

Biological Model of Dengue Spread with Non-Markovian Properties



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Abstract A fatal and infectious called Dengue found in the tropical zone of the world is a mosquito-borne and caused by four viruses namely Den 1-Den 4. The transmission is achieved from one person to another via a bite of female adult Aedes mosquitoes. The dynamic of spread does not really follow the Markovian process therefore does have memory effect, thus can well be described by using nonlocal differential operators with non-singular and non-local kernel as these operators have a crossover from exponential decay law to power law as waiting time distribution. In this chapter, we reverted the classical model to fractional model by using the concept of recently established fractional differential operators known as the Caputo-Fabrizio derivative. To include into mathematical system the memory and the crossover effects. The new model was subjected to analysis of existence and uniqueness of the system solution to insure the well posedness of the modified system. Due to the complexity of the new system, a newly introduced numerical scheme was used to solve the system and some numerical simulations were performed to see the effect of the Mittag-Leffler law that brings the crossover effect.

Keywords Caputo-Fabrizio derivative · Dengue model · Fractional differential equations · Existence and uniqueness · Fixed point theorem

1 Introduction

Dengue disease is a common arboviral disease in tropical regions of the world. It is transferral to humans by the bite of Aedes mosquitoes. There are four types of virus which is denoted by one, two, three, and four. The bites of the Aedes mosquitoes

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is the reason of the viruses that transferral to humans. If a person infected once in the life by these one of the four serotypes of viruses will never get infected by that serotype again but loses immunity to other three stereotypes of the viruses [8]. There are lots of Bio-mathematical models have been proposed to recognize the transferral dynamics of these type of infectious diseases. In recent years, modeling has become a valuable tool in the analysis of dengue disease transferral dynamics and to determine the factors that influence the spread of disease to support control measures. Many researchers have proposed [5–8, 13, 14, 16, 18] epidemic model [10] to study the transferral dynamics of dengue disease.

There is no specific medicine to cure dengue disease. Awareness programs can be helpful in reducing the prevalence of the disease. Different Bio-mathematical models have been proposed to study the impact of awareness in controlling dengue and these type diseases. Prevention of mosquitoes bites is one of the ways to prevent dengue disease. The mosquitoes bite humans during day and night when lights are on. So, to get rid of mosquitoes bite, people can use mosquito repellents and nets. If infected hosts feel they have symptoms of the disease and approach the doctor in time for the supportive treatment, they can recover fast. This type of awareness can help controlling the disease. Another way of controlling dengue is destroying larval breeding sites of mosquitoes and killing them. Spray of insecticides may be applied to control larvae or adult mosquitoes which can transmit dengue viruses. This type of biological model have two properties as we observed Markovian and Non-Markovian. In dengue spread model does not really follow the Markovian process therefore does have memory effect, thus can well be described using the concept of nonlocal differential operators with non local and non singular kernel as these operators have a crossover from exponential decay law to power law as waiting time distribution.

Fc is applied in various directions of Bio-mathematics, physics, signal-processing, fluid-mechanics, visco-elasticity, finance, electro-chemistry and in many more. In the branch of fc, we study fractional integral and fractional derivative as an important aspects. Recently, many researcher and scientists have studied various type of issues in this special branch [1–3, 9]. The Caputo-Fabrizio derivative brought new weapons into applied mathematics to model complex real-world problems more accurately. Caputo-Fabrizio derivative is give the result of non-Markovian process. In the RL derivative the kernal inside it is gives the result for power law but Caputo-Fabrizio shows the result for exponential decay.

The main objective of this chapter is to discuss fractional Caputo-Fabrizio derivative for the mathematical system to finding the crossover effects and memory effect Also by using fixed point theorem we are finding the details of the uniqueness and exactness and of the solution. The development of this article is as follows. In Sect. 2, we discuss the Caputo-Fabrizio and AB derivative. In Sect. 3, the mathematical portion of fractional dengue spread model and also by applying CF derivative we find the approximate solution. In Sect. 4, by using fixed point theorem, we proved the uniqueness and existence of system of solutions in Sect. 6, Numerical Solution are discuss and in the last Sect. 7 we presented concluding remarks.

2 Preliminaries

Some definitions and properties of the fractional derivative are presented here.

Definition 2.1 Let f be a function not necessarily differentiable, and κ be a real number such that $0 < \kappa \leq 1$, then the Riemann-Liouville derivative with κ order with power law is given as [15]

$${}^{RL}D_t^\kappa [f(t)] = \frac{1}{\Gamma(1-\kappa)} \frac{d}{dt} \int_0^t (t-y)^{-\kappa} f(y) dy. \tag{2.1}$$

Definition 2.2 Let $f \in H^1(a, b)$, $b > a$, $\kappa \in [0, 1]$ then the new Caputo derivative of fractional order is given by:

$$D_t^\kappa (f(t)) = \frac{M(\kappa)}{(1-\kappa)} \int_a^t f'(x) \exp\left[-\kappa \frac{t-x}{1-\kappa}\right] dx. \tag{2.2}$$

where $M(\kappa)$ is a normalization function such that $M(0) = M(1) = 1$ [4]. But, if the function does not belong to $H_1(a, b)$ then, the derivative can be reformulated as

$$D_t^\kappa (f(t)) = \frac{M(\kappa)}{(1-\kappa)} \int_a^t (f(t) - f(x)) \exp\left[-\kappa \frac{t-x}{1-\kappa}\right] dx. \tag{2.3}$$

Remark 2.1 The authors remarked that, if $\sigma = \frac{1-\kappa}{\kappa} \in [0, \infty)$, $\kappa = \frac{1}{1+\sigma} \in [0, 1]$, then Eq. (2.1) assumes the form

$$D_t^\kappa (f(t)) = \frac{N(\sigma)}{(\sigma)} \int_a^t f'(x) \exp\left[-\frac{t-x}{\sigma}\right] dx, \quad N(0) = N(\infty) = 1 \tag{2.4}$$

In Addition,

$$\lim_{\sigma \rightarrow 0} \frac{1}{\sigma} \exp\left[-\frac{t-x}{\sigma}\right] = \delta(x-t) \tag{2.5}$$

Now after the introduction of a new derivative, the associate anti-derivative becomes important, the associated integral of the new Caputo derivative with fractional order was proposed by Losada and Nieto [11].

Definition 2.3 [11] Let $0 < \kappa < 1$. The fractional integral of order κ of a function f is defined by

$$I_{\kappa}^t(f(t)) = \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)}f(t) + \frac{2\kappa}{(2 - \kappa)M(\kappa)}\int_0^t f(s)ds, t \geq 0. \tag{2.6}$$

Remark 2.2 Note that, according to above definition, the fractional integral of Caputo type of function of order $0 < \kappa < 1$ is an average between function f and its integral of order one. This therefore imposes

$$\frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)}f(t) + \frac{2\kappa}{(2 - \kappa)M(\kappa)} = 1 \tag{2.7}$$

The above expression yields an explicit formula for

$$M(\kappa) = \frac{2}{2 - \kappa}, 0 \leq \kappa \leq 1 \tag{2.8}$$

Because of the above, Losada and Nieto proposed that the new Caputo derivative of order $0 < \kappa < 1$ can be reformulated as

$$D_{\kappa}^t(f(t)) = \frac{1}{1 - \kappa}\int_a^t f'(x)\exp\left[-\kappa\frac{t - x}{1 - \kappa}\right]dx. \tag{2.9}$$

3 Model Description

In the given model, total host human population, N_h . We divided this human population into four parts: R_h (recovered), I_h (infectious), E_h (exposed), S_h (susceptible) and total vector (mosquito) population, also we divide N_v into three parts: I_v (infectious), E_v (exposed), S_v (susceptible). We assume that the fraction u_1 of susceptible hosts use mosquito repellents to avoid mosquitoes bite. So, the fraction $(1 - u_1)$ of susceptible hosts interact with infectious mosquitoes. The fraction u_2 of infectious hosts seek for the timely supportive treatment and recover fast by the rate r_h ($r > 1$). The fraction $r_1 u_2$ (r_1 is the proportionality constant) of infectious hosts use mosquito repellents to avoid mosquitoes bite. u_3 is a control variable that represents the eradication effort of insecticide spraying. That follows that mortality rate of mosquito population increases at a rate $r_2 u_3$ (r_2 is the proportionality constant) and also it is assume that recruitment rate of this is reduced by a factor of $1 - u_3$.

In this section, we describes the geometry of dengue disease together with control measures. The system of differential equations which shows the present SEIR-SEI vector host model is given in [13].

$$\begin{aligned}
 \frac{dS_h}{dt} &= \mu_h N_h - (1 - u_1) \frac{b\beta_h}{N_h} S_h I_v - \mu_h S_h \\
 \frac{dE_h}{dt} &= (1 - u_1) \frac{b\beta_h}{N_h} S_h I_v - (v_h + \mu_h) E_h \\
 \frac{dI_h}{dt} &= v_h E_h - [ru_2\gamma_h + (1 - u_2)\gamma_h + \mu_h] I_h \\
 \frac{dR_h}{dt} &= [ru_2\gamma_h + (1 - u_2)\gamma_h] I_h - \mu_h R_h \\
 \frac{dS_v}{dt} &= (1 - u_3)\pi_v - (1 - r_1u_2) \frac{b\beta_h v}{N_h} S_v I_h - (r_2u_3 + \mu_v) S_v \\
 \frac{dE_v}{dt} &= (1 - r_1u_2) \frac{b\beta_h v}{N_h} S_v I_h - (r_2u_3 + v_v + \mu_v) E_v \\
 \frac{dI_v}{dt} &= v_v E_v - (r_2u_3 + \mu_v) I_v
 \end{aligned}
 \tag{3.1}$$

The parameters of the model are given in the following table.

Symbols	Description
μ_h	Death rate of host population
v_h	Host's incubation rate
γ_h	Recovery rate of host population
β_h	Transmission probability from vector to host
π_v	Vector population recruitment rate
μ_v	Vector population death rate
v_v	Vector's incubation rate
β_v	Host to vector the transmission probability
b	Rate (biting) of vector

Total host population, $N_h = R_h + I_h + E_h + S_h$, total vector population, $N_v = I_v + E_v + S_v$.

$$\frac{dN_h}{dt} = 0 \quad \text{and} \quad \frac{dN_v}{dt} = (1 - u_3)\pi_v - (r_2u_3 + \mu_v)N_v.$$

So, N_h remains constant and N_v approaches the equilibrium $(1 - u_3)\pi_v / (r_2u_3 + \mu_v)$ as $t \rightarrow \infty$. Introducing the proportions

$$\begin{aligned}
 s_v &= \frac{S_v}{(1 - u_3)\pi_v / (r_2u_3 + \mu_v)}, \quad s_h = \frac{S_h}{N_h}, \quad e_h = \frac{E_h}{N_h}, \quad i_h = \frac{I_h}{N_h}, \quad r_h = \frac{R_h}{N_h}, \\
 e_v &= \frac{E_v}{(1 - u_3)\pi_v v / (r_2u_3 + \mu_v v)}, \quad i_v = \frac{I_v}{(1 - u_3)\pi_v / (r_2u_3 + \mu_v v)}
 \end{aligned}$$

Since $s_v = 1 - e_v - i_v$ and $r_h = 1 - s_h - e_h - i_h$ the system of Eq. (3.1) is the equivalent written by five dimensional non-linear system of ODEs:

$$\begin{aligned}
\frac{ds_h}{dt} &= \mu_h(1 - s_h) - \alpha s_h i_v \\
\frac{de_h}{dt} &= \alpha s_h i_v - \beta e_h \\
\frac{di_h}{dt} &= v_h e_h - \gamma i_h \\
\frac{de_v}{dt} &= \delta s_v i_h - (\epsilon + v_v) e_v \\
\frac{di_v}{dt} &= v_v e_v - \epsilon i_v
\end{aligned} \tag{3.2}$$

Here,

$$\begin{aligned}
\alpha &= \frac{b\beta_h\pi_v(1 - u_1)(1 - u_3)}{N_h(r_2u_3 + \mu v)}, \quad \beta = v_h + \mu_h, \quad \gamma = ru_2\gamma_h + (1 - u_2)\gamma_h + \mu_h, \\
\delta &= (1 - r_1u_2)b\beta_v, \quad \epsilon = r_2u_3 + \mu v.
\end{aligned}$$

Due to Markovian process, this system is exponentially stable with no memory. Thus, to include the memory effect into this bio-mathematical model, we introduced Caputo-Fabrizio arbitrarily ordered derivative to moderate this system by non Markovian process as given by

$$\begin{aligned}
{}_0^{CF} D_t^\kappa s_h &= \mu_h(1 - s_h) - \alpha s_h i_v \\
{}_0^{CF} D_t^\kappa e_h &= \alpha s_h i_v - \beta e_h \\
{}_0^{CF} D_t^\kappa i_h &= v_h e_h - \gamma i_h \\
{}_0^{CF} D_t^\kappa e_v &= \delta s_v i_h - (\epsilon + v_v) e_v \\
{}_0^{CF} D_t^\kappa i_v &= v_v e_v - \epsilon i_v
\end{aligned} \tag{3.3}$$

These come with the initial conditions

$$i_v(0) = \delta_5, \quad e_v(0) = \delta_4, \quad i_h(0) = \delta_3, \quad e_h(0) = \delta_2, \quad s_h(0) = \delta_1. \tag{3.4}$$

4 Uniqueness and Existence of a System of Solutions of Dengue Models with Non-Markovian Properties

In this section investigate numerical result of fractional model based on CF derivative. We discuss the uniqueness and existence of the solutions by fixed point theorem. For this we apply the fractional integral operator due to Nieto and Losada [11] on Eq. (3.3), to examine the existence of the system of solutions. We obtain

$$\begin{aligned}
 s_h(t) - s_h(0) &= {}_0^CF I_t^\kappa \{ \mu_h(1 - s_h) - \alpha s_h i_v \} \\
 e_h(t) - e_h(0) &= {}_0^CF I_t^\kappa \{ \alpha s_h i_v - \beta e_h \} \\
 i_h(t) - i_h(0) &= {}_0^CF I_t^\kappa \{ v_h e_h - \gamma i_h \} \\
 e_v(t) - e_v(0) &= {}_0^CF I_t^\kappa \{ \delta s_v i_h - (\epsilon + v_v) e_v \} \\
 i_v(t) - i_v(0) &= {}_0^CF I_t^\kappa \{ v_v e_v - \epsilon i_v \}
 \end{aligned}
 \tag{4.1}$$

By using the equation discussed by Nieto and Losada [11], we have

$$\begin{aligned}
 i_v(t) - i_v(0) &= \frac{2\kappa}{(2 - \kappa)M(\kappa)} \int_0^t \{ v_v(y) e_v(y) - \epsilon i_v(y) \} dy \\
 &\quad + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \{ v_v(t) e_v(t) - \epsilon i_v(t) \} \\
 e_v(t) - e_v(0) &= \frac{2\kappa}{(2 - \kappa)M(\kappa)} \int_0^t \{ \delta s_v(y) i_h(y) - (\epsilon + v_v(y)) e_v(y) \} dy \\
 &\quad + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \{ \delta s_v(t) i_h(t) - (\epsilon + v_v(t)) e_v(t) \} \\
 i_h(t) - i_h(0) &= + \frac{2\kappa}{(2 - \kappa)M(\kappa)} \int_0^t \{ v_h(y) e_h(y) - \gamma i_h(y) \} dy \\
 &\quad + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \{ v_h(t) e_h(t) - \gamma i_h(t) \} \\
 e_h(t) - e_h(0) &= \frac{2\kappa}{(2 - \kappa)M(\kappa)} \int_0^t \{ \alpha s_h(y) i_v(y) - \beta e_h(y) \} dy \\
 &\quad + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \{ \alpha s_h(t) i_v(t) - \beta e_h(t) \} \\
 s_h(t) - s_h(0) &= \frac{2\kappa}{(2 - \kappa)M(\kappa)} \int_0^t \{ \mu_h(1 - s_h(y)) - \alpha s_h(y) i_v(y) \} dy \\
 &\quad + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \{ \mu_h(1 - s_h(t)) - \alpha s_h(t) i_v(t) \}
 \end{aligned}
 \tag{4.2}$$

So we can write for clarity

$$\begin{aligned}
 Z_1(t, s_h) &= \mu_h(1 - s_h(t)) - \alpha s_h(t) i_v(t), \\
 Z_2(t, e_h) &= \mu_h(1 - s_h(y)) - \alpha s_h(y) i_v(y) \\
 Z_3(t, i_h) &= v_h(t) e_h(t) - \gamma i_h(t)
 \end{aligned}$$

$$\begin{aligned} Z_4(t, e_v) &= \delta s_v(t) i_h(t) - (\epsilon + v_v(t)) e_v(t) \\ Z_5(t, i_v) &= v_v(t) e_v(t) - \epsilon i_v(t) \end{aligned} \tag{4.3}$$

Theorem 4.1 *If the following inequality holds then The kernels Z_1, Z_2, Z_3, Z_4 and Z_5 satisfy the Lipschitz condition and contraction.*

$$0 < a_1 + \beta b_1 \leq 1.$$

Proof Starting with the kernel Z_1 . Let two function is s_{h_1} and s_{h_2} then we get the following:

$$\|Z_1(t, s_h) - Z_1(t, s_{h_1})\| = \| -\mu_h \{s_h(t) - s_{h_1}(t)\} - \alpha \{s_h(t) - s_{h_1}(t)\} i_v(t) \|. \tag{4.4}$$

Now using the triangular inequality (4.4), we have

$$\begin{aligned} \|Z_1(t, s_h) - Z_1(t, s_{h_1})\| &\leq \| \alpha \{s_h(t) - s_{h_1}(t)\} i_v(t) \| + \| \mu_h \{s_h(t) - s_{h_1}(t)\} \| \\ &\leq \|s_h(t) - s_{h_1}(t)\| \{a_1 + b_1 \|i_v(t)\|\} \\ &\leq \{a_1 + b_1 \beta\} \|s_h(t) - s_{h_1}(t)\| \leq \gamma_1 \|s_h(t) - s_{h_1}(t)\| \end{aligned} \tag{4.5}$$

Taking $\gamma_1 = a_1 + \beta b_1$ here the $\beta = i_v(t)$ are bounded functions, then we have

$$\|Z_5(t, i_v) - Z_1(t, i_{v_1})\| = \gamma_5 \|i_v(t) - i_{v_1}(t)\| \tag{4.6}$$

Hence, the Lipschitz condition is satisfied for Z_1 , and if additionally $0 < (a_1 + \beta b_1 \leq 1)$, this condition is satisfy then it gives us a contraction for Z_1 .

Similarly all the cases II, II, III and IV satisfy the Lipschitz condition as follows:

$$\begin{aligned} \|Z_4(t, e_v) - Z_1(t, e_{v_1})\| &= \gamma_4 \|e_v(t) - e_{v_1}(t)\|, \\ \|Z_3(t, i_h) - Z_1(t, i_{h_1})\| &= \gamma_3 \|i_h(t) - i_{h_1}(t)\|, \\ \|Z_2(t, e_h) - Z_1(t, e_{h_1})\| &= \gamma_2 \|e_h(t) - e_{h_1}(t)\|, \\ \|Z_1(t, s_h) - Z_1(t, s_{h_1})\| &= \gamma_1 \|s_h(t) - s_{h_1}(t)\|. \end{aligned} \tag{4.7}$$

when we consider the kernels, the Eq. (4.2) becomes

$$\begin{aligned} i_v(t) &= \frac{2\kappa}{(2 - \kappa)M(\kappa)} \int_0^t (Z_5(y, i_v)) dy + i_v(0) + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} Z_5(t, i_v) \\ e_v(t) &= \frac{2\kappa}{(2 - \kappa)M(\kappa)} \int_0^t (Z_4(y, e_v)) dy + e_v(0) + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} Z_4(t, e_v), \end{aligned}$$

$$\begin{aligned}
 i_h(t) &= \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_3(y, i_h)) dy + i_h(0) + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} Z_3(t, i_h), \\
 e_h(t) &= \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_2(y, e_h)) dy + e_h(0) + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} Z_2(t, e_h), \\
 s_h(t) &= \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_1(y, s_h)) dy + s_h(0) + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} Z_1(t, s_h). \quad (4.8)
 \end{aligned}$$

Now, presenting the following recursive formula:

$$\begin{aligned}
 i_{v_n}(t) &= \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_5(y, i_{v_{n-1}})) dy + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} Z_5(t, i_{v_{n-1}}) \\
 e_{v_n}(t) &= \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_4(y, e_{v_{n-1}})) dy + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} Z_4(t, e_{v_{n-1}}) \\
 i_{h_n}(t) &= \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_3(y, i_{h_{n-1}})) dy + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} Z_3(t, i_{h_{n-1}}), \quad (4.9) \\
 e_{h_n}(t) &= \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_2(y, e_{h_{n-1}})) dy + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} Z_2(t, e_{h_{n-1}}), \\
 s_{h_n}(t) &= \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_1(y, s_{h_{n-1}})) dy + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} Z_1(t, s_{h_{n-1}}),
 \end{aligned}$$

and the initial conditions are gives as below:

$$i_{v_0}(t) = i_v(0), \quad e_{v_0}(t) = e_v(0), \quad i_{h_0}(t) = i_h(0), \quad e_{h_0}(t) = e_h(0), \quad s_{h_0}(t) = s_h(0). \quad (4.10)$$

Now, difference between the successive terms are presented as follow:

$$\begin{aligned}
 \varsigma_n(t) = i_{v_n}(t) - i_{v_{n-1}}(t) &= \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_5(y, i_{v_{n-1}}) - Z_5(y, i_{v_{n-2}})) dy \\
 &\quad + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} (Z_5(t, i_{v_{n-1}}) - Z_5(t, i_{v_{n-2}})) \\
 \chi_n(t) = e_{v_n}(t) - e_{v_{n-1}}(t) &= \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_4(y, e_{v_{n-1}}) - Z_4(y, e_{v_{n-2}})) dy
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} (Z_4(t, e_{v_{n-1}}) - Z_4(t, e_{v_{n-2}})), \\
 \xi_n(t) = i_{h_n}(t) - i_{h_{n-1}}(t) & = \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_3(y, i_{h_{n-1}}) - Z_3(y, i_{h_{n-2}})) dy \\
 & + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} (Z_3(t, i_{h_{n-1}}) - Z_3(t, i_{h_{n-2}})), \\
 \psi_n(t) = e_{h_n}(t) - e_{h_{n-1}}(t) & = \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_2(y, e_{h_{n-1}}) - Z_2(y, e_{h_{n-2}})) dy \\
 & + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} (Z_2(t, e_{h_{n-1}}) - Z_2(t, e_{h_{n-2}})) \\
 \phi_n(t) = s_{h_n}(t) - s_{h_{n-1}}(t) & = \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_1(y, s_{h_{n-1}}) - Z_1(y, s_{h_{n-2}})) dy \\
 & + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} (Z_1(t, s_{h_{n-1}}) - Z_1(t, s_{h_{n-2}})) \quad (4.11)
 \end{aligned}$$

Noticing that

$$\begin{aligned}
 s_{h_n}(t) & = \sum_{i=0}^n \phi_i(t), \\
 e_{h_n}(t) & = \sum_{i=0}^n \psi_i(t), \\
 i_{h_n}(t) & = \sum_{i=0}^n \xi_i(t), \\
 e_{v_n}(t) & = \sum_{i=0}^n \chi_i(t), \\
 i_{v_n}(t) & = \sum_{i=0}^n \varsigma_n(t).
 \end{aligned} \tag{4.12}$$

Step by step we get

$$\begin{aligned}
 \|\phi_n(t)\| & = \|s_{h_n}(t) - s_{h_{n-1}}(t)\| \\
 & = \left\| \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_1(y, s_{h_{n-1}}) - Z_1(y, s_{h_{n-2}})) dy \right. \\
 & \quad \left. + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} (Z_1(t, s_{h_{n-1}}) - Z_1(t, s_{h_{n-2}})) \right\| \quad (4.13)
 \end{aligned}$$

Employing the triangular inequality, Eq. (4.13) reduces to

$$\begin{aligned} \|s_{h_n}(t) - s_{h_{n-1}}(t)\| &\leq \frac{2\kappa}{(2 - \kappa)M(\kappa)} \left\| \int_0^t (Z_1(y, s_{h_{n-1}}) - Z_1(y, s_{h_{n-2}})) dy \right\| \\ &+ \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \|(Z_1(t, s_{h_{n-1}}) - Z_1(t, s_{h_{n-2}}))\|. \end{aligned} \tag{4.14}$$

The Lipschitz condition is satisfy with the kernel, we have

$$\begin{aligned} \|s_{h_n}(t) - s_{h_{n-1}}(t)\| &\leq \frac{2\kappa}{(2 - \kappa)M(\kappa)} \gamma_1 \int_0^t \|s_{h_{n-1}} - s_{h_{n-2}} dy\| \\ &+ \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \gamma_1 \|s_{h_{n-1}} - s_{h_{n-2}}\|, \end{aligned} \tag{4.15}$$

then we get

$$\|\phi_n(t)\| \leq \frac{2\kappa}{(2 - \kappa)M(\kappa)} \gamma_1 \int_0^t \|\phi_{n-1}(y)\| dy + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \gamma_1 \|\phi_{n-1}(t)\|. \tag{4.16}$$

Similarly, the following results are obtained by us:

$$\begin{aligned} \|\varsigma_n(t)\| &\leq \frac{2\kappa}{(2 - \kappa)M(\kappa)} \gamma_5 \int_0^t \|\varsigma_{n-1}(y)\| dy + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \gamma_5 \|\varsigma_{n-1}(t)\|, \\ \|\chi_n(t)\| &\leq \frac{2\kappa}{(2 - \kappa)M(\kappa)} \gamma_4 \int_0^t \|\chi_{n-1}(y)\| dy + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \gamma_4 \|\chi_{n-1}(t)\|, \\ \|\xi_n(t)\| &\leq \frac{2\kappa}{(2 - \kappa)M(\kappa)} \gamma_3 \int_0^t \|\xi_{n-1}(y)\| dy + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \gamma_3 \|\xi_{n-1}(t)\|, \\ \|\psi_n(t)\| &\leq \frac{2\kappa}{(2 - \kappa)M(\kappa)} \gamma_2 \int_0^t \|\psi_{n-1}(y)\| dy + \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} \gamma_2 \|\psi_{n-1}(t)\|. \end{aligned} \tag{4.17}$$

Now we are presenting the subsequent theorem by consideration of the above results, □

Theorem 4.2 *The fractional dengue Models (3.3) with Non-Markovian Properties has a system of solutions under the conditions that we can find t_0 such that*

$$\frac{2\kappa}{(2-\kappa)M(\kappa)}\gamma_1 t_0 + \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)}\gamma_1 \leq 1$$

Proof Here first we considered that the functions $i_v(t)$, $e_v(t)$, $i_h(t)$, $e_h(t)$, $s_h(t)$ are bounded and Also, we prove that Lipschitz condition is satisfy with the kernels and hence on consideration of the results of Eqs. (4.16) and (4.17) and by employing the recursive method, we derive the relation as follows:

$$\begin{aligned} \|\phi_n(t)\| &\leq \|s_h(0)\| \left[\left(\frac{2(1-\kappa)}{(2-\kappa)M(\kappa)}\gamma_1 \right) + \left(\frac{2\kappa}{(2-\kappa)M(\kappa)}\gamma_1 t \right) \right]^n, \\ \|\psi_n(t)\| &\leq \|e_h(0)\| \left[\left(\frac{2(1-\kappa)}{(2-\kappa)M(\kappa)}\gamma_2 \right) + \left(\frac{2\kappa}{(2-\kappa)M(\kappa)}\gamma_2 t \right) \right]^n, \\ \|\xi_n(t)\| &\leq \|i_h(0)\| \left[\left(\frac{2(1-\kappa)}{(2-\kappa)M(\kappa)}\gamma_3 \right) + \left(\frac{2\kappa}{(2-\kappa)M(\kappa)}\gamma_3 t \right) \right]^n, \\ \|\chi_n(t)\| &\leq \|e_v(0)\| \left[\left(\frac{2(1-\kappa)}{(2-\kappa)M(\kappa)}\gamma_4 \right) + \left(\frac{2\kappa}{(2-\kappa)M(\kappa)}\gamma_4 t \right) \right]^n, \\ \|\zeta_n(t)\| &\leq \|i_v(0)\| \left[\left(\frac{2(1-\kappa)}{(2-\kappa)M(\kappa)}\gamma_5 \right) + \left(\frac{2\kappa}{(2-\kappa)M(\kappa)}\gamma_5 t \right) \right]^n. \end{aligned} \quad (4.18)$$

Therefore, the system of functions (4.12) is smooth and exists. However, to show that the above functions are the system of solutions of the given system of Eq. (3.3), we assume that

$$\begin{aligned} i_v(t) - i_v(0) &= i_v(t) - F_{v_n}(t) \\ e_v(t) - e_v(0) &= e_{v_n}(t) - E_{v_n}(t), \\ i_h(t) - i_h(0) &= i_{h_n}(t) - D_{h_n}(t), \\ e_h(t) - e_h(0) &= e_{h_n}(t) - C_{h_n}(t), \\ s_h(t) - s_h(0) &= s_{h_n}(t) - B_{h_n}(t). \end{aligned} \quad (4.19)$$

So, we have

$$\begin{aligned} \|B_{h_n}(t)\| &= \left\| \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} (Z(t, s_h) - Z(t, s_{h_{n-1}})) \right. \\ &\quad \left. + \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z(y, s_h) - Z(y, s_{h_{n-1}})) dy \right\| \\ &\leq \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} \| (Z(t, s_h) - Z(t, s_{h_{n-1}})) \| \\ &\quad + \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t \| (Z(y, s_h) - Z(y, s_{h_{n-1}})) \| dy \\ &\leq \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} \gamma_1 \|s_h - s_{h_{n-1}}\| \end{aligned}$$

$$+ \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t \gamma_1 \|s_h - s_{h_{n-1}}\| t. \tag{4.20}$$

On using this process recursively, it yields

$$\|B_{h_n}(t)\| \leq \left(\frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} + \frac{2\kappa}{(2-\kappa)M(\kappa)} t \right)^{n+1} \gamma_1^{n+1} \alpha. \tag{4.21}$$

On taking the limit on Eq. (4.21) as $n \rightarrow \infty$, we get

$$\|B_{h_n}(t)\| \rightarrow 0.$$

Similarly, we get

$$\|F_{v_n}(t) \rightarrow 0\|, \|E_{v_n}(t) \rightarrow 0\|, \|D_{h_n}(t) \rightarrow 0\|, \text{ and } \|C_{h_n}(t)\| \rightarrow 0.$$

Hence existence is verified. □

Now, On proving the uniqueness of a system of solutions of Eq. (3.3)

Let there exist another system of solutions of (3.3) $s_{h_1}(t), e_{h_1}(t), i_{h_1}(t), v_{v_1}(t)$ and $i_{v_1}(t)$ then

$$\begin{aligned} s_h(t) - s_{h_1}(t) &= \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} (Z_1(t, s_h) - Z_1(t, s_{h_1})) \\ &+ \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t (Z_1(y, s_h) - Z_1(y, s_{h_1})) dy. \end{aligned} \tag{4.22}$$

On Eq. (4.22), if we applying norm then we get,

$$\begin{aligned} \|s_h(t) - s_{h_1}(t)\| &\leq \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} \|(Z_1(t, s_h) - Z_1(t, s_{h_1}))\| \\ &+ \frac{2\kappa}{(2-\kappa)M(\kappa)} \int_0^t \|(Z_1(y, s_h) - Z_1(y, s_{h_1}))\| dy. \end{aligned} \tag{4.23}$$

From employing the Lipschitz conditions of the kernel, we have

$$\|s_h(t) - s_{h_1}(t)\| \leq \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} \gamma_1 \|s_h - s_{h_1}\| + \frac{2\kappa}{(2-\kappa)M(\kappa)} \gamma_1 t \|(s_h - s_{h_1})\|. \tag{4.24}$$

It gives

$$\|s_h(t) - s_{h_1}(t)\| \left(1 - \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} \gamma_1 - \frac{2\kappa}{(2-\kappa)M(\kappa)} \gamma_1 t \right) \leq 0. \tag{4.25}$$

Theorem 4.3 *The system of Eq. (3.3) has a unique system of solutions if the following condition holds:*

$$\|s_h(t) - s_{h_1}(t)\| \left(1 - \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)}\gamma_1 - \frac{2\kappa}{(2 - \kappa)M(\kappa)}\gamma_1 t \right) \geq 0. \tag{4.26}$$

Proof If the condition holds (4.26), then

$$\|s_h(t) - s_{h_1}(t)\| \left(1 - \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)}\gamma_1 - \frac{2\kappa}{(2 - \kappa)M(\kappa)}\gamma_1 t \right) \leq 0, \tag{4.27}$$

then we have

$$\|s_h(t) - s_{h_1}(t)\| = 0.$$

Then we get

$$s_h(t) = s_{h_1}(t) \tag{4.28}$$

Similarly, we have

$$\begin{aligned} i_v(t) &= i_{v_1}(t), \\ e_v(t) &= e_{v_1}(t), \\ i_h(t) &= i_{h_1}(t), \\ e_{h_1}(t) &= e_{h_1}(t). \end{aligned} \tag{4.29}$$

Therefore, this verified the uniqueness of the system of solutions of Eq. (3.3). \square

5 Numerical Solution

In this section, we construct a numerical scheme for fractional model based on the CF derivative. On applying this scheme we first consider the following non-linear fractional ODE:

$$\begin{cases} {}_0^C D_t^\kappa u(t) = f(t, u(t)) \\ u(0) = u_0 \end{cases} \tag{5.1}$$

On applying the fundamental theorem of fc The above eq can be converted to a fractional integral equation:

$$u(t) - u(0) = \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} f(t, u(t)) + \frac{2\kappa}{(2 - \kappa)M(\kappa)} \int_0^t f(\tau, u(\tau)) d\tau, \tag{5.2}$$

At a given point t_{n+1} , $n = 0, 1, 2, \dots$ we reformulated the above equation as

$$u(t_{n+1}) - u(t_0) = \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} [f(t_{n+1}) - f(t_n)] + \frac{2\kappa}{(2 - \kappa)M(\kappa)} \int_{t_n}^{t_{n+1}} f(\tau, u(\tau)) d\tau \tag{5.3}$$

The second step is approximation of our numerical scheme of the function $f(t, u(t))$. Thus we approximate $f(t, u(t))$ by using the well-known Lagrange interpolation polynomial to obtain following result for the interval $[t_n, t_{n+1}]$,

$$P(\tau) (\approx f(\tau, u(\tau))) = \left\{ \frac{(\tau - t_{n-1})}{(t_n - t_{n-1})} \right\} f(t_n, u_n) + \left\{ \frac{(\tau - t_n)}{(t_{n-1} - t_n)} \right\} f(t_{n-1}, u_{n-1}) \tag{5.4}$$

$$P(\tau) (\approx f(\tau, u(\tau))) = \left\{ \frac{(\tau - t_{n-1})}{(t_n - t_{n-1})} \right\} f_n + \left\{ \frac{(\tau - t_n)}{(t_{n-1} - t_n)} \right\} f_{n-1} \tag{5.5}$$

The above approximation can included in Eq. (5.3) to produce

$$u(t_{n+1}) - u(t_0) = \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} [f(t_{n+1}) - f(t_n)] + \frac{2\kappa}{(2 - \kappa)M(\kappa)} \int_{t_n}^{t_{n+1}} \left[\left\{ \frac{(\tau - t_{n-1})}{(t_n - t_{n-1})} \right\} f_n + \left\{ \frac{(\tau - t_n)}{(t_{n-1} - t_n)} \right\} f_{n-1} \right] d\tau \tag{5.6}$$

thus, after some simplifications and integrating, the following equation is obtained:

$$u_{n+1} - u_n = \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} [f_{n+1} - f_n] + \frac{2\kappa}{(2 - \kappa)M(\kappa)} h \left[\frac{3}{2} f_n - \frac{1}{2} f_{n-1} \right] \tag{5.7}$$

Now for finding the numerical solution of fractional model based on the CF derivative. For the Eq. (3.3) we get the solution

$$\begin{aligned} s_{h_{n+1}} - s_{h_n} &= \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} [\mu_h(1 - s_{h_{n+1}}) - \alpha s_{h_{n+1}} i_{v_{n+1}} - \mu_h(1 - s_{h_n}) + \alpha s_{h_n} i_{v_n}] \\ &\quad + \frac{2\kappa h}{(2 - \kappa)M(\kappa)} \left[\frac{3}{2} [\mu_h(1 - s_{h_n}) - \alpha s_{h_n} i_{v_n}] - \frac{1}{2} [\mu_h(1 - s_{h_{n-1}}) - \alpha s_{h_{n-1}} i_{v_{n-1}}] \right] \\ e_{h_{n+1}} - e_{h_n} &= \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} [\alpha s_{h_{n+1}} i_{v_{n+1}} - \beta e_{h_{n+1}} - \alpha s_{h_n} i_{v_n} + \beta e_{h_n}] \\ &\quad + \frac{2\kappa h}{(2 - \kappa)M(\kappa)} \left[\frac{3}{2} [\alpha s_{h_n} i_{v_n} - \beta e_{h_n}] - \frac{1}{2} [\alpha s_{h_{n-1}} i_{v_{n-1}} - \beta e_{h_{n-1}}] \right] \\ i_{h_{n+1}} - i_{h_n} &= \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} [v_{h_{n+1}} e_{h_{n+1}} - \gamma i_{h_{n+1}} - v_{h_n} e_{h_n} + \gamma i_{h_n}] \\ &\quad + \frac{2\kappa h}{(2 - \kappa)M(\kappa)} \left[\frac{3}{2} [v_{h_n} e_{h_n} - \gamma i_{h_n}] - \frac{1}{2} [v_{h_{n-1}} e_{h_{n-1}} - \gamma i_{h_{n-1}}] \right] \\ e_{v_{n+1}} - e_{v_n} &= \frac{2(1 - \kappa)}{(2 - \kappa)M(\kappa)} [\delta s_{v_{n+1}} i_{h_{n+1}} - (\epsilon + v_{v_{n+1}}) e_{v_{n+1}} - \delta s_{v_n} i_{h_n} + (\epsilon + v_{v_n}) e_{v_n}] \end{aligned}$$

$$\begin{aligned}
 & + \frac{2\kappa h}{(2-\kappa)M(\kappa)} \left[\frac{3}{2} [\delta s_{v_n} i_{h_n} - (\epsilon + v_{v_n}) e_{v_n}] - \frac{1}{2} [\delta s_{v_{n-1}} i_{h_{n-1}} - (\epsilon + v_{v_{n-1}}) e_{v_{n-1}}] \right] \\
 i_{v_{n+1}} - i_{v_n} = & \frac{2(1-\kappa)}{(2-\kappa)M(\kappa)} [v_{v_{n+1}} e_{v_{n+1}} - \epsilon i_{v_{n+1}} - v_{v_n} e_{v_n} + \epsilon i_{v_n}] \\
 & + \frac{2\kappa h}{(2-\kappa)M(\kappa)} \left[\frac{3}{2} [v_{v_n} e_{v_n} - \epsilon i_{v_n}] - \frac{1}{2} [v_{v_{n-1}} e_{v_{n-1}} - \epsilon i_{v_{n-1}}] \right] \tag{5.8}
 \end{aligned}$$

6 Numerical Simulation

In this part, By using the proposed numerical scheme of the model for different values of fractional order we present the numerical simulation. The numerical simulations are shown in Figs. 1, 2, 3, 4 and 5. Figure 1 is considered κ to be 1, Fig. 2 is considered κ to be 0.75, Fig. 3 is considered κ to be 0.55, in Fig. 4 is considered κ to be 0.35 and finally Fig. 5 is considered κ to be 0.15.

To achieve our numerical simulation the following initial conditions and parameters were used [17].

$N_h = 5,071,126$, $\pi_v = 2,500,000$, $v_h = 0.1667$, $\mu_h = 0.0045$, $\mu_v = 0.02941$, $\gamma_h = 0.328833$, $b\beta_h = 0.75$, $b\beta_v = 0.375$, $v_v = 0.1428$.

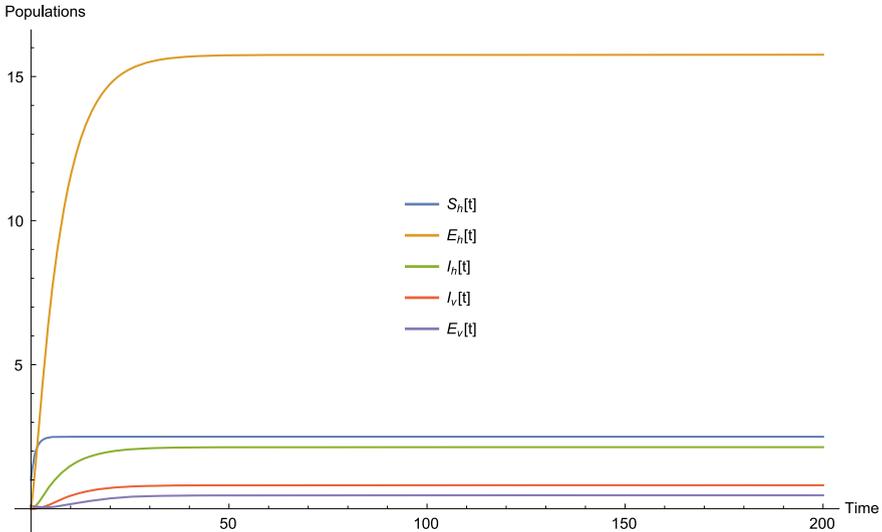


Fig. 1 For $\kappa = 1$

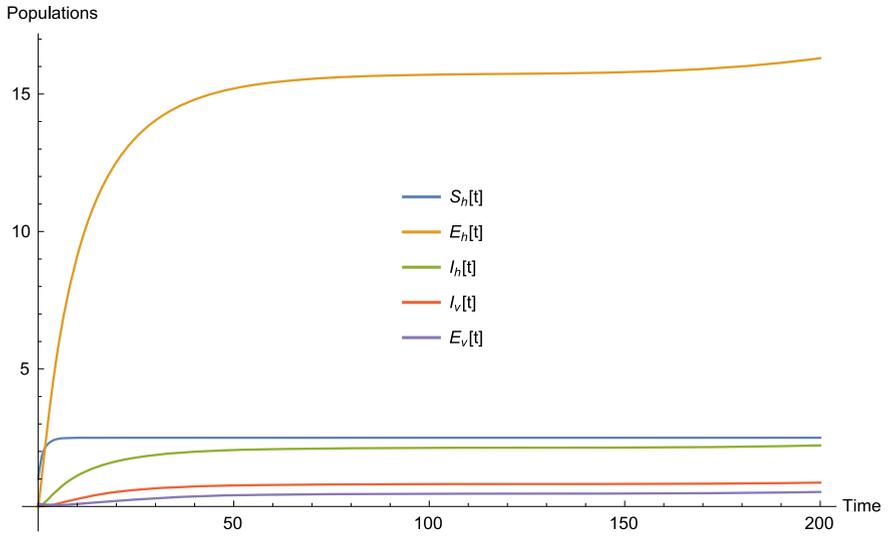


Fig. 2 For $\kappa = 0.75$

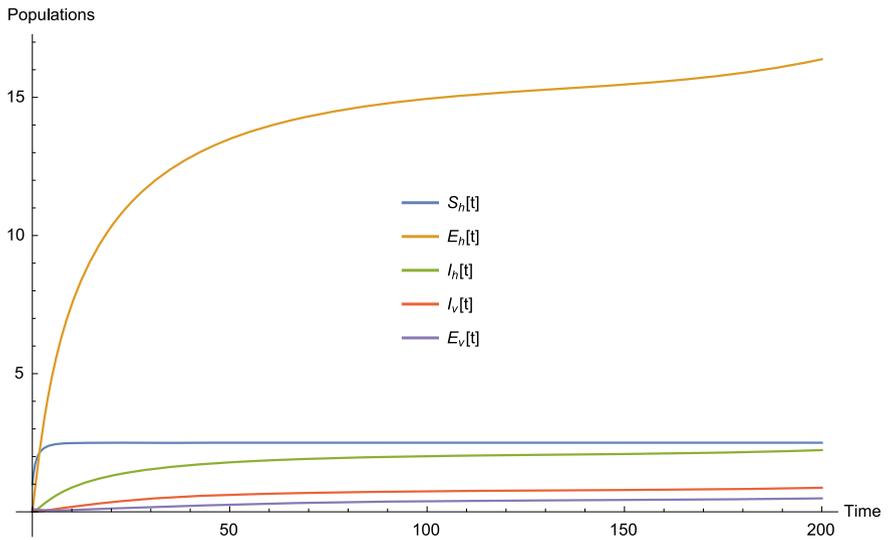


Fig. 3 For $\kappa = 0.55$

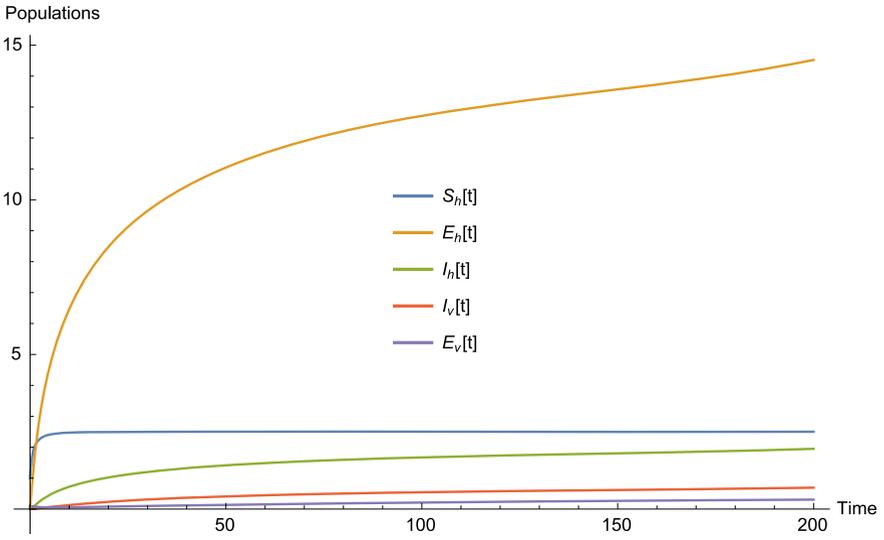


Fig. 4 For $\kappa = 0.35$

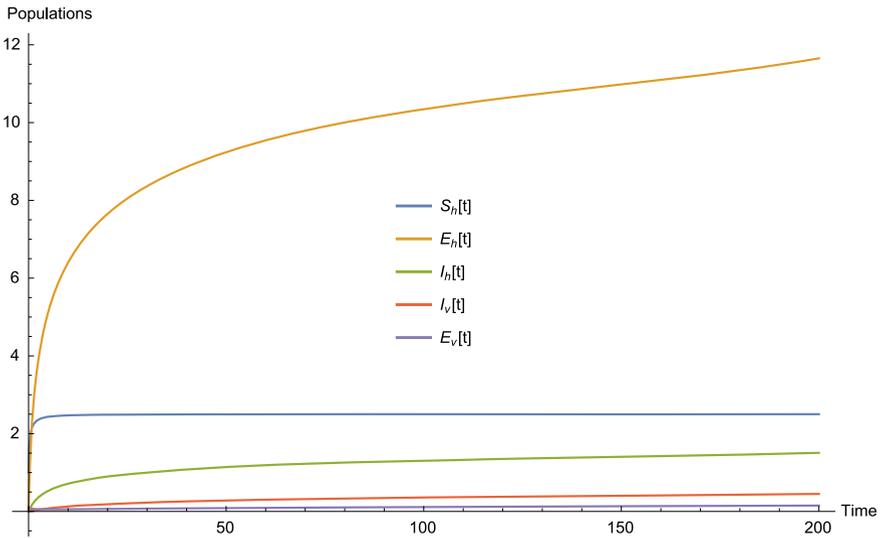


Fig. 5 For $\kappa = 0.15$

7 Conclusion

Although the dynamics spread of Dengue fever has in the attention of many researchers in the same field of applied mathematics in biology, it is worth noting that, still there is no attention has been given to modeling the spread with a differential operator having non-Markovian properties but the associated evolution equation having Markovian properties. If we consider the recent development in fractional differentiation and integration, a derivative with non-local kernel and non-singular was suggested by Caputo and Fabrizio and posses several properties that one observed in many problems occurring in biological modeling. We these properties, we devoted our paper to the discussion and analysis underpinning the dynamical spread of Dengue in given population. We provided a motivation to underpin why this operator is used for this model, then, we presented a detailed analysis of uniqueness and existence and the exact solution using the fixed-point theorem in Banach space. With the aim of improving the accuracy of numerical scheme, a new method was suggested by Toufit and Atangana [19] and was found to be highly accurate and very easier to implement. We used this numerical scheme to solve the new model with fading memory induces by the exponential kernel and presented numerical simulation.

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