



Radar Adaptive Sidelobe Cancellation Technique Based on Spatial Filtering

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Abstract. The electromagnetic environment of radar operation is increasingly complex, and active interference will have a great impact on radar performance. Side-lobe cancellation technology is an effective means to eliminate interference by auxiliary antennas. This paper introduces an adaptive beamforming algorithm to form the cancellation weight based on the secondary antenna. The weight convergence speed of several algorithms is analyzed, and the cancellation ability is analyzed, and a normalized least mean square algorithm is proposed.

Keywords: Sidelobe cancellation · Adaptive sidelobe cancellation · Least mean square algorithm

1 Introduction

The space electromagnetic environment under the process of informationization has become increasingly complex, and the struggle for electromagnetic space has been unprecedentedly intensified, which has had a profound impact on military activities. The interference of ground clutter and various active interferences bring continuous challenges to the development of radar systems at this stage [1].

The sharp deterioration of the electromagnetic environment in which radars operate is a serious challenge for modern radar systems. In order to extract targets from strong ground clutter and interference, the radar also has anti-resistance such as adaptive interference suppression and frequency agility to obtain lower antenna side lobes, low intercept rate performance and high maneuverability. In order to adapt to the complex and varied application environment, the radar system must have higher mobility and flexibility, and lower development and maintenance costs [2]. Therefore, a new radar anti-jamming technology that can well balance the above requirements is needed. It is a good choice regardless of the anti-jamming effect of the side-lobe cancellation technology or its implementation cost.

The function of the sidelobe cancellation system is to cancel the sidelobe interference. It sets a certain number of auxiliary antennas around the main antenna of the radar to form an adaptive array with the main antenna. The adaptive weighting of the auxiliary array makes the synthesis of the entire sidelobe cancellation system. The zero point of the receiving pattern adaptively aligns the interference direction to achieve the purpose of suppressing interference. In the sidelobe cancellation system, the weighting coefficient of the main antenna is always 1, and the weighting coefficient of the

auxiliary array is determined by the adaptive algorithm. Therefore, the sidelobe cancellation system is a special case of the adaptive beamforming system.

The adaptive algorithm used in the radar sidelobe cancellation system can be divided into open-loop algorithm and closed-loop algorithm. The open-loop algorithm has a large amount of computation and engineering implementation is difficult. This paper studies the closed-loop adaptive algorithm. The closed-loop algorithm is mainly based on the Wiener filtering algorithm with minimum gradient descent. The optimal solution is obtained according to the selected performance surface function. The adaptive algorithm obtained by this principle is the least mean square algorithm (LMS). However, its convergence speed and error characteristics are difficult to balance, and high convergence speed will bring about a large steady-state error. The sampling matrix inversion algorithm (SMI) extended by this algorithm can improve the convergence speed and the contradiction of steady state error, but the amount of calculation becomes larger. In addition, considering the constraint of the dispersion degree of the correlation matrix caused by the gradient steepest descent principle adopted by LM and SMI algorithm, this paper uses the conjugate gradient method (CGM) to improve. Each iteration of the orthogonal path, constantly updated to seek the optimal solution. This paper proposes a normalized LMS algorithm to improve the contradiction between the convergence speed and steady state error of a typical LMS algorithm.

2 Radar Sidelobe Cancellation Technology

The active interference of the radar can be accessed not only from the main lobe of the antenna but also from the side lobes of the antenna. One of the ways to deal with co-channel interference from side lobes into the receiver is to use very low side lobes. However, the development of low sidelobe antennas is extremely difficult and costly, and only the newly developed radar antennas have lower sidelobe levels. Moreover, with the advancement of interference technology, the effective power of interference increases continuously, and it is difficult to effectively suppress strong sidelobe interference only by the extremely low sidelobe antenna. The most effective way to deal with sidelobe interference is the sidelobe adaptive cancellation. The pattern of the antenna has spatial filtering characteristics, that is, "space selection". The main lobe is equivalent to "passband", and the side lobes are equivalent to "stopband". If the auxiliary antenna is additionally added, the signal it receives is made. The sum is weighted to form a new spatial filtering characteristic, further eliminating the interference signal received by the side lobes of the main antenna [3].

The adaptive sidelobe cancellation achieves a nulling at the interference angle of the sidelobe position to suppress strong interference signals entering the antenna by the side lobes. Generally, an adaptive canceller is composed of a high-gain radar main antenna and a plurality of low-gain auxiliary antennas (the gain of the auxiliary antenna is equivalent to the first side lobe gain of the main antenna), and the adaptive processor is based on the main and auxiliary antennas. The received signal calculates a set of weight coefficients, adjusts the amplitude and phase of the auxiliary antenna, and adaptively forms a zero point in the active interference direction to achieve the purpose of suppressing active interference. The sidelobe cancellation schematic is shown in the Fig. 1.

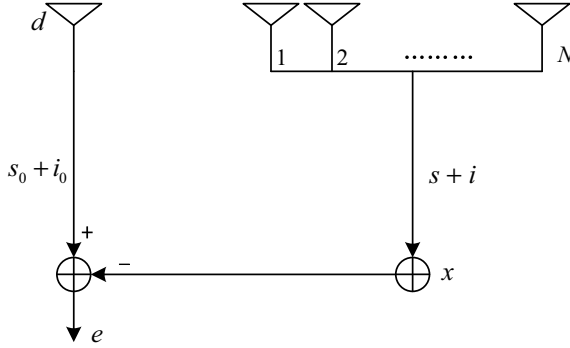


Fig. 1. Schematic diagram of sidelobe cancellation

In the schematic diagram, k represents the moment, and $d(k)$ represents the main lobe signal, which includes the useful signal $s_0(k)$ and the interference signal $i_0(k)$ received by the radar main lobe [4].

For N auxiliary antennas, the signal received by the auxiliary antenna is an $N \times 1$ dimensional matrix $x(k)$, where the useful signal and the interference signal are respectively $s(k)$ and $i(k)$, adaptive side lobes. The weight vector obtained by the cancellation algorithm can be expressed as $N \times 1$ dimensional matrices $\omega(k)$, the auxiliary antenna output $e(k)$ in the cancellation system can be expressed as:

$$e(k) = d(k) - \omega^H(k)x(k) \quad (1)$$

since the mean value of the useful signal is zero, that is, $e(k)$ is the cancellation residual, and the representation pair Ability to dissipate.

3 Adaptive Cancellation Algorithm

3.1 Least Mean Square Algorithm

The purpose of the least mean square algorithm is to minimize the square of the average error. Through algebraic calculation, multiple iterations are used to obtain the convergence coefficient of the antenna array to achieve the desired antenna array performance [5]. The mean square error expression is:

$$|e(k)|^2 = |d(k)|^2 - 2d(k)\omega^H x(k) + \omega^H(k)x(k)x^H(k)\omega(k) \quad (2)$$

For the quadric surface formed by the objective function, the iterative relationship of weights is obtained through its performance function, and the optimal search based on gradient information is realized. We use the steepest descent method (the opposite direction of the gradient) to iteratively obtain the gradient, and then derive the weight change relationship [6].

The recursive relationship of weights is:

$$\omega(k+1) = \omega(k) + \frac{1}{2} \mu [-\nabla(k)] \tag{3}$$

$$\omega(k+1) = \omega(k) - \mu [R_{xx}\omega - r] = w(k) + \mu e^*(k)x(k) \tag{4}$$

When the gradient vector is zero, that is, when the Wiener solution is reached, the following relationship is obtained:

$$E[e(k)x(k)] = 0 \tag{5}$$

The implementation steps of the least mean square algorithm are:

- (1) Weight initial setting, vector $\omega(k)$ is initially $N \times 1$ all zero matrix
- (2) Define the auxiliary antenna to receive the signal:

$$x(k) = vS \times s(k) + vS \times i(k) + n \tag{6}$$

vS stands for $N \times 1$ -dimensional steering vector and n stands for noise.

- (3) Define the auxiliary antenna output signal: $y(k) = w^H(k)x(k)$
- (4) Weight update: $\omega(k+1) = \omega(k) - \mu [R_{xx}\omega - r] = w(k) + \mu e^*(k)x(k)$

3.2 Sampling Matrix Inversion Algorithm

A significant disadvantage of the least mean square algorithm is that it must undergo multiple iterations before reaching a stable convergence. We use the sampling matrix inversion algorithm to perform time-correlated estimation of the K -sampled array matrix. It does not require iterative calculations and is an adaptive algorithm based on the Maximum Signal to Interference and Noise Ratio (SINR) criterion [7]. For this block adaptation method, the k th block within the K samples is sampled: $X_K(k)$.

$$X_K(k) = \begin{bmatrix} x_1(1+kK) & x_1(2+kK) & x_1(K+kK) \\ x_2(1+kK) & x_2(2+kK) & x_2(K+kK) \\ x_M(1+kK) & x_M(2+kK) & x_M(K+kK) \end{bmatrix} \tag{7}$$

The implementation steps of the sampling matrix inversion algorithm are:

- (1) Weight initial setting, vector $\omega(k)$ is initially $N \times 1$ all zero matrix
- (2) Define the auxiliary antenna to receive the signal:

$$x(k) = vS \times s(k) + vS \times i(k) + n \tag{8}$$

vS stands for $N \times 1$ -dimensional steering vector and n stands for noise.

(3) Calculating the sampling covariance matrix:

$$R_{xx}(k) = \frac{1}{K} X_K(k) X_K^H(k) \quad (9)$$

(4) Computational correlation matrix:

$$r = \frac{1}{K} d^*(k) X_K(k) \quad (10)$$

(5) Calculate the weight vector:

$$\omega_{SMI}(k) = R_{xx}^{-1}(k) r(k) = [X_K(k) X_K^H(k)]^{-1} d^*(k) X_K(k) \quad (11)$$

3.3 Conjugate Gradient Algorithm

In view of the convergence of the correlation matrix dispersion degree on the convergence speed caused by the steepest descent method in the previous algorithms, we use the conjugate gradient method to improve. The orthogonal path of each iteration is continuously updated to seek the optimal solution, because the orthogonal search direction of the CGM algorithm converges the fastest [8]. The goal of the CGM algorithm is to minimize the quadratic cost function by multiple iterations:

$$J(\omega) = \frac{1}{2} \omega^H A \omega - d^H \omega \quad (12)$$

A is a $K \times N$ -dimensional matrix and represents K -sampling of the N -element auxiliary antenna. The gradient value of the cost function is:

$$\nabla J(\omega) = A \omega - d \quad (13)$$

The vector is updated by defining the form of the residual to reduce the number of iterations:

$$r(1) = -J'(\omega(1)) = d - A \omega(1) \quad (14)$$

Define the conjugate direction of the iteration by the residual:

$$D(1) = A^H r(1) \quad (15)$$

The weight iteration relationship is:

$$\omega(k+1) = \omega(k) - \mu(k)D(k) \quad (16)$$

The step size is selected as:

$$\mu(k) = \frac{r^H(k)AA^Hr(k)}{D^H(k)A^HAD(k)} \quad (17)$$

The update of the residual vector and the direction vector can be expressed as:

$$r(k+1) = r(k) + \mu(k)AD(k) \quad (18)$$

$$D(k+1) = A^Hr(k+1) - \alpha(k)D(k) \quad (19)$$

$$\alpha(k) = \frac{r^H(k+1)AA^Hr(k+1)}{r^H(k+1)AA^Hr(k)} \quad (20)$$

3.4 Normalized Least Mean Square Algorithm

Normalized LMS (NLMS) is an improved algorithm based on the typical LMS algorithm, which aims to avoid the interference caused by gradient noise amplification, and adaptively adjust the tracking step size to make the tracking effect, The iterative speed and error variation is better than the typical LMS algorithm with a constant step size. The basic idea is to give a larger step size in the tracking phase, so that the signal converges faster, but after convergence, in order to prevent the steady-state error caused by the excessive step size, the step size is adaptively adjusted through the whole process to achieve fast convergence. Stable after convergence [9]. The basic idea is to adjust the step size of the algorithm according to the input signal. The input signal is proportional to the steady-state error, and the step size is inversely proportional to the steady-state error. The normalized LMS algorithm normalizes the step size by the square norm of the input signal to obtain the step size that changes with the signal to improve the performance of the LMS algorithm [10].

The variable step size LMS algorithm step size can be expressed as:

$$\omega(k+1) = \omega(k) + \mu(k)e^*(k)x(k) \quad (21)$$

In order to achieve fast convergence, it is necessary to select the step value appropriately, reduce the instantaneous square error, and use the instantaneous square error as a simple estimate of the mean square error MSE, which is also the basic idea of the LMS algorithm.

In order to speed up the convergence, it is appropriate to minimize the squared error, obtain the partial derivative of the variable coefficient, and make it zero, and find:

$$\mu(k) = \frac{1}{x^T(k)x(k)} \tag{22}$$

The resulting step value may cause a negative value of the instantaneous error variation. In order to control the offset, considering the derivative of the instantaneous square error is not equal to the mean square error MSE derivative value, the normalized LMS algorithm is modified as follows:

$$\omega(k+1) = \omega(k) + \frac{\mu}{\gamma + x^T(k)x(k)} e(k)x(k) \tag{23}$$

where μ is called a fixed convergence factor and its purpose is to control the amount of offset. The parameter γ is set to avoid the $x^T(k)x(k)$ is too small and the step value is too large.

4 Simulation and Performance Analysis

The simulation analysis in this paper is based on the linear arrangement of 8 auxiliary antennas. The array element spacing is 0.5λ , and the desired signal arrival angle is 0° . The interference signal has a wave azimuth angle of 30° and a useful signal mean of zero. The simulation analysis analyzes the convergence speed and weight stability of LMS algorithm, SMI algorithm and CGM algorithm. Finally, the normalized LMS algorithm proposed by the simulation compares the convergence performance of typical LMS algorithm (Fig. 2).

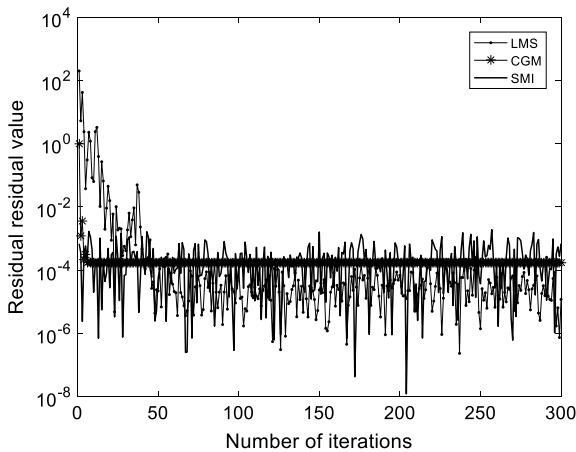


Fig. 2. Algorithm cancellation comparison chart

The convergence of the LMS is achieved through continuous iteration, but the convergence speed is slower, and the offset is also large after convergence. The SMI

algorithm performs time-correlation estimation through the antenna array correlation matrix of the sampling point to obtain the optimal weight and direction graph. It does not need to be iterated, so the algorithm is fast, but the inverse operation is performed on a large amount of data, and the hardware implementation is very complicated. The algorithm performance of CGM algorithm is very superior, with fast convergence speed and high stability. However, the algorithm is very complicated due to the iterative update of weights by conjugate gradient method.

The contradiction between the convergence speed and the steady-state error, the steps of 0.1, $5e-3$ and $2e-3$ are selected for comparison. Through Fig. 3, we can observe that the step value of 0.1 has the fastest convergence rate, but the convergence value is very obvious at the convergence value after convergence. The step value $2e-3$ has the slowest convergence rate and is stable after convergence. The step value $5e-3$ convergence speed and stability are in between.

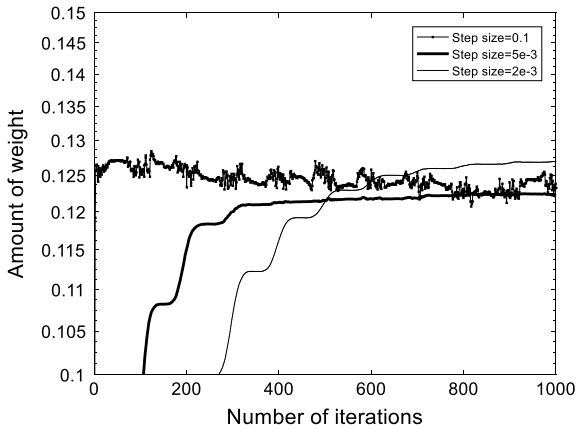


Fig. 3. Weight magnitude iteration graph

In the Fig. 4, the normalized LMS algorithm dynamically changes the step size in the weight update formula. Because the fixed step value is too large, the convergence speed is fast but there is a large steady-state error value after convergence, and the step value is selected to be small. It is stable after convergence but the convergence speed is very slow. The improved algorithm performs normalization calculation based on the error value and signal value of the current point. As the iteration proceeds, the step size in the weight update formula is changed. In the early iteration, the larger step size is used to increase the convergence speed. As the convergence continues, the normalized step value becomes smaller to maintain a small steady state error.

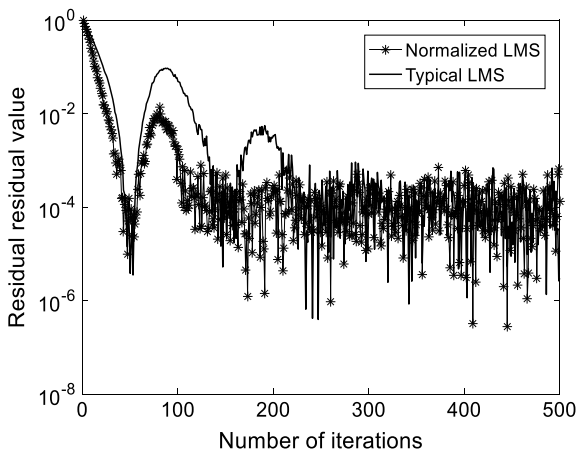


Fig. 4. Weight convergence speed characteristic diagram

5 Conclusion

In this paper, we study the radar sidelobe cancellation technology for active interference and analyze the principle of sidelobe cancellation. An adaptive cancellation algorithm based on auxiliary antenna is introduced to analyze the performance characteristics of several adaptive algorithms. LMS algorithm with excellent performance and complexity is improved. NLMS algorithm is proposed to verify the normalization. LMS algorithm can solve the contradiction between convergence speed and steady state error.

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