Fractional Order Sliding-Mode Controller for Quadcopter

Om Veer Dhakad and Vivek Kumar

Abstract This paper proposed the concept of fractional order sliding-mode control (FOSMC) for chattering reduction and reduced error converging time for Quadcopter. The controller is designed to control over the six degrees of freedom of the Quadcopter and enhances the stability as compared to PI-based control. In PI-based controllers the high transient overshoots deteriorate the stability of Quadcopter system. The FOSMC improves both transient and steady-state behavior of the Quadcopter motion. The FOSMC controller is highly robust controller as it rejects system uncertainties and disturbances drastically. FOSMC controller provides chattering free response as chattering is considered the main drawback in present conversional sliding-mode controls. Chattering is low amplitude noise present at high frequencies. For vertical take-off and landing (VTOL), an ERROR sliding surface is considered and then a mathematical analysis is carried out under the sliding-mode control law. The input unit step and ramp disturbances are considered to check the robustness of the controller. The Lyapunov-based stability criterion is used to check stability of the controller. The controller is tested by simulation in Simulink MATLAB. The FOSMC is not just gives better execution time with minimum control input, in addition, it shows strong, notwithstanding outer load unsettling influence and parameter varieties.

Keywords Fractional order sliding-mode control (FOSMC) · Advance control theory · Simulink MATLAB · Quadcopter (UAV)

1 Introduction

The control of Quadcopters or UAVs (unmanned aerial vehicles) has been a subject of significant intrigue over the previous decades. Be that as it may, the control out-

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line for a quadrotor UAV is not a basic assignment. Most importantly, as a turning wing flying machine and a characteristic nonlinear framework, the quadrotor is open-circle instable, which requires a quick control reaction and an expansive task go [1]. It has been broadly utilized as a part of an assortment of circumstances including reconnaissance, fire flighting, ecological observing, et cetera [2]. As a smaller scale helicopter, the quadrotor UAV pulls in extraordinary consideration from military and common applications because of its exceptional focal points, for example, straight-forward structure, vertical taking off and landing (VTOL), and quick moving [3]. Aside from the control of UAV, I&I is additionally connected in visual servoing, control of pendulum on truck, and numerous other mechatronic frameworks [4]. To control a small scale quadrotor UAV and beat the vulnerabilities related with the thrust and drag coefficients, Fujimoto et al. disentangled the dynamic model and built up a versatile controller by means of the I&I philosophy. In any case, just numerical recreation comes about are introduced in these works, and no test comes about with the I&I versatile outline are given, [5]. An I&I strategy is utilized to assess the obscure mass of a VTOL vehicle, and it ensures that the estimation focalizes to its actual esteem, [6]. This approach, not quite the same as the great versatile strategy, does not require the LP condition, nor does it summon assurance comparability. It likewise disentangles the solidness investigation by giving cross terms in the Lyapunov work [7]. As of late, another versatile control configuration utilizing the drenching and invariance (I&I) approach is the first master postured in [8] and after that further created by numerous scientists [4–7]. In any case, the exemplary versatile control strategy dependably requires a direct parameterization (LP) condition, and now and then, the controller's singularities will appear [9]. The asymptotic steadiness (AS) of the shut circle framework is demonstrated by means of a hypothesis of fell frameworks. To make up for the parametric vulnerabilities, the versatile nonlinear control technique is an appropriate decision and has been generally used [10]. The dynamic arrangement of the quadrotor is isolated into the inward circle and the external circle subsystem. Such a plan is not troublesome for usage, however, the strength of the shut circle framework cannot be effectively ensured. To defeat this downside, an inward and external circle-based flight controller is proposed in [11]. A novel vigorous back stepping-construct controller that is based with respect to an integral sliding-mode approach is proposed for an underactuated quadrotor. Despite the fact that the outline method of the back stepping plan is clear and the evidence of the strength is standardized, the control picks up are difficult for tuning [12]. In this way, lately, more nonlinear hearty control plans have been produced. With respect to the underactuated property, different control procedures have been created for the quadrotor and other under-actuated mechatronic frameworks, for example, a wheeled portable robot, a submerged vehicle, and overhead cranes [13, 14]. Second, the quadrotor UAV has six degrees of opportunity, yet it has just four control inputs accessible; this is known as an underactuated property, which additionally prompts solid couplings between the dynamic states. Third, the estimation of some parameters related to the dynamic model, for example, the inertial minutes and streamlined coefficients, cannot be estimated or gotten precisely. Besides, the quadrotor UAV is exceptionally delicate to outer aggravations, for example, twist blast because of its little size and weight. To take care of these issues, diverse control procedures have been created. Since the dynamic model of the quadrotor can be linearized, at that point some conventional straight control can be utilized to settle the quadrotor in a little range around the balance. While considering the nonlinearity and a vast task run, the execution of the straight controller might be corrupted strategies [15, 16]. A control plot utilizing RISE input and neural system feedforward is connected on a rotorcraft-based UAV, and the semi-global asymptotic following of the disposition and elevation states is demonstrated by a Lyapunov-based dependability investigation, [17]. The RISE criticism control can make up for outside aggravations and demonstrating vulnerabilities while ensuring semi-global asymptotic outcomes in the feeling of a persistent control input, [18]. A general sliding-mode control (SMC) is produced for a class of unverifiable under-actuated frameworks and after that used to settle a quadrotor helicopter. Be that as it may, a downside of the SMC is the babbling issue, which may deteriorate the control execution for functional usage. To accomplish a nonstop control methodology, a powerful necessary of the signum of the mistake (RISE) - based controller is first exhibited in our past work in [17] and additionally created by other researchers [18, 19].

In this paper, in light of the previously mentioned control difficulties for a quadcopter, a novel nonlinear hearty following control conspire is proposed. In the first place, to comprehend the under actuation and the couplings, the control framework is separated into two sections: the inward circle (state of mind circle) controller and the external circle (position circle) controller. Second, to meet the necessities for vigorousness and quick reaction of mentality control, the RISE technique is controlled the inward circle subsystem. At long last, so as to address the obscure parameters and decrease the trouble of control outline, an I&I-based versatile control plot is used for the external circle subsystem control plan. The proposed technique yields the principal I&I, RISE, and internal and external circle-based control comes about for the underactuated quadcopter UAV within the sight of parametric vulnerabilities and obscure outer unsettling influences. Specifically, in spite of the trouble of strength examination for the internal/external circle approach, the proposed controller ensures the asymptotic following the shut circle framework through the Lyapunov-based security investigation. Besides, to approve the execution of the proposed outline, the non-straight powerful versatile control configuration is checked by the ongoing investigations on equipment on the up and up recreation (HILS) quadcopter test bed. To our best learning, a couple of past works has shown ongoing exploratory outcomes for the I&I approach on a quadcopter UAV framework.

This paper is presented in the following sections after the introduction part as covered in Sect. 1, Sect. 2 describes the Quadcopter Dynamics, mathematical Controller Designing is brought in Sect. 3, Sect. 4 explains about the sliding-mode Fractional Calculation, the Results & Discussions is placed in Sect. 5 of the paper and finally Sect. 6 Concludes the finding of the proposed controller.

2 Quadcopter Dynamics

The quadcopter schematic is shown in Fig. 1. The thrusts produced by the four rotors are signified by $q_i(t)$, $i = 1, 2, 3, 4$, individually. Let $J = \{x_J, y_J, z_J\}$ speak to one side-hand idleness outline with z_J being the vertical heading toward the sky. The body settled casing, which is signified by $S = \{x_s, y_s, z_s\}$, is situated at the focal point of gravity of the airplane. The Euclidean position and Euler point of the Quadcopter regarding the casing $\{J\}$ are described by $\in(t) = [x(t), y(t), z(t)]^{\mathrm{T}} \in R^3$ and $\zeta(t) = [\varphi(t), \theta(t), \psi(t)]^{\mathrm{T}} \in R^3$ (Fig. 2).

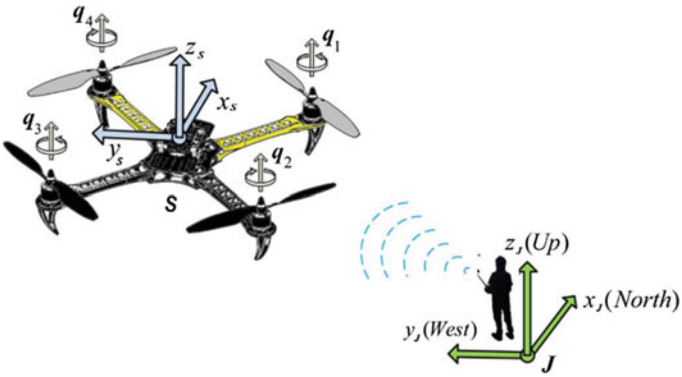

Fig. 1 Quadcopter schematic

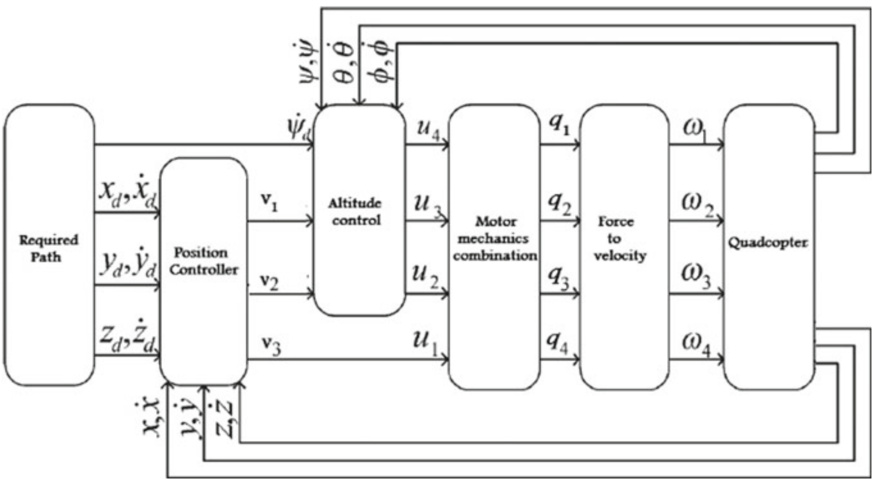

Fig. 2 Fractional controller block diagram

The dynamic model shown in $\{J\}$ is given as

$$
\begin{aligned}
J_1\ddot{\varphi} &= -C_4 l\dot{\varphi} + b_1(t) + l\tau_1 \\
J_2\ddot{\theta} &= -C_5 l\dot{\theta} + b_2(t) + l\tau_2 \\
J_3\ddot{\psi} &= -C_6\dot{\psi} + b_3(t) + k\tau_3 \\
m\ddot{x} &= -C_1\dot{x} + (\cos\psi \sin\theta \cos\varphi + \sin\psi \sin\varphi)u_t \\
m\ddot{y} &= -C_2\dot{y} + (\sin\psi \sin\theta \cos\varphi - \cos\psi \sin\varphi)u_t \\
m\ddot{z} &= -C_3\dot{z} - mg + (\cos\theta \cos\varphi)u_t
\end{aligned} \tag{1}
$$

where $m \in R^+$ signifies the mass, $J_i \in R^+$ for $i = 1, 2, 3$ are the MOI (Moments of Inertia), g is the acceleration due to gravity, $C_i \in R^+$ for $i = 1, \ldots, 6$ describes the aerodynamic damping coefficient, $l \in R^+$ indicates the separation between the epicenter of the Quadcopter and the rotor pivot, and $b_i(t) \in R$ for $i = 1, 2, 3$ speaks to the obscure time-fluctuating outer unsettling influences. In(1), $\tau_1(t)$, $\tau_2(t)$, $\tau_3(t)$ and $u_t(t)$ mean the aggregate thrust and three rotational powers delivered by the four rotors, and $k \in R^+$ shows a invariant force to moment factor.

3 Controller Design

Comment 1 The general control objective is to outline the control input signs to let the quadrotor track time-changing wanted directions $[x_d(t), y_d(t), z_d(t), \psi_d(t)]^{\mathrm{T}}$.

Comment 2 The framework parameters J_i (for $i = 1, 2$, and 3), C_j ($j = 1, \ldots, 6$), l, and k are positive unknown constants. The rotor thrusts $[q_1, q_2, q_3, q_4]^{\mathrm{T}}$ and the connection between the control input signals $[\tau_1, \tau_2, \tau_3, u_t]^{\mathrm{T}}$ is given by

$$
\begin{aligned}
&\tau_1 = -q_1 - q_2 + q_3 + q_4 \ \tau_2 = -q_1 + q_2 + q_3 - q_4 \\
&\tau_3 = -q_1 + q_2 - q_3 + q_4 \ u_t = q_1 + q_2 + q_3 + q_4
\end{aligned} \tag{2}
$$

The speed of rotation for each motor ω_i for $i = 1, 2, 3, 4$ can be calculated in view of the estimation of the lifting force q_i. Now simplifying the above mathematical dynamic model from Eq. (1) we get

$$
\begin{aligned}
&m\ddot{\in} + C_{\in}\dot{\in} + mge_3 = \mathrm{u}_t {}_S^J R e_3 \\
&J\ddot{\zeta} + C_\zeta\dot{\zeta} - b = \tau
\end{aligned} \tag{3}
$$

where $C_{\in} = \text{diagonal}\{C_1, C_2, C_3\} \in R^{3\times3}$, $e_3 = [0, 0, 1]^{\mathrm{T}} \in R^3$, $J = \text{diagonal}\{J_1/l, J_2/l, J_3/k\} \in R^{3\times3}$, $C_\zeta = \text{diagonal}\{C_4, C_5, C_6/k\} \in R^{3\times3}$, $b = [(b_1(t)/l), (b_2(t)/l), (b_3(t)/c)]^{\mathrm{T}} \in R^3$, $\tau = [\tau_1, \tau_2, \tau_3]^{\mathrm{T}} \in R^3$, ${}_S^J R(\eta) \in \mathrm{SO}(3)$ signifies the rotation matrix between the inertial frame $\{J\}$ and the body-fixed-frame $\{S\}$.

4 Fractional Calculations

As specified before, considering the sliding surface as follows:

$$s_1 = \dot{€} + D^{\alpha}\lambda_{\mathrm{RL}}\big(\mathrm{sign}(e_1)^r\big) \tag{4}$$

$$s_2 = \dot{\zeta} + D^{\alpha}\lambda_{\mathrm{RL}}\big(\mathrm{sign}(e_2)^r\big) \tag{5}$$

Assumptions

$$€_1 = \int € + €$$
$$€_1 = € - \dot{€}$$
$$\dot{€}_1 = \dot{€} - \ddot{€}$$

Similarly, $\dot{\zeta}_1 = \dot{\zeta} - \ddot{\zeta}$
Let, $\dot{s}_1 = -C_1\mathrm{sign}(s_1)$ and $\dot{s}_2 = -C_2\mathrm{sign}(s_2)$
Again $s_1 = C_J \int \dot{€} - C\dot{€}$ and $s_2 = C_J \int \dot{\zeta} - C\dot{\zeta}$
where $\mathrm{Sign}(p) = \begin{cases} -1, & p < 0 \\ 0, & p = 0 \\ 1, & p > 0 \end{cases}$, p is variable
Now from Eqs. (4), (5) and from above expressions

$$-C_1\mathrm{sign}(s_1) = \dot{s}_1 = C_J\dot{€} + \ddot{€}$$

And from Eq. (3) we get

$$-C_1\mathrm{sign}(s_1) = C_J\dot{€} - \frac{C_€\dot{€}}{m} - ge_3 + \frac{u_t{}_S^J Re_3}{m}$$
$$\text{Or, } -C_1\mathrm{sign}(s_1) = \left(C_J - \frac{C_€}{m}\right)\dot{€} - ge_3 + \frac{u_t{}_S^J Re_3}{m} \tag{6}$$

But $\dot{€} = -C_2\mathrm{sign}(s^1) - €$
Therefore, Eq. (6) becomes

$$-C_1\mathrm{sign}(s_1) = \left\{\left(C_J - \frac{C_€}{m}\right)\big(-C_2\mathrm{sign}(s^1) - €\big)\right\} - ge_3 + \frac{u_t{}_S^J Re_3}{m}$$

Now simplifying for $u_t{}_S^J$

$$u_t{}_S^J = \frac{\left[-\left\{\left(C_J - \frac{C_€}{m}\right)\big(-C_2\mathrm{sign}(s^1) - €\big)\right\} + ge_3 - C_1\mathrm{sign}(s_1)\right]Re_3^T}{m} \tag{7}$$

Now calculating the values of τ_1, τ_2 and τ_3, and from assumptions

$$-C_2 \text{sign}\,(s_2) = \dot{s}_2 = C_J \dot{\zeta} + \ddot{\zeta}$$

And from Eq. (3)

$$-C_2\,\text{sign}\,(s_2) = C_J \dot{\zeta} + \frac{\tau_1 + b_1 - C_\zeta \dot{\zeta}}{J_1}$$

$$\text{Or, } -C_2\,\text{sign}\,(s_2) = \left(\frac{C_J - C_\zeta}{J_1}\right)\dot{\zeta} + \frac{\tau_1 + b_1}{J_1} \tag{8}$$

But $\dot{\zeta} = -C_4\,\text{sign}(s^2) - \zeta$
Therefore, Eq. (8) becomes

$$-C_2\,\text{sign}(s_2) = \left(\frac{C_J - C_\zeta}{J_1}\right)\left(-C_4\,\text{sign}(s^2) - \zeta\right) + \frac{\tau_1 + b_1}{J_1}$$

Now simplifying for τ_1

$$\tau_1 = J_1\left[-C_2\,\text{sign}(s_2) - \left(\frac{C_J - C_\zeta}{J_1}\right)\left(-C_4\,\text{sign}(s^2) - \zeta\right)\right] - b_1 \tag{9}$$

Similarly,

$$\tau_2 = J_2[-C_2\,\text{sign}(s_2) - \left(\frac{C_J - C_\zeta}{J_2}\right)\left(-C_5\,\text{sign}(s^2) - \zeta\right)] - b_2 \tag{10}$$

$$\tau_3 = J_3[-C_2\,\text{sign}(s_2) - \left(\frac{C_J - C_\zeta}{J_3}\right)\left(-C_6\,\text{sign}(s^2) - \zeta\right)] - b_3 \tag{11}$$

4.1 Lyapunov Stability of System

Considering M as a Lyapunov Stability Function for stability analysis of sliding surface, for stable system M should be positive, i.e., $M > 0$, for error sliding surface "s"

$$M = \frac{1}{2}s^2$$

It means that $M > 0$, due to sliding surface (s) is a squared term it will give always positive value.

Now, according to the stability criteria $\dot{M}$ should be negative

$$\dot{M} = s\dot{s}$$

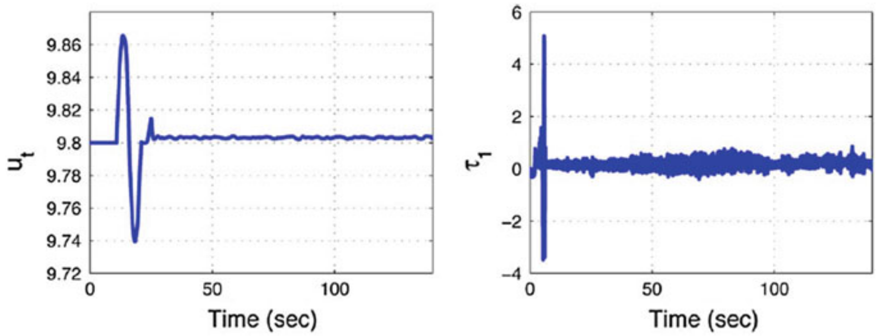

Fig. 3 Total thrust force u_t and rotational force τ_1 versus time

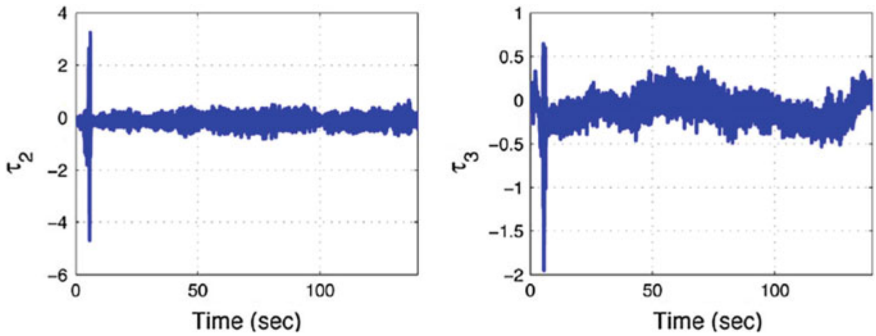

Fig. 4 Rotational forces τ_2 and τ_3 versus time

Or,

$$\dot{M} = s(-C \operatorname{sign}(s))$$

where $C > 0$ for $t > 0$, it means that $\dot{M} < 0$.

Lyapunov Stability is satisfied.

5 Results and Discussions

The FOSMC (Error) controller is executed by Simulink MATLAB simulator. The following images show the behavior of the quadcopter to various points. In Fig. 3 the control input thrust force and rotational force parameters obtained in Eqs. (7) and (9), i.e., u_t and τ_1 by using fractional order method are shown with respect to time. The graph shows that an initial variation is more but within the finite time or very less time system attain the desired values of u_t and τ_1.

In Fig. 4 the control input rotational forces parameters obtained in Eq. (10) and (11), i.e., τ_2 and τ_3 by using fractional order method are shown with respect to time. The graph shows that an initial variation is more but within the finite time or very less time system attain the desired values of τ_2 and τ_3.

In Fig. 5 the variation of Euler angles φ, θ and ψ with time is shown. Initially $\varphi = 0, \theta = 0$ and $\psi = 0$ and the desired values are $\varphi = 0, \theta = 0$ and $\psi = \pi/6.28$. The graph shows that the initial variations is more but within the finite time or very less time system attain the desired values of Euler angles φ, θ and ψ. In Fig. 6 the

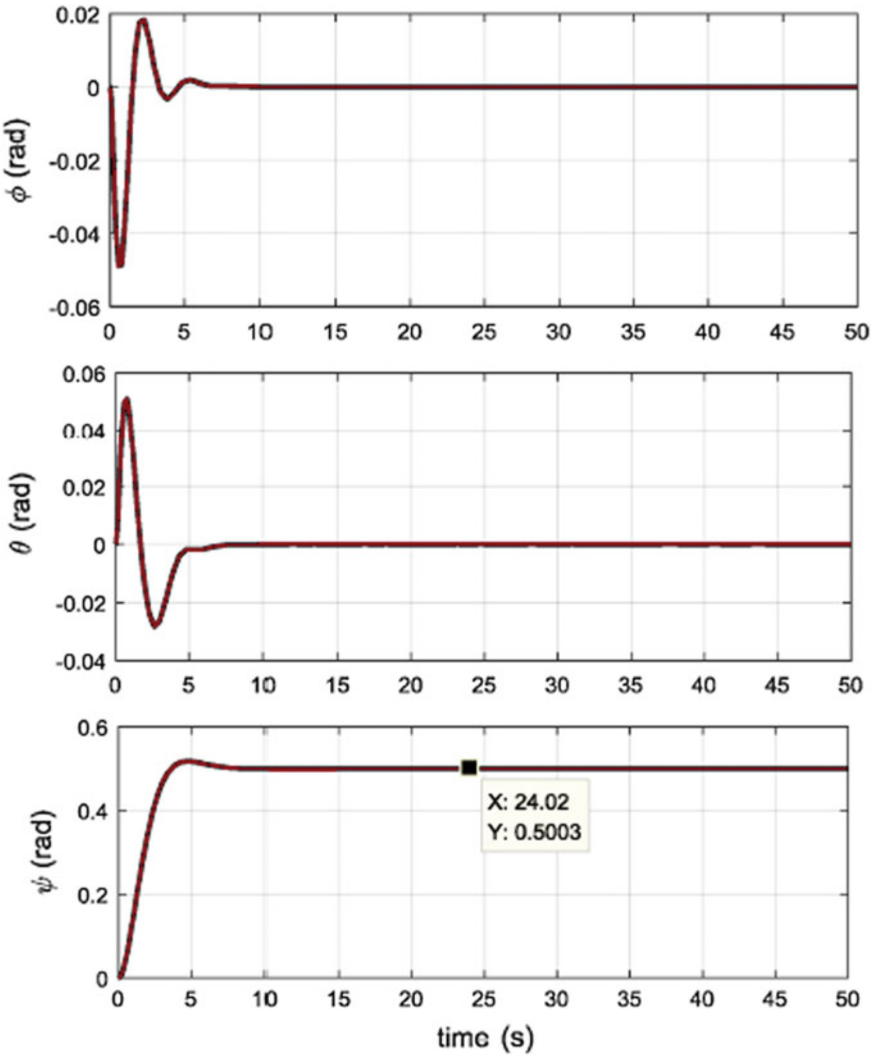

Fig. 5 Euler angles φ, θ and ψ versus time

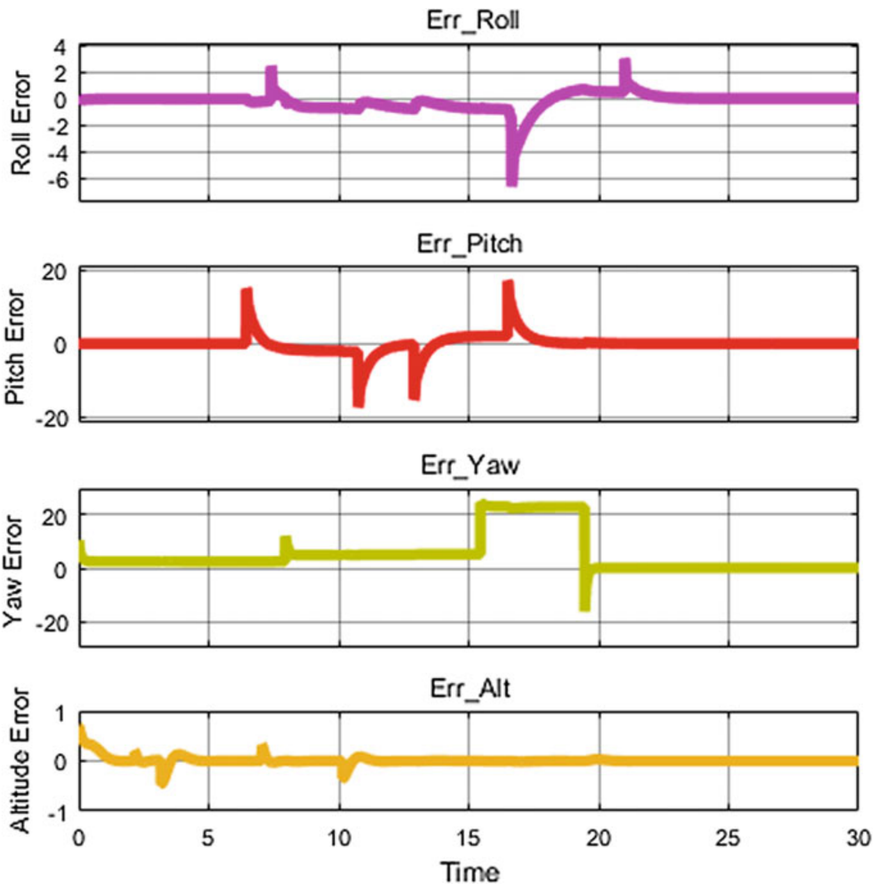

Fig. 6 Error analysis

error in roll, pitch, yaw and altitude is analyzed with time. The variation in error is very less which shows the system stability. In Fig. 7 the variation of error in chattering form is shown for the order of time is 10^{-2} s.

6 Conclusions

As per the results obtained from the simulation, it is verified that the FOSMC control is robust and chattering free. The obtained as per the Fig. 7 is of very small amplitude and can be ignored. The controller shows full control over the six degrees of freedom. It is observed from the mathematical calculations that the FOSMC control variable 'α' can be modeled to tune appropriate error amplitudes and corresponding converging

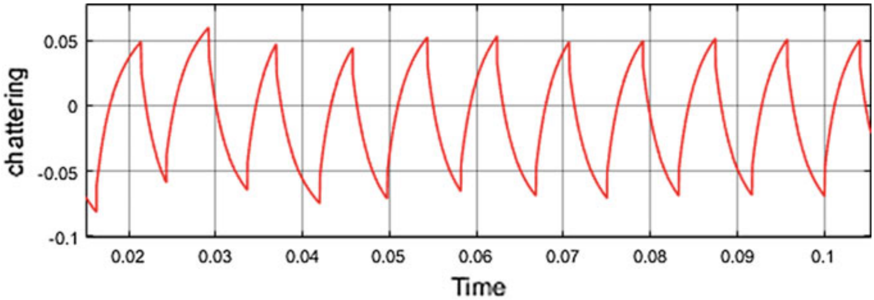

Fig. 7 Chattering

time. It is also shown by the results that the controller improves initial transients and steady-state performance of Quadcopter. The error analysis graphs show finite time converge of parametric errors as per Lyapunov Stability criterion.

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