Unsupervised Hyperspectral Band Selection Method Based on Low-Rank Representation

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Abstract. In order to reduce the spectral redundancy of hyperspectral remote sensing images and reduce the computational complexity of subsequent processing, an unsupervised hyperspectral image band selection algorithm based on low-rank representation (LRBS) was proposed in this paper. First, a low-rank representation of the hyperspectral image is proposed and a low-rank coefficient matrix is obtained. Then, each column of the low-rank coefficient is used as a vertex of the graph to perform spectral clustering. Lastly, we use the fixed initial k-means cluster centers for clustering to get the salient band of each cluster. The experimental simulation results show that the bands selected by LRBS algorithm can improve the classification accuracy and have better performance than other methods.

Keywords: Band selection (BS) · Low-rank representation (LRR) · Hyperspectral image classification

1 Introduction

HYPERSPECTRAL imaging (HSI) is one of the most powerful technologies to remotely detect and recognize the material properties of the object in the interest scene for the broadband wavelength and high spectral resolution [1–3] However, a large number of spectral bands pose a great challenge for information extraction. For example, in hyperspectral image classification, the classification accuracy decreases with the increase of the number of bands relative to a small number of tag samples. Therefore, it is very necessary to reduce the amount of data and save resources. Two methods are selected for dimension reduction: feature extraction and band selection. Feature extraction uses mapping methods to convert raw data to fewer new features including principal component analysis (PCA) and independent component analysis (ICA) [4]. However, feature extraction will change the original data to a certain degree. Different from feature extraction, bands selection selects proper band subsets from the original data set [5]. It not only preserves the original features and physical meaning of remote sensing data but also reduces the size of the data to achieve the goal of dimensionality reduction.

Recently, some hyperspectral BS methods have been proposed for hyperspectral classification and target detection. Weiwei sunpresented a novel symmetric sparse representation (SSR) method to solve the band selection problem in hyperspectral imagery classification [6]. H Su et al. proposed a particle swarm optimization (PSO)-based system to select bands and determine the optimal number of bands to be selected simultaneously [7]. Low-rank representation (LRR) is a new tool for robustly and efficiently processing high-dimensional data. Xiaotao Wang proposed a predimension-reduction algorithm that couples weighted low-rank representation (WLRR) with a skinny intrinsic mode functions (IMFs) dictionary for hyperspectral image (HSI) classification. Alex Sumarsono et al. introduced the LRR model into the hyperspectral image and used it to estimate the number of subspaces [8–11]. However, these low-rank-based methods do not make full use of the resulting low-rank information.

In this paper, we developed an unsupervised BS approach, to be called low-rank band selection (LRBS) for hyperspectral image processing where no *a priori* knowledge available to be used. The organization of this paper is organized as follows. In Sect. 2, the proposed method is specifically described. Section 3 presents the extensive experiments and conclusions are drawn in Sect. 4.

2   LRBS Method

2.1   The LRR Model

A hyperspectral image is denoted as $X = (x_1, x_2, \cdots, x_L)^T \in R^{L \times n}$, where L is the number of bands, n is the total number of pixels, and $x_i = (x_{i,1}, x_{i,2}, \cdots, x_{i,n})^T$ is the ith spectral band. The HSI data includes the low-rank target parts and the noise part. Therefore, the following model is defined as low-rank representation of HSI:

$$\min \Gamma(\mathbf{Z}) + \lambda \|\mathbf{E}\|_{1,2}, \quad st. \quad X = DZ + E, \tag{1}$$

where $\Gamma(\bullet)$ is the coefficient of the low-rank matrix, D is the dictionary of HSI, E denotes the noise, λ is the balance parameter of the two parts of the equation. You can get λ by

$$\lambda = \frac{1}{\sqrt{\log L}} \tag{2}$$

The hyperspectral image contains redundancy and noise information. The singular value decomposition is used to filter the image noise to get the data dictionary in this paper. The specific approach is as follows: First, singular value decomposition is performed on $X = U\Sigma V$, and V is right singular matrix with the size of n*m, U is left

singular matrix with the size of L*m, Σ is semi-positive definite m*m order diagonal matrix whose diagonal elements are singular value. The data dictionary is calculated by the following equation:

$$D = U * \Sigma * V'$$ (3)

2.2 Solution of the LRR Model

The rank minimization problem is known to be NP-hard, and the most popular choice is to replace rank with the nuclear norm to make the minimization to be convex relaxation problem. Therefore, the Eq. (1) can be written as follow:

$$\min \|Z\|_* + \lambda \|E\|_{1,2}, \quad s.t. \quad X = DZ + E,$$ (4)

where $\|Z\|_*$ is the nuclear norm of Z.

The augmented Lagrange multiplier (ALM) method is quite prevalent in the LRR-related works [12]. In this section, inexact-ALM is extended to handle the proposed LRR problem in (4). By introducing the intermediate variable J, it can be rewritten in the following equivalent form

$$\min \|J\|_* + \lambda \|E\|_{1,2} \quad s.t. \quad X = DZ + E, Z = J$$ (5)

With the aid of the Lagrange multipliers Y1 and Y2, (5) can be redefined in the final optimization problem (6)

$$\min_{Z,E,J,Y_1,Y_2} \|Z\|_* + \lambda \|E\|_{1,2} + <Y_1, X - DZ - E>$$
$$+ <Y_2, Z - J> + \frac{\mu}{2}(\|X - DZ - E\|_F^2 + \|Z - J\|_F^2)$$ (6)

In (6), each unknown variable of Z, E, J, Y_1, and Y_2 is optimized in iterative fashion that is to successively optimize one variable with the other fixed variables. The solving procedure is summarized in Algorithm 1. In each iteration, the variables J and E are updated by the singular value thresholding operator and the $l_{2,1}$ norm thresholding operator. As for the variable Z, it has a closed solution for each column shown in step 2.2. The steps of algorithm is described as bellow.

Algorithm 1 Solving Problem by Inexact ALM

Input: data matrix X, dictionary D, parameter λ, μ

Output: Z, E

Step 1. Initialize:

$Z = J = 0, E = 0, Y_1 = 0, Y_2 = 0, \mu = 10^{-4}, \mu_{\max} = 10^6, \rho = 1.1, \varepsilon = 10^{-6}$

Step 2. While not converged $X - DZ - E \rightarrow 0$ do：

1) Fix the other variables and update J by

$$J = \arg\min \frac{1}{\mu}\|J\|_* + \frac{1}{2}\| J - (Z + Y_2/\mu) \|_F^2$$

2) Fix the others and update Z by

$$Z = (D^T D + E)^{-1}(D^T X - D^T E + J + \frac{Y_1 D^T}{\mu} - \frac{Y_2}{\mu})$$

3) Fix the others and update E by

$$E = \arg\min \frac{\lambda}{\mu}\| E \|_{2,1} + \frac{1}{2}\|E - (X - DZ + Y_1/\mu)$$

4) Update the multipliers

$$Y_1 = Y_1 + \mu\left(X - DZ - E\right), Y_2 = Y_2 + \mu(Z - J)$$

5) Update the parameter μ by $\mu = \min\left(\mu_{\max}, \rho\mu\right)$

6) Check the convergence conditions

End while

2.3   The LRBS Algorithm

This section describes the proposed band selection algorithm based on low-rank representation (LRBS). We use the technique of spectral clustering algorithm to compute the eigenvectors of graph Laplacian which weight is composed of the low-rank matrix. The algorithm implementation needs to use K-means to cluster a data set into some subsets. The number of categories k in k-means is obtained by virtual dimensionality (VD) algorithm. Different from the classic K-means, in this paper, we utilized a fixed initial cluster centers instead of the random initial ones.

Assume that $\{\mathbf{b}_l\}_{l=1}^L$ is a set of band number of a hyperspectral image cube where $\mathbf{b}_l$ is the l th spectral band represented by a column vector, $\mathbf{b}_l = (b_{l1}, b_{l2}, \cdots, b_{lN})^T$ and $\{b_{li}\}_{i=1}^N$ is the set of all N pixels in the l th band image. The implementation steps of the BS method proposed in the paper is listed as following.

Algorithm 2 The LRBS algorithm
Input: All the hyperspectral bands
Output: The selected bands
Initialize: k (obtained by virtual dimensionality(VD) algorithm)
Step 1: Divide all the bands are into N subsets, calculate the euclidean distances between the bands of each set,
Step2: Find the band b_1 with the largest correlation value in the cluster center set.
Step 3: Find the band b_n with the largest value different from b_1 in the left band subsets.
Step 4: k=k+1; if k<p, go to step 2.
Step 5: Using k-means algorithm with the initial center set to cluster k subsets.
Step 6: k=1; calculate the center of the subset.
Step 7: Find the band b_n with the smallest value with the subset center, put b_n in the set $\{b_s\}$.
Step 8: k=k+1; if k<k, go to step 6.

3   Hyperspectral Image Experiments

In this section, we conducted a series of experiments to evaluate the classification performance of the selected bands by different BS methods. The compared algorithms include uniform BS (UBS), minimum estimated abundance covariance (MEAC), multigraph determinantal point process (MDPP), dominant set extraction BS (DSEBS), and the proposed BS method. The experiment platform is listed as follows: the machine operating system used is Windows10, the machine configuration is: CPU frequency 2.50 GHz, running memory 8 GB software environment: MATLAB R2014a. The three hyperspectral datasets we used in the experiment include Purdue University's Indiana Indian Pines image, Salinas Valley image, and the image of University of Pavia.

Table 1 shows the specific band subsets selected using different methods on three datasets. Tables 2, 3, and 4 show the classification accuracy of the bands selected using different methods for hyperspectral classification. It can be seen from Table 2 that the band subsets selected by the LRSB method have higher classification accuracy than other classifications in subsequent classifications such as 1st, 2nd, 6th, 7th, 8th, 9th, and 14th classes in Purdue's data. It can also be seen from Tables 3 and 4 that the band classification selected by the LRBS method generally achieves higher classification accuracy in image classification. Therefore, the experiment shows that the LRBS algorithm provided in this paper can indeed select a subset of frequency bands that perform better.

Table 1. Bands selected by UBS, MEAC, MDPP, DSEBS, LRBS

Data	Methods	Selected bands
Purdue Indian Pines (18 bands)	UBS	1, 14, 27, 40, 53, 66, 79, 92, 105, 118, 131, 144, 157, 170, 183, 196, 209, 220
	MEAC	159, 3, 92, 96, 82, 36, 39, 55, 41, 1, 2, 33, 206, 38, 163, 17, 204, 9
	MDPP	10, 39, 59, 75, 79, 85, 92, 130, 140, 146, 147, 149, 150, 152, 160, 164, 175, 193
	DSEBS	42, 129, 97, 131, 174, 16, 176, 177, 172, 43, 192, 193, 98, 171, 99, 132, 40, 33
	LRBS	72, 35, 31, 147, 167, 128, 112, 65, 108, 109, 137, 189, 155, 162, 170, 182, 200, 210
Salinas (21 bands)	UBS	1, 12, 23, 34, 45, 56, 67, 78, 89, 100, 111, 122, 133, 144, 155, 166, 177, 188, 199, 210, 224
	MEAC	107, 148, 203, 149, 5, 8, 105, 3, 28, 12, 18, 10, 44, 36, 25, 17, 51, 32, 110, 68, 58
	MDPP	1, 8, 11, 22, 27, 28, 50, 57, 58, 65, 90, 99, 105, 119, 123, 134, 142, 157, 175, 191, 204
	DSEBS	99, 101, 16, 119, 177, 112, 44, 46, 120, 47, 131, 175, 196, 121, 17, 102, 174, 180, 187, 135, 42
	LRBS	51, 16, 66, 36, 43, 129, 45, 76, 89, 98, 109, 118, 125, 137, 142, 159, 165, 207, 184, 196, 203
Univ. of Pavia (14 bands)	UBS	1, 9, 17, 25, 33, 41, 49, 57, 65, 73, 81, 89, 97, 103
	MEAC	1, 23, 24, 40, 42, 58, 56, 59, 48, 31, 47, 83, 25, 54
	MDPP	2, 23, 44, 46, 50, 62, 66, 73, 89, 91, 92, 93, 96, 102
	DSEBS	86, 102, 64, 20, 21, 63, 65, 6, 19, 22, 7, 66, 95, 67
	LRBS	70, 14, 20, 27, 1, 34, 47, 51, 62, 64, 68, 91, 99, 94

4   Conclusion

This paper proposed an unsupervised low-rank representation of the band selection method. First, the low-rank representation of the hyperspectral image is solved using an inexact -ALM algorithm to obtain a low-rank coefficient matrix. Each column of the low-rank coefficient is used as a vertex of the graph to perform spectral clustering to determine the initial k-means. The clustering center is clustered to finally obtain a band that satisfies the conditions. Experiments on three HSI datasets show that the frequency band selected by the LRBS algorithm can select a better band subset for image classification.

Table 2. P_D, P_{OA}, P_R calculated from the classification results for Purdue's data

Class	Full Bands	UBS	MEAC	MDPP	DSEBS	LRBS
	P_D	P_D	P_D	P_D	P_D	P_D
1	95.65	95.65	93.48	95.65	95.65	100
2	96.01	97.13	93.07	96.08	96.99	94.47
3	96.99	96.51	96.27	97.35	97.23	96.99
4	98.73	98.73	98.31	99.58	98.31	98.31
5	89.44	90.68	91.51	92.34	93.58	94.62
6	97.12	97.67	97.40	96.71	97.12	98.08
7	100	100	100	100	100	100
8	98.78	98.54	99.16	97.49	97.91	99.16
9	100	100	90.00	100	100	100
10	93.93	91.98	93.31	94.65	93.00	95.37
11	94.70	96.13	94.55	95.48	95.85	95.44
12	95.45	94.94	96.29	96.80	97.30	96.80
13	98.54	98.54	99.02	97.56	96.59	98.54
14	93.52	94.15	94.78	94.70	94.55	96.28
15	90.67	95.60	92.49	96.89	93.52	94.30
16	98.92	98.92	98.92	98.92	98.92	98.92
POA	95.09	95.69	94.91	95.89	95.88	**96.09**
PR	97.61	97.90	97.52	98.00	97.99	**98.09**

Table 3. P_D, P_{OA}, P_R calculated from the classification results for Salinas

Class	Full Bands	UBS	MEAC	MDPP	DSEBS	LRBS
	P_D	P_D	P_D	P_D	P_D	P_D
1	95.52	97.16	97.71	97.76	97.16	97.56
2	98.42	98.85	98.44	97.99	99.17	98.95
3	93.78	95.5	94.03	93.98	95.65	95.85
4	95.62	94.69	94.33	97.49	94.74	95.19
5	96.9	96.45	95.19	95.22	96.9	95.44
6	98.79	98.59	98.56	98.79	98.56	97.83
7	98.63	98.21	98.18	97.99	97.65	97.54
8	96.69	95.81	97.4	95.23	96.11	97.14
9	95.87	95.6	94.74	95.29	95.73	95.92
10	96.67	96.37	96.34	96.46	97.25	96.71
11	97.75	97.85	91.1	97.75	98.31	97.1
12	97.15	96.16	95.54	97.46	97.66	96.63
13	96.51	96.94	93.35	96.4	95.63	96.4
14	95.89	98.14	97.66	97.01	98.04	98.88
15	94	95.27	96.52	95.42	95.25	96.42
16	93.3	96.07	93.86	95.07	95.02	95.52
POA	96.37	96.49	96.45	96.25	96.63	**96.86**
PR	98.23	98.29	98.27	98.17	98.36	**98.47**

Table 4. PD, POA, PR calculated from the classification results for University of Pavia

Class	Full Bands	UBS	MEAC	MDPP	DSEBS	LRBS
	P_D	P_D	P_D	P_D	P_D	P_D
1	86.42	87.67	86.44	87.97	87.71	77.61
2	73.34	84.38	83.33	82.14	84.63	88.04
3	79.85	78.9	76.66	76.17	79.22	87.18
4	98.81	97.84	98.88	96.95	97.99	77.1
5	91.49	89.93	91.33	93.32	87.11	98.29
6	89.1	91.35	82.78	87.53	87.44	93.26
7	81.1	83.32	76.26	75.64	76.34	89.62
8	78.46	79.09	79.51	79.83	79.3	82.4
9	77.24	75.86	76.32	74.01	76.17	75.53
P_{OA}	84.32	85.19	83.85	84.33	84.25	**85.82**
P_R	96.76	96.93	96.64	96.76	96.75	**97.07**

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