldots, [\tau_i(b_j, B_j)]_{M+1,Q_i}\end{matrix}\right.$$
$$\left.\begin{matrix}(a_j,\ \alpha_j)_{1,N}, \tau_j(a_j,\ \alpha_j)_{N+1,P_i;r} \\ (1-u-v-\lambda k_1-\mu k_1-\lambda' k_2-\mu' k_2, s+t)\end{matrix}\right], \tag{3.1}$$

applicable under the conditions as available from (2.1).

(B) For the Jacobi polynomials ([9], Eq. (4.3.2), p. 68 and [3], p. 158), our result (2.1) yields the following result by setting

$$S_n^1[x] = P_n^{(\alpha',\beta')}(1-2x) \text{ in which case}$$

$$m_1 = 1, \quad A_{n_1,k_1} = \binom{n_1+k_1}{n_1}\frac{(\alpha'+\beta'+n_1+1)_{k_1}}{(\alpha'+1)_{k_1}}$$

and also taking

$$m_2 = 1, \quad A_{n_2,k_2} = \binom{n_2 + k_2}{n_2} \frac{\left(\alpha'' + \beta'' + n_2 + 1\right)_{k_2}}{(\alpha'' + 1)_{k_2}},$$

we obtain

$$\int_a^b (x-a)^{u-1}(b-x)^{v-1}(x-c)^{-u-v}$$

$$.\aleph^{M,N}_{P_i,Q_i,\tau_i;r}\left[z\left(\frac{x-a}{x-c}\right)^s\left(\frac{b-x}{x-c}\right)^t \Big| \begin{matrix} (a_j, A_j)_{1,N}, \ldots, [\tau_i(a_j, A_j)]_{N+1,P_i} \\ (b_j, B_j)_{1,M}, \ldots, [\tau_i(b_j, B_j)]_{M+1,Q_i} \end{matrix}\right]$$

$$.P^{(\alpha',\beta')}_{n_1}\left[1 - 2z_1\left(\frac{x-a}{x-c}\right)^\lambda\left(\frac{b-x}{x-c}\right)^\mu\right]$$

$$.P^{(\alpha'',\beta'')}_{n_2}\left[1 - 2z_2\left(\frac{x-a}{x-c}\right)^{\lambda'}\left(\frac{b-x}{x-c}\right)^{\mu'}\right]dx$$

$$= \sum_{k_1=0}^{[n_1]}\sum_{k_2=0}^{[n2]}\binom{n_1+\alpha'}{n_1-k_1}\binom{n_2+\alpha''}{n_2-k_2}(-z_1)^{k_1}(-z_2)^{k_2}$$

$$.\binom{\alpha'+\beta'+n_1+k_1}{k_1}\binom{\alpha''+\beta''+n_2+k_2}{k_2}$$

$$.(b-a)^{u+v+(\lambda+\mu)k_1+\left(\lambda'+\mu'\right)k_2-1}(b-c)^{-u-\lambda k_1-\lambda' k_2}(a-c)^{-v-\mu k_1-\mu' k_2}$$

$$.\aleph^{M,\,N+2}_{P_i+2,\;Q_i+1,\;\tau_i;r}\left[z\left(\frac{b-a}{b-c}\right)^s\left(\frac{b-a}{a-c}\right)^t \Big| \begin{matrix} \left(1-u-\lambda k_1-\lambda' k_2, s\right), \left(1-v-\mu k_1-\mu' k_2, t\right) \\ (b_j, B_j)_{1,M}, \ldots, [\tau_i(b_j, B_j)]_{M+1,Q_i} \end{matrix}\right.$$

$$\left. \begin{matrix} \left(a_j,\ \alpha_j\right)_{1,N}, \tau_j\left(a_j,\ \alpha_j\right)_{N+1,P_i;r} \\ \left(1-u-v-\lambda k_1-\mu k_1-\lambda' k_2-\mu' k_2,\ s+t\right) \end{matrix}\right], \tag{3.2}$$

valid under the conditions as obtainable from (2.1).

(C) For the Laguerre polynomials ([9], Eq. (5.1.6), p. 10 and [3], p. 158), we have the following interesting consequence of our result (2.1), by setting

$$S^1_n[x] \to L^{(\alpha')}_n(x) \text{ in which case}$$

$$m_1 = 1, \quad A_{n_1,k_1} = \binom{n_1+\alpha'}{n_1}\frac{1}{(\alpha'+1)_{k_1}}$$

and also taking

$$m_2 = 1, \quad A_{n_2,k_2} = \binom{n_2 + \alpha''}{n_2} \frac{1}{(\alpha'' + 1)_{k_2}},$$

we get

$$\int_a^b (x-a)^{u-1}(b-x)^{v-1}(x-c)^{-u-v}$$
$$.\aleph^{M,N}_{P_i,Q_i,\tau_i;r}\left[z\left(\frac{x-a}{x-c}\right)^s\left(\frac{b-x}{x-c}\right)^t \middle| \begin{matrix}(a_j,A_j)_{1,N},\ldots,[\tau_i(a_j,A_j)]_{N+1,P_i}\\(b_j,B_j)_{1,M},\ldots,[\tau_i(b_j,B_j)]_{M+1,Q_i}\end{matrix}\right]$$
$$.L^{(\alpha')}_{n_1}\left[z_1\left(\frac{x-a}{x-c}\right)^{\lambda}\left(\frac{b-x}{x-c}\right)^{\mu}\right]\ .L^{(\alpha'')}_{n_2}\left[z_2\left(\frac{x-a}{x-c}\right)^{\lambda'}\left(\frac{b-x}{x-c}\right)^{\mu'}\right]dx$$
$$=\sum_{k_1=0}^{[n_1]}\sum_{k_2=0}^{[n_2]}\frac{(-n_1)_{k_1}(-n_2)_{k_2}}{k_1!k_2!}\binom{n_1+\alpha'}{n_1}\frac{1}{(\alpha'+1)k_1}\binom{n_2+\alpha''}{n_2}\frac{1}{(\alpha''+1)K_2}z_1^{k_1}z_2^{k_2}$$
$$.(b-a)^{u+v+(\lambda+\mu)k_1+(\lambda'+\mu')k_2-1}(b-c)^{-u-\lambda k_1-\lambda' k_2}(a-c)^{-v-\mu k_1-\mu' k_2}$$
$$.\aleph^{M,N+2}_{P_i+2,Q_i+1,\tau_i;r}\left[z\left(\tfrac{b-a}{b-c}\right)^s\left(\tfrac{b-a}{a-c}\right)^t \middle| \begin{matrix}(1-u-\lambda k_1-\lambda' k_2,s),\ (1-v-\mu k_1-\mu' k_2,t)\\(b_j,B_j)_{1,M},\ldots,[\tau_i(b_j,B_j)]_{M+1,Q_i}\end{matrix}\right.$$
$$\left.\begin{matrix}(a_j,\alpha_j)_{1,N},\ \tau_j(a_j,\alpha_j)_{N+1,P_i;r}\\(1-u-v-\lambda k_1-\mu k_1-\lambda' k_2-\mu' k_2,\ s+t)\end{matrix}\right], \tag{3.3}$$

suitable under the conditions as required sufficiently for (2.1).

(D) Letting $n_2 \to 0$ in (2.1), we have

$$\int_a^b (x-a)^{u-1}(b-x)^{v-1}(x-c)^{-u-v}$$
$$.\aleph^{M,N}_{P_i,Q_i,\tau_i;r}\left[z\left(\frac{x-a}{x-c}\right)^s\left(\frac{b-x}{x-c}\right)^t \middle| \begin{matrix}(a_j,A_j)_{1,N},\ldots,[\tau_i(a_j,A_j)]_{N+1,P_i}\\(b_j,B_j)_{1,M},\ldots,[\tau_i(b_j,B_j)]_{M+1,Q_i}\end{matrix}\right]$$
$$.S^{m_1}_{n_1}\left[z_1\left(\frac{x-a}{x-c}\right)^{\lambda}\left(\frac{b-x}{x-c}\right)^{\mu}\right]dx$$
$$=\sum_{k_1=0}^{[n_1/m_1]}\frac{(-n_1)_{m_1k_1}}{k_1!}A_{n_1,k_1}z_1^{k_1}.(b-a)^{u+v+(\lambda+\mu)k_1-1}(b-c)^{-u-\lambda k_1}(a-c)^{-v-\mu k_1}$$

$$.\aleph^{M,N+2}_{P_i+2,Q_i+1,\tau_i;r}\left[z\left(\frac{b-a}{b-c}\right)^s\left(\frac{b-a}{a-c}\right)^t\middle|\begin{matrix}(1-u-\lambda k_1-\lambda' k_2, s),(1-v-\mu k_1-\mu' k_2, t)\\(b_j, B_j)_{1,M},\ldots,[\tau_i(b_j, B_j)]_{M+1,Q_i}\end{matrix}\right.$$
$$\left.\begin{matrix}(a_j,\alpha_j)_{1,N},\tau_j(a_j,\alpha_j)_{N+1,P_i;r}\\(1-u-v-\lambda k_1-\mu k_1-\lambda' k_2-\mu' k_2,\ s+t)\end{matrix}\right], \tag{3.4}$$

valid under the conditions as essential for (2.1).

(E) Taking $\tau_i \to 1$ in (2.1), the I-function given by Saxena [2, 3] is obtained from Aleph function and the main integral (2.1) converts in the following form:

$$\int_a^b (x-a)^{u-1}(b-x)^{v-1}(x-c)^{-u-v}$$
$$.I^{M,N}_{P_i,Q_i;r}\left[z\left(\frac{x-a}{x-c}\right)^s\left(\frac{b-x}{x-c}\right)^t\middle|\begin{matrix}(a_j,A_j)_{1,N},(a_j,A_j)_{N+1,P_i}\\(b_j,B_j)_{1,M},(b_j,B_j)_{M+1,Q_i}\end{matrix}\right]$$
$$.S^{m_1}_{n_1}\left[z_1\left(\frac{x-a}{x-c}\right)^{\lambda}\left(\frac{b-x}{x-c}\right)^{\mu}\right]S^{m_2}_{n_2}\left[z_2\left(\frac{x-a}{x-c}\right)^{\lambda'}\left(\frac{b-x}{x-c}\right)^{\mu'}\right]dx$$
$$=\sum_{k_1=0}^{[n_1/m_1]}\sum_{k_2=0}^{[n_2/m_2]}\frac{(-n_1)_{m_1k_2}(-n_2)_{m_2k_2}}{k_1!k_2!}A_{n_1,k_1}A_{n_2,k_2}z_1^{k_1}z_2^{k_2}$$
$$.(b-a)^{u+v+(\lambda+\mu)k_1+(\lambda'+\mu')k_2-1}(b-c)^{-u-\lambda k_1-\lambda' k_2}(a-c)^{-v-\mu k_1-\mu' k_2}$$
$$.I^{M,N+2}_{P_i+2,Q_i+1;r}\left[z\left(\frac{b-a}{b-c}\right)^s\left(\frac{b-a}{a-c}\right)^t\middle|\begin{matrix}(1-u-\lambda k_1-\lambda' k_2, s),(1-v-\mu K_1-\mu' K_2, t)\\(b_j,B_j)_{1,M},(b_j,B_j)]_{M+1,Q_i}\end{matrix}\right.$$
$$\left.\begin{matrix}(a_j,\alpha_j)_{1,N},(a_j,\alpha_j)_{N+1,P_i;r}\\(1-u-v-\lambda k_1-\mu k_1-\lambda' k_2-\mu' k_2,\ s+t)\end{matrix}\right], \tag{3.5}$$

valid under the conditions as required sufficiently for (2.1).

(F) If we take $\tau_i \to 1$ and $r = 1$ in (2.1), the Aleph function reduces to Fox's H-function [1] and the main integral takes the following form:

$$\int_a^b (x-a)^{u-1}(b-x)^{v-1}(x-c)^{-u-v}\quad .H^{M,N}_{P,Q}\left[z\left(\frac{x-a}{x-c}\right)^s\left(\frac{b-x}{x-c}\right)^t\middle|\begin{matrix}(a_j,A_j)\\(b_j,B_j)\end{matrix}\right]$$
$$.S^{m_1}_{n_1}\left[z_1\left(\frac{x-a}{x-c}\right)^{\lambda}\left(\frac{b-x}{x-c}\right)^{\mu}\right]S^{m_2}_{n_2}\left[z_2\left(\frac{x-a}{x-c}\right)^{\lambda'}\left(\frac{b-x}{x-c}\right)^{\mu'}\right]dx$$

$$= \sum_{k_1=0}^{[n_1/m_1]} \sum_{k_2=0}^{[n_2/m_2]} \frac{(-n_1)_{m_1 k_2} (-n_2)_{m_2 k_2}}{k_1! k_2!} A_{n_1,k_1} A_{n_2,k_2} z_1^{k_1} z_2^{k_2}$$

$$.(b-a)^{u+v+(\lambda+\mu)k_1+(\lambda'+\mu')k_2-1} (b-c)^{-u-\lambda k_1-\lambda' k_2} (a-c)^{-v-\mu k_1-\mu' k_2}$$

$$.H^{M,N+2}_{P+2,Q+1,}\left[z\left(\frac{b-a}{b-c}\right)^{s} \left(\frac{b-a}{a-c}\right)^{t} \middle| \begin{matrix} (1-u-\lambda k_1-\lambda' k_2, s), (1-v-\mu k_1-\mu' k_2, t)(a_1,\alpha_1), (a_p,\alpha_p) \\ (b_1,B_1), (b_q,B_q)(1-u-v-\lambda k_1-\mu K_1-\lambda' k_2-\mu' k_2, s+t) \end{matrix} \right], \tag{3.6}$$

valid under the conditions as required sufficiently for (2.1).

The significance of outcomes lies in its various generalizations. In perspective of the generality of the function and polynomials of very broad nature involved in the results, our results encompass several particular cases of interest scattered hitherto in the literature.

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