Glimpses from the *Āryabhaṭasiddhānta* *

1 Introduction

In a paper entitled "Āryabhaṭa I's astronomy with midnight day-reckoning" published by me nine years ago in the *Gaṇita* (Vol. 18, No. I, 1967), I had adduced concrete and conclusive evidence to show that Āryabhaṭa I, the celebrated author of the *Āryabhaṭīya*, wrote one more work on astronomy which was known as *Āryabhaṭasiddhānta.* Whereas in the *Āryabhaṭīya* the day was reckoned from one sunrise to the next, in the *Āryabhaṭasiddhānta* the day was reckoned from one midnight to the next. This latter work of Āryabhaṭa which adopted midnight day-reckoning was first mentioned by Brahmagupta (628 AD) of Bhinmal in Rajasthan, who was so much impressed by its wide popularity that he epitomised the teachings of this work in his calendrical work bearing the title "Food prepared with sugar candy" (*Khaṇḍakhādyaka*). The notable points of difference of this work of Āryabhaṭa I from his other work (viz. the *Āryabhaṭīya*) were recorded by his scholiast Bhāskara I (629 AD) hailing from Valabhī in Gujarat, in Ch. vii of his *Mahābhāskarīya.* The above work of Āryabhaṭa was also mentioned by Varāhamihira (died 587 AD) of Kāpitthaka near Ujjain, Govindasvāmī (ninth century) of Kerala, Mallikārjuna Sūri (1178 AD) of Veṅgī in Āndhra, Maithila Caṇḍeśvara of Benaras, Rāmakṛṣṇa Ārādhya (1472 AD) of Āndhra, Bhūdhara (1572 AD) of Kampil in Uttar Pradesh, and Tamma Yajvā (1599 AD) of Ahobila in Āndhra. This work of Āryabhaṭa was famous for its description of the astronomical instruments particularly the water clocks, and has been remembered by the commentators of the *Sūryasiddhānta* while commenting on the *Yantrādhyāya* of that work. The commentator Rāmakṛṣṇa has even quoted as many as 34 verses from that work. These verses were discussed by me in the said paper. But this is not all that is known regarding that work. The above mentioned commentators of the *Sūryasiddhānta* have given some more information regarding the contents of that work which is otherwise unknown to us. The object of the present paper is to throw light on this information.

* K. S. Shukla, *Indian Journal of History of Science*, Vol. 12, No. 2 (1977), pp. 181–186.

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2 Time from shadow

Mallikārjuna Sūri (1178 AD) states the following method for finding time from the gnomonic shadow and ascribes it to the *Āryabhaṭasiddhānta*:

> When the Sun is in the signs of Scorpion, (Sagittarius), Capricorn or Aquarius, then, as a rule, and elsewhere too, if it is within two *ghaṭīs* before or after noon, the measure of the gnomonic shadow in terms of digits (*aṅgulas*) is equīvalent to time in terms of *ghaṭīs*. At that time one might get an approximate estimate of time in the manner stated in the *Āryabhaṭasiddhānta.* If you ask how, then proceed like this: If it is $\frac{1}{2}$ of a *ghaṭī*, or 1 *ghaṭī*, or $1\frac{1}{2}$ *ghaṭīs*, or any number of *ghaṭīs* not exceeding two before noon, then (having constructed a circle on level ground and having drawn the east-west and north-south lines through its centre) set up a gnomon of 9 digits on the line directed towards the east from the centre of the circle in such a way that the tip of the shadow may fall on the north-south line passing through the centre of the circle. Then if the distance between the centre of the circle and the foot of the gnomon is $\frac{1}{2}$ of a digit, it would indicate that $\frac{1}{2}$ of a *ghaṭī* is to elapse before noon. If the distance is one digit, it is 1 *ghaṭī* before noon, and if $1\frac{1}{2}$ digits, then it is $1\frac{1}{2}$ *ghaṭīs* before noon.
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> If the desired time is (within 2 *ghaṭīs*) after noon, then one should set up a gnomon of 9 digits on the line going towards the west from the centre. If the distance between the centre and the gnomon is $\frac{1}{2}$ of a digit, it is $\frac{1}{2}$ of a *ghaṭī* past noon; if 1 digit, it is 1 *ghaṭī* past noon; and if $1\frac{1}{2}$ digits, it is $1\frac{1}{2}$ *ghaṭīs* past noon. But this method would work only when the gnomon is set up in such a way that the tip of its shadow falls on the north-south line (passing through the centre).[1]

Tamma Yajvā,[2] too, has ascribed this method to the *Āryabhaṭasiddhānta.*

3 Possibility of an eclipse

Mallikārjuna Sūri informs us that the possibility of an eclipse was discussed in the *Āryabhaṭasiddhānta* in the following way:

> Adding 6 signs to the Sun at a *parva* (full moon or new moon), one gets the Earth's shadow. This is the eclipser of the Moon. When it is equal to the Moon's node, we have a total eclipse of

[1] Mallikārjuna's commentary on *Sūryasiddhānta*, iii. 35.
[2] In his commentary on *Sūryasiddhānta*, iii. 35.

the Moon. A solar eclipse will also be total provided the Moon's latitude corrected for parallax happens to be zero at that time. Even when the Moon's latitude exists, a partial eclipse of the Sun will be possible provided the parallax in latitude is less than half the sum of the diameters of the eclipsed and eclipsing bodies. Thus, at places where the equinoctial midday shadow is 1 digit, parallax in latitude is always less than half the sum of the eclipsed and eclipsing bodies. Where the equinoctial midday shadow is 5 digits, there the parallax in latitude is sometimes less and sometimes equal to half the sum of the diameters of the eclipsed and eclipsing bodies. When the parallax in latitude amounts to half the sum of the eclipsed and eclipsing bodies, then, if the Moon's latitude is zero, a solar eclipse does not occur. Where the equinoctial midday shadow is 9 digits, there the parallax in latitude is sometimes less than, sometimes equal to, and sometimes greater than half the sum of the diameters of the eclipsed and eclipsing bodies. When the former is equal to or greater than the latter, a solar eclipse is not possible provided the Moon's latitude (at new Moon) is zero. But all this happens only when the longitude of the eclipsed body is equal to that of the Moon's node.

When the distance of the Shadow or the Sun from the Moon's node is 12° and also if the Moon's velocity per day amounts to 12°20′, a lunar eclipse certainly does not occur anywhere. When this distance is less than 12° and the Moon's velocity greater than 12°20′, then there is a possibility of a lunar eclipse. But when the distance is 13° and the Moon's velocity 13°20′, even then a lunar eclipse is impossible. When the distance is less than that (and the Moon's velocity greater than 13°20′), there is a possibility of a lunar eclipse. Again, when the distance of the Shadow or the Sun from the Moon's ascending node exceeds 14° and also if the Moon's velocity is 14°20′, a lunar eclipse is impossible. (In fact) when the distance is 14°, a lunar eclipse is always impossible. In that case, the Moon's velocity does not play any role. Hence, one should proceed to calculate a lunar eclipse only when the said distance is less than 14°.

In the case of a solar eclipse too, at places where the equinoctial midday shadow is 1 digit and the distance of the Earth's shadow or the Sun from the Moon's ascending node amounts to 14°, a solar eclipse is impossible. At those very places, if the said distance is less than 14°, there is a possibility of a solar eclipse. Where the equinoctial midday shadow is 5 digits and the said distance

> is 16°, a solar eclipse is impossible. But if the said distance is less than 16°, there is a possibility of a solar eclipse. In a place where the equinoctial midday shadow is 9 digits and the Sun is at the last point of the sign Gemini, the length of the day amounts to 36 *ghaṭīs*; and when at the end of the sign Sagittarius, the day amounts to 24 *ghaṭīs*. Where the equinoctial midday shadow exceeds 9 digits, there is no habitation. Hence, knowledge of occurrence of eclipses for those places is of no use. People do not live beyond 600 *yojanas* from the equator. The region lying north of that is inaccessible to man.
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> All this has been explained in detail in the *Āryabhaṭasiddhānta*.[3]

Tamma Yajvā and Rāmakṛṣṇa Ārādhya too have included the above discussion of the possibility of an eclipse in their commentaries on the *Sūryasiddhānta*.[4]

4 Lord of the *parva* (New Moon or Full Moon)

Mallikārjuna Sūri says:

> Although the method of finding the lord of the *parva* is not discussed here, but this topic having been discussed in the *Āryabhaṭasiddhānta* and also because it is necessary at this place it is being stated here. The sum of the revolutions performed since the beginning of creation by the Sun and the Moon's ascending node at the desired *parva*, multiplied by 2, gives the number of *parvas* elapsed since the time of creation. (These are presided over by the seven lords Brahmā, Indra, Śakra, Kubera, Varuṇa, Agni, and Yama in the serial order). Dividing the number of *parvas* elapsed by 7, we get the number of complete cycles of the *parvas*. The number that is obtained as the remainder, counted with Brahmā, gives the lord of the current *parva*.[5]

Tamma Yajvā has also quoted this rule.[6]

It is noteworthy that Mallikārjuna quotes the above rule for the reason that it was stated in the *Āryabhaṭasiddhānta*. This probably suggests that the *Āryabhaṭasiddhānta* was an important work of the *Sūryasiddhānta* school.

[3] Mallikārjuna's commentary on *Sūryasiddhānta*, iv. 6.
[4] iv. 6.
[5] Mallikārjuna's commentary on *Sūryasiddhānta*, iv. 7–8.
[6] See Tamma Yajvā's commentary on *Sūryasiddhānta*, i. 67.

5 Observation of the planets

The *Sūryasiddhānta*, the *Brāhmasphuṭasiddhānta*, the *Śisyadhīvṛddhida* and other works on Hindu astronomy describe the method of observing the planets. According to Mallikārjuna Sūri, the same method occurred in the *Āryabhaṭa-siddhānta* also. It might be explained briefly as follows:[7]

> First of all calculation was made of the length of the gnomonic shadow cast by the planet. The length of this gnomon was taken to be equal to the height of the observer's eye. Then two bamboos were set up vertically near the gnomon in the direction contrary to the shadow and a pipe (*yaṣṭi*) of 5 cubits in length was tied to these bamboos in the direction of the hypotenuse of shadow, in such a way that one end of the pipe was just at the top of the gnomon. This done, the planet was seen by the observer through the pipe by placing his eye at the top of the gnomon.
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> Sometimes the planet was seen indirectly in water, oil, or mirror placed at the tip of the shadow. For this purpose another gnomon of equal length was set up in the direction of the shadow at a distance equal to double the shadow. The same method was used for the observation of the conjunction of two planets. When the two planets were in close conjunction, only one pipe was used. But when they were separated by a distance, two pipes were used, one directed towards one planet and the other towards the other. One end of each pipe was at the top of the gnomon, so that the observer could see both the planets with his eye at the top of the gnomon.
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> This method was used also to see the Moon at its first visibility, the elevation of the lunar horns and the eclipses of the Sun and the Moon.

6 Distances of Mercury and Venus at rising or setting

Mallikārjuna Sūri writes:

> According to the *Āryabhaṭasiddhānta*, the time-degrees of the heliocentric distance of Venus at the time of its rising or setting are 9 when Venus is in swift motion, and 7 when in retrograde motion. The corresponding time-degrees of Mercury are 13 for swift motion and 12 for retrograde motion.[8]

[7]See Mallikārjuna's commentary on *Sūryasiddhānta*, vii. 12.
[8]Mallikārjuna's commentary on *Sūryasiddhānta*, ix. 9–11.

It might be mentioned here that the *Khaṇḍakhādyaka* gives 9 time-degrees for Venus and 13 time-degrees for Mercury which correspond to their swift motion. Those for retrograde motion are not given. From the above statement of Mallikārjuna Sūri we find that the *Āryabhaṭasiddhānta* contained time-degrees for swift as well as retrograde motions of Mercury and Venus.

7 Rising and setting of Canopus (*Agastya*)

Mallikārjuna Sūri writes:

> Canopus sets when the Sun's longitude amounts to 2 signs minus the local latitude. It rises when the Sun's longitude is 6 signs minus that. Thus we have stated here the view of the *Āryabhaṭasiddhānta* as an alternative method.[9]

That this method really belonged to the *Āryabhaṭasiddhānta* is confirmed by its occurrence in the *Pūrva-Khaṇḍakhādyaka*. See Bina Chatterjee's edition, Vol. 2, p. 147, lines 9–10.

8 The shadow instruments

Tamma Yajvā describes the shadow instruments of the *Āryabhaṭasiddhānta* as follows:

> We now describe the (shadow) instruments described there (i.e. in the *Āryabhaṭasiddhānta*). The gnomon (*śaṅku*) is of three kinds. The first is cylindrical in shape, 12 digits high, and 2 digits in diameter. The second is conical in shape, 2 digits in diameter at the base, 12 digits in height, pointed at the top, and having two horizontal holes at the bottom and top (separated by a distance of 12 digits) with two nails fixed to them. This gnomon having been set up vertically, a thread of 12 digits in length should be suspended between the two nails. The third gnomon is cylindrical in shape with small diameter, and its height is 12 digits. This (last) gnomon is fit for use by all people. The *yaṣṭi-yantra* (i.e. the pipe) is a smooth cylindrical pipe which is as many digits in length as there are degrees in a radian (i.e. 57). The *dhanur-yantra* (i.e. the semi-circle) is a semi-circle whose radius has as many digits as there are degrees in a radian, and whose circumference is graduated with the marks of degrees, and which is furnished with the chord and the arrow. The *cakra-yantra* (i.e. the circle)

[9]Mallikārjuna's commentary on *Sūryasiddhānta*, ix. 17.

> is a circle (or hoop) whose radius is as many digits in length as there are degrees in a radian, and whose circumference bears 360 marks of degrees as well as two holes one at each equinoctial point. The *chatra-yantra* (i.e. the umbrella) is constructed like the *cakra-yantra* with a vertical rod at its centre. The rod should be made as big as there are digits in a radian.[10]

9 The water-clocks

Describing the water-clocks of the *Āryabhaṭasiddhānta*, Tamma Yajvā says:

> Now I give the method of knowing time as taught by Āryabhaṭa and others. First of all one should construct a high cylindrical pillar with a (uniform cylindrical) cavity inside and a hole at the bottom, and fill it with water. Then, keeping an eye on the clock, allow the water to flow out through the hole at the bottom. Now divide the digits of the height of the pillar by the *ghaṭīs* in which the water has flown out (of the pillar): this would give the digits corresponding to one *ghaṭī*. The pillar should then be surmounted by the effigy of a peacock, man or monkey, or of a tortoise or some bird constructed out of bamboo pieces and leather. (A needle should then be inserted horizontally through the belly of the effigy; one end of a cord should be tied to it in the middle, some portion of the cord should be wrapped on it in coils, and the other end of the cord should be tied to a sling carrying a dry hollow gourd filled with some mercury; and the sling containing gourd should then be put on water in the cavity of the pillar underneath, so that, as the water flows out of the cavity through the hole at the bottom, the gourd goes down and the needle rotates.) A cord should also be hung at the end of the needle which is made to pass through the belly of the bird, for the knowledge of time. The number of coils that the thread makes on the needle indicates the number of *ghaṭīs* elapsed. Or, tie a cord to the sling carrying a dry gourd full of mercury and throw the sling into the cylindrical cavity full of water underneath the effigy of the man or peacock etc., and let the other end of the cord go out of the mouth of the man etc. and hang there. This outer cord should be made to pass through 60 beads, separated by equal intervals of one *ghaṭī*. This done, as the water flows out of the cavity, the beads enter the mouth of the man etc., one by one, at the end of every *ghaṭī*. In this way one

[10] Tamma Yajvā's commentary on *Sūryasiddhānta*, xiii, 20–21.

might have a proper knowledge of time.[11]

The above description of the shadow and water instruments closely agrees with that given in the verses cited by Rāmakṛṣṇa Ārādhya in his commentary on the *Sūryasiddhānta*. For details one might refer to *Gaṇita*, Vol. 18, No. 1, pp. 83–105.

[11] Tamma Yajvā's commentary on *Sūryasiddhānta*, xiii. 22–25.