

Chapter 2

Fertility Dynamics with Family Bargaining



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2.1 Introduction

Statistical discrimination might lead to shorter and less frequent labor market participation of women than would otherwise be measured.¹ Such is apparently the case especially in Japan, as emphasized by Kawaguchi (2008) and Yamaguchi (2017). It is statistically plausible that women are likely to withdraw from the labor market when they marry or have children. Therefore, women tend to be given only light and easy work and thereby earn a lower wage income. When capital accumulation is low, physical labor is considerably important for goods production. Men have more physical strength than women. Therefore, the male wage rate is high. Although biological differences exist between women and men, it has been recognized that statistical gender discrimination might not be a necessary consequence of biological gender differences.

Discrimination is asserted only as a consequence of a self-fulfilling prophecy, as argued by Kawaguchi (2008) and Yamaguchi (2017). Consequently, unlike men, women must bear disproportionately heavy burdens of housework, child-bearing, and child-rearing at home even while working in the labor market. These circumstances motivate women and men to negotiate their family decisions such as family labor supply, childbearing, and child-rearing. This paper presents analyses of the effects of family bargaining on fertility dynamics along with economic development. It has been acknowledged that the persistently declining fertility rate has turned upward at

¹Schwab (1986) defines statistical discrimination as an employer's action according to a stereotype such as women quit more frequently than men. Although early arguments presented in the literature (e.g., Arrow 1972) implicitly assume that statistical discrimination must be efficient in producing output, Schwab (1986) shows that they might be inefficient. This paper does not address issues of efficiency.

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higher income levels in economically developed countries since around 2000 (Apps and Rees 2004; Day 2012).

In a pioneering paper, Galor and Weil (1996) report an analysis of fertility and childrearing portioning between genders in a neoclassical growth model, assuming biological gender differences. Day (2016) develops the model to present the possibility of a fertility rebound in a Galor and Weil (1996) type model by incorporating market childcare services.² In contrast to these works in unitary models, Kemnitz and Thum (2015) present an analysis of the effects of gender wage gaps on fertility in a collective model of family decisions. However, they do not consider the dynamics. Yakita (2018a) reports results of an analysis of fertility dynamics along with economic development in a unitary model without family bargaining, assuming the production of market childcare services following Day (2012). As described herein, the analyses of fertility dynamics described by Yakita (2018a) are extended with consideration of family bargaining in a collective model.

Setting up of the model follows the pattern of a collective model of Komura (2013), who assumes heterogeneous preferences for having children. The strong bargaining power of a spouse with a stronger preference for rearing children is expected to raise the fertility rate. If the bargaining power depends on the relative wage rate and if the wage rate depends on capital accumulation, then economic development powered by capital accumulation influences the bargaining power distribution, which consequently affects fertility dynamics.

The main results are the following. As the relative female wage rate rises along with economic development, the fertility rate rebounds in spite of the increased opportunity cost of child-rearing time at home if women have strong preferences for having children. Efficient provision of market childcare services reinforces the upward dynamics of fertility. It also helps alleviate gender discrimination.

The structure of the paper is the following. The next section sets up a growth model. Section 2.3 presents the model dynamics. The last section concludes the paper.

2.2 Model

This paper presents consideration of a collective model of family used by Komura (2013), who emphasizes gender-based difference in preferences for having children. Couples might choose between rearing their children at home and purchasing market childcare services. An underlying assumption for this paper is that, because of existing statistical discrimination, only women rear their children at home. Men work fulltime and inelastically in the labor market, although women choose to allocate their time between child rearing at home and market labor.

²Apps and Rees (2004) and Hirazawa and Yakita (2009) also assume the availability of childcare outside the home.

For analytical purposes, child rearing at home and market childcare services are assumed to be perfect substitutes. This assumption is useful to clarify the effects of market childcare availability on family decision making. Market childcare service production uses only female labor with material goods inputs, as described by Day (2016) and Yakita (2018a). Market childcare services are provided as club goods such as kindergartens or childcare centers, where a childcare worker can care for more than one child at a time.

Goods production technology can be represented as a production function with capital, mental labor, and physical labor with constant returns to scale, as described by Galor and Weil (1996).

2.2.1 Households

In this model³, a family comprises a woman and a man, i.e., a couple. Individuals live for two periods: first a young working-period and then an old retired-period. The length of lifetime is certain. The length of each period is normalized to one. To avoid the matching issue in marriage, equal members of women and men are assumed. In addition, a couple is assumed to rear the same number of girls and boys. Each couple has a common utility over consumption and having children.

The utility function of a couple in period t is⁴

$$u_t = [\gamma^f \theta + \gamma^m (1 - \theta)] \ln n_t + \rho \ln c_{t+1}^2, \quad (2.1)$$

where c_{t+1} represents second-period family consumption and n_t denotes the number of girl-boy pairs.⁵ $\rho \in (0, 1)$ stands for the time preference factor. $\gamma^i > 0$ is the utility weight of person i on having children. The relative magnitude of the utility weight of women might be greater than, equal to, or less than that of men.⁶ Also, $\theta = \theta(w^f/w^m)$ denotes the bargaining power of a women in a family, where w_t^f and w_t^m respectively represent the wage rates of women and men.⁷ An important assumption is that $\theta(0) = 0$, $\theta(1) = 1/2$ and $\theta'(w^f/w^m) > 0$ for $0 < w^f/w^m \leq 1$.⁸

³This section is attributable to Kemnitz and Thum (2015).

⁴Komura (2013) assumes a similar family utility function.

⁵For analytical simplicity, first-period consumption of the family is omitted here. The period of childhood is not addressed explicitly in this paper.

⁶Actually, the ideal numbers of children of women and men vary among countries. The OECD Family Database (<http://www.oecd.org/social/family/database.htm>. Accessed 12 January 2017) reported that the average personal ideal numbers of children for women and men 15 years old and older in the mid-2000s were, respectively, 2.58 and 2.50 in France, 2.48 and 2.48 in Japan, 2.42 and 2.45 in UK, 1.96 and 2.17 in Germany, and 2.12 and 2.04 in Italy.

⁷For the problem to be economically meaningful, $[\gamma^f \theta + \gamma^m (1 - \theta)] < \rho$ is assumed.

⁸Bargaining power can instead be assumed to depend on the relative wage income. Kemnitz and Thum (2015) use the relative wage income.

Letting P_t be the price of external childcare, the first-period budget constraint of the couple can be written as

$$w_t^m + (1 - \tilde{l}_t)w_t^f = P_t x_t + s_t, \quad (2.2)$$

where \tilde{l}_t denotes the (internal) child-rearing time of a wife at home, x_t represents the amount of external childcare purchased, and s_t denotes savings for their retirement. The second-period budget constraint is

$$r_{t+1}s_t = c_{t+1}^2, \quad (2.3)$$

where r_{t+1} stands for the gross interest rate in period $t + 1$. Letting $1/\varphi$ be a required time input to rear a pair of children, the total time input necessary to rear n_t pairs of children is given as $n_t/\varphi = \tilde{l}_t + x_t$, where $0 \leq \tilde{l}_t \leq 1$ and $x_t \geq 0$. The assumption of perfect substitutability is not made merely for simplicity but also for clarifying the role of the availability of external childcare services outside the home. This formulation enables us to consider a situation in which parents can choose either internal or external childcare services.⁹

Deconstruction of the utility maximization problem confronted by a couple must be done to two steps: cost minimization of child bearing and utility maximization. The cost of rearing n_t pairs of children is given as $C = w_t^f \tilde{l}_t + P_t x_t = (w_t^f - P_t)\tilde{l}_t + P_t(n_t/\varphi)$. By minimizing the cost for rearing n_t pairs of children subject to time constraint $0 \leq \tilde{l}_t \leq 1$, one can obtain the following cost function:

$$C(n_t) = \begin{cases} (i) \frac{P_t n_t}{\varphi} & \text{when } w_t^f > P_t \\ (ii) \frac{n_t}{\varphi} w_t^f & \text{when } w_t^f \leq P_t \text{ and } \frac{n_t}{\varphi} < 1 \\ (iii) w_t^f + P_t(\frac{n_t}{\varphi} - 1) & \text{when } w_t^f \leq P_t \text{ and } \frac{n_t}{\varphi} \geq 1 \end{cases} \quad (2.4)$$

In case (i), the couple chooses $\tilde{l}_t = 0$ and purchases $x_t = n_t/\varphi$ of external childcare to have n_t pairs of children. The wife supplies fulltime labor to the market. In case (ii), the wife allocates the time endowment between internal childcare $0 < \tilde{l}_t = n_t/\varphi < 1$ and market labor $0 < 1 - \tilde{l}_t < 1$. In case (iii), the couple chooses to rear n_t pairs of children at home; the wife does not supply labor to the market, i.e., $\tilde{l}_t = 1$. In cases (ii) and (iii), they are unwilling to purchase external childcare, i.e., $x_t = 0$.

As the second step, the couple chooses the number of children n_t and consumption c_{t+1} to maximize family utility subject to the following intertemporal budget constraint:

$$w_t^m + w_t^f = \frac{c_{t+1}}{r_{t+1}} + C(n_t). \quad (2.5)$$

The first-order conditions for utility maximization are

⁹Kemnitz and Thum (2015) assume the same formulation. Under a logarithmic utility function, however, it is infeasible for utility maximization to have $\tilde{l}_t = 0$ and $x_t = 0$ simultaneously.

$$\rho/c_{t+1} = \lambda_t/r_{t+1} \text{ and } \bar{\gamma}/n_t = \lambda_t C'(n_t) \quad (2.6)$$

and budget constraint (2.5), where $\bar{\gamma} \equiv \gamma^f \theta + \gamma^m (1 - \theta)$. Using cost function (2.4), and from (2.5) and (2.6), one obtains the following solutions. In case (i) in which $w_t^f > P_t$, one obtains

$$n_t = \frac{\bar{\gamma}\varphi}{\bar{\gamma} + \rho} \frac{w_t^f + w_t^m}{P_t} \text{ and } s_t = \frac{\rho}{\bar{\gamma} + \rho} (w_t^f + w_t^m) \quad (2.7)$$

For case (ii) in which $w_t^f \leq P_t$ and $n_t/\varphi < 1$, we have

$$n_t = \frac{\bar{\gamma}\varphi}{\bar{\gamma} + \rho} \frac{w_t^f + w_t^m}{w_t^f} \text{ and } s_t = \frac{\rho}{\bar{\gamma} + \rho} (w_t^f + w_t^m). \quad (2.8)$$

For optimal plan (2.8) to be consistent with condition $n_t/\varphi < 1$, one must have $w_t^f/w_t^m > \bar{\gamma}/\rho$. For case (iii), in which $w_t^f \leq P_t$ and $n_t/\varphi \geq 1$, if the optimal plans are interior solutions to the problem, then we obtain $n_t = \frac{\bar{\gamma}\varphi}{\bar{\gamma} + \rho} \frac{P_t + w_t^m}{P_t}$. However, for n_t in the optimization condition to be consistent with condition $n_t/\varphi \geq 1$, one must have $P_t/w_t^m \leq \bar{\gamma}/\rho$. Therefore, $w_t^f/w_t^m \leq \bar{\gamma}/\rho$ because $w_t^f \leq P_t$. Couples are not willing to purchase external childcare in case (iii). Therefore, a corner solution is obtained¹⁰:

$$n_t = \varphi \text{ and } s_t = w_t^m. \quad (2.9)$$

2.2.2 Goods Production Sector

The consumption goods production technology is assumed to be given by the following constant-returns-to-scale production function.¹¹

$$Y_t = F(K_t, L_t) + bL_t^m, \quad (2.10)$$

where Y_t represents aggregate output, K_t signifies aggregate physical capital, L_t denotes aggregate labor, and L_t^m expresses aggregate male labor in period t . $F(K, L)$ is a linear-homogeneous function in physical capital K and labor L ; and $b > 0$ is the constant marginal productivity of male physical labor. Capital accumulation presumably increases labor productivity: $F_{LK} > 0$. Aggregate labor L_t is the sum of non-physical labor of female and male workers, $L_t = L_t^m + L_t^{fY}$, where L_t^{fY} denotes the female non-physical labor used in the goods production sector.

¹⁰Here, $w_t^m/P_t < (\bar{\gamma} + \rho)/\bar{\gamma}$ is assumed.

¹¹The production function is assumed by Galor and Weil (1996) and by Kimura and Yasui (2010).

Letting N_t be the number of couples in period t , then the production function can be rewritten in percouple terms as

$$Y_t/N_t = F(k_t, 1 + l_t^{fY}) + b, \quad (2.11)$$

where $k_t = K_t/N_t$, $L_t^m/N_t = 1$ and $l_t^{fY} = L_t^{fY}/N_t$. The zero-profit conditions are

$$F_K(k_t, 1 + l_t^{fY}) = r_{t+1}, \quad (2.12)$$

$$F_L(k_t, 1 + l_t^{fY}) = w_t^f, \text{ and } F_L(k_t, 1 + l_t^{fY}) + b = w_t^m. \quad (2.13)$$

The gender wage gap is given as $(w_t^m - w_t^f)/w_t^f = b/F_L(k_t, 1 + l_t^{fY})$. It is noteworthy that even if women do not supply market labor as in case (iii), i.e., even when $l_t^{fY} = 0$, then the *potential* female wage rate can be given as the marginal product.

2.2.3 Childcare Production Sector

The production of childcare presumably requires goods input $B > 0$ per unit of childcare output and also that it requires female labor. The goods inputs include equipment and facilities for child rearing such as childcare centers and kindergartens. Therefore, it can be assumed to depend on the level of childcare service output. As the number of children cared for in the sector increases, the degree to which the input of goods must care for them also increases. The childcare production technology can be written as $X_t = \mu L_t^{fX}$, where X_t represents the aggregate product of childcare, L_t^{fX} stands for the labor employed, and μ stands for the labor productivity in the sector. Presumably, $\mu > 1$ because each childcare worker is expected to care for more than one child at a time.¹² This assumption means that the childcare services are a kind of club good. Goods input B is necessary to maintain the club goods property for the childcare production output size.

Profit of the childcare production sector is given as

$$\pi_t^X = P_t X_t - w_t^f L_t^{fX} - B X_t, \quad (2.14)$$

where $B X_t$ stands for the total goods cost. The zero-profit condition in this sector can be written as

$$P_t \mu - w_t^f - B \mu = 0. \quad (2.15)$$

¹²Japanese national standards for the number of children per childcare worker at childcare centers are about three for children under age one and about six for children of 1 and 2 years old.

Therefore, $P_t \stackrel{\geq}{<} w_t^f$ as $\frac{\mu B}{\mu-1} \stackrel{\geq}{<} w_t^f$. When the female wage rate is sufficiently low relative to the per-unit goods cost, the price of external childcare is higher than the female wage rate, and *vice versa*. It is noteworthy that the services will not be produced in cases (ii) and (iii) because external childcare services are not demanded.

2.3 Dynamics

2.3.1 Market Equilibrium

The dynamics of the system is examined for each of three cases, as determined by the capital market equilibrium

$$K_{t+1} = s_t N_t, \quad (2.16)$$

where s_t is given as (2.7), (2.8), and (2.9) for each case.

The other important market is the labor market. Assuming that men are employed full time, the equilibrium condition is explainable by female labor. In case (i), because both spouses of each couple supply full-time labor to the market and purchase external childcare, one has $L_t^f = L_t^m = N_t$ and $\tilde{l}_t = 0$, where L_t^f denotes the female labor force. Couples do not purchase external childcare in cases (ii) and (iii). Therefore, $L_t^{fX} = 0$. Wives supply market part-time labor in case (ii), although they do not supply market labor in case (iii). Therefore,

$$L_t^f = N_t = L_t^{fY} + L_t^{fX}, \text{ and } L_t^{fY}, L_t^{fX} > 0 \text{ in case (i),} \quad (2.17a)$$

$$L_t^f = (1 - \tilde{l}_t)N_t = L_t^{fY} \text{ in case (ii), and} \quad (2.17b)$$

$$L_t^f = 0 \text{ in case (iii).} \quad (2.17c)$$

The aggregate labor used in goods production is $L_t = L_t^m + L_t^{fY}$. The amount of labor employed in external childcare production is L_t^{fX} .

Finally, the equilibrium condition in the childcare market is $X_t = n_t N_t$ in case (i).¹³ Childcare services production is linear in female labor. Therefore, the supply of childcare is determined to be equal to the demand, as expressed by the right-hand of the equilibrium condition.

An examination of the dynamics of each case is presented below.

Case (i) From (2.7), (2.13), (2.15), and the definition of the bargaining power,

¹³In case (i), from (2.7), and (2.13), one has $n_t = n(l_t^{fY}; k_t)$. The equilibrium condition in the external child-care market can be rewritten as $n_t = \mu(1 - l_t^{fY})$ using the labor market equilibrium condition (2.17a). Therefore, one can obtain $l_t^{fY} = l^{fY}(k_t)$. Consequently, variables in period t , w_t^f , w_t^m , r_{t+1} , P_t , l_t^{fY} , l_t^{fX} , Y_t , X_t , n_t , and s_t , are determined for a given k_t .

$$n_t = \frac{\bar{\gamma}\varphi}{\bar{\gamma} + \rho} \frac{2F_L(k_t, 2 - n_t/\mu\varphi) + b}{F_L(k_t, 2 - n_t/\mu\varphi)/\mu + B}, \quad (2.18)$$

from which one obtains $n_t = n(k_t)$. Differentiating (2.18) with respect to k_t yields

$$\frac{dn_t}{dk_t} = \frac{[2 - \frac{\bar{\gamma}+\rho}{\bar{\gamma}\varphi\mu}n_t + (\gamma^f - \gamma^m) \frac{b+2F_L}{(b+F_L)^2} \frac{\rho b\theta'}{\rho+\bar{\gamma}}] F_{LK}}{\frac{\bar{\gamma}+\rho}{\bar{\gamma}\varphi} (\frac{F_L}{\mu} + B) + \frac{F_{LL}}{\varphi\mu} [2 - \frac{\bar{\gamma}+\rho}{\bar{\gamma}\varphi\mu}n_t + (\gamma^f - \gamma^m) \frac{b+2F_L}{(b+F_L)^2} \frac{\rho b\theta'}{\rho+\bar{\gamma}}]}. \quad (2.19)$$

The sign of dn_t/dk_t depends on the signs of $F_L + F_{LL}(2 - \frac{n_t}{\varphi\mu})$, $2 - \frac{(\bar{\gamma}+\rho)n_t}{\bar{\gamma}\varphi\mu}$, and $(\gamma^f - \gamma^m) \frac{b+2F_L}{(b+F_L)^2} \frac{\rho b\theta'}{\rho+\bar{\gamma}}$.¹⁴ One can demonstrate that $F_L + F_{LL}(2 - \frac{n_t}{\varphi\mu}) > 0$ holds if $F(K, L)$ is the CES function. Using (2.15) and $w_t^m = w_t^f + b$, one can obtain $\partial n_t / \partial w_t^f = P_t [2 - \frac{(\bar{\gamma}+\rho)n_t}{\bar{\gamma}\varphi\mu}] > 0$ because children are normal goods. It seems plausible in this case that parents consider price changes when purchasing external childcare services. Unless γ^f is too small to make the numerator of the right-hand side of (2.19) negative, one can plausibly obtain $dn_t/dk_t > 0$, as reported by Yakita (2018a). If $\gamma^f > \gamma^m$ ($\gamma^f < \gamma^m$), then $dn_t/dk_t > 0$ is greater (smaller) than in the case of $\gamma^f = \gamma^m$.¹⁵

The equilibrium condition in the capital market can be written as

$$n(k_t)k_{t+1} = \frac{\rho}{\bar{\gamma} + \rho} [2F_L(k_t, 2 - \frac{n_t}{\varphi\mu}) + b], \quad (2.20)$$

from which one obtains

$$\frac{dk_{t+1}}{dk_t} = \frac{\rho}{\bar{\gamma}\varphi\mu} [1 - (\gamma^f - \gamma^m) \frac{(F_L + \mu B)b\theta'}{(F_L + b)^2}] (F_{LK} - F_{LL} \frac{1}{\varphi\mu} \frac{dn_t}{dk_t}), \quad (2.21)$$

and

$$\frac{dw_t^f}{dk_t} = (F_{LK} - F_{LL} \frac{1}{\varphi\mu} \frac{dn_t}{dk_t}) > 0. \quad (2.22)$$

Although $dk_{t+1}/dk_t > 0$ if $\gamma^f \leq \gamma^m$, the sign of the right-hand side of (2.21) cannot be determined *a priori* if $\gamma^f > \gamma^m$. For analytical convenience, the effect of the change in the bargaining power on the fertility rate is assumed to be not so great as to hamper capital accumulation, i.e., $dk_{t+1}/dk_t > 0$ even when $\gamma^f > \gamma^m$.

Lemma 2.1 *When women supply all their time to the labor market and couples purchase market childcare, wage rates and the number of children increase as capital accumulates if the women's utility weight of having children is greater than that of men.*

¹⁴Presumably, the denominator of the right-hand side of (2.19) is positive for the continuity of n_t with respect to k_t .

¹⁵One might have $dn_t/dk_t < 0$ if γ^f is sufficiently small relative to γ^m , but the denominator of the right-hand side of (2.19) remains positive. Nevertheless, such a case is implausible.

Case (ii) From (2.13), n_t in (2.8) can be rewritten as

$$n_t = \frac{\bar{\gamma}\varphi}{\bar{\gamma} + \rho} \left[2 + \frac{b}{F_L(k_t, 2 - n_t/\varphi)} \right], \quad (2.23)$$

from which one can obtain $n_t = n(k_t)$. One can then readily demonstrate that

$$\frac{dn_t}{dk_t} = \frac{-[1 - (\gamma^f - \gamma^m) \frac{F_L \rho \theta' (2F_L + b)}{\bar{\gamma}(\rho + \bar{\gamma})(F_L + b)^2}] \varphi F_{LK}}{\frac{\bar{\gamma} + \rho}{\bar{\gamma}} \frac{(F_L)^2}{b} - [1 - (\gamma^f - \gamma^m) \frac{F_L \rho \theta' (2F_L + b)}{\bar{\gamma}(\rho + \bar{\gamma})(F_L + b)^2}] F_{LL}}. \quad (2.24)$$

In case (ii), wives allocate their time between internal child-rearing ($\tilde{l}_t = n_t/\varphi < 1$) and market (part-time) labor ($1 - \tilde{l}_t$). Because couples do not demand external child-care, one obtains $l_t^{fY} = 1 - \tilde{l}_t = 1 - n(k_t)/\varphi$, where capital accumulation demands more female labor in goods production, i.e., $dl_t^{fY}/dk_t > 0$.

The equilibrium condition in the capital market is given as

$$k_{t+1} = \frac{\rho}{\bar{\gamma}\varphi} F_L[k_t, 1 + l^{fY}(k_t)], \quad (2.25)$$

from which one can obtain

$$\frac{dk_{t+1}}{dk_t} = \frac{\rho}{\bar{\gamma}\varphi} \left[1 - \frac{\gamma^f - \gamma^m}{\bar{\gamma}} \frac{b F_L \theta'}{(F_L + b)^2} \right] \frac{dw_t^f}{dk_t} \quad (2.26)$$

and

$$\begin{aligned} \frac{dw_t^f}{dk_t} &= F_{LK} + F_{LL} \frac{dl_t^{fY}}{dk_t} = F_{LK} + F_{LL} \left(-\frac{dn_t}{dk_t} \right) \\ &= \frac{\frac{\bar{\gamma} + \rho}{\bar{\gamma}} \frac{(F_L)^2}{b} F_{LK}}{\frac{\bar{\gamma} + \rho}{\bar{\gamma}} \frac{(F_L)^2}{b} - [1 - (\gamma^f - \gamma^m) \frac{F_L \rho \theta' (2F_L + b)}{\bar{\gamma}(\rho + \bar{\gamma})(F_L + b)^2}] F_{LL}} > 0. \end{aligned} \quad (2.27)$$

The denominator of the right-hand side of (2.27) is assumed to be positive for the continuity of the wage rate with respect to k_t . For analytical purposes, $dk_{t+1}/dk_t > 0$ is assumed, as in the previous case.

If $\gamma^f \leq \gamma^m$, then we have $dn_t/dk_t < 0$ from (2.24). Conversely, if $\gamma^f > \gamma^m$, then

$$1 - \frac{\gamma^f - \gamma^m}{\bar{\gamma}} \frac{F_L \theta'}{(F_L + b)^2} b \geq 1 - \frac{\gamma^f - \gamma^m}{\bar{\gamma}} \frac{F_L \theta'}{(F_L + b)^2} \frac{\rho}{\rho + \bar{\gamma}} (2F_L + b) \quad (2.28)$$

because one can safely assume that $s_t (= \rho(2F_L + b)/(\rho + \bar{\gamma})) \geq b$. If the right-hand side of (2.28) is positive, then $dn_t/dk_t < 0$ and $dk_{t+1}/dk_t > 0$.¹⁶ However, if

¹⁶The denominator on the right-hand side of (2.24) is assumed for the continuity of n_t with respect to k_t , as in the previous case.

savings are sufficiently great or if the wage rate is sufficiently high that the right-hand side is negative, then one might have $dn_t/dk_t > 0$ and $dk_{t+1}/dk_t > 0$. As capital accumulates, the wage rates become higher (see (2.27)). Thereby, $dn_t/dk_t > 0$ might hold.

Lemma 2.2 *When women partly supply their time to the labor market and women care for a child at home, (a) the wage rates increase but the number of children decreases as capital accumulates if the women's utility weight of having children is smaller than that of men; and (b) the wage rates and the fertility rate might increase as capital accumulates if the women's utility weight related to having children is greater than that of men.*

Case (iii) From (2.9) and (2.13),

$$k_{t+1} = \frac{1}{\varphi} [F_L(k_t, 1) + b], \quad (2.29)$$

from which one can obtain

$$\frac{dk_{t+1}}{dk_t} = \frac{1}{\varphi} F_{LK} > 0, \quad (2.30)$$

and¹⁷

$$\frac{dw_t^f}{dk_t} = F_{LK} > 0. \quad (2.31)$$

Lemma 2.3 *The time paths of variables are independent of the utility weights that spouses assign to having children.*

2.3.2 Dynamics of the Development Path

As explained in this section, the time paths of the wage rates and the fertility rate are examined, assuming that the initial per-couple capital stock is sufficiently small. For expositional purposes, it is assumed that the economy does not fall into a trap of long-term equilibrium in case (ii) or case (iii). Moreover, a gender wage gap is assumed to exist, satisfying $\frac{b\bar{\gamma}(\mu-1)}{\mu(\rho-\bar{\gamma})} < B$ for given utility weights γ^f and γ^m .¹⁸ If the inequality of the assumption holds strictly, then some range of the female wage

¹⁷Although no women work in the market, the potential female wage rate is given as the marginal product of labor in the goods production sector.

¹⁸If this assumption is not satisfied, then the fertility rate might not decrease with the female wage rate. However, economically developed and even economically developing countries have sustained decreased fertility during recent decades. Therefore, this assumption is apparently not only plausible, but realistic.

rate exists, within which women choose to rear children at home rather than incur the high costs of external childcare.

Presuming first that the initial female wage rate is sufficiently low to satisfy $w_t^f/w_t^m \leq \bar{\gamma}/\rho$ or, equivalently, $w_t^f \leq \bar{\gamma}b/(\rho - \bar{\gamma})$, then the economy is in case (iii) in which the fertility rate is constant i.e., $n_t = \varphi$. The female wage rate is too low for women to supply labor to the market. If the marginal utility of having children is still high at $\tilde{l}_t = 1$, i.e., if the women are in the corner solution, then the couple foregoes the utility that might derive from having more children.

From (2.30) and (2.31), it is readily apparent that capital accumulates and the female wage rate rises as time passes. Increases in the female wage rate narrow the gender wage gap, thereby increasing the bargaining power of women. If the female utility weight related to having children is lower than that of men, i.e., if $\gamma^f \leq \gamma^m$, then the reduced gender wage gap moves the economy from case (iii) to case (ii) in which $w_t^f/w_t^m > \bar{\gamma}/\rho$. However, it is not necessarily the case that $\gamma^f > \gamma^m$. Although the economy moves to case (ii) as time passes if $1 > (\gamma^f - \gamma^m)\theta'/\rho$, the economy stays in case (iii) if $1 \leq (\gamma^f - \gamma^m)\theta'/\rho$.¹⁹ When $1 < (\gamma^f - \gamma^m)\theta'/\rho$, $\bar{\gamma}/\rho$ becomes greater than the gender wage gap w_t^f/w_t^m at $w_t^f/w_t^m = \bar{\gamma}/\rho$ but the female wage rate rises concomitantly with capital accumulation.²⁰

When $1 > (\gamma^f - \gamma^m)\theta'/\rho$ at $w_t^f/w_t^m = \bar{\gamma}/\rho$, the female wage rate becomes sufficiently high to satisfy $w_t^f/w_t^m > \bar{\gamma}/\rho$ or, equivalently, $w_t^f > \bar{\gamma}b/(\rho - \bar{\gamma})$. Then, the economy goes into the phase of case (ii). At $w_t^f(k_t) = b\bar{\gamma}/(\rho - \bar{\gamma})$, one has $n_t = \varphi$ and $k_{t+1} = \rho b/[\varphi(\rho - \bar{\gamma})]$. Therefore, the paths of n_t and k_{t+1} are continuous in k_t at $w_t^f(k_t) = b\bar{\gamma}/(\rho - \bar{\gamma})$ if $1 > (\gamma^f - \gamma^m)\theta'/\rho$.

In the phase of case (ii), if the utility weight of women on having children is not so great, then the fertility rate decreases along with capital accumulation (see (2.24)). Having too many children raises the opportunity cost of rearing children at home. Therefore, couples reduce the number of children until the marginal utility of having children equals the opportunity cost. Women increase the labor supply to the market, consequently reducing the internal child-rearing time at home. The increased female wage income increases per-couple capital, which in turn increases the wage rate, as might be apparent from (2.26) and (2.27). However, unless the female wage rate becomes higher than the price of external childcare, couples are not willing to purchase external childcare.

By contrast, if the women's utility weight assigned to having children is sufficiently great, then one cannot rule out the possibility of a positive relation between fertility and capital accumulation. The fertility rate might rise as the economy accumulates capital and wage rates increase. In such a case, although the opportunity cost of child-rearing time at home increases, women prefer to have more children and therefore increase the internal child rearing time without purchasing market childcare services. The intuition underlying this result is the income effect. Therefore,

¹⁹In this case, the long-term per-couple capital stock is given as $k_{(iii)} = (1/\varphi)[F_L(k_{(iii)}, 1) + b]$, where $F_L(k_{(iii)}, 1) < b\bar{\gamma}/(\rho - \bar{\gamma})$ and the fertility rate is $n_{(iii)} = \varphi$.

²⁰In this case, one cannot rule out the possibility that the economy might move to case (i) while skipping case (ii).

whether the fertility rate decreases in case (ii) depends on the relative utility weight that women assign to having children. Capital accumulation raises the wage rates of both women and men throughout the phase of case (ii).

After the female wage rate becomes sufficiently high to satisfy $w_t^f \geq \mu B/(\mu - 1)$, the economy moves into the phase of case (i), in which $w_t^f \geq P_t$.²¹ Substituting $w_t^f(k_t) = \mu B/(\mu - 1)$ into (2.18), (2.20), (2.23), and (2.25), the time paths of n_t and k_{t+1} can be shown to be continuous in k_t at $w_t^f(k_t) = \mu B/(\mu - 1)$, at which per couple capital level, the values of the fertility rate and per couple capital stock are $n_t = \bar{\gamma}\varphi[2 + b(\mu - 1)/(\mu B)]/(\bar{\gamma} + \rho)$ and $k_{t+1} = \rho\mu B/[\bar{\gamma}\varphi(\mu - 1)]$ for both cases (ii) and (i). Therefore, the time paths are also continuous in k_t at $w_t^f(k_t) = \mu B/(\mu - 1)$. At this wage rate, women start to supply fulltime labor to the labor market while purchasing external childcare to rear their children. The female market labor is allocated between goods production and the external childcare production sector to equate the female wage rates in both sectors, although mothers rear their children at home immediately before the wage rate satisfies $w_t^f = P_t$.

The fertility rate increases with the per-couple capital stock, i.e., $dn_t/dk_t > 0$, although the magnitude of the effect of capital accumulation on fertility depends on the relative utility weights of women and men on having children (see (2.19)). The greater the women's utility weight is, the greater the effect of capital accumulation on fertility becomes.²² The female wage rate increases with the per-couple capital stock (see (2.22)). Therefore, the fertility rate rises through the income effect as the female wage rate increases. As might be readily apparent from (2.15), the price of external childcare becomes lower relative to the female wage rate as the female wage rate increases. For that reason, couples can afford to purchase more external childcare as the female wage rate rises.

Because per-couple capital stock and the female wage rate remain constant in the long-term equilibrium, the fertility rate also becomes constant in the long term, as might be apparent from (2.20). If the stability condition is satisfied, then one has the long-term per-couple capital satisfying²³

$$k = \frac{\rho}{\bar{\gamma}\varphi\mu} \{F_L[k, 1 + l^{JY}(k)] + \mu B\}, \quad (2.32)$$

and the long-term fertility rate satisfying

$$n = \frac{\bar{\gamma}\varphi}{\bar{\gamma} + \rho} \frac{2F_L(k, 2 - n/\mu\varphi) + b}{F_L(k, 2 - n/\mu\varphi)/\mu + B}. \quad (2.33)$$

²¹The possibility cannot be ruled out, *a priori*, that the economy is trapped in a lower long-term equilibrium in case (ii), in which the per-couple capital stock is given as $k_{(ii)} = (\rho/\bar{\gamma}\varphi)F_L[k_{(ii)}, 1 + l^{JY}(k_{(ii)})]$, where $F_L[k_{(ii)}, 1 + l^{JY}(k_{(ii)})] < \mu B/(\mu - 1)$. In this case, the economy does not have a fertility rebound.

²²The possibility that the effect is negative when the women's utility weight is too small relative to that of men cannot be ruled out theoretically. Nevertheless, it is apparently implausible.

²³The stability condition is given as $dk_{t+1}/dk_t < 1$.

The gender wage gap remains even in the long term, but the gap is smaller when the long-term percouple capital stock is greater.

From (2.32) and (2.33), one obtains^{24, 25}

$$\frac{dn}{d\gamma^f} \underset{<}{\overset{\geq}{=}} 0, \quad \frac{dk}{d\gamma^f} < 0, \quad \text{and} \quad \frac{dw^f}{d\gamma^f} = F_{LK} \frac{dk}{d\gamma^f} - \frac{F_{LL}}{\bar{\gamma}\varphi} \frac{dn}{d\gamma^f} \underset{<}{\overset{\geq}{=}} 0. \quad (2.34)$$

Although the long-term fertility rate might be higher than, equal to, or lower than φ , a higher women's utility weight assigned to having children is not necessarily associated with a higher fertility rate. A high utility weight of women lowers the per-couple capital stock. The intuition underlying these results is that having more children increases the cost of external child rearing, which in turn reduces capital formation and the savings of couples. The lower wage rate deriving from the reduced capital stock might discourage couples from having more children.

Theorem 2.1 (a) *If the preference of women for having children is weaker than that of men, then the fertility rate turns upward when couples purchase market childcare at a price that is lower than the female wage rate. By contrast, (b) if the preference of women for having children is sufficiently stronger than that of men, then a fertility rebound might occur even when mothers care for their children at home.*

The time path of the fertility rate is presented in Fig. 2.1, where the horizontal line shows the level of per-couple capital stock.²⁶ The path of the fertility rate shows a reverse-J-shape or U-shape in the female wage rate (and per-couple capital stock).²⁷ It is noteworthy that if the utility weight assigned by women to having children is sufficiently great, then the fertility rebound might occur even when couples do not purchase market childcare services (Fig. 2.1b). The wage income of couples increases as capital accumulates. The increased income might induce couples to increase the number of children and spend more of the mothers' time on child-rearing at home, thereby reducing the female market labor supply through the income effect. With the increased bargaining power of women, the income effect on having children overwhelms the negative effects of the increased opportunity cost. This case contrasts against the result obtained with a unitary model such as that presented by Yakita (2018a), in which the fertility rate declines steadily in the phase of case (ii).

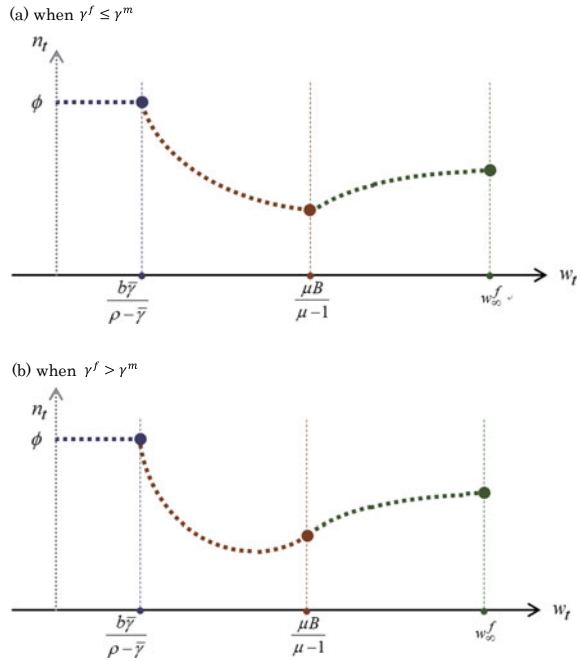
²⁴See Appendix.

²⁵Presumably, (2.21) has a positive sign.

²⁶For these figures, a gender wage gap satisfying $[b\bar{\gamma}(\mu - 1)]/[\mu(\rho - \bar{\gamma})] < B$, i.e., case (ii) is assumed to exist.

²⁷Figure 1 presents the case in which mothers care for their children at home in case (ii), i.e., $\tilde{l}_t < 1$, even when the female wage rate approaches $w_t^f(k_t) = \mu B/(\mu - 1)$. If $\tilde{l}_t = 1$ at $w_t^f(k_t) = \mu B/(\mu - 1)$, then the time path might have a flat part correspondingly.

Fig. 2.1 Change in the fertility rate



2.4 Conclusion

A collective model was used to demonstrate that changes in relative bargaining power of women and men occurring along with economic development strongly influence fertility dynamics. Bargaining power is assumed to depend on the gender wage gap, which derives from labor productivity differences between women and men. A fertility rise can occur when couples start to purchase market childcare services and substitute it for internal child-rearing by mothers at home. This result has been reported in the literature (Kemnitz and Thum 2015; Yakita 2018a), but it is noteworthy that another possibility of fertility rebound can be presented.

The fertility rebound might occur even without couples' purchasing market childcare services if the women's utility weight related to having children is sufficiently high. The increased wage income induces couples to have more children and to reduce the mothers' labor supply to the market, which reflects the high preference of mothers for having children by increasing the bargaining power of women. Assuming the existence of a long-term equilibrium, one can also show that a higher preference of women for having children does not necessarily raise and possibly lowers long-term fertility as a consequence of family bargaining, *ceteris paribus*. These results are novel. Moreover, they contrast against the result reported by Kemnitz and Thum (2015) that the family bargaining systematically biases fertility downward.

Results of the present analyses also imply that the gender wage *income* gap stems only from biological differences between genders in the long term. Efficient provision of market childcare might eliminate, or at least alleviate, statistical discrimination between genders in the long term, with both parents increasingly bearing the burdens of housework and child-bearing as equally as they can.²⁸

Neither human capital formation nor the parents' preference for the quality of children has been considered. Parents might spend their income on human capital investment in children as well as childcare services.²⁹ Simultaneous consideration of the respective effects of preferences for the quantity and quality on the fertility dynamics remains as a subject for future study.

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Appendix

Effects of the female utility weight related to long-term fertility and the wage rate

From (2.32) and (2.33),

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} dk/d\gamma^f \\ dn/d\gamma^f \end{pmatrix} = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}, \quad (2.35)$$

where

$$\begin{aligned} a_{11} &= 1 - \frac{\rho}{\varphi\mu\bar{\gamma}} \left[1 - \frac{(2F_L + b)b\theta'}{(F_L + b)^2} (\gamma^f - \gamma^m) \right] F_{LK} \\ a_{12} &= \frac{\rho}{\bar{\gamma}\rho\varphi} \left[1 - \frac{(2F_L + b)b\theta'}{(F_L + b)^2} (\gamma^f - \gamma^m) \right] \frac{F_{LL}}{\mu\varphi} \\ a_{21} &= - \left[2 - \frac{\bar{\gamma} + \rho}{\bar{\gamma}\varphi\mu} + \frac{(2F_L + b)\rho b\theta'}{(\bar{\gamma} + \rho)(F_L + b)} (\gamma^f - \gamma^m) \right] F_{LK}, \\ a_{22} &= \frac{\bar{\gamma} + \rho}{\bar{\gamma}\varphi\mu} (F_L + \mu B) + \left[2 - \frac{\bar{\gamma} + \rho}{\bar{\gamma}\varphi\mu} + \frac{(2F_L + b)\rho b\theta'}{(\bar{\gamma} + \rho)(F_L + b)} (\gamma^f - \gamma^m) \right] \frac{F_{LL}}{\varphi\mu} \\ g_1 &= - \frac{\rho}{\bar{\gamma}^2\varphi\mu} (F_L + \mu B)\theta < 0, \\ g_2 &= \frac{\rho\varphi\mu}{(\bar{\gamma} + \rho)^2} \frac{2F_L + b}{F_L + \mu B} \theta > 0. \end{aligned}$$

²⁸The Human Development Report by United Nations (<http://hdr.undp.org/en/data>. Accessed 21 December 2017) shows a negative relation between the Human Development Index and the Gender Inequality Index during 2000–2015.

²⁹Yakita (2018b) presents an analysis of the consequences of family Nash bargaining on fertility and human capital accumulation by considering both the quantity and quality of children.

From (2.35) one can obtain

$$\frac{dn}{d\gamma^f} = H^{-1}(a_{21}g_1 - a_{11}g_2)$$

and

$$\frac{dk}{d\gamma^f} = H^{-1}(a_{22}g_1 - a_{12}g_2) \quad (2.36)$$

where $H = a_{11}a_{22} - a_{12}a_{21} > 0$ from the stability condition, $dk_{t+1}/dk_t < 1$. From the assumption of $0 < dk_{t+1}/dk_t$, it follows that $a_{12} > 0$. When $dn_t/dk_t > 0$, $a_{21} < 0$. a_{22} is the denominator on the right-hand side of (2.19), which can be assumed to be positive for the continuity of the solution with respect to $\gamma^f - \gamma^m$. Presumably, $\partial k_{t+1}/\partial k_t > 0$, which is plausible.

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