# The Nonlinear Flow Analytical Model and Its Field Use

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**Abstract.** The nonlinear flow exists in ultra-low-permeability reservoir. It is important to establish an analytical model, which can be used in revealing nonlinear flow rule and improving producing status in field. According to the potential superposition principle, the mathematical model to predict the pressure gradient is established, and then, the nonlinear flow model is established by combining the pressure gradient boundary getting from the indoor nonlinear flow, and at last nonlinear flow area is determined in plane by increasing virtual production wells. It shows three features in the application of T14 block in Hailar oilfield. Firstly, the high value zone of cross-well pressure gradient is mainly concentrated in the area of 20–50 m near the oil–water well. Secondly, the pressure gradient is obviously affected by well spacing, and its lowest value is inclined to one side of the oil well. At last, the nonlinear flow zone is the main zone in ultra-low-permeability reservoir, and the area percentage is 60%, which is the main cause of the quickly production decline. The nonlinear flow area can be determined using the dynamic data. It can overcome two problems including parameter selection and extension in oilfield.

**Keywords:** Ultra-low permeability · Nonlinear flow · Analytical model · Pressure gradient · Quasilinear flow



This paper was prepared for presentation at the 2018 International Field Exploration & Development Conference in Xi'an, China, 18–20 September 2018.


J. Lin (ed.), *Proceedings of the International Field Exploration and Development Conference 2018*, Springer Series in Geomechanics and Geoengineering, https://doi.org/10.1007/978-981-13-7127-1_1

## 1 Introduction

The core displacement experiment in laboratory and the oilfield development practice show that nonlinear flow exists in ultra-low-permeability reservoirs [1–5]. Numerical reservoir simulation is the main method to study nonlinear flow at present. The numerical simulation software of three-phase nonlinear flow is developed [6–11]. It is considered that most of the flow regions in low-permeability reservoirs are in the nonlinear flow range. Quasilinear flow occurs only in a small area near the wellhole. However, many parameters required for modeling are often difficult to be determined accurately (especially when non-Darcy flow is considered in ultra-low-permeability reservoirs), coupled with the complexity of reservoir numerical simulation (currently mainstream commercial software does not take into account non-Darcy flow), so there are some difficulties in the field application for engineers and technicians. In addition, some scholars put forward some mathematical models to describe nonlinear flow based on laboratory experimental data [12–14]. However, how to obtain the analytical solution and how to apply it to the oil field to match the actual dynamic characteristics are not been reported in the reference.

In this paper, the nonlinear flow model of injection-production well group is studied from the point of view of establishing equation based on the actual production dynamic data of oil field, and it is applied in typical blocks.

## 2 The Nonlinear Flow Analytical Model

The key to the establishment of nonlinear flow model is the prediction of pressure gradient. An equation for calculating the displacement pressure gradient of water flooding at the midpoint of the mainstream line of one source (water injection well) one confluence (oil well) with constant output is proposed [15], but this method is only suitable for the case of two wells with the same injection rate and oil production rate. The distribution equation of the pressure gradient between injection and production wells with one source and one confluence is derived [16, 17], but in the course of oilfield development, the number of wells in an injection-production well group is rarely one, generally more than two.

### 2.1 Assumed Condition

To simplify the process and facilitate the establishment of mathematical models, set the following assumptions: ① The reservoir is stratified, and the longitudinal fluid movement and material exchange are ignored; ② the reservoir is homogeneous and equal in thickness, and the difference of fluid distribution on the plane is not considered; ③ no consideration of compressibility of porous media and liquids; ④ it is steady flow; ⑤ the flow process isothermal.

## 2.2 Potential of Plane Radial Flow

In reference [18], the concept of apparent permeability is introduced and the starting pressure gradient is considered, and when the pressure gradient is greater than the minimum starting pressure gradient, the equation of low velocity non-Darcy flow is:

$$v = -0.0864\frac{k'}{\mu}gradP \tag{1}$$

In the equation: $v$ is flow velocity, m/d; $k'$ is apparent permeability, $10^{-3}$ $\mu m^2$; $\mu$ is underground fluid viscosity, mPa s; $P$ is formation pressure at any point in an infinite formation, MPa; $GradP$ is the pressure gradient here, MPa/m.

The following is based on the dynamic and static data of the field, and $\frac{k'}{\mu}$ is obtained.

According to the definition of potential:

$$\psi = \frac{k'}{\mu}P \tag{2}$$

In the equation: $\psi$ is the potential, when the distance from it to the well is $r$, it has stress implication.

The equation for calculating the potential of a point in infinite plane is:

$$\psi = \frac{5.787q}{\pi h}\ln r + \mathrm{C} \tag{3}$$

In the equation: C is constant, determined by boundary conditions.

Suppose at a certain time $T_0$, an injection-production well group is in production in the infinite layer (Fig. 1), the average reservoir thickness of each well in the well group is $h$, $m$; $W_1$ is a water injection well, the well location coordinates are $(x_1, y_1)$, the

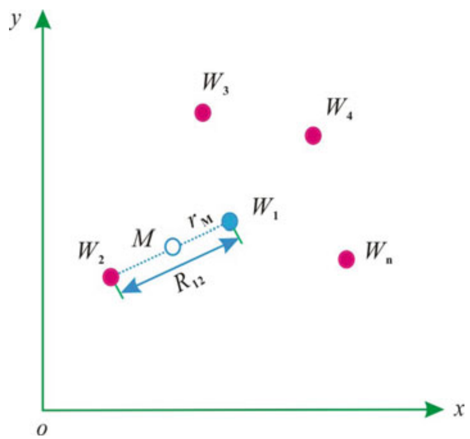


**Fig. 1.** The well location figure of nonlinear flow model

bottom hole pressure is $P_1$, MPa, the injection rate is $q_1$, t/d, $W_2W_3,\ldots W_n$ is oil production well, the well location coordinates are ($x_2$, $y_2$), ($x_3$, $y_3$), …, ($x_n$, $y_n$), the bottom hole pressure is $P_2$, $P_3$, …, $P_n$, MPa, the production is $q_2$, $q_3$, …, $q_n$, t/d, and $R_{12}$ is the interval between injection well $W_1$ and production of well $W_2$, m.

On the main line of water injection well $W_1$ and any oil production well ($W_2$ well here), the distance between any point M and the water injection well $W_1$ is $r_M$, The coordinate of the point M is ($x$, $y$). According to Eq. (3) and the principle of superposition, the potential of M point is as follows:

$$\psi_M = \frac{5.787}{\pi h}\left[\begin{array}{l} -q_1 \ln r_M + q_2 \ln(R_{12} - r_M) \\ + \sum_{i=3}^{n}\left(q_i \ln\sqrt{(x - x_i)^2 + (y - y_i)^2}\right) \end{array}\right] + C \tag{4}$$

If the well coordinates, fluid production, and water injection are known, according to Eq. (2), based on the wellbore potential of water injection well $W_1$ and any oil production well ($W_2$ well here), the equation to calculate $\frac{k'}{\mu}$ is as follows:

$$\frac{k'}{\mu} = \frac{5.787}{\pi h(P_{\inf} - P_{wf})} \times \left[(q_1 + q_2)\ln\frac{R_{21}}{r_w} + \sum_{i=3}^{n} q_i \ln\frac{R_{i1}}{R_{i2}}\right] \tag{5}$$

In the equation: $P_{\inf}$ is the bottom hole pressure of water injection well, MPa; $r_W$ is well radius, m; $P_{wf}$ is bottom well flow pressure, MPa; $R_{i1}$ is the well spacing between $W_1$ well and $W_i$ well, $m$; $R_{i2}$ is the well spacing between $W_2$ well and $W_i$ well, $m$.

Therefore, at a certain time, $\frac{k'}{\mu}$ of whole reservoir can be calculated and fixed, its value is related to the time and the dynamic and static parameters of water injection.

The distribution of the pressure gradient along the main stream is derived below.

The coordinates of any point M on the main stream are unknown. It is necessary to found the function of well point coordinate and distance from injection well; the relationship of $r_M$ and coordinates is obtained by coordinate transformation:

$$x = x_1 \pm \frac{1}{\sqrt{1 + a^2}} r_M \tag{6}$$

$$y = a\left(x_1 \pm \frac{1}{\sqrt{1 + a^2}} r_M\right) + b \tag{7}$$

In the equation: $a$ is a straight line slope, $a = \frac{y_1 - y_2}{x_1 - x_2}$; $b$ is a straight line intercept, $b = y_1 - \frac{y_1 - y_2}{x_1 - x_2} x_1$.

In the Eqs. (6) and (7), if $x_1 > x_2$, that is, when the water injection well is located on the right side of the production well, it is negative, conversely, it is positive. Equations (2), (6), and (7) are substituted into Eq. (4); the pressure gradient of any point M on the mainstream line between injection and production wells is gained by derivation as follows:

$$|gradP| = \frac{5.787\mu}{\pi hk'}\left\{\frac{q_1}{r_M} - \frac{q_2}{(R_{12} - r_M)} - \sum_{i=3}^{n}\frac{q_i\left(\left|x_1 \pm \frac{1}{\sqrt{1+a^2}}r_M - x_i\right| + \left|a^2x_1 \pm \frac{a^2}{\sqrt{1+a^2}}r_M + ab - ay_i\right|\right)}{\sqrt{1+a^2}\left[\left(x_1 \pm \frac{1}{\sqrt{1+a^2}}r_M - x_i\right)^2 + \left(a\left(x_1 \pm \frac{1}{\sqrt{1+a^2}}r_M\right) + b - y_i\right)^2\right]}\right\} \tag{8}$$

The pressure gradient distribution on the main stream of water injection well and production well can be calculated according to Eq. (8).

Equation (5) is substituted into Eq. (8); the pressure gradient at any point on the mainstream line can be obtained. Combined with nonlinear flow definition, the model of nonlinear flow can be obtained:

$$\lambda_{\min} < \frac{(P_{\inf} - P_{wf})}{(q_1+q_2)\ln\frac{R_{21}}{r_w} + \sum_{i=3}^{n} q_i \ln\frac{R_{i1}}{R_{i2}}} \left\{\frac{q_1}{r_M} - \frac{q_2}{(R_{12} - r_M)} - \sum_{i=3}^{n}\frac{q_i\left(\left|x_1 \pm \frac{1}{\sqrt{1+a^2}}r_M - x_i\right| + \left|a^2x_1 \pm \frac{a^2}{\sqrt{1+a^2}}r_M + ab - ay_i\right|\right)}{\sqrt{1+a^2}\left[\left(x_1 \pm \frac{1}{\sqrt{1+a^2}}r_M - x_i\right)^2 + \left(a\left(x_1 \pm \frac{1}{\sqrt{1+a^2}}r_M\right) + b - y_i\right)^2\right]}\right\} < \lambda_{\max} \tag{9}$$

In the equation: $\lambda_{\min}$ is maximum starting pressure gradient, MPa/m. $\lambda_{\max}$ is maximum starting pressure gradient, MPa/.

### 2.3 Determination Process of Nonlinear Flow Zone

The process of determining the nonlinear flow zone can be divided into three steps:

The first is to determine the pressure gradient between injection and production wells according to the analytical model of nonlinear flow; the second is to determine the maximum and minimum starting pressure gradient in the laboratory, or to determine it based on the regression between permeability and experimental data. The third is the deployment of virtual production wells with a production rate of 0 in fault boundaries or areas with low well control, the calculation of the cross-well pressure gradient is calculated, the main aim is to increase data point, and the determination of nonlinear flow region in plane can be coupled at last.

## 3 Oil Field Application

### 3.1 Overview of the Block

T14 block is located in the east of A oil field in Hai Ta Basin. A semi-enclosed fault block reservoir with east, north, and west sides blocked by faults and open to the south side. The fault is a reverse normal fault, and the reservoir is mainly developed in

T formation. The fault block is 0.82 km in length and 0.75 km in width. The formation dip angle is 11.0°, the reservoir permeability is $8.5 \times 10^{-3}$ μm$^2$, and the formation crude oil viscosity is 2.7 mPa s. Block T14 has two well groups A56-54 and T236-216, which are separated by D4 fault and are in low water cut stage (Table 1).

### 3.2 Variation Characteristics of Pressure Gradient in Mainstream Line

Firstly, the maximum starting pressure gradient of T14 block is 0.06 MPa/m, the minimum starting pressure gradient of T14 block is 0.003 MPa/m, which are measured by laboratory experiments, so the pressure gradient range of nonlinear flow zone is 0.003–0.06 MPa/m. Furthermore, according to the method in Sect. 2.2, the pressure gradient distribution between wells in block T14 is determined (Fig. 2). The pressure gradient distribution in the mainstream line between injection and production wells has the following two aspects:

Firstly, the high value of pressure gradient between wells is mainly concentrated in a very small area near oil and water wells. The maximum pressure gradient is mainly within 50 m of injection wells, and the average value is 0.337 MPa/m; the next is within 20 m near the oil well, and the average value is 0.230 MPa/m. The pressure gradient in the rest region of the wells is small, and the average value is 0.034 MPa/m (Table 2).

Secondly, the pressure gradient is obviously affected by well spacing, and the minimum pressure gradient is biased toward the oil production well side. The greater the well spacing becomes, the lower the pressure gradient becomes, and it has more difficulty to put the reservoir into production. For example, the injection well spacing of A56-50 well is maximum and reaches 288 m, which leads to the lowest pressure gradient at the same relative position between oil and water wells. The minimum pressure gradient between the wells is reached at 190 m from the injection well, and the average value is 0.0151 MPa/m. However, this point is not located in the middle of oil and water wells, but is skewed to oil wells and distributed unsymmetrically.

### 3.3 Nonlinear Flow Plane Distribution

The pressure gradient of two typical well groups A56-54 and T236-216 in T14 block has been calculated. In order to determine the pressure gradient of the well-less control area near the fault, five additional virtual wells are increased. The range of nonlinear flow zone is determined by coupling on the plane (Fig. 3). The main flow zone between oil and water wells is nonlinear flow, and the area ratio is 60.1%. Following the law of nonlinear flow, the flow velocity is small, which is the main reason for the rapid decline of the fault block (the initial annual natural decline rate is 17.6%). The next step is to improve the development effect by well pattern encryption. The quasilinear flow zone is very low, mainly within 50 m around the injection well and 20 m around the oil well, and the controlled area ratio is 4.7%. The flow in this area follows the law of quasilinear flow, and the flow velocity is great. In the area of the back of oil–water well line, the pressure gradient is usually lower than the minimum starting pressure gradient, it is mainly dead oil area, and the area ratio is 35.2%. The crude oil in the region is difficult to flow, it is the remaining oil-rich region, and a digging potential well can be deployed near the fault.

**Table 1.** The basic information of T14 block

| Well group | Well number | Well type | Bottom hole pressure/Mpa | Water injection rate or production of fluid (t/d) | Water cut (%) | Distance between injection well and production well (m) | Distance between minimum pressure gradient point and injection well (m) | Apparent mobility (mD/mPa s) | Average perforation thickness (m) | Minimum starting pressure gradient (MPa/m) | Maximum starting pressure gradient (MPa/m) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A56-54 | A56-54 | Injection well | 31.2 | 32.0 | | | | | 26.9 | 0.003 | 0.060 |
| | T14 | Production well | 3.8 | 7.0 | 7.1 | 208 | 150 | 0.930 | | | |
| | A56-52 | production well | 2.0 | 8.8 | 6.0 | 189 | 120 | 1.004 | | | |
| T236-216 | T216 | Injection well | 31.3 | 42.0 | | | | | 36.8 | 0.003 | 0.060 |
| | A52-50 | Production well | 2.1 | 4.6 | 3.3 | 201 | 120 | 1.397 | | | |
| | A56-50 | Production well | 6.7 | 12.3 | 1.2 | 288 | 190 | 1.319 | | | |

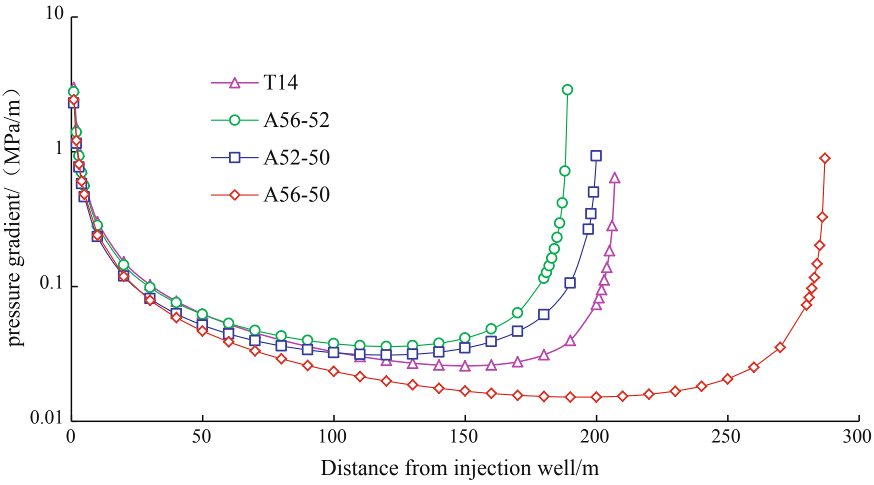


**Fig. 2.** The distribution of pressure gradient of T14 block

**Table 2.** The cross-well fluidity and pressure gradient of T14 block

| Well group | Well number | Bottom hole flow pressure (MPa) | Bottom flow pressure of water injection well (MPa) | Cross-well apparent permeability to viscosity ratio (mD/mPa s) | Average pressure gradient (MPa/m) | | |
|---|---|---|---|---|---|---|---|
| | | | | | Near water injection well $r_M \leq 50$ m | Remaining regions $50 < r_M < R - 20$ m | Near production well $r_M \geq R - 20$ m |
| A56-54 | T14 | 3.8 | 31.2 | 0.930 | 0.384 | 0.034 | 0.195 |
| | A56-52 | 2.0 | | 1.004 | 0.359 | 0.042 | 0.252 |
| T236-216 | A52-50 | 4.9 | 31.3 | 1.397 | 0.297 | 0.037 | 0.258 |
| | A56-50 | 1.3 | | 1.319 | 0.307 | 0.021 | 0.216 |
| Average | | | | 1.163 | 0.337 | 0.034 | 0.230 |

*Remark R* is injection-production spacing, m
$r_M$ is the distance from the well, m

## 4 Conclusion

(1) According to the superposition principle of potential, a nonlinear analytical model of flow in ultra-low-permeability reservoirs is established, which is under the condition of simultaneous production of multiple wells with unequal production rate. According to the production dynamic data, the pressure gradient distribution on the mainstream line between production wells is determined. By adding virtual

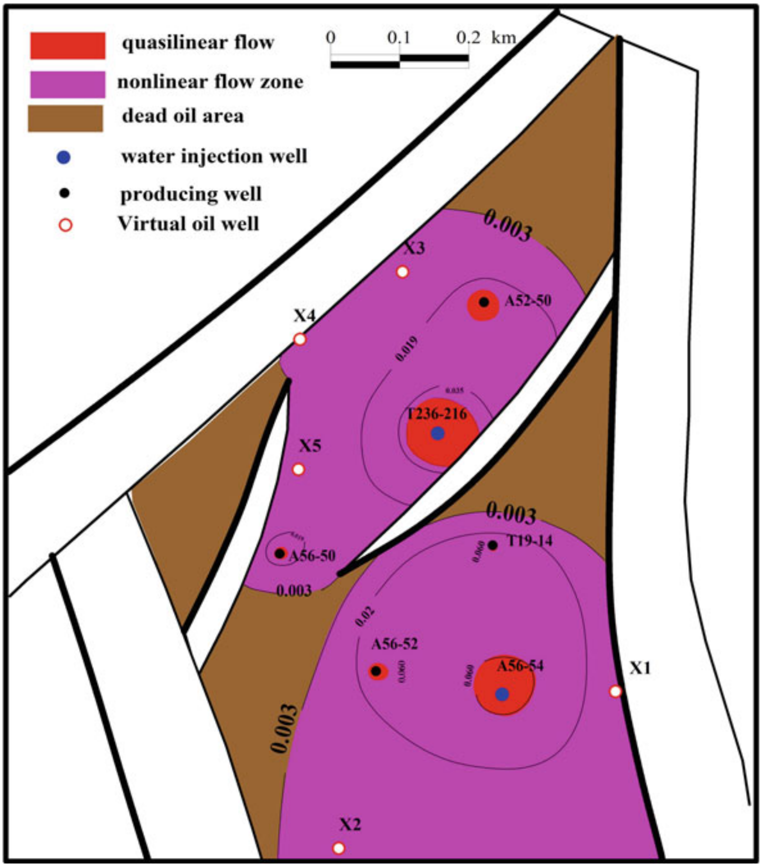


**Fig. 3.** The distribution of nonlinear flow zone

wells, the prediction accuracy of pressure gradient near faults and areas with low well control degree can be improved. Finally, the nonlinear flow area can be determined by coupling the pressure gradient of each point on the plane.

(2) The new method has been applied in the field of T14 block. There are three aspects about the pressure gradient and nonlinear flow characteristics. Firstly, the high value of the inter-well pressure gradient is mainly concentrated in the small area near the oil–water well; secondly, the pressure gradient is obviously affected by the well spacing, and its lowest value is inclined to one side of the oil well; thirdly, the nonlinear flow zone in ultra-low-permeability reservoirs is the main reason for the rapid decline of production.

**Acknowledgements.** The authors thank Li Bin at Hailar oilfield for providing the data of T14 block.

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