A Comparison of 1D and 2D Spatial Variability in Probabilistic Slope Stability Analysis

Rubi Chakraborty and Arindam Dey

Abstract Slope stability analysis is a highly challenging task in geotechnical engineering as the influence of uncertainty involved in geotechnical properties on failure behavior of slopes is inevitable. Traditional deterministic slope stability approach, based on a single factor of safety (FoS) parameter, cannot explicitly encounter the uncertainties involved in geotechnical properties and failure mechanism, leading to erroneous results of slope stability. Hence, slope stability practice is highly persuadable to probabilistic treatment, which allows quantification of the uncertainty and rationally integrating the same into the analysis. The present study investigates the influence of inherent spatial variation of soil domain in probabilistic slope stability analysis. To accomplish this, a hypothetical slope is analyzed, considering 1D spatial variation, with the aid of GeoStudio 2007, using Morgenstern-Price limit equilibrium method (LEM) coupled with Monte Carlo simulation (MCS). The results are compared with those of Griffiths et al. (2007), wherein 2D random field for soil shear strength was considered and the analysis was carried out with the help of random finite element method (RFEM). The influence of correlation lengths on the probabilities of failure is compared. The results reveal that the probability of failure highly depends on spatial variation of soil property in both the methods. When correlation length is small, the failure probability is essentially zero; failure probability increases rapidly for intermediate correlation lengths, and for large correlation lengths, the failure probability becomes constant. It is also found that combining LEM with one-dimensional random field gives lower probabilities of failure than RFEM, as RFEM is more efficient in simulating the field uncertainty. Moreover, RFEM can search and identify the weakest path through the soil domain for the failure to occur, whereas LEM presumes a predefined failure plane.

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1 Introduction

Several issues of uncertainty in slope stability analysis originate mainly due to geological anomalies, inherent spatial variability of soil properties, lack of data availability, uncertainty in potential failure surface, simplifying approximations adopted in geotechnical modeling as well as human errors in design and construction. Soil structure is the result of a complex combination of various geological, environmental, and physical–chemical processes. Soil heterogeneity is classified into two main classes: lithological heterogeneity (described as the presence of a different lithology within a uniform soil domain or one soil layer overlying another layer of soil) and inherent spatial soil variability (variation of soil properties from one point to another in space within one layer). The latter uncertainty originates due to different depositional conditions and loading histories (Elkateb et al. 2003, Lloret-Cabot et al. 2014). A one-dimensional random field showing the cone tip resistance measured during a cone penetration test (CPT) is shown in Fig. 1 as an example of spatial variation of soil. Among all the uncertainties, soil heterogeneity plays a crucial role in assessment of failure mechanism of slope and this paper mainly focuses on the influence of inherent spatial variability of failure probability of a soil slope. The conventional way to tackle with spatial variability in geotechnical engineering was to depend upon high safety factor, experience, and engineering judgement. However, Morgenstern (2000) reported several case histories where relying entirely on high safety factor lead to poor predictions in 70% of the cases. Therefore, it has been realized that more reliable tools to incorporate geotechnical uncertainty are necessary. In order to incorporate uncertainty in geotechnical practice, limit equilibrium analysis in combination with Monte Carlo simulation (MCS) technique was introduced. Later on, the stochastic finite element method was introduced in combination with MCS to incorporate soil spatial variability into a numerical analysis.

The contemporary studies on spatial variability in geotechnical engineering mainly focuses on the inherent soil variability within a homogeneous soil layer (Griffiths and Fenton 2000; Griffiths et al. 2004; Ji et al. 2011; Hicks et al. 2014; Li et al. 2014; Jiang et al. 2014; Li et al. 2015; Jamshidi and Alaie 2015). Ji et al. (2011) observed that failure probability can be overestimated when spatial variation of soil is not considered; i.e., a slope will be predicted comparatively safer when spatial variability is ignored. Allahverdizadeh et al. (2015) stated that there exists a critical correlation length value which results in an unconservative design due to underestimation of failure probability. This critical spatial correlation length resulting in maximum failure probability was shown to $0.5H$ to H, where H is the height of the slope. Other researchers (e.g., Griffiths and Fenton 2000) have also reported this critical spatial correlation length. Chakraborty and Dey (2017) showed that there exists a threshold correlation length beyond which the probability of failure

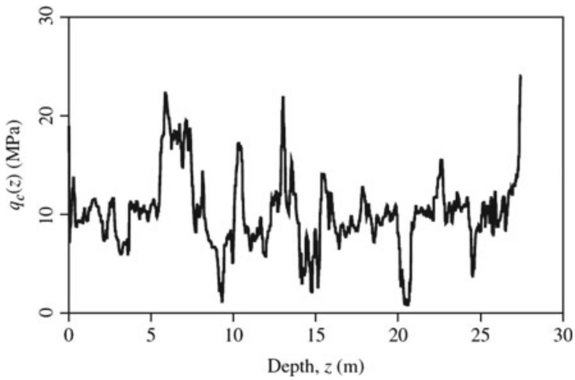

Fig. 1 Variation of cone tip resistance (q_c) with depth using a cone penetrometer (Griffiths and Fenton 2007)

is underestimated by conventional probabilistic analysis without considering spatial variation. In this paper, a hypothetical slope is analyzed, considering 1D spatial variation of soil properties, with the aid of GeoStudio 2007. The Morgenstern-Price limit equilibrium method (LEM) is coupled with MCS technique to assess the probability of slope failure for different spatial variabilities of soil domain. The results are compared with those of Griffiths et al. (2007), where 2D random field for soil shear strength was considered and analyzed with the help of random finite element method (RFEM). The probabilities of failure for different correlation lengths are compared. It is found that combining LEM with one-dimensional random field underestimates the probability of failure in comparison with that obtained from RFEM.

2 LEM-Based Probabilistic Approach Considering 1D Spatial Variation

A 2(H):1(V) slope having 60 m length and 15 m height (Fig. 2) is analyzed considering 1D spatial variation with the aid of GeoStudio 2007, using Morgenstern-Price limit equilibrium method (LEM) coupled with Monte Carlo simulation (MCS). The mean cohesion (c) of 5 kPa and angle of internal friction (ϕ) of 20° are considered with a standard deviation of 1.5 kPa and 6.22°, respectively. A correlation coefficient of 0.5 is considered between c and ϕ. The soil properties are taken as per Griffiths et al. (2007) for further comparison. In a probabilistic approach, soil properties are considered to be random variables and the failure probability of (P_f) is computed. To execute this, the Monte Carlo simulation (MCS) method is generally preferred for its simplicity and ease of computation. MCS simply generates a series of trial values of the random variables and evaluates the *limit state function* for each trial value. A *limit state function,* generally denoted by $g(x)$, is a mathematical function that determines whether the system is in failure or not. The limit state function is universally (though arbitrarily) defined such that $g(x) < 0$ indicates a failed state and $g(x) > 0$ indicates

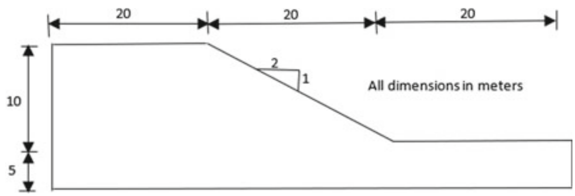

Fig. 2 Slope geometry considered in the present study

a safe state. The number of failures that occur among all the realizations of random variables is counted, and the probability of failure is approximated as

$$P_\mathrm{f} = \frac{n_\mathrm{f}}{N} \tag{1}$$

where n_f is the number of failures and N is the total number of realizations.

2.1 Basic Probability Principles

This section briefs the basic probability concepts required for the present study. A continuous random variable, for example, undrained shear strength, S, can be expressed by probability density function (PDF), usually denoted as $f_s(s)$. Normal or Gaussian distribution function is generally used in geotechnical problem. Figure 3a illustrates a PDF for normally distributed random variable undrained shear strength S. The equation for the normal probability density function is

$$f_s(s) = \frac{1}{\sqrt{2\pi\sigma_s}} \mathrm{e}^{-(s-\mu_s)/2\sigma_s^2} \tag{2}$$

where μ_s is the mean value of S, and σ_s is the standard deviation of s.

Normal distribution is most commonly used to characterize geotechnical properties, because the sum of random variables tends to be in normal distribution as per the central limit theorem. For example, it is a common practice to characterize the distribution of cohesion using the normal distribution, as the cohesive strength of soil is defined as the sum of electro-chemical interactions at its molecular level. However, the disadvantage of using normal distribution is that it allows negative values. A simple technique to avoid this limitation is to characterize geotechnical properties by a nonnegative distribution, such as lognormal distributions.

The cumulative distribution function is related to the probability density function (Fig. 3b), usually denoted as $F_s(s)$. The cumulative distribution function (CDF) gives the probability that the random variable S having a value less than s, i.e.,

$$P(S < s) = F_s(s) \tag{3}$$

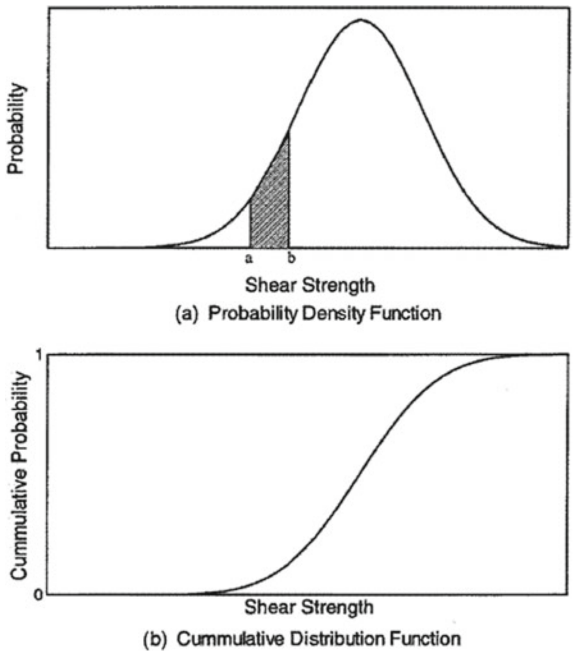

Fig. 3 **a**. Probability density function; **b**. cumulative density function (Kitch 1994)

While the PDF or CDF leads to a complete characterization of a random variable, it is convenient to have some simpler descriptors for random variables. The most obvious characteristic of a random variable is the mean, denoted by

$$\mu_s = \int_{-\propto}^{\propto} s f_s(\mathrm{s})\mathrm{d}s \tag{4}$$

The mean, also known as the expected value of a random variable, is denoted by $E[S]$. Further, the next most important characteristic is the variability of a random variable; i.e., how much does the value of the random variable differ from its mean. The measure of the average deviation is called the variance, σ_s^2, and is defined as

$$\sigma_s^2 = E[(s - \mu_s)^2] = \int_{-\propto}^{\propto} (s - \mu_s)^2 f_s(s)\mathrm{d}s \tag{5}$$

The square root of the variance is known as the standard deviation, denoted as σ_s, and is thus defined as $\sigma_s = \sqrt{\sigma_s^2}$.

While comparing two different random variables with different mean values, it is often required to normalize the standard deviation of a random variable by its mean value. This quantity is called the coefficient of variation, (COV) defined as

$$\mathrm{COV} = \frac{\sigma_s}{\mu_s} \tag{6}$$

Most geotechnical engineering problems are functions of more than one variable. For example, the drained shear strength of a soil is a function of both cohesion, c', and the friction angle, ϕ'. If the cohesion (c') and friction angle (ϕ') are both random variables, then the joint probability distribution should be considered for a particular range of soil cohesion and friction angle values. When two random variables are considered together, there is often a statistical relationship between them. The correlation between two random variables is measured by the covariance of the two variables. Given two random variables, X and Y, the covariance of X and Y, denoted as Cov $[X, Y]$, is defined as

$$\mathrm{Cov}[X, Y] = E[(X - \mu_X)(Y - \mu_Y)] \tag{7}$$

As with the variance, there is a normalized version of the covariance, which is called the correlation coefficient, ρ, and is defined as

$$\rho_{X,Y} = \frac{\mathrm{Cov}[X, Y]}{\sigma_X \sigma_Y} \tag{8}$$

where σ_X and σ_Y are standard deviations of random variable X and Y, respectively.

Therefore, based on the above discussion, in this study, a lognormal PDF is assigned for soil shear strength parameters (cohesion, c, and angle of internal friction,ϕ) with a correlation coefficient of 0.5 having a one-dimensional spatial variation. In this study, it is assumed that the soil property varies only in horizontal direction within the slope soil domain and the vertical spatial variation is ignored. To simulate 1D random field in SLOPE/W 2007, the soil shear strength properties (cohesion, c, and angle of internal friction,ϕ) are sampled at specified distances to evaluate the probabilities of failure for different correlation lengths. The pattern of spatial variability in soil property is aptly represented by the correlation distance or scale of fluctuation (Vanmarcke 1977), which is further described in detail in Sect. 2.2.

2.2 *Scale of Fluctuation or Correlation Length*

The scale of fluctuation (SOF) represents the spatial range over which the soil property shows a relatively strong correlation in space. An approximate method of estimating the SOF is presented by Vanmarcke (1977) as

$$\delta_v = 0.8\overline{d} \tag{9}$$

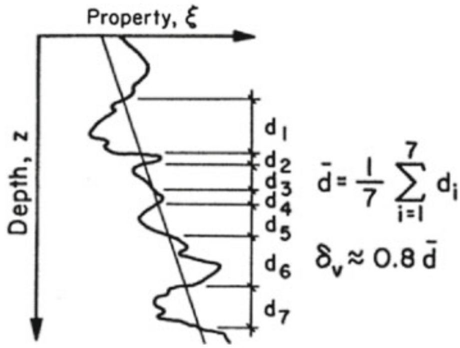

Fig. 4 Estimation of vertical scale of fluctuation (Vanmarcke 1977)

where δ_v is the vertical scale of fluctuation and $\overline{d}$ is the average distance between intersections of the fluctuating component with its trend line, as shown in Fig. 4.

A large correlation length reflects smooth variation of properties, whereas a small length means erratic variability as shown in Fig. 5. Estimation of correlation distance is illustrated in DeGroot (1996) as well as Lacasse and Nadim (1996). In this study to generate 1D random field in horizontal direction, soil properties are sampled for specified distances to estimate failure probability for different correlation lengths. When soil domain of 60 m length is sampled for specified distance, for example 25 m, there will be two complete sampling distances of 25 m and a partial sampling distance of 10 m. A partial sampling distance is considered to be correlated with the immediate preceding sampling section. The correlation coefficient between two soil sections can be estimated as follows (Vanmarcke 1983):

$$\rho(\Delta Z, \Delta Z^{'}) = \frac{Z_0^2\Gamma(Z_0) - Z_1^2\Gamma(Z_1) + Z_2^2\Gamma(Z_2) - Z_3^2\Gamma(Z_3)}{2\Delta Z\Delta Z[\Gamma(\Delta Z)\Gamma(\Delta Z^{'})]^{0.5}} \tag{10}$$

where

$\Delta Z, \Delta Z' =$ the length between two sections
$\mathrm{Z}_0 =$ the distance between the two sections,
$\mathrm{Z}_1 = \Delta Z + \mathrm{Z}_0$,
$\mathrm{Z}_2 = \Delta Z + \mathrm{Z}_0 + \Delta Z^{'}$
$\mathrm{Z}_3 = \Delta Z^{'} + \mathrm{Z}_0$ and
Γ is a dimensionless variance function and can be approximated as (Vanmarcke 1983):

$$\Gamma(Z) = 1.0 \text{ when } Z \le \delta \text{, and}$$
$$\Gamma(Z) = \frac{\delta}{Z} \text{ when } Z > \delta \tag{11}$$

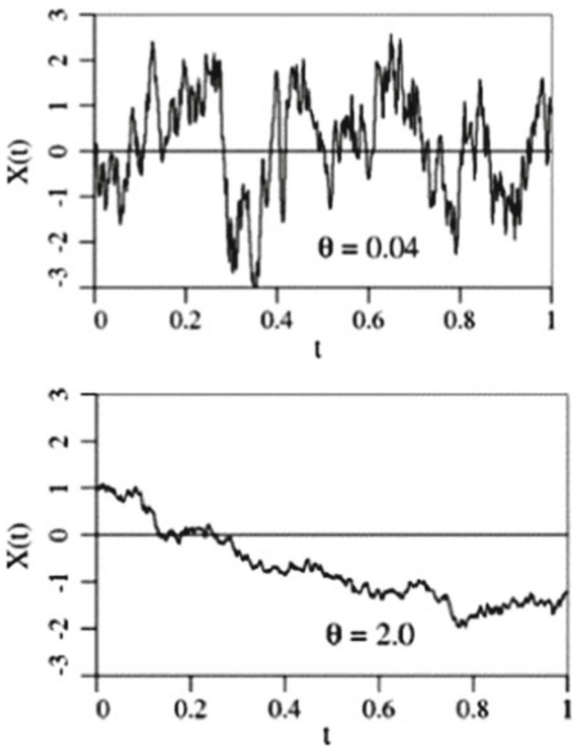

Fig. 5 Simulation of *X(t)* for different scales of fluctuation (Griffiths and Fenton 2007)

Here, δ is the scale of fluctuation. To incorporate spatial variation of soil property in SLOPE/W, the value of scale of fluctuation is decided by the user-defined desired sampling distance along the slip surface.

A legitimate criticism of these LEM-based probabilistic approaches, however, is that they are not capable to encounter spatial variation in the soil domain properly and are inextricably linking with conventional slope stability approaches. To overcome these disadvantages, a more realistic and advanced approach called the random finite element method (RFEM, Fenton and Griffiths 1993) is recommended, wherein the random field theory is combined with the deterministic finite element analysis, as described in Sect. 3.

3 Random Finite Element Method (RFEM)

A more advanced method of probabilistic analysis in geotechnical engineering for the incorporation of spatial variability of soil, called the 'random finite element method' (RFEM), (Fenton and Griffiths, 1993; Paice 1997; Griffiths and Fenton 2000), was developed in 1990s. RFEM is a completely different approach for failure probability

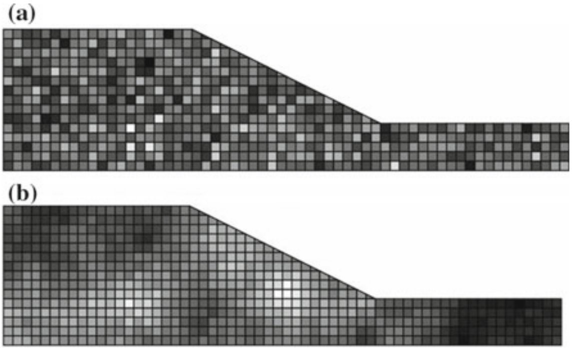

Fig. 6 A typical 2D random filed for slope with **a**. Small correlation length; **b** high correlation length (Griffiths et al. 2007)

prediction. In RFEM, a soil property is considered as a random variable at any location within a soil domain since the soil property is an uncertain quantity at every point of the soil domain. The pattern of variability of soil is characterized by the correlation distance or scale of fluctuation as described in Sect. 2.2. A typical two-dimensional random field with low and high correlation length is shown in Fig. 6.

Generally, all engineering properties are the result of local average of some sort. Uncertainty of the average shear strength along the slip surface is a more accurate measure of uncertainty than the strength at discreet locations. Because failure in slope is more probable when the average shear strength along the failure surface is less than the applied shear stress rather than because of the presence of some local weak zones. Therefore, geotechnical engineers are much more interested in studying the behavior of random field considering the average properties of the field, rather than the properties at discreet locations. Consider the moving local average defined as

$$X_T(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} X(\xi)\mathrm{d}\xi \tag{12}$$

where $X_T(t)$ is the local average of $X(t)$ over a window of width T centered at t (Fig. 7). The upper plot (in the figure) is averaged over the moving window length T to get the lower plot. Local averaging reduces the contribution of the high-frequency components and thus attenuates the variance. The variance of any property at their discrete points is always less than the variance spatially averaged over a particular domain. As the extent of the length (T, over which the soil property being averaged) increases, the corresponding variance decreases.

In RFEM, each random variable is characterized by a PDF. RFEM also correlates the random variables at adjoining locations. A joint probability distribution function characterizes the set of random variables at all locations within the soil domain, which are considered as a random field. RFEM combines nonlinear finite element methods with random field theory. It essentially overlays a random field upon a

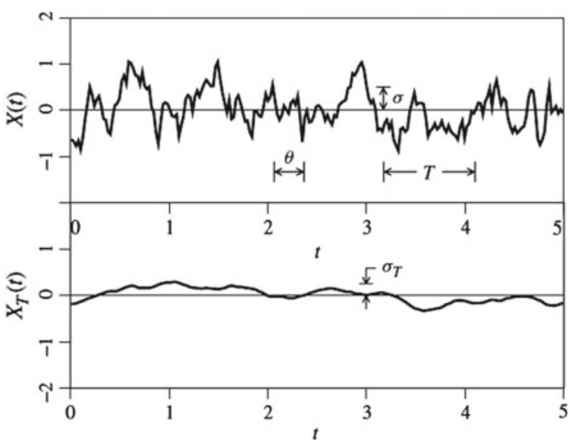

Fig. 7 Effect of local averaging on variance (Fenton and Griffiths 2008)

finite element mesh, resulting in each mesh behaving as a random variable. RFEM completely accounts for spatial correlation and averaging. It does not presume the shape or position of the critical slip surface of failure as it can identify the actual weakest failure path through the soil mass.

If the mean and covariance of a random field vary with position, mathematically it is difficult to estimate the governing joint PDF from the real data. Therefore, further simplifications of stationary or statistically homogeneous random field are required. A stationary random field signifies that the joint PDF governing the random field is invariant in space; i.e., the CDF, mean, and variance are constant at any location within the random field. Rather than their absolute locations within the random field, the covariance of two random variables depends only on their separation distance. Under the simplifying assumptions of stationary random field, a random field can be aptly characterized if the mean, variance, and their rate of variation in space are known. The rate of variation can be characterized by the second moment of the joint PDF governing the random field, equivalently using covariance function, variance function, or spectral density function.

4 Results and Discussions

In this study, the failure probabilities of the hypothetical slope considering 1D random field, based on traditional probabilistic approach, are compared with the probabilities of failure considering 2D random field (Griffiths et al. 2007) for different correlation lengths (Fig. 8). It is observed that combining LEM with one-dimensional random fields underestimates failure probabilities of slope structure as compared to RFEM. Because the 2D random field can simulate field uncertainty better than 1D spatial variation, hence increasing the probability of failure. Moreover, LEM method presumes the failure mechanism using deterministic methods (using Morgenstern-Price

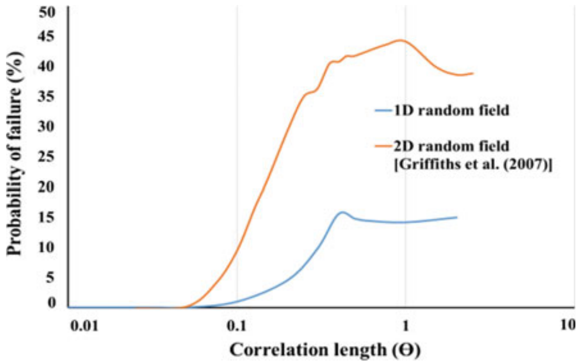

Fig. 8 Comparison of effect of 1D and 2D spatial variation of soil on the probability of failure

method in the present study), whereas the RFEM allows the failure surface to originate through the weakest path in the soil layers in a particular realization. When correlation length is small, the failure probability is essentially zero; failure probability increases rapidly for intermediate correlation lengths, and for large correlation lengths, the failure probability becomes constant. The local averaging gets maximized for a very small value of correlation length. Therefore, for small correlation length, the soil properties tend to take their mean values, and for each realization, the soil domain becomes essentially homogeneous. Considering the mean values of soil properties result in a safe slope (having FoS greater than unity), the failure probability is always zero. When the correlation length is very large, the soil for the entire domain becomes strongly correlated. Hence, within each realization, the slope becomes essentially homogeneous, but different from one realization to the next. Whereas, the slope is not homogeneous for intermediate values of correlation length. RFEM is able to search out the weakest path through the soil mass, and hence, the anomalies, such as locations of weak areas, govern the failure probability.

5 Conclusions

The paper highlights the contrast between the results obtained by traditional LEM-based probabilistic approach and a more advanced random finite element method (RFEM) for stability analysis of slope. Both the methods evaluate the failure probability of slope as opposed to the deterministic FoS measure of slope safety. It is noticed from the results that the probability of failure highly depends on spatial variation of soil property, as failure probability changes with different correlation lengths in both the methods. However, depending on the value of the correlation length, the methods show a significant difference in results. It is observed that the LEM probabilistic approach considering one-dimensional random fields underestimates the probabilities of failure in comparison with RFEM. The reason behind this is 2D random field can simulate field uncertainty better than 1D spatial variation, hence increasing the probability of failure. Moreover, LEM method presumes the failure mechanism by the means of deterministic methods, while the RFEM allows the failure plane to develop through the weakest path in the soil layers. Therefore, traditional LEM-based probabilistic approach leads to unconservative predictions of slope failure probability and may end up producing erroneous design solutions. Hence, the incorporation of spatial variation properly with the aid of more advanced techniques (RFEM) is recommended.

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