An Improved SSI Approach for Structural Modal Identification

Jun Li, Gao Fan, Hong Hao and Han Li

Abstract This paper proposes an improved Stochastic Subspace Identification (SSI) approach for structural modal identification. The automated interpretation of the SSI output, namely the stabilization diagram, is conducted based on the defined hard criteria and the used clustering techniques. To validate the performance of the proposed approach, numerical studies on a steel frame model are conducted and the ambient vibration measurements contaminated with a significant noise effect are used for identification. Experimental testing data are also used for the structural modal identification with the proposed approach. The accuracy and robustness of the proposed approach are demonstrated by the successful identification of modal parameters.

Keywords SSI · Long-term health monitoring · Automated · Clustering · Operational modal identification

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1 Introduction

In the past decades, there has been a significant development in the vibration-based long-term structural health monitoring (SHM) of the civil structures. A noticeable increase of SHM applications can be found on the long-span bridges [12], high-rise buildings [13] and ancient structures [1]. These long-term SHM systems can keep collecting the structural vibration data under ambient or forced excitations and report the abnormal conditions of structures in almost real time to prevent the casualties and property losses. Modal parameters including natural frequency, mode shape and damping ratio are often involved as condition indicators as they are sensitive to damage and can be extracted from the vibration data directly. When developing the long-term SHM system for the purpose of structural condition monitoring, a large amount of vibration data need to be processed to identify the modal parameters and reflect the structural conditions. In this situation, the conventional modal identification methods requiring the engineers' interactions and manual tuning is impractical for engineering applications, due to the substantial amount of human resource requirement and a significant labor time demand. As a result, the development and application of the automated modal identification approaches have attracted a remarkable amount of attention.

Structural modal identification using the measured vibration responses from structures under the operational or natural sourced excitations is known as Operational Modal Analysis (OMA). Many OMA methods have been proposed and applied to the practical engineering structures such as frequency domain decomposition (FDD) [9], Eigensystem Realization Algorithm (ERA) [14] and Stochastic Subspace Identification (SSI) (Van Overschee and De Moor 1999). Among these methods, SSI has a clear mathematical background and has been applied for the modal identification of civil engineering structures. Traditionally, the modal analysis by using SSI involves the identification of mode candidates in a range of system orders. The interpretation of the SSI output, namely the stabilization diagram, is based on the empirical observations and engineers' experiences to distinguish the physical modes and the spurious modes. The automated methods are becoming attractive since they can contribute to reducing the engineers' involvement in the interpretation process of the stabilization diagram. Previous studies such as [2] and [3] have shown the capability of using clustering techniques to distinguish the physical modes and the spurious modes in automated modal identification. Two major streams of the clustering techniques are applied. The first one is the K-means clustering [7] and another one is hierarchical clustering [6]. K-means clustering can group the modes into two clusters, where one for physical modes and another one for spurious modes. Hierarchical clustering lists all the modes from the SSI output as a single group and keeps merging two groups with minimum distance as one until the minimum distance less than the predefined termination threshold.

This paper proposes an improved SSI approach used for automated structural modal identification. The automated interpretation of the SSI output contains three procedures: (1) Hard criteria used to remove the certain spurious clusters; (2) Modes clustering to gather physical modes and filter out small groups; (3) Clusters merging to further remove spurious modes. The innovation of this study is to develop a novel way to define and update the clustering criteria based on the statistical properties of grouped modes' modal parameters. The effectiveness and performance of the proposed approach are investigated based on numerical and experimental studies on a seven-story steel frame structure.

2 Methodology

2.1 SSI Parameters Selection

The improvement in the proposed approach is developed based on the traditional data-driven SSI method [11]. For structural modal identification, properly selecting the model parameters in SSI, such as half the number of block rows of the Hankel matrix i and the range of model order [*nmin*, *nmax*] is unneglectable for an accurate prediction. Based on the mathematical background of SSI, the identification may not be successful if a signal with i sampling points is shorter than one period of the fundamental mode. Therefore, many researchers tend to select a large i value to ensure that the first mode can be reliably identified. However, a large i value will enlarge the size of the Hankel matrix and also raise the number of spurious modes which lead to an increase in the computation burden. Hence, i should be carefully selected based on $i \geq T_f/dt$, where T_f is the first fundamental period and dt is the sampling time interval. Rather than selecting the physical modes for a single model order, the SSI methods tend to identify the modes in a range of model orders to form a stabilization diagram. The physical modes shall be observed with consistent modal parameters across a number of model orders, so as to distinguish with spurious modes. The range of system model order is also limited to prevent failure in identification or expansion in computational cost. The lower bound *nmin* is usually chosen as twice as the number of the expected modes in the frequency range of interest. On the other hand, *nmax* is limited as $nmax < i \times l$, where l is the number of the measurement channels.

2.2 Automated Interpretation of SSI Output

2.2.1 Hard Criteria

This procedure is defined to eliminate some certain spurious modes and enhance the efficiency and accuracy of modal identification. The hard criteria cannot remove all the spurious modes but it is possible to decrease the number of spurious modes. Damping ratio, Modal Phase Collinearity (MPC) [8] as well as the Mean Phase

Deviation (MPD) [10] are selected as the hard criteria. In practical, most of the civil structures have damping ratios ξ larger than 0 and less than 10%. Identified modes have damping ratios out of this range can be considered as spurious modes and removed from the system modes matrix. The threshold values for MPC and MPD are dependent on the testing environment and the structure properties under consideration. For structures with clear linear behaviors, threshold values for MPC and MPD are defined as 0.7 and 0.3 respectively. For structures with complex behaviors or under unstable experimental conditions, 0.3 and 0.7 for MPC and MPD are defined.

2.2.2 Modes Clustering

The clustering procedure based on the statistics of modes is the main innovation contribution of this paper. The clustering procedure can be concluded as the following seven steps:

(1) Conduct the data-driven SSI with the selected model parameters and form the SSI output as a matrix with a random model order. The reason for randomizing the model order is because, during the study, some criteria for physical-modes clusters are converged as a very tiny value such as 10^{-5} or even 10^{-6} due to the similarities of the modes in adjacent order.

(2) Scan the matrix from the first to the last row and run a pointer from the first to the last element. Once the scanning pointer examines the first mode that does not belong to any cluster, a new cluster C_k is created and this mode is stored as the first element of this cluster.

(3) Scan the successive row of the matrix and find the mode with the minimum distance to the cluster. The distance between cluster *Ck* and mode is defined as:

$$D = D_f + D_\phi = \frac{\left|\overline{f_k} - f_i\right|}{\overline{f_k}} + 1 - MAC(\overline{\phi_k}, \phi_i) \tag{1}$$

where $\overline{f_k}$ and $\overline{\phi_k}$ is the mean frequency and mode shape of the cluster *Ck*. f_i and ϕ_i is the mode frequency and mode shape of *i*th mode in the successive row.

(4) Extract the modal parameters and MPC of the mode with the minimum distance $m_{min.}$ If the total number of the mode is less than 10% of the total model order, the following criteria relating with the frequency, damping ratio, MAC value and MPC are used to judge whether this mode belongs to *Ck*

$$\begin{array}{ll} \frac{\left|\overline{f_k} - f_m\right|}{\overline{f_k}} \le \mathrm{T}_f; & \frac{|\mathrm{median}(\xi_k) - \xi_m|}{\xi_i} \le \mathrm{T}_d; \\ 1 - MAC(\phi_k, \phi_m) \le \mathrm{T}_{MAC}; & \frac{|MPC_k - MPC_m|}{MPC_k} \le \mathrm{T}_{MPC}; \end{array} \tag{2}$$

These constant thresholds are selected based on the conventional criteria of stabilization diagram.T_f and T_{MAC} are set as 1 and 2%. Since the relatively scattering observation for damping ratio and MPC, 10–20% for T_d and 5–10% for T_{MPC} are recommended depending on the signal to noise ratio.

(5) Once the number of modes collected in *Ck* is more than 10% of the total model order, the constant criteria based on the 99.7% rules of their corresponding difference vectors are updated. The difference vector is computed as a difference between each element in the cluster to the mean value of the cluster. The criteria are then updated as

$$T = \mu + 3\sigma \tag{3}$$

(6) Continue scanning the rest of the rows using the updated criteria. If a new mode is collected in the *Ck*, the criteria are updated again follows Step 5. Once the scanning pointer reaches the last row, stop the clustering process for cluster *Ck* and return to the beginning to start the process for the next cluster $Ck + 1$.

(7) When all the modes in the system mode matrix are collected in clusters, finish the clustering process and remove the clusters contain less than 10% of the total model order.

2.2.3 Clusters Merging

After the above clustering procedure, the number of spurious modes is greatly reduced and most of the physical modes are collected into large clusters. However, due to the existing noise and measurement error, the physical modes representing the same mode may group into two or three clusters. Meanwhile, as 10% of total model order is a small threshold, some relative stable spurious mode may remain in the system mode matrix. To further improve the robustness of the proposed approach, the clusters merging procedure is developed. This procedure is designed in term of the 99% confident interval of frequency, damping ratio and MPC of clusters. The clusters merging procedure can be concluded as

1. Compare every two clusters and pick up the larger one.
2. Compute the 99% mean value confident interval of each criterion.
3. Determine whether these two clusters should be merged by evaluating if the mean values of frequency, damping ratio and MPC of the smaller cluster are within the confident intervals of the large cluster. If yes, the mode shapes between two clusters are further evaluated by comparing their MAC value and the mean of the internal MAC value between each mode shape and the mean mode shape. If the MAC value is larger than the mean MAC value, these two clusters are merged.
4. Continue the merging process until every two clusters are compared and processed.

3 Numerical Studies

Numerical studies are conducted on a steel frame structural model designed based on a laboratory seven-storey frame structure, as elaborated in [4]. The finite element (FE) model and the laboratory structure are shown in Fig. 1a, b, respectively. The column and beam in every storey are modelled with three and four elements respectively using two nodes planar beam elements. Every node in the two-dimensional coordinate system consists of three degrees-of-freedom (DOFs), which are the translational displacements in x and y directions and the rotational displacement in the x-y plane. Mass blocks are simulated by adding weights to the corresponding nodes of the FE model as lumped masses. The FE model of the frame includes totally 70 planar elements and 65 nodes. With 3 DOFs at each node, the model has 195 DOFs in total. Two fixed supports at node 1 and 65 are modelled by restraining the corresponding DOFs. The ambient excitation is simulated by applying a zero mean and 0.005 standard deviation white noise excitation at both supports simultaneously. The dynamic responses of the numerical model are computed using the Newmark-beta method. The sampling frequency is 500 Hz, and 100 s of dynamic data are generated. Acceleration responses along the x-direction at node 4, 7, 10, 13, 16, 19, 22 are selected for the modal analysis because they are the most representative nodes for the mode shape. The noise in the measurements is added into the measured dynamic response signal$_m$ as

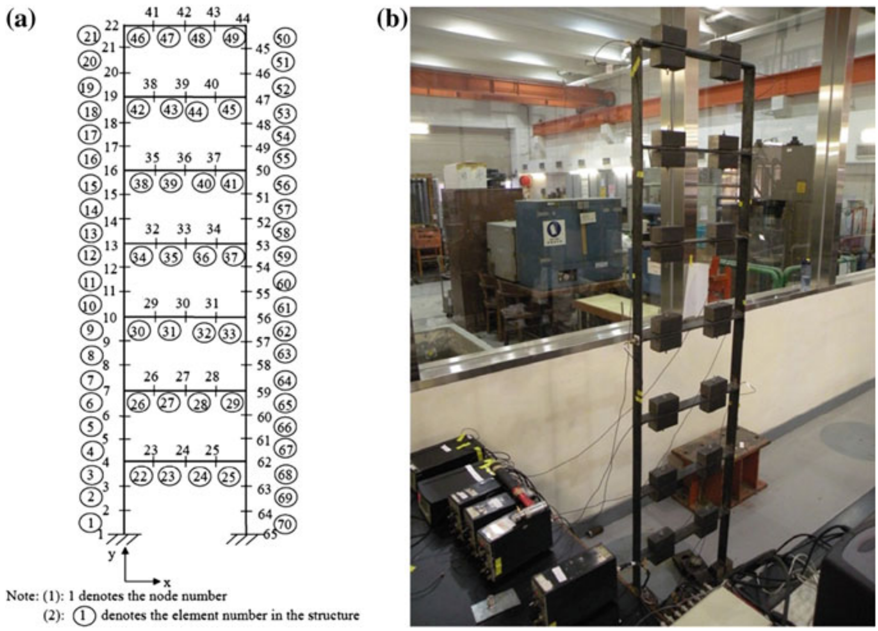

Fig. 1 **a** The FE model of the frame structure, **b** the laboratory steel frame model

$$\text{signal}_{noise} = \text{signal}_m + N_p * Noise \tag{4}$$

where *Np* is the noise level associated with the measured signal. *Noise* is a vector and assumed as a normally distributed random vector with zero mean and unit standard deviation of the dynamic signal [5]. In this study, 20% noise is added. Such a large noise always leads to a serious effect on the modal analysis.

The vibration data are pre-processed and down-sampled to 100 Hz. Model parameter *i* is selected as 50 and the range of model order is selected as 30–120. Since the numerical simulation and laboratory tests have a less noise effect and a minor synchronization issue, the thresholds are selected as 0.7 for MPC and 0.3 For MPD. The results after conducting each procedure are separately presented in Fig. 2a–c. The singular values of the spectral matrix derived using FDD method are shown behind the stabilization diagram for the comparison purpose. Figure 2a shows the remaining modes after hard criteria removal described in Sect. 2.2.1. The original stabilization diagram is not shown here but in fact, this procedure significantly removes certain spurious modes. Figure 2b shows the remaining modes after the proposed clustering procedure. Comparing Fig. 2a, b, the spurious modes representing the same structural mode are effectively removed or separated as two small clusters. Meanwhile, due to the noise effect, some physical modes representing the same mode are grouped as two or three relatively small clusters (but larger than the 10% threshold). As shown in Fig. 2c, the clustering merging procedure successfully eliminates all the spurious modes, and the physical modes representing the same mode are merged as one large cluster. The number of modes in each physical modes related cluster is much more than 50% threshold. In addition, two spurious modes clusters at around 12.03 Hz are also merged as shown in Fig. 3. These two clusters are merged because the scattered modes are separately grouped into other small clusters, only limited number of modes with relatively close properties are grouped in these two clusters. However, the merged cluster is finally deleted from the system modes matrix as the size is far from the 50% threshold.

4 Experimental Study

An experimental study is further performed to evaluate the effectiveness of the proposed approach on the practical application using the laboratory frame structure, as shown in Fig. 1b. The frame structure is under ambient vibration in the laboratory. 30 min accelerations with 2000 Hz sampling frequency on each node of the left column are used for the modal identification. A Fast Fourier Transfer of the time domain vibration data is conducted to process the signal to the frequency domain. From the frequency spectrum, the frequency of interest is found under 30 Hz. To filter out the high frequency noise, the sampling rate is down-sampled to 100 Hz followed a 50 Hz low-pass filter. The same model parameters for SSI as those in the numerical study are selected. By processing the SSI output progressively, the remaining clusters and the clustering results are shown in Figs. 4a–c. In contrast

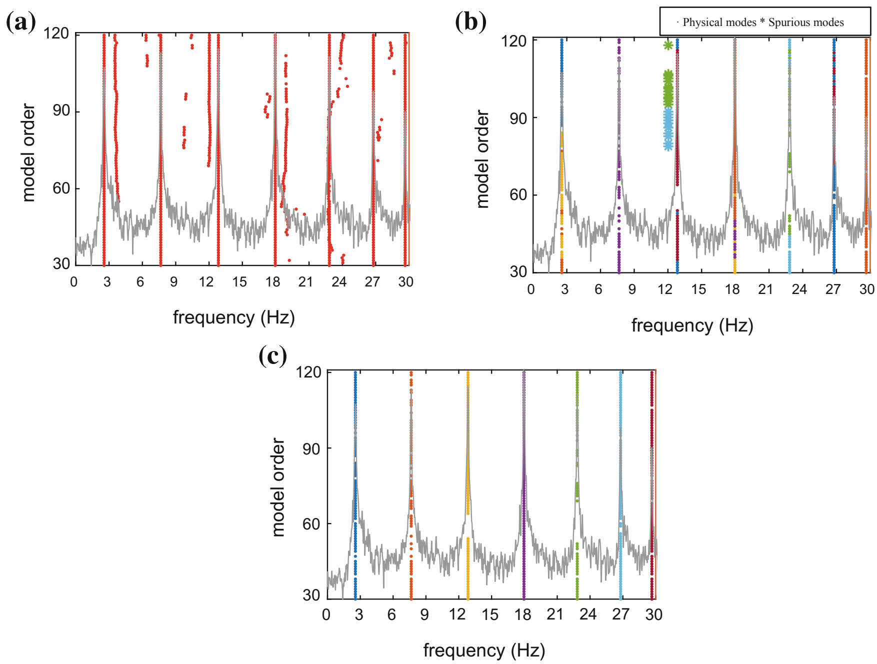

Fig. 2 Modal identification results in the numerical study by the proposed approach: **a** hard criteria, **b** modes clustering, **c** clusters merging

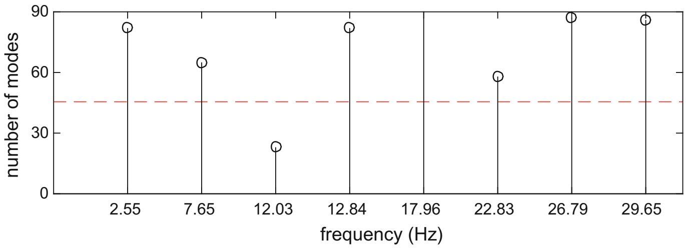

Fig. 3 The number of modes for each cluster after clusters merging procedure

with the numerical study, more spurious modes remain in the stabilization diagram and more confusing peaks in the first singular value spectrum can be found, as shown in Fig. 4a. The existing spurious modes with relatively stable modal parameters can be a challenge for the operational modal identification. The results obtained from the proposed approach demonstrate that a strong robustness on processing test data with low signal to noise ratio. As shown in Fig. 4b, most spurious modes are removed and the remaining spurious modes are separately grouped in small clusters. The good separation of spurious modes clusters is because the spurious modes representing the same mode may be consistent in one or two modal parameters. However, when

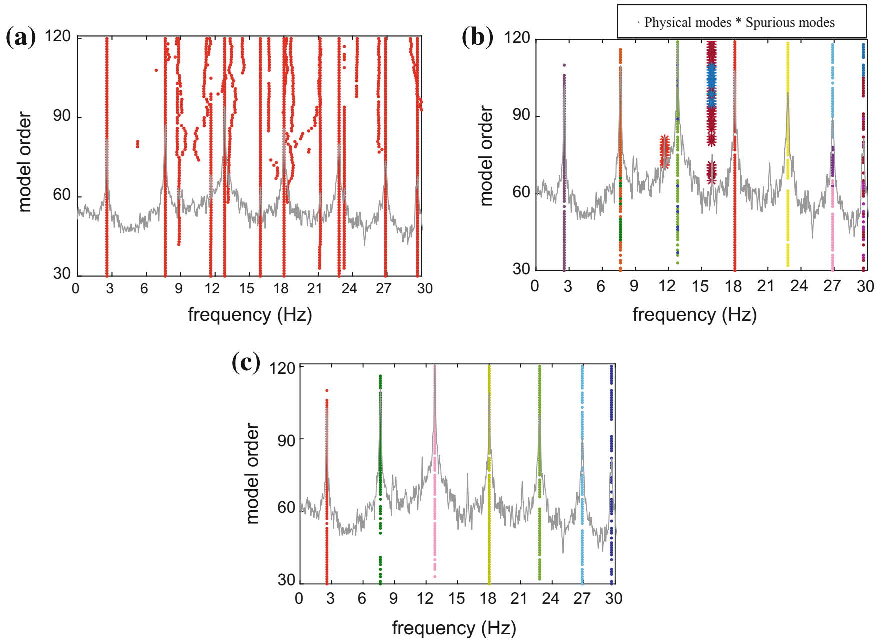

Fig. 4 Modal identification results in the experimental study by the proposed approach: **a** hard criteria, **b** modes clustering, **c** clusters merging

all the parameters mentioned in Sect. 2.2.2 are considered, they are distinguished by the self-adapted thresholds. The final seven clusters are shown in Fig. 4c, where all the spurious modes clusters are filtered and the physical modes clusters are kept with highly consistent modal parameters, indicating the accuracy and performance of the proposed approach. Figure 5 shows the identified frequencies and the corresponding mode shapes of the laboratory frame. These identification results are well matched with those in a previous study [4] with a maximum relative error of less than 1%.

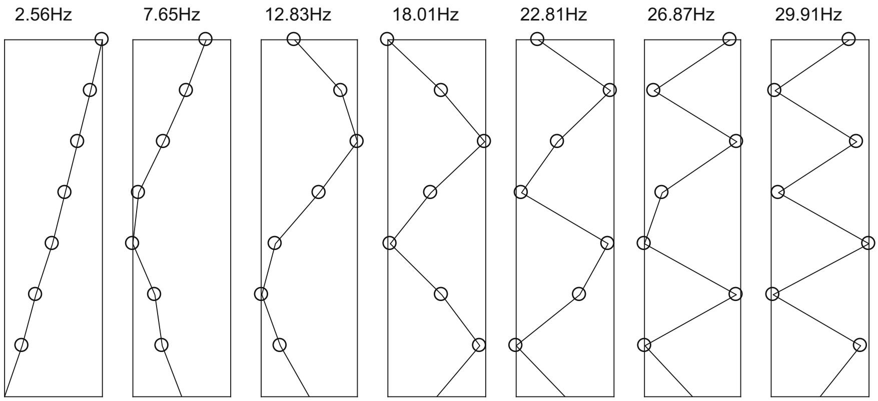

Fig. 5 Identified mode shapes of the laboratory frame

5 Conclusions

The proposed automated modal analysis approach is presented with an improved performance than the traditional data-driven SSI in the modal identification of a steel frame structure. Numerical and experimental studies are conducted. The results demonstrate that a robust performance to tolerate the noise effect and a better performance to remove the spurious modes are achieved. With the minor requirement on the users' manual tuning, the proposed approach could be effective and promising in processing the long-term monitoring data of civil engineering structures.

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