Efficient Algorithm for Frequency Estimation Used in Structural Damage Detection

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Abstract. In damage detection processes, the accuracy of estimating the eigenfrequencies of structures is crucial because the frequencies are not highly sensitive to damage. This paper analyses the accuracy of the Discrete Fourier Transform when estimating the frequency and amplitude of sine waves, identifies its limitations and proposes an algorithm to significantly improve the attained results. Standard methods used to evaluate the eigenfrequencies fail because the results depend on the position of the spectral lines, which are related to the acquisition time. Frequently, interpolation involving the amplitude peaks displayed on several spectral lines located around the maximizer is employed to improve the frequency readability. The estimated results are improved indeed, but the achieved precision still depends on the acquisition time. We develop an algorithm that uses the maximizer of signals with different time lengths, which are obtained from the original acquired signal by cropping. The three selected maximizer are used for parabolic interpolation input data. The maximum of the regression curve represents a precise estimate of the amplitude, associated with the true frequency of the targeted harmonic component. The efficiency of the algorithm is demonstrated for harmonic and multi-harmonic signals.

Keywords: Signal processing · Accurate frequency estimation · Discrete Fourier Transform · Interpolation · Excel VBA

1 Introduction

Damage detection using modal parameters extracted from vibration signals gained the attention of numerous researchers and practitioners in the last decades [1–5]. The most common parameter is the eigenfrequency because, in the market, simple and robust equipment is available to measure it.

However, there is a problem when using this modal parameter, namely the low sensitivity of the frequency change due to the damage [6], which makes the accurate

M. A. Wahab (Ed.): DAMAS 2019, LNME, pp. 283–300, 2020.
https://doi.org/10.1007/978-981-13-8331-1_20

frequency estimation a necessity. It is also important to ensure the repeatability of the results because measurement confidence guarantees early stage damage detection.

Most beam-like structures have large intervals between their harmonics, so it is easy to discern between consecutive frequencies. Observing incipient damage is still difficult because in an early state the damage produces a small frequency drop [7]. If involving standard frequency evaluation, the frequencies of the signal components are calculated and displayed in the spectrum at equidistantly distributed lines. The position of these lines depends on the signal length so that the frequency drop is observed just if the damage grows enough to produce such shift of the frequency that determines the maximizer to move to the anterior line in the spectrum [8]. Even if no structural change occurs, it is possible that for a different signal length the maximizer moves to a neighbor spectral line. As a consequence, an apparent frequency decrease or increase can be suggested without the structure being subject of deterioration [9]. This is why there is a need for using advanced frequency estimation algorithms that provide exact information regarding the frequency even if it has the value between two spectral lines.

This paper presents a review of some actual interpolation methods used to increase the precision of the frequency estimation and proposes a new algorithm implemented in MS Excel-VBA that can precisely indicate the frequency components of a signal irrespective to the signal length taken for the analysis.

2 Review of the Main Actual Interpolation Methods

2.1 Discrete Fourier Transform

Let us consider an analog signal $x(t)$ representing the vibration response of a structure. To give computers the possibility to transmit, store, and process the signal, it must be converted in a digital signal $x(n)$. This is a sequence of N samples $(x_0, \ldots x_n, \ldots x_{N-1})$ representing the values of the analog signal captured in N equidistantly taken time intervals. The distance between two successive samples is referred to as time resolution τ and depends on the sampling rate F_S that is defined as the number of samples taken in one second. For the digital signal $x(n)$, the relation between its length t_S, number of samples and the sampling rate is:

$$t_S = (N-1)\tau = \frac{N-1}{f_S} \tag{1}$$

Discrete Fourier Transform (DFT) find a set of sinusoids, which can be added together to reconstruct the original signal $x(n)$. The period T_1 of the first sinusoid is equal to the length in time of the analyzed signal. For this period, the fundamental frequency is $f_1 = 1/T_1$. The reconstructed signal component having this frequency is displayed in the spectrum as the first spectral line.

The next sinusoid fit the signal length twice, hence $t_S = 2T_2$, so that the second component is displayed on the spectral line where the frequency is $f_2 = 2f_1$. For the general case, we can write the frequency of the k-th harmonic component $f_k = kf_1$, where k is referred to as the spectral line number. The distance between the spectral lines is constantly f_1. Therefore this value is known as the frequency resolution and is denoted with Δf. For $k = 0$, this is not sinusoid but the DC value. If the signal contains no continuous component, the amplitude value displayed at $f_0 = 0$ Hz should be null, but the DFT is not always able to ensure it. In fact, DFT creates, for each spectral line, a sequence of values:

$$X_k = \sum_{n=0}^{N-1} x_n [\cos(-2\pi kn/N) + i \sin(-2\pi kn/N)] \tag{2}$$

If the number of spectral lines is $k = 0 \ldots N-1$, thus equal to the number of samples of the time-domain signal, it results in a number of N linearly independent equations with N unknowns and it is possible to calculate the real and imaginary parts of the coefficients X_k, as:

$$\mathrm{Re}X_k = \frac{2}{N} \sum_{n=0}^{N-1} x_n [\cos(-2\pi kn/N)] \tag{3}$$

$$\mathrm{Im}X_k = \frac{2}{N} \sum_{n=0}^{N-1} x_n [\sin(-2\pi kn/N)] \tag{4}$$

Hence, the find the absolute values of the coefficients using the relation:

$$X_k = \sqrt{(\mathrm{Re}X_k)^2 + (\mathrm{Im}X_k)^2} \tag{5}$$

DFT represented by these coefficients contains N spectral lines, from which it is sufficient to display only half of them, due to symmetry [10]. It is known that the precision achieved by frequency evaluation using DFT depends on the signal acquisition time. Two favorable cases exist: (1) it is possible to acquire a signal in an extremely long time in order to obtain a fine frequency resolution, or (2) the signal length fit a whole number of periods T for the targeted frequency. For damage detection, the fulfillment of the first condition is problematic because vibration signals acquired as a structural response are rapidly damped, especially in the case of higher frequencies [11]. Consequently, the estimated frequencies have a significant deviation from the true values. Because the period of the targeted component is not known, we can acquire the whole cycles just by accident. The problem in estimating frequencies if this condition is not met is the occurrence of spectral leakage, which introduces errors in the estimated frequencies [12]. This aspect is detailed below.

Let us consider a discrete signal with period T and frequency f, acquired in the time t_S. If the acquisition time does not contain a whole number k of cycles with length T but is a little bit longer, which is usually the case, we can write:

$$t_S = T(k + \delta) \tag{6}$$

where δ is a fraction of one cycle. It follows:

$$\frac{1}{t_S} = \frac{1}{T(k+\delta)} \tag{7}$$

Or, taking the inverse of the two fractions, results:

$$f = \Delta f(k+\delta) \tag{8}$$

Hence, the true frequency falls between two spectral lines. Some researchers prefer to work with the normalized frequency, which is obtained by dividing the frequency at the frequency resolution. For the general case, the normalized frequency is:

$$\bar{f} = \frac{f}{\Delta f} = k+\delta \tag{9}$$

From Eq. (9) it can easily be deduced that for the frequency that is the value corresponding to a spectral line, the normalized frequency value is even number of the spectral line. This is illustrated in DFT representation in Fig. 1, where on the spectral line k is plotted against the amplitude of the harmonic signal component A_k. If A_k is the biggest value in a given frequency bandwidth, it is known as the maximizer. Figure 1 shows also the two neighbor spectral lines $k-1$ and $k+1$ and the two amplitudes A_{k-1} and A_{k-1} displayed at these lines.

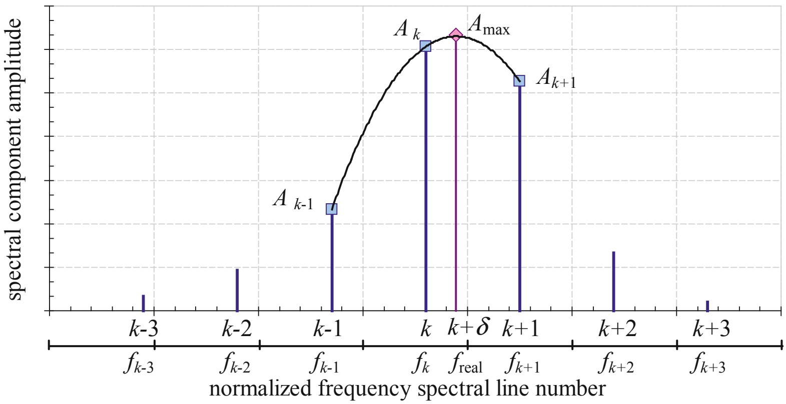

Fig. 1. DFT output for a sinusoid (frequency f = 5 Hz) by standard evaluation.

One can observe in Fig. 1 that the frequency $f_{real} \neq k\Delta f$ is not correctly estimated by the frequency associated to the maximizer A_k. Some interpolation methods were developed to find the frequency between two spectral lines. An analysis of the results achieved by using these interpolation methods is given in next subsection.

2.2 Determining the Precision of the Main Actual Interpolation Methods

So far as we know, the frequency values estimated with standard methods, e.g. DFT, provide results directly linked to the position of the spectral lines of one spectrum obtained for a single time length, which is in most cases the acquisition time. Finding f_{real} at an inter-line position relies on finding the regression curve fitting to several points obtained in the DFT. If just two points in the spectrum are considered, a method to weight their influence is employed. In all cases, finding the correction term δ ensuring a corrected frequency f_{corr} as close as possible to f_{real} is targeted. This can be made either by using at the abscissa the spectral line numbers or the frequencies. In the first case the corrected spectral line number is estimated as:

$$k_{\text{corr}} = k + \delta \tag{10}$$

Or, for the latter case the corrected frequency is found from Eq. (9).

The estimation of f_{corr} employing the correction coefficient δ is referred to as the *fine frequency estimation*, as opposed to the *coarse frequency estimation* performed by directly locating the DFT maximum [13].

Interpolation Based on Two Points in the Spectrum.

Grandke developed a method [14] that involves the peak A_k and the largest neighbor amplitude of DFT achieved from a signal windowed by a Hann window. In the spectral representation in Fig. 15 the maximizer largest neighbor is A_{k-1}. It is also possible to get the larges neighbor at the spectral line A_{k+1}. In both cases, following steps are performed to calculate the corrected frequency. First the ratios

$$\alpha^{-} = \frac{A_{k-1}}{A_k} \text{ or } \alpha^{+} = \frac{A_{k+1}}{A_k} \tag{11}$$

are calculated I function of the bigger neighbor of the maximizer. Afterwards, the correction term is calculated as:

$$\delta^{-} = \frac{2\alpha^{-} - 1}{\alpha^{-} + 1} \text{ or } \delta^{+} = \frac{2\alpha^{+} - 1}{\alpha^{+} + 1} \tag{12}$$

Finally, the frequency result as:

$$f_{\text{corr}} = (k + \delta^{-})\Delta f \text{ or } f_{\text{corr}} = (k + 1 + \delta^{+})\Delta f \tag{13}$$

Quinn proposed a method [15] that directly uses the DFT of the signal. Because no windowing is employed, the method is much faster as that proposed by Grandke. The method implies both neighbor amplitudes of the maximizer and request two interpolations, each of them involving just two amplitudes. The following ratios are calculated for Quinn's estimator:

$$\alpha = \frac{A_{k-1}}{A_k} \text{ and } \alpha = \frac{A_{k+1}}{A_k} \tag{14}$$

Next, the correction terms are calculated as:

$$\delta_1 = \frac{\alpha_1}{1-\alpha_1} \text{ and } \delta_2 = -\frac{\alpha_2}{1-\alpha_2} \tag{15}$$

Finally, the frequency result as:

$$f_{\text{corr}} = (k + \delta_{1,2})\Delta f \tag{16}$$

If $|\delta_1| < |\delta_2|$, then the correction term δ_2 is chosen in Eq. (16), else δ_1 is chosen.

Jain et al. [16] proposed an interpolation method similar to that proposed by Quinn, but the correction coefficients were calculated in a different way. For $A_{k-1} > A_{k+1}$, the correction term is calculated from the relations:

$$\alpha_1 = \frac{A_k}{A_{k-1}} \text{ and } \delta_1 = \frac{\alpha_1}{1+\alpha_1} \tag{17}$$

and the frequency results from the relation:

$$f_{\text{corr}} = (k - 1 + \delta_1)\Delta f \tag{18}$$

If $A_{k-1} \leq A_{k+1}$, the correction term is calculated from the relations:

$$\alpha_2 = \frac{A_{k+1}}{A_k} \text{ and } \delta_2 = -\frac{\alpha_2}{1-\alpha_2} \tag{19}$$

and the corrected frequency is derived as:

$$f_{\text{corr}} = (k + \delta_2)\Delta f \tag{20}$$

The tests are performed on a signal with the frequency $f_{\text{real}} = 4.89\,\text{Hz}$, the amplitude $A_{\text{real}} = 1\,\text{mm/s}^2$ and the initial time length $t_S = 1.5\,\text{s}$. The original signal is generated using $N_S = 301$ samples by a sampling rate $f_S = 200\,\text{Hz}$, resulting a time resolution $\Delta t = 0.005\,\text{s}$. The efficiency of the interpolation methods is tested for eleven time lengths achieved by stepwise truncating the original sinusoidal signal. To obtain the ten shorter signals, $N_{S-it} = 8$ samples (i.e. $t_{S-it} = 0.05\,\text{s}$) are removed at each step. The results achieved employing the three above described methods are presented in Fig. 2.

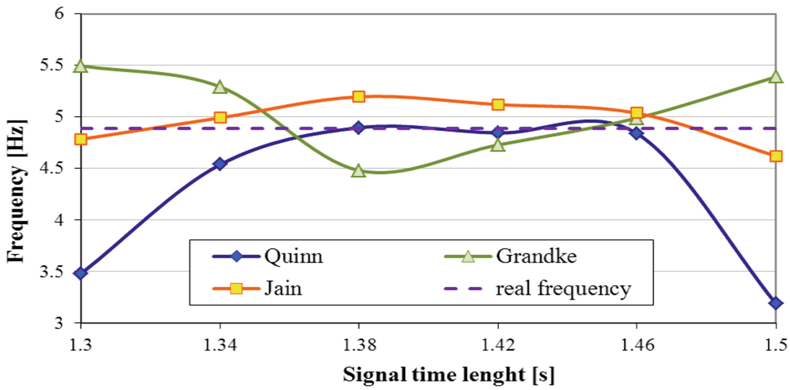

Fig. 2. Frequency estimation tests employing interpolation methods based on two points.

Interpolation Based on Three Points in the Spectrum.
The next analyzed methods involve three amplitudes at interpolation. Ding [17] proposed a barycentric method, where the correction term δ results from the relation:

$$\delta = \frac{A_{k+1} - A_{k-1}}{A_{k-1} + A_k + A_{k+1}} \tag{21}$$

Instead, Voglewede [18] proposed a correction term that involves the quadratic method. The correction term is found from the relation:

$$\delta = \frac{A_{k+1} - A_{k-1}}{2(2A_k - A_{k-1} - A_{k+1})} \tag{22}$$

A quite similar quadratic estimator of the correction term is introduced by Jacobsen in [19], which results from the relation:

$$\delta_{\mathrm{Jac}} = \frac{A_{k+1} - A_{k-1}}{2A_k - A_{k-1} - A_{k+1}} \tag{23}$$

For all these methods, the corrected frequency is calculated as:

$$f_{\mathrm{corr}} = (k + \delta)\Delta f \tag{24}$$

Tests are performed by involving the same signal and procedure as previously described. The results are presented in Fig. 3.

From Figs. 2 and 3, it can be seen that the errors in frequency estimation still depend on the acquisition time, i.e. the closer the time length t_S to a multiple of periods T, the higher the precision of the estimation is. However, the errors are significant and unpredictable, so this approach is not recommended to evaluate the eigenfrequencies of structures for damage detection purposes.

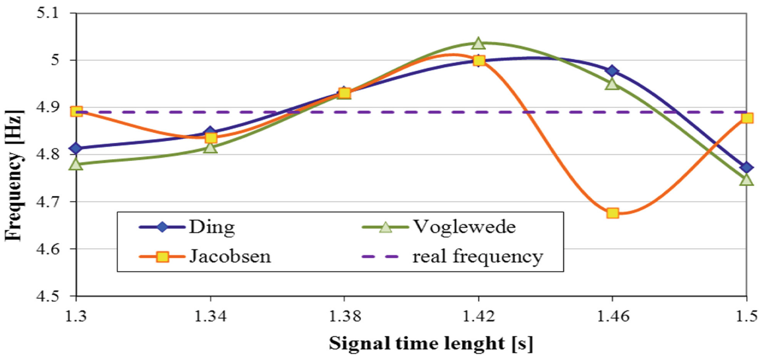

Fig. 3. Frequency estimation tests employing interpolation methods based on two points.

3 The Proposed Frequency Estimation Method

3.1 Description of the Propose Frequency Estimation Method

The weakness of the existing interpolation methods mainly consist in the fact that the points on which the interpolation is made come from the same DFT. We develop a new frequency estimation method based on three points achieved from three different DFTs that are points of a sinc function [20]. The peak values distributed of this function is close to a polynomial one, so that a second-order polynomial interpolation is made. To obtain the three points for interpolation, the method involves shortening the signal in time domain and calculating the DFT at iteration. In the following sub-sections we present the algorithm implemented in a program developed by the authors in MS-VBA, which is used for the accurate frequency estimation.

Signal Generation.
The test signal is generated involving the “SignalGeneration” sheet, but it is also possible to import real measured signals. To evaluate the accuracy of the results, in the paper we use signals generated with known frequencies. It is possible to create signals composed by up to three harmonic components whit defined amplitudes and frequencies. The operator can also impose the number of samples N_S and time resolution Δt. The signal generated or taken directly from the acquisition system is transferred to the “DFT” sheet.

Standard DFT Analysis.
After it is created or imported, the signal is transferred from to the “DFT” sheet. Here, it is represented in a chart, as that shown in Fig. 4a. By click on the “DFT calculation” button, the real and imaginary coefficients are generated according to Eqs. (3) and (4), which are further used to calculate the complex module with Eq. (5). These values are displayed in a spectral representation as that shown in Fig. 4b. In these calculations, the ratio $2/N_S$ is not considered, because we intend obtaining dissimilar amplitudes in the frequency domain representations for the truncated signals. The amplitudes attain in this way higher values for longer signals in the time domain, i.e. bigger number of samples contained in the signal.

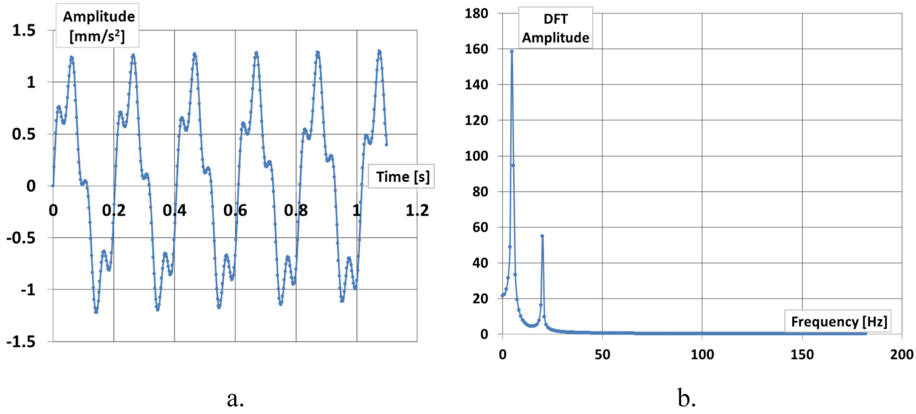

Fig. 4. Generated harmonic signal and its standard

Setting of the Number of Peaks to be Considered in the DFTs Obtained After Signal Truncation.

After DFT is calculated, a window is displayed that request setting the number m of peaks to be extracted from DFTs calculated after each signal truncation. In the normal case, because the frequencies of a beam are not close together and leakage does not significantly affect the amplitudes of the neighbor harmonic components, the selected number should be the number of the components observed in DFT. But, if the structure is excited with a frequency close to the targeted frequency component, it achieves here the highest amplitudes and the number of peaks can be set as one. A procedure to excite the structure in such way is described in [20].

After entering the value of m, the signal is truncated by 2 samples at iteration, until the half signal length remains. DFTs are calculated for each truncated signal. From these spectra, the number of peaks set before is selected and displayed in an overlapped spectrum, as shown in Fig. 5.

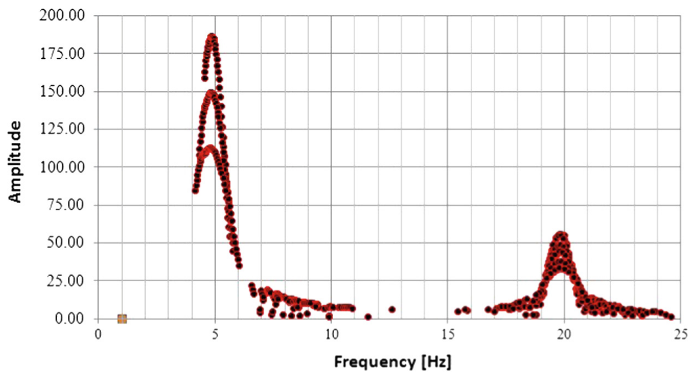

Fig. 5. Overlapped DFT for the cropped signals

Each curve in Fig. 5 represents the peak amplitudes for DFTs that correspond to a value of m. Note that, because we didn't consider the value $2/N_S$ when calculating the complex coefficients, the amplitudes of these curves differ. Dissimilar happens if considering $2/N_S$. In this case, the amplitudes of the curves are equal, but do not permit identifying the points to be used for interpolation.

Selection of the Frequency Bandwidth of Interest.
The next approach is setting the frequency bandwidth of interest. This should include the targeted frequency and frame it as narrow as possible. An indication of the coarse estimated frequency is found in the overlapped spectrum displayed in Fig. 5. This information permits selecting the lowest and highest value for the bandwidth. The values are typed in two windows which open one after the other. As a consequence, the limits of the overlapped spectrum are set, and it displays the results for just one harmonic component, as shown in Fig. 6.

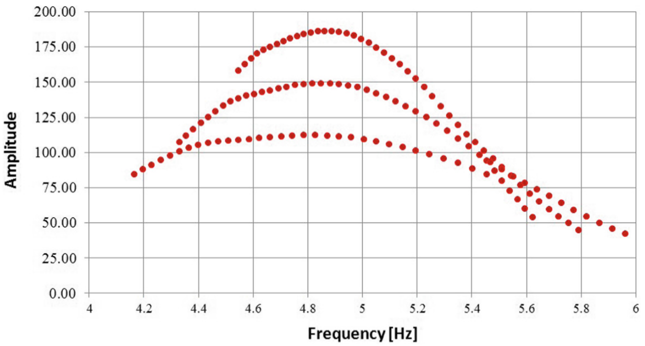

Fig. 6. Zoom on the overlapped DFT displaying the selected bandwidth.

Selection of the Number of Cycles of the Targeted Harmonic Component.
Next, a window opens, showing the number of cycles k for which the curve with the highest amplitude (see Fig. 6) is calculated. It requests setting the number of cycles for which the amplitudes are selected for interpolation. Here, the number of maximum integer cycles should be specified in the window. This is the number displayed in the window if the highest curve has a clear maximum, else the number should be reduced with one. For the specified number, the software selects all frequency-amplitude points and displays them graphically in Fig. 7

Performing interpolation and obtaining the estimated frequency.
From the set of selected values, the maximizer and the two neighbors are found, for which a second order polynomial regression curve is calculated. For this curve, the maximum if found analytically. The estimated frequency is now found as the abscissa of the maximum. The three values extracted as maximizer from the three DFTs and the maximum found by interpolation are shown in Fig. 8. In a similar way is found the estimated frequency for the second harmonic component that constitutes the original signal. Finally, the estimated amplitude is calculated using the constant $2/N_{S\text{-max}}$, and is

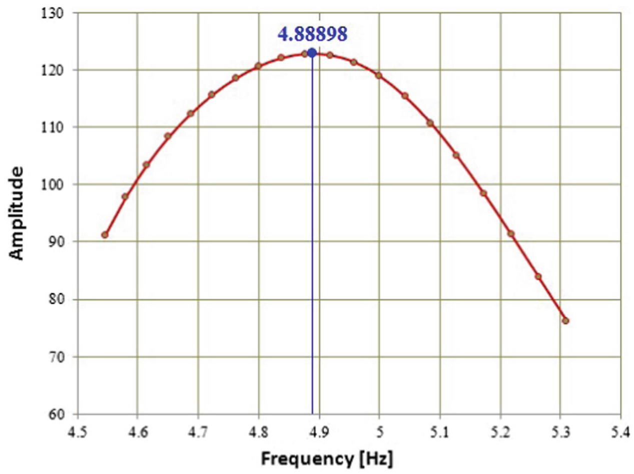

Fig. 7. Overlapped DFT for the cropped signals

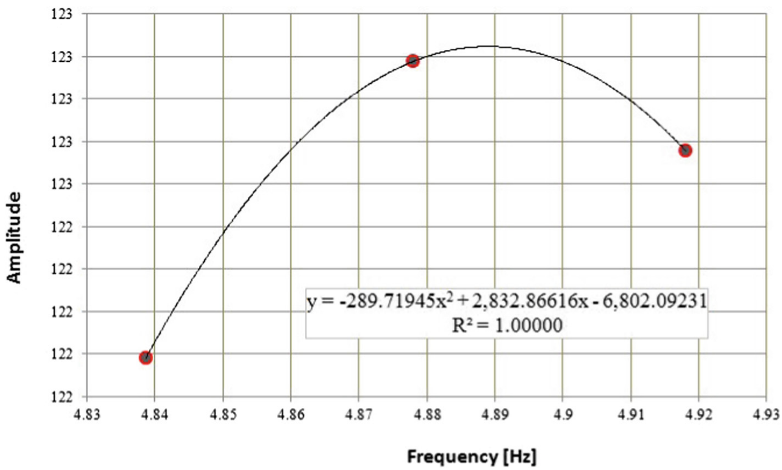

Fig. 8. The interpolation performed for three points that belong to three DFT

displayed along with the estimated amplitude. $N_{S\text{-max}}$ represents the number of samples contained in the truncated signal which ensures the maximizer in Fig. 7.

Overlapping DFTs ensure a fine frequency resolution. Because the interpolation is made by using three amplitudes from different DFTs, all close to the real amplitude, the results are expected to be accurate. However, a slight deviation to the right is noticed, because the distribution of the maximizer is not symmetric but follows a pseudo-sinc function [21].

3.2 Numerical Tests to Validate the Precision of the Frequency Estimation

To highlight the accuracy of the contrived method and the subsequent developed program, we made simulation on signals with known harmonic components. This facilitates comparing the results achieved from the frequency estimation with that of the generated frequency. The frequency estimation results are also compared with those obtained from DFT of the original signal and simulations made by applying the interpolation methods presented in Sect. 2.

Signal Containing One Harmonic Component.

The original signal with the frequency $f = 4.89$ Hz is again considered in this subsection, generated in the same conditions as the signal tested with known interpolation methods presented in Sect. 2, and for the same time lengths. The results, obtained involving the developed algorithm, are presented in Table 1.

Table 1. Data used for tests and the achieved results for the signal with $f = 4.89$ Hz.

t_S [s]	Δf [Hz]	N_S	m	f_{min}	f_{max}	k	f_{corr} [Hz]	A_{corr} [mm/s^2]
1.50	0.666667	301	1	4	6	6	4.8889816	0.9945186
1.46	0.684932	293	1	4	6	6	4.8889816	0.9945186
1.42	0.704225	285	1	4	6	6	4.8889816	0.9945186
1.38	0.724638	277	1	4	6	6	4.8889816	0.9945186
1.34	0.746269	269	1	4	6	6	4.8889816	0.9945186
1.30	0.769231	261	1	4	6	6	4.8889816	0.9945186

From Table 1 and Fig. 9 one can observe that the frequencies estimated with the proposed method do not depend on the acquisition time and are definitely more accurate as these obtained by actual frequency estimators which use interpolation methods. The error is found to be 0.02%, which totally fulfill the requirements for early damage detection. Another advantage of the proposed method consists in the fact that it makes possible to estimate the amplitude, which is impossible when current frequency estimation methods based on interpolation are used.

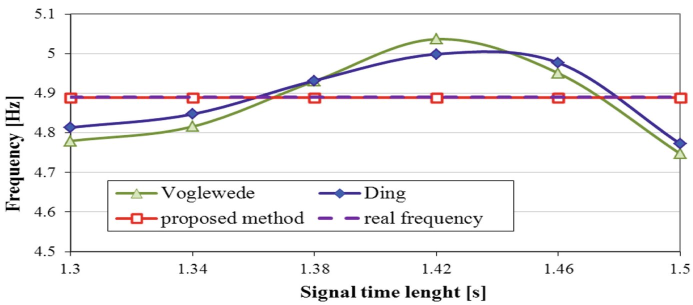

Fig. 9. Frequency estimation results achieved by employing the known methods in comparison with those achieved by the proposed method

Signal Containing Two Harmonic Components.
To show the method works if the analyzed signal contains more harmonic components, we performed tests for a signal simulating the first two out of plane vibration modes of a cantilever beam. The first component is again the sinusoidal signal with the frequency f_1 = 4.89 Hz and the amplitude $A_1 = 1\,\mathrm{mm/s^2}$. For the second harmonic component we choose the frequency f_2 = 28.75 Hz, which is expected for the second harmonic of an Euler–Bernoulli cantilever beam. The amplitude is considered $A_2 = 0.3\,\mathrm{mm/s^2}$. The two components are found separately.

First, the fundamental frequency is estimated. We use the same truncation strategy as for the single component signal. Accurate frequency estimation is possible, the results presented in Table 2 sustaining this conclusion. The error is still 0.02%.

Table 2. Data used for tests and the achieved results for the fundamental frequency.

t_S [s]	Δf [Hz]	N_S	m	$f_{\min}$	$f_{\max}$	k	f_{corr} [Hz]	A_{corr} [mm/s^2]
1.50	0.666667	301	2	4	6	6	4.888632	0.995148
1.46	0.684932	293	2	4	6	6	4.888632	0.995148
1.42	0.704225	285	2	4	6	6	4.888632	0.995148
1.38	0.724638	277	2	4	6	6	4.888632	0.995148
1.34	0.746269	269	2	4	6	6	4.888632	0.995148
1.30	0.769231	261	2	4	6	6	4.888632	0.995148

We did not succeed to find the second harmonic with the strategy described for the previous tests. This happened because the number of samples per cycle was too low. For higher frequencies, the algorithm requests a minimum number of samples per cycle, which is found to be 20 for an accurate estimation. Therefore, we reduced the time resolution from Δt = 0.005 s to Δt^* = 0.002 s and maintained unaltered the number of samples of the original signal, that are N_S = 301. The shortened signals are obtained by extracting 8 samples by iteration. The results are presented in Table 3. One can observe that the frequency is well estimated, the maximum error being 0.127 Hz. This shows that, even for signals containing more harmonics, the frequencies can be estimated well even for the harmonics with small amplitude.

Table 3. Data used for tests and the achieved results for the second harmonic.

t_S [s]	Δf [Hz]	N_S	m	$f_{\min}$	$f_{\max}$	k	f_{corr} [Hz]	A_{corr} [mm/s^2]
0.600	0.666666667	301	10	27	30	16	28.83255	0.319129
0.584	0.684931507	293	10	27	30	16	28.83255	0.319129
0.568	0.704225352	285	10	27	30	16	28.83255	0.319129
0.552	0.724637681	277	10	27	30	15	28.83457	0.306852
0.536	0.746268657	269	10	27	30	15	28.83457	0.306852
0.520	0.769230769	261	10	27	30	14	28.87619	0.285024

The next approach was testing if a small frequency drop, as that occurring in early damage state, can be accurately quantified. The original signal is that described above, hence it is generated with N_S = 301 samples and time resolution Δt = 0.002 s, resulting the signal time length t_S = 0.6 s. It has two harmonic components that have the frequency f_1 = 4.89 Hz and the amplitude $A_1 = 1\,\mathrm{mm/s^2}$, respectively the frequency f_2 = 28.75 Hz and the amplitude $A_2 = 0.3\,\mathrm{mm/s^2}$. To see if small frequency changes are observable and possible to be quantified accurately, we simulated five small frequency drops Δf_2 and obtained five reduced frequencies $f_{2-\mathrm{R}}$. Afterward, we estimated the frequency for each generated signal and compared the generated frequency drop with that obtained by estimation. The absolute error is also calculated and used to appreciate the accuracy of the performed frequency estimations.

Table 4. Estimator's sensitivity analysis to frequency changes for the second harmonic.

Generated data			Estimated data			Error [%]
f_2 [Hz]	$f_{2-\mathrm{R}}$ [Hz]	Δf_2 [Hz]	f_2 [Hz]	$f_{2-\mathrm{R}}$ [Hz]	Δf_2 [Hz]	
28.75	28.73	0.02000	28.83255	28.81252	0.02003	0.32
28.75	28.71	0.04000	28.83255	28.79265	0.03990	0.32
28.75	28.69	0.06000	28.83255	28.77289	0.05966	0.32
28.75	28.6056	0.14440	28.83255	28.68969	0.14286	0.31
28.75	28.60	0.15000	28.83255	28.68412	0.14843	0.31
28.75	27.50	1.25000	28.83255	27.59033	1.24222	0.29

From Table 4 one can observe that, even for the component with the smaller amplitude, the proposed frequency estimator is able to find fine frequency changes. The accuracy permitted observing the differences between the generated frequency drops 28.6056 Hz and 2.6 Hz and even quantifying this difference. On the other hand, the bigger frequency changes are also estimated with accuracy, which is proved by the results in last row in Table 4.

Table 5. RSFs obtained from generated signals and estimations for the second harmonic.

Generated data			Estimated data			Error [%]
f_2 [Hz]	$f_{2-\mathrm{R}}$ [Hz]	RFS [%]	f_2 [Hz]	$f_{2-\mathrm{R}}$ [Hz]	RFS [%]	
28.75	28.73	0.069565	28.83255	28.81252	0.069467	0.14
28.75	28.71	0.139130	28.83255	28.79265	0.138396	0.52
28.75	28.69	0.208695	28.83255	28.77289	0.206925	0.84
28.75	28.6056	0.502260	28.83255	28.68969	0.495485	1.34
28.75	28.60	0.521739	28.83255	28.68412	0.514813	1.32
28.75	27.50	4.347826	28.83255	27.59033	4.308380	0.90

Damage detection methods developed by the authors [22–25] make use of the Relative Frequency Shift (RFS), which is calculated as:

$$\mathrm{RFS}_i = \Delta\bar{f}_i = \frac{f_i - f_{i-\mathrm{R}}}{f_i} \tag{24}$$

where f_i is the frequency of the healthy beam and $f_{i-\mathrm{R}}$ that of the damaged beam for the i-th vibration mode. Usually we express this values in percent, thus these are multiplied by 100. The RSF values for the frequency drops are presented in Table 5. It clearly results that the estimated frequency values permit finding correct RFSs.

The question is if the method works for low frequencies, being known here are the biggest problems in observing frequency changes. This happen mainly because the frequency drops due to damage in early stage, for the first vibration mode, are much smaller as the half of the frequency resolution. As a consequence, the damage occurrence is observable just until the frequency drop approaches $\Delta f/2$. For a signal with the time length $t_S = 1.5$ s results $\Delta f = 0.667$ Hz, so that a frequency drop $\Delta f_1 = 0.02$ Hz will not be observed. The change in the spectrum consists, in this case, in an amplitude alteration because another distribution on the same spectral lines results. Therefore, if the original signal's frequency $f_1 = 4.89$ Hz drops to $f_{1-\mathrm{R}} = 4.87$ Hz the change will not be noticed. By employing the proposed method to test if the frequency drop $\Delta f_1 = 0.02$ Hz is observed, we obtained the results presented in Table 6. We conclude this small change is observable and the achieved results permit calculating a reliable RFS. Another test, made for a bigger frequency drop which is still not observable by applying the standard frequency evaluation, lead to the same conclusion. The results are also presented in Table 6.

Table 6. RSFs obtained from generated signals and estimations for the first harmonic.

Generated data		Estimated data					
f_1 [Hz]	$f_{1\text{-R}}$ [Hz]	Δf_1 [Hz]	RFS [%]	f_1 [Hz]	$f_{1\text{-R}}$ [Hz]	Δf_1 [Hz]	RFS [%]
4.89	4.87	0.02	0.4089	4.88863	4.86946	0.01917	0.3921
4.89	4.79	0.10	2.0449	4.88863	4.78471	0.10391	2.1256

3.3 Recommendation

After performing a series of simulations, we found out the settings that should be imposed on the acquisition system and the developed software to allow evaluating the highest and the lowest frequency of interest. These are following:

– The length in time of the original signal should cover at least six and a half periods of the fundamental frequency, i.e. $t_S > 6.5T_1$. This condition refers to the lowest frequency and aims ensuring a symmetric distribution of the peaks in the overlapped spectrum;
– The time resolution of the original signal should ensure twenty samples for each period of the highest frequency of interest, i.e. $T_{\max} > 20\Delta t$. This condition refers to the highest frequency and aims achieving a dense overlapped spectrum around the presumed inter-spectral line;

- The number of maxima m, searched when plotting the overlapped spectrum, can be set much bigger as the presumed harmonics. This condition refers to the harmonic with low amplitude for which ensuring sufficient maxima is necessary.
- The number k of cycles selected when estimating the frequency of the original signal should be maintained in all further estimations for the damaged beam. This refers to all harmonic components and aims to guarantee the repeatability of results.
- The number k of cycles for a given component should be estimated from the signal length t_S and the coarse estimated frequency of that component and should be taken in such way to ensure sufficient time for truncation, so the condition $t_S - kT_i > 0.3\,T_i$ and $t_S - kT_i < 0.6\,T_i$ should be met. This condition refers to all harmonic components and aims ensuring conditions to keep k unchanged when lower frequencies are assessed.

It is sometimes challenging to fulfill simultaneously the first two conditions, as a substantial number of samples are needed. In such cases, the measurements should be focused on acquiring signals to be processed to evaluate low frequencies. Afterward, shorter signals with improved time resolution should address the high frequencies.

4 Conclusion

Accurate frequency estimation is crucial for the detection of damage in early stage. Current frequency estimation methods, both the standard estimation by DFT as well as estimations involving interpolation fail in detecting small frequency changes. This prevents structural changes from being observed at an early stage. To overcome this limitation, we developed a frequency estimation method and software based on it, which have shown that even a small frequency drop can be estimated.

Tests performed with this software have shown it can separately evaluate the frequencies of a signal composed by more harmonics with different amplitudes. Differences between the results obtained for the fundamental frequency when it was estimated from the harmonic signal and from the composed signal are insignificant (less than 0.0003 Hz). It was also shown that the harmonics with low amplitude from multi-tone signals can be accurately estimated. Repeatability was assured for all estimations. When simulating a slight frequency drop, the software was able to identify it. It was observed a difference even if the generated frequencies are 28.6056 Hz and 28.6 Hz. For all frequency changes we succeed to calculate the Relative Frequency Shifts, irrespective to how small the frequency drop was.

Future research will focus on the complete automation of the estimation process, which will be based on the features extracted through the coarse analysis of the signal in its incipient phase.

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