10 Computer-Aided Diagnosis of Epilepsy Using Bispectrum of EEG Signals

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Abstract

This chapter aims to analyze the dynamics of brain activity from the electroencephalogram (EEG) signals and classify the seizures that are responsible for epilepsy. Hence, seizure classification is the primary task of this article. In this study, a nonlinear higher-order spectral method is proposed that significantly explore the underlying dynamics of nonstationary EEG signals. Various statistical parameters are measured from the principal region of the higher-order spectra that are subjected to the data reduction technique of the locality sensitive discriminant analysis (LSDA). The LSDA maps the measured features at higher dimensional space and ranks them according to the probability of discrimination. The ranked features are then used as inputs to the support vector machine (SVM) classifier with radial basis function kernel. The proposed algorithm is simulated on the web-available Bonn university database that achieved excellent seizures classification accuracy.

Keywords

EEG signals · Higher-order spectra · Locality sensitive discriminant analysis · Support vector machine · Radial basis kernel

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10.1 Introduction

The World Health Organization (WHO) report states that more than 2% of the world population are suffering from epilepsy each year worldwide; out of them, nearly 90% of people belong to the developing countries [90]. The reasons may be lack of awareness, cure, diet, neutrinos, vitamins supplements, etc. It can be observed that the proper diet plan and meditation cure most types of seizures.

The human brain commands the human nervous system that controls most of the human activities. It is a cluster of billions of nerve cells or neurons that are connected via a very complex network. Usually, the information exchange among the neurons takes place through electrical and chemical impulse signals that have a swift response time. It collects information from the sensory organs and transfers to the muscles.When the neurons clusters are hyper-synchronously fired at a time, this sudden surge of electrical activity causes involuntary and temporary disturbance of regular neural activity which can lead to seizures, or in other sense, anything that disturbs the regular pattern of neuron activity can lead to seizures. The recurrent unprovoked seizures are known as epilepsy [10].

The abnormality in brain activity is not always a sign of a seizure. It may happen that those persons are diagnosed with some specific syndrome. Otherwise, in some conditions, a seizure-affected person may continue to show standard electrical activity patterns. Hence, the seizures can be classified depending on the starting area of the disturbance inside the brain and how far it spreads. A single seizure may occur due to high fever or head injury, and this does not indicate that a person has epilepsy. Only when a person suffers from two or more seizures, they may be considered as symptoms of epilepsy. Although the seizures are the predominant symptom of epilepsy, but having a seizure does not necessarily mean that a person is affected by epilepsy [57, 90].

Epilepsy can happen from a variety of causes such as brain tumors, traumatic brain injury, strokes, heart attacks, genetically, abnormal blood vessel formation, etc., which do not lead to epileptic seizures. In most cases, the seizures are controlled with diet, exercise, and medication as many types of seizures do not cause brain destruction. When the seizures are occurring very frequently, that may cause brain damage that leads to epilepsy. Those seizures cannot be controlled with medication [10, 57]. The area of the brain where seizures occur first is known as the focus area or epileptic zone area. By performing surgery, the doctors remove the defined focus area of the brain where seizures are originated. Before performing surgery, the epileptic person is monitored intensively to pinpoint the exact location in the brain as the focus area [33].

There are numerous neuroimaging techniques that are used to explore the dynamics of brain activities at the pre-surgical stage. The computed tomography (CT), single-photon emission computed tomography (SPECT), positron emission tomography (PET), functional magnetic resonance imaging (fMRI), diffusion optical tomography (DOT), magnetic resonance imaging (MRI), magnetoencephalogram (MEG), and more are employed to reveal the structural abnormalities of the brain. These techniques map the damaged areas or abnormalities that may be the origin of seizures. Despite of rapid advancement of the neuroimaging techniques, the analysis of electroencephalogram (EEG) signals is the most frequently used technique for diagnosis of neurological diseases [33, 90].

The EEG signals are recorded in two ways, namely, extracranial and intracranial. The extracranial EEG signals are recorded by noninvasive electrodes placed on the scalp with the international 10/20 system, whereas the small and deep penetrating electrodes are used to record the intracranial signals. The intracranial EEG signals having highly nonstationary and nonlinear behavior including impulses and sharp spikes as compared to the extracranial EEG signals. The prediction, detection, and occurrences of epilepsy by visualization of lengthy EEG signals are a soporific and time-consuming process for Neuro-experts. Hence, the prime challenge is to develop a signal processing algorithm which can extract substantial information about the nonlinear dynamics of the brain's electrical activities. In this chapter, a computer-aided nonlinear higher-order statistics (HOS)-based method has been proposed that leads to an excellent seizure classification accuracy on the Bonn university EEG database [7].

The layout of the present chapter is as follows: Sect. 10.2 contains the literature study about the existing seizures classification algorithms. The studied database is briefly illustrated in Sect 10.3. Section 10.4 contains the proposed methodology which includes brief descriptions about higher-order spectra, feature extraction, dimension reduction algorithm, and the support vector machine (SVM) classifier. The discussion on the achieved results is given in Sect. 10.5, whereas the conclusion is included in Sect. 10.6.

10.2 Related Work

The literature reveals that many research papers have been published on automated classification of epileptic seizure based on the EEG signals.

Earlier the linear prediction (LP) technique is used for seizure classification. The LP algorithm computes the future values which depend on the present and past values behaviors. Pradhan et al. [63] used the LP algorithm for data compression, storage, and transmission of EEG signals. It is relevant to extract the significant features that reveal the underlying nonlinearity and complexity of the EEG signals. Altunay et al. [6] computed the LP error for the epileptic EEG signal detection. In the series of LP-based EEG signal classification algorithm, Joshi et al. [34] proposed the fractional LP method. The energy of the original signal and LP fractal model error are used as measurements for the classification of ictal and seizure-free EEG signals. However, the proposed features on the LP algorithm are not suitable to capture the required information about the nonstationary EEG signals, which yields a low classification accuracy. Some model-based algorithms are also proposed for the EEG signals classification [36, 37, 83]. The autoregressive (AR) model deals with the prediction of the future values based on the present and previous values, which also helps to compute the transfer function. The AR coefficients of the EEG signals can be used as inputs to different classifiers. Ubeyli et al. [83] used the least square-SVM (LS-SVM) classifier to classify the EEG signal based on AR coefficients.

Usually, the local binary pattern (LBP) is applied for image classification or in pattern recognition. It extracts some suitable patterns that explore the pattern characteristics [40]. Kaya et al. [35] extracted the uniform and non-uniform 1D-LBP features from the studied signals and employed those attributes as inputs to different classifiers to evaluate the performance efficiency. The authors used five different classifiers, namely, the BayesNet, artificial neural network (ANN), functional tree (FT), SVM, and logistic regression (LR). They got maximum classification accuracy with the BayesNet classifier. Tiwari et al. [80] employed the 1D-LBP for automated epilepsy detection. They detect the key-points features of the EEG signals by applying the difference of Gaussian (DoG) filter at multiple scales. The 1D-LBP algorithm is applied at these detected key-points, and the histogram of the LBPs are employed as features. The SVM classifier with radial basis (RBF) kernels is used to distinguish the seizure classes. The proposed algorithm achieved excellent classification accuracy in both binary and three seizures classes classification cases.

Many researchers proposed the spectral analysis techniques to explore the nonlinear dynamics of EEG signals. Polat and Gunes [60] used the non-parametric spectral analysis technique of EEG signals for the purpose of epilepsy detection. The spectral estimation had been performed with various methods, and the spectral coefficients are discriminated with the decision tree classifier. They extended their work with same parameters and classified them with the artificial immune recognition systems [61]. Faust et al. [19] proposed the frequency domain parametric method for automatic identification of epileptic seizure based on EEG signals.

The frequency domain techniques alone are not often suitable for analysis of non-stationary signals. As it does not reveal the time and frequency information simulteneously. To overcome this problem, many researchers proposed the time-frequency (TF) analysis methods that allow an analysis of both parameters (time and frequency) in a single frame. The TF techniques are used by numerous authors [32, 65, 69, 75, 81, 82] for analysis of the EEG signals vis-a-vis the brain activities. The TF methods decompose the signal at time and frequency axes simultaneously. Samiee et al. [65] proposed the rational discrete short-time Fourier transform (DSTFT) for epileptic seizure classification. They measured five absolute values of the statistical parameters (standard deviation, mean, median, minimum value, maximum value) from the decomposed DSTFT coefficients. They achieved 98.1% classification accuracy with the multilayer perceptron (MLP) classifier to classify sezures and non-sezures EEG signals. In the DSTFT, a fixed-length window does not provide an accurate analysis of time in frequency frame and vice versa. Sharma and Pachori [69] proposed the improved eigenvalue decomposition of the Hankel matrix and Hilbert transform (IEVDHM-HT) technique for the TF representation. The studied signal is decomposed into sub-signals by using the IEVDHM with a specific criterion such as eigenvalue selection, number of iterations, etc. The Hilbert transform (HT) algorithm is applied on each sub-band to extract the instantaneous amplitude and frequency parameters to represent the studied signal on the TF plane. The Rényi entropy is measured from the TF matrix, and it is used as features for seizure classification. They achieved 99.41% classification accuracy for seizure-free and seizure classes when the length of the studied signals is 4080 points. Fu et al. [21] used the Hilbert marginal spectrum (HMS) analysis technique for seizure classification. The HMS is derived from the empirical mode decomposition (EMD) method which decompose the non-stationary multicomponent signals into a group of sub-band signals known as the intrinsic mode functions (IMFs). The spectral entropies and energy are measured from these sub-bands and used as inputs to the SVM classifier for EEG signal classification.

Iscan et al. [32] combine time domain and frequency domain attributes for seizures classification. The cross-correlation (CC) and the power spectral density (PSD) are computed from the studied signals. The four statistical features, namely, the peak value, equivalent width, centroid and mean square abscissa, are measured from the CC of the analysis signals, while the total power is computed for each EEG sub-band signal (δ, θ α, β, γ). Hence, the combined features are used for the classification of normal and epileptic EEG signals. Various classifiers, namely, the Naive Bayes (NB), k-nearest neighbor (KNN), SVM, least square support vector machine (LS-SVM), Parzen, Fisher discriminant analysis (FDA), quadratic classifiers, binary decision tree (BDT), and nearest mean, are used to classify the labeled combined attributes. They achieved 94.94%, 97.97%, 88.88%, 100%, 91.91%, 89.89%, 100%, 98.98%, and 91.91% classification accuracy, respectively. Tzallas et al. [81, 82] used the time-frequency (TF) analysis techniques for automatic EEG signal classification for different problems such as seizure classification with the ANN [81] and epileptic seizure detection [82].

With regard to the TF resolution property, the wavelet transform (WT) is a powerful tool for investigation of non-stationary signals. It provides a time-varying decomposition of EEG signals which makes possible to capture the transient features of the studied signals. Hence, various wavelet-based techniques and their variants are proposed for exploration of nonlinear dynamics of the EEG signals. Subasi used the discrete wavelet transform (DWT) for the EEG signals classification. The mixture of the expert model is used as a classifier to classify the seizures-labeled wavelet coefficients [77]. They extended their work and applied various data dimensionality reduction techniques such as the principal component analysis (PCA), independent component analysis (ICA), and linear discriminative analysis (LDA) for reducing the dimensionality of the wavelet coefficients. The resultant wavelet coefficient matrix is fed to the SVM classifier to discriminate seizures classes [78]. Acharya et al. [1] computed different nonlinear features from the DWT coefficients that are evaluated by the SVM classifier. The multifractal analysis technique and WT had been proposed for classification of EEG signals [84], while the phase-space reconstruction (PSR) with Euclidean distance (on wavelet coefficients) methods is used to distinguish healthy and epileptic EEG signals [43]. Kumar et al. [39], Guo et al. [26], and Ocak et al. [50] computed the approximation entropy (ApEn) from the wavelet coefficients. The ANN and the optimal threshold value classifiers are used to classify the computed features. The results obtained in [50] reveal that the detail coefficients of the first level provided the best classification accuracy of 96.65% using the ApEn as a feature.

Guo et al. [24, 25] proposed various measurements for the seizures-labeled EEG signal classification. The relative wavelet energy [24] and line length [25] attributes are measured from the decomposed signals that are classified by the ANN classifier.

The wavelet packet entropy in [86] and the mixed band wavelet chaos neural network in [22] are used for epileptic seizure detection. Orhan et al. [51] proposed the DWT for EEG signal decomposition into sub-bands. The K-mean clustering techniques are employed to separate the features according to the labeled classes. These labeled clustered data are classified using the MLP neural network (MPNN) model. Zhou et al. [95] decomposed the EEG signals by using the DWT method up to three, four, and five levels of decomposition. The different features, namely, lacunarity and fluctuation indexes, are measured from these decomposed scaled signals. The lacunarity is a scale-dependent feature that measures heterogeneity, while fluctuation index reflects the intensity fluctuation of EEG signals. The Bayesian linear discriminant analysis classifier is used for labeled feature classification. The proposed method achieved an excellent classification accuracy of the order of 96.67% for intracranial EEG signal classification.

Many authors proposed multiple variants of the WT, namely, the multiwavelet transform [26], orthogonal wavelet filter banks [11, 12], tunable Q-wavelet transform [14, 29, 58, 68], wavelet packet decomposition (WPD) [4], dual-tree complex wavelet transformation (DTCWT) [17, 59], empirical wavelet transform [13], flexible analytic wavelet transform [70], and new frequency slice wavelet transform [79], etc., for seizure classification based on the EEG signals. Many variants of the WT extract relevant information from the nonlinear and nonstationary EEG signals. However, the obtained results may vary with a change of the basis wavelet; also the computational complexity increases with an increase of decomposition levels.

Similar to the WT, the EMD is another nonlinear algorithm that identifies the subtle information regarding seizure classification and it is easy to use [5, 8, 20, 44, 52, 53, 67]. The EMD is a data decomposition method that decomposes an arbitrary signal into the IMFs. It is a data-dependent TF analysis method that is characterized by the Hilbert-Huang transform (HHT) algorithm [31]. Fu et al. [20] used the HHT-based TF representation (TFR) method for seizure and non-seizure EEG signal classification. The TFR is simulated as a time-frequency image (TFI), and the segmentation of these images has been done based on the frequency bands of rhythms presented in the EEG signals. The different statistical features up to fourth order are measured from the pixel intensity of the TFI. These attributes are evaluated by the SVM classifier with RBF kernel function. They reported 99.125% classification accuracy with the θ-rhythm of EEG signals. Alam et al. [5] computed the same features as in [20] directly from the IMFs. The collected features are classified by the ANN classifier that yields 100% classification accuracy. Bajaj and Pachori [8] proposed the EMD-based algorithm for epileptic seizure detection. The instantaneous signal area is computed from the trace of the windowed IMFs. They achieved 90% sensitivities with 24.25% error rate detection. Pachori [53] uses the EMD algorithm for ictal and seizure-free EEG signal classification, where the decomposed IMFs are transformed in terms of the Fourier Bessel (FB) expansion, and the mean frequency of FB expansion is considered as features.

Pachori et al. [54–56] extracted different features from IMFs and inputted them into various classifiers to classify labeled EEG signals. The second-order difference plot (SODP) is measured from the IMFs, and the area of 95% confidence ellipse (measured from the SODP) is considered as a feature [55]. They obtained 97.72% classification accuracy to discriminate epileptic seizure from seizure-free EEG signals. They extended their work in [56], where they measured two areas, first one corresponding to the SODP, while the other for the graph obtained when the IMFs of the analytic signal are represented in the complex plane. Sharma et al. [67] classify the epileptic seizures based on the phase-space representation (PSR) of IMFs. Oweis et al. [52] used the HHT algorithm to decompose the EEG signal into the IMFs, and the instantaneous frequency and amplitude are computed from each decomposed signal. They observed that the starting three or four modes are highly determinant by using the hypothesis testing. The hypothesis testing used the t-test with two different p-values that allow reducing the insignificant feature space. The proposed algorithm is evaluated on 25 healthy and 25 seizure EEG signals. The ANN is used as a classifier that yields 94% classification accuracy, while the 80% accuracy is obtained with the multivariate empirical mode decomposition (MEMD) method. However, the EMD does not correctly extract the low-energy components from the nonlinear EEG signals; also, the EMD suffered from mode mixing problem that can be temporarily reduced by the ensemble empirical mode decomposition (EEMD) [91]. As compared to the EMD and EEMD, the complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN) gives an accurate reconstruction of the EEG signal without mode mixing that allows a better spectral separation of the mode functions. Hassan et al. [28] proposed the CEEMDAN method for decomposition of the EEG signals into the IMFs. The different statistical features, namely, the average, variance, and kurtosis are computed from these IMFs. The ANN classifier is used to separate the labeled classes. They achieved the classification accuracy of order 98.87% for normal (eye open) and non-seizure including normal eye closed and 86.37% for three categories, e.g., seizure, seizure-free, and healthy EEG signals. The variational mode decomposition (VMD) algorithm is proposed in [94] for automated seizure detection based on the EEG signals. The VMD can extract relevant low-energy component information from the raw EEG signals by decomposing it into a fixed number of band-limited IMFs (BLIMFs).

To explore the subtle information about the underlying nonlinear dynamics from the EEG signals, many authors used various nonlinear parameters extraction methods with different algorithms. The recurrence quantification analysis (RQA) visualizes the recurrence behavior of the phase-space trajectory of dynamical systems. Acharya et al. [3] used the RQA for automatic epilepsy EEG signal detection. They extended their work with another nonlinear parameter called the Hurst parameter [2]. The Hurst parameter customarily used to measure the long-term memory of the nonlinear signal. The short-term maximum Lyapunov exponent (STLmax) that measures the rate of divergence, is used to know the dynamic behavior of the EEG signals [23, 45]. To measure the complexity of the studied signals, various entropies, such as the ApEn, Shannon entropy, Fuzzy entropy Renyi entropy, log energy entropy, and more are used. Srinivasan et al. [76] and Zhang et al. [93] used the ApEn as a feature, and the labeled features are classified by the ANN and the SVM classifiers, respectively. Guo et al. [25]

proposed the line length features derived from the DWT coefficients. The line length features are used to capture the changes in the waveform dimensionality. These are very sensitivity for signal amplitude and frequency variations. The EEG signals are decomposed up to five levels of decomposition with the DWT corresponding to various frequency bands. The line length features are measured from each sub-signal and classified by using the ANN classifier. The proposed algorithm achieved 99.60% for normal (eye open) and seizure, 97.75% for seizure and seizure-free including eye open, and 97.77% for seizure and non-seizure classification accuracy for different EEG signal sets.

Wang et al. [87] proposed the adaptive learning algorithm, whereas Shin et al. [73] used the sparse representation model for EEG signal classification. Besides the single-scale entropy measurement, the multiscale entropy is also registered in literature for the EEG signal based seizures classification. The single-scale entropy measurement is not able to quantify the underlying dynamics of the nonlinear signals typically when the signals are generated from the biological system. Labate et al. [41] proposed the multiscale entropy as a feature for measuring the complexity in Alzheimer's disease. They extended the same concept for the EEG signal classification with the multiscale permutation entropy. The features are classified by the SVM classifier with various kernel functions such as linear, RBF, and sigmoidal, and they obtained 78%, 95%, and 91% classification accuracy, respectively [42]. Guo et al. [27] proposed the genetic programming (GP) algorithm for the EEG signal classification. The GP automatically extracted the features from the original feature database. It not only improves the performance of discrimination of the classifier but at the same time reduces the dimensionality of feature space. The DWT is applied on the raw EEG signals, and the mean, standard deviation, energy, curve length, and skewness are measured as original features from each sub-band signal that are fed to the GP algorithm. The GP generated the new dimension features having a substantial probability of discrimination capability. These GP generated features are classified with the KNN classifier which results in 99.20% classification accuracy. There are various learning based algorithms, namely, the random forest, extreme machine learning [92], ANN [88], recurrent neural network (RNN) [23], etc., that are also used for the EEG signal classification.

10.3 Database

In the study, the web-available EEG signal database has been considered that is recorded in the University of Bonn, Germany [7]. The database is the collection of five sets of EEG signals, namely, Z, O, N, F, and S. Each set contains 100 single-channel EEG time series of 23.6 sec duration. These signals are the subset of 128-channel EEG signals that are selected by the neuro experts. All these signals are digitalized at 173.61 Hz sampling rate using 12-bit resolution. A band-limited filter (0.53–40 Hz) is used to remove the artifact due to muscle activities. The EEG time series sets Z and O are recorded extracranially on five healthy persons during the relaxed state with eyes open (set Z) and eyes closed (set O) condition, respectively. However, the remaining time series sets N, F, and S are recorded intracranially from five epileptic patients in their pre-surgical evaluation of epilepsy. Sets N and F consist of seizure-free time series. The signals of set N are collected from the hippocampal formation of the opposite hemisphere of the brain, while the set F time series are recorded from the epileptogenic zone, whereas the intracranial time series of set S are collected from the epileptic zone during seizure activity. All signals are collected using the standard 10–20 electrode placement scheme.

In brief, the different EEG signals groups can be summarized as:

O – Normal signal set, persons are relaxed and in the awake state with eyes closed.
Z – Normal signal set, persons are relaxed and in the awake state with eyes open.
N – Seizure-free signal set recorded from the hippocampal formation of the opposite hemisphere of the brain.
F – Seizure-free signal set recorded from the epileptogenic zone.
S – Signal set collected from the epileptic zone during seizure activity.

In this chapter, the three classification problems have been considered as follows:

Case 1. Seizure (S) versus normal eyes-open (Z)
Case 2. Seizure (S) versus all non-seizure (FNZO)
Case 3. Seizure (S) versus seizure-free (FN) versus normal (ZO)

Figure 10.1 shows the EEG time series from each set. The Z and O are the healthy, F and N are the seizure-free, while S represents the seizure EEG signal, respectively. In the broad scene, S is known as the seizure EEG signal, while the remaining (ZOFN) is known as the non-seizure EEG signals.

10.4 Proposed Methodology

In this study, a nonlinear higher-order spectra (HOS) based algorithm is proposed for the seizure-labeled EEG signal classification. Figure 10.2 depicts the block diagram representation of the proposed algorithm. The EEG signals are highly nonlinear in nature. Hence, to extract the relevant information from such signals, the higher-order spectruma, namely, bispectrum, is used. The bispectrum reveals the nonlinearity present in EEG signals and may better illustrate the dynamics of the nervous system. It helps to the diagnosis of brain diseases by understanding of neurological processes. The bispectrum contains the six symmetries along with various directions. Thus, the principal region has been extracted to avoid the irrelevant mathematical calculation. The numerous features are extracted from the principal region that capture the underlying dynamics of the studied signals. As the studied dataset contains five sets of different classes, each consisting of a hundred long EEG time series,hence, the dimension of the extracted features is much high; also the extracted features may contain some irrelevant features that may degrade the algorithm

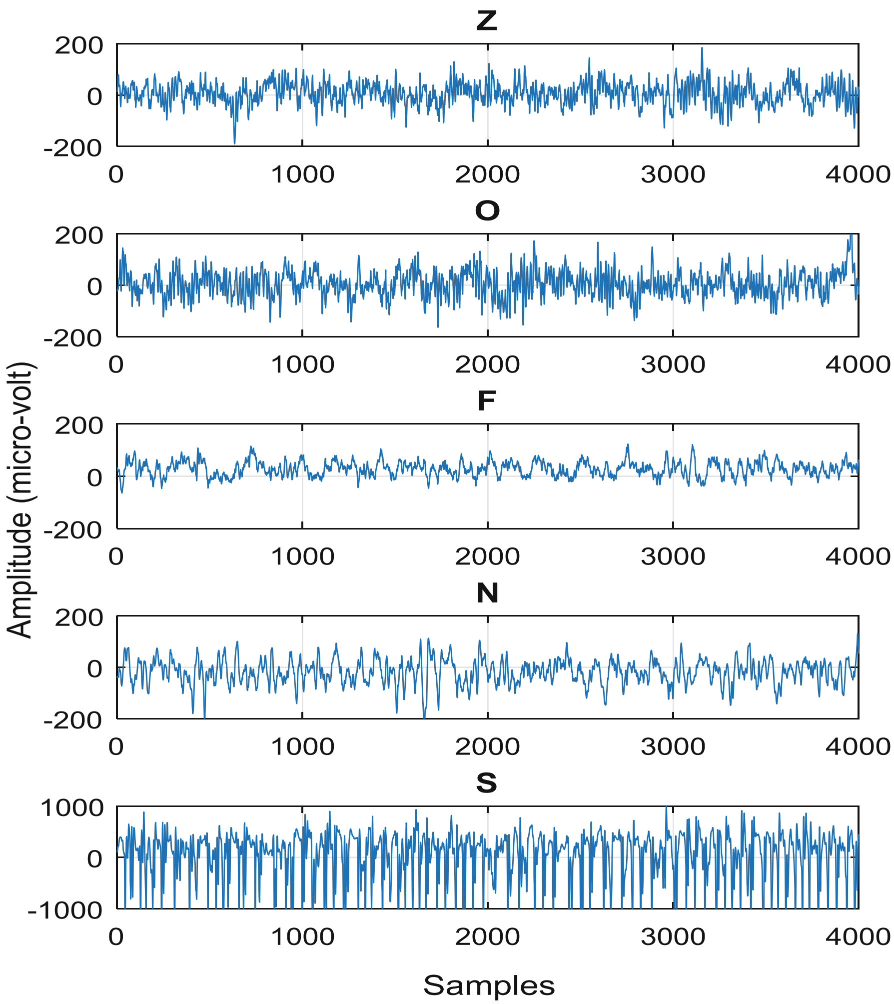

Fig. 10.1 Signals from each set of database

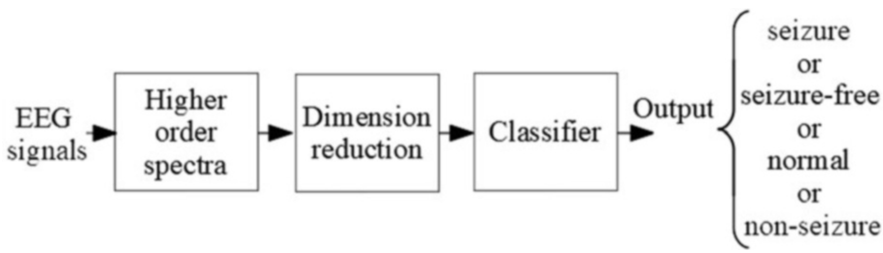

Fig. 10.2 Block representation of proposed algorithm for epileptic seizure detection

performance. A dimensionality reduction technique known as the locality sensitive discriminant analysis (LSDA) is used to moderate feature space dimensionality. It ranked the features according to the probability of separation. These ranked features are then subjected to the supervised learning SVM classifier with RBF kernel.

10.4.1 Higher-Order Spectra

The nonlinearity analysis of a system operating under a random input has been quite extensively done for many years [15]. The HOS are initially generated for the ergodic random processes, and later, it has been expanded to the deterministic signals [15, 64]. In the real-time signal processing, the researchers are mainly dealing with the nonlinear, non-Gaussian, and nonstationary signals. The second- or lower-order statistics do not adequately analyze these types of signals. Also, the considered noise in signal processing is assumed to be Gaussian distributed and additive. The literature reveals that the HOS of order higher than two is commonly used for the analysis of nonstationary and non-Gaussian signal [15, 48, 64]. The HOS can reveal in terms of moments and cumulant functions. The cumulant function is defined as the logarithm of the moment function. For a zero mean signal, both are equal up to order three. The HOS have some unique properties that help for the analysis of non-Gaussian signals; such as, the higher-order (three or more) cumulants of the Gaussian distribution are equal to zero [47, 49]. Figure 10.3 illustrates the bispectrum of each set of the EEG signals and the principal region of bispectrum. The third-order cumulant (ToC) is infinitely differentiable and convex, that allows analysis of the non-minimum-phase and phase couple signals; also, it is scale-invariant. Hence, the ToC of a non-Gaussian signal with an additive uncorrelated Gaussian noise filters out the Gaussian noise part, and represents only the ToC of the signal [47]. Sharma et al. [71] used the residual-based higher-order statistics algorithm for the classification of focal and nonfocal EEG signals. The proposed method measured the disturbance in the labeled EEG signals with various statistical attributes. They extended their work and proposed a center slice algorithm of higher-order statistics for automated glaucoma detection based on fundus images [72].

Let $y(\tau)$ be a zero mean random process where $\boldsymbol{\tau} = 1, 2, \ldots L$. The ToC can be represented as follows [47]:

$$\begin{aligned} C_y^3(a,b) &= E[y(\tau)^*\, y(\tau+a)\, y(\tau+b)] \\ &= E[y(\tau)\, y(\tau+a)^*\, y(\tau+b)] \\ &= E[y(\tau)\, y(\tau+a)\, y(\tau+b)^*] \end{aligned} \tag{10.1}$$

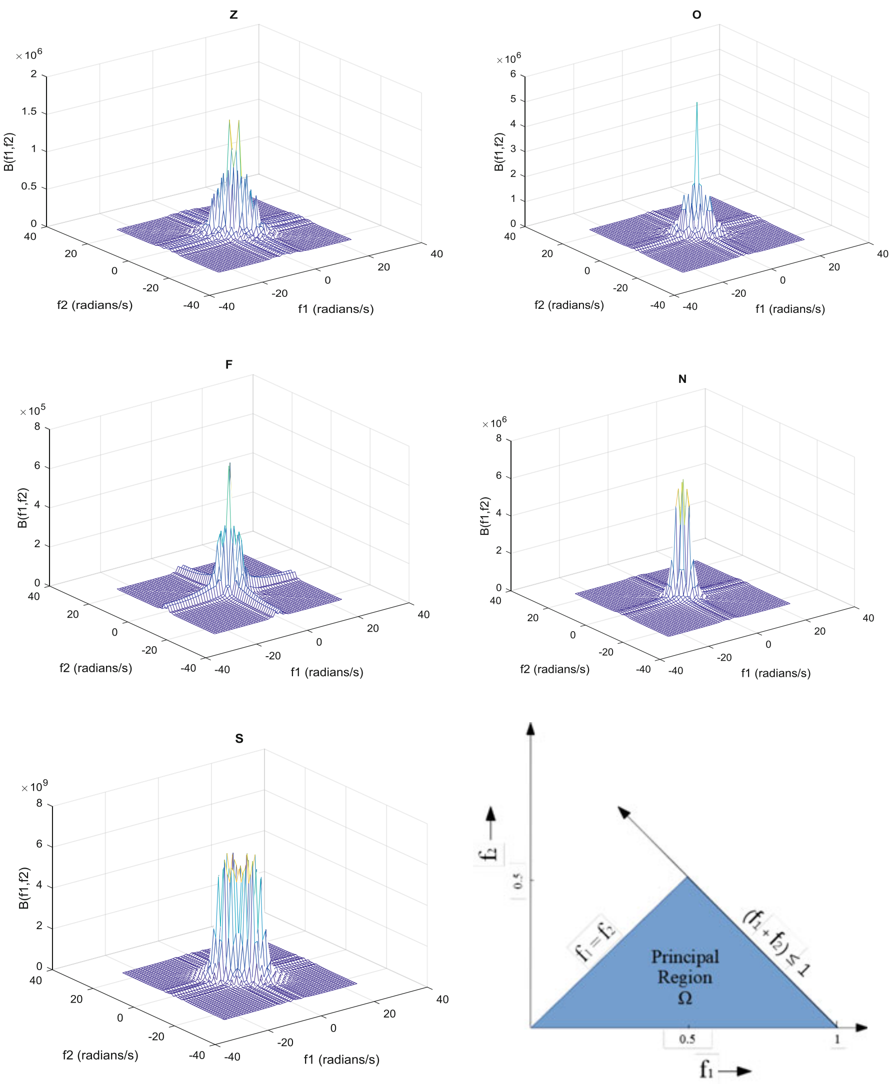

Fig. 10.3 Bispectrum magnitude corresponding to the various set of EEG signals and the principal region of the bispectrum

where $\boldsymbol{E}$ and "$*$" denote the expectation operator and complex conjugate, respectively. The Fourier transform of ToC is defined as bispectrum [49] and is given below:

$$B(\zeta_1, \zeta_2) = FT\left[C_y^3(a, b)\right] \tag{10.2}$$

$$or$$

$$B(\zeta_1, \zeta_2) = E[Y(\zeta_1)^* \, Y(\zeta_2) \, Y(\zeta_1 + \zeta_2)]$$

where *FT* denotes the Fourier transform, $Y(\zeta)$ is the discrete-time Fourier transform of $y(\tau)$, and $\zeta_1$and $\zeta_2$are the normalized frequency components that lie between 0 and 1. The bispectrum of a real signal is uniquely defined with the triangle $\mathbf{0 \leq \zeta_2 \leq \zeta_1 \leq (\zeta_1 + \zeta_2) \leq 1}$, known as the principal domain [64].

10.4.2 Feature Extraction

The suitable feature extraction is an essential part of any classification algorithm. Otherwise, it may lead to underperformance of the classifier. To explore the characteristics of EEG signals, various parameters, namely, the maximum value, energy, mean, variance, normalized bispectrum entropy (NbE) up to order three, interquartile distance (IqD), and spectral flatness are measured from the principal domain of the bispectrum [38, 66, 74]. These statistical features are very significant and less computationally complex.

Let z_i be the random variable that denotes the elements of bispectrum principal domain, and let $p(z_i)$; $i = 0, 1, \ldots, \eta - 1$ be the probability of occurrence of z_i, where η is the number of distinct random values. Then, the features are defined as follow:

$$Maximum\ value\ of\ z_i = Max\ (\mathrm{z}_i) \tag{10.3}$$

$$Energy = \sum_{i=1}^{\eta} |z_i|^2 \tag{10.4}$$

$$Mean = \sum_{i=1}^{\eta} z_i \, p(z_i) \tag{10.5}$$

$$Variance = \sum_{i=1}^{\eta} (z_i\text{-Mean})^2 \, p(z_i) \tag{10.6}$$

$$NbE1 = -\sum_i p_i \, \log(p_i); \quad p_i = \frac{|B(\zeta_1, \zeta_2)|}{\sum_\Omega |B(\zeta_1, \zeta_2)|} \tag{10.7}$$

$$NbE2 = -\sum_i q_i \, \log(\mathrm{q}_i); \quad \mathrm{q}_i = \frac{|B(\zeta_1, \zeta_2)|^2}{\sum_\Omega |B(\zeta_1, \zeta_2)|^2} \tag{10.8}$$

$$NbE3 = -\sum_i r_i \, \log(\mathrm{r}_i); \quad \mathrm{r}_i = \frac{|B(\zeta_1, \zeta_2)|^3}{\sum_\Omega |B(\zeta_1, \zeta_2)|^3} \tag{10.9}$$

The IqD is a measure of variability, based on dividing the dataset into quartiles [74] and can be expressed as follows:

$$IqD = \gamma - \nu \tag{10.10}$$

where $P(\gamma) = \frac{3}{4}$, $P(\nu) = \frac{1}{4}$, and P is the sample cumulative distribution function of the diagonal of the center slice.

$$Spectral\ flatness = \frac{GM(z_i)}{AM(z_i)} \tag{10.11}$$

where GM and AM represent geometric and arithmetic mean, respectively [74].

10.4.3 Locality Sensitive Discriminant Analysis (LSDA)

The LSDA is a supervised dimensional reduction technique. It is based on the concept of maximizing the local margin between different classes while minimizing the local margin within the classes. It splits the data into the within-class and between-class graphs according to its class labels using the nearest neighbor graph (NNG) [16]. The transformation or mapping from the original feature space to a new feature space is allowed to maximize the probability of separation among different classes. Hence, each separated class data is mapped by using a linear transformation matrix such that it preserves the local as well as discriminant neighborhood information. Let l data points $\{x_1, x_2, \ldots, x_l\} \subset R^n$ be sampled from a large dataset. Assume that $N_s(x_i)$ is a group of neighbors having the same label, while $N_d(x_j)$ is a group of neighbors having different labels such that $N_s(x_i) \cap N_d(x_j) = \phi$; *where* ϕ *is the null space* and $N_s(x_i) \cup N_d(x_j) = N(x_l)$. The LSDA ranked the resultant mapped features and arrange them from the highest probability of discrimination to the lowest probability of discrimination.

10.4.4 Support Vector Machine

The SVM is a most popular supervised machine learning technique commonly used for data classification and regression. It maximizes the distance between the support vectors such that the probability of data points error becomes minimum. The slack variable is introduced for compensating the data points error. The support vector of a class represents the boundary that passes through the far most data point that is well separated from another class data [18, 30, 85]. The hyperplane is a virtual perpendicular plane between the support vectors. The SVM is the most popular machine learning algorithm having high generalization performance without any additional a priori knowledge even in high dimensional input space. Different linear or nonlinear kernels are used to map the old feature into a new feature space to achieve the lowest probability of misclassification error [89].

Let X_j be a training vector such that $X_j \in R^n$; $j = 1, 2, \ldots, l$ in two classes, labeled as$y \in \{+1, -1\}^t$. The SVM separates the training vector such as [18]:

$$\begin{aligned} &\min_{\omega, b, \xi} \quad \frac{1}{2}\omega^T\omega \;+\; \mathrm{C}\sum_{j=1}^{l}\xi_j \\ &\textit{subject to} \quad \mathrm{y}_j\left(\omega^T\phi\left(X_j\right) + \mathrm{b}\right) \geq 1\text{-}\xi_j \end{aligned} \tag{10.12}$$

where ϕ is defined as mapping space with an error cost,$C \succ 0$, $\mathrm{X}_j \geq 0$. The above equation is a constraint optimization problem that can be solved with the Lagrangian method. Hence, the optimization function can be written as follows:

$$\begin{aligned} &\min_{\lambda} \quad \mathrm{f}(\lambda) = \frac{1}{2}\lambda^T Q\lambda \text{ - } \mathrm{e}^T\lambda \\ &\textit{subject to} \quad 0 \leq \lambda_j \leq \mathrm{C}; \quad \mathrm{y}^T\lambda = 0 \end{aligned} \tag{10.13}$$

where $Q_{jk} \equiv y_j \; \mathrm{y}_k \; \phi(X_j)^T \; \phi(X_k)$ and e is the vector of all ones. Let $K(X_j, X_k) = \phi(X_j)^T \; \phi(X_k)$ be the kernel function; then, the generalized function of the SVM classifier can be defined as [85]:

$$\beta_\kappa = sign\left[\sum_{j\in l}\lambda_j \; \mathrm{y}_j \; K\left(X_j, X_k\right) + \mathrm{b}\right] \tag{10.14}$$

where λ_j is a nonzero Lagrange multiplier and b denotes the bias.

10.5 Results and Discussion

In this study, a nonlinear third-order spectrum, namely, bispectrum algorithm, is proposed for seizures classification based on the EEG signals. To explore the seizure dynamics, the EEG signals undergo the bispectrum analysis, and the bispectrum of the EEG signal reveals that significant alteration takes place with the EEG signals classes. The magnitude of the bispectrum of seizure-EEG signal (S) is more spread than other classes bispectrum magnitude, also its magnitude is of the order of 10^9 which is high as compared to the bispectrum magnitude graph of seizure-free and normal EEG signals. It can be illustrated that spectra magnitude of standard EEG signal has significant high peaks at a higher frequency as compared to the spectra of seizure-free EEG signals.

From the principal region of bispectrum-magnitude of the EEG signals, several statistical features are computed to extract the relevant information about the brain electrical activities. In this study, two binary-classes classification and one three-classes classification have been performed. The extracted features are subjected to the LSDA algorithm to reduce the feature matrix dimension and also to rank the features from the higher value to lower value subjected to the probability of discrimination. The ranked features are fed to the SVM classifier with RBF and ten-fold cross-validation strategy. The SVM classifier with RBF kernels achieved the maximum accuracy in all cases which are 98.67% for S-Z, 96.30% for S-FN-ZO, and 97.53% for S-FNZO.

In literature, various algorithms are listed for seizure classification. Table 10.1 illustrates various seizure classification methods to investigate brain electrical activities. Peker et al. [59] used the DT-CWT for decomposition of the EEG signals into low-dimensional wavelet coefficients. These features are discriminated with the complex-valued neural network. Their proposed technique is suited for binary as well as three-classes classification. They achieved 100% for S-Z, 99.15% for S-FNOZ classes, and 98.28% and 99.30% classification accuracy for three-classes, namely, S-FN-ZO and S-F-Z, respectively.

Bao et al. [9] employed the power spectral features, Higuchi fractal dimension, Petrosian fractal dimension, and Hjorth parameters as features that were classified with probabilistic neural network (PNN) classifier. They registered 97.00% classification accuracy for differentiating S-from NF-labeled signals. The LP method for seizure classification is used by Joshi et al. [34] and Altunay et al. [6]. The LP energy is used as a feature in [6], while the fractional LP error energy and fractal model signal energy are computed from the EEG signals to be used as features in [34]. They obtained 94% and 95.33% classification accuracy for seizures and seizure-free (S-NF) datasets with the threshold value classifier and the SVM classifier, respectively. The permutation entropy (PE) is used as a measure to distinguish the epileptic EEG signals from the seizure-free EEG signals [46]. They classified subsets S and N with the SVM classifier to the obtained classification accuracy of order 88.83%. Polat and Gunes [60] proposed the non-parametric spectral estimation method for the EEG signal classification. They reported 98.72% classification accuracy with the decision tree (DT) classifier, while the same group achieved a maximum classification accuracy of 100% when the principal component analysis (PCA) algorithm is implemented for dimension reduction of the fast Fourier transform (FFT) coefficient with the artificial immune recognition system classifier [61]. In [62], the authors proposed the AR model for EEG signal representation. The PSD vector is considered as a feature. The distance-based data reduction (DBDR) algorithm is applied to reduce the dimension of the attribute vectors. The obtained features are classified with the C4.5 decision tree classifier. They achieved 99.32% classification accuracy for S-O datasets. Tiwari et al. [80] proposed the 1D-LBP algorithm to extract the relevant information for seizure classification. They obtained 99.45% for S-NF, 99.31% for S-FNZO, and 98.80% for ZO-FN-S classification accuracy when the sparse LBP features are classified by the SVM classifier. Kaya et al. [35] proposed the uniform and non-uniform 1D-LBP feature extraction method from the raw EEG signals. These extracted features are applied to different classifiers to evaluate algorithm performance. They obtained 99.50% classification accuracy with the functional tree (FT) classifier for S-O classes classification.

Table 10.1 A comparison of performances of the various methods for seizure classification

References	Methods and features	Classifier	Classes	Accuracy (%)
Peker et al. [59]	DT-CWT	Complex-valued neural networks	S-Z	100
			S-NFZO	99.15
			S-F-Z	99.30
			S-FN-ZO	98.28
Bao et al. [9]	Power spectral features, Petrosian fractal dimension and Higuchi fractal dimension, Hjorth parameters	PNN	S-NF	97
Altunay et al. [6]	LP error energy	Thresholding	S-NF	94.00
Joshi et al. [34]	FLP error energy and signal energy	SVM	S-NF	95.33
Nicolaou and Georgiou [46].	Permutation entropy	SVM	S-N	88.83
Polat and Gunes [62]	AR	C4.5 decision tree classifier	S-O	99.32
Polat and Gunes [60]	FFT	DT	S-Z	98.72
Polat and Gunes [61]	FFT	The artificial immune recognition system	S-Z	100
Tiwari et al. [80]	1D LBP	SVM	S-NF	99.45
			S-FNZO	99.31
			S-FN-ZO	98.80
Kaya et al. [35]	1D LBP	Functional tree	S-O	99.50
Sharma and Pachori [68]	TQWT and FD	LS-SVM	S-FNZO	99.60
			S-Z	100
			S-O	100
			S-ZO	100
			S-FN	99.67
Bhattacharyya et al. [14]	TQWT-based multiscale KNN entropy	SVM	S-FN-ZO	98.60
			S-Z	100
			S-FNZO	99
Ubeyli [83]	Burg AR	LS-SVM	S-O	99.56
Bhati et at. [11]	Three-band synthesis wavelet filter bank	MLPNN	S-NF	99.66
Subasi [77]	DWT	A mixture of expert model	S-Z	95.00

(continued)

Table 10.1 (continued)

References	Methods and features	Classifier	Classes	Accuracy (%)
Subasi and Gursoy [78]	DWT-PCA	SVM	S-Z	98.75
	DWT-ICA			99.50
	DWT-LDA			100
Acharya et al. [4]	WPD-PCA	GMM	S-NFZO	99.31
Tzallas et al. [81]	TF analysis	ANN	S-Z	100
			S-NFZO	97.73
			S-F-Z	99.28
			S-FN-ZO	97.72
Samiee et al. [65]	DSTFT	MLP	S-Z	99.30
			S-NFZO	98.10
Srinivasan et al. [75]	Time and frequency features	RNN	S-Z	99.60
Orhan et al. [51]	DWT, k-means clustering	MLPNN	S-Z	100
			S-NFZO	99.60
			S-F-Z	96.67
			S-FN-ZO	95.60
Guo et al. [27].	Genetic algorithm	KNN	S-Z	99.20
			S-F-Z	93.50
			S-FN-ZO	93.50
Ocak [50]	DWT	ApEn optimal threshold value	S-ZNF	96.65
Sharma and Pachori [67]	95% confidence area measure of 2D PSR of IMFs, IQR of Euclidian distances of 3D PSR of IMFs	LS-SVM	S-NF	98.67
Fu et al. [20]	Hilbert-Huang transform	SVM	S-Z	99.13
Proposed work	Higher-order spectra	SVM	S-Z	98.67
			S-FN-ZO	96.30
			S-FNZO	97.56

The TQWT is proposed in Sharma and Pachori [68] and Bhattacharyya et al. [14] for the seizure EEG signal classification. Sharma and Pachori [68] computed the signal fractal dimension as a feature on each sub-band. These features are subjected to the LS-SVM classifier for the seizure classification. They achieved 100% classification accuracy for the individual and combine normal classes with seizure class (S-O, S-Z, and S-ZO), and 99.67% classification accuracy for S-NF while 99.60% for the binary class (S-FNZO).

Bhattacharyya et al. [14] measured the multiscale KNN entropy as features for seizure classification on the TQWT decomposed sub-bands. The multiscale entropy measures the complexity of multivariate EEG signals over different scales. These mixed labeled features are classified by the SVM classifier and obtained 98.60% to 100% classification for different seizure classes. The Burg AR coefficients are used for the EEG signal classification [83]. They reported 99.56% classification accuracy. Bhati et al. [11] proposed a TF localized three-band wavelet filter-bank technique for the EEG signal classification. The proposed technique is implemented by using the semidefinite relaxation and nonlinear least square method. They obtained classification accuracy of the order of 99.66% for seizure and seizure-free (S-NF) classes with the MLP neural network (MLPNN) classifier.

Subasi [77] used the DWT for EEG signal decomposition. The author computed four statistical features. Three out of four features, namely, the mean, average power, and the standard deviation, are computed from each sub-band, while the fourth feature is the ratio of the absolute mean values of the adjacent sub-bands. These four statistical features from the DWT coefficients were classified by a modular neural network called the mixture of experts (MEs) and reported classification accuracy of order 95.00%. The author extended the wavelet-based signal classification technique [78]. In this study, the different data dimension reduction techniques such as the principal component analysis (PCA), independent component analysis (ICA), and linear discriminant analysis (LDA) are used to reduce the dimension of the wavelet coefficients, and the resultant coefficients were classified by the SVM classifier. They achieved 98.85% using PCA, 99.5% using ICA, and 100% using LDA, classification accuracy with the SVM classifier. Acharya et al. [4] used the wavelet packet decomposition (WPD) algorithm for the seizure classification. The method decomposed the EEG signals into higher and lower frequency bands simultaneously, and the PCA is simulated on computed feature vectors from each sub-band. For the binary (S-NFZO) seizure classification, they reported 99.31% classification accuracy with the Gaussian mixture model classifier.

Tzallas et al. [81] used the TF analysis technique for automated EEG signal classification. The fractional energy is computed as features from the selected segments obtained with the TF-EEG signal decomposition method. The ANN classifier is used to classify the computed features, and has obtained from 97.72% to 100% classification accuracy for separating different seizure classes. Samiee at al. [65] used the DSTFT for EEG signal decomposition on the TF axis. They computed different statistical features, namely, the average, median, maximum, standard deviation, and minimum, from the rational DSTFT coefficients. They obtained 98.10% for S-NFZO and 99.30% for S-Z, the order of accuracy using the MLP classifier. The TF feature-based classification methodology is also proposed in Srinivasan et al. [75]. They achieved classification accuracy of 99.60% for S-Z classes separation with the recurrent neural network (RNN) classifier. Orhan et al. [51] measured various nonlinear parameters from the DWT coefficients that capture the subtle information from the EEG signals. They reported 96.67% to 100%

accuracy with the MLPNN classifier for separating the different classes of seizures. The authors recorded 99.20% to 99.50% classification accuracy with the features extracted from the genetic programming, while these features are subjected to the KNN classifier to separating the seizures [27]. The ApEn used to compute the complexity of the nonlinear signal. Ocak et al. [50] computed the ApEn from the DWT coefficients for the S-ZNF seizure classification. They achieved 96.65% classification accuracy with the optimal threshold value-based classification algorithm.

The EMD method-based seizure classification algorithms are listed in the literature. Sharma et al. [67] used the EMD algorithm that utilized the Hilbert-Huang transform for signal decomposition into IMFs for complexity computation at high scales. They measured 95% confidence area of 2D and 3D PSR of each IMF. These computed areas are used as features to register 98.67% classification accuracy for binary (S-NF) seizure classification with the LS-SVM classifier. Fu et al. [20] registered 99.13% binary (S-Z) classification accuracy with the EMD algorithm and the SVM classifier.

10.6 Conclusion

As epilepsy occurs, due to episodic nature of the seizures, the detention and prediction of the seizures with visualization of the EEG signals by Neuro-experts are a very tedious and time-consuming process. The EEG signals are practically very lengthy. Hence, the prime challenge is to develop signal processing algorithms to explore the underlying behavior of the nonstationary and nonlinear EEG signals. Many nonlinear classification methods have been proposed that are able to capture the significant underlying information about the dynamics of the brain’s electrical activities. In this study, an elegant and convenient nonlinear HOS algorithm is proposed to extract relevant information from the EEG signals. The various statistical features are measured from the bispectrum of the EEG signals. These extracted measurements are input to the LSDA for discrimination of features according to the class label. The SVM with the different kernels (to maximize the probability of separation) are used for signal classification. The proposed method has achieved an excellent classification accuracy to classify seizures in various categories. The achieved results show the effectiveness of the proposed algorithm.

References

1. Acharya UR (2013) Automated diagnosis of epileptic electroencephalogram using independent component analysis and discrete wavelet transform for different electroencephalogram durations. Proc Inst Mech Eng H J Eng Med 227(3):234–244
2. Acharya UR, Chua CK, Lim TC, Dorithy, Suri JS (2009) Automatic identification of epileptic EEG signals using nonlinear parameters. J Mech Med Biol 9(04):539–553

3. Acharya UR, Sree SV, Chattopadhyay S, Yu W, Ang PCA (2011) Application of recurrence quantification analysis for the automated identification of epileptic EEG signals. Int J Neural Syst 21(03):199–211
4. Acharya UR, Sree SV, Alvin APC, Suri JS (2012) Use of principal component analysis for automatic classification of epileptic EEG activities in wavelet framework. Exp Syst Appl 39 (10):9072–9078
5. Alam SS, Bhuiyan MIH (2013) Detection of seizure and epilepsy using higher order statistics in the EMD domain. IEEE J Biomed Health Inform 17(2):312–318
6. Altunay S, Telatar Z, Erogul O (2010) Epileptic EEG detection using the linear prediction error energy. Expert Syst Appl 37(8):5661–5665
7. Andrzejak RG, Lehnertz K, Mormann F, Rieke C, David P, Elger CE (2001) Indications of nonlinear deterministic and finite-dimensional structures in time series of brain electrical activity: dependence on recording region and brain state. Phys Rev E 64(6):061907
8. Bajaj V, Pachori RB (2013) Epileptic seizure detection based on the instantaneous area of analytic intrinsic mode functions of EEG signals. Biomed Eng Lett 3(1):17–21
9. Bao FS, Lie DYC, Zhang Y (2008) A new approach to automated epileptic diagnosis using EEG and probabilistic neural network. In: Tools with artificial intelligence, 2008. ICTAI'08. 20th IEEE international conference on, vol 2. IEEE, Dayton, pp 482–486
10. Bassett DS, Sporns O (2017) Network neuroscience. Nat Neurosci 20(3):353
11. Bhati D, Pachori RB, Gadre VM (2017a) A novel approach for time-frequency localization of scaling functions and design of three-band biorthogonal linear phase wavelet filter banks. Digital Signal Processing 69:309–322
12. Bhati D, Sharma M, Pachori RB, Gadre VM (2017b) Time-frequency localized three-band biorthogonal wavelet filter bank using semidefinite relaxation and nonlinear least squares with epileptic seizure EEG signal classification. Digital Signal Processing 62:259–273
13. Bhattacharyya A, Pachori RB (2017) A multivariate approach for patient-specific EEG seizure detection using empirical wavelet transform. IEEE Trans Biomed Eng 64(9):2003–2015
14. Bhattacharyya A, Pachori RB, Upadhyay A, Acharya UR (2017) Tunable wavelet transform based multiscale entropy measure for automated classification of epileptic EEG signals. Appl Sci 7(4):385
15. Brillinger DR (1965) An introduction to polyspectra. Ann Math Stat 36:1351–1374
16. Cai D, He X, Zhou K, Han J, Bao H (2007) Locality sensitive discriminant analysis. IJCAI 2007:1713–1726
17. Chen G (2014) Automatic EEG seizure detection using dual-tree complex wavelet-Fourier features. Exp Syst Appl 41(5):2391–2394
18. Cortes C, Vapnik V (1995) Support-vector networks. Mach Learn 20(3):273–297
19. Faust O, Acharya UR, Min LC, Sputh BH (2010) Automatic identification of epileptic and background EEG signals using frequency domain parameters. Int J Neural Syst 20(02):159–176
20. Fu K, Qu J, Chai Y, Dong Y (2014) Classification of seizure based on the time-frequency image of EEG signals using HHT and SVM. Biomed Signal Process Control 13:15–22
21. Fu K, Qu J, Chai Y, Zou T (2015) Hilbert marginal spectrum analysis for automatic seizure detection in EEG signals. Biomed Signal Process Control 18:179–185
22. Ghosh-Dastidar S, Adeli H, Dadmehr N (2007) Mixed-band wavelet-chaos neural network methodology for epilepsy and epileptic seizure detection. IEEE Trans Biomed Eng 54 (9):1545–1551
23. Güler NF, Ubeyli ED, Güler I (2005) Recurrent neural networks employing Lyapunov exponents for EEG signals classification. Expert Systems with Applications 29(3):506–514
24. Guo L, Rivero D, Seoane JA, Pazos A (2009) Classification of EEG signals using relative wavelet energy and artificial neural networks. In: Proceedings of the first ACM/SIGEVO Summit on Genetic and Evolutionary Computation. ACM, New York, pp 177–184
25. Guo L, Rivero D, Dorado J, Rabunal JR, Pazos A (2010a) Automatic epileptic seizure detection in EEG based on line length feature and artificial neural networks. J Neurosci Methods 191 (1):101–109

26. Guo L, Rivero D, Pazos A (2010b) Epileptic seizure detection using multiwavelet transform based approximate entropy and artificial neural networks. J Neurosci Methods 193(1):156–163
27. Guo L, Rivero D, Dorado J, Munteanu CR, Pazos A (2011) Automatic feature extraction using genetic programming: An application to epileptic EEG classification. Expert Syst Appl 38 (8):10425–10436
28. Hassan AR, Haque MA (2015) Epilepsy and seizure detection using statistical features in the complete ensemble empirical mode decomposition domain. In: TENCON 2015-2015 IEEE Region 10 Conference. IEEE, pp 1–6. https://doi.org/10.1109/ICMEW.2016.7574707-8
29. Hassan AR, Siuly S, Zhang Y (2016) Epileptic seizure detection in EEG signals using tunable-q factor wavelet transform and bootstrap aggregating. Comput Methods Progr Biomed 137:247–259
30. Hsu CW, Chang CC, Lin CJ et al (2003) A practical guide to support vector classification. University of National Taiwan, Taipei
31. Huang NE, Shen Z, Long SR, Wu MC, Shih HH, Zheng Q, Yen NC, Tung CC, Liu HH (1998) The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. In: Proceedings of the Royal Society of London A: mathematical, physical and engineering sciences, vol 454. The Royal Society, London, pp 903–995
32. Iscan Z, Dokur Z, Demiralp T (2011) Classification of electroencephalogram signals with combined time and frequency features. Expert Syst Appl 38(8):10499–10505
33. Jirsa VK, Stacey WC, Quilichini PP, Ivanov AI, Bernard C (2014) On the nature of seizure dynamics. Brain 137(8):2210–2230
34. Joshi V, Pachori RB, Vijesh A (2014) Classification of ictal and seizure-free EEG signals using fractional linear prediction. Biomed Signal Process Control 9:1–5
35. Kaya Y, Uyar M, Tekin R, Yıldırım S (2014) 1d-local binary pattern based feature extraction for the classification of epileptic EEG signals. Appl Math Comput 243:209–219
36. Khamis H, Mohamed A, Simpson S (2009) Seizure state detection of temporal lobe seizures by autoregressive spectral analysis of scalp EEG. Clin Neurophysiol 120(8):1479–1488
37. Kim SH, Faloutsos C, Yang HJ (2013) Coercively adjusted autoregression model for forecasting in epilepsy EEG. Comput Math Methods Med 2013. https://doi.org/10.1155/2013/545613
38. Kreyszig E (2011) Advanced engineering mathematics. J. Willey & Sons Inc, New York, London
39. Kumar Y, Dewal M, Anand R (2014) Epileptic seizures detection in EEG using DWT-based ApEn and artificial neural network. Signal, Image, Video Process 8(7):1323–1334
40. Kumar TS, Kanhangad V, Pachori RB (2015) Classification of seizure and seizure-free EEG signals using local binary patterns. Biomed Signal Process Control 15:33–40
41. Labate D, La Foresta F, Morabito G, Palamara I, Morabito FC (2013a) Entropic measures of EEG complexity in Alzheimer's disease through a multivariate multiscale approach. IEEE Sensors J 13(9):3284–3292
42. Labate D, Palamara I, Mammone N, Morabito G, La Foresta F, Morabito FC (2013b) SVM classification of epileptic EEG recordings through multiscale permutation entropy. In: Neural Networks (IJCNN), The 2013 International Joint Conference on. IEEE, San Diego, pp 1–5
43. Lee SH, Lim JS, Kim JK, Yang J, Lee Y (2014) Classification of normal and epileptic seizure EEG signals using wavelet transform, phase-space reconstruction, and Euclidean distance. Comput Methods Progr Biomed 116(1):10–25
44. Li S, Zhou W, Yuan Q, Geng S, Cai D (2013) Feature extraction and recognition of ictal EEG using EMD and SVM. Comput Biol Med 43(7):807–816
45. Mammone N, Principe JC, Morabito FC, Shiau DS, Sackellares JC (2010) Visualization and modeling of STLmax topographic brain activity maps. J Neurosci Methods 189(2):281–294
46. Nicolaou N, Georgiou J (2012) Detection of epileptic electroencephalogram based on permutation entropy and support vector machines. Expert Syst Appl 39(1):202–209
47. Nikias CL, Mendel JM (1993) Signal processing with higher-order spectra. IEEE Signal Process Mag 10(3):10–37

48. Nikias CL, Petropulu AP (1993) Higher-order spectra analysis: a nonlinear signal processing framework. PTR Prentice Hall, Englewood Cliffs
49. Nikias CL, Raghuveer MR (1987) Bispectrum estimation: A digital signal processing framework. Proc IEEE 75(7):869–891
50. Ocak H (2009) Automatic detection of epileptic seizures in EEG using discrete wavelet transform and approximate entropy. Expert Syst Appl 36(2):2027–2036
51. Orhan U, Hekim M, Ozer M (2011) Eeg signals classification using the K-means clustering and a multilayer perceptron neural network model. Expert Syst Appl 38(10):13475–13481
52. Oweis RJ, Abdulhay EW (2011) Seizure classification in EEG signals utilizing Hilbert-Huang transform. Biomed Eng Online 10(1):38
53. Pachori RB (2008) Discrimination between ictal and seizure-free EEG signals using empirical mode decomposition. Res Lett Signal Process 2008:14
54. Pachori RB, Bajaj V (2011) Analysis of normal and epileptic seizure EEG signals using empirical mode decomposition. Comput Methods Prog Biomed 104(3):373–381
55. Pachori RB, Patidar S (2014) Epileptic seizure classification in EEG signals using a second-order difference plot of intrinsic mode functions. Comput Methods Prog Biomed 113 (2):494–502
56. Pachori RB, Sharma R, Patidar S (2015) Classification of normal and epileptic seizure EEG signals based on empirical mode decomposition. In: Complex system modeling and control through intelligent soft computations. Springer, Cham, pp 367–388
57. Pati S, Alexopoulos AV (2010) Pharmacoresistant epilepsy: from pathogenesis to current and emerging therapies. Cleve Clin J Med 77(7):457–567
58. Patidar S, Panigrahi T (2017) Detection of epileptic seizure using kraskov entropy applied on tunable-q wavelet transform of EEG signals. Biomed Signal Process Control 34:74–80
59. Peker M, Sen B, Delen D (2016) A novel method for automated diagnosis of epilepsy using complex-valued classifiers. IEEE J Biomed Health Inform 20(1):108–118
60. Polat K, Güneş S (2007) Classification of epileptiform EEG using a hybrid system based on decision tree classifier and fast Fourier transform. Appl Math Comput 187(2):1017–1026
61. Polat K, Güneş S (2008a) Artificial immune recognition system with fuzzy resource allocation mechanism classifier, principal component analysis, and FFT method based new hybrid automated identification system for classification of EEG signals. Expert Systems with Applications 34(3):2039–2048
62. Polat K, Güneş S (2008b) A novel data reduction method: distance-based data reduction and its application to classification of epileptiform EEG signals. Appl Math Comput 200(1):10–27
63. Pradhan N, Dutt DN (1994) Data compression by a linear prediction for storage and transmission of EEG signals. Int J Bio-Med Comput 35(3):207–217
64. Rosenblatt M, Van Ness JW et al (1965) Estimation of the bispectrum. Ann Math Stat 36 (4):1120–1136
65. Samiee K, Kovacs P, Gabbouj M (2015) Epileptic seizure classification of EEG time-series using rational discrete short-time Fourier transform. IEEE Trans Biomed Eng 62(2):541–552
66. Shannon C (1948) A mathematical theory of communication. Bell Syst Tech 27:379–423
67. Sharma R, Pachori RB (2015) Classification of epileptic seizures in EEG signals based on phase space representation of intrinsic mode functions. Expert Syst Appl 42(3):1106–1117
68. Sharma M, Pachori RB (2017a) A novel approach to detect epileptic seizures using a combination of tunable-q wavelet transform and fractal dimension. J Mech Med Biol 17(07):1740003
69. Sharma RR, Pachori RB (2017b) Time-frequency representation using IEVDHM-HT with application to classification of epileptic EEG signals. IET Sci Meas Technol 12(1):72–82
70. Sharma M, Pachori RB, Acharya UR (2017) A new approach to characterize epileptic seizures using analytic time-frequency flexible wavelet transform and fractal dimension. Pattern Recogn Lett 94:172–179
71. Sharma R, Sircar P, Pachori RB (2019a) A new technique for classification of focal and nonfocal EEG signals using higher-order spectra. J Mech Med Biol 19(01):1940010

72. Sharma R, Sircar P, Pachori RB, Bhandary SV, Acharya UR (2019b) Automated glaucoma detection using a center slice of higher order statistics. J Mech Med Biol 19(01):1940011
73. Shin Y, Lee S, Ahn M, Cho H, Jun SC, Lee HN (2015) Noise robustness analysis of sparse representation based classification method for nonstationary EEG signal classification. Biomed Signal Process Control 21:8–18
74. Spiegel RM, Murray R, Stephens LJ (1999) Theory and problems of statistics, 3rd edn. McGraw-Hill, New York
75. Srinivasan V, Eswaran C, Sriraam N (2005) Artificial neural network based epileptic detection using time-domain and frequency-domain features. J Med Syst 29(6):647–660
76. Srinivasan V, Eswaran C, Sriraam N (2007) Approximate entropy-based epileptic EEG detection using artificial neural networks. IEEE Trans Inf Technol Biomed 11(3):288–295
77. Subasi A (2007) Eeg signal classification using wavelet feature extraction and amixture of expert model. Expert Syst Appl 32(4):1084–1093
78. Subasi A, Gursoy MI (2010) Eeg signal classification using PCA, ICA, LDA and support vector machines. Expert Syst Appl 37(12):8659–8666
79. Tao Z, Wan-Zhong C, Ming-Yang L (2016) Automatic seizure detection of electroencephalogram signals based on frequency slice wavelet transform and support vector machine. Acta Phys Sin 65(3):038703
80. Tiwari AK, Pachori RB, Kanhangad V, Panigrahi BK (2017) Automated diagnosis of epilepsy using key-point based local binary pattern of EEG signals. IEEE J Biomed Health Inform 21 (4):888–896
81. Tzallas AT, Tsipouras MG, Fotiadis DI (2007) Automatic seizure detection based on time-frequency analysis and artificial neural networks. Comput Intell Neurosci 2007:80510
82. Tzallas AT, Tsipouras MG, Fotiadis DI (2009) Epileptic seizure detection in EEGs using time-frequency analysis. IEEE Trans Inf Technol Biomed 13(5):703–710
83. Ubeyli ED (2010) Least squares support vector machine employing model-based methods coefficients for the analysis of EEG signals. Expert Syst Appl 37(1):233–239
84. Uthayakumar R, Easwaramoorthy D (2013) Epileptic seizure detection in EEG signals using multifractal analysis and wavelet transform. Fractals 21(02):1350011
85. Vapnik V (2013) The nature of statistical learning theory. Springer Science &Business Media, New York
86. Wang D, Miao D, Xie C (2011) Best basis-based wavelet packet entropy feature extraction and hierarchical EEG classification for epileptic detection. Expert Syst Appl 38(11):14314–14320
87. Wang S, Chaovalitwongse WA, Wong S (2013) Online seizure prediction using an adaptive learning approach. IEEE Trans Knowl Data Eng 25(12):2854–2866
88. Webber W, Lesser RP, Richardson RT, Wilson K (1996) An approach to seizure detection using an artificial neural network (ann). Electroencephal Clin Neurophysiol 98(4):250–272
89. Weston J, Watkins C et al (1999) Support vector machines for multi-class pattern recognition. Esann 99:219–224
90. WHO (2015) Epilepsy report by world health organization. URL http://www.who.int/en/news-room/fact-sheets/detail/epilepsy
91. Wu Z, Huang NE (2009) Ensemble empirical mode decomposition: a noise-assisted data analysis method. Adv Adapt Data Anal 1(01):1–41
92. Yuan Q, Zhou W, Li S, Cai D (2011) Epileptic EEG classification based on extreme learning machine and nonlinear features. Epilepsy Res 96(1-2):29–38
93. Zhang Z, Zhou Y, Chen Z, Tian X, Du S, Huang R (2013) Approximate entropy and support vector machines for electroencephalogram signal classification. Neural Regen Res 8(20):18–44
94. Zhang T, Chen W, Li M (2017) Ar based quadratic feature extraction in the VMD domain for the automated seizure detection of EEG using random forest classifier. Biomed Signal Process Control 31:550–559
95. Zhou W, Liu Y, Yuan Q, Li X (2013) Epileptic seizure detection using lacunarity and Bayesian linear discriminant analysis in intracranial EEG. IEEE Trans Biomed Eng 60(12):3375–3381