Chapter 9
Problems in Real-World Context and Mathematical Modelling

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Abstract This chapter discusses teacher education efforts and analyses the research outcomes in the domain of solving problems in real-world contexts, particularly in the field of mathematical modelling among other tasks situated in real-world contexts in Singapore mathematics classrooms. The first part of this chapter begins with an understanding of "applications and modelling" from the perspective of the Singapore school mathematics curriculum framework. The second part of this chapter reports on the efforts made in supporting applications and modelling in teacher education through professional development opportunities. This chapter continues with a discussion of findings from local research in solving problems in real-world contexts (applications and/or modelling) carried out with students in primary and secondary schools to add to the repertoire of knowledge in this domain. Challenges are surfaced in the light of the preceding sections with implications for teacher education and research with the acknowledgement that there is still some distance to go to know more about applications and modelling and actualizing the curriculum in a more holistic sense through teacher education and the implementing of modelling lessons. This chapter discusses the way forward in supporting the advancement of mathematical modelling in the mathematics curriculum.

Keywords Applications and mathematical modelling · Interdisciplinary project work · Teacher education · Professional development

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9.1 Perspective of Applications and Modelling from the Singapore School Mathematics Curriculum Framework

Applications and modelling is a relatively new domain under the process component of the Singapore school mathematics curriculum framework (MOE 2006a, b, 2012a). The curriculum document articulates that applications and mathematical modelling tasks are crucial in helping students draw connections between school mathematics and the real world, enhance understanding of mathematical knowledge and skills, and develop mathematical competencies (MOE 2012a, p. 15). *Connections* in the document refer to the "ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and the real world" (p. 15).

Stillman et al. (2008) noted the difference between applications tasks and mathematical modelling activities. While both are problems situated within real-world contexts, the nature of use of mathematical knowledge and skills in each is different. An applications task involves the teacher selecting real-world situations for students to apply predetermined mathematics learned in class. Such tasks can be open-ended in terms of solutions and answers as long as students can draw appropriate connections and interpretations of mathematics they know within the real-world situation presented in the problem context. One adaptation of an applications task could be what the Singapore Ministry of Education refers to as "Problems in Real-World Contexts" (PRWC). In PRWC tasks, students solve a multi-part mathematics problem where the stem of the problem presents the context and key variables (MOE 2015). Each question part that follows requires students to apply the mathematics they know to find an answer. Real-world interpretation of the problem context and, often, assumption making are involved during sense-making in the solution process. For the purpose of assessment, PRWC tasks are often open-ended to a limited extent. Since October 2016, PRWC tasks have been incorporated into the GCE "O" levels mathematics examinations. Teachers have started to familiarize themselves with PRWC task design for teaching and learning in secondary mathematics according to the assessment guidelines provided by the Singapore Ministry of Education (see MOE 2015). Other Singapore literature on the use of PRWC for teaching, learning, and assessment is still limited to date (e.g. Yeo et al. 2018).

In contrast, a mathematical modelling activity starts with a real-world problem situation where modellers (or problem solvers) use mathematical lenses to solve the problem. In doing so, different modellers may apply different mathematics to develop a mathematical model to solve the problem or even use mathematics new to them in the process. In this regard, a mathematical modelling activity is truly open-ended, possibly right from the beginning during problem posing where modellers may craft their own mathematical problem from the real-world situation. Literature records different types of modelling activities from various theoretical stances (see Kaiser and Sriraman 2006). The Singapore school mathematics curriculum defines mathematical modelling as "the process of formulating and improving a mathemat-

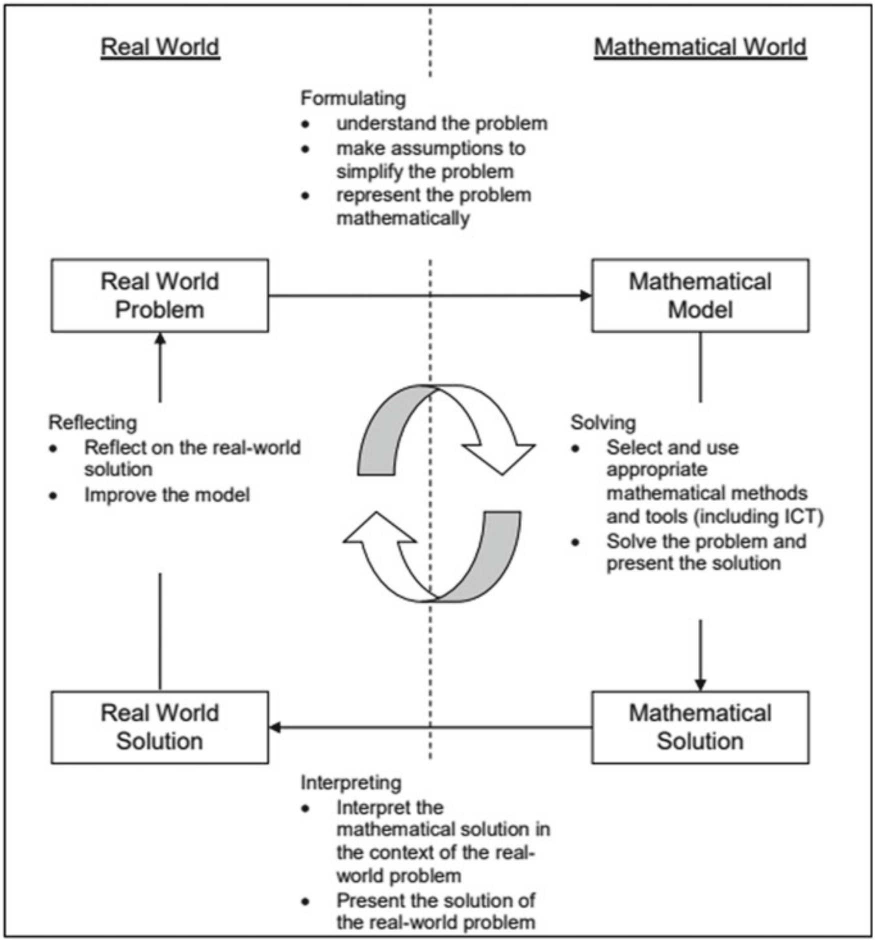

Fig. 9.1 A simplified diagram of the mathematical modelling process (MOE 2012b)

ical model to represent and solve real-world problems". A mathematical model is a "mathematical representation or idealisation of a real-world situation" which can be presented in numerical, algebraic, geometrical, or statistical forms (MOE 2012b, p. 2). As outlined in various key modelling literature in this established field (e.g. Blum et al. 2007; Lesh and Doerr 2003; Niss 2010; Stillman et al. 2016), four stages in the modelling process are also recognized in the Singapore mathematics curriculum documents (MOE 2015): real-world problem, mathematical model, mathematical solution, and real-world solution. The Singapore curriculum adapted the generic mathematical modelling process by incorporating connecting *elements*, namely formulating, solving, interpreting, and reflecting between the various modelling stages as shown in Fig. 9.1 (MOE 2012b).

The expected modelling competencies of students when they attempt modelling activities in Singapore schools are articulated for each element and for each stage of the modelling process. The *formulating* element is activated when the modeller crafts a mathematical model to represent the real-world problem as a start of the solution process. It is expected that the modeller attempts to understand the real-world context and lists the assumptions involved as the initial model evolves. Key mathematical variables and the relationships between these variables are also defined along with the parameters for model development. The *solving* element involves the modeller selecting and using appropriate mathematical methods and tools to solve the mathematical problem. The *interpreting* element requires modellers to interpret the mathematical solution in the context of the real-world problem by checking if they have indeed addressed the problem and whether their solution makes sense within the real-world constraints within the context. Finally, the *reflecting* element calls for modellers to reflect on the reasonableness of their solutions, assumptions made, and appropriateness of mathematics applied. Modellers can investigate more strategies to be used as part of the solution process and even examine the applicability and generalizability of the mathematical models developed on other similar contexts. Limitations of the models can be discussed, leading to the development of more sophisticated mathematical models for the real-world problem.

9.2 Teacher Education on Mathematical Modelling

Applications in the mathematics curriculum have been part of the students' problem-solving endeavours in school and therefore is a familiar content area. On the other hand, although mathematical modelling was incorporated into the Singapore mathematics curriculum framework since 2007, teacher development programmes on mathematical modelling have only begun in earnest in 2009. To date, mathematical modelling activities have been implemented in many Singapore secondary schools and all pre-university institutions (i.e. junior colleges). Such activities range from simple classroom discussions of real-world problems resulting in the use of selected elements from the modelling stages mentioned above to large-scale inter-school modelling challenges held in Singapore schools or overseas (see Kwek and Ko 2011; Lee and Ng 2015; Ng and Lee 2012). The following subsections highlight the inception of teacher education on mathematical modelling through mass outreach efforts and subsequently in the form of in-service professional development courses. Research efforts in teacher development with the help of emerging frameworks are also discussed.

9.2.1 Mathematics Teachers Conference

On 4 June 2009, the Association of Mathematics Educators (AME) in collaboration with the Mathematics and Mathematics Education (MME) Academic Group organized a Mathematics Teachers Conference (MTC). The theme of the conference was Mathematical Applications and Modelling with international experts presenting the keynote addresses and plenaries. In addition, there were workshops on various aspects of applications and modelling intended for the primary schools, secondary schools, and junior colleges. Two booklets written by MME AG authors were specifically prepared for the MTC, one for primary teachers entitled *Applications and Modelling for the Primary Mathematics Classroom* (Dindyal 2009) and the other for secondary teachers entitled *Mathematical Modelling in the Secondary and Junior College Classroom* (Ang 2009). These booklets contained several examples that focused on some of the concerns of the teachers about the approaches to use for teaching applications and modelling in schools and included several modelling examples for the teachers to use in their teaching.

9.2.2 Mathematical Modelling Outreach

The Mathematics and Mathematics Education (MME) Academic Group at the National Institute of Education (NIE) organized a Mathematical Modelling Outreach (MMO) event on June 2010. The event was attended by at least 320 participants, including teachers and students from 31 primary and secondary schools in Singapore, Australia, and Indonesia. The event involved primary and secondary school students working with inter-school discussion groups on mathematical modelling activities facilitated by pre-service teachers. Students were seen to have benefitted from the modelling activities in various ways:

(i) They learned new concepts while working on the modelling activities.
(ii) They learned to be innovative and were not be constrained by the limits of the modelling task.
(iii) They performed at different levels.
(iv) They worked collaboratively on the task and thereby developed better social skills.
(v) They learned to manage meaningful experiences with complex systems.

Various professional development programmes were also conducted for the accompanying teachers by MME teacher educators (see Ng and Lee 2012) and internationally renowned researchers in the field. Such an event was inspired by similar ones held in Australia (Brown et al. 2015) and Germany (Kaiser and Schwarz 2010). The event culminated in a publication of an academic book "Mathematical Modelling—From Theory to Practice" (Lee and Ng 2015)—a collection of thoughts and ideas shared and deliberated by the presenters, both international and local. Learning gleaned from the MMO was presented at the ICTMA 15 Conference in the form of a symposium consisting of three papers which were later published (Chan 2013; Lee 2013; Ng 2013). The papers provided insights into teacher professional development on mathematical modelling from the Singapore perspective, covering areas that included conceptions, task design, and facilitation. In addition, the tasks that were designed and employed at this event have also been collated and consolidated as a task book for teachers' use (Ng and Lee 2012). The book features simple and enriching modelling tasks, and each task is discussed in detail with prompting questions for students as well as teachers' notes for facilitation. It serves as a rich resource to support teachers' address of mathematical modelling in the Singapore mathematics classrooms.

9.2.3 Promoting a Framework of Instruction for Mathematical Modelling

A research project was started in 2015 to design and test strategies that will enable secondary mathematics teachers develop competencies needed in the teaching of mathematics modelling through the use of a new proposed instructional framework (Ang 2015; Lee and Ang 2015). The project led to a seminar that included talks by both international and local experts on mathematical modelling. The 3-hour seminar was attended by 75 participants, including teachers from 17 schools as well as officers from the relevant units of the Ministry of Education. One of the collaborators of the project took the project further by completing a Ph.D. study on "Professional Development for Teachers of Mathematical Modelling in Singapore" (Tan 2015), further contributing to the expertise in teacher education on mathematical modelling. The tried-and-tested mathematical modelling instructional framework (Ang 2015) is now made more accessible through a digital platform (www.mathmodelling.sg) allowing teachers not only access to relevant online resources for the teaching and learning of mathematical modelling but also to be able to contribute to the collection of resources.

9.2.4 In-Service Professional Development Courses

Teacher education on mathematical modelling turned towards the provision of in-service training because of the pedagogical demands on the teachers in facilitating modelling activities. Recently, the Ministry of Education rolled out intensive mathematical modelling professional development courses for in-service mathematics teachers representing the majority of Singapore secondary schools. These courses focus on helping teachers understand what mathematical modelling is, how individual elements of mathematical modelling can be activated within a mathematics classroom, and how to design and incorporate simple modelling activities in mathematics classrooms. To complement these courses, the Ministry of Education commissioned another in-service course on mathematical modelling conducted by MME so that secondary mathematics teachers who are interested in experiencing the full modelling cycle and those who lead mathematical modelling events in their schools can fully immerse in the potentials of modelling activities. The MME in-service course allows teachers to experience being a modeller during real-world problem-solving facilitated by the course instructor before unpacking the modelling stages and elements in more detail. Modelling task design involving full modelling cycles and rubrics for evaluating students' modelling attempts is also discussed.

9.3 Research Involving Teachers in Mathematical Modelling

Several case studies have been carried out in primary schools with respect to building teachers' capacity in the designing and facilitating of modelling activities. Case studies allow the researchers to understand more deeply the teachers' thinking and facilitative actions that led to the successful completion of their own as well as the students' modelling endeavours. Research designs based on an adaptation of the multi-tiered teaching experiment design (Lesh and Kelly 2000) alongside a design experiment method (Dolk et al. 2010) formed the theoretical basis in framing the teacher development process at the primary school level. The multi-tiered teaching experiment (Fig. 9.2) was a three-tiered design that enabled the researcher (Tier 1) to collaborate with the teachers (Tier 2) in the discussion and design of the modelling tasks as well have the teacher to facilitate modelling activities to have an understanding of the features of the modelling task as well as the experience of going through the modelling process. They would also get to see how the students (Tier 3) worked on the modelling task and generated models. The incorporation of design experiment methodology, in particular, during the retrospective analysis phase, enabled the researchers (Tier 3) to guide the analysis and interpretation of data within an interactive cycle to find out the teacher's (Tier 2) rationales during reflection sessions as to why certain instructional approaches were taken, which then formed the basis for feedback and knowledge for the next cycle of facilitation of students' (Tier 3) modelling endeavours. The main principle underlying the multi-tiered teaching experiment framework was to seek corroboration through triangulation where non-prescriptive conditions were provided for the development of new conceptions of participants' experiences, interactions were structured to test and refine constructs, tools were provided to facilitate the construction of models, and the formative feedback and consensus building were integrated into the learning process.

Findings from Chan et al. (2015) on the enactment of the research designs through interviews and retrospective analyses sessions with a novice teacher–modeller revealed that the teacher development process had enabled him to have a better understanding of the different phases and the iterative nature of the modelling pro-

Tier 3 - Researchers	* Development of conceptual framework (model) to develop teachers' knowledge and capacity in facilitating modelling tasks in two cycles. This involved creating learning situations for teachers and students through describing, explaining, predicting teachers' and students' behaviours. * Researchers collaborate with teachers to test and review modelling activity. * Researchers reflect on their own evolving knowledge of the participants' learning experiences for the development of tools to scaffold teachers.
Tier 2 - Teachers	* Teachers collaborate with researchers to test and review modelling activity. * Teachers review feedback for designing own modelling tasks. * Teachers reflect on their own evolving knowledge of the students' learning experiences for the development of tools to scaffold their learning.
Tier 1 - Students	* Students engage in model-eliciting tasks in small groups where they will be involved in constructing and refining models that reveal their interpretation of the problem situation. They will describe, represent, explain, justify and document their mathematical constructions.

Fig. 9.2 A three-tiered teaching experimental framework

cess. The interaction between the tiers (Tiers 1 and 2) played a crucial part in enabling the teacher–modeller to complete the modelling experience successfully. Moreover, the knowledge acquired through the modelling experience would serve to help him become familiar with the students' evolving ways of thinking about important ideas and abilities that he would want the students to develop (Tier 3). As well, the interaction between the teacher–modeller and the researchers (Tiers 2 and 1) was also seen as a model development process for putting the theoretical framework into practice and reviewing how each party was learning through the express-test-revise cycles of the multi-tiered teaching experiment.

In another study involving a novice teacher–modeller, Ng et al. (2013) reported that three teacher competencies surfaced as crucial in the incorporation of mathematical modelling during the retrospective analysis phase of the study, and these were striking an appropriate balance between questioning and listening during facilitation of student discussions, use of metacognitive strategies, and (c) fostering the setting of assumptions in the modelling process. It was inferred that developing these competencies would pave the way for cultivating a pattern of listening–observing–questioning behaviours for a better understanding of students' thinking and interpretation of the real-world problem, overcoming blockages through metacognitive strategies as well as learning to work within parameters by being aware of the need for and setting assumptions when solving real-world problems. There were two follow-up studies on the same teacher–modeller where the teacher's reflections were sought to determine the critical moments of learning and her perceptions of the modelling experience, respectively. For the former, the teacher reported that the deliberate focus in the use of questions to refine students' models, encourage articulation of student ideas in self-evaluation of the models, and clarify and understand student reasoning were core to helping the students in the successful completion of the modelling activity (Ng et al. 2012). It revealed how the retrospective analysis phase, employed in conjunction with the other phases, could serve as a platform for eliciting critical moments of learning for the teacher, building upon the careful selection and discussion of appropriate stimuli. For the latter, based on the teacher's reflections about the modelling experience, the teacher found the modelling activity to be mathematically rich and that it provided a platform for the identification of key variables and their relationships, relating school-based mathematical knowledge and skills to the real-world experience, and justification of mathematical models developed (Seto et al. 2012). In the teacher's view, carrying out modelling activities would make for a more student-centred mathematics lesson that allowed for richer communication of ideas as they would have to justify the models and make their thinking visible compared to a regular mathematics class.

The case studies above also led the researchers to examine the effective use of videos in developing teacher competencies (Ng et al. 2015). It was noted in that using videos as a means for stimulated recall was sufficiently effective in eliciting teacher reflection towards identifying crucial competencies for self-development as one went through the playback instances in the negotiation of meaning and sense-making. Ng and her associates (2015) stressed the importance of timely and appropriate teacher scaffolding when facilitating student group discussions during mathematical modelling activities, in particular the need for teacher to listen in during student talk and mediating between teacher questions and prompts. The findings from Ng's research with her colleagues extended those from Blomhøj and Kjeldsen (2006) as well as Julie and Mudaly (2007) which can be summarized into three interrelated dilemmas faced by teachers when facilitating modelling activities: (a) balancing between a holistic and a reductionist approach, (b) using modelling as a vehicle versus as content, and (c) providing sufficient and appropriate student autonomy during the modelling process.

On developing teachers in the teaching of mathematical modelling at the secondary school level, Tan and Ang (2016) designed a School-Based Development Programme (SBDP) (Fig. 9.3) aimed at influencing teachers' knowledge and resources, goals and orientations in planning, designing, and enacting modelling learning experiences. The SBDP framework sought to help teachers acquire knowledge of modelling and modelling instruction (content) and take them through the transformative learning cycle of modelling (process) within the school context and culture (context). Teachers began with the planning and designing of modelling lessons adopted from Ang's (2015) framework which put forth a set of decision procedures in scaffolding novice teachers towards translating their modelling ideas into a series of modelling learning tasks pitched at different levels of learning experiences. The enactment of the teachers' lessons was video-recorded, and modelling issues encountered were analysed and discussed with the researcher. The cognitive dissonance that surfaced from the teachers' reflections was used to revise and reorganize their learning experiences in anticipation of the enactment of the next modelling activity.

Findings from Tan's case study of the enactment of a teacher participant in the SBDP revealed that the teacher was able to plan and structure developmentally appropriate modelling learning experiences for their students as evidenced by factoring increasing demands on students' modelling skills and competencies in the planning of the students' modelling experiences. Through focused group discussions, the development of the teacher's knowledge of the modelling task solution space

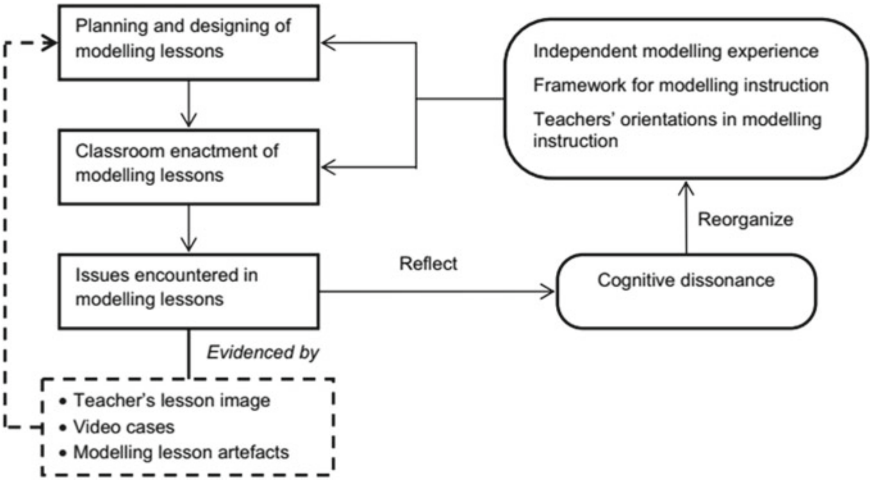

Fig. 9.3 School-based development programme for mathematical modelling (Tan and Ang 2016)

was heightened and it was inferred that the formation of the lesson image in the SBPD programme served as an important resource to support their enactment of modelling instruction practice. The patterns of the teacher's instruction were found to be driven by goals to develop students' modelling competencies as evidenced from the various lesson episodes, and the teaching orientations were mainly viewed as "modelling as content".

9.4 Research Involving Students in Solving Real-World Problems

Research involving school students in solving problems in real-world contexts in the Singapore classrooms took place only in recent years. We take real-world contexts to mean problems that include modelling activities, interdisciplinary project work as well as word problems that include authentic data. Though limited, the research involving primary school students covered different aspects such as the students' mathematical reasoning, competencies, attitudes, problem-solving, and empowerment. Most of the researches were qualitative case studies of selected groups of Primary 5 students to determine what they were capable of during mathematical modelling except for the research on attitudes which included both quantitative and qualitative aspects. As these studies were carried out in conjunction with research on developing teachers' capacity in mathematical modelling as well, the studies therefore embraced the multi-tiered teaching experiment design framework similar to what had been discussed in the previous section. One other study reported in this section is that of engaging secondary school students in interdisciplinary project work which has a history in terms of developing students to make connections with the real world.

9.4.1 Research on Students' Mathematical Modelling in Schools

A recent case study on fostering students' mathematical reasoning through engaging a modelling activity showed that students generated several models (seen as systems) in their decision-making process of selecting school swimmers for the national swimming meet (Chan et al. 2016). It was found that students came up with a point-and-elimination system where they awarded points to winners of various races and eliminated participants who exceeded a certain time in several races to narrow the number of selection choices; students also used averages to find the average time of the participants as a means to validate their findings through the point-and-elimination system. In generating these models, it was found that there appeared to be a cyclical pattern during the modelling endeavour where students exhibited reasoning behaviours such as comparing and analysing data, drawing logical conclusions, justifying decisions and procedures, explaining the mathematical concept, and then back to comparing and analysing data and so on with the testing of hypothesis surfacing whenever the teacher attempted to scaffold and extend their thinking. These reasoning aspects are highly valued as part of reformed pedagogy in mathematics education. In this respect, mathematical modelling is found to empower students not just in terms of eliciting their mathematical reasoning but has also provided opportunities for students to engage in exploration, metacognitive thinking, making decision, and interpretations (Chan et al. 2017).

It can be said that all primary school students in the research were novice modellers. It was not unexpected that they faced some difficulties in making assumptions as a modelling competence during their modelling endeavours. This was revealed in a study on assessing students' mathematical modelling competencies (Chan et al. 2012). The modelling task required students to plan the most efficient route for a teacher to travel from her home to her new school, and various bus service routes were given in an authentic map. In this regard, making a fair comparison of the various bus services plying different routes was an essential aspect of assumption making. Some students articulated that "all buses start and end at the same time" and "we assume that there are no junctions" which were seen as examples of flawed assumptions. Flawed assumptions could impede successful completion of the modelling activity. Learning to make valid assumptions is a crucial aspect of mathematical modelling as it acts as a bridge that connects the real world and the mathematical world. As such, there was a need for teacher–facilitators to scaffold the students' thinking which in turn encourage revisions of thinking and generating better models. On the other hand, students were found to be able to mathematize the problem situations, make interpretations, and justify why their model worked.

From the perspective of viewing modelling as problem-solving, it was found that students go through stages of describing the modelling task by breaking down task information, making geometrical considerations through manipulation, making predictions through revising their solutions when they were tasked to build a box with the greatest volume (Chan 2009). The modelling process forced students to manifest problem-solving behaviours, formulate important relationships between variables, and assess the appropriateness and limitations of the models and communicate their results.

One case study reported by Ang (2013) involved Secondary 3 students who worked on designing a possible layout for a car park with at least 100 parking spaces while leaving as much space as possible to the remainder of a field for other activities. From the written feedback, there were students who found the modelling activity refreshing and the experience enabled them to apply ideas of trigonometry in real-world situations. Others found the experience stressful, while one student preferred classroom time to be teaching time instead of doing projects. Video observations showed that the modelling activity was still very much guided by the teacher through promptings who later revealed that he was concerned that the students would not be able to arrive at the expected solution. Ang stressed the need for a strong framework to guide teachers who otherwise might have the tendency to over-facilitate.

One other research carried out with Primary 6 students was to find out their perception of problem-based learning after the completion of a series of six modelling activities (Chan 2011). The questionnaire showed positive responses in the attributes of interest, perseverance, and confidence. The mixed-ability students were found to have slightly more positive attitudes as they registered higher but statistically not significant mean scores in the three areas mentioned than high-ability students. It suggests that the use of modelling activities could impact how learning environments might be designed towards shaping desirable learning behaviours.

9.4.2 Research on Interdisciplinary Project Work

Research into real-world problem-solving is an established field. Singapore's journey into this could be said to have started sometime in the late 1990s where Interdisciplinary Project Work (IPW) was implemented nation-wide in all primary and secondary schools as well as pre-university colleges (MOE 1999). IPW are real-world problems which are designed to incorporate at least two content subjects as knowledge anchors in the solution process. It could be perceived that an IPW involving mathematics is somewhat similar to an applications task and could be seen as the prelude that set the stage for targeted incorporation of applications and mathematical modelling in the Singapore mathematics curriculum framework in later years of curriculum revisions (evidenced in MOE 2006a, b, 2012a). Deliberate timetabling changes were made then in schools so that student groups could spend up to 10 weeks working on one IPW, meeting with two or more facilitating teachers from different subject specializations during a common curriculum time slot. The final outcome expected from the student groups would be that of a solution to the problem and their oral presentation of their solution process, examining the content knowledge and skills applied, group collaboration efforts, communication of ideas, and independent learning and research. A key goal of using IPW in schools is the recognition of interdisciplinarity in real-world problem-solving where different content knowledge and skills come together in appropriate ways for real-world decision-making in connection to the context.

Research into the use of IPW involving mathematics has been and still is limited. Ng (2009) investigated secondary students' mathematical knowledge application, their use of metacognitive strategies, and their perception of interconnectedness in an IPW involving mathematics, science, and design and technology as anchor subjects. As reported in Ng (2011), students' application of mathematical knowledge in real-world problems was at times purely mathematical and devoid of real-world interpretations. Their solution process could be technically correct in terms of computations but lacks sense-making in real-life application purposes. Ng (2010) also noted incidences of partial metacognitive blindness in group problem-solving attempts during IPW where a dominant group member brushed aside metacognitive monitoring behaviours of others, resulting in incorrect mathematical outcomes. Ng et al. (2007) presented the development of two new scales through factor analysis for measuring students' perceptions of interconnectedness during real-world problem-solving. It was revealed that Express students tend to perceive the interconnectedness of mathematics (within mathematics, between mathematics and other subjects, and between mathematics and the real-world) more than Normal Academic students and are likely to make efforts at making connections such as engaging in using mathematics for inter-subject learning.

9.4.3 Research on Students' Solving Problems Using Authentic Information

Cheng's (2013a) study focused on solving problems in real-world context. She asserted that such problems for young children at the primary level can be designed to embed the three stages of pre-task, actual task, and extension task. The problem posing activities in Cheng (2013b) required primary students to use a variety of real-life situations to construct their own word problems. The students were also required to solve the problems that they constructed. The problems posed by the students suggested gaps in their understanding of fraction addition and multiplication concepts. For example, the inappropriate context was found in the problems posed by students suggesting that students' understanding of fraction concepts can be deepened through classroom discussion of the choice and appropriateness of the real-life context used in fraction word problems.

In another study, Cheng and Toh (2015) reported on the advantages of designing and using mathematical problems in real-world context for both teachers and young children. The teachers would deepen their own mathematical knowledge for teaching in areas such as knowledge of the curriculum, task design, and children's thinking. The students would be able to develop their mathematical processes and computational skills and become more flexible in their thinking. In addition, real-world problems provide the opportunity for students to acquire twenty-first-century competencies through critical and inventive thinking. Cheng and Toh also reported the challenges that teachers faced in using real-world problems with young children and cautioned against dressing the context too thickly such that the opportunities to unpack the mathematics are compromised.

9.5 Challenges and the Way Forward

Mathematical applications and modelling (MAM) are relatively new additions in the school mathematics curriculum in Singapore (MOE 2006a, b). Changes in the curriculum always bring to the fore some implementation issues. How well a curriculum meets its goals ultimately depends on the level of preparation of the teachers using this curriculum and the extent to which these teachers are confident to implement that curriculum in the classroom. Applications and modelling are not located within specific content domains, but rather it is expected that they cut across different areas in the curriculum. Conversely, mathematics teachers are more confident with problem-solving (see Kaur and Dindyal 2010). This is to be expected, as problem-solving is the central organising idea of the local mathematics curriculum since the 1990s, and over time, teachers have developed greater expertise in working with problem-solving as compared to MAM. While the syllabus documents from MOE have provided some implementation guidelines, it is not sufficient without having the teachers to actually go through some form of professional development courses in order for them to be ready to take on such activities with the students. The last few years have seen the inception of teacher education efforts towards advancing MAM. Nonetheless, challenges with the MAM persist.

First, let us look at *applications* in the mathematics curriculum. This is not new, as applications have been emphasized in the local curriculum in various ways and more so through its emphasis on problem-solving. Applications are not meant only for the more able students. There are aspects of applications that can be tailored to the needs of even low-performing students. However, a "teacher who is himself or herself confident in the applications of mathematics and has a strong content background with a sound pedagogical content knowledge has better chances of implementing the applications of mathematics in his or her classroom" (Dindyal and Kaur 2010, p. 328). Generally, most mathematics teachers are familiar with the idea of applications in mathematics and they do understand that this involves higher-order thinking. It is also easier for the mathematics teachers to search and adapt applications tasks for use in classrooms with students of various abilities.

Second, regarding modelling, the issues are more pronounced. Blum et al. (2007) have stated that:

> Yet while applications and modelling play more important roles in many countries' classrooms than in the past, there still exists a substantial gap between the ideas expressed in educational debate and innovative curricula on the one hand, and everyday teaching practice on the other. In particular, genuine modelling activities are still rare in mathematics classrooms. (p. xi)

Mathematical modelling details in the 2006 mathematics syllabus were scant, and understandably, the lack of awareness and understanding in this domain was a logical reason why mathematical modelling did not quite take off. In 2012, the production of the mathematical modelling resource kit by MOE (MOE 2012b) detailed essential aspects of mathematical modelling such as its definition, process, benefits, and facilitation guides alongside a series of modelling examples and students' samples solutions. The booklet was meant as a starter kit to support secondary mathematics teachers in the implementation of mathematical modelling. While the booklet may enhance the awareness of mathematical modelling, the challenge still remains with respect to how to design and facilitate modelling activities, assess performance and how to address issues that surface. Teachers need practical information about orchestrating their lessons. Ang (2010), primarily from a secondary perspective, identified three main challenges for implementing mathematical modelling in Singapore schools. First, teachers in Singapore have not had much exposure to mathematical modelling, although they are otherwise very well trained. Very few primary mathematics teachers in Singapore are specialists in the teaching of mathematics, and most of them do not have a mathematics degree, although they teach mathematics. It is quite challenging of the typical mathematics teacher to be well versed in sourcing for modelling tasks, modifying and implementing these tasks for use in the classroom. Even at the secondary level, where the mathematics teachers have followed post-secondary-level mathematics courses, the situation is not much better. Second, students are driven by assessments, and if mathematical modelling does not form an assessable component, then students are not motivated to engage in modelling activities. And third, there is a perceived lack of resources for teachers to use in the classroom. These issues were echoed by Chan et al. (2015) for teachers at the primary level. Chan et al. also highlighted the paucity of research in mathematical modelling at the local level. Although in recent years outreach platforms like the MTC and the MMO have been carried out, it would be too simplistic to assume that issues about the implementation of mathematical modelling by teachers in schools would be resolved. While teacher education and skilling of teachers in this domain have begun, the professional development work is still very much in its infancy, and it is reasonable to say that there is still some way to go before teachers get to have a deeper sense of awareness, appreciation, and confidence in implementing mathematical modelling activities in the classroom. There needs to be a sustained effort in preparing pre-service teachers and providing professional development opportunities to in-service teachers.

The following suggestions could be worth considering as the way forward in advancing teacher education and professional development efforts in the field of mathematical modelling:

(a) Problem-solving seen from a modelling perspective. While teachers are considered to be conversant in teaching mathematical problem-solving from the perspective of imparting heuristics skills for solving problems using definitive procedures, the notion of problem-solving needs to evolve towards one that is closer to solving unstructured problems where the problem solver has to develop a more productive way of thinking through cycles of expressing, testing, and revising of solutions (Lesh and Zawojewski 2007). Such cycles convey a more realistic process of problem-solving that depict what scientists and engineers do in generating models and conceptual tools towards problem resolution instead of one seen as the direct mappings between the structure of the problem situation and the structure of a symbolic expression that leads to only one way of interpreting the problem (Lesh and Doerr 2003). This perspective, which is synonymous to a modelling perspective, suggests the modelling process as is a non-trivial and thought-revealing problem-solving process. In this regard, teachers will also need to learn to get into a different pedagogical mode with less of frontal prescriptive teaching and more of being facilitators and mediators of learning.
(b) Adopting a developmental framework for instruction. It has been highlighted that having an awareness of mathematical modelling is not enough. Ang (2015) called for a practical instructional framework grounded in design principles that will help and guide teachers in preparing modelling lessons, activities, and learning experiences in the classroom. There is a variety of frameworks in the literature, and some of these arise out of different modelling interpretations and research agendas. For the local curriculum, and especially since mathematical modelling is relatively young, the adoption of a sound and user-friendly framework within any teacher development course on mathematical modelling is a welcome move.
(c) Researcher–teacher collaboration. Having the necessary resources (e.g. syllabus documents, literature, frameworks) and attending mathematical modelling courses are helpful means to acquire the knowledge in the related field. However, teacher competence in mathematical modelling can only be strengthened over time through facilitating more modelling activities (Chan et al. 2012). A worthwhile endeavour to deepen one's competence is for teachers to work with experts through the adoption of established research frameworks like those of the multi-tiered teaching experiment framework (Lesh and Kelly 2000) or the design research methodology (Dolk et al. 2010). Adopting such design frameworks provides affordances for close collaboration for exploring, adapting, and refining lesson plans in bringing about intended learning. As Gravemeijer and van Eerde (2015, p. 523) put it, they help "researchers and teachers experience the project as a collective effort in which they together analyse video footage, student work, and other data to decide on the next steps" leading towards ownership and understanding.
(d) Tried-and-tested materials. The recent research findings in local classrooms highlighted earlier pointed to positive learning outcomes, particularly with respect to what students are capable of when confronted with modelling activ-

ities. While some may argue that the research carried out are case studies and are not generalizable to be indicative of positive learning of the masses, they nonetheless have provided useful information and may be seen as encouraging signs with respect to students' displaying problem-solving and reasoning behaviours valued in reformed pedagogy. One way forward is to translate research materials and findings into instructional materials as part of professional development efforts (e.g. Ng and Lee 2012). The research–practice nexus may be an influential way in helping teachers to appreciate mathematical modelling more and at the same time give them the confidence to carry out similar modelling activities with the knowledge of what to expect using tried-and-tested materials.

(e) Greater balance in assessing process and product of learning outcomes. While there have been some discussions to downplay the emphasis on grade chasing in assessment, it remains a challenge since performance in high-stake examinations is still an important criterion in student selection purposes. In order not to relegate the solving of real-world or modelling problems as a side dish to be carried out as time fillers after examinations, the values and benefits of engaging students in such activities need to be seen as upholding student learning and even performances through integrating them within lessons. Findings from qualitative research that show the value of what students go through as mathematical processes can play a greater role in heightening awareness with respect to eliciting those processes that are valued in mathematics education.

(f) Professional learning communities (PLCs). Many education institutions worldwide are leveraging on innovative ways for teachers to network and develop themselves professionally. The Ministry of Education has acknowledged that PLCs form an integral part of in-service teacher professional development (MOE 2014). In this light, an interest group that focuses on mathematical modelling may be set up at the school cluster or zonal level for teachers to work and learn collaboratively by sharing resources and experiences in this domain. Ang from NIE in 2017 launched an online mathematical modelling resource centre (www.mathmodelling.sg) through a funded research project in support of advancing the field of mathematical modelling for teachers. There is a prospective advantage that incorporating PLCs with digital technology using such online resources may prove to be a flexible way for professional growth.

9.6 Conclusion

The central aim of the Singapore mathematics curriculum is mathematical problem-solving. Professional development of teachers and research have been focused on numerous aspects of problem-solving in the last 30 years. In the last decade, there has been increased attention paid to solving real-world problems, in particular, with a focus on the mathematical modelling aspect of the process component of the mathematics curriculum framework. With the recognition that problem-solving abilities in today's world are more than just about applying a heuristic or getting the right answer, there is a need to expose and equip students to complex situations where they would have to describe, explain, analyse, construct, manipulate, and interpret those situations and to develop competencies that are valued in twenty-first-century economies. Many countries have embraced Science, Technology, Engineering and Mathematics (STEM) instructional approaches as a means to engage students learning in real-world situations. Through expanding the idea of problem-solving to incorporate applications and modelling, the Singapore mathematics curriculum has poised itself in being relevant by implementing reformed pedagogies and developing twenty-first-century skills with applications and modelling during a time of change.

This chapter has outlined the teacher development as well as the research efforts carried out in advancing the cause of applications and modelling in Singapore schools. Locally, this domain is still in its infancy and what has been implemented has left a trial of documentation for reflections and refinements. There is still much to learn in this domain as there are challenges to overcome. Nonetheless, it has been a meaningful start in the last 10 years to reinforce the importance of solving problems in real-world situations as a fundamental process of mathematical literacy.

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