

## On three stanzas from the $Pa \tilde{n} casiddh \bar{a} ntik \bar{a}^*$

1. We will here consider three stanzas from the  $Pa\tilde{n}casiddh\bar{a}ntik\bar{a}$  of Varāhamihira (c. 550 AD), edited by G. Thibaut and S. Dvivedī (1889 AD). These stanzas were examined by us while comparing the astronomical constants of the midnight day-reckoning of Āryabhaṭa I (499 AD), as given by his follower Bhāskara I (629 AD), with those of the old  $S\bar{u}ryasiddh\bar{a}nta$ , as summarised by Varāhamihira. This comparison revealed to us that the astronomical constants ascribed to Āryabhaṭa I's midnight day-reckoning were in general agreement with those found in Varāhamihira's version of the  $S\bar{u}ryasiddh\bar{a}nta$ . The differences were, however, found to exist as regards the distances from the Sun at which the planets become visible and as regards the distances and diameters of the Sun and the Moon. It was soon discovered that the differences were not real but were due to the emendations made in the traditional text of the Pañcasiddhāntikā by the editors.

2. Of the above-mentioned three stanzas, one is stanza 12 of the seventeenth chapter. It states the distances of the planets from the Sun at which they rise heliacally, and runs as follows:

Traditional text

स्फुटदिनकरांतरांशा-श्वन्द्रादीनां च दर्शनीज्ञेयाः । विशतिरूनावसुशशि शिखिमुनिनवरुद्रेदियैः क्रमशः ॥

Text as emended by Thibaut and Dvived $\bar{\mathrm{i}}$ 

स्फुटदिनकरान्तरांशा-श्वन्द्रादीनां च दर्शने ज्ञेयाः । विंशतिरूना वसुशशि-शिखिमुनिनवकेन्द्रियैः क्रमशः ॥

The emended version, translated by Thibaut, is as follows:

The degrees of the distances from the sun at which the true planets become visible are 12 for the moon, 19 for Mars, 17 for Mercury, 13 for Jupiter, 11 for Venus, 15 for Saturn.

<sup>\*</sup> K. S. Shukla, *Ganita*, Vol. 5, No. 2 (1954), pp. 129–136.

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	Distance according to			
Planet	Modern	Āryabhaṭa I	Brahmagupta	The above
	$S \bar{u} ry a siddh ar{a} nt a$	and		emended
		Bhāskara I		text
Moon	$12^{\circ}$	$12^{\circ}$	$12^{\circ}$	$12^{\circ}$
Mars	$17^{\circ}$	$17^{\circ}$	$17^{\circ}$	$19^{\circ}$
Mercury	$12^{\circ}$ to $14^{\circ}$	$13^{\circ}$	$13^{\circ}$	$17^{\circ}$
Jupiter	$11^{\circ}$	11°	11°	$13^{\circ}$
Venus				
(when direct)	$10^{\circ}$	9°	$10^{\circ}$	11°
Saturn	$15^{\circ}$	$15^{\circ}$	$15^{\circ}$	$15^{\circ}$

Table 1: Distances from the Sun at which the planets become visible.

The constants given in this stanza and those given in the modern  $S\bar{u}rya-siddh\bar{u}nta$  and by Āryabhaṭa I and Brahmagupta (628 AD) are exhibited in Table 1.

The table shows that the constants given in the emended text differ from those given by the other Hindu authorities in the case of Mars, Mercury, Jupiter, and Venus and that the differences are such as to throw doubt in the correctness of the emended text. It appears from the comparison of the last three columns that the error, if any, in the emended text lies between the words giving the constants for the Moon and Mars and between the words giving the constants for Venus and Saturn. Note that the constants for Mars, Mercury, and Jupiter in the second and third columns have shifted bodily by one space downwards in the last column.

Let us now examine the traditional text to see whether it gives any clue to the above discrepancy. We observe that

- (i) it is inconsistent with the subject matter, as the number of constants mentioned there is seven, whereas the number of planets to which those constants correspond is only six; and
- (ii) it is metrically defective, as there are 14 syllables in place of 12 in the third quarter.

Turning to the emended text, we find that Thibaut and Dvivedī have got rid of the above defects of the traditional text by replacing the word rudra

(meaning 11) by the suffix ka. And this drastic change, made in the traditional text, is indeed the cause of the whole trouble.

The most plauseworthy emendation of the text at this place would be the deletion of the superfluous word  $\dot{s}a\dot{s}i$  (meaning Moon or 1). With this emendation the stanza would run

स्फुटदिनकरान्तरांशा-श्चन्द्रादीनां च दर्शने ज्ञेयाः । विंशतिरूना वसुशिखि-मुनिनवरुद्रेन्द्रियैः क्रमशः ॥

and mean

The degrees of the true distances from the Sun at which the Moon and others become visible are 12, 17, 13, 11, 9, and 15 respectively.

One may easily see that these constants are exactly the same as prescribed by  $\bar{A}$ ryabhaṭa I, and also not much different from those occurring in the modern  $S\bar{u}ryasiddh\bar{a}nta$ .

3. The other two stanzas are stanzas 15 and 16 of the ninth chapter. They deal with the distances and diameters of the Sun and the Moon and run as follows:

Traditional Text

मुनिकृतगुणेन्द्रियघ्नः स्फुटकर्णः खकृतभाजितोऽर्कस्य । कक्षेति चन्द्रकरणों दृघ्नः कक्षा शशांकस्य ॥ खखवसुखमुनींद्रविषया भानोः खकृतर्तुसुगुणाः शशिनः । तात्कालिकमानार्थं स्फुटकक्षाभ्यां पृथग्विभजेत् ॥

Text as emended by Thibaut and Dvivedī

मुनिकृतगुणेन्द्रियघ्नः स्फुटकर्णः खार्कभाजितोऽर्कस्य । कक्षेति चन्द्रकर्णो-ऽग्निघ्नः कक्षा शशाङ्कस्य ॥ खवसुखमुनीन्द्रविषया भानोः खकृतर्तुसुरगुणाः शशिनः । तात्कालिकमानार्थं स्फुटकक्षाभ्यां पृथग्विभजेत् ॥

The emended text, translated by Thibaut, runs:

The true hypotenuse multiplied by 5347 and divided by 120 gives the  $kaksh\bar{a}$  of the sun; the true hypotenuse of the moon multiplied by 3 gives the  $kaksh\bar{a}$  of the moon. Take 5147080 for the sun and 333640 for the moon and, in order to find their (apparent) dimensions for a given time, divide those two quantities separately by the true distances in *yojanas*.

In the notes that follow this translation, Thibaut interprets these stanzas as containing the following formulae:

Sun's true distance in yojanas

 $=\frac{5347\times(\text{Sun's mean distance in mins.})}{120};$ 

Moon's true distance in yojanas

 $= 3 \times (Moon's mean distance in mins.);$ 

Sun's true diameter in minutes

$$=\frac{5147080}{\text{Sun's true distance in yojanas}};$$

Moon's true diameter in minutes

 $= \frac{333640}{\text{Moon's true distance in } yojanas}$ 

Both Thibaut and Dvivedī derive the first two formulae by assuming 5347 and 360 to be the mean distances in *yojanas* of the Sun and the Moon respectively. But this assumption does not agree with the numbers used in the last two formulae, as they yield 962.6 and 926.8 minutes for the mean diameters of the Sun and the Moon respectively, which is wrong. These numbers are about 30 times greater than the real diameters. Thibaut and Dvivedī, therefore, prescribe the division by 30 of the diameters obtained by the application of the third and fourth formulae. Dvivedī thinks that this division by 30 has been omitted in the text probably because, in the time of Varāhamihira, this operation was obligatory by convention. Thibaut is, however, doubtful of the correctness of the text, and writes:

But for some reason or other their text—provided it be correct—does not mention the division by 30.

Thibaut and Dvivedī's assumption that the numbers 5347 and 360 denote the distances (in *yojanas*) of the Sun and Moon respectively is incompatible with their assumption in the next chapter<sup>1</sup> of the number 146 for the diameter (in *yojanas*) of the Sun. The last mentioned number should have been

<sup>&</sup>lt;sup>1</sup>See Thibaut's and Dvivedī's notes on  $Pa\tilde{n}casiddhantika$ , x. 1.

**Table 2:** Mean distances and diameters (in yojanas) of the Sun and Moonaccording to the modern  $S\bar{u}ryasiddh\bar{a}nta$  and the midnight day-reckoning of Āryabhaṭa I

	Modern Sūryasiddhānta		Midnight day-reckoning of Āryabhața I	
-	Distance	Diameter	Distance	Diameter
Sun	689378	6500	689358	6480
Moon	51566	480	51566	480

(a) Actual

	(	(b)	$\mathbf{As}$	abraded	by	42.97
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_	Modern $S\bar{u}ryasiddh\bar{a}nta$		Midnight day-reckoning of Āryabhața I	
	Distance	Diameter	Distance	Diameter
Sun	16043	151	16040	150.8
Moon	1200	11.4	1200	11.4

more appropriately taken to be 151 *yojanas*. Table 2 shows that the correct distances of the Sun and the Moon conforming to the diameter 151 *yojanas* of the Sun are 16040 and 1200 *yojanas* and not 5347 and 360 *yojanas* as assumed by Thibaut and Dvivedī.

The inconsistencies in the interpretation of Thibaut and Dvivedī are due, as in the previous case, to the changes made by them in the traditional text. For example, *khakṛta* (meaning 40) has been changed into  $kh\bar{a}rka$  (meaning 120), and dr has been changed into agni.

Such drastic changes are not necessary; emendation of the obvious clerical errors is enough to secure mathematically correct meaning. With these minor corrections, the text reads:

मुनिकृतगुणेन्द्रियघ्नः स्फुटकर्णः खकृतभाजितोऽर्कस्य । कक्षेति चन्द्रकर्णो दिग्घ्नः कक्षा शशाङ्कस्य ॥ खवसुखमुनीन्दुविषया भानोः खकृतर्तु(व)सुगुणाः शशिनः । तात्कालिकमानार्थं स्फुटकक्षाभ्यां पृथग्विभजेत् ॥

This gives the following four formulae:

Sun's true distance in yojanas

 $=\frac{5347\times(\text{Sun's true distance in minutes})}{40};$ 

Moon's true distance in yojanas

 $= 10 \times (Moon's true distance in minutes);$ 

Sun's true diameter in minutes

 $=\frac{517080}{\text{Sun's true distance in yojanas}};$ 

Moon's true diameter in minutes

38640

 $= \frac{30040}{\text{Moon's true distance in yojanas}}.$ 

These may be derived as follows:

Assuming 16040 and 1200 yojanas to be the distances of the Sun and Moon respectively,<sup>2</sup> we have

Sun's true distance in  $yojanas = \frac{16040 \times (Sun's true distance in minutes)}{120}$ ,

120 being the value of the radius (in minutes) used in the  $Pa\tilde{n}casiddh\bar{a}ntik\bar{a}$ . Thus,

Sun's true distance in  $yojanas = \frac{5347 \times (Sun's true distance in minutes)}{40}$ .

Similarly,

Moon's true distance in  $yojanas = \frac{1200 \times (Moon's true distance in mins.)}{120}$ = 10 × (Moon's true distance in mins.).

Now assuming that the Sun's mean diameter is  $32\frac{95}{401}$  minutes and the Moon's mean diameter 32.2 minutes, we have

Sun's true diameter in minutes

 $= \frac{(\text{Sun's mean diameter in minutes}) \times (\text{Sun's mean distance in yojanas})}{\text{Sun's true distance in yojanas}}$  $= \frac{517080}{\text{Sun's true distance in yojanas}}.$ 

 $^2 \mathrm{See}$  Table 2b.

Similarly,

Moon's true diameter in minutes =	38640
moon's true diameter in innutes –	Moon's true distance in <i>yojanas</i>

(ed. The following note is given as a footnote to the above equation in the original:) The word 'yojana' has been used above in the general sense of a 'linear unit'. It should not be confused with the terrestrial yojana of the  $Pa\tilde{n}casiddh\bar{a}ntik\bar{a}$  which is equal to 7.8 miles approximately.