Mathematical Modelling of Poor Nutrition in the Human Life Cycle

Ebenezer Bonyah, Kojo Ababio and Patience Pokuaa Gambrah

Abstract Nutrition is very crucial in the survival of human race and more importantly the development of a child from the womb to adulthood. In some instances, the age of the individuals determines the kind of nutrients required. Therefore, the human cycle has something to do with the nutrients obtained. We formulate a mathematical model as a system of non-linear ordinary differential equations to investigate the effects of poor nutrition from conception to adulthood using the poor pregnant woman nutrient status. The steady states are studied and R_0 of poor nutrition in the society are calculated. To keep the society healthy and free of malnutrition, malnourished pregnant females are encouraged to eat foods that contain all the nutrients needed for development. The model is supported with numerical simulation.

Keywords Nutrition · Reproduction number · Pregnant women · Steady states · Conception

1 Introduction

Malnutrition means basically an individual who is over or under nutrition. The World Food Programme (WFP) classifies malnutrition as a condition where the physical function of the human body cannot be performed such as normal growth, pregnancy, recovery from injury and diseases [1, 2].

From conception through pregnancy, birth, childhood, adolescence and adulthood, nutrition plays a vital role in every stage which supports health and wellness and improving the quality of life. Good nutrition for pregnant women plays an important

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role in giving birth to a healthy baby and also ensuring good health status of the nursing mother [2]. The effects of poor nutrition begin in the womb, continues well into childhood, adulthood and cycles across generations [3, 4]. When a pregnant woman is malnourished in essential elements such as potassium, calcium, iodine, and others the unborn baby begins to face challenges in the proper development which could affect such individual to the adulthood. There is a positive correlation between the health status of the pregnant mother and the health status of the child to be born. In many cases, the medical health practitioners are able to detect and advice appropriately the food and other activities necessary for the health of the child [5]. A malnourished pregnant woman is provided with micro-nutrients in order to improve the nutrient status which will lead to healthy pregnancy. Anaemia is very dangerous for pregnant woman and all effort must be put in place to reduce or prevent which will bring about health growth with the right birth weights. Good birth weights depict the healthy status of the baby which will lead to the proper development of the child [3].

About one billion and nine hundred million (1.9 billion) adults worldwide are deemed overweight, while four hundred and sixty-two (462) millionaire also known to be underweight [6–10]. It has been found from studies that approximately 41 million children under the age of five (5) years are considered overweight or obese. In addition, 159 million children in the entire world are found to be stunted and 50 million also identified as wasted. Adding to this burden, are the 528 million or 29% of women of reproductive age around the world affected by anaemia, for which approximately half would be amenable to iron supplementation [11].

Nutrition has been identified to be a major factor in every human stages of development from conception to adulthood. For one to become healthy in a society, good nutrition is therefore required especially for pregnant women since the life cycle starts from conception through pregnancy. Malnutrition is the most serious and common health problem that occurs when a person's diet does not contain the proper amounts of nutrients. Mathematical modelling has become an indispensable tool in investigating many scientific processes in the world including social, health, economic etc. that addresses challenges in the absence of real data in the society.

Nita et al. [12], formulated a mathematical model in order to analyze transmission dynamics of malnutrition and underweight individuals in pregnant women in the society. They calculated the basic reproduction number R_0 at the equilibrium state of the model which decided the existence of malnutrition and underweight in the society. Local stability, global stability and numerical simulation were done for this model. Their result suggested that, to live a better and healthy life, one must consume healthy and nutritive food. They further suggested that in future work, deciding (optimum) dosage of nutrients at the early stage and incorporate different layers of the society to have more realistic analysis.

Nita et al. [13], proposed a transmission model of poor nutrition in the human life cycle to study the spread of poor nutrition at different stages of life from a malnutrited pregnant female. They modelled the sample fertile female population using the application of SEIR model constructed as a system of non-linear ordinary differential equations for the various compartments. They calculated the basic reproduction number R_0 at an endemic equilibrium point which decided the existence of poor nutrition in the society. Local stability, global stability and numerical simulation were done for this model. Their results suggested that the transmission rate of healthy pregnant female giving birth to low weight babies due to pregnancy complications contribute largely to making a poor nutritional life cycle in the society.

Senelani et al. [14], constructed a mathematical model to explore the effect of malnutrition on the spread of cholera. In their study, both nourished and malnourished individuals were included in the model as those susceptible and infected of cholera respectively. The sensitivity analysis carried out in their study revealed that an increase in the number of individuals susceptible to cholera as a result of malnutrition led to a higher number of cholera infected individuals in a community. They concluded that nutritional related matters should be attended to immediately so as to improve the nutrition status of the rural communities affected by cholera. Diana [15], developed a model hinged on the first law of thermodynamics which basically focused on controlling and managing weight variations in the human body. In this work, the authors assumed the human body as an open system which accommodates input into the system in the form of food. They partitioned the human population into resting metabolic rate, non-exercise activity thermogenesis and dietary induced thermogenesis [15]. As a result of interactions between the various compartments a set of nonlinear ordinary differential equations were obtained. Their study quantified a metabolic adaptation because of caloric restriction that seek to defend baseline body weight.

Carson et al. [16], also constructed a model that focused on a general description with respect to general body weight, a given time interval and how the body will behave. The available data suggested that there is no clear distinction between body composition and mass, and an invariant manifold. For a constant food intake rate with the corresponding physical activity level as well as the body weight all will lead to a steady state and this matches with a unique body weight.

Dumitru et al. [17], proposed a mathematical model for poor nutrition in life cycle in humans. They used the Caputo, Atangana-Baleanu and Fabrizio derivatives on the model to investigate poor pregnant women nutrient status. They calculated the basic reproduction number R_0 at an endemic equilibrium points which decided the existence of poor nutrition in the society. The proposed model was examined in fractional derivatives sense via Caputo Fabrizio, Atanagan-Baleanu and Caputo. Comparative numerical analysis of these operators was extensively carried out and showed that Caputo and Atangana-Baleanu derivative in all alpha values produced similar results. The Fabrizio Caputo operator converged quickly as compared to the other two operator and therefore more efficient.

Milinda et al. [18], investigated nutrition status which concentrated on undernourished children in a malarial formulated model. Logistic regression was employed to explore mortality rate of malaria infection. They found out that insecticide-treated bed nets given to under-nutritioned children led to fewer malaria deaths related cases. The authors suggested that free bed net can be given to the vulnerable in the communities.

Several mathematical models on epidemiology have been formulated and analyzed, however, there are few models constructed on nutrition related issues such as underweight and overweight. The formulation and analysis of nutrition related model would go a long way to provide some qualitative information on this serious health issue.

The main aim of this work is to present a modified model on poor nutrition in the human life cycle and analyze to present some useful qualitative information for decision making process.

2 Model Formulation

In this section, we assume that malnutrition is transmissible and can be treated. Based on the above assumption, ‘we formulate mathematical model from conception through pregnancy, birth, childhood and adolescence. The total population is denoted by $N(t), \forall t > 0$. The population is divided into five compartments.

2.1 Model Diagram

The proportion of babies from malnutrited pregnant female being low weight is β_1 and high weight is $(1 - \beta_1)$. When low weight babies are not given proper breast feeding or formula feeding for the first 6 months and medical care, they grow to become child undergrowth at rate of δ. With good nutrition and health care, high weight baby and under growth children grow to become healthy adolescents at rates η and γ respectively. The induced mortality rate for malnutrition pregnant female and low birth weight is denoted by α_1 and natural mortality rate is μ. The recruitment rate into malnutrited pregnant female is Λ (Table 1).

2.2 Model Equation

The nonlinear differential equations below describes lack of nutrition from one compartment to other in Fig. 1.

$$\frac{dF_{MP}}{dt} = \Lambda - \beta_1 F_{MP} B_{LW} - (1 - \beta_1) F_{MP} B_{HW} - (\alpha_1 + \mu) F_{MP}$$

$$\frac{dB_{LW}}{dt} = \beta_1 F_{MP} B_{LW} - (\delta + \alpha_1 + \mu) B_{LW}$$

$$\frac{dB_{HW}}{dt} = (1 - \beta_1) F_{MP} B_{HW} - (\eta + \mu) B_{HW} \tag{2.2.1}$$

Table 1 The table gives detailed explanation of the model parameter and variable

Variables	Description
N	Sample size of female in fertile stage
F_{MP}	Malnutrited pregnant female
B_{LW}	Low weight baby
B_{HW}	High weight baby
C_U	Child undergrowth
A_H	Adolescent healthy growth
Parameters	Description
Λ	Recruitment rate into malnutrited pregnant female
β_1	Proportion of babies from F_{MP} being low weight baby
$(1-\beta_1)$	Proportion of babies from F_{MP} being high weight baby
α_1	Induced death rate of F_{MP} and low birth weight
δ	Rate at which individuals from B_{LW} moves to C_U compartment
η	Rate at which individuals from B_{HW} grows to A_H compartment
γ	Rate at which individuals from C_U grows to A_H compartment
μ	Natural death rate

$$\frac{dC_U}{dt} = \delta B_{LW} - (\gamma + \mu)C_U$$

$$\frac{dA_H}{dt} = \eta B_{HW} + \gamma C_U - \mu A_H$$

The total population at time t is represented by $N(t)$, $N(t) = F_{MP} + B_{LW} + B_{HW} + C_U + A_H$,

$$\text{Then, } \frac{dN}{dt} = \Lambda - \alpha_1(F_{MP} + B_{LW}) - \mu N. \tag{2.2.2}$$

2.3 Model Analysis

In this section, we verify some basic properties and perform stability analysis of the system (2.2.1).

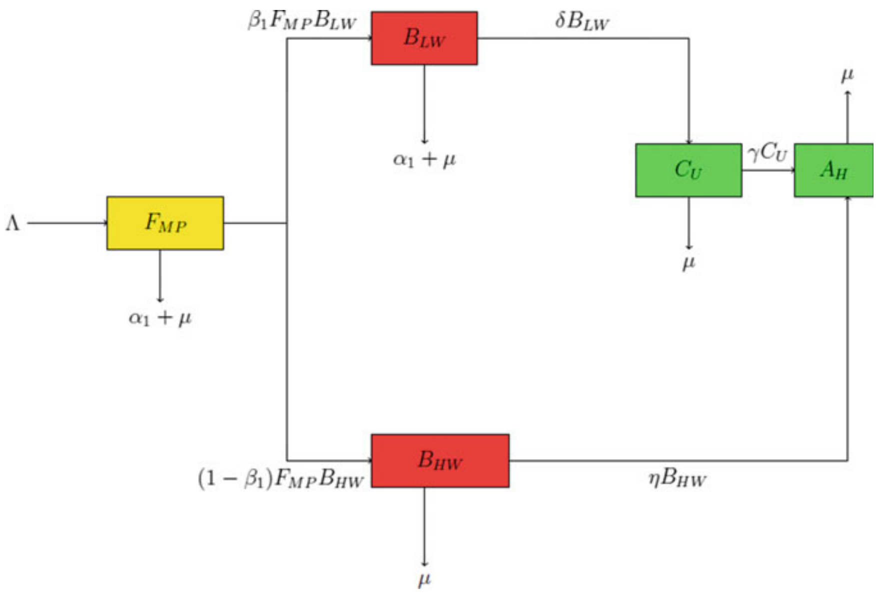


Fig. 1 Compartmental model for the transmission of poor nutrition where arrows with head means moving out or coming in of a compartment

2.3.1 Boundedness of the Solution

Lemma 2.3.1 The closed set $\phi = \left\{(F_{MP}, B_{LW}, B_{HW}, C_U, A_H) \in \mathbb{R}^5_+ : N \leq \frac{\Lambda_H}{\mu}\right\}$ is positively invariant with respect to model (2.2.1).

Proof Assuming $(F_{MP}, B_{LW}, B_{HW}, C_U, A_H) \in \mathbb{R}^5_+$ for all $t > 0$, we want to prove that the region ϕ is positively invariant so that it becomes sufficient to look at the dynamics of the system (2.2.1). From Eq. (2.2.2), we have the rate at which the total population changes over time as:

$$\frac{dN}{dt} = \Lambda - \alpha_1(F_{MP} + B_{LW}) - \mu N$$

In the absence of malnutrition, this equation can be rewritten as

$$\frac{dN}{dt} + \mu N = \Lambda \tag{2.3.1}$$

Solving the differential equation using the integrating factor, we obtain

$$N(t) = \frac{\Lambda}{\mu} + Ke^{-\mu t}$$

Using initial conditions, $t = 0$, $N(0)$, we have

$$N(t) = \frac{\Lambda}{\mu} + \left(N(0) - \frac{\Lambda}{\mu}\right)e^{-\mu t}$$

$$N(t) = N(0)e^{-\mu t} + \frac{\Lambda}{\mu}\left(1 - e^{-\mu t}\right) \tag{2.3.2}$$

as $t \to \infty$:

$$\lim_{t\to\infty} N(t) = \frac{\Lambda}{\mu}$$

if $N(0) \leq \frac{\Lambda}{\mu}$, then we have $N(t) = \frac{\Lambda}{\mu}$, $\forall t > 0$ as $t \to \infty$.

Also, if $N(0) > \frac{\Lambda}{\mu}$, then the solutions $(F_{MP}(t), B_{LW}(t), B_{HW}(t), C_U(t), A_H(t))$ in the region ϕ is positively invariant. We conclude from this theorem that it is sufficient to deal with the dynamics of system (2.2.1) in ϕ. Based on that, the model can be assume to be epidemiologically well-posed for mathematical analysis [19].

2.4 Analysis of Malnutrition Free Steady State

Equation 2.4.1 Steady state. At steady state, we assume the population is constant over time. We determine the steady state of malnutrition free by putting the right hand side of system (2.2.1) to zero.

$$\begin{aligned}
\Lambda - \beta_1 F_{MP}B_{LW} - (1-\beta_1)F_{MP}B_{HW} - (\alpha_1 + \mu)F_{MP} &= 0 \\
\beta_1 F_{MP}B_{LW} - (\delta + \alpha_1 + \mu)B_{LW} &= 0 \\
(1-\beta_1)F_{MP}B_{HW} - (\eta + \mu)B_{HW} &= 0 \\
\delta B_{LW} - (\gamma + \mu)C_U &= 0 \\
\eta B_{HW} + \gamma C_U - \mu A_H &= 0
\end{aligned} \tag{2.4.1}$$

Therefore the malnutrition free equilibrium (MFE) is given by

$$E_0 = \left(F_{MP}^0, 0, 0, 0, 0\right) = \left(\frac{\Lambda}{(\mu + \alpha_1)}, 0, 0, 0, 0\right)$$

Solving the equations, we obtained endemic state of the system.

2.5 The Basic Reproduction Number

The basic reproduction number R_{LW}, which provides some useful information on the spread of disease was computed in this work [20]. The next generation matrix approach was employed to drive the threshold R_{0LW} which is given by:

$$R_{LW} = \rho\left(FV^{-1}\right)$$
$$R_{LW} = \frac{\beta_1 \Lambda}{(\mu + \alpha_1)(\delta + \alpha_1 + \mu)} + \frac{(1 - \beta_1)\Lambda}{(\mu + \alpha_1)(\eta + \mu)}$$

2.6 Stability Analysis of Steady States

Let

$$\phi = \left\{(F_{MP}, B_{LW}, B_{HW}, C_U, A_H) \in \mathbb{R}^5_+ : N \leq \frac{\Lambda_H}{\mu}\right\}$$

2.6.1 Local Stability of Malnutrition Free Steady State

Theorem 2.6.2 *The malnutrition free equilibrium* (E_0) *is locally asymptotically stable if* $R_{WL} < 1$ *and unstable if* $R_{WL} > 1$.

Proof The Jacobian matrix of the system (2.2.1) is given by

$$J = \begin{bmatrix} -(\alpha_1 + \mu) - \beta_1 B_{LW} - (1 - \beta_1) B_{HW} & -\beta_1 F_{MP} & -(1 - \beta_1) F_{MP} & 0 & 0 \\ \beta_1 B_{LW} & {}_p\beta_1 F_{MP} - (\delta + \alpha_1 + \mu) & 0 & 0 & 0 \\ (1 - \beta_1) B_{HW} & 0 & (1 - \beta_1) F_{MP} - (\eta + \mu) & 0 & 0 \\ 0 & \delta & 0 & -(\gamma + \mu) & 0 \\ 0 & 0 & \eta & \gamma & -\mu \end{bmatrix}$$

Evaluating at the malnutrition free equilibrium point gives

$$J(E_0) = \begin{bmatrix} -(\alpha_1 + \mu) & -\frac{\beta_1 \Lambda}{(\mu + \alpha_1)} & -\frac{(1-\beta_1)\Lambda}{(\mu+\alpha_1)} & 0 & 0 \\ 0 & \frac{\beta_1 \Lambda}{(\mu+\alpha_1)} - (\delta + \alpha_1 + \mu) & 0 & 0 & 0 \\ 0 & 0 & \frac{(1-\beta_1)\Lambda}{(\mu+\alpha_1)} - (\eta + \mu) & 0 & 0 \\ 0 & \delta & 0 & -(\gamma + \mu) & 0 \\ 0 & 0 & \eta & \gamma & -\mu \end{bmatrix}$$

From the Jacobian matrix we obtain the eigenvalues as follows:

$$\lambda_1 = -(\alpha_1 + \mu), \lambda_2 = \frac{(\mu + \alpha_1)(\delta + \alpha_1 + \mu) - \beta_1 \Lambda}{(\mu + \alpha_1)}, \lambda_3 = \frac{(\mu + \alpha_1)(\eta + \mu) + (1 - \beta_1)\Lambda}{(\mu + \alpha_1)},$$
$$\lambda_4 = -(\gamma + \mu) \text{ and } \lambda_5 = -\mu$$

Since all the eigenvalues are negative then $R_{WL} < 1$. So we conclude that malnutrition free is locally asymptotically stable if $R_{WL} > 1$.

2.6.2 Global Stability of the Malnutrition Free Steady State

Theorem 2.6.4 *The disease steady-state free E_0 whenever it exists, is globally asymptotically stable if $R_{0WL} \leq 1$ when all solutions of system (2.2.1) in $\mathbb{R}^5$ are bounded.*

Proof The proof requires that a suitable Lyapunov function is chosen by taking into account the infective classes of the non-linear ordinary differential equations of the system (2.2.1).

$$V(t) = c_1\left(B_{LW} - B_{LW}^0 - B_{LW}^0 \ln \frac{B_{LW}}{B_{LW}^0}\right) + c_2\left(B_{HW} - B_{HW}^0 - B_{HW}^0 \ln \frac{B_{HW}}{B_{HW}^0}\right)$$

where c_1 and c_2 are non-negative constant to be determined. Then V is C^1 on the interior of ϕ, E_0 is global minimum of V on ϕ, and $V\left(B_{LW}^0, B_{HW}^0\right) = 0$. The time derivative of $V(t)$ computed along solutions of (2.2.1) is $\dot{V}(t) = c_1 \frac{dB_{LW}}{dt} + c_2 \frac{dB_{HW}}{dt}$

$\dot{V}(t) = c_1(\delta + \alpha_1 + \mu)(R_{WL} - 1)B_{LW} + c_2(\eta + \mu)(R_0 - 1)B_{HW} \leq 0$, if $R_0 \leq 1$

Now $\dot{V}(t)$ is negative if $R_0 < 1$ and $\dot{V}(t) = 0 \Leftrightarrow B_{LW} = B_{HW} = 0$, if $R_{WL} = 1$. Therefore, the malnutrition free equilibrium is globally asymptotically stable if $R_0 \leq 1$.

2.7 Stability of Endemic Steady State

$$\Lambda - \beta_1 F_{MP} B_{LW} - (1 - \beta_1) F_{MP} B_{HW} - (\alpha_1 + \mu) F_{MP} = 0$$

$$\beta_1 F_{MP} B_{LW} - (\delta + \alpha_1 + \mu) B_{LW} = 0$$

$$(1 - \beta_1) F_{MP} B_{HW} - (\eta + \mu) B_{HW} = 0 \tag{2.7.1}$$

$$\delta B_{LW} - (\gamma + \mu) C_U = 0$$

$$\eta B_{HW} + \gamma C_U - \mu A_H = 0$$

Solving the Eq. (2.7.1), we obtained an endemic equilibrium point
$E_1^* = \left(F_{MP}^*, B_{LW}^*, B_{HW}^*, C_U^*, A_H^*\right)$ and $E_2^* = \left(F_{MP}^*, B_{LW}^*, B_{HW}^*, C_U^*, A_H^*\right)$

where

$$E_1^* = \left[\frac{(\delta+\mu+\alpha_1)}{\beta_1}, \frac{(\mu+\alpha_1)}{\beta_1}(R_{LW}-1), 0, \frac{(\mu+\alpha_1)\delta}{\beta_1(\mu+\gamma)}(R_{LW}-1), \delta\frac{(\mu+\alpha_1)}{\beta_1(\mu+\gamma)}(R_{LW}-1)\right] \text{ and}$$
$$E_2^* = \left[\frac{\eta+\mu}{(1-\beta_1)}, 0, \frac{(\mu+\alpha_1)}{(1-\beta_1)}(R_{HW}-1), 0, \frac{\eta(\mu+\alpha_1)\delta}{\mu(1-\beta_1)}(R_{HW}-1)\right].$$

Theorem 2.7.1 *The malnutrition endemic equilibrium is locally asymptotically stable if $R_{WL} > 1$ and unstable if $R_{WL} \leq 1$*

Proof Evaluating the Jacobian matrix at the endemic equilibrium points gives

$$J(E_1^*) = \begin{bmatrix} -(\mu+\alpha_1)+(\mu+\alpha_1)(1-R_{LW}) & -(\delta+\mu+\alpha_1) & -\frac{(1-\beta_1)(\delta+\mu+\alpha_1)}{\beta_1} & 0 & 0 \\ -(\mu+\alpha_1)(1-R_{LW}) & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{(1-\beta_1)(\delta+\mu+\alpha_1)}{\beta_1}-(\eta+\mu) & 0 & 0 \\ 0 & \delta & 0 & -(\gamma+\mu) & 0 \\ 0 & 0 & \eta & \gamma & -\mu \end{bmatrix}$$

and

$$J(E_2^*) = \begin{bmatrix} -(\mu+\alpha_1)+(\mu+\alpha_1)(1-R_{HW}) & -\frac{\beta_1(\eta+\mu)}{(1-\beta_1)} & -(\eta+\mu) & 0 & 0 \\ 0 & \frac{\beta_1(\eta+\mu)}{(1-\beta_1)}-(\delta+\mu+\alpha_1) & 0 & 0 & 0 \\ -(\mu+\alpha_1)(1-R_{HW}) & 0 & \frac{(1-\beta_1)(\delta+\mu+\alpha_1)}{\beta_1}-(\eta+\mu) & 0 & 0 \\ 0 & \delta & 0 & -(\gamma+\mu) & 0 \\ 0 & 0 & \eta & \gamma & -\mu \end{bmatrix}$$

where
$R_{LW} = -\frac{\beta_1\Lambda}{\mu(\delta+\mu+\alpha_1)}$ and $R_{HW} = \frac{(1-\beta_1)\Lambda}{\mu(\eta+\mu)}$
The trace of $J(E_1^*)$ is
$tr(J(E_1^*)) = -(\alpha_1+\mu)+(\alpha_1+\mu)(1-R_{LW})+\frac{(1-\beta_1)(\delta+\alpha_1+\mu)}{\beta_1}-(\eta+\mu)-(\gamma+\mu)-\mu < 0$ if
$\frac{(1-\beta_1)(\delta+\alpha_1+\mu)}{\beta_1} < (\eta+\mu)$ and determinant is given by

$$\det(J(E_1^*)) = -\left[(\delta+\alpha_1+\mu)(\alpha_1+\mu)(1-R_{LW})\left((\eta+\mu)-\frac{(1-\beta_1)(\delta+\alpha_1+\mu)}{\beta_1}\right)(\gamma+\mu)\mu\right] > 0$$

If $R_{LW} > 1$. Also the trace of $J(E_2^*)$ is
$tr(J(E_2^*)) = -(\alpha_1+\mu)+(\alpha_1+\mu)(1-R_{HW})+\frac{\beta_1(\eta+\mu)}{(1-\beta_1)}-(\delta+\alpha_1+\mu)-(\gamma+\mu)-\mu < 0$ if
$\frac{\beta_1(\eta+\mu)}{(1-\beta_1)} < (\delta+\alpha_1+\mu)$ and determinant is given by
$\det(J(E_2^*)) = -\left[(\eta+\mu)(\alpha_1+\mu)(1-R_{HW})\left(\frac{\beta_1(\eta+\mu)}{(1-\beta_1)}-(\delta+\alpha_1+\mu)\right)(\gamma+\mu)\mu\right] > 0$ if $R_{HW} > 1$.

Since the trace is negative and the determinant is positive then $R_{WL} > 1$ in both cases. We then conclude that malnutrition at endemic equilibrium is locally asymptotically stable whenever $R_{WL} > 1$.

3 Numerical Simulation

In this section, we will estimate the parameters in the model based on literature values, perform sensitivity analysis and finally we will conduct numerical simulations.

3.1 Parameter Estimation

The estimation of parameters has been a major challenge in the validation of epidemiological modelling. In this section we tried to estimate some of the parameter values of system (2.2.1).

Some estimated assumptions were considered in order make purposeful of illustrations in tracking the dynamics of malnutrition. For unavailability of data, we used literature values as indicated in the Table 2 based on model system Eq. (2.2.1). The following initial conditions were used for the purpose of numerical simulation $F_{MP} = 30$, $B_{LW} = 20$, $B_{HW} = 20$, $C_U = 10$ and $A_H = 35$. The parameter values used for the work is given in Table 2.

3.2 Sensitivity Analysis

Sensitivity analysis seeks to present to characterization of the uncertainty of parameters with regard to a given model. It offers the opportunity to have information on the effect of a particular parameter in the modeling processes [21].

We performed sensitivity analysis on R_{LW} with respect to the parameter value so that vital parameter values influence can be measured for the malnutrition model Eq. (2.2.1). For one to increase or reduce a parameter it is essential for one to have some relative information regarding the human morbidity and mortality in relation to the transmission dynamics of malnutrition. value In determining how best to reduce human mortality and morbidity due to malnutrition. According to Chintnis et al. [22], sensitivity analysis is commonly used to determine the robustness of

Table 2 The value of the parameters of the model

Parameter	Range	Sources
Λ	(0, 0.35)	[12, 13]
β_1	(0, 1)	Assumed
α_1	0.1	[12]
δ	0.006	[13]
η	0.013	Assumed
γ	(0, 1)	Assumed
μ	0.3	[12]

Table 3 Sensitivity index for malnutrition

R_0	Parameter	Sensitivity index
R_{0LW}	Λ	1
	β_1	1
	α_1	−0.46739
	δ	−0.1304347
	μ	−0.1304347
R_{0HW}	Λ	1
	β_1	−0.0012014
	α_1	−0.25
	η	−0.0415335
	μ	−1.708466

model predictions to parameter values. Thus, we are concerned with parameters that would significantly affect the model's basic reproduction number which is usually responsible for the spread of the phenomenon. Sensitivity analysis allows the measure of the relative variation in a state variable when there is a parameter variation.

Definition 3.2.1 The normalized forward sensitivity index of a variable u, which depends differentially on a parameter, p, is defined as: $r_p^u = \frac{\partial u}{\partial p} \times \frac{p}{u}$.

Since the reproduction number R_0 is a differentiable function of the parameters, the sensitivity index may alternatively be defined using partial derivatives as: $S = \frac{\partial R_0}{\partial \rho} \times \frac{\rho}{R_0}$, where ρ is the parameter of interest.

From Table 3, it depicts that R_{LW} was sensitive to β_1 and Λ. When each one of them increases making other parameters fixed, their values rose up the since they have positive indices. The most sensitive parameter observed was Λ which has an effect on malnutrited pregnant female population as well as the infected compartment.

4 Results

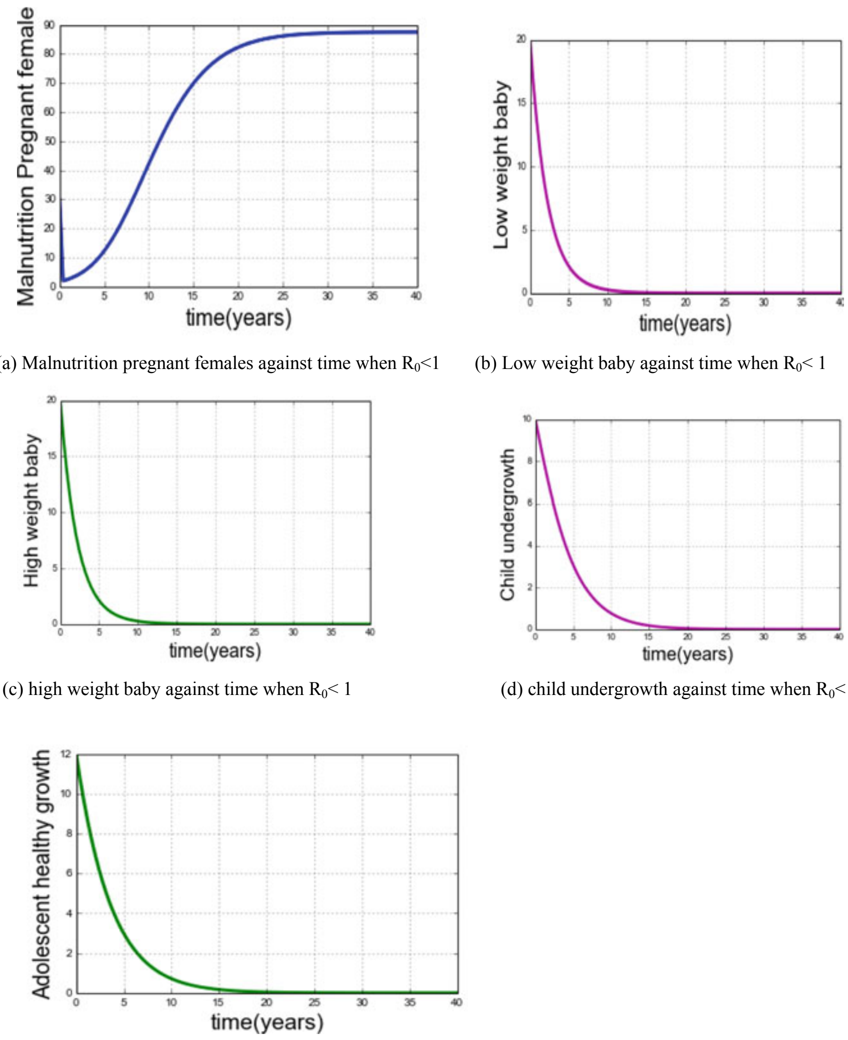
4.1 Simulation Results

From Fig. 2a, we observed that as time increases, the malnutrition pregnant female population increases to a maximum point and approaches the carrying capacity. That is the upper bound of the population.

Also, Fig. 2b the low weight baby converge to the equilibrium point zero as time increases. Thus low weight baby dies out from the population with time.

In Fig. 2c, we observe that as time increases, the high weight baby converged to equilibrium point zero. That is high weight baby dies out from the population.

Furthermore, Fig. 2d shows that child undergrowth will converge to the equilibrium point zero with time. Thus child undergrowth dies out from the population.



(a) Malnutrition pregnant females against time when $R_0<1$

(b) Low weight baby against time when $R_0< 1$

(c) high weight baby against time when $R_0< 1$

(d) child undergrowth against time when $R_0< 1$

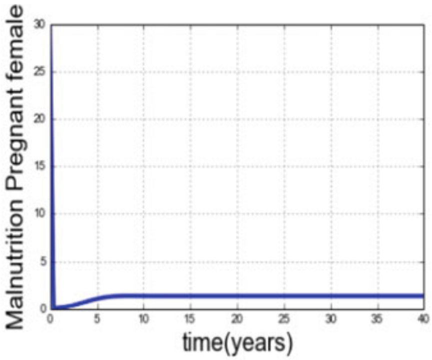
(e) Healthy adolescence growth against time when $R_0< 1$

Fig. 2 Simulation results for malnutrition free with $R_0 < 1$

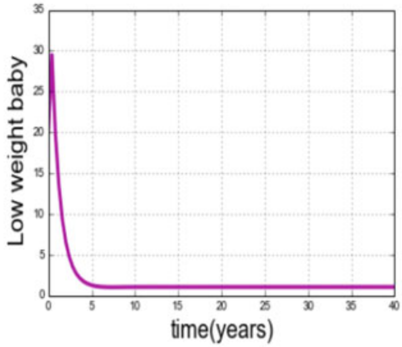
Figure 2e shows that healthy adolescent growth will converge to equilibrium point zero with time. That is healthy adolescent growth dies out from the population.

Again, we observed from Fig. 2b–e that all the populations converge to zero as time evolves. This shows that malnutrition can be minimized with time. Therefore, local stability of the malnutrition free state holds as shown in Theorem 2.6.2.

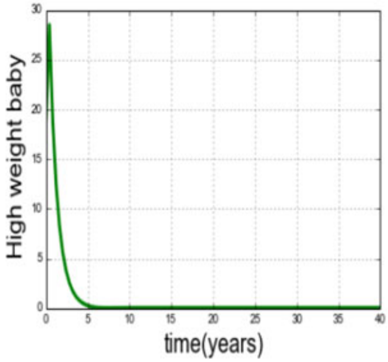
In Fig. 3a, it is observed that as time increases, the malnutrition pregnant female population decreases which shows that there is movement to another compartment. Thus, malnutrition pregnant female still exists in the population. In Fig. 3b, the low



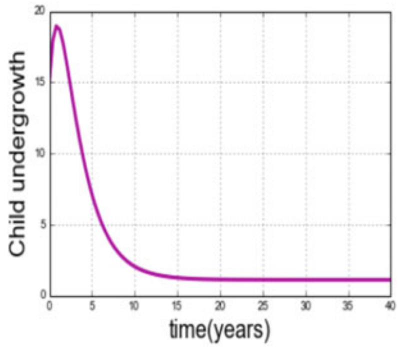
(a) malnutrition pregnant females against time



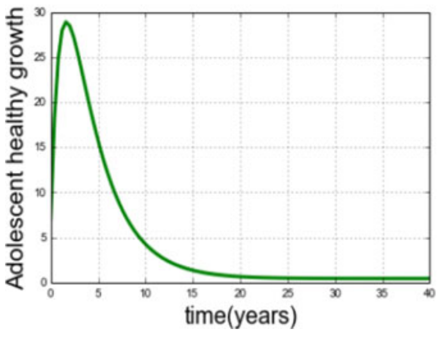
(b) A plot of low weight baby against time



(c) A plot of high weight baby against time



(d) A plot of child under growth against time



(e) A plot of healthy adolescence growth against time

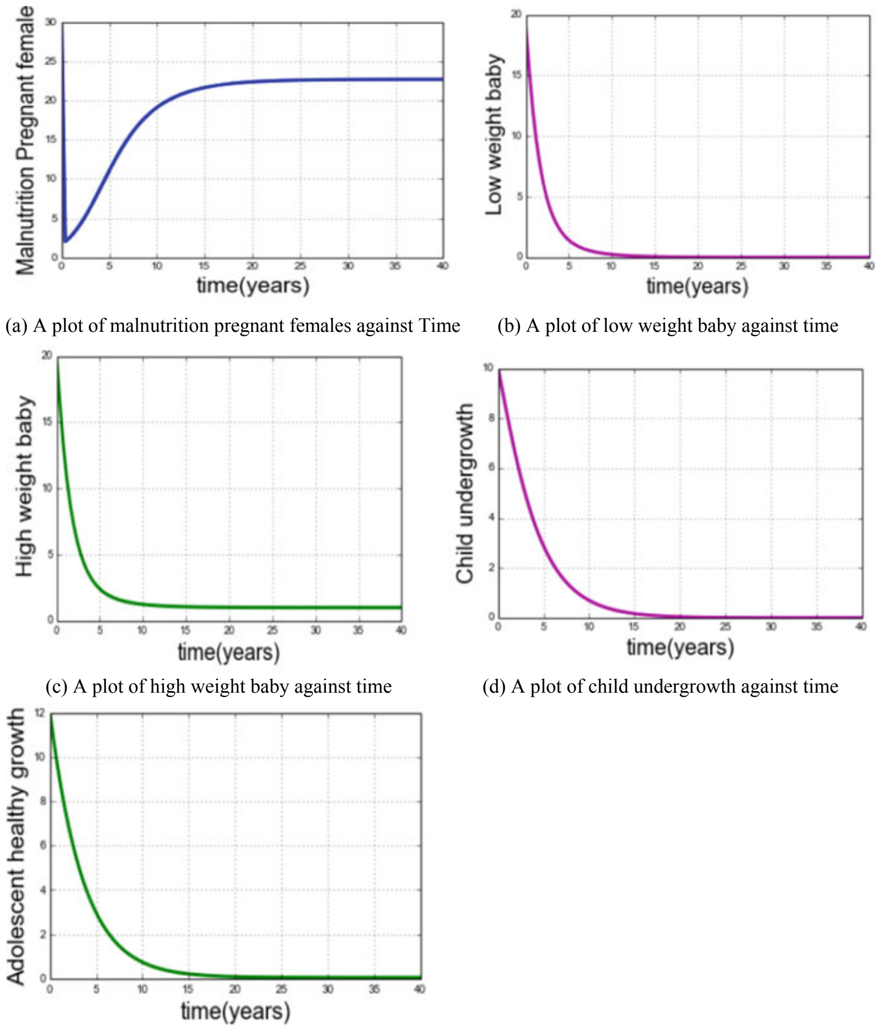
Fig. 3 Simulation results for endemic steady states E_1^*

weight baby decreases to equilibrium point as time increases. Thus, low weight baby still exists in the population.

Also, Fig. 2c, we observe that as time increases, the high weight baby converged to equilibrium point zero with time. This confirms the results on the steady state E_1^*.

Furthermore, Fig. 3d shows that child undergrowth decreases to the equilibrium point as time increases. Thus, child undergrowth will still exist in the population.

From Fig. 4a, we observe that as time increases, the malnutrition pregnant female population reduces which shows that there is movement to another compartment. Thus, malnutrition pregnant female still exists in the population.



(a) A plot of malnutrition pregnant females against Time

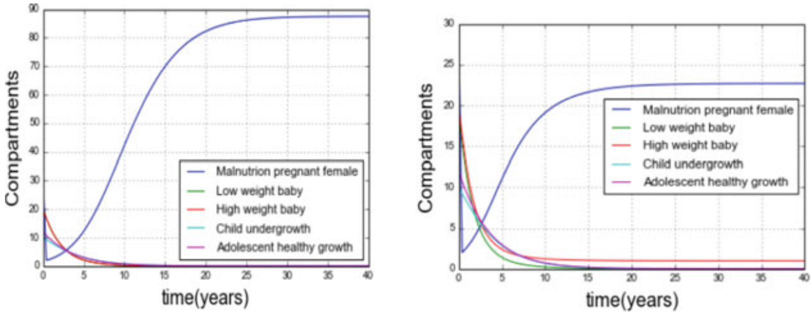
(b) A plot of low weight baby against time

(c) A plot of high weight baby against time

(d) A plot of child undergrowth against time

(e) A plot of healthy adolescence growth against time

Fig. 4 Simulation results for endemic steady states E_2^*



(a) A plot of all the compartments with $R_{0LW} < 1$ (b) A plot of all the compartments with $R_{0LW} > 1$

Fig. 5 Simulation results for $R_{0LW} < 1$ and $R_{0LW} > 1$

In 4b, d, we observe that as time increases, the low weight baby and child undergrowth converged to equilibrium point zero with time. This confirms the results on our steady state E_2^*. Also, Fig. 4c, we observe that as time increases, the high weight baby decreases to equilibrium point. That is high weight baby will still exist in the population with time. Furthermore, from 3e and 4e, the healthy adolescent population will converge to zero as time increases.

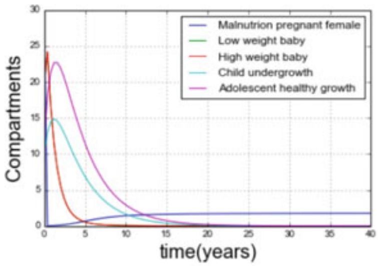
Figure 5 shows how severe malnutrition at individual compartments. When $R_{0LW} < 1$ all the infected compartments converge to zero. Thus, effect of malnutrition dies out from the population at malnutrition free.

Also, when $R_{0LW} > 1$, three of the compartments converge to zero (dies out from the population).

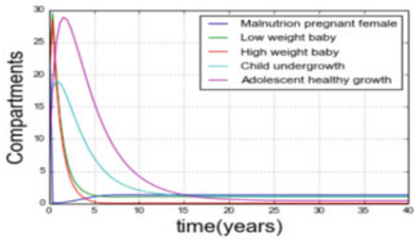
This confirms the steady states where both low weight babies and child undergrowth were found to be zero. Malnutrition is minimized but cannot be eradicated. Therefore, the existence malnutrition in the population.

From Fig. 6a, when $R_{0LW} < 1$ and $R_{0HW} < 1$, we observe malnutrition free equilibrium point E_0. That is malnutrition is stable. Also, in Fig. 6b when $R_{0HW} < 1$ and $R_{0LW} > 1$, E_0 will be unstable and the endemic equilibrium E_2^* will be stable. This means that high weigh babies and healthy adolescent will die out. However, Fig. 6c shows the case where E_0 will be unstable and the endemic equilibrium point E_1^* will be stable.

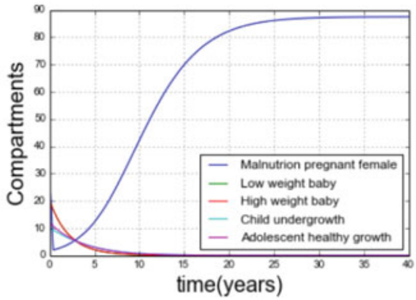
From Fig. 6b, c we observe that malnutrition cannot be eradicated but can be minimized. Hence, malnutrition at endemic equilibrium is locally stable as shown in Theorem 2.7.1.



(a) all the compartments with $R_{0LW} < 1$ and $R_{0HW} < 1$



(b) all the compartments with $R_{0HW} < 1$ and $R_{0LW} > 1$



(c) A plot of all the compartments with $R_{0LW} < 1$ and $R_{0HW} > 1$

Fig. 6 Simulation results for malnutrition free and endemic steady states

5 Conclusion

In this paper, we have successfully studied the effect of poor nutrition in the human life cycle. The effect of poor nutrition begins in the womb, continues well into childhood, adulthood and cycles across generations. We formulated a mathematical model from the conception to adulthood using the pregnant women nutrient status. The basic reproduction number R_0 was calculated. This serves as a threshold to which malnutrition will die out when $R_0 < 1$ or will persist when $R_0 > 1$. The model is supported with numerical simulation. We had a multiple steady state, which was written in terms of the reproduction number for low weight babies and high weight babies. The analysis on the steady state suggested that both malnutrition free and endemic equilibrium are both locally stable. Results from numerical simulations for all the compartments showed that malnutrition will die out locally and become unstable globally at the endemic state. Malnutrition can be minimized for a period but cannot be eradicated completely from the society. This suggests that good nutrition is very important in every stage of human development in the society. Good nutrition will reduce the rate at which fertile female becomes malnourished in the society. The pregnant women nutritional status should be improved to give birth to healthy baby which will grow eventually to become healthy adolescent. Literature values and assumed parameters were used because of the unavailability of data on malnutrition. In future work, this model can be improved to consider all stages throughout the human life cycle.

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