Chapter 6
Singapore's Participation in International Benchmark Studies—TIMSS, PISA and TEDS-M

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Abstract Large-scale international assessments of schooling effects attempt to provide comparative data for participating countries. Two such assessments are the Trends in International Mathematics and Science Study (TIMSS) conducted by the International Association for the Evaluation of Educational Achievement (IEA) and the Programme for International Student Assessment (PISA) conducted by the Organization for Economic Co-operation and Development (OECD). Singapore has participated in TIMSS since 1995 and PISA since 2009. These studies use student outcomes as measures of school effectiveness and educational achievement. They focus on student achievement mainly in three school subjects: mathematics, science and language. Other international studies like the Teacher Education and Development Study in Mathematics (TEDS-M) also provide comparative data on teachers of mathematics and related matters. Singapore participated in TEDS-M. The results of TEDS-M were available in 2012. This chapter presents snapshots of significant data and findings of Singapore's participation in TIMSS 2015, PISA 2009 and 2015 and TEDS-M. For TIMSS 2015, it focuses on the performance of Singapore students and their engagement and attitudes for mathematics. For PISA 2009 and 2015, it focuses on the performance of Singapore students and their exposure to mathematics content and their drive and motivation to learn mathematics. For TEDS-M, it focuses on the national contexts and policies for teacher education and nature of mathematics teacher education programmes in Singapore. It also examines the performance of future teachers from Singapore in mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) and their beliefs and perceptions of opportunities to learn. The chapter concludes with possible reasons about the commendable performance of Singapore students in TIMSS and PISA.

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6.1 Introduction

Large-scale international assessments of schooling effects attempt to provide comparative data for participating countries. Two such assessments are the Trends in International Mathematics and Science Study (TIMSS) conducted by the International Association for the Evaluation of Educational Achievement (IEA) and the Programme for International Student Assessment (PISA) conducted by the Organization for Economic Co-operation and Development (OECD). Singapore has participated in both of them. These studies use student outcomes as measures of school effectiveness and educational achievement. They focus on student achievement mainly in three school subjects: mathematics, science and language. Singapore participates in TIMSS and PISA for four main purposes, which according to Kaur (2013b) are as follows:

- to benchmark the outcomes of schooling, vis-à-vis the education system against international standards;
- to learn from educational systems that are excelling;
- to update school curriculum and keep abreast of global advances; and
- to contribute towards the development of excellence in education internationally.

Other international studies like the Teacher Education and Development Study in Mathematics (TEDS-M) also provide comparative data on teachers of mathematics and related matters. Singapore participated in TEDS-M.

This chapter presents snapshots of significant data and findings of Singapore's participation in TIMSS 2015 (Mullis et al. 2016), PISA 2009 (OECD 2010a), PISA 2012 (OECD 2013a), PISA 2015 (OECD 2015) and TEDS-M (Tatto et al. 2012). For TIMSS 2015, it focuses on the performance of Singapore students and their engagement and attitudes for mathematics (Mullis et al. 2016). For PISA it focuses on the performance of overall Singapore students in PISA 2009, 2012 and 2015 and specifically for PISA 2012 the performance of students from Singapore on some released sample items and students' motivation to learn mathematics (OECD 2013a). For TEDS-M, it focuses on the national contexts and policies for teacher education and nature of mathematics teacher education programmes in Singapore. It also examines the performance of future teachers from Singapore in mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) and their beliefs and perceptions of opportunities to learn (Tatto et al. 2012). The chapter concludes with possible reasons about the commendable performance of Singapore students in TIMSS and PISA.

6.2 Trends in International Mathematics and Science Study (TIMSS)

Trends in International Mathematics and Science Study (TIMSS) is a series of international mathematics and science assessments conducted every four years by the International Association for the Evaluation of Educational Achievement (IEA). TIMSS is designed to provide trends in fourth- and eighth-grade mathematics and science achievement in an international context. TIMSS 2015 was the sixth and most recent cycle of assessment. Forty-six countries participated at the eighth-grade level, and 56 countries participated at the fourth-grade level. Data were collected from representative samples of students, in participating countries, at the respective grade levels. However, the teacher participants may not constitute representative samples as they were the teachers of the students. The TIMSS 2015 International Results in Mathematics (Mullis et al. 2016) contain analysis of data that spans from achievement of participants to home environment that supports mathematics and science achievement, school resources for teaching mathematics and science, school climate, teacher preparation and classroom instruction. Singapore has participated in all the six cycles of TIMSS so far. Several publications have focused on the performance of Singapore's students in TIMSS 1995, 1999, 2003, 2007 and 2011 (Kaur 2005, 2009a, b, 2013a; Boey 2009; Kaur et al. 2012, 2013).

In this section, we focus on the performance of Singapore students and their engagement and attitudes for mathematics in TIMSS 2015. The data and findings reported in this chapter are drawn from the respective international mathematics reports of TIMSS 2015 (Mullis et al. 2016), TIMSS 2011 (Mullis et al. 2012) and TIMSS 2007 (Mullis et al. 2008). All of these reports are available at the IEA TIMSS and PIRLS International Study Centre website (http://timssandpirls.bc.edu).

6.2.1 *Performance of Singapore Students in TIMSS 2015*

The performance of Singapore students in TIMSS in the six cycles held so far has been consistently outstanding and captured the attention of many educators and politicians worldwide. Table 6.1 shows the rank of Singapore in the last six cycles of TIMSS for both grades 4 and 8.

The international benchmarks presented as part of the TIMSS data help to provide participating countries with a distribution of the performance of their students in an international setting. For a country, the proportions of students reaching these benchmarks are perhaps telling of certain strengths and weaknesses of mathematics education programmes of the country. The benchmarks delineate performance at four points of the performance scale. Characteristics of students at each of the benchmarks are shown in Fig. 6.1.

Students who participated in TIMSS 2015 at the grade 8 level are from the same cohort of grade 4 students who participated in TIMSS 2011. Similarly, the 8th graders

Advanced International Benchmark - 625	
Grade 4 *Students can apply their understanding and knowledge in a variety of relatively complex situations and explain their reasoning*. They solve a variety of multi-step word problems involving whole numbers. Students at this level show an increasing understanding of fractions and decimals. They can apply knowledge of a range of two- and three-dimensional shapes in a variety of situations. They can interpret and represent data to solve multi-step problems.	Grade 8 *Students can apply and reason in a variety of problem situations, solve linear equations, and make generalizations*. They can solve a variety of fraction, proportion, and percent problems and justify their conclusions. Students can use their knowledge of geometric figures to solve a wide range of problems about area. They demonstrate understanding of the meaning of averages and can solve problems involving expected values.
High International Benchmark - 550	
Grade 4 *Students can apply their understanding and knowledge to solve problems.* They can solve word problems involving operations with whole numbers, simple fractions, and two-place decimals. Students demonstrate understanding of geometric properties of shapes and of angles that are less than or greater than a right angle. Students can interpret and use data in tables and a variety of graphs to solve problems.	Grade 8 *Students can apply their understanding and knowledge in a variety of relatively complex situations*. They can use information to solve problems involving different types of numbers and operations. They can relate fractions, decimals, and percentages to each other. Students at this level show basic procedural knowledge related to algebraic expressions. They can solve a variety of problems with angles including those involving triangles, parallel lines, rectangles, and similar figures. Students can interpret data in a variety of graphs and solve simple problems involving outcomes and probabilities.
Intermediate International Benchmark	
Grade 4 *Students can apply basic mathematical knowledge in simple situations*. They demonstrate an understanding of whole numbers and some understanding of fractions and decimals. Students can relate two- and three-dimensional shapes and identify and raw shapes with simple properties. They can read and interpret bar graphs and tables.	Grade 8 *Students can apply basic mathematical knowledge in a variety of situations.* They can solve problems involving negative numbers, decimals, percentages, and proportions. Students have some knowledge of linear expressions and two- and three-dimensional shapes. They can read and interpret data in graphs and tables. They have some basic knowledge of chance.
Low International Benchmark	
Grade 4 *Students have some basic mathematical knowledge*. They can add and subtract whole numbers, have some understanding of multiplication by one-digit numbers, and can solve simple word problems. They have some knowledge of simple fractions, geometric shapes, and measurement. Students can read and complete simple bar graphs and tables.	Grade 8 *Students have some knowledge of whole numbers and basic graphs*.

Fig. 6.1 Descriptions of the TIMSS 2015 International Benchmarks. *Source* Mullis et al. (2016) Exhibits 2.1 and 2.8

Table 6.1 Ranking of Singapore's students for Mathematics in TIMSS

TIMSS	Rank	
	Grade 4	Grade 8
1995	1	1
1999	–	1
2003	1	1
2007	2	3
2011	1	2
2015	1	1

Source http://timssandpirls.bc.edu

Table 6.2 Percentage of Singapore students in last three cycles of TIMSS at the respective benchmarks for mathematics achievement

TIMSS	Grade	International benchmarks			
		Advanced (625)	High (550)	Intermediate (475)	Low (400)
2015	4	50 (2.1)	80 (1.7)	93 (0.9)	99 (0.3)
2011	4	43 (2.0)	78 (1.4)	94 (0.7)	99 (0.2)
2007	4	41 (2.1)	74 (1.7)	92 (0.9)	98 (0.3)
2015	8	54 (1.8)	81 (1.5)	94 (0.9)	99 (0.2)
2011	8	48 (2.0)	78 (1.8)	92 (1.1)	99 (0.3)
2007	8	40 (1.9)	70 (2.0)	88 (1.4)	97 (0.6)

( )—standard errors
Source Mullis et al. (2016) Exhibits 2.3 and 2.10

in TIMSS 2011 were from the same cohort of 4th graders in TIMSS 2007. Table 6.2 shows the percentage of grade 4 and 8 students from Singapore at the benchmarks for the past three cycles of TIMSS, namely TIMSS 2007, TIMSS 2011 and TIMSS 2015.

It is apparent from Table 6.2 that as the cohorts of students progressed from 4th to 8th grade, higher proportions of the students reached the advanced international benchmark. 41% of grade 4 students at the advanced international benchmark in TIMSS 2007 compared to 48% grade 8 at the same benchmark in TIMSS 2011 and 43% grade 4 at the advanced international benchmark in TIMSS 2011 compared to 54% grade 8 at the same benchmark in TIMSS 2015. Table 6.2 also shows that percentages of grade 4 and 8 students reaching the high and advanced benchmarks have steadily increased over the last three cycles of TIMSS. The periodic revisions of the school mathematics curriculum from the year 2000 onwards placing heightened emphasis on problem-solving and mathematical processes such as thinking skills and reasoning appear to have contributed towards improved student learning of mathematics (Ministry of Education 2016a).

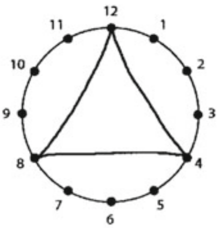
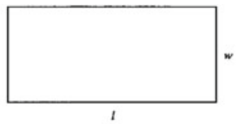
Grade 4	Grade 8
Content domain: Geometric Shapes and Measures Cognitive domain: Reasoning Description: Draws a specified geometric shape by connecting dots on a circle In the circle, draw a triangle with all sides the same length.  What points did you connect? 12, 4, 8, 12 Percent full credit: 64 (1.8) International average: 58 (0.3)	Content domain: Algebra Cognitive domain: Applying Description: Identifies the formula that represents a situation involving area  The shape above is a rectangle, with length *l*, and width *w*. If the length is doubled and the width stays the same, which formula gives the area (*A*) of the new rectangle? Ⓐ $A = 2l + 2w$ Ⓑ $A = 2l + 4w$ ● $A = 2lw$ Ⓓ $A = 4lw$ Percent full credit: 82 (1.6) International average: 51 (0.3)

Fig. 6.2 Examples of items of the High International Benchmark Level. *Source* Mullis et al. (2016) Exhibits 2.6.3 and 2.13.2

However, for the low international benchmark level, the proportion of students reaching it improved by 1% from 2007 to 2011 but remained the same at 99% from 2011 to 2015. These findings have been of concern to policy makers and educators in Singapore. It may be said that the revisions of the curriculum have had limited impact on these students. Since 2013, teachers of low attainers in mathematics have received additional support in the form of resources and self-development (see Chap. 13 for details).

Figures 6.2 and 6.3 show items of the High International Benchmark and Advanced International Benchmark Levels, respectively, for TIMSS 2015. For each item in the figures, the per cent correct for Singapore and the international average are stated. The grade 4 item, shown in Fig. 6.2, is a non-routine and challenging one for 4th graders in Singapore. Study of circles and equilateral triangles is beyond the scope of the mathematics curriculum in grade 4. As such, Singapore's 4th graders performed reasonably well on the item. Their counterparts from Republic of Korea (76%) and Japan (73%) did better than them. The grade 8 item, shown in Fig. 6.2, may be said to be a routine one for 8th graders in Singapore schools. Students from Singapore were ranked the best for the item.

Figure 6.3 shows items of the Advanced International Benchmark Level. In the figure, the grade 4 item is a non-routine one for Singapore's 4th graders. The multi-step word problem is a higher-order thinking task. Nevertheless, the students performed reasonably well on it and were ranked fourth. Their counterparts from Republic of Korea (77%), Hong Kong SAR (71%) and Japan (66%) did better than them.

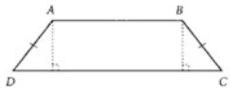
Grade 4	Grade 8
Content domain: Number Cognitive domain: Reasoning Description: Solves a multi-step reason problem involving division Sally has 12 lengths of wire, 40 round beads, and 48 flat beads. She uses 1 length of wire, 10 round beads, and 8 flat beads to make 1 bracelet. If Sally makes all her bracelets the same, how many bracelets can she make? Ⓐ 40 Ⓑ 12 Ⓒ 5 ● 4 Percent full credit: 65 (2.1) International average: 37 (0.3)	Content domain: Geometry Cognitive domain: Reasoning Description: Uses the Pythagorean theorem in finding the perimeter of a trapezoid  *ABCD* is a trapezoid with *AB* = 10 cm and *CD* = 16 cm. *AD* = *BC*. The distance between the parallel lines, *AB* and *CD*, is 4 cm. What is its perimeter? ● 36 cm Ⓑ 34 cm Ⓒ 32 cm Ⓓ 30 cm Percent full credit: 68 (1.8) International average: 32 (0.3)

Fig. 6.3 Examples of items of the Advanced International Benchmark Level. *Source* Mullis et al. (2016) Exhibits 2.7.1 and 2.14.4

The grade 8 item may be said to be a routine one for 8th graders in Singapore schools. They were ranked second to Chinese Taipei (72%) for the item.

6.2.2 *Engagement and Attitudes of Singapore Students in TIMSS 2015*

As part of TIMSS 2015, students completed tests on mathematics and science and also a student questionnaire that collected data on students' views about their mathematics instruction and attitudes towards mathematics. Grade 4 students were asked to indicate their degrees of agreement to statements on the *Students' Views on Engaging Teaching in Mathematics Lessons Scale, Students Like Learning Mathematics Scale* and *Students Confident in Mathematics Scale*. The grade 8 students were asked to indicate their degrees of agreement to statements on four scales, the same three scales as the grade 4 and the Students Value Mathematics Scale. In this section, we present data for both grades 4 and 8 for the three common scales that were part of their student questionnaires.

6.2.2.1 Students' Views on Engaging Teaching in Mathematics Lessons

The student questionnaire asked students about how engaging their mathematics lessons were. Students were scored according to their degree of agreement with ten

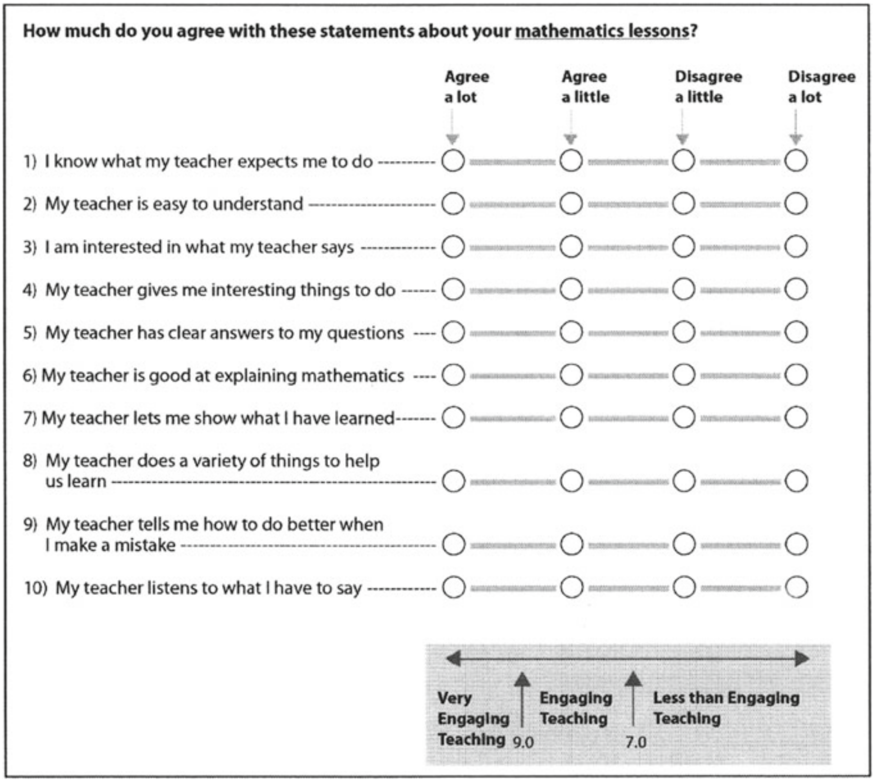


Fig. 6.4 Engaging Teaching in Mathematics Lessons Scale. *Source* Mullis et al. (2016) Exhibits 10.1 and 10.2

statements on the *Students' Views on Engaging Teaching in Mathematics Lessons* Scale shown in Fig. 6.4.

Students who experienced Very Engaging Teaching in mathematics lessons had a score on the scale of at least 9.0, which corresponds to their "agreeing a lot" with five of the ten statements and "agreeing a little" with the other five, on average. Students who experienced teaching that was Less than Engaging had a score no higher than 7.0, which corresponds to their "disagreeing a little" with five of the ten statements and "agreeing a little" with the other five, on average. All other students experienced Engaging Teaching in mathematics lessons. Table 6.3 shows students' views on Engaging Teaching in Mathematics Lessons for students from grades 4 and 8 from Singapore and the international averages in TIMSS 2015.

The 4th graders' average scale score for views on Engaging Teaching in Mathematics Lessons ranged from 11.2 for Bulgaria to 8.2 for Japan, while that for 8th graders ranged from 11.2 for Jordon to 8.4 for Republic of Korea. It is apparent from Table 6.3 that 55% of 4th graders from Singapore found their mathematics lessons very engaging. Just like their peers from the top-performing countries, this result is

Table 6.3 Students' views on Engaging Teaching in Mathematics Lessons

Grade/Country	Very Engaging Teaching		Engaging Teaching		Less than Engaging Teaching		Average scale score
	Per cent of students	Average achievement	Per cent of students	Average achievement	Per cent of students	Average achievement	
Grade 4							
Singapore	55 (1.0)	625 (4.0)	37 (0.7)	613 (4.3)	7 (0.5)	592 (6.7)	9.3 (0.04)
International average	68 (0.2)	510 (0.4)	26 (0.1)	498 (0.6)	5 (0.1)	481 (1.2)	
Grade 8							
Singapore	33 (1.0)	633 (3.6)	52 (0.8)	620 (3.4)	16 (0.8)	596 (6.3)	9.7 (0.04)
International average	43 (0.2)	494 (0.7)	41 (0.2)	478 (0.6)	17 (0.2)	464 (0.9)	

( )—standard errors
Source Mullis et al. (2016) Exhibits 10.1 and 10.2

lower than the international average of 68%, which contrasts with their achievement on the test items. Also for the 8th graders, 33% found their mathematics lessons very engaging compared to the international average of 43%. As the data collected represent students' perceptions, it appears that more 4th graders compared with 8th graders in Singapore mathematics lessons perceived that their mathematics lessons were very engaging. A perception of an engaging lesson may be one where students use manipulatives or carry out activities such as measuring lengths and volumes. Such lessons are more prevalent in the primary school than secondary school mathematics lessons in Singapore. It is noteworthy that the percentages of students at both grade levels are close to the international averages for "Less than Engaging Teaching" though the average achievements of the students from Singapore are much higher than the international averages. Such a finding prompts one to speculate if achievement on the test items is solely an outcome of "teaching" during mathematics lessons.

6.2.2.2 Students Like Learning Mathematics

The student questionnaire also asked students about their liking of learning mathematics. Students were scored according to their degree of agreement with nine statements on the *Students Like Learning Mathematics* Scale shown in Fig. 6.5.

Students who very much Like Learning Mathematics had a score of at least 10.1, which corresponds to their "agreeing a lot" with five of the nine statements and "agreeing a little" with the other four, on average. Students who do not Like Learning Mathematics had a score no higher than 8.3, which corresponds to their "disagreeing

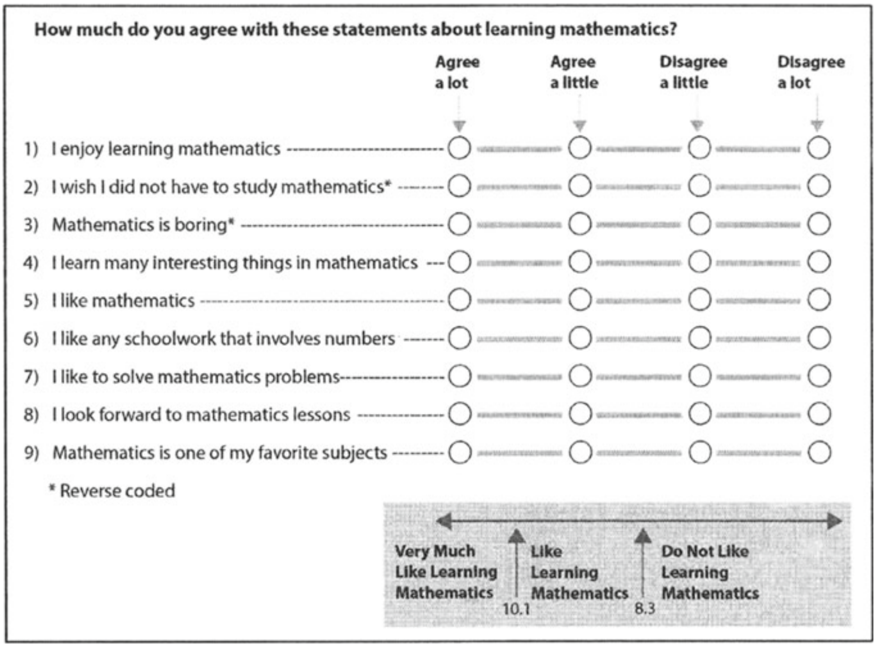


Fig. 6.5 Like Learning Mathematics Scale. *Source* Mullis et al. (2016) Exhibits 10.3 and 10.4

a little" with five of the nine statements and "agreeing a little" with the other four, on average. All other students Like Learning Mathematics. Table 6.4 shows the data for students' Like Learning Mathematics for students from grades 4 and 8 from Singapore and the international averages in TIMSS 2015.

The 4th graders' average scale score for Like Learning Mathematics ranged from 11.3 for Turkey to 8.9 for Republic of Korea, while that for 8th graders ranged from 11.4 for Botswana to 8.7 for Slovenia. It is apparent from Table 6.4 that only 39% of 4th graders from Singapore very much like learning mathematics. Just like their peers from the top-performing countries, this result is lower than the international average of 46%, which again contrasts with their achievement on the test items. However, for the 8th graders 24% very much like learning of mathematics and this result was marginally higher than the international average of 22% unlike that for the other top-performing countries. The push for mastery in the learning of mathematics in Singapore schools may have produced good achievement scores but certainly have not provided all students with enjoyment that translated into feelings of "like".

Table 6.4 Students Like Learning Mathematics

Grade/Country	Very much Like Learning Mathematics		Like Learning Mathematics		Do not Like Learning Mathematics		Average scale score
	Per cent of students	Average achievement	Per cent of students	Average achievement	Per cent of students	Average achievement	
Grade 4							
Singapore	39 (0.8)	640 (4.1)	38 (0.7)	611 (4.1)	23 (0.8)	591 (4.5)	9.6 (0.03)
International average	46 (0.2)	521 (0.5)	35 (0.1)	495 (0.5)	19 (0.1)	483 (0.8)	
Grade 8							
Singapore	24 (0.7)	654 (3.2)	42 (0.8)	625 (3.5)	33 (0.8)	592 (4.3)	10.1 (0.03)
International average	22 (0.1)	518 (0.8)	39 (0.1)	485 (0.6)	38 (0.2)	462 (0.6)	

( )—standard errors
Source Mullis et al. (2016) Exhibits 10.3 and 10.4

6.2.2.3 Students Confident in Mathematics

The student questionnaire also asked students about their confidence in mathematics. Students were scored according to their degree of agreement with nine statements on the *Students Confident in Mathematics* Scale shown in Fig. 6.6.

Students Very Confident in Mathematics had a score of at least 10.6, which corresponds to their "agreeing a lot" with five of the nine statements and "agreeing a little" with the other four, on average. Students who were Not Confident in Mathematics had a score no higher than 8.5, which corresponds to their "disagreeing a little" with five of the nine statements and "agreeing a little" with the other four, on average. All other students were Confident in Mathematics. Table 6.5 shows the data for students' Confidence in Mathematics for students from Singapore and the international averages in TIMSS 2015.

The 4th graders' average scale score for Confident in Mathematics ranged from 10.6 for Kazakhstan to 8.9 for Chinese Taipei, while that for 8th graders ranged from 10.7 for Israel to 9.1 for both Thailand and Chinese Taipei. It is apparent from Table 6.5 that 19% of 4th graders from Singapore reported that they were very confident in mathematics. Just like their peers from the top-performing countries, this result is lower than the international average of 32% despite their commendable achievement on the test items. For 8th graders, the international average was 14% for students claiming that they were very confident in mathematics and the per cent for the same was marginally lower, i.e. 13%, for Singapore students. Asian students, including those from Singapore, are always modest in making claims of achievement.

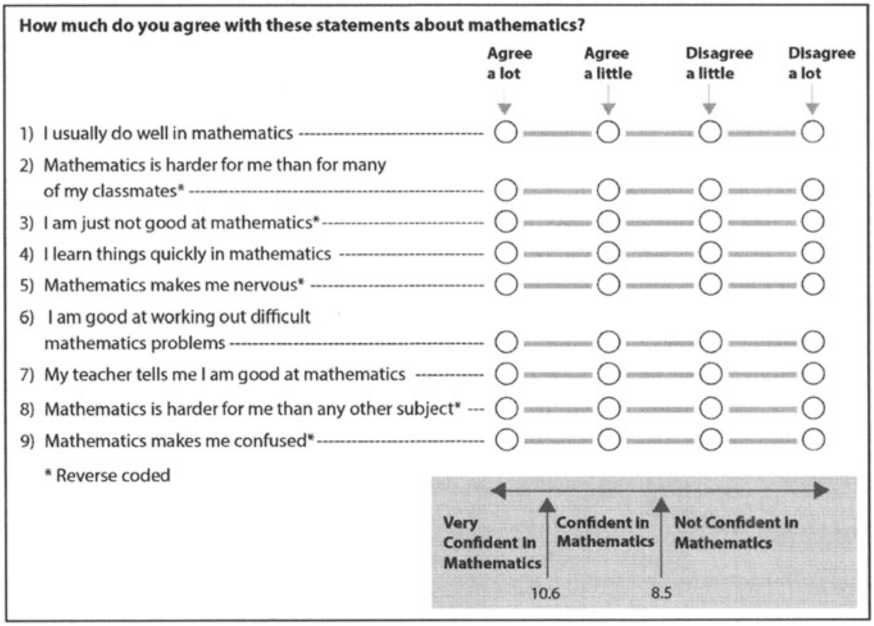


Fig. 6.6 Students Confident in Mathematics Scale. *Source* Mullis et al. (2016) Exhibits 10.5 and 10.6

Table 6.5 Students Confident in Mathematics

Grade/Country	Very Confident in Mathematics		Confident in Mathematics		Not Confident in Mathematics		Average scale score
	Per cent of students	Average achievement	Per cent of students	Average achievement	Per cent of students	Average achievement	
Grade 4							
Singapore	19 (0.8)	681 (3.6)	42 (0.6)	633 (3.6)	39 (1.1)	572 (4.0)	9.2 (0.05)
International average	32 (0.1)	546 (0.5)	45 (0.1)	502 (0.5)	23 (0.1)	460 (0.6)	
Grade 8							
Singapore	13 (0.5)	675 (3.0)	41 (0.7)	642 (2.8)	46 (0.8)	588 (4.0)	9.7 (0.04)
International average	14 (0.1)	554 (0.8)	43 (0.1)	494 (0.6)	43 (0.2)	449 (0.6)	

( )—standard errors
Source Mullis et al. (2016) Exhibits 10.5 and 10.6

Therefore, it is not alarming that students from the top 5 education systems in Asia both at grades 4 and 8 do not agree a lot or agree a little with the nine statements in Fig. 6.6.

6.3 Programme for International Student Assessment (PISA)

Programme for International Student Assessment (PISA) was launched by the OECD in 1997. It aims to evaluate education systems worldwide every three years by assessing 15-year-olds' competencies in the key subjects: reading, mathematics and science. Most importantly, the PISA assessments focus on literacy and the use of knowledge by participants. Although in every cycle, all the three subjects are assessed, only one of the subjects is the focus. For example in PISA 2009, reading was the focus; in PISA 2012, mathematics was the focus; and in PISA 2015, science was the focus. Initially, participants of PISA were OECD countries, but at present, non-OECD countries like Singapore and economies like Shanghai are also participating. More than 70 economies participated in PISA 2009. Singapore participated in PISA for the first time in 2009. PISA collects data from students and their school leaders. After every cycle of PISA, the myriad analysis of the data is publically available for everyone through the OECD web pages (http://www.oecd.org/pisa/) and also in the form of reports such as *PISA 2009 Results: What Students Know and Can Do* (OECD 2010a); *PISA 2009 Results: Overcoming Social Background* (OECD 2010b); and *PISA 2009 Results: Learning to Learn* (OECD 2010c).

6.3.1 Performance of Singapore Students in PISA

As one of the world's best-performing school education systems in a 2007 Mckinsey study of teachers (Barber and Mourshed 2007), Singapore has been among the top-performing countries in PISA for the last three cycles. Table 6.6 shows that Singapore has moved up rapidly in PISA overall rankings from fifth in 2009 to first in 2015. It was noted that the results of the 2015 and past PISA cycles reflected the deliberate curricular shifts made over the years towards a greater emphasis on higher-order, critical thinking skills, and pedagogical shifts in moving learning beyond content to mastery and application of skills to solve authentic problems in various contexts (Ministry of Education 2016b).

The PISA 2012 focused on mathematics. Singapore ranked second with a mean score of 573 points that was significantly lower than Shanghai, China, and significantly higher than Hong Kong that ranked third. For PISA 2012, Table 6.7 shows that on average across OECD countries, 13% of students were top performers in mathematics with proficiency Level 5 or 6. These students have capacity of develop-

Table 6.6 Global features of Singapore performance in PISA 2009, 2012 and 2015

Year	Focus	Rank (overall)	Mathematics		Reading	Science
			Average score	Rank	Average score	Average score
2009	Reading	5	562	2	526	542
2012	Mathematics	2	573	2	542	551
2015	Science	1	564	1	535	556

Source OECD (2009, 2012, 2015)

Table 6.7 Percentage of students from Singapore and the OECD average in PISA 2012 at each level of mathematics proficiency

Country	Rank	Average	International Benchmarks				
			Above level 2 (420)	Above level 3 (482)	Above level 4 (545)	Above level 5 (607)	Above level 6 (669)
Singapore	2	573 (1.3)	91.7	79.5	62.0	40.0	19.0
OECD average		490 (0.4)	77.0	54.5	30.8	12.6	3.3

( )—standard errors
Source OECD (2012)

ing and working with model for complex situations, and they can work strategically using broad, well-developed thinking and reasoning skills (OECD 2013a). Two-fifths (40%) of students from Singapore were at these levels. On the other side, 23% of students in OECD countries did not achieve Level 2 in PISA mathematics. Level 2 is the baseline level on the mathematics proficiency scale that is required for full participation in modern society (OECD 2013a). The percentage of low achievers who were below Level 2 was 8.3% for Singapore.

6.3.2 Students Performance on Mathematics Released Sample Items of PISA 2012

For mathematics, PISA assesses mathematical literacy that is defined as an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens (OECD 2013d, p. 17).

The PISA mathematics assessment framework has three dimensions, which are:

1. Processes (three categories and seven fundamental capabilities)

Categories (i) formulating situations mathematically, (ii) employing mathematical concepts, facts, procedures and reasoning and (iii) interpreting, applying and evaluating mathematical outcomes.

Fundamental mathematical capabilities (i) communicating, (ii) mathematizing, (iii) representation, (iv) reasoning and argument, (v) devising strategies for problem-solving, (vi) using symbolic, formal and technical language and operations and (vii) using mathematical tools.

2. Content (four overarching ideas)

(i) quantity, (ii) space and shape, (iii) change and relationships and (iv) uncertainty and data.

3. Contexts (four categories)

(i) personal, (ii) occupational, (iii) societal and (iv) scientific (OECD 2013d, p. 18).

This section presents two examples, Drip Rate and Revolving Door, with accompanying released sample items from the PISA 2012. These items illustrate the dimensions of the PISA assessment framework and also highlight the performance of students from Singapore.

Figure 6.7 shows Example 1 (Drip Rate), comprising items (Questions 1 and 3) categorized as change and relationships. The key to Question 1 lies in students being able to relate the change in drip rate to the change in time, given the variables drop factor and volume are held constant. This question intends to model the change and relationships with appropriate algebra functions, as well as interpreting symbolic representations of relationships. A form of proportional reasoning is needed. This is a question at mathematics proficiency Level 5, and the challenge is that it requires students to give a brief explanation of the effect of specified change to one variable on a second variable if other variables remain constant. In particular, students' explanation needs to describe both the direction of the effect (i.e. getting smaller) and its size (i.e. 50%).

On average across OECD, less than one-quarter of the students answered this question correctly. Only 33.42% of the students from Singapore could state both the direction and size of the effect correctly and obtained full credit. Another 26.97% of the Singapore students could state either the direction or the size of the effect, but not both, and obtained partial credit.

Question 3 requires students to transpose an equation to find expression for volume v so as to obtain the required result by substituting values of two variables into the expression. This is a question at Level 5 proficiency. The question also makes certain demand on interpreting formula linking three variables in a medical context and translating from natural language to symbolic language. Students from Singapore did well with 63.86% obtaining full credit.

Figure 6.8 shows Example 2 (Revolving Door) with three accompanying released items. The first two questions are space and shape items. Question 1 is a proficiency Level 3 question. It requires some basic factual knowledge about circle geometry and

Example 1: Drip Rate

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients. Nurses need to calculate the drip rate, D, in drops per minute for infusions. They use the formula $D = \frac{dv}{60n}$ where
d is the drop factor measured in drops per millilitre (mL)
v is the volume in mL of the infusion
n is the number of hours the infusion is required to run.

Question 1:
Description: Explain the effect that doubling one variable in a formula has on the resulting value if other variables are held constant.
Content area: change and relationships; Context: Occupational; Process: Employ

A nurse wants to double the time an infusion runs for.
Describe precisely how *D* changes if n is **doubled** but d and *v* do not change.

	Full credit	Partial credit
Singapore	33.42% (1.1)	26.35% (0.98)
OECD average	16.32% (0.18)	11.82% (0.14)

( ) – standard errors

Question 3:
Description: Transpose an equation and substitute two given values
Content area: Change and relationships; Context: Occupational; Process: Employ

Nurses also need to calculate the volume of infusion, v, from the drip rate, D. An infusion with a drip rate of 50 drops per minute has to be given to a patient for 3 hours. For this infusion the drop factor is 25 drops per millilitre.

What is the volume in mL of the infusion?
Volume of the infusion: ……………mL

	Full credit
Singapore	63.86% (1.2)
OECD average	25.70% (0.21)

( ) – standard errors

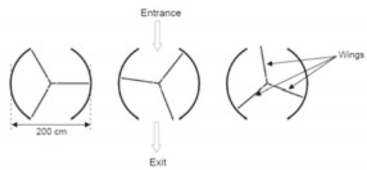
Fig. 6.7 Drip Rate example with accompanying released items and Singapore students' achievement in PISA 2012. *Source* OECD (2013c, pp. 6–8); OECD (2012)

spatial understanding of the diagrams. Students need to recognize the relevance of the information about equal sectors in order to find the central angle of a sector of a circle. The performance of students from Singapore on this question was commendable (76%), which was far above the OECD average (58%).

Question 2 requires students to interpret a geometrical model in a real-life situation and then calculate the length of an arc. It requires substantial geometry reasoning about the design features of revolving door that enable it to perform its function as a doorway while maintaining a sealed space that prevents air flowing between the entrance and exit. This is a novel question and it requires some creative thought, not just the application of any textbook knowledge they would have learnt. Classified as formulate for the dimension process, this item draws very heavily on the fundamental mathematical capability of reasoning and argument, because the problem in the real situation has to be carefully analysed and transformed into a mathematical problem

Example 2: Revolving Door

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



Question 1:
Description: Compute the central angle of a sector of a circle.
Content area: Space and shape; Context: Scientific; Process: Employ.

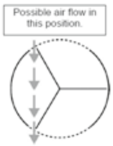
What is the size in degrees of the angle formed by two door wings?
Size of the angle: ……………… °

	Full credit
Singapore	75.72% (1.1)
OECD average	57.67% (0.25)

( ) – standard errors

Question 2:
Description: Interpret a geometrical model of real life situation to calculate the length of an arc.
Content area: Space and shape; Context: Scientific; Process: Formulate

The two door openings (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.
What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?



Maximum arc length: ……………… cm

	Full credit
Singapore	13.17% (0.81)
OECD average	3.47% (0.08)

( ) – standard errors

Question 3:
Description: Identify information and construct an (implicit) quantitative model to solve the problem.
Content area: Quantity; Context: Scientific; Process: Formulate

The door makes 4 complete rotations in a minute. There is room for a maximum of two people in each of the three door sectors.
What is the maximum number of people that can enter the building through the door in 30 minutes? (A) 60 (B) 180 (C) 240 (D) 720

	Full credit
Singapore	59.30% (0.99)
OECD average	46.42% (0.24)

( ) – standard errors

Fig. 6.8 Revolving Door example with accompanying released items and Singapore students' achievement in PISA 2012. *Source* OECD (2013c, pp. 33–35); OECD (2012) in geometric terms and then back again to the contextual situation of the problem. This question was one of the most challenging questions in the PISA 2012 test and it belongs to the upper end of Level 6 on the mathematics proficiency scale. Less than 15% of the students from Singapore were able to complete this question correctly. On average across OECD countries, only 3.5% of the students answered this question correctly.

Question 3 addresses a different type of challenge, involving rates and proportional reasoning, and it lies at mathematics proficiency Level 4. Students are required to identify relevant information and construct an implicit quantitative model to solve the problem. The content category of the question is quantity category because of the way in which the multiple relevant quantities have to be combined by number operations to produce the required number of persons to enter in 30 min. The question also makes considerable demand on the formulating process. A student needs to understand the real-world problem so as to assemble the data provided in the right way. Students from Singapore did reasonably well on this item with 59.3% obtaining full credit. On average across OECD countries, almost half of the students answered this question correctly.

6.3.3 Singapore Students' Exposure to Mathematics Content and Their Drive and Motivation to Learn Mathematics in PISA 2012

As part of PISA 2012, each student took a two-hour handwritten test on reading, mathematics and science (with a focus on mathematics). The tests were a mixture of open-ended and multiple-choice questions that were organized in groups based on a passage setting out a real-life situation. Following the cognitive test, students spend nearly one more hour answering a questionnaire about themselves, their family and home, general aspects of learning mathematics, problem-solving experiences, and specific aspects of learning mathematics as in 2012 the focus of PISA was mathematics.

6.3.3.1 Students' Exposure to Mathematics Content

Research shows that students' exposure to subject content in school, known as "opportunity to learn", is associated with student performance (Schmidt et al. 2001; Sykes et al. 2009). The PISA 2012 questionnaire asked students how often they encountered various types of mathematics problems or tasks during their time at school and also how familiar they were with mathematical concepts such as exponential function, divisor, quadratic function, proper number, linear equation, vectors, complex number, rational number, radicals, subjunctive scaling, polygon, declarative fraction, congruent figure, cosine, arithmetic mean and probability. Responses

Table 6.8 Index of Singapore students' Exposure to Mathematics Content in PISA 2012

Country	Exposure to word problems	Exposure to formal mathematics	Exposure to applied mathematics
Singapore	1.56 (0.016)	2.23 (0.010)	2.00 (0.010)
OECD average	1.87 (0.003)	1.70 (0.003)	1.92 (0.002)

( )—standard errors
Source OECD (2014) Table 1.3.1

to the questionnaire were used to create three categories: exposure to word problems, exposure to formal mathematics and exposure to applied mathematics and respective indices created (OECD 2014). The values of these indices range from 0 to 3, with 0 corresponding to no exposure and 3 to frequent exposure.

Table 6.8 shows Singapore students' indices for Exposure to Mathematics Content and the corresponding OECD averages. Singapore stood out among all PISA participating countries as having the strongest relationship between the index of exposure to formal mathematics and students' mathematics performance (OECD 2014, p. 153). This result suggests that opportunities to learn formal mathematics are associated with PISA performance. Furthermore, exposure to more advanced mathematics content, such as algebra and geometry, seems to be related to high performance on the PISA mathematics performance. Exposure to word problems, which are usually represented in textbooks as applications of mathematics, is also related to performance, but was found to be less strong when compared to the OECD average. From the index of exposure to applied mathematics, it is apparent that students in Singapore are exposed to a wide range of problems (including with real-world contexts) to solve during their study of mathematics. In this way, students learn to apply mathematics in varying contexts and develop necessary skills for future use.

6.3.3.2 Students' Drive and Motivation to Learn Mathematics

In PISA 2012, students' perseverance, openness to problem-solving, and students' intrinsic and instrumental motivation to learn mathematics were measured to assess Students' Drive and Motivation to Learn Mathematics (OECD 2013b). Perseverance and Openness to Problem-Solving are two new scaled indices in 2012 PISA. They were developed in recognition of the increasing importance of problem-solving in the cognitive part of the assessment. Based on students' self-reports, PISA results show that drive and motivation are essential for students' to realize their potential. Students' Perseverance was gauged by their responses to the five statements shown in Table 6.9. Students responded with one of the following: "very much like me", "mostly like me", "somewhat like me", "not much like me" or "not at all like me". Across OECD countries, 56% of students indicated that they do not give up easily when confronted with a problem, 49% indicated that they remain interested in the tasks that they start, and 44% indicated that they continue working on tasks until

Table 6.9 Items measuring students' Perseverance and students' Openness to Problem-Solving

How well does each of the following statements below describe you?		Percentage of students	
Students' perseverance		OECD average	Singapore
a	When confronted with a problem, I give up easily[b]	56.0 (0.2)	61.8 (0.7)
b	I put off difficult problems[b]	36.9 (0.1)	43.8 (0.7)
c	I remain interested in tasks that I start[a]	48.9 (0.2)	57.9 (0.8)
d	I continue working on tasks until everything is perfect[a]	43.8 (0.2)	61.1 (0.9)
e	When confronted with a problem I do more than what is expected of me[a]	34.5 (0.1)	45.3 (0.9)
Openness to problem-solving			
a	I can handle a lot of information[a]	53.0 (0.2)	44.2 (0.9)
b	I am quick to understand things[a]	56.6 (0.1)	50.4 (0.9)
c	I seek explanations for things[a]	60.7 (0.2)	68.5 (0.7)
d	I can easily link facts together[a]	56.7 (0.2)	52.4 (1.0)
e	I like to solve complex problems[a]	33.1 (0.1)	39.1 (0.9)

( )—standard errors
[a]Percentage of students who reported that the statements describe someone "very much like me" or "mostly like me"
[b]Percentage of students who reported that the statements describe someone "not much like me" or "not at all like me"
Source OECD (2013b) Tables III.3.1a and III.3.2a

everything is perfect. The percentage of Singapore students showing perseverance for each individual statement is higher than the international average, with 62% of students indicating that they do not give up easily when confronted with a problem, 58% indicating that they remain interested in the tasks that they start, and 61% indicating that they continue working on tasks until everything is perfect.

PISA 2012 also measured students' Openness to Problem-Solving through their responses to the five statements shown in Table 6.9. The questions asked students about the extent to which they feel they resemble someone who can handle a lot of information, is quick to understand things, seeks explanations for things, can easily link facts together and likes to solve complex problems.

Student's responses to each question could range from: the statement describing someone "very much like me", "mostly like me", "somewhat like me", "not much

Table 6.10 Index of Singapore students' Perseverance and Openness to Problem-Solving in PISA 2012

	Index of students' Perseverance				Index of students' Openness to Problem-Solving			
	All students	By proficiency level			All students	By proficiency level		
		Below level 2	Level 4	Level 5 or 6		Below level 2	Level 4	Level 5 or 6
Singapore	0.29 (0.02)	0.09 (0.04)	0.35 (0.03)	0.36 (0.03)	0.01 (0.02)	−0.30 (0.06)	0.02 (0.03)	0.20 (0.03)
OECD average	0.00 (0.00)	−0.28 (0.01)	0.19 (0.01)	0.43 (0.01)	0.00 (0.00)	−0.40 (0.01)	0.29 (0.01)	0.70 (0.01)

( )—standard errors
Source OECD (2013b) Tables III.3.1d, III.3.2d and III.3.8

like me" or "not at all like me". Across OECD countries, 53% of students indicated that they can handle a lot of information, 57% reported that they are quick to understand things, and 61% reported that they seek explanation for things, 57% reported that they can easily link facts together, and only 33% indicated that they like to solve complex problems. Singapore students showed higher intention in seeking explanation for things and solving complex problems than the international average with 69% reporting that they seek explanation for things and 39% indicating that they like to solve complex problems, but showed lower self-belief of being able to handling lots of information, quickly understanding things and easily linking facts together.

The responses to the items in Table 6.9 were used to create the index of students' Perseverance and index of students' Openness to Problem-Solving. The indices were standardized to have a mean of 0 and a standard deviation of 1 across the OECD countries and other economies and countries that participated in PISA 2012. Table 6.10 shows the indices for perseverance and openness to problem-solving for Singapore students in PISA 2012. The mean index of perseverance ranged from 0.77 for Kazakhstan to −0.59 for Japan. Singapore had an index of 0.29, which was the best among the top-performing East Asian countries/economies that participated in PISA 2012.

It is apparent from Table 6.10 that students with proficiency Level 5 or 6 reported higher levels of perseverance than those with lower proficiency levels, which indicates a strong association between perseverance and mathematics performance in terms of proficiency level achieved in PISA 2012. However, the perseverance index of Singapore students with proficiency Level 5 or 6 was lower than the international average of 0.43.

The index of students' openness to problem-solving ranged from 0.62 for Jordan and Montenegro to −0.73 for Japan. Table 6.10 shows that index for Singapore students was 0.01 just above the OECD average. There appears to be generally an inverse relationship between openness to problem-solving and mathematics performance among students who participated in PISA 2012. Singapore students with proficiency Level 5 or 6 reported relatively higher levels of openness to problem-solving than those with lower proficiency levels. For Levels 4, 5 and 6 of proficiency,

Table 6.11 Items measuring Intrinsic and Instrumental Motivation to Learn Mathematics

Thinking about your views on mathematics: to what extent do you agree with the following statements?		Percentage of students*	
Intrinsic motivation to learn mathematics (mathematics interest)		OECD average	Singapore
a	I enjoy reading about mathematics	30.6 (0.2)	68.1 (0.9)
b	I look forward to my mathematics lessons.	36.2 (0.2)	76.8 (0.8)
c	I do mathematics because I enjoy it	38.1 (0.2)	72.2 (0.8)
d	I am interested in the things I learn in mathematics	53.1 (0.2)	77.1 (0.8)
Instrumental motivation for mathematics			
a	Making an effort in mathematics is worth it because it will help me in the work that I want to do later on	75.0 (0.1)	90.4 (0.6)
b	Learning mathematics is worthwhile for me because it will improve my career prospects and chances	78.2 (0.1)	88.2 (0.6)
c	Mathematics is an important subject for me because I need it for what I want to study later on	66.3 (0.2)	87.4 (0.6)
d	I will learn many things in mathematics that will help me get a job	70.5 (0.2)	85.5 (0.7)

( )—standard errors
*Percentage of students who “strongly agree” or “agree”
Source OECD (2013b) Tables III.3.4a and III.3.5a

their indices of openness to problem-solving were lower than the OECD average. It is interesting to note that for PISA 2012, creative problem-solving students from Singapore were ranked first and yet their perceptions of openness to problem-solving suggest that they do not have attributes of good problem solvers. This mismatch could be attributed to their inability to self-assess their abilities or sheer over modesty, as often portrayed by Asian students.

PISA measures students’ Intrinsic Motivation to Learn Mathematics and Instrumental Motivation to Learn Mathematics through their responses “strongly agree”, “agree”, “disagree” or “strongly disagree” with the statements shown in Table 6.11.

As shown in Table 6.11, on average across OECD countries, students who participated in PISA 2012 have shown relatively low levels of intrinsic motivation to learn mathematics. Only 31% of students indicated that they agree or strongly agree that they enjoy reading about mathematics, 36% reported that they look forward to their mathematics lessons, 38% reported that they do mathematics because they enjoy it, and 53% reported that they are interested in the things they learn in mathematics. However, Singapore students seem to have high levels of intrinsic motivation to learn mathematics, with 68% of students indicating that they enjoy reading about mathematics, 77% indicating that they look forward to their mathematics lessons, 72%

Table 6.12 Index of Singapore students' Intrinsic Motivation to Learn Mathematics and Instrumental Motivation to Learn Mathematics in PISA 2012

	Index of students' Intrinsic Motivation to Learn Mathematics				Index of students' Instrumental Motivation to Learn Mathematics			
	All students	By proficiency level			All students	By proficiency level		
		Below level 2	Level 4	Level 5 or 6		Below level 2	Level 4	Level 5 or 6
Singapore	0.84 (0.02)	0.60 (0.06)	0.88 (0.03)	0.88 (0.02)	0.40 (0.02)	0.38 (0.06)	0.44 (0.03)	0.34 (0.03)
OECD average	0.00 (0.00)	−0.21 (0.01)	0.17 (0.01)	0.51 (0.01)	0.00 (0.00)	−0.22 (0.01)	0.15 (0.01)	0.40 (0.01)

( )—standard errors
Source OECD (2013b) Tables III.3.4d, III.3.5d and III.3.8

reporting that they do mathematics because they enjoy it, and 77% reporting that they are interested in the things they learn in mathematics.

From Table 6.11, it is also apparent that students who participated in PISA 2012 appreciate the instrumental value of mathematics. On average across OECD countries, 75% of students responded that they agree or strongly agree that making an effort in mathematics is worthwhile because it will help them in the work that they want to do later on in life. 78% of students responded that learning mathematics will improve their career prospects, and 71% of students believed that learning many things in mathematics will help them get a job. Likewise, Singapore students have also shown very high levels of instrumental motivation to learn mathematics, with 90% of students responding that they agree or strongly agree that making an effort in mathematics is worth it because it will help them in the work that they want to do later on, 88% of students responding that learning mathematics is worthwhile because it will improve their career, 87% reporting that mathematics is an important subject because they need it for what they want to study later on, and 86% believing that many things they learnt in mathematics will help them get a job.

The responses were used to create standardized indices, with mean of 0 and standard deviation of 1, for students' Intrinsic Motivation and Instrumental Motivation to Learn Mathematics. The index for students' intrinsic motivation to learn mathematics ranged from 0.96 for Albania to −0.35 for Austria, while that for instrumental motivation ranged from 0.56 for Peru to −0.57 for Romania. Table 6.12 shows the indices for Singapore students' intrinsic motivation and instrumental motivation to learn mathematics and the corresponding OECD averages in PISA 2012. For intrinsic motivation to learn mathematics, Singapore had an index of 0.84. For instrumental motivation, the index was 0.40. Both indices were the highest compared with the other top-performing East Asian countries/economies in PISA 2012.

From Table 6.12, it is also apparent that students with proficiency Level 5 or 6 showed significantly higher index of intrinsic motivation than those with proficiency level below 2. The results suggest an association between students' intrinsic motivation and mathematics performance in terms of proficiency level achieved in PISA 2012. However, for instrumental motivation to learn, it appears that Singapore is an exception as students at all proficiency levels show high indices of instrumental motivation to learn mathematics.

6.4 Teacher Education and Development Study in Mathematics (TEDS-M)

TEDS-M is the first international comparative study on the training of future mathematics teachers carried out by IEA. Seventeen countries including Singapore participated in the study. Singapore participated in the study to compare teacher education at the National Institute of Education (NIE), the sole teacher education institute in Singapore, and performance of NIE student teachers in mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) against international benchmarks.

TEDS-M was a survey study that used specific questionnaires to collect data from educators and future mathematics teachers. The theoretical framework of the study is detailed in Tatto et al. (2008). The study comprises three components. Component 1 is about the national contexts and policies for teacher education. The national research coordinators of the participating countries provided country reports explaining these contexts and policies. Component 2 is specific to the nature of mathematics teacher education programmes. Coordinators of the institutes that were sampled in each country completed the Institution questionnaire. Educators from the institutes did the same for the Educator questionnaire that sought their beliefs about pedagogy and activities offered by their courses for future mathematics teachers. Information about the school mathematics curricula and mathematics teacher education courses was also collected and analysed.

Component 3 examines the outcomes of teacher education in terms of the performance of future mathematics teachers in MCK and MPCK and their beliefs and perceptions of opportunities to learn (OTL) about mathematics and pedagogy. The mathematics for teaching test comprised MCK and MPCK items. The MCK items covered four content knowledge domains (Number, Geometry, Algebra and Data) and three cognitive domains (Knowing, Applying and Reasoning). These domains are based on the corresponding domains used in the TIMSS 2007 framework (Mullis et al. 2007). The MPCK items measured three types of mathematics knowledge for teaching: mathematical curricular knowledge; knowledge of planning for mathematics teaching and learning (pre-active); and enacting mathematics for teaching and learning (interactive). The test comprised 24 items and 30 items for the primary and lower secondary future mathematics teachers, respectively, and teachers had 60 min to complete it. The beliefs and OTL survey comprised 53 Likert-type items, and teachers had 30 min to complete it. The survey sought their beliefs about the nature of mathematics, learning mathematics and mathematics achievement and perceptions about content and skills relating to seven broad areas hypothesized to influence knowledge for teaching mathematics: tertiary-level mathematics, school-level mathematics, mathematics education pedagogy, general pedagogy, teaching diverse students, learning through school-based experiences, and coherence of their teacher education programme.

As NIE is the sole teacher education institute in Singapore, it provided a census sample to represent Singapore. Altogether 380 primary (263 primary generalist + 117 primary math specialist) and 393 (142 lower secondary + 251 upper secondary) secondary future mathematics teachers completed the TEDS-M tests and surveys. Seventy-seven NIE mathematicians, mathematics educators and teacher educators who taught at least one course to the future teachers participating in TEDS-M also completed the Educator questionnaire in 2007.

The TEDS-M international report detailing the data and findings related to the three components was published in 2012 (Tatto et al. 2012). Several publications by Wong Khoon Yoong, who was the National Research Coordinator for Singapore, and his colleagues provide us with insights about findings that are Singapore centric (Wong et al. 2011, 2012a, b, 2013a, b, c; 2014). In the following sections, we draw on the international report and also publications by Wong and colleagues and present a brief overview of findings that provide us with a glimpse of where mathematics teacher education sits in the international arena and how future mathematics teachers rank in the same.

6.4.1 National Contexts and Policies for Teacher Education

Wong et al. (2012) noted that teacher education policies varied widely across the 17 countries that participated in TEDS-M and it was not possible to draw definitive implications about the effects of these policies on the performance of future teachers. Furthermore in several countries, including Singapore, policies have changed since the country reports were submitted in 2008. Nevertheless, Tatto et al. (2012) reported that in both Chinese Taipei and Singapore where future mathematics teachers scored high on the TEDS-M tests:

- there were strong controls over the number of entrants accepted into teacher education programmes;
- there were specific policies to ensure that teaching is an attractive career; and
- teacher education programmes were able to recruit able high school graduates.

6.4.2 Nature of Mathematics Teacher Education Programmes

The primary mathematics education programmes were classified along a generalist–specialist continuum. In Singapore, data were collected in November 2007 and May/June 2008, from four different types of pre-service programmes for primary teachers at the National Institute of Education (NIE):

- Diploma in Education, Dip Ed (A) or Dip Ed (C);
- Bachelor of Arts with Education, BA (Ed) (C-series);
- Bachelor of Science with Education, B.Sc. (Ed) (C-series);
- Postgraduate Diploma in Education (Primary), PGDE (P) (A) or PGDE (P) (C).

At the time of the study, the Dip Ed and PGDE (P) programmes offered two options: option A covered two teaching subjects (one of which was mathematics) and option C covered three teaching subjects (one of which was mathematics). The C-series Bachelor programmes trained only primary school teachers and covered four teaching subjects, including mathematics. Teachers who were training to teach two subjects were classified as primary mathematics specialists, while those who were training to teach more than two subjects were classified as generalists.

Secondary mathematics teacher education programmes covered either lower secondary up to grade 10 or upper secondary up to grade 12. In Singapore, data were collected in November 2007 and May/June 2008 from two cohorts of the Postgraduate Diploma in Education (PGDE) (Secondary) programme. Future mathematics teachers in the programme prepared to teach either lower secondary mathematics or all secondary mathematics. Those preparing to teach lower secondary mathematics are generally weaker in mathematics compared to those who are preparing to teach all secondary mathematics. In TEDS-M, the lower secondary mathematics teachers from Singapore were classified as those preparing to teach lower secondary to grade 10, while those preparing to teach all secondary mathematics were classified as upper secondary up to grade 12. It is reported by Wong et al. (2012) that Singapore and Chinese Taipei had the highest requirements for the mathematics courses that future teachers must complete in order to enter the professional component of their teacher education programmes. However, secondary future teachers in Chinese Taipei and Russia were prepared to teach only one subject, while those in NIE were prepared to teach one major and one minor subject.

6.4.3 Performance of Future Teachers in MCK and MPCK and Their Beliefs and Perceptions of Opportunity to Learn

Future mathematics teachers from Singapore performed well on the MCK and MPCK tests and Singapore ranked among the top countries. Table 6.13 gives an overview

Table 6.13 Ranking and score of NIE student teachers

	Primary generalist	Primary math specialist	Lower secondary	Upper secondary
N	263	117	142	251
MCK	2 (586)	2 (600)	1 (544)	3 (587)
MPCK	2 (588)	1 (604)	2 (539)	4 (562)

NB. International mean = 500
Source of data Tatto et al. (2012, pp. 139, 143, 147, 150)

of their performance and Table 6.14 gives a detailed breakdown of the same by programmes of study at NIE. From the tables, it is apparent that among the primary student teachers at NIE, those who were trained to teach only two subjects performed better than those who were trained to teach more than two subjects. The secondary student teachers who were trained to teach upper secondary performed better than those who were trained to teach lower secondary. This result was expected. Table 6.14 shows that for future primary teachers when the performance is analysed by NIE programmes, student teachers in the BSc (Ed) programme in fact topped both tests in the Primary Generalist group. One reason could be some student teachers in this programme were doing undergraduate mathematics as their academic subject. Among the six NIE programmes for future primary mathematics teachers in Table 6.14, the performance of student teachers in Dip Ed (C) was the lowest for both tests. This is not unexpected because the Diploma programme admits student teachers not qualified for the Bachelor programmes.

6.4.3.1 Primary MCK and MPCK Test Items

As an illustration of the performance of future primary teachers from Singapore, we consider two released items, one from MCK and the other from MPCK. Figure 6.9 shows an item MFC 204 of the MCK Geometry-Knowing domain. This item requires knowledge of the relationships among quadrilaterals. For example, a square is both a rectangle and a rhombus. Wong et al. (2012b) reported that 66% of NIE student teachers had chosen the correct option C. This is slightly higher than the international level of 64%. As these geometric relationships have been covered in the Subject Knowledge (SK) courses at NIE, they expected the student teachers "to perform better in this task than the result reported here". A better performance entails a deeper understanding of these relationships. An approach worth considering is to reinforce student teachers' ability to differentiate the defining properties of quadrilaterals from the other properties. Knowing the roots of these relationships should help in the understanding of *why* a square is both a rectangle and a rhombus, for example.

Figure 6.10 shows two items of the MPCK Enacting domain: MFC 208A at the intermediate level and MFC 208B at the advanced level. MFC 208A tests the ability to recognize the two common misconceptions that multiplication will always produce

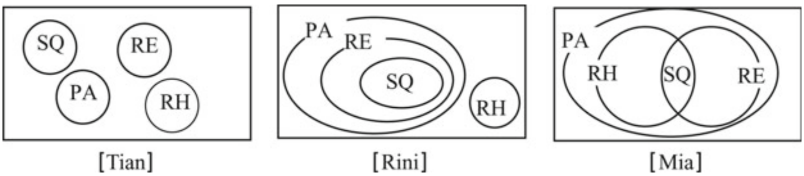
Table 6.14 Performance of Singapore student teachers in MCK and MPCK tests

Programme group/country	MCK		MPCK	
	Rank	Mean	Rank	Mean
Primary generalist (Grade 6 maximum)				
Singapore (All)	2	586	2	588
B.Sc. (Ed)		625		626
PGDE (P) (C)		593		596
BA (Ed)		586		587
Dip. Ed (C)		567		568
Primary mathematics specialist				
Singapore (All)	2	599	1	603
PGDE (P) (A)		600		601
Dip Ed (A)		598		607
Lower secondary				
Singapore	1	544	2	539
Upper secondary				
Singapore	3	587	4	562
International mean = 500; *standard deviation* = 100				

Source of data Wong et al. (2012b, p. 300); Tatto et al. (2012 pp. 139, 143, 147, 150)

MFC 204

Three students have drawn the following Venn diagrams showing the relationships between four quadrilaterals: Rectangles (RE), Parallelograms (PA), Rhombuses (RH), and Squares (SQ).



Which student's diagram is correct? (A) [Tian] (B) [Rini] (C) [Mia]

Fig. 6.9 Released item (MFC 204) of the MCK Geometry-Knowing domain (Wong et al. 2013a, p. 300)

a larger product and division will always make a number smaller. MFC 208B tests the competency to "translate" an abstract operation into a visual model to help pupils correct these misconceptions.

It is reported in Wong et al. (2012) that 67% of NIE student teachers could state at least one misconception in MFC 208A. Although this is much higher than the corresponding international performance of 41%, "that about one-third … could not recognize these misconceptions, giving irrelevant responses" was still "truly

[Jeremy] notices that when he enters 0.2 × 6 into a calculator his answer is smaller than 6, and when he enters 6 ÷ 0.2 he gets a number greater than 6. He is puzzled by this, and asks his teacher for a new calculator!
MFC 208A (a) What is [Jeremy's] most likely misconception?
MFC 208B (b) Draw a visual representation that the teacher could use to model 0.2 × 6 to help [Jeremy] understand WHY the answer is what it is?

Fig. 6.10 Released items (MFC 208A and 208B) of the MCK Enacting domain (Wong et al. 2012b, p. 302)

MFC 610D
Determine whether the following is an irrational number always, sometimes or never?
D. Result of dividing 22 by 7.

MFC 705A
We know that there is only one point on the real number line that satisfies the equation $3x = 6$, namely $x = 2$.
A. Suppose now that we consider this same equation in the plane, with coordinates x and y. What does this set of points that satisfy the equation $3x = 6$ look like in this setting?

□ one point □ one line □ one plane □ other

Fig. 6.11 Released MCK items (MFC 610D and MFC 705A) of the Number-Knowing and Geometry-Knowing domains, respectively (Wong et al. 2012a, p. 3; 2012b, pp. 14, 22)

surprising" to them. This is because these two misconceptions are covered in the NIE Curriculum Studies (CS) courses. Overall, only 23% of NIE student teachers could answer both parts of MFC 208 correctly. For mathematics educators at NIE, a question worth pondering is thus how student teachers' ability to recognize and deal with misconceptions can be strengthened through their CS courses.

6.4.3.2 Secondary MCK and MPCK Test Items

As an illustration of the performance of future secondary teachers from Singapore, we consider some released items, two from MCK and one from MPCK. Figure 6.11 shows two MCK items that NIE student teachers found rather difficult though both belonged to the domain of knowing.

The performance of NIE student teachers on items MFC 610D and MFC 705A is reported in Wong et al. (2012a). For item MFC 610D, 66% of NIE student teachers knew that the result of diving 22 by 7 is never an irrational number, whereas 31% thought it is always an irrational number, probably not realizing that 22/7 is only an approximation for π. The corresponding international averages were 40 and 51%, respectively. The performance of NIE student teachers on item MFC 705A was also weak. About 69% knew that the solution to the equation $3x = 6$ in the plane is a line, but 27% thought it was a point thinking that the solution $x = 2$, which give a single value. The corresponding international averages were 58 and 33%, respectively. The

MFC 712C A mathematics teacher wants to show some students how to prove the quadratic formula. Determine whether the following types of knowledge is needed in order to understand a proof of this result. C. How to complete the square of a trinomial. □ needed □ not needed

Fig. 6.12 Released MPCK item (MFC 712C) of the Algebra-Planning domain (Wong et al. 2012a, p. 3; 2013a, p. 30)

performance of NIE student teachers on both of the above items suggests weak conceptual understanding of some basic mathematics.

Figure 6.12 shows a MPCK item that student teachers at NIE found difficult too. The item belongs to the content domain—Algebra—and type of mathematics knowledge for teaching—planning. Students' performance of the item is reported in Wong et al. (2012a). In proving the quadratic formula, only 37% knew that the proof requires the knowledge to complete the square of a trinomial; this result was much worse than the international average of 55%. As noted by Wong and colleagues, two major reasons may account for this poor result. First, the term trinomial is rarely used in Singapore textbooks, and second, some secondary teachers do not teach this formula using "complete the square" approach. As such, student teachers who had not encountered this proof during their school days may not have the opportunity to learn it their post-secondary mathematics courses.

6.4.3.3 Singapore Student Teachers' Beliefs and Perceptions of Opportunity to Learn

In Wong et al. (2011), the outcomes of a questionnaire on the reasons for student teachers to become a teacher and their beliefs about teaching as a lifetime career were presented. Drawing on their findings, among the nine reasons given in the questionnaire for becoming a teacher, the two most important reasons were "I like working with young people" and "I want to have an influence on the next generation". In line with these reasons, 86% of primary and 82% of secondary student teachers at NIE either expected teaching as a lifetime career or believed in this possibility. These levels of commitment are higher than the corresponding international levels of 72 and 77%. These findings are consistent with the "career-based" model of teacher employment in Singapore. The student teachers at NIE have a distinctive status as employees of the Ministry of Education, and Wong et al. (2013c) noted that "this distinctive system of 'paying' people to be trained as teachers is indicative of a career-based system in its fullest sense".

Wong et al. (2012a) also noted that NIE student teachers and educators generally endorsed the conceptual approaches to learning mathematics compared to the procedural ones. 78% of the educators believed that mathematics should be learned through student activity, compared to 72% of the primary and 66% of the secondary student teachers. Student teachers who held conceptual orientations tended to have higher MCK and MPCK scores compared to those with procedural or fixed ability beliefs. Wong and colleagues reported that on a scale of 0–1, the coverage of mathematics education pedagogy in NIE programmes ranged from 0.68 to 0.72 and this was similar to the international mean. However, the coverage of general pedagogy was in the range of 0.57–0.65 and this was low when compared to Russia, Switzerland and the USA. NIE student teachers also scored below the international mean for opportunities to learn about teaching diverse students, and they rated "rarely having opportunities" to read about research on mathematics and mathematics education; write mathematical proofs; and develop research projects to test teaching strategies for pupils of diverse abilities. A significant difference between the perceptions of student teachers and educators at NIE was that educators felt that they had provided fairly frequent opportunities for the student teachers to engage in interactive learning experiences, such as to ask questions, participate in class discussion, work in groups and make presentations to the class. However, with the exception of group work, the student teachers rated the other three interactive experiences lower than the educators. Nevertheless, 90% of NIE students rated their programmes as effective or very effective in preparing them to teach mathematics.

6.4.4 What Are the Implications of the Main Findings of the TEDS-M for Educators in Singapore?

The findings of comparative studies like TEDS-M (Tatto et al. 2012) provide participating countries with ample scope to make comparisons with other participating countries and glean valuable insights. Drawing on the data and findings of Tatto et al. (2012), Wong et al. (2011), five key implications arising from the findings of TEDS-M were noted by Wong and colleagues for educators in Singapore (Wong et al. 2011). They are as follows.

6.4.4.1 Recruit Future Mathematics Teachers with Strong Mathematics Background

The results of the TEDS-M study in general (Tatto et al. 2012) and Singapore's data (Wong et al. 2011) in particular affirm that teachers with sound mathematical knowledge demonstrate high performance in MCK and MPCK, which are necessary requisites for them to teach mathematics competently in schools. Hence, it is important that the Ministry of Education in Singapore continue to recruit future mathematics teachers with strong entry qualifications.

6.4.4.2 Stress Sound Grounding in Mathematics-Related Knowledge in NIE Programmes

Generally, the good performance of NIE future teachers in MCK and MPCK affirms a strong grounding in mathematics-related knowledge that has been acquired while undergoing study at the NIE to be a mathematics teacher. Although generally the performance of the future teachers was commendable, there were differences in performance across the different programmes (see Table 6.14). Therefore, there is a need to look at the structure of these programmes and make revisions that would help future teachers of mathematics learn more mathematics while preparing to teach mathematics at NIE. Compared to some other countries that also participated in TEDS-M, future mathematics teachers at NIE reported relatively low coverage of validation/structuring/ abstracting topics such as Boolean algebra, mathematical induction, logical connectives and linear space. This could be an area for consideration at least for the Bachelor Degree curriculum as these topics are important for the development of mathematical thinking.

Another gap is the relatively low attention given to teaching mathematics to diverse students. Paying attention to the teaching of students with diverse backgrounds is important as it is in line with differentiated instruction in schools advocated by the Ministry of Education in Singapore. Yet another area that warrants attention is the general agreement among educators and future teachers about the low frequency requiring future teachers to read about research in mathematics and mathematics education. Given the recent trends towards evidence-based practices, it is imperative that educators engage future teachers to read, discuss and experiment with researched practices.

6.4.4.3 Align Opportunities to Learn from the Perceptions of Educators and Future Teachers

Some mismatches were found between perceptions of opportunities to learn some components of the NIE programmes as reported by the educators and future teachers. One significant area to probe further is the frequency of using interactive learning experiences such as future teachers asking questions and discussions during lessons. Periodic surveys like the one used by TEDS-M for opportunities to learn by educators at NIE may help them keep NIE programmes relevant and prepare future teachers who are also ready for the rapidly changing learning spaces of the future.

6.4.4.4 Strengthen Commitment to Teaching as a Lifetime Career

Although NIE student teachers had expressed more favourable commitment to teaching as a lifetime career when compared to their international counterparts, there were 20% of first-career future teachers who were not fully committed. Steps should be taken while these future teachers are at NIE to acquaint them with the various challenges and achievements of being a teacher.

6.4.4.5 Learn from Other High-Performing Countries

Chinese Taipei and Russia performed better than Singapore in MCK and MPCK for some groups of future mathematics teachers. When they did perform better, the differences in scores were much larger than those of NIE student teachers. It is valuable for educators at NIE to learn about teacher education systems of these two countries through study visits and research collaborations.

6.5 Conclusion—Why Singapore Students Do Well in TIMSS and PISA?

This chapter has put forth the performance in mathematics for both students and future mathematics teachers in Singapore, in international benchmark studies TIMSS, PISA and TEDS-M. As noted by Barber and Mourshed (2007) in the McKinsey report "The quality of an Education System cannot exceed the quality of its teachers" (p. 16), it is apparent from the findings of the TEDS-M study that mathematics teachers in Singapore are one of the contributory factors for the commendable performance of their students in Mathematics. An analogy to Barber and Mourshed's claim that the quality of teachers in any education system is significantly dependent on the quality of teacher educators in that system is also supported by the findings of the TEDS-M for Singapore. Therefore, it appears that the quality of both mathematics teacher educators and mathematics teachers partly explains the performance of Singapore students in TIMSS and PISA.

Since the introduction of the New Education System (NES) in 1979 (Goh and The Education Study Team 1979), Singapore has dedicatedly pursed the vision of a high-quality education system that devotes attention and resources not only to high achievers, but also to lower level achievers. In line with the vision, the school mathematics curriculum has undergone periodic revisions since the 1980s, to remain relevant and keep abreast of development in the world around. It has been detailed in Chap. 2 how the education system has developed so far and in tandem how the school mathematics curriculum has also evolved into one that provides for every child in school. The curriculum lays a solid foundation in mathematics for all students in elementary grades, which seems to play a core role in students' later success. From upper primary onwards, students are assigned specialist teachers in mathematics. From upper secondary onwards, a range of specialized mathematics courses at higher levels are available for students who are interested to build up their strengths. It is apparent that the government invests wholeheartedly in education. One may say that the school levels the playing field for all students. Students who are lacking in progress are identified almost immediately and helped to overcome difficulties and allowed to achieve.

Since 1981, the curriculum has adopted the Concrete-Pictorial-Abstract approach to the teaching and learning of mathematics. This approach provides students with the necessary learning experiences and meaningful contexts, using concrete hands-on materials and pictorial representations to construct abstract mathematical knowledge. The system-wide guides of the intended curriculum issued by the Curriculum Planning and Development Division of the Ministry of Education place emphasis on the scope and sequence of topics taught at the respective grade levels. It makes clear the nature of the spiral curriculum and the student-centric learning experiences necessary for the acquisition of deep mathematical knowledge. The principles of teaching and phases of learning detailed in the guides make apparent that deep conceptual knowledge and procedural fluency must be the goals of mathematics instruction. Students must through exploration, clarification, practice and application over time represent mathematical concepts in multiple ways and apply them to solve problems in unfamiliar situations. Therefore, it also appears that the education system and school mathematics curriculum contribute in part towards the success of Singapore's students in TIMSS and PISA.

Singapore students' strong drive and motivation to learn mathematics are key to their performance in the subject. In addition, the high expectations of students by teachers and parents certainly impact their performance. In short we may say that society, in Singapore, as a whole places a premium on education.

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