The seminal contribution of K. S. Shukla to our understanding of Indian astronomy and mathematics *

Kripa Shankar Shukla[1] was born on June 12, 1918 in Lucknow. He completed his undergraduate and postgraduate studies in mathematics at Allahabad University. In 1941, he joined the research programme on Indian mathematics at the Department of Mathematics, Lucknow University, to work with Prof. Avadhesh Narayan Singh (1905–1954). Prof. Singh, the renowned collaborator of Bibhutibhusan Datta (1888–1958), had joined Lucknow University in 1928. He initiated a research programme on the study of Indian astronomy and mathematics at the University in 1939. He managed to collect a number of manuscripts of important source-works and also attracted many researchers to work with him.

Shukla's first paper, published in 1945, presented a comprehensive survey of the second correction (due to evection) for the Moon in Indian Astronomy. In 1955, Shukla was awarded the D.Litt. degree from Lucknow University for his thesis on "Astronomy in the Seventh Century India: Bhāskara I and His Works". Dr. Shukla became the worthy successor of Prof. Singh to lead the research programme on Indian astronomy and mathematics at Lucknow University. Though he retired as Professor of Mathematics in 1979, he continued to guide researchers and work relentlessly to publish a number of outstanding articles and books—which included an edition and translation of *Vaṭeśvara-siddhānta* (c. 904), the largest known Indian astronomical work with over 1400 verses, brought out by Indian National Science Academy in two volumes during 1985–86. Prof. Shukla passed away on June 22, 2007.

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* M. D. Srinivas, 2018 (To appear in *Gaṇita-Bhāratī*).

[1]For a detailed biography of Prof. K. S. Shukla along with a list of his publications, see: R. C. Gupta, "Dr. Kripa Shankar Shukla, Veteran Historian of Hindu Astronomy and Mathematics", *Gaṇita-Bhāratī*, 20 (1998), pp. 1–7. Also, Yukio Ohashi, "Kripa Shankar Shukla (1918–2007)", *Indian Journal of History of Science*, 43 (2008), pp. 475–485.

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1 Publications of K. S. Shukla on Indian astronomy and mathematics[2]

Prof. Shukla was famous as a great teacher and expositor of astronomy and mathematics. In the 1950s he wrote popular textbooks on trigonometry and algebra and also published a Hindi translation of Part I of the History of Hindu Mathematics by B. B. Datta and A. N. Singh. It was indeed very unfortunate that there was no course on History of Mathematics or Indian Mathematics taught at the Lucknow University, notwithstanding the presence of a great scholar and teacher such as Prof. Shukla on its faculty.[3] However, Prof. Shukla guided several researchers in their work on Indian astronomy and mathematics. Amongst those who worked with Shukla for their Ph.D. Degree, are the well known scholars, Parmanand Singh (who worked on the *Gaṇitakaumudī* of Nārāyaṇa Paṇḍita), and the Japanese scholar Yukio Ohashi (who worked on the history of astronomical instruments in India).

Shukla brought out landmark editions of twelve important source-works of Indian astronomy and mathematics. Some of them were published from Lucknow University under the Hindu Astronomical and Mathematical Texts Series. Following is a list of the source-works published by Shukla.

1. *Sūryasiddhānta* with commentary of Parameśvara, ed. by K. S. Shukla, Lucknow University, Lucknow 1957.
2. *Pāṭīgaṇita* of Śrīdharācārya, ed. and tr. with notes by K. S. Shukla, Lucknow University, Lucknow 1959.
3. *Mahābhāskarīya* of Bhāskara I, ed. and tr. with notes by K. S. Shukla, Lucknow University, Lucknow 1960.
4. *Laghubhāskarīya* of Bhāskara I, ed. and tr. with notes by K. S. Shukla, Lucknow University, Lucknow 1963.
5. *Dhīkoṭidakaraṇa* of Śrīpati, ed. and tr. with notes by K. S. Shukla, Akhila Bharatiya Sanskrit Parishad, Lucknow 1969.
6. *Bījagaṇitāvataṃsa* of Nārāyaṇa Paṇḍita, ed. by K. S. Shukla, Akhila Bharatiya Sanskrit Parishad, Lucknow 1970.
7. *Āryabhaṭīya* of Āryabhaṭa, ed. and tr. with notes by K. S. Shukla and K. V. Sarma, Indian National Science Academy, New Delhi 1976.
8. *Āryabhaṭīya* of Āryabhaṭa with the commentary of Bhāskara I, ed. by K. S. Shukla, Indian National Science Academy, New Delhi 1976.

[2] For an insightful overview of the publications of Prof. Shukla on Indian astronomy, see: Yukio Ohashi, "Prof. K. S. Shukla's Contribution to the Study of Hindu Astronomy", *Gaṇita-Bhāratī*, 17 (1995), pp. 29–44.

[3] See R. C. Gupta (1998), p. 3.

9. *Karaṇaratna* of Devācārya, ed. and tr. with Notes by K. S. Shukla, Lucknow University, Lucknow 1979.
10. *Vaṭeśvarasiddhānta* and *Gola* of Vaṭeśvara, ed. and tr. with notes by K. S. Shukla, 2 Volumes, Indian National Science Academy, New Delhi 1985–86.
11. *Laghumānasa* of Mañjula, ed. and tr. with notes by K. S. Shukla, Indian National Science Academy, New Delhi 1990.
12. *Gaṇitapañcaviṃśī*, ed. and tr. by K. S. Shukla, *Indian Journal of History of Science*, 52.4 (2017), pp. S1–S22.

As may be seen from the above list, most of these editions also include lucid English translations and detailed mathematical explanatory notes. This is indeed one of the greatest contributions of Prof. Shukla, since till the 1960s there had been very few editions of the classical source-works of Indian astronomy which also included a translation as well as explanatory notes. As regards the source-works of Indian mathematics, there were the well known translations, along with explanatory notes, authored by Colebrooke[4] and Rangacarya[5] of the mathematics chapters of *Brāhmasphuṭasiddhānta* of Brahmagupta, the *Līlāvatī* and *Bījagaṇita* of Bhāskarācārya and the *Gaṇitasārasaṅgraha* of Mahāvīra. As regards Indian astronomy, while a number of source-works were published by Sudhākara Dvivedi and other scholars, the only texts which were translated into English,[6] along with explanatory notes, were the *Sūryasiddhānta* by Burgess,[7] *Pañcasiddhāntikā* of Varāhamihira by Thibaut,[8] *Āryabhaṭīya* by Sengupta[9] and Clark,[10] and the *Khaṇḍakhādyaka* of Brahmagupta by Sengupta.[11]

The scholarly world is highly indebted to Prof. Shukla for having taken great pains to publish lucid translations, along with detailed mathematical explanatory notes, of some of the most important source-works of Indian astronomy, including works of all the three categories, namely, *Siddhānta*, *Tantra* and

[4] H. T. Colebrooke, *Algebra with Arithmetic and Mensuration from the Sanscrit of Brahmegupta and Bhāscara*, John Murray, London 1817.

[5] M. Rangacarya, *The Gaṇitasārasaṅgraha of Mahāvīrācārya with English Translation and Notes*, Government Press, Madras 1912.

[6] There were also translations of *Sūryasiddhānta*, *Āryabhaṭīya*, *Siddhāntaśiromaṇi* of Bhāskara II and *Grahalāghava* of Gaṇeśa into various Indian languages, some of which also included explanatory notes.

[7] E. Burgess, *Translation of the Sūryasiddhānta*, The American Oriental Society, New Haven 1860.

[8] G. Thibaut and Sudhakar Dvivedi, The *Pañcasiddhāntikā*, Medical Hall Press, Benares 1889.

[9] P. C. Sengupta, "The *Āryabhaṭīyam*", Journal of Department of Letters of Calcutta University, 16 (1927), pp. 1–56.

[10] W. E. Clark, *The Āryabhaṭīya of Āryabhaṭa*, University of Chicago Press, Chicago 1930.

[11] P. C. Sengupta, *The Khaṇḍakhādyaka*, University of Calcutta, Calcutta 1934.

Karaṇa, and covering the entire classical *Siddhāntic* period from Āryabhaṭa (c. 499) to Śrīpati (c. 1039). His explanatory notes often include summaries of important discussions found in various commentaries, and also detailed references to similar results or procedures contained in other important texts. Shukla's editions and translations have therefore acquired the status of canonical textbooks which can be profitably used by all those interested in a serious study of Indian astronomical tradition. In collaboration with the renowned scholar Samarendra Nath Sen (1918–1992), Prof. Shukla has also edited a pioneering History of Indian Astronomy in 1985, which continues to be the standard reference work on the subject.[12]

Prof. Shukla has also written over 40 important research articles, which have ushered in an entirely new perspective on the historiography of Indian astronomy and mathematics. We may here make a mention of just a few of his seminal contributions:

(i) Clear exposition of various aspects of the Vasiṣṭha, Romaka and Pauliśa *Siddhāntas* as summarised in *Pañcasiddhāntikā* of Varāhamihira.

(ii) Correction of the faulty readings and translations of some of the crucial verses giving the number of civil days and other parameters of a *yuga*, as presented in the 1978 edition of *Yavanajātaka* by David Pingree.

(iii) Discovery of the verses of *Āryabhaṭasiddhānta* dealing with *yantras* (instruments).

(iv) Correct explanation of the *manda-saṃskāra* (equation of centre) in Indian astronomy, including the computation of the *aviśiṣṭa-mandakarṇa* (iterated *manda*-hypotenuse) and its significance.

(v) Correct explanation of the second lunar correction (incorporating the evection correction) as presented by Mañjulācārya.

(vi) Discovery of the verses of Acārya Jayadeva on the *cakravāla* method for solving quadratic indeterminate equations.

(vii) Detailed exposition of the study of magic squares in Indian mathematics.

(viii) Publication of a revised and updated version of Part III of the 'History of Hindu Mathematics' by B. B. Datta and A. N. Singh.

In what follows, we shall present some highlights of the seminal contribution of Prof. Shukla in relation to items iv, ii and viii of the above list (in that order).

[12] S. N. Sen and K. S. Shukla, (eds.), *History of Indian Astronomy*, Indian National Science Academy, New Delhi 1985 (2nd Revised Edition 2000).

2 Explaining the correct formulation of the *manda-saṃskāra* (equation of centre) in Indian astronomy

In his landmark translations of *Mahābhāskarīya* and *Laghubhāskarīya*, published in 1960 and 1963 respectively, Prof. Shukla explained the correct formulation of the *manda-saṃskāra* or the equation of centre, as expounded by Bhāskara I (c. 629). This corrected a longstanding misconception, as the equation of centre in Indian astronomy was totally misconstrued by modern scholarship for nearly two centuries. In what follows, we shall summarise the formulation of *manda-saṃskāra* or the equation of centre, and the computation of the *manda-karṇa* or the *manda*-hypotenuse, as expounded by Bhāskara in Chapter IV of *Mahābhāskarīya*, following closely the lucid exposition of Prof. Shukla. We shall discuss the *manda-saṃskāra* formulated in terms of an epicycle model.[13]

In Figure 1, O is the centre of the earth, P_0 the mean planet and U the mandocca or the *manda*-apsis. $OP_0 = R$, is the radius of the concentric. P_1 is on the epicycle centred at P_0, with radius equal to the tabulated epicycle radius r_0,[14] such that P_0P_1 is parallel to OU. P_1 is also on the eccentric circle with O' as the centre, where O' is along OU, such that $OO' = r_0$. The true radius of the epicycle r is different from the tabulated radius r_0. Hence, P_1 is not the *manda-sphuṭa* or the true *manda*-corrected planet. Similarly, $OP_1 = K_0$, is only the *sakṛt-karṇa* or the initial hypotenuse and not the true hypotenuse.

The *manda-sphuṭa* or the true *manda*-corrected planet is at P (along P_0P_1) such that $P_0P = r$, which is the true epicycle radius. Correspondingly the true *manda*-hypotenuse is given by $OP = K$. The main feature of this model is that the true epicycle radius r and the true *manda*-hypotenuse K are related by

$$r = \frac{r_0}{R}K. \tag{1}$$

Let θ_0 be the longitude of the mean planet P_0, θ_u the longitude of the *mandocca* U, and θ_{ms} the longitude of the *manda-sphuṭa* or the *manda*-corrected

[13] *Mahābhāskarīya* of Bhāskara I, ed. and tr. with Notes by K. S. Shukla, Lucknow University, Lucknow 1960. The equation of centre for the Sun and the Moon is discussed, following both the epicycle and eccentric-circle models, on pp. 110–119 and pp. 122–126, respectively. The equation of centre for the planets is similar and discussed later on pp. 134–144.

[14] The values of the *manda* and *śīghra* epicycle radii are presented in Chapter VII of *Mahābhāskarīya* (ibid. pp. 206–7). It is important to note that, except in the case of the Sun and the Moon, even these tabulated epicycles are not constant, but vary with the anomaly. Their extreme values are given at the beginning of the odd and even quadrants, and in between they have a periodic variation. Only in the case of the Sun and the Moon, this factor does not come into play, and we can treat the tabulated epicycle as a constant.

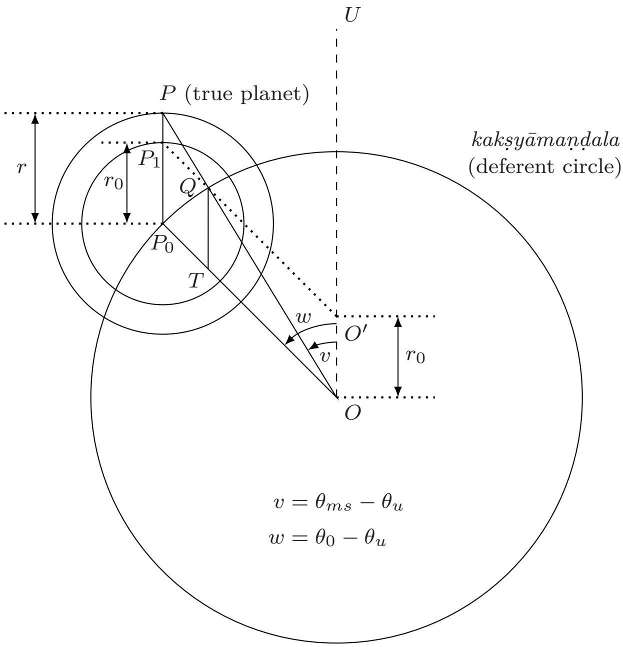

Figure 1: *Manda*-correction and the iterated-*manda*-hypotenuse.

planet P. Hence, the mean anomaly is given by the angle $P_0OU = \theta_0–\theta_u$, and the true anomaly is given by the angle $POU = \theta_{ms}–\theta_u$, and the angle $P_0OP = \theta_0–\theta_{ms}$. From Figure 1, we can easily see that the *manda*-correction for the longitude will be

$$R\sin(\theta_{ms}–\theta_0) = -\left(\frac{r}{K}\right) R\sin(\theta_0–\theta_u). \tag{2}$$

Now, by applying the condition (1), we get the final form of the *manda*-correction

$$R\sin(\theta_{ms}–\theta_0) = -\left(\frac{r_0}{R}\right) R\sin(\theta_0–\theta_u). \tag{3}$$

Thus, the *manda*-correction or the equation of centre (3) involves only the ratio of the mean epicycle radius r_0 and the radius R of the concentric. It does not involve the initial hypotenuse K_0 or even the true hypotenuse K.

The important point to be realised is that the *karṇa* or hypotenuse does not appear in the equation of centre in Indian astronomy—unlike in the case of the *śīghra*-correction (or the so called equation of conjunction) where the correction crucially depends on the *śīghra-karṇa* or the *śīghra*-hypotenuse—because the *manda*-epicycle, in Indian planetary theory, is assumed to be variable and varies in the same way as *karṇa* as shown in relation (1). It is for this reason that the *karṇa* gets replaced by just the radius of the concentric in the equation of centre—not because of any approximation that the radius does not differ too much from the hypotenuse in the *manda-saṃskāra.*

While it may not make its appearance in the equation of centre, the *manda-karṇa* is still very important. For instance, it gives the true distance in the case of the Sun and the Moon. Now, in order to determine both r and K, Bhāskara presents the following iterative process. To start with, the initial hypotenuse (*sakṛt-karṇa*), K_0, is computed in the usual way in terms of the mean anomaly $(\theta_0 – \theta_u)$, using the mean epicycle radius r_0:

$$K_0 = OP_1 = \sqrt{[R\sin(\theta_0 – \theta_u)]^2 + [R\cos(\theta_0 – \theta_u) + r_0]^2}. \tag{4}$$

Then the next approximation to the epicycle radius, r_1, is computed using

$$r_1 = \frac{r_0}{R} K_0. \tag{5}$$

From r_1 the corresponding hypotenuse K_1 is computed using

$$K_1 = \sqrt{[R\sin(\theta_0 – \theta_u)]^2 + [R\cos(\theta_0 – \theta_u) + r_1]^2}. \tag{6}$$

And, from K_1, the next approximation r_2 is computed using

$$r_2 = \frac{r_0}{R} K_1. \tag{7}$$

And so on, till there is no appreciable difference between successive results (*aviśeṣa*), which means that, for some m

$$r_{m+1} = \frac{r_0}{R} K_m \approx r_m. \tag{8}$$

Then, it can be seen right away that the iterated radius r_m and the associated hypotenuse K_m, are such that

$$r_m = \frac{r_0}{R} K_m. \tag{9}$$

In other words, they very nearly satisfy the relation (1) that characterises the true epicycle r and the corresponding true hypotenuse K.

In his explanatory notes to the *Mahābhāskarīya*, Shukla explains how the above iteration process actually converges to the true radius r and the true hypotenuse K, satisfying the relation (1). He also identifies the geometrical location P of the *manda*-corrected planet in the following manner. In Figure 1, let O' be the point on OU such that $OO' = r_0$. Let the line $O'P_1$ intersect the concentric at Q. Then, the P the true or *manda*-corrected planet is located at the intersection of the lines OQ and P_0P_1, extended if necessary. Now, draw the line QT parallel to P_0P_1, where T is located on OP_0. From the similar triangles OQT and OPP_0, we get

$$\frac{QT}{OQ} = \frac{P_0P}{OP}. \tag{10}$$

Since, $QT = P_0P_1 = r_0$, $P_0P = r$, $OQ = R$, and $OP = K$, equation (10) reduces to relation (1) as required.

Later, in another landmark article published in 1973,[15] Shukla explains how the above formulation of the *manda-saṃskāra* is the one followed almost universally by all schools of Indian astronomy, except for a few astronomers such as Pṛthūdakasvāmi (c. 860) and some seventeenth century astronomers who had not understood the traditional formulation. Shukla first presents detailed quotations from the astronomers of the school of Āryabhaṭa (such as Lalla (c. 750), Govindasvāmi (c. 800), Sūryadevayajvā (c. 1200) and the later Kerala astronomers Parameśvara (c. 1430), Nīlakaṇṭha Somayājī (c. 1500) and Putumana Somayājī (c. 1600)) to show that all of them clearly hold the view that:

(i) The *manda*-hypotenuse does not appear in the equation of centre because the radius of the epicycle and the hypotenuse vary according to the relation (1) mentioned above,

(ii) The true epicycle radius and the true hypotenuse may be found by an iterative process such as the one discussed in *Mahābhāskarīya*.

Shukla next presents quotations from Brahmagupta (c. 628), Śrīpati (c. 1039), Bhāskara II (c. 1150) and the *Ādityapratāpa-siddhānta* to show that they also subscribe to the same formulation of the *manda-saṃskāra* as outlined above. He then refers to the view of Caturveda Pṛthūdakasvāmi (c. 860) the commentator of Brahmagupta, that the hypotenuse is not used in the *manda*-correction because the difference between the radius of the concentric and the hypotenuse is small so that the latter is approximated by the radius itself.[16]

> अतः स्वल्पान्तरत्वात् कर्णो मन्दकर्मणि न कार्यः इति।
> So, there being little difference in the result, the hypotenuse-proportion should not be used in the *manda-saṃskāra*.

[15] K. S. Shukla, "Use of Hypotenuse in the Computation of the Equation of Centre Under the Epicyclic Theory in the School of Āryabhaṭa???", *Indian Journal of History of Science*, 8 (1973), pp. 44–57. The provocative title of the paper is due to the fact that it was written in response to an erroneous claim made by the renowned scholar T. S. Kuppanna Shastri (1900–1982) in his article "The School of Āryabhaṭa and the Peculiarities Thereof" (*Indian Journal of History of Science*, 4 (1969), pp. 126–134).

[16] Shukla (1973), p. 52, citing Pṛthūdaka's commentary on *Brāhmasphuṭasiddhānta* XXI.29.

Shukla also discusses the refutation of the above view of Pṛthūdakasvāmi by Bhāskara II in his *Vāsanābhāṣya* on *Siddhāntaśiromaṇi*.[17] Shukla then considers the case of the *Sūryasiddhānta* and remarks:[18]

> The method prescribed in the *Sūryasiddhānta* for finding the equation of the centre is exactly the same as given by the exponents of the schools of Āryabhaṭa I and Brahmagupta and there is no use of the hypotenuse-proportion. The author of the *Sūryasiddhānta* has not even taken the trouble of finding the *manda* hypotenuse. So it may be presumed that the views of the author of the *Sūryasiddhānta* on the omission of the use of the hypotenuse in finding the equation of the centre were similar to those obtaining in the schools of Āryabhaṭa and Brahmagupta.

Shukla perhaps forgot to mention in this context the important fact that his own 1957 edition of the *Sūryasiddhānta* with the commentary of Parameśvara has a verse (verse IV.2 of the Chapter IV dealing with lunar eclipse) which states that the distance of the Sun or the Moon is proportional to the corresponding "*manda-śravaṇa*" or the *manda*-hypotenuse. And the commentator Parameśvara glosses *manda-śravaṇa* as "*mandasphuṭasiddhakarṇaḥ*", the hypotenuse determined by the location of the true *manda*-corrected planet. Parameśvara also notes that these distances are used for computing diameters of the Sun and the Moon. Shukla notes that this verse is not found in other versions of *Sūryasiddhānta*. All the versions however present an alternate rule for computing the diameters as being inversely proportional to the *sphuṭabhukti* or the true velocity.[19]

In this context, we may also mention that some of the astronomers in North India in the seventeenth century seem to have failed to comprehend the traditional formulation of the *manda-saṃskāra* as expounded by Bhāskara I, Brahmagupta and others. We can see this for instance in the commentary *Gūḍharthaprakaśaka* of Raṅganātha on *Sūryasiddhānta* which was composed in the year 1603. While explaining the verse II.39, which merely prescribes that the radius of the concentric should be the denominator in the expression for the equation of centre, Raṅganātha seems to be following Pṛthūdakasvāmi when he argues that:[20]

> मन्दकर्णस्य त्रिज्यासन्नत्वेन स्वल्पान्तरेण त्रिज्यातुल्यत्वेनाङ्गीकारात्।
> [The hypotenuse is not used in the *manda-saṃskāra*] because the

[17] *Ibid.* pp. 52–3.
[18] *Ibid.* p. 54.
[19] *Sūryasiddhānta* with commentary of Parameśvara, ed. by K. S. Shukla, Lucknow University, Lucknow 1957, p. 58.
[20] *Sūryasiddhānta* with commentary *Gūḍharthaprakaśaka* of Raṅganātha, ed. By F. E. Hall and Bāpū Deva Śāstrin, Baptist Mission Press, Calcutta 1859, pp. 77–8.

> *manda*-hypotenuse is close to the radius [of the concentric] and it can be accepted to be the equal to the radius with a slight difference.

Shukla mentions in his 1973 paper that Munīśvara (c. 1646), the son of Raṅganātha, and his famous rival Kamalākara (c. 1658) also did not follow the traditional view on *mandasaṃskāra*. Instead of considering a variable epicycle, they seem to have advocated the use of just the tabulated epicycle and also division by the *sakṛt-karṇa* or the first hypotenuse (K_0 of equation (4)).

In conclusion Shukla notes:[21]

> From what has been said above it is clear that the hypotenuse has not been used in Hindu astronomy in the computation of the equation of the centre under the epicyclic theory. It is also obvious that with the single exception of Caturvedācārya Pṛthūdaka all the Hindu astronomers are unanimous in their views regarding the cause of omission of the hypotenuse. According to all of them the *manda* epicycles stated in the works on Hindu astronomy correspond to the radius of the planet's mean orbit and are therefore false.
>
> Since the *manda* epicycle stated in the Hindu works corresponded to the radius of the planet's mean orbit, the true *manda* epicycle corresponding to the planet's true distance (in the case of the Sun and the Moon) or true-mean distance (in the case of the planets Mars, etc.) was obtained by the process of iteration. The planet's true or true-mean distance (*manda-karṇa*) was also like wise obtained by the process of iteration.
>
> Direct methods for obtaining the true *manda-karṇa* or true *manda* epicycle were also known to later astronomers. Mādhava (c. 1340–1425) is said to have given the following formula for the true *manda-karṇa*:[22]
>
> $$\text{true } \textit{manda-karṇa} \text{ (or iterated } \textit{manda-karṇa}) = \frac{R^2}{\sqrt{R^2 - (\textit{bhujāphala})^2} \;\tilde{+}\; \textit{koṭiphala}},$$
>
> $\sim$ or $+$ sign being taken according as the planet is in the half orbit beginning with the anomalistic sign Capricorn or in that sign beginning with the anomalistic sign Cancer.

[21] Shukla (1973), p. 54.

[22] The reference is to Nīlakaṇṭha's commentary on *Āryabhaṭīya*, III.17–21, and *Tantrasaṅgraha* II.44.

The exact analytical expression of Mādhava for the iterated-*manda-karṇa*, mentioned above by Shukla, can be recast in the form

$$K = \frac{R^2}{\sqrt{R^2 - [r_0 \sin(\theta_0 - \theta_u)]^2} - r_0 \cos(\theta_0 - \theta_u)}. \tag{11}$$

Here, it may be noted the above result (11) can be easily derived[23] by using the similarity of the triangles OQT and OPP_0 in Figure 1. We have, $OP = OQ \times \frac{OP_0}{OT}$, which may be recast in the form:

$$K = \frac{R^2}{R_v}, \tag{12}$$

where, $OT = R_v$ is the so called inverse hypotenuse or *viparīta-karṇa*, which can easily be shown to be given by

$$R_v = \sqrt{R^2 - [r_0 \sin(\theta_0 - \theta_u)]^2} - r_0 \cos(\theta_0 - \theta_u). \tag{13}$$

Using Mādhava's exact expression for the iterated-*manda-karṇa*, we can also obtain the exact equation satisfied by the orbit of a planet which is moving on a variable epicycle as specified in the *manda-saṃskāra*. It is seen that the orbit is no longer an eccentric circle but a general oval figure.

3 How modern scholarship has misconstrued the equation of centre in Indian astronomy

Shukla's detailed explanation of the *manda-saṃskāra* was indeed path-breaking since, for nearly two centuries, modern scholarship had totally misinterpreted this and other aspects of Indian planetary theory. One of the earliest accounts of Indian planetary theory was the 1790 article of Samuel Davis (1760–1819), which was largely based on *Sūryasiddhānta* and its commentary *Gūḍhārthaprakāśaka* of Raṅganātha. While discussing the equation of centre for the Sun and the Moon, Davis remarks that while the hypotenuse is used in Indian astronomy for computing the retrogressions of planets (through the equation of conjunction or *śīghra-saṃskāra*), they do not do so while computing the equation of centre. He cites the commentator (Raṅganātha, whom

[23] For a detailed discussion of Mādhava's exact expression for the iterated *manda*-hypotenuse, see: *Tantrasaṅgraha of Nīlakaṇṭha Somayājī*, ed. and tr., with notes by K. Ramasubramanian and M. S. Sriram, Hindustan Book Agency, New Delhi 2011, pp. 96–107, pp. 496–7. Also, *Madhyamānayanaprakāraḥ of Mādhava*, ed. and tr. with notes by U. K. V. Sarma, R. Venketeswara Pai and K. Ramasubramanian, *Indian Journal of History of Science*, 46.1 (2011), pp. T1–T29.

we have cited earlier) as attributing this to the small difference between the hypotenuse and the radius of the concentric:[24]

> It is, however, only in computing the retrogradations and other particulars respecting the planets Mercury, Venus, Mars, Jupiter, and Saturn, where circles greatly excentric are to be considered, that the Hindus find the length of the *carṇa*, or hypotenuse ...; in other cases, as for the anomalistic equations of the sun and the moon, they are satisfied to take ..., their difference, as the commentator on the [*Sūrya*] *Siddhānta* observes, being inconsiderable.

The next major discussion on Indian planetary theories is found in the 1816 article of Henry Thomas Colebrooke (1765–1837), who had access to many more source-works of Indian astronomy. Colebrooke first reiterates what Davis had noted regarding the equation of centre based on his study of the *Sūryasiddhānta*. Colebrooke then notes that Brahmagupta and Bhāskara II held a different view that the reason why the hypotenuse does not appear in the equation of centre is not due to any approximation being made, but because the epicycle itself varies with the hypotenuse. However, at the same time, Colebrooke also mentions that the commentators of Brahmagupta (Pṛthūdakasvāmi) and Bhāskara (Munīśvara) do not agree with this view:[25]

> The Hindus, who have not any of Ptolemy's additions to Hipparchus, have introduced a different modification of the hypothesis, for they give an oval form to the excentric or the equivalent epicycle, as well as to the planet's proper epicycle. That is they assume that the axis of the epicycle is greater at the end of the (*sama*) even quadrants of anomaly ..., and least at the end of the (*viśama*) odd quadrants ...
>
> A further difference of theory, though not of practice, occurs among the Hindu astronomers ... A reference to Mr. Davis essay ... will render intelligible what has been already said and what now remains to be explained. It is there observed ... that for the anomalistic motion of the sun and moon they are satisfied to take ... the sine of the mean anomaly reduced to its dimensions in the epicycle in parts of the radius of the concentric, equal to the sine of the anomalistic equation. The reason is subjoined: 'The differ-

[24] S. Davis "On the Astronomical Computations of the Hindus", *Asiatic researches*, 2 (1790), pp. 225–287. The quotation appears on page 251 and refers to a diagram on page facing 249.

[25] H. T. Colebrooke, "On the notions of the Hindu Astronomers concerning the Precession of the Equinoxes and the Motions of the Planets", *Asiatic researches*, 16 (1816), pp. 209–250. The quotation appears on pp. 235–238.

> ence as the commentator on the *Sūrya Siddhānta* observes being inconsiderable.'
>
> Most of the commentators on the *Sūrya Siddhānta* assign that reason; but some of them adopt Brahmegupta's explanation. This astronomer maintains that, the operation of finding the *carṇa* is rightly omitted ... His hypothesis as briefly intimated by himself, and as explained by Bhāscara, supposes the epicycle, which represents the excentric, to be augmented in proportion which *carṇa* (or the distance of the planet's place from the earth's centre) bears to the radius of the concentric; and it is on this account, and not as a mere approximation that the finding of the *carṇa*, with subsequent operation to which it is applicable is dispensed with.
>
> The scholiast of Brahmegupta objects to his author's doctrine on this point, that upon the same principle, the process of finding the *carṇa* ... should in like manner be omitted in the proper epicycle of the five minor planets; and he concludes therefore, that the omission of that process has no other ground, but the very inconsiderable difference of the result in the instance of a small epicycle. For as remarked by another author ([Munīśvara] in the *Marīci*[26] [commentary on *Siddhāntaśiromaṇi*]), treating on the same subject, the equation itself and its sine are very small near the line of the apsides; and at a distance from that line, the *carṇa* and the radius approach to equality.

The first English translation, along with detailed explanation, of an Indian astronomical text appeared nearly fifty years later. The translation of *Sūryasiddhānta* due to Ebenezer Burgess (1805–1870) (as revised by William Dwight Whitney (1827–1894)) was published in 1860. This again noted that the equation of centre in Indian astronomy was based on the approximation that the hypotenuse was nearly equal to the radius. It also claimed that the *manda*-corrected planet was not on any epicycle or eccentric, but was always located on the concentric itself:[27]

> The world wide difference between the spirit of the Hindu astronomy and that of the Greek ... the one is purely scientific, devising

[26] The reference is to the following statement in Munīśvara's commentary *Marīci* (on *Siddhāntasiromaṇi*, *Golādhyāya*, *Chedyakādhikara*, verses 36–37): "तथा च यत्र परमफलासन्नं फलं तत्र कर्णस्य त्रिज्यासन्नत्वेनाल्पान्तरं यत्र च कर्णस्य बह्वन्तरेण त्रिज्यातोऽधिकत्वं न्यूनत्वं वा तत्र फलस्यैवाल्पत्वेनाल्पान्तरत्वमिति भावः।" (*Siddhāntaśiromaṇi* of Bhāskarācārya, *Vāsanābhāṣya* and *Marīci* by Munīśvara, ed., Dattātreya Viṣṇu Āpaṭe, Volume I, Ānandāśrama Press, Poona 1943, p. 190).

[27] E. Burgess, *Translation of the Sūryasiddhānta*, The American Oriental Society, New Haven 1860, p. 48, pp. 64–5.

> methods for representing and calculating the observed motions and attempting nothing further; the other is not content without fabricating a fantastic and absurd theory respecting the superhuman powers which occasion the movements with which it is dealing. The Hindu method has this convenient peculiarity, that it absolves from all necessity of adapting the disturbing forces to one another, and making them form one consistent system, capable of geometrical representation and mathematical demonstration; it regards the planets as actually moving in circular orbits, and the whole apparatus of epicycles . . . as only a device for estimating the amount of the force and of its resulting motion, exerted at any given point by the disturbing cause . . .
>
> Now as the dimensions of the epicycle in all cases are small, . . . may be without any considerable error may be assumed to be equal to . . .; this assumption is accordingly made and . . . gives the equation concerned.

Nearly a hundred years later, in 1956, when translations of the *Pañcasiddhāntikā*, *Āryabhaṭīya* and the *Khaṇḍakhādyaka* had also became available, the renowned historian of astronomy Otto Neugebauer (1899–1990), presented an analysis of the "Hindu Planetary Theory". He noted that:

> Ignoring the theory of latitude the model which forms the basis for the methods followed, e.g., by the *Sūryasiddhānta*, or in the *Khaṇḍakhādyaka*, is an eccentric epicycle. A model of this type (*cf.* Figure 2) is determined by the radius r of the epicycle, the eccentricity e, and the longitude λ_A of the apogee A' of the deferent of radius R.

Neugebauer also discussed the four-step process of combining the *manda* and the *śīghra-saṃskāras* (equations of centre and conjunction) and found that it was an interesting way of combining these equations—especially when they were given in the form of tables—which is different from the Ptolemaic method of interpolation between extreme values. However, as regards the equation of centre, he repeated what was by then the standard view that it was an approximation:[28]

> Hindu astronomy, however, operates in the case of the correction . . . for eccentricity with an approximate formula . . . There is no reason to treat the effect of the eccentricity with so much less

[28]O. Neugebauer, "The Transmission of Planetary Theories in Ancient and medieval Astronomy", *Scripta Mathematica*, 22 (1956), pp. 165–192. The quotations appear on pages pp. 176–180.

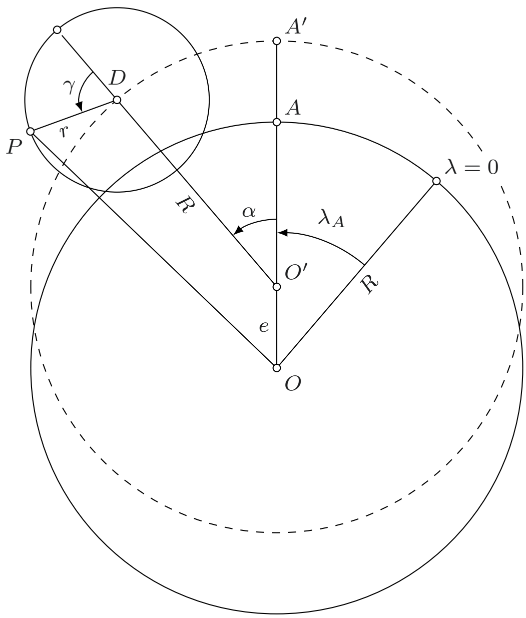

Figure 2: Eccentric epicycle model of Neugebauer.

> accuracy than the effect of the anomaly, except for the fact that usually e [the eccentricity] is smaller than r [the ratio of the radius of the *śīghra* epicycle to that of the concentric]. It may be that in the course of the historical development of planetary theory greater emphasis was attached to the phenomena caused by the anomaly than to those due to the eccentricity, but we know so little about the history of planetary theories that we hardly have any choice except to register the facts.

Starting from the late 1950s, David Pingree (1933–2005), another distinguished scholar of history of exact sciences and a junior collaborator of Neugebauer (whom he succeeded as Professor of History of mathematics at Brown University), brought out a number of studies of Indian astrology and astronomy. Pingree was a reputed scholar of Sanskrit, Akkadian, Arabic and of course Greek and Latin. One of Pingree's main concerns was the transmission of theories and techniques of exact sciences, especially between Mesopotamia, Greece and India in ancient times. Hence, even while a large number of source-works of Indian astronomy—of the classical *Siddhāntic* period (c. 500–1200) and of the medieval Kerala School—had become available (including some of the classic works of Shukla), Pingree, in his studies of Indian planetary theories, seems to have focussed mainly on ancient texts such as the *Paitāmaha-*

siddhānta of *Viṣṇudharmottarapurāṇa*, *Yavanajātaka*, *Pañcasiddhāntikā* and the *Sūryasiddhānta*. It is important to keep in mind that, except for the *Sūryasiddhānta* (which, in any case, is considered to be a later text), all the other texts relied upon by Pingree are in the form of brief summaries that too available only in parts, and the available manuscripts are such that the text had to be substantially emended at several places.

Amidst the large corpus of writings by Pingree on Indian astronomy, we shall here focus only on his analysis of the *manda-saṃskāra* or the equation of centre and some related issues. In 1971, Pingree wrote a paper "On the Greek Origin of the Indian Planetary Model Employing a Double Epicycle". Here, Pingree claims that the "common Indian model for the motion of the star planets" was a "double epicycle model", which "involves two concentric epicycles" and reaffirms the old view of Burgess and Whitney that the planet always moved on a concentric or a deferent circle:[29]

> It is my intention here to investigate the Greek background of the common Indian model for the star planets which involves two concentric epicycles.
>
> In the *Paitāmaha-siddhānta* of the *Viṣṇudharmottarapurāṇa*, which is our earliest extent exponent of the Indian double epicycle model (it was probably composed in the first half of the fifty century AD) the pattern was set for all later texts ...
>
> These two epicycles must be regarded simply as devices for calculating the amounts of the equations by which the mean planet on its concentric orbit is displaced to its true position. This interpretation is confirmed by the explanation offered in early texts of the mechanics of the unequal motions of the planets: demons stationed at the *manda* and *śighra* points on their respective epicycles pull at the planets with chords of wind.[30] The computation of the total effect of these two independent forces upon the mean planet varies somewhat from one school (*pakṣa*) of astronomers to another, or even from astronomer to astronomer within a *pakṣa*; but the fundamental concept remains clear: the planet is always situated on the circumference of a deferent circle concentric with the centre of the earth while two epicycles (one each for the [case of] Sun and Moon) revolve about it.

It is not clear whether any geometric model—not to mention a model where the planets are moving on the concentric—can be inferred at all from the available *Paitāmaha-siddhānta*. What is indisputable is that the vast literature on

[29] D. Pingree, "On the Greek Origin of the Indian Planetary Model Employing a Double Epicycle", *Journal of History of Astronomy*, 2 (1971), pp. 80–85.

[30] The reference here is to *Sūryasiddhānta* II.2.

Siddhāntic astronomy, starting from the *Āryabhaṭīya*, clearly talks of the true *manda*-corrected planet being located on the epicycle or the eccentric. Pingree however thinks that these planetary models were "seldom ... used in computation":[31]

> Āryabhaṭa ... correctly describes an eccentric-epicyclic model and indicates the different directions a planet must travel on an epicycle to produce the differing effects of the equation of the anomaly and the equation of the centre. Though a number of later Indian astronomers acquainted with the *Āryabhaṭīya* or derivative texts of the *Āryapakṣa* refer to the eccentric model, it seems seldom to have been used in computation.

Pingree further claims that his version of the Indian double epicycle model "fits most closely into the attempts of Peripatetics [a group of Greek philosophers owing allegiance to Aristotle] in the late first and second century to preserve concentricity while explaining some of the phenomena."[32]

Pingree also notes that:[33]

> The Indians had to still take into account the problem of the varying distances of the Sun and the Moon whose computation is essential for the prediction of eclipse magnitudes. These distances they made to vary with the true instantaneous velocity of the luminaries.[34] Thereby, of course, as was inevitable, strict concentricity is lost. This fact, however, does not militate against the theory of the peripatetic origin of the Indian double-epicycle model.

In some of his later review articles also, Pingree has reiterated his conception of the Indian planetary model being a double epicycle model with two concentric epicycles (see Figure 3).[35] It continues to be cited in the literature as "the planetary model of the Indian tradition".[36]

Following up on his concentric double-epicycle model, Pingree wrote an article on 'concentric with equant' in 1974, where he makes the even more fantastic claim that the verses IV. 9–12 and IV. 19–21 of *Mahābhāskarīya* give the procedure for computing the motion of a body moving along a circle

[31] *Ibid.* p. 81.
[32] *Ibid.* p. 83.
[33] *Ibid.* p. 84.
[34] The reference here is to *Paitāmaha-siddhānta* V.3–4.
[35] See for instance, D. Pingree, "Mathematical Astronomy in India", in C. G. Gillespie (ed.), *Dictionary of Scientific Biography*, Vol. XV, New York 1978, pp. 533–633. Also, D. Pingree, "Astronomy in India", in C. Walker (ed.), *Astronomy before Telescope*, British Museum Press, London 1996, pp. 123–42.
[36] See for instance, T. Knudsen, *The Siddhāntasundara of Jñanarāja*, Johns Hopkins University Press, Baltimore 2014, pp. 184–5.

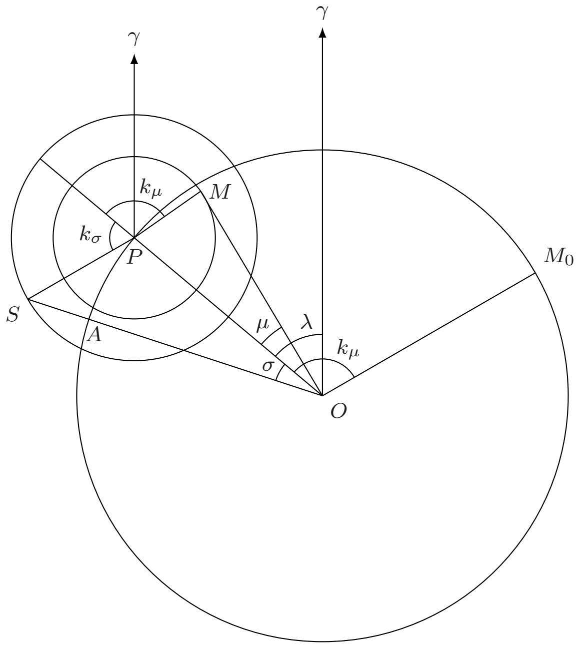

Figure 3: Double epicycle model of Pingree.

but executing uniform motion with respect to a point (equant) displaced from the centre of the circle:[37]

> One purpose of the present article is to point out that a procedure for solving a concentric with equant is described in ... the *Mahābhāskarīya* ...; its second purpose is to suggest a pre-Ptolemaic, Peripataetic origin of the model, and therefore of the equant as well.
>
> In *Mahābhāskarīya* IV. 19–21, is found the method of computing the effect of a concenrtric with equant by means of an eccentric with varying eccentricity. ...
>
> In IV. 9–12, Bhāskara gives an equivalent solution employing an epicycle of varying radius.

It was indeed observed by B. L. van der Waerden and I. V. M. Krishna Rav in the 1950s (whose work has also been cited by Pingree) that the expression for the equation of centre used in Indian astronomy is the same as what would

[37] D. Pingree, "Concentric with Equant", Archives Internationalles d'histoire des Sciences, 24 (1974), pp. 26–28.

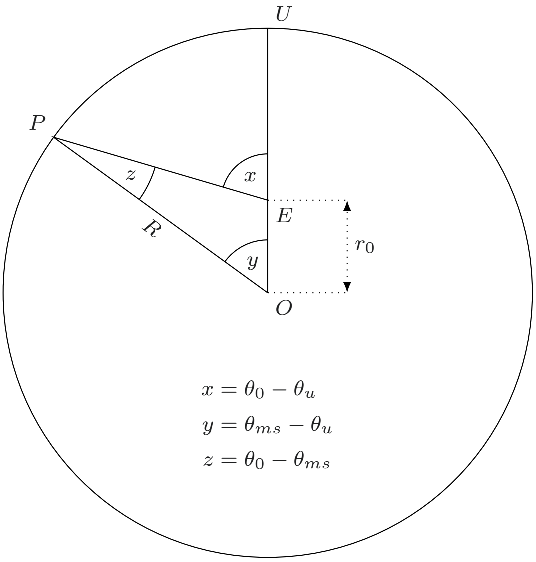

Figure 4: Concentric with equant.

be obtained in the case of a body moving under the hypothesis of 'motion along a concentric with equant'.[38]

In Figure 4, the planet P is moving on the circle with centre O. U is the *ucca* or the apsis and E is the equant point on OU such that $OE = r_0$, the tabulated epicycle radius. If the planet moves uniformly as seen from the equant point, then the angle PEU is the mean anomaly $\theta_0 - \theta_u$, and the angle POU is the true anomaly $\theta_{ms} - \theta_u$, and it can easily be seen that the equation of centre will have the same form as given by equation (3):

$$R \sin(\theta_{ms} - \theta_0) = -\left(\frac{r_0}{R}\right) R \sin(\theta_0 - \theta_u).$$

However, this equivalence is only with respect to the computation of the longitudes of the planets and not their geocentric orbits which also involves the variation of their distance from the centre of the earth. In fact, what Bhāskara is describing in verses IV. 9–12 and IV. 19–21 of *Mahābhāskarīya* is an iterative method to compute the varying *manda-karṇa* or the hypotenuse

[38] See B. L. van der Waerden, "Tamil Astronomy", *Centaurus*, 4 (1956), pp. 221–234, and the references cited there. The title of the paper is due to the fact that the investigations of van der Waerden and Krishna Rav were aimed at understanding the *vākyas* giving the longitudes of the Sun and Moon. It so happens that the *vākya* system of South India had been wrongly characterised as "Tamil Astronomy" by Neugebauer in 1952 (O. Neugebauer, "Tamil Astronomy: A Study in the History of Astronomy in India", *Osiris*, 10 (1952), pp. 252–276).

drawn from the centre of the concentric to the planet on either an epicycle (of variable radius) or an eccentric (of variable eccentricity). That *manda-karṇa*, as the commentator Govindasvāmi notes, is *grahaghanabhūmadhyāntaram*,[39] the distance between the planet and the centre of the earth. There is thus no way that the *manda-saṃskāra* of Indian Astronomy can be conflated with the 'concentric with equant' model of planetary motion—irrespective of whether such a model was known to the Peripatetics (as Pingree suspects) or not.

Pingree, however, has reiterated his claim that, in *Mahābhaskarīya*, "the epicyclic and eccentric models are considered and both are used to solve the concentric with equant model by iteration" in his review article of 1978.[40] Again we find this being echoed in the current literature in statements such as:[41]

> In the early Indian texts the anomalies of the Sun and Moon are both modelled with concentric equant (the Earth is at the centre of the deferent). . . .
>
> Bhāskara explains the equivalence of the concentric equant and an oscillating eccentric model by computing one from the other.

It is indeed unfortunate that such distorted views—concerning the formulation of *manda-saṃskāra* and the meaning of *manda-karṇa*—continue to prevail amongst the scholars studying Indian astronomy, notwithstanding the fact that these issues have been dealt with very clearly and conclusively in the books and articles of Prof. Shukla.

4 Correcting the verses giving the *yuga* parameters in Pingree's edition of *Yavanajātaka*

In 1989, Prof. Shukla wrote a seminal article[42] where he examined and corrected the text and translation of about ten verses, which presented the basic

[39] *Mahābhaskarīya with Bhāṣya of Govindasvāmi and Siddhāntadīpikā of Parameśvara*, ed. by T. S. Kuppanna Sastri, Government Oriental Manuscripts Library, Madras 1957, p. 190.

[40] D. Pingree (1978), p. 593.

[41] D. Duke, "Were Planetary Models of India Strongly Influenced by Greek Astronomy?" in J. M. Steele, ed., *The Circulation of Astronomical Knowledge in the Ancient World*, Brill, Leiden 2018, pp. 559–575. The quotations appear on pages pp. 562, 570–1. It may also be noted that in this and some of his earlier articles Duke has shown that the four-step process used in Indian planetary models gives a better approximation to the Ptolemaic model with the equant, than is the case with a simple eccentric-epicycle model.

[42] K. S. Shukla,"The Yuga of the *Yavanajātaka*: David Pingree's Text and Translation Reviewed", *Indian Journal of History of Science*, 34 (1989), pp. 211–223.

parameters of the yuga, in chapter 79 of the famous critical edition and translation of *Yavanajātaka* of Sphujidhvaja published by Prof. Pingree in 1978.[43]

The publication of *Yavanajātaka* of Sphujidhvaja, was an important milestone that established Prof. Pingree as a leading scholar of history of astrology and astronomy in the ancient world. In his preface, Pingree recounts the importance of the work, how hard he had to work for getting the manuscript and editing and translating it over nearly two decades, and about his confidence that his "main conclusions are unassailable":[44]

> Sphujidhvaja first attracted my attention over twenty years ago, when I read the brief account of the *Yavanajātaka* given by Mahāmahopādhyāya Haraprasād Śāstri ... I spent the academic year 1957–58 in India ... In December of 1957, I travelled to Nepal to attempt to see the manuscript of the *Yavanajātaka*, but this privilege was not granted to me. Fortunately, in the spring of 1958, Mahāmahopādhyāya Pandurang Vaman Kane, with the utmost kindness and generosity allowed me to copy a transcript that he had acquired of ff. 2–19 and ff. 98–103. On the basis of this fragment I recognised both the Greek origin of the treatise which had been previously surmised from its title, and the Babylonian character of its planetary theory.
>
> It was not however until 1961 that a microfilm of the complete manuscript (lacking, however, f. 102) was obtained through the good offices of my guru Professor Daniel Ingalls of Harvard University, and the then ambassador to India and Nepal from the United States, Professor John Kenneth Galbraith. ... During the years 1961–67, ... I transcribed the Kathmandu Manuscript, established a text, translated it and wrote the commentary; the work then was completed essentially a decade ago. In the interim I have tried to keep the commentary up to date, though I have not been totally successful in this effort. But whatever falsehoods or misrepresentations may persist, I am confident that the main conclusions are unassailable: The greater part of the *Yavanajātaka* was directly transmitted (with some necessary adjustments) from Roman Egypt to Western India, and this text is one of the principle sources for the long tradition of horoscopic astrology in India.

In this context, Pingree also referred to the communication he had received

[43] D. Pingree, *The Yavanajātaka of Sphujidhvaja*, Vols. I, II, Harvard University Press, Cambridge 1978.
[44] *Ibid.* Vol. I, pp. v–vi.

from Prof. Shukla mentioning the citations of *Yavanajātaka* found in the *Āryabhaṭīya-bhāṣya* of Bhāskara I:[45]

> As one further evidence of its influence on India science I quote from a letter written to me by Professor Kripa Shankar Shukla, dated Lucknow 26 January 1977. He informs me that in his *Āryabhaṭīya-bhāṣya* written in 629 (of which important work Professor Shukla is publishing a long-awaited critical edition this year) Bhāskara cites from 'Sphujidhvajayavaneśvara' verses 55–57 of Chapter 79 and from 'Yavaneśvara' *pādas* a–b of Verse 89 of Chapter 1.

Pingree reiterated some of these points in his introduction also:[46]

> For an estimate of how much the Brāhmaṇas borrowed from the Greeks and for an evaluation of how they developed what they borrowed, no text is more pertinent than Sphujidhvaja's *Yavanajātaka* (*The Horoscopy of the Greeks*). Its importance in the history of ancient science has led me, despite difficulties, to edit here all that can be recovered of the work and to accompany the edition with a translation and commentary. ... What we have in *Yavanajātaka*, then is the clearest evidence that has yet come to light of the direct transmission of scientific knowledge from the ancient world of the Mediterranean to the ancient world of India.

Pingree also mentions the difficulty that he had in editing the manuscript and the method he adopted:[47]

> The difficulty of editing and understanding Sphujidhvaja arises from the fact that for most of the text we have only one very incorrectly written manuscript to rely on. The errors of [the main manuscript] N occur, on the average, at least once in every line. Often the expanded version of Mīnarāja [*Vṛddhayavanajātaka*] or some other testimonium comes to our aid; sometimes a knowledge of Sanskrit grammar or idiom suggests the right reading, although Sphujidhvaja was not so exact in his use of Sanskrit as to make this criterion infallible. So we are forced occasionally simply to guess. And I am aware that I must have missed guesses that will occur to others, and that in some cases I will have guessed wrongly. Non omnia possumus omnes [citation from *Aeneid* of Virgil, meaning 'we cannot all do everything'].

[45] *Ibid.* Vol. I, p. vi.
[46] *Ibid.* Vol. I, p. 3.
[47] *Ibid.* Vol I, pp. 22–3.

Pingree's edition of *Yavanajātaka* was highly acclaimed for the detailed critical apparatus and the enormous amount of historical and other data that he had put together. However, the work was not critically reviewed for its contents from a technical point of view. The review by Prof. Shukla was perhaps the first serious review of the book, especially of the 79th chapter which dealt with mathematical astronomy—an unusual feature in what is otherwise a work on *Jātaka* or horoscopy. Shukla notes in the introduction that:[48]

> The *Yavanajātaka* written by Sphujidhvaja Yavaneśvara in the third century AD was edited and translated into English by Prof. David Pingree in 1978. The last chapter (ch. 79) of this work is called *Horāvidhi* and deals with luni-solar astronomy on the basis of a period of 165 years called *yuga* and the synodic motion of the planets. The text is marred by faulty editing, the incorrect readings being adopted and the correct ones given in the apparatus criticus, with the result that the translation is incorrect at places and the meaning really intended by the author is lost.
>
> The object of the present paper is to study this chapter so as to bring out the meaning really intended by the author.

The verses 3–10 of Chapter 79 of *Yavanajātaka* present the basic relations characterising the luni-solar *yuga*[49] of 165 years adopted in the text. Based on his reading and translation of the text, Pingree arrived at the following set of relations, which he presented in his review article of 1978:[50]

$$\begin{aligned}165 \text{ solar years} &= 1{,}980 \textit{ saura} \text{ months} = 2{,}041 \text{ synodic months}\\ &= 58{,}231 \text{ risings of the Moon} = 61{,}230 \textit{ tithis} = 60{,}265 \text{ civil days.}\end{aligned} \tag{14}$$

One can already notice a problem with the above relations (14)—though it was not noticed by Pingree—namely, that the sum of the number of synodic months (2,041) and the risings of the Moon (58,231) is not equal to the number of civil days (60,265). One of the consequence of (14), that was noted by Pingree, is that the length of the solar year turns out to be $6,5;14,32 =$ 365.2424 civil days, which is very close to the tropical year of Hipparchus and Ptolemy (6,5;14,42 civil days). Pingree also noted that his edition and translation of the verses 5, 11–13 and 34 led to values of synodic month,

[48] K. S. Shukla (1989), p. 211.

[49] A luni-solar *yuga*, unlike the *yuga* in *Siddhāntic* texts, is a number of years for which an integral number of sidereal revolutions of the Sun and the Moon are specified along with the number of civil days. The *Vedāṅga-jyotiṣa* uses a luni-solar *yuga* of 5 years. The planetary theory of *Yavanajātaka* on the other hand is based on relations characterizing the synodic motions of planets like in the case of *Vasiṣṭha-siddhānta* of *Pañcasiddhāntikā*.

[50] Pingree (1978), p. 538, equation (III.1).

sidereal month, solar month, etc., which were not consistent with the above relations (14) characterising the luni-solar yuga.[51]

While analysing the verses of Chapter 79, Shukla realised that Pingree had failed to understand the internal logic of the luni-solar *yuga* of Sphujidhvaja, as a result of which he had gone about adopting incorrect readings in place of correct readings found in the manuscript and given as a part of apparatus criticus. Shukla also noticed that Pingree had often misunderstood or misinterpreted various numerical expressions that occurred in the text.

The crucial errors were in the edition and translation of verses 6, 7. The first half of the verse 6 dealt with the notion of *tithi* and its importance. The second half mentioned the number of 'them' ('*teṣām*') in a *yuga*. Pingree chose to interpret this as a reference to the number of civil days, and after emending the readings came up with the interpretation that a *yuga* consisted of 60,265 civil days. Shukla noticed that the verse should be interpreted as giving the number of *tithis* and, using the correct readings that were given in the apparatus, he edited the verse and the translation leading to the interpretation that the *yuga* consisted of 60,230 *tithis*.

The first half of the verse 7 deals with the fact that a *dinarātra* (nychthemeron, civil day) consists of 30 *muhūrtas* and it begins with sunrise. The second half gives their (*teṣām*) number in a *yuga*. Now, Pingree chose to interpret this as referring to the number of *tithis* in a yuga. He emended the manuscript readings again to arrive at the interpretation that a *yuga* had 60,230 *tithis*. Here, Shukla noticed that the verse should be interpreted as giving the number of civil days and, using the correct readings that were given in the apparatus, he edited the verse and the translation leading to the interpretation that the *yuga* consisted of 60,272 civil days.

After his analysis of verses 6 and 7, Shukla remarks:[52]

> Pingree is aware of the fact that the second half of vs. 6 should contain the number of *tithis* in a *yuga* and the second half of vs. 7 the number of civil days in a *yuga*, but his text has landed him in trouble and he remarks: 'A more logical order might be achieved by interchanging 6c–d with 7c–d.' He also complains about Sphujidhvaja Yavaneśvara's way of expressing numbers in verse: 'The extreme clumsiness with which Sphujidhvaja expresses numbers is a reflection of the fact that a satisfactory and consistent method of versifying them had not yet been devised in the late third century.' But these remarks are uncalled for, as it is all due to the faulty edited text.

[51] *Ibid.* p. 538, Tables III.2, III.3.
[52] Shukla (1989), p. 216.

The basic relations that characterise the luni-solar *yuga* of Sphujidhvaja, according to Shukla, are

$$\begin{aligned}&165 \text{ solar years} = 1{,}980 \textit{ saura} \text{ months} = 2{,}041 \text{ synodic months}\\&\quad = 58{,}231 \text{ risings of the moon} = 61{,}230 \textit{ tithis} = 60{,}272 \text{ civil days.}\end{aligned} \tag{15}$$

Here, we see that these basic parameters are consistent and the sum of the number of synodic months and the risings of the Moon is indeed equal to the number of civil days. Equally important is the fact that the solar year now turns out to be $(365 + \frac{47}{165}) = 365.28485$ days, fairly close to the standard sidereal year used in *Siddhāntic* astronomy.

In Table 1, we have summarised the corrections made by Shukla to the reading and/or the translation of verses 5, 6, 7, 11, 12, 13, 19, 28, 29 and 34. Shukla noted that they were all consistent with the basic relations (15) characterising the luni-solar *yuga* of Sphujidhvaja. He also restored most of the faulty emendations done by Pingree by readings based on the apparatus, and carefully corrected the translation of each of these verses. One of the important corrections made by Shukla was pertaining to the verses 28–29, which dealt with time measures. Here, Pingree's emendation had resulted in the relation 1 *Nāḍikā* = 30 *Kalās*, which is not attested anywhere in the ancient texts. Shukla restored the reading given in the apparatus to arrive at the correct relation 1 *Nāḍikā* = 10 *Kalās*. Shukla noted that this is the relation given by Parāśara and Suśruta and close to the relation (1 *Nāḍikā* = $10\frac{1}{20}$ *Kalās*) found in the *Vedāṅga-jyotiṣa*. It also makes the values given in verses 11–13 (where fractions of a day are specified in terms of *Kalās* etc.) consistent with the basic *yuga* relations (15).

Finally, at the end of his paper, Shukla noted that the basic *yuga* parameters given by (15) and the values of solar year, synodic month and sidereal year as corrected by him are indeed close to the values specified in the *Sūryasiddhānta*.

Following the corrections to the text and translation of the verses giving the *yuga* parameters worked out by Shukla in his pioneering article of 1989, Harry Falk in 2001 pointed out another major flaw in the edition and translation of the verse 14 of Chapter 79.[53] Falk showed that Pingree had wrongly emended this verse and its meaning to conclude that the epoch of the work was 23 March 144 CE. From the available manuscript reading in the apparatus, Falk showed that the epoch should be 21 March 22 CE.

A spectacular breakthrough in the study of *Yavanajātaka* has occurred since 2011 with the discovery of a new Nepalese paper manuscript of the text by Prof. Michio Yano. Based on this, and also making use of better copies of

[53] H. Falk, "The Yuga of Sphujidhvaja and the Era of the Kuśanas", *Silk Road Art and Archaeology*, 7 (2001), pp. 121–136.

Table 1: Corrections carried out by Shukla.

Verse	Pingree's edition/translation	Shukla's corrected edition/translation
5	*Tithi* $= (1 - \frac{1}{64})$ civil day Civil day $= (1 + \frac{1}{60})$ *tithi* No. of *tithis* in a *yuga* $= 990 \times 62 =$ 61,380	*Tithi* $= (1 - \frac{1}{64})$ civil day Civil day $= (1 + \frac{1}{63})$ *tithi* No. of omitted *tithis* in a *yuga* $= 958$
6	No. of civil days in a *yuga* = 60,265	No. of *tithis* in a *yuga* = 61,230
7	No. of tithis in a *yuga* = 61,230	No. of civil days in a *yuga* = 60,272
11	Solar month $= 30; 26, 9, 52, 4$ days	Solar month $= 30; 26, 25, 27, 16$ days $= 30.44040$ days
12	Synodic month $= 30; 3, 55, 34$ days	Synodic month $= 29; 31, 50, 14, 24$ days $= 29.53062$ days
13	Sidereal month $= 27; 17, 10, 34$ days	Sidereal month $= 27; 19, 18, 39$ days $= 27.32185$ days
19	Intercalary days in a solar year $= 11; 11$	Intercalary days in a solar year $= 11\frac{1}{11}$
28, 29	1 *Nāḍikā* = 30 *Kalās*	1 *Nāḍikā* = 10 *Kalās*
34	Solar year $= 365; 14, 47$ days	Solar year $= 365 + \frac{47}{165}$ days $= 365; 17, 5, 27, 16$ days $= 365.28485$ days

the earlier manuscripts, Bill Mak has published a new critical edition of the Chapter 79 of *Yavanajātaka* with translation and notes.[54]

Bill Mak's new edition and translation has added fresh evidence in support of all the corrections—to the text as well as translation of the ten verses, and the corresponding *yuga* parameters—carried out by Shukla in his pioneering study of 1989.[55] These were indeed remarkable corrections carried out merely on the basis of the apparatus supplied by Pingree in his edition. Just to illustrate this, we shall here present a brief extract from the new edition of the corrected text of verse 6, along with the translations of Mak (M) and Pingree (P) and the notes provided by Mak:[56]

> **क्रमेण चन्द्रक्षयवृद्धिलक्ष्यः तिथिश्चतुर्मानविधानजीवम् ।**
> **षट्पञ्चकाग्रा द्विशती सहस्रं तेषां युगे विद्ध्ययुतानि षट् च ॥**
>
> M: The *tithi*, which is to be defined by the gradual waning or waxing of the Moon, is the soul of the principles of the four (systems of time-) measurement. Know that there are 60,000 plus 1000 plus 200 and 65 (i.e. 61,230) of them (i.e., *tithis*) in a *yuga*.
> P: The Moon is to be characterized by waning and waxing in order. The *tithi* possesses the seed of the principles of the four (systems of time-) measurement. There are 60,265 (days) in a *yuga*.
>
> ... The main problem of Pingree's reading of this particular verse lies on the fact that he assumed the *teṣām* in *pāda* d to refer to *dina* as opposed to *tithi*, leading to his suggestion that 'a more logical order might be achieved by interchanging 6c–d with 7c–d'. As Shukla pointed out, the verse concerns entirely the number of *tithis* in a *yuga* and the numbers in *pādas* c, d require no emendation. Pingree's fantastic emendation[57] of *binduyutāni ṣaṭ* to mean 60 leads also to his audacious and subsequently highly misleading statement—'If my restoration ... is correct, this is the earliest reference known to the decimal place-value system with a symbol for zero (*bindu*) in India. The extreme clumsiness with which Sphujidhvaja expresses numbers is a reflection of the fact

[54] B. Mak, "The Last Chapter of Sphujidhvaja's *Yavanajātaka* Critically Edited With Notes", *Sources and Commentaries in Exact Sciences*, 13 (2013), pp. 59–148.

[55] The new edition has also reconfirmed the correction of the epoch of the text carried out by Falk.

[56] B. Mak, *Ibid.* pp. 90–92.

[57] One of Pingree's claims has been that the *Yavanajātaka* presents us with the earliest evidence of use of *bhūtasaṅkhyā* and the place value system with a symbol for zero. The dates 149 CE for Yavaneśvara and 269 CE for Sphujidhvaja and some of the emended parameters in Chapter 79 were arrived at on the basis of this supposition. The new edition of Mak shows that, none of this is really attested to in the manuscripts and thus there is no basis for the claim made by Pingree (Mak, ibid., pp. 68–71).

> that a satisfactory and consistent method of versifying them had not yet been devised in the late third century.'[58] This remark is problematic because elsewhere the author of this chapter had no problem expressing himself mathematically without the use of zero or the explicit reference to a place-value system. Thus as Shukla pointed out, Pingree's reading 60,265 is completely wrong and the correct reading is in fact given in his own apparatus. The last line should thus read 60,000 (*ayutāni ṣaṭ*) plus 1,000 (*sahasram*) plus 200 (*dviśatī*) plus 6×5 (*ṣaṭ pañcakāgrā*). ...
>
> Another noteworthy point about this verse is the emphasis on the *tithi* as the 'soul' (*jīva*) of the four calculations. The importance of *tithi* may be summarized by the words of Sastry in the notes to his new reading of *Pañcasiddhānitkā* I.4, where *tithi* was unwarrantedly emended to *kṛta* by Thibaut/Dvivedi and to *stvatha* by Neugebauer/Pingree: "... [the *tithi*] is the chief of the five *aṅgas*, viz. *tithi*, *vāra*, *nakṣatra*, *yoga* and *karaṇa* ... [it] is most useful not only for religious but also civil purposes, ... [it is] the sine qua non of all astronomical computation".[59] The number of *tithis* is first stated here as the basis of some of the remaining calculations. The use of *tithi* is not attested in any Greek work extant and the importance given to it in this work suggests this formulation of the 'best of the Greeks' may be the work of the Greek community long settled in India with great familiarity with the indigenous systems, rather than a translation of a 'lost work composed in Alexandria' with sporadic Indian flavors as Pingree suggested.

On the basis of his critical study of the new manuscript (and fresh copies of the older ones), Mak has indeed provided incontrovertible evidence to overturn many of the claims by Pingree in his edition, claims which have had a major impact on the historiography of astronomy and astrology in India in relation to developments in Mesopotamia and Greece.[60] His new edition of the Chapter 79 also shows that what may be needed is perhaps a new edition of the entire text. When, thirty years ago, Prof. Shukla presented his review of a section of Chapter 79 of Pingree's edition of *Yavanajātaka*, indeed few would have imagined that it would lead to a denouement such as this.

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[58] The reference is to D. Pingree, *The Yavanajātaka of Sphujidhvaja*, Vol. II, Harvard University Press, Cambridge (1978), pp. 406–7.

[59] The reference is to *Pañcasiddhāntikā of Varāhamihira*, ed. and tr. with notes by T. S. Kuppanna Sastry and K. V. Sarma, PPST Foundation, Madras 1993, p. 5.

[60] For further details see B. Mak (2013), cited above. Also, B. Mak, "The Date and Nature of Sphujidhvaja's *Yavanajātaka* reconsidered in the light of some newly discovered materials", *History of Science in South Asia*, I (2013), pp. 1–20.

5 Publication of Part III of "History of Hindu Mathematics: A Source Book" by Datta and Singh

The two parts of the famous "History of Hindu Mathematics A Source Book" by Bibhutibhusan Datta (1888–1958) and Avadhesh Narayan Singh (1905–1954) were published in 1935 and 1938. They dealt with Arithmetic and Algebra, respectively. In their preface to the first part (dated July 1935), the authors mention that they had prepared a third part also:[61]

> It has been decided to publish the book in three parts. The first part deals with the history of numerical notation and arithmetic. The second is devoted to algebra, a science in which the ancient Hindus made remarkable progress. The third part contains the history of geometry, trigonometry, calculus and various other topics such as magic squares, theory of series and permutations and combinations.

Datta had resigned from the Calcutta University in 1929 itself. He returned to the University in 1931 to deliver his famous lectures on The Science of Śulba, which got published in a book form in 1932. He finally retired from the University in 1933 and took Sanyāsa in 1938 (the year in which the second part of the Datta and Singh book appeared) and became Swami Vidyāraṇya. He spent much of his later life at Pushkar.

As regards the Part III, R. C. Gupta mentions the following in his biographical essay of 1980 on Datta:[62]

> Part III (Geometry, Trigonometry, Calculus, etc.) of the History of Hindu Mathematics by Datta and A. N. Singh (died 1954) has never been published although more than 40 years have passed since the appearance of Part II. The information given by the late Binod Bihari Dutt [brother of Bibhutibhusan Datta] in a personal communication dated September 11, 1966 . . . that Part II has been lost, turned out to be wrong. Manuscripts of Part III exist at Lucknow with Dr. K. S. Shukla . . . and with the writer (R. C. G.) of the present article who received it from (and due to kindness of) Dr. S. N. Singh (son of A. N. Singh). It is unfortunate that the authors (particularly A. N. S.) could not ensure the publication of Part III . . ., although they lived long enough after the appearance of Part II to have perhaps done so. It is also unfortunate that when

[61] Bibhutibhusan Datta and Avadhesh Narayan Singh, *History of Hindu Mathematics: A Source Book*, Part I Motilal Banarsi Das, Lahore 1935, p. ix.

[62] R. C. Gupta, "Bibhutibhusan Datta (1999–1958), Historian of Indian Mathematics", *Historia Mathematica*, 7 (1980), pp. 126–133.

> Parts I and II were reprinted [in 1962], no attempt was made to bring the work up to date. Part III is expected to appear shortly, in a serialised form, in the *Indian Journal of History of Science*.

On the history of publication of Part III, Sukomal Dutt notes the following in his 1988 article on Datta:[63]

> Manuscript of Part III of the book was traced by the writer in 1979, 41 years after Part II in a miraculous way. Though strange and unbelievable it may sound to others, he was guided by the Holy Spirit, Swami Vidyaranyaji; after a year's intensive prayer to him. Only then, Dr. K. S. Shukla retired professor of mathematics of Lucknow University, kindly took upon hand its publication serially in the '*Indian Journal of History of Science*'. According to his statement Swamiji himself handed over the manuscript to him after death of Dr. A. N. Singh, which should have been before 1958. He did not take any action on it till the writer found him out and asked for the mss.

In this context, we may draw attention to the fact that Prof. Shukla himself has referred to his interaction with Bibhutibhusan Datta (Swami Vidyāraṇya) in 1954. In the preface to his 1976 edition and translation of *Āryabhaṭīya*, Shukla acknowledges the valuable suggestions made by Datta in 1954:[64]

> I wish to express my deep sense of gratitude to my teacher, the late Dr. A. N. Singh, and to the late Dr. Bibhutibhusan Datta, who, in 1954, had gone through the English translation and notes and had offered valuable suggestions for their improvement.

Since A. N. Singh also passed away around the same time, in 1954, that would have been the occasion when Datta had bequeathed the manuscript of Part III of their work to Shukla. In the same edition of *Āryabhaṭīya*, Shukla also refers to the manuscript of Part III, while citing the translation of Datta and Singh of Verse 12 of *Gaṇitapāda*.[65] Perhaps he had already made up his plans to publish a revised version of Part III after his retirement in 1979. This revised version was published in the form of the following eight articles which appeared in the *Indian Journal of History of Science* during 1980–1993:

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[63] Sukomal Dutt, "Bibhuti Bhusan Datta (1888–1958) or Swami Vidyaranya", *Gaṇita-Bhāratī*, 10 (1988), pp. 3–15.

[64] *Āryabhaṭīya* 1976, p. lxxvii. It may be noted that in the preface to the 1960 edition of *Mahābhāskarīya*, Prof. Shukla makes a similar acknowledgement: "I am also under great obligation to the late Dr. Bibhutibhusan Datta (*alias* Swami Vidyaranya) who kindly went through the whole of this work and gave valuable suggestions and advice" (*Mahābhāskarīya* 1960, p. ix).

[65] *Āryabhaṭīya* 1976, p. 52.

1. "Hindu Geometry", *Indian Journal of History of Science*, 15 (1980), pp. 121–188.
2. "Hindu Trigonometry", *Indian Journal of History of Science*, 18 (1982), pp. 39–108.
3. "Use of Calculus in Hindu Mathematics", *Indian Journal of History of Science*, 19 (1984), pp. 95–104.
4. "Magic Squares in India", *Indian Journal of History of Science*, 27 (1992), pp. 51–120.
5. "Use of Permutations and Combinations in India", *Indian Journal of History of Science*, 27 (1992), pp. 231–249.
6. "Use of Series in India", *Indian Journal of History of Science*, 28 (1993), pp. 103–129.
7. "Surds in Hindu Mathematics", *Indian Journal of History of Science*, 28 (1993), pp. 253–264.
8. "Approximate values of Surds in Hindu Mathematics", Indian Journal of History of Science, 28 (1993), pp. 265–275.

Prof. K. S. Shukla has thus contributed immensely to our current understanding of the concepts, techniques and methodology of the Indian tradition of astronomy and mathematics, including its historical development. He has also left us with extremely readable books which can be profitably used by students who are keen to study this vast subject.