Low-Complexity Robust Adaptive Beamforming Based on Covariance Matrix Reconstruction

Zhiwei Guan[1], Heping Shi[1](✉), Lizhu Zhang[1], and Ning Ma[2]

[1] School of Automobile and Transportation, Tianjin University of Technology and Education (TUTE), Tianjin 300222, China
shiheping@tju.edu.cn
[2] School of Electronic Engineering, Tianjin University of Technology and Education (TUTE), Tianjin 300222, China

Abstract. A new beamformer based on covariance matrix reconstruction is introduced. The essence of the new approach is to eliminate the desired signal component in the sample covariance matrix and thus complex integral operation is avoided in the procession of covariance matrix reconstruction. Besides, the actual array steering vector is estimated by a new technique. Contrary to other reference beamformers, simulation results demonstrate the effectiveness of our proposed method.

Keywords: Robust adaptive beamforming · Covariance matrix reconstruction · Steering vector estimation

1 Introduction

Adaptive beamforming has been one of the most important research areas in sensor array signal processing for decades, which has possessed many applications such as wireless communication and microphone array speech processing. It is well known that conventional adaptive beamforming such as standard Capon beamforming needs to assume exact knowledge of desired signal (DS). It will suffer significant performance degradation when the information of DS is not known accurately.

Numerous robust adaptive beamforming (RAB) techniques have been proposed to deal with this problem [1, 2]. For instance, diagonal loading is known to be a popular approach, however, it is difficult to choose the optimal loading factor. In [3], a new beamformer named worst-case performance optimization based on the second-order cone programming (SOCP) problem was reported. However, to solve this problem, specific optimization toolbox such as CVX is needed.

In order to achieve robustness against large look direction mismatch, a new method is introduced in this paper. In [4], the interference-plus-noise (IPN) covariance matrix was reconstructed by using the Capon spectrum to integrate over an angular sector separated from the direction of the desired signal. However, this caused complex integral operation. To reduce the computational complexity, the IPN covariance matrix is reconstructed by utilizing a new approach in this paper. The essence of the new

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approach is to wipe up the desired signal component in the sample covariance matrix and thus complex integral operation can be avoided in the procession of covariance matrix reconstruction. Furthermore, unlike traditional uncertainty set based robust beamformers, it is not necessary to use a large size of the uncertainty set against direction-of-arrival (DOA) mismatch in the proposed algorithm.

2 Array Signal Models

Consider a uniform linear antenna array with N omnidirectional equispaced antenna sensor elements receiving multiple far-field narrowband signals. Let $\mathbf{n}(k)$ be a $N*1$ complex vector representing the array observation at the kth snapshot

$$\mathbf{x}(k) = \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k) \tag{1}$$

where $\mathbf{s}(k) = s(k)\mathbf{d}(\theta_s)$, $\mathbf{i}(k)$ and $\mathbf{n}(k)$ respectively represent the statistic independent desired signal, interference and noise.

The beamformer output is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k) \tag{2}$$

where $\mathbf{w} = [w_1, \ldots, w_N]^T$ is the complex weight vector of the antenna array, and $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose operator.

In general, we introduce the output signal-to-interference-plus-noise ratio (SINR) to measure the beamformer performance. Assuming the desired signal steering vector is $\mathbf{a}(\theta_0)$, θ_0 is the DOA of the DS.

$$SINR = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{a}(\theta_0)|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}} \tag{3}$$

where σ_s^2 denotes the DS power and the $N*N$ ideal IPN covariance matrix $\mathbf{R}_{i+n}$ can be acquired from $\mathbf{R}_{i+n} = E\{[\mathbf{i}(k) + \mathbf{n}(k)][\mathbf{i}(k) + \mathbf{n}(k)]^H\}$.

The standard Capon beamformer can be obtained by solving the following optimization problem:

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a}(\theta_0) = 1 \tag{4}$$

The solution to (4) optimization problem is

$$\mathbf{w} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0) \mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_0)} \tag{5}$$

Note that the perfectly known array covariance matrix $\mathbf{R}_{i+n}$ is unavailable and it is often replaced by the sample covariance matrix

$$\mathbf{R}_x = (1/K)\sum_{k=1}^{K}\mathbf{x}(k)\mathbf{x}(k)^H \tag{6}$$

K is the number of snapshots. As K increases, the matrix $\mathbf{R}_x$ will converge to the theoretical covariance matrix $\mathbf{R} = \sigma^2\mathbf{a}(\theta_0)\mathbf{a}(\theta_0)^H + \mathbf{R}_{i+n}$, that is to say, the desired signal steering vector lies in the sample covariance matrix $\mathbf{R}_x$. The DS may be treated as an interference signal, which leads to DS "self-cancelation", and thus the array system performance is affected. Therefore, it is necessary to eliminate the DS component in the sample covariance matrix $\mathbf{R}_x$.

3 The Proposed Algorithm

In the proposed method, instead of using the spatial distribution, the IPN covariance matrix is reconstructed by regulating sample covariance matrix, and the desired signal steering vector is estimated with the help of the parallelogram rule of vectors.

3.1 IPN Covariance Matrix Reconstruction

The essence of the proposed IPN covariance matrix reconstruction technique is to wipe out the desired signal component in the sample covariance matrix. To achieve this goal, the sample covariance matrix $\mathbf{R}_x$ is decomposed as follows:

$$\mathbf{R}_x = \sum_{i=1}^{N}\lambda_i\mathbf{e}_i\mathbf{e}_i^H = \mathbf{E}_s\mathbf{\Lambda}_s\mathbf{E}_s^H + \mathbf{E}_n\mathbf{\Lambda}_n\mathbf{E}_n^H \tag{7}$$

where $\{\lambda_i, i = 1,\ldots,N\}$ is the eigenvalues of matrix $\mathbf{R}_x$ in descending order, $\mathbf{e}_i$ is the eigenvector associated with λ_i. $\mathbf{E}_s = [\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_{M+1}]$ and $\mathbf{E}_n = [\mathbf{e}_{M+2}, \ldots, \mathbf{e}_N]$. $\mathbf{\Lambda}_s = diag\{\lambda_1, \lambda_2, \ldots, \lambda_{M+1}\}$ and $\mathbf{\Lambda}_n = diag\{\lambda_{M+2}, \ldots, \lambda_N\}$ is a diagonal matrix. M is the number of interference.

As mentioned above, the DS component in the sample covariance matrix will reduce the beamformer performance. To minimize the impact of this component, refer to the idea of diagonal loading, the signal-plus-interference subspace matrix can be processed as follows:

$$\hat{\Lambda}_s = [\lambda_1 - 10\sigma_n^2, \lambda_2 - 10\sigma_n^2, \ldots, \lambda_{M+1} - 10\sigma_n^2] \tag{8}$$

However, nothing is done with other eigenvalues which are associated with the noise subspace. That is to say, all the eigenvalues associated with the signal-plus-interference subspace minus a small amount and the eigenvalues associated with the noise subspace remain the original sample. Although the above processing will also have an effect on the interference signal, this operation can better reduce the impact of the DS component. The presence of the DS is critical to the performance of beamformer. Next, we recombine the adjusted matrix and get the reconstructed IPN covariance matrix.

$$\mathbf{R}_{re} = \mathbf{E}_s\hat{\Lambda}_s\mathbf{E}_s^H + \mathbf{E}_n\mathbf{\Lambda}_n\mathbf{E}_n^H \tag{9}$$

3.2 Desired Signal Steering Vector Estimation

In practical applications, for the actual DS steering vector, the presumed DS steering vector $\mathbf{a}(\theta_0)$ always exists mismatch. In [5], $\sqrt{N}\mathbf{e}_t$ is used as the estimation of the actual DS steering vector, where the eigenvalue vector $\mathbf{e}_t$ is most similar to the DS steering vector $\mathbf{a}(\theta_0)$. Simulation results have demonstrated that this estimation is also deviated from the actual value to a certain extent. In this paper, the actual DS steering vector is estimated as follows:

$$\widehat{\mathbf{a}} = \sqrt{N}\mathbf{e}_t + \mathbf{\Delta} \tag{10}$$

where $\mathbf{\Delta}$ denotes the mismatch vector with an expression of $\mathbf{\Delta} = \mathbf{a}(\theta_0) - \mathbf{e}_t$. That is to say, the actual DS steering vector can be expressed as the difference between the presumed DS steering vector $\mathbf{a}(\theta_0)$ and estimation of the actual DS steering in [3].

From a complexity point of view, the main computational cost in this algorithm is the eigendecomposition operation on the sample covariance matrix $\mathbf{R}_x$, which has a complexity of $O(N^3)$. Compared to the other RAB methods, the presented method does not need an integral operation or any optimization toolbox, so it can be applied to practical engineering easily.

4 Simulation Results

Computer simulations are performed using a 10-element uniform linear array with elements spaced a half wavelength apart. Three independent narrowband signal sources are present in the directions of 0°, 30°, and 50°, respectively. The first signal is the desired signal, and the other two are interferences. The noise is presumed to be white noise with unit covariance.

Three other RAB approaches are compared with the proposed method in terms of the array output signal-to-interference-plus-noise ratio (SINR): the traditional diagonal loading beamformer; the beamformer using worst-case performance optimization (WCB) [3], and the beamformer based on steering vector estimation with as little as possible prior information (API) [4]. The angular location of the signal of interest is presumed to be $\Theta = [\theta_0 - 5°, \theta_0 + 5°]$ in the beamformer of [3]. For each scenario, 100 independent Monte Carlo trials are used.

In the first example, we set $\theta_0 = 2°$, that is to say, there is a 2° look direction error. In this example, the SNR equals 10 dB. The output SINR against the number of snapshots is investigated and the corresponding results are shown in Fig. 1.

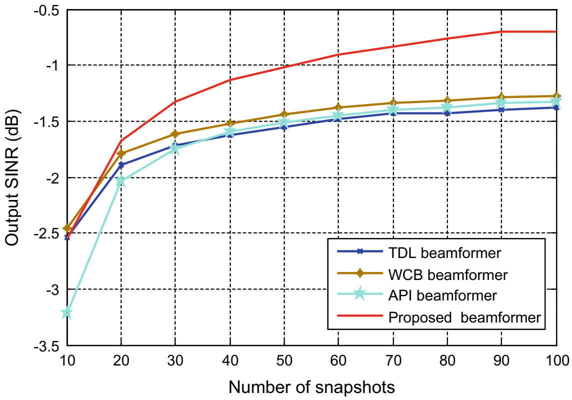

Fig. 1. Output SINR of beamformers against the number of snapshots

It is found that the proposed approach enjoys the best performance among the RAB methods tested and converges to a relatively high level with fewer snapshots. The performance improvement is a direct result of the DS elimination and the ASV estimation.

5 Conclusion

A new low-complexity RAB approach is presented in this letter. Based on the sample covariance matrix, in the proposed method, the IPN covariance matrix is reconstructed and the actual ASV is estimated using a desired signal steering vector and the eigenvectors of the sample covariance matrix. In contrast to other RAB approaches, the proposed method does not involve any optimization and integral operation, and it can achieve better performance.

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