Random Vibration Damage Detection for a Composite Beam Under Varying Non-measurable Conditions: Assessment of Statistical Time Series Robust Methods

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Abstract. The problem of vibration-response-only damage detection for a composite beam under variable and non-measurable Environmental and Operational Conditions (EOCs) is considered via three unsupervised Statistical Time Series (STS) type robust detection methods. These include three versions of a novel Functional Model (FM) based method, a Multiple Model (MM) based method, and a Principal Component Analysis (PCA) based method. Performance assessment is based on hundreds of inspection experiments under temperature ranging from 0 to 28 °C and tightening torque ranging from 1 to 4 Nm. The results confirm the methods' high effectiveness, with a version of the FM based method and the MM based method achieving ideal performance, characterized by 100% correct detection rate for 0% false alarm rate.

Keywords: Robust damage detection · Structural Health Monitoring (SHM) · Vibration based methods · Unsupervised methods · Composite structures · Uncertainty

1 Introduction

Random vibration based Structural Health Monitoring (SHM) has rapidly progressed over the past several years, reaching high levels of technological maturity [1–3]. Statistical Time Series (STS) type methods [4], which employ corresponding models of the structural dynamics, are popular as they offer various advantages, including the exclusive use of data-based stochastic models which may be quite compact, only partially describing the dynamics.

Yet, a major challenge relating to effective damage diagnosis under variable Environmental and Operating Conditions (EOCs) still remains. The fundamental reason behind it has to do with the fact that variable EOCs may affect the underlying structural dynamics to a degree that may be similar or even greater

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than that caused by damage, and as such changes are at the core of damage diagnosis, the latter may become highly challenging and ineffective [5]. Overcoming this challenge requires the development of *robust* diagnosis methods, that are methods capable of 'separating', to the extent possible, the effects of variable EOCs from those of damage on the structural dynamics [5,6].

This generally requires the modeling of the considered dynamics under variable EOCs and uncertainty. Such modeling may assume various forms and be broadly classified—along with corresponding methods—as 'explicit' or 'implicit'. *'Implicit'* methods include Principal Component Analysis (PCA) [7] and Factor Analysis (FA) [8] based methods, while *'explicit'* methods model the dynamics (for instance the 'healthy' structural dynamics in the context of damage detection) via explicit deterministic or stochastic modeling techniques and include Multiple Model (MM) [9], Random Coefficient (RC) model based [10], and the newly introduced Functional Model (FM) based methods [11–15]. Although assessment of individual methods are available, systematic and critical comparative assessments are still scarce in the literature.

This study <u>*aims*</u> at contributing in this direction via the systematic and critical comparison of three distinct STS robust methods for damage *detection* which are based on *random vibration response signals*, that is:

(a) Versions of the Functional Model (FM) based method introduced by the authors and co-workers in a series of recent conference papers [12–15],
(b) a Multiple Model (MM) based method [9], and,
(c) a Principal Component Analysis (PCA) based method [9].

A new Residual Variance (FM-RV) based version of the FM based method is presently introduced and included in the assessment, along with two Residual Uncorrelatedness versions, one using the Portmanteau test (FM-RU-P version) [15] and one using the Peña-Rodríguez Test (FM-RU-PR version) [12].

All methods are *unsupervised* in nature, implying that random vibration signals only from the healthy structure are employed for their training in the baseline phase. Moreover, the variable EOCs are assumed *non-measurable* under the methods' normal (diagnostic) operation; yet, the FM method assumes their availability in the baseline (training) phase. Also, as only response signals are employed, the methods are based on transmittance type dynamics obtained through stochastic data-based parametric models of the AutoRegressive with eXogenous excitation (ARX) type [15–17].

It is also noted that the damage detection problem is tackled in a *batch mode*, implying that the methods operate on *short duration batches* of signal records collected periodically or on demand over time, with diagnostic decision making implemented at the end of a complete batch; not at each time instant, as it would be the case with *sequential* methods (for instance see [18]).

The comparative critical assessment of the methods is based on an experimental procedure employing a composite structure, which represents the topology of a commercial Unmanned Aerial Vehicle (UAV) boom that operates under variations in Environmental (temperature ranging from 0 to 28 °C) and Operating (tightening torque ranging from 1 to 4 Nm, simulating assembly variability)

Conditions (EOCs). Damage is simulated via the attachment of a small (12.6 g) mass on the beam, while additional uncertainty is introduced via the occasional attachment of a special adhesive tape. Damage detection is assessed in a systematic and statistically reliable way employing hundreds of inspection experiments, with the results presented in terms of Receiver Operating Characteristic (ROC) curves [19, pp. 34–35].

The rest of this article is organized as follows: The experimental set-up is presented in Sect. 2, the STS type robust damage detection methods are reviewed in Sect. 3, and their experimental assessment is presented in Sect. 4. Concluding remarks are finally summarized in Sect. 5.

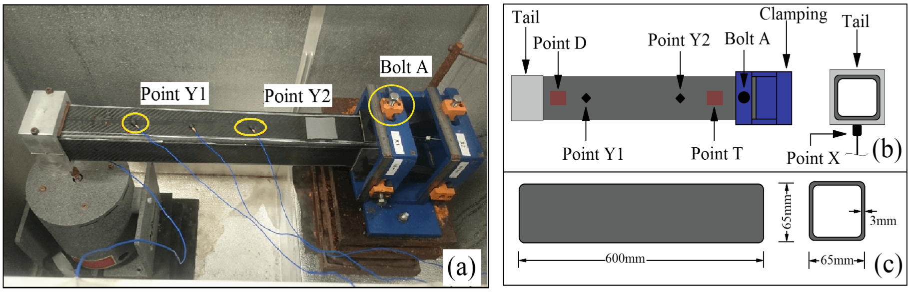

Fig. 1. The beam and the experimental set-up. **a** photo of the set-up. **b** schematic of the beam with the damage position (Point D), adhesive tape position (Point T), excitation position (Point X), and vibration measurement positions (Points Y1 and Y2). **c** geometrical details [12].

2 The Experimental Set-Up

The Structure, the Varying EOCs, and the Damage Scenario. The experiments are based on a lab-scale composite beam (further details in [12]), representing the topology of the main part of a commercial Unmanned Aerial Vehicle (UAV) boom. The beam is clamped at one end, simulating its connection to the fuselage, while its free end is attached to an aluminum mass representing part of the aircraft tail (Fig. 1(a)). The beam is placed in a freezer for temperature variation in the range [0–28] °C, while the tightening torque of Bolt A (Fig. 1(b)) is changed from 1 up to 4 Nm simulating assembly variability. The considered damage scenario is simulated via the attachment of a small, 12.6 g, mass at Point D (Fig. 1(b)) on the beam. Additional uncertainty is introduced via a piece of special (reinforced by plastic mesh) adhesive tape, placed (in certain experiments) at Point T on the surface of the beam (Fig. 1(b)). This serves to simulate potential material and/or manufacturing variability, such as variation in resin, fiber orientation, and so on, among nominally identical composite beams, thus exploring the methods' robustness to unknown uncertainty factors.

Table 1. Experimental details [12].

Structural state	Temperature range (°C)	Torque (Nm)	No. of exps
Baseline (training) phase			
Healthy	Set A := [0° – 28°] with a step of 2°	1, 2, 3, 4	60*
Inspection (diagnosis) phase			
Healthy (H)	A & $\{3°, 21°\}$	1	51◇
	A & $\{9°, 19°, 25°\}$	2	54◇
	A & $\{15°, 25°\}$	3	51◇
	A & $\{3°, 9°, 15°, 19°, 21°\}$	4	60◇
Healthy (H_1) (tape)	[0° 1° 3° 7° 9° 10° 14° ... 15° 17° 21° 23° 25° 27°]	$1, 3, 4$	117◇
Damaged (D) (12.6 g mass)	-//-	-//-	-//-

* 1 experiment per temperature and torque value.
◇ 3 experiments per temperature and torque value.
Sampling frequency $f_s = 4\ 654.5$ Hz; signal length 2 500 samples (0.54 s).
BWD [5–2 327.25] Hz.

The Vibration Signals. Vibration experiments are performed using an electromechanical shaker applying a random, low frequency and band limited, white Gaussian force vertically at Point X, while the acceleration response signals at Points Y1 and Y2 on the beam are acquired through lightweight accelerometers (Fig. 1(a),(b)). Details on the experiments are provided in Table 1; also in [12]. Each measured vibration response signal is sample mean corrected and normalized by its own sample standard deviation.

Preliminary Analysis: Effects of Damage on the Uncertain Dynamics. Welch-based estimates [20, pp. 186–187] of the Transmittance Function (TF) [17] magnitude with the healthy and damaged beams under various EOCs are presented in Fig. 2. Significant variability is observed in the healthy dynamics, which, to a certain extent 'masks' the effects of damage.

3 The Statistical Time Series (STS) Type Robust Damage Detection Methods

Three Statistical Time Series (STS) type robust damage detection methods are employed and assessed. As aforementioned, they are all *unsupervised*, implying that signals obtained only from the healthy structure are employed in the baseline (training) phase, and do not require measurement of the EOC variability factors during their operation (in the inspection phase), although the first method assumes that such measurements are available in the preliminary baseline (training) phase. As the force excitation is assumed non-measurable, all three methods are based on the vibration response signals measured at Points Y1 and Y2, specifically on the corresponding transmittance dynamics. The three methods include: (a) The recently introduced Functional Model (FM) based method,

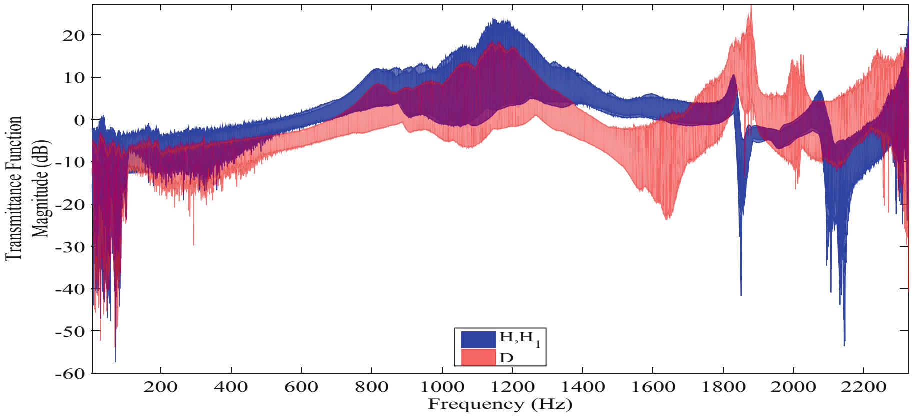

Fig. 2. Effects of damage on the transmittance dynamics under varying conditions: Welch-based transmittance function magnitude estimates based on 333 experiments with the healthy structure (H, H_1) and 117 experiments with the damaged structure. (Point Y1 to Point Y2 transmittance; Estimation based on $N = 100\,000$ sample long signals, Hamming windowing, segment length of 8 192 samples, 95% overlap, MATLAB function: `tfestimate.m`) [12].

with two versions making use of the Residual Uncorrelatedness (versions FM-RU-P and FM-RU-PR) and a third, new version, making use of the Residual Variance (version FM-RV); (b) a Multiple Model (MM) based method; and (c) a Principal Component Analysis (PCA) based method. Brief accounts of the methods are provided below.

3.1 The Functional Model (FM) Based Method

The cornerstone of the FM based method is the proper representation of the healthy structural dynamics under any EOCs in a parameter space, referred to as the '*healthy subspace*'. This subspace is constructed in the *baseline phase* using signal records obtained from controlled experiments (allowing—only in this phase—for measurement of the EOCs) and a data-based Functional Model (FM) [13–15,21].

Baseline (Training) Phase: Off-Line 'Healthy Subspace' Construction. The FM employed—for the healthy subspace construction—in this study is a Vector-dependent Functionally Pooled AutoRegressive with eXogenous excitation (VFP-ARX) model that incorporates the varying temperature and tightening torque through a 2-dimensional *operating (scheduling) parameter vector*[1] $\boldsymbol{k}$. The determination of this model is based on a total number of M controlled experiments under a specific combination of temperature and torque

[1] Vector/matrix quantities are designated by bold face lower/upper characters, respectively.

values. Thus, the *operating parameter vector* $\boldsymbol{k} = [k_l \, k_m]^T$ $(l = 1, 2, \ldots M_1$, $m = 1, 2, \ldots M_2)$ is employed, where k_l and k_m designate variable temperature and torque, respectively, and the subscript discretization index.

The complete set of baseline experiments then provides $M = M_1 \times M_2$ random vibration response signal pairs $x_{\boldsymbol{k}}[t], y_{\boldsymbol{k}}[t]$ with each signal being N samples long. Based on this set, a VFP-ARX$(n_a, n_b)_p$ model of the form [22,23]:

$$y_{\boldsymbol{k}}[t] + \sum_{i=1}^{n_a} a_i(\boldsymbol{k}) \cdot y_{\boldsymbol{k}}[t-i] = \sum_{i=0}^{n_b} b_i(\boldsymbol{k}) \cdot x_{\boldsymbol{k}}[t-i] + e_{\boldsymbol{k}}[t] \tag{1a}$$

$$e_{\boldsymbol{k}}[t] \sim iid\, \mathcal{N}\big(0, \sigma_e^2(\boldsymbol{k})\big) \quad \boldsymbol{k} \in \mathbb{R}^2 \tag{1b}$$

$$a_i(\boldsymbol{k}) = \sum_{j=1}^{p} a_{i,j} \cdot G_j(\boldsymbol{k}), \quad b_i(\boldsymbol{k}) = \sum_{j=1}^{p} b_{i,j} \cdot G_j(\boldsymbol{k}), \quad \sigma_e^2(\boldsymbol{k}) = \sum_{j=1}^{q} s_j \cdot G_j(\boldsymbol{k}) \tag{1c}$$

is obtained. n_a, n_b designate the AR and X orders, respectively, *iid* identically independently distributed, $\mathcal{N}$ Gaussian distribution, $t = 1, 2, 3, \ldots$ the normalized (by the sampling period) discrete time, and $e_{\boldsymbol{k}}[t]$ the innovations (model residual) signal under conditions $\boldsymbol{k}$, which is assumed to be zero-mean, white (serially uncorrelated) with variance $\sigma_e^2(\boldsymbol{k})$ and potentially cross-correlated with their counterparts corresponding to different EOCs (different $\boldsymbol{k}$'s). As indicated by (1c), the AR and X parameters and variance $\sigma_e^2(\boldsymbol{k})$ are modeled as explicit functions of $\boldsymbol{k}$, by using p- and *q-dimensional* functional subspaces, respectively, spanned by the mutually independent functions $G_j(\boldsymbol{k})$. These form one functional subspace basis for the model parameters and one for the residual sequence variance, both consisting of bivariate polynomials obtained as tensor products from typical univariate polynomials such as Legendre, Chebyshev and so on (details in [22]). The constants $a_{i,j}$, $b_{i,j}$, s_j, designate the AR, X and $\sigma_e^2(\boldsymbol{k})$ coefficients of projection, respectively. $a_{i,j}$ and $b_{i,j}$ are estimated based on Ordinary Least Squares (OLS) [22]. Then the residual variance is estimated[2] as [22]:

$$\widehat{\sigma}_{e_o}^2(\boldsymbol{k}, \widehat{\boldsymbol{\theta}}) = \frac{1}{N} \sum_{t=1}^{N} e_{\boldsymbol{k}}^2[t, \widehat{\boldsymbol{\theta}}] \quad \forall\ \boldsymbol{k} = [k_l \, k_m]^T \ (l = 1, 2, \ldots M_1,\ m = 1, 2, \ldots M_2) \tag{2}$$

with the subscript o designating the structure under its *healthy* state (baseline phase) and $\boldsymbol{\theta} = [a_{1,1} \ldots a_{n_a,p} \ \ b_{0,1} \ldots b_{n_b,p}]^T$. Based on the above estimates and (1c), it may be written:

$$\begin{aligned}
\widehat{\sigma}_{e_o}^2(\boldsymbol{k}_1) &= s_1 G_1(\boldsymbol{k}_1) + s_2 G_2(\boldsymbol{k}_1) + \cdots + s_q G_q(\boldsymbol{k}_1) \\
\widehat{\sigma}_{e_o}^2(\boldsymbol{k}_2) &= s_1 G_1(\boldsymbol{k}_2) + s_2 G_2(\boldsymbol{k}_2) + \cdots + s_q G_q(\boldsymbol{k}_2) \\
&\ \ \vdots \qquad\qquad\qquad\qquad \vdots \\
\widehat{\sigma}_{e_o}^2(\boldsymbol{k}_M) &= s_1 G_1(\boldsymbol{k}_M) + s_2 G_2(\boldsymbol{k}_M) + \cdots + s_q G_q(\boldsymbol{k}_M)
\end{aligned}$$

[2] Estimators/estimates are designated by a hat.

Stacking the above in matrix form leads to:

$$\begin{bmatrix} \widehat{\sigma}^2_{e_o}(\boldsymbol{k}_1) \\ \widehat{\sigma}^2_{e_o}(\boldsymbol{k}_2) \\ \vdots \\ \widehat{\sigma}^2_{e_o}(\boldsymbol{k}_M) \end{bmatrix} = \begin{bmatrix} G_1(\boldsymbol{k}_1) & G_2(\boldsymbol{k}_1) & \dots & G_q(\boldsymbol{k}_1) \\ G_1(\boldsymbol{k}_2) & G_2(\boldsymbol{k}_2) & \dots & G_q(\boldsymbol{k}_2) \\ \vdots & \vdots & \ddots & \vdots \\ G_1(\boldsymbol{k}_M) & G_2(\boldsymbol{k}_M) & \dots & G_q(\boldsymbol{k}_M) \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_q \end{bmatrix} \Leftrightarrow \boldsymbol{\sigma}_{e_o} = \boldsymbol{G} \cdot \boldsymbol{s} \tag{3}$$

based on which $\boldsymbol{s}$ is estimated as (^{T} designating matrix transposition):

$$\widehat{\boldsymbol{s}} = [\boldsymbol{G}^T \cdot \boldsymbol{G}]^{-1} \cdot [\boldsymbol{G}^T \cdot \boldsymbol{\sigma}_{e_o}] \tag{4}$$

The determination of the VFP-ARX model orders and its AR and X parameters functional subspace dimensionality p, for a given basis function family, is based on standard procedures using a Genetic Algorithm (GA) for the minimization of the Bayesian Information Criterion (BIC) and the Residual Sum of Squares (RSS), while model validation is based on formal verification of the residual sequence uncorrelatedness (whiteness) hypothesis corresponding to the M experiments used in model estimation. Full details on VFP-ARX model estimation and validation are provided in [22,23]. A similar procedure with a GA is also used for the determination of the residual variance functional subspace dimensionality q.

Inspection (Diagnosis) Phase: On-Line Damage Detection. Damage detection is achieved by examining—using a fresh random vibration signal pair[3] $x_u[t], y_u[t]$ obtained under unknown EOCs (that is unknown current value of $\boldsymbol{k}$)—whether or not the structural dynamics reside within the 'healthy subspace', so that the structure may or may not, respectively, be declared as healthy. This is essentially equivalent to examining whether or not the current signal pair $x_u[t], y_u[t]$ is 'consistent' with the available VFP-ARX model expressing the healthy subspace. This consistency examination may be realized in two steps [24]:

<u>*Step 1*</u>: Employ the VFP-ARX model of the baseline phase to estimate the unknown EOCs vector $\boldsymbol{k}$ that 'best' (according to a proper criterion) expresses the current signal pair. That is, given the current random vibration response signal pair $x_u[t], y_u[t]$, the estimates $\widehat{\boldsymbol{k}}$ and $\widehat{\sigma}^2_{e_u}(\widehat{\boldsymbol{k}})$ (innovations variance) are obtained using the equations of VFP-ARX model (1a) and (1c), as follows[4]:

$$\widehat{\boldsymbol{k}} = \arg\min_{\boldsymbol{k}} \sum_{t=1}^{N} e_u^2[t, \boldsymbol{k}], \quad \widehat{\sigma}^2_{e_u}(\widehat{\boldsymbol{k}}) = \frac{1}{N} \sum_{t=1}^{N} e_u^2[t, \widehat{\boldsymbol{k}}] \tag{5}$$

The estimate of $\boldsymbol{k}$ is obtained based on a GA algorithm followed by nonlinear refinement using Sequential Quadratic Programming [22,23].

[3] The subscript u designates the structure in an unknown health state.
[4] $e_u[t, \boldsymbol{k}]$ corresponds to the residual $e_{\boldsymbol{k}}[t]$ in (1a).

Step 2: 'Consistency' with the healthy subspace is confirmed through the *successful validation* of the model corresponding to $\widehat{\boldsymbol{k}}$. This may be achieved via two model residual based schemes, resulting in corresponding versions of the method: (a) A Residual Uncorrelatedness (RU) based version, and, (b) a Residual Variance (RV) based version that is presently postulated, motivated by [4].

(2a) The Residual Uncorrelatedness (RU) Based Versions. The residual $e_u[t, \widehat{\boldsymbol{k}}]$ whiteness testing, at a user selected risk level, may be achieved via two distinct tests, the Portmanteau Test or the Peña-Rodríguez Test, thus giving rise to the FM-RU-P or the FM-RU-PR versions of the method, respectively.

(i) The Portmanteau Test based (FM-RU-P) version: Residual whiteness is examined via the hypothesis test:

$$\begin{array}{lll} H_o : \rho[\tau] = 0 & \tau = 1, 2, \ldots, h & \text{(null hypothesis – healthy structure)} \\ H_1 : \rho[\tau] \neq 0 & \text{for some } \tau & \text{(alternative hypothesis – damaged structure)} \end{array} \tag{6}$$

where $\rho[\tau]$ designates the normalized autocovariance of the residual sequence at lag τ. The Q statistic below follows chi-square (χ^2) distribution with h degrees of freedom under the null (H_o) hypothesis of a valid model[5], that is [4]:

$$Q := N(N+2) \sum_{\tau=1}^{h} (N-\tau)^{-1} \widehat{\rho}^2[\tau] \sim \chi^2(h) \tag{7}$$

where h is the (user selected) maximum lag. The null hypothesis is then accepted, at a (user selected) risk level α (probability of rejecting H_o even though it is correct) as follows:

$$\begin{array}{rcl} Q < \chi^2_{1-\alpha}(h) & \Rightarrow & \text{(null hypothesis - healthy structure)} \\ \text{Else} & \Rightarrow & \text{(alternative hypothesis - damaged structure)} \end{array} \tag{8}$$

with $\chi^2_{1-\alpha}(h)$ designating the chi-square distribution's $(1-\alpha)$ critical point.

(ii) The Peña-Rodríguez Test based (FM-RU-PR) version: The partial autocorrelation $\pi_{e_u}[\tau]$ is examined via this testing procedure, according to which under the H_o hypothesis the D statistic below follows a standard normal distribution [25]:

$$D := (\lambda/\beta)^{-1/\zeta} (\zeta/\sqrt{\lambda}) \left(Q^{1/\zeta} - (\lambda/\beta)^{1/\zeta} \left(1 - \frac{1}{2\lambda} \left(\frac{\zeta - 1}{\zeta^2} \right) \right) \right) \sim \mathcal{N}(0,1) \tag{9}$$

with:

$$Q = -N \sum_{\tau=1}^{h} \frac{h+1-\tau}{h+1} \log \left(1 - \frac{N+2}{N-\rho} \, \widehat{\pi}^2_{e_u}[\tau] \right)$$

[5] In which case the estimated innovations (residual) series $e_u[t, \widehat{\boldsymbol{k}}]$ is uncorrelated (white).

$$\beta = \frac{3(h+1)(h-2(n_a+n_b))}{2h(2h+1)-12(h+1)(n_a+n_b)}, \ \lambda = \frac{3(h+1)h^2}{2(2h(2h+1)-12(h+1))}$$
$$\zeta = \left(1 - \frac{2(h/2-(n_a+n_b))(h^2/(4(h+1))-(n_a+n_b))}{3(h(2h+1)/(6(h+1))-(n_a+n_b))^2}\right)^{-1}$$

where $\widehat{\pi}_{e_u}[\tau]$ designates the estimated partial autocorrelation of the residual series $e_u[t, \widehat{\boldsymbol{k}}]$ at lag $\tau = 1, 2, \ldots, h$.

Thus the following test is used at the α risk level as follows:

$$\begin{array}{ll} |\mathrm{D}| \leq Z_{1-a} \Rightarrow & H_o \text{ is accepted} \\ \text{Else} \qquad\quad \Rightarrow & H_1 \text{ is accepted} \end{array} \tag{10}$$

with Z_{1-a} designating the standard normal distribution' s $(1-a)$ critical point.

(2b) The Residual Variance (RV) Based Version. In this version damage detection is based on the fact that the variance $\sigma^2_{e_u}(\boldsymbol{k})$ becomes minimal, specifically equal to $\sigma^2_{e_o}(\boldsymbol{k})$ (see (2)), if and only if the current structure is healthy. Thus, the following hypothesis testing problem is constructed:

$$\begin{array}{ll} H_o : \sigma^2_{e_o}(\boldsymbol{k}) = \sigma^2_{e_u}(\boldsymbol{k}) & \text{(null hypothesis – healthy structure)} \\ H_1 : \sigma^2_{e_o}(\boldsymbol{k}) < \sigma^2_{e_u}(\boldsymbol{k}) & \text{(alternative hypothesis – damaged structure)} \end{array} \tag{11}$$

The F statistic below follows F distribution with $(N_u, N_o - d)$ degrees of freedom (N_o and N_u designate the number of samples used in estimating the residual variance in the healthy and current states, typically $N_o = N_u = N$; and d designates the dimensionality of vector $\boldsymbol{\theta}$) [4]:

$$\text{Under } H_o : \ F = \frac{\widehat{\sigma}^2_{e_u}(\widehat{\boldsymbol{k}})}{\widehat{\sigma}^2_{e_o}(\widehat{\boldsymbol{k}})} \sim F(N_u, N_o - d) \tag{12}$$

where $\widehat{\sigma}^2_{e_o}(\widehat{\boldsymbol{k}})$ is obtained via (1c) using the projection coefficients estimates as obtained by (4) and the q basis functions $G_j(\boldsymbol{k})$. The following test is then constructed at the α risk level:

$$\begin{array}{ll} F \leq f_{1-\alpha}(N_u, N_o - d) \Rightarrow & H_o \text{ is accepted (healthy structure)} \\ \text{Else} \qquad\qquad\qquad\quad\;\; \Rightarrow & H_1 \text{ is accepted (damaged structure)} \end{array} \tag{13}$$

with $f_{1-\alpha}(N_u, N_o - d)$ designating the corresponding F distribution's $1-\alpha$ critical point.

It is worth stressing that in the case where the structure is declared as healthy, the FM method also provides *precise estimates* of the current EOCs through $\widehat{\boldsymbol{k}}$.

3.2 A Multiple Model (MM) Based Method

In this method a Multiple Model (MM) representation is employed for modeling the healthy structural dynamics under variable EOCs. This representation consists of a set of conventional ARX-type models along with the Gaussian probability density functions of their estimated parameter vectors (also see [26]).

Baseline (Training) Phase. The M pairs of vibration response signals (also used in the FM based method) obtained under different EOCs are employed for the determination of the MM representation ('healthy set' of models) m_o, based on M transmittance $ARX(n_a, n_b)$ models [17], $m_{o,i}$ ($i = 1, \ldots, M$), with corresponding parameter vectors $\{\boldsymbol{a}_o\} = \{\boldsymbol{a}_{o,1}, \ldots, \boldsymbol{a}_{o,M}\}$. Each estimated vector is (asymptotically, that is as the signal length $N \to \infty$) associated with a Gaussian probability density function, with mean equal to each point estimate $\boldsymbol{\alpha}_{o,i}$ and estimated covariance $\boldsymbol{\Sigma}_{o,i}$.

Inspection (Diagnosis) Phase. Once a fresh pair of vibration response signals is obtained from the structure in unknown health state, the objective is to decide whether or not its dynamics is adequately represented by the MM representation m_o, in which case the structure is declared as healthy, otherwise as damaged. Towards this end, a fresh transmittance function ARX model m_u (with parameter vector $\boldsymbol{a}_u$) and of same orders as those in m_o, is estimated. A distance metric $D(m_o, m_u)$ between m_o and m_u is then obtained (details in [9]), which is presently defined as:

$$D(m_o, m_u) := \sum_{k=1}^{M} d(m_{o,k}, m_u) \tag{14}$$

with $d(m_{o,k}, m_u)$ designating the Kullback–Leibler (KL) divergence (pseudo–distance) [27, pp. 756–758] between the standard ARX models m_o and m_u.

The structure is then declared as healthy if and only if $D(m_o, m_u)$ is smaller than a user–selected threshold; else it is declared as damaged.

3.3 A Principal Component Analysis (PCA) Based Method

This method (referred to as U-PCA-ARX) employs Principal Component Analysis (PCA) on the parameter vector of the transmittance ARX based representation of the structural dynamics [9].

Baseline (Training) Phase. Like in the previous method, M transmittance ARX(n_a, n_b) models [17], with corresponding parameter vectors $\boldsymbol{a}_o = \{\boldsymbol{a}_{o,1}, \ldots, \boldsymbol{a}_{o,M}\}$, are estimated and their sample mean and covariance matrix are then obtained. Each vector is centered by subtracting its sample mean. The Singular Value Decomposition of the covariance matrix is subsequently performed as $\boldsymbol{P} = \boldsymbol{U}\,\boldsymbol{S}^2\,\boldsymbol{U}^T$, where $\boldsymbol{S}^2$ is a diagonal matrix that contains the positive eigenvalues in decreasing order, while $\boldsymbol{U}$ is a real unitary matrix including the corresponding eigenvectors.

The first n eigenvectors of $\boldsymbol{U}$, which explain a certain, user–selected, fraction γ (%) of the total parameter vector variability, are dropped, assuming that they are mainly affected by the varying EOCs. The last m eigenvectors (presumably associated with damage) are stacked in a matrix $\boldsymbol{U}_m$ (containing m columns) that is used by the PCA to transform the (centered) parameter vectors $\{\boldsymbol{a}_o\} = \{\boldsymbol{a}_{o,1}, \ldots, \boldsymbol{a}_{o,M}\}$ ('healthy set' of parameters), into a reduced m-dimensional space [9]. It is noted that, as with all PCA–based methods, the detection performance may be significantly affected by the selected γ [9].

Inspection (Diagnosis) Phase. A new (transmittance) ARX model m_u of the same order as those of the baseline phase and parameter vector $\boldsymbol{a}_u$ is estimated based on a pair of vibration acceleration signals acquired from the current, unknown health state of the beam. The parameter vector's sample mean obtained in the baseline phase is then used to center $\boldsymbol{a}_u$, which is subsequently transformed, as $\bar{\boldsymbol{a}}_u = \boldsymbol{U}_m^T \boldsymbol{a}_u$, into the m-dimensional space. The Euclidean norm of $\bar{\boldsymbol{a}}_u$ is then obtained:

$$D := ||\bar{\boldsymbol{a}}_u||_{l_2} \tag{15}$$

and the structure is declared as healthy if and only if D is smaller than a user–selected threshold; else it is declared as damaged [9].

4 Experimental Assessment of the Methods

The experimental assessment of the three Statistical Time Series robust damage detection methods is based on 117 experiments with the damaged structure and 333 experiments with the healthy structure, all under various temperature and torque conditions which are different from those used in the baseline (training) phase (see Table 1). The comparative assessment of the methods' damage detection performance is based on Receiver Operating Characteristic (ROC) curves [19, pp. 34–35], each one representing the true positive rate (percentage of correct damage detections), versus the false positive rate (percentage of false alarms) for varying detection thresholds.

4.1 Baseline (Training) Phase

A transmittance (function) ARX model is initially obtained based on a pair of vibration-response signals from points Y1 and Y2 (Fig. 1(a)) under certain values of temperature (0°C) and tightening torque (2 Nm) from the structure under healthy state, and a standard identification procedure [20, pp. 203–205]. This includes model order selection based on the Bayesian Information Criterion (BIC) and the Residual Sum of Squares/Signal Sum of Squares (RSS/SSS), as well as model parameter estimation via OLS (MATLAB function: arx.m). This leads to an ARX(70,70) model characterized by zero delay, that is $b_0 \neq 0$ in the exogenous polynomial.

The Functional Model (FM) Based Method. Maintaining the AR and X orders and the zero delay for the VFP-ARX model, its AR and X parameters functional subspace is determined using $M = 60$ experiments and a GA based optimization procedure [12]. This procedure leads to a VFP-ARX(70,70)$_{30}$

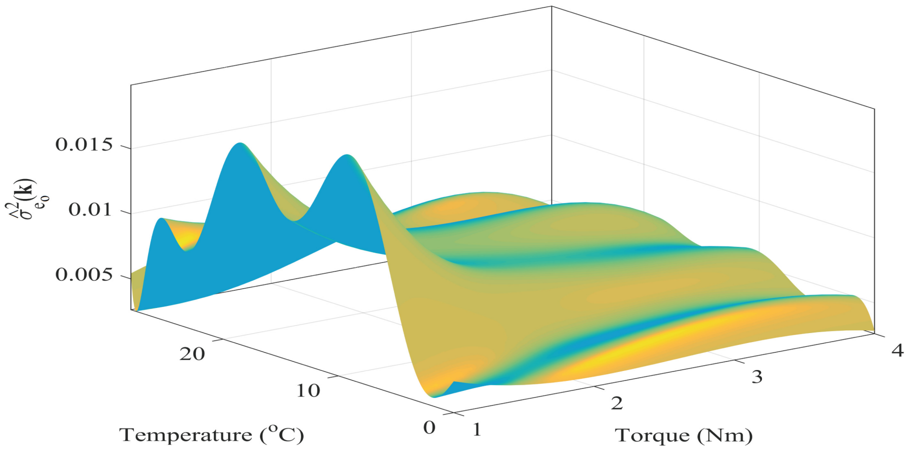

Fig. 3. Functional Model (FM) based method: innovations variance $\widehat{\sigma}^2_{e_o}(\boldsymbol{k})$ estimate as a function of temperature and torque.

model with functional subspace spanned by $p = 30$ bivariate Shifted Legendre polynomials. Then, the dimensionality of the residual variance functional subspace is selected as $q = 30$, using the 60 variance estimates (corresponding to the above experiments) obtained by (3), and bivariate Chebyshev type II polynomials. The functional dependence of the residual variance $\widehat{\sigma}^2_{e_o}(\boldsymbol{k})$ with respect to temperature and torque is presented in Fig. 3.

Table 2. Details on the detection methods.

The FM based method				
Selected model	*No. of train. experim.*	*Samples Per Param. (SPP)*	*Condition number*	*Whiteness testing maximum lag h*
VFP-ARX$(70,70)_{30}$	60	70.92	1.73×10^7	Portmanteau: 10 Peña-Rodríguez: 210
The MM and PCA based methods				
Selected model	*No. of train. experim.*	*Samples Per Param. (SPP)*	*Condition number*	*No. of rejected components/γ (PCA based method)*
ARX$(70,70)$	60	35.46	1.10×10^5	2 / 45.5%

The Multiple Model (MM) Based Method. Using the available vibration response signals (the exact same 60 experiments as previously reported), the 'healthy set' m_o that consists of 60 transmittance function ARX(70,70) models ($m_{o,i}$, $i = 1,\ldots,60$) is constructed and the corresponding parameter vectors $\{\boldsymbol{a}_o\} = \{\boldsymbol{a}_{o,1}, \ldots, \boldsymbol{a}_{o,60}\}$ and covariance matrices are obtained. Estimation details are provided in Table 2.

The PCA Based Method. This is also based on the same 60 transmittance function ARX(70,70) models, $m_{o,i}$ ($i = 1, \ldots, 60$), previously reported. The obtained model parameters are centered by subtracting their sample mean, which leads to the 'healthy set' of parameters. The dimensionality of the sample covariance matrix is selected equal to 50×50, and it is constructed using the first 25 AR and first 25 X parameters from the 60 transmittance ARX models. Based on the fact that the number of variability (uncertainty) sources affecting the structural dynamics is two (temperature and tightening torque), the eigenvectors (principal components) which are removed are $n = 2$, which leads to $m = 48$ and $\gamma = 45.5\%$ (details in Table 2).

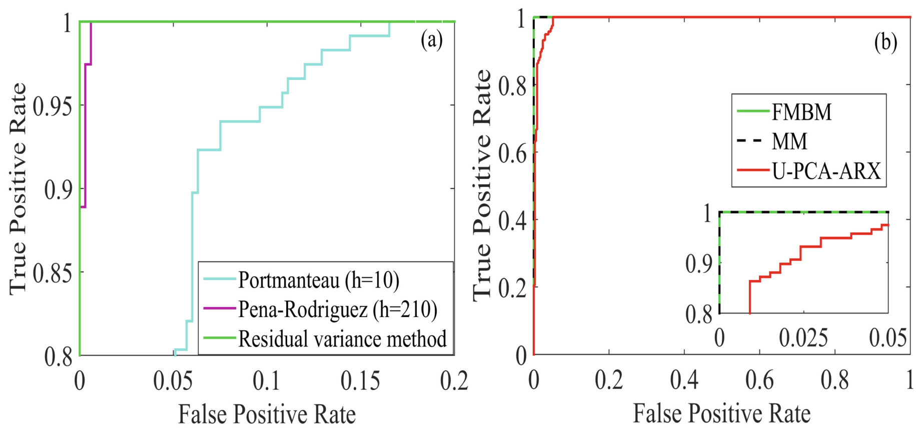

Fig. 4. Comparative damage detection performance assessment: ROC curves for the **a** three versions of the FM based method, and, **b** the FM based (RV version), the MM based, and the PCA based methods. (333 experiments with the healthy and 117 with the damaged structure.)

4.2 Inspection (Diagnosis) Phase

Based on the results of Fig. 4(a), it is evident that the Functional Model Residual Variance (FM-RV) version outperforms the two Residual Uncorrelatedness (FM-RU) versions, with the ROC curve being ideal (reaching the point (0,1)), implying the achievement of 100% correct detection rate with 0% false alarm rate. Among the two RU versions, the FM-RU-PR ($h = 210$) reaches somewhat lower, but still very good performance, while the FM-RU-P lags behind significantly.

A comparison of the best (FM-RV) FM method version with the MM and PCA based methods is provided in Fig. 4(b). Evidently, the MM based method achieves perfect performance as well, with the ROC coinciding with that of the FM-RV based method. On the other hand, the PCA based method achieves the lowest performance, characterized by 100% correct detection rate for false alarm rate greater than 8% (or 97% correct detection rate for 5% false alarm rate).

5 Concluding Remarks

The problem of vibration-response-only damage detection for a composite beam under variable and non-measurable Environmental and Operational Conditions (EOCs) was considered via three unsupervised, batch, Statistical Time Series (STS) type robust detection methods. These included a novel Functional Model (FM) based method in three distinct versions (FM-RU-P, FM-RU-PR, FM-RV), a Multiple Model (MM) based method, and a Principal Component Analysis (PCA) based method. Performance assessment was based on a total of 450 inspection experiments (333 with the healthy and 117 with the damaged structure), which were all excluded from the methods' baseline (training) phase. The main conclusions of the study may be summarized as follows:

(a) Despite the variations in Environmental (temperature ranging from 0 to 28 °C) and Operating (tightening torque ranging from 1 to 4 Nm) Conditions (EOCs), vibration-response-only damage detection was possible via the STS robust methods, with ideal performance—corresponding to 100% correct detection rate for 0% false alarm rate—achieved.
(b) Ideal performance was achieved by the Functional Model (FM) based and the Multiple Model (MM) based methods.
(c) The FM based method requires explicit knowledge of the precise EOC magnitudes in the baseline phase; this is not the case with the other methods.
(d) Of the three versions of the FM based method considered, the presently introduced, Residual Variance (RV) based version, achieved the best (ideal) performance.

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