Chapter 5
Beyond School Mathematics

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Abstract In this chapter, we unfold a two-sided painting of the tertiary mathematics education landscape in Singapore. One side displays how the education system in Singapore prepares her students for further learning of mathematics at university, while the other portraits how mathematics is taught at the tertiary level in the mathematics department of some Singapore-based universities. Regarding the pre-university mathematics education at 'A'-level, we examine some of the major syllabus changes for Mathematics, making sense of these changes through the analytical lens of curriculum orientation. In passing, we also looked at the H3 Mathematics curriculum and its implementation, and the niche school NUS High School of Mathematics and Science. For the tertiary mathematics education, we rely on the collective wisdom of seven mathematics professors who have rich experience in teaching undergraduate mathematics from the top four local universities. The story of what goes beyond school mathematics in Singapore brings forth an important message, that is, tertiary mathematics education is responsive to shifts in educational policies occurring at schools—one which is unique of Singapore.

Keywords Tertiary mathematics · A-level mathematics syllabi · Curriculum orientation · H3 Mathematics · Niche-area schools · Undergraduate mathematics courses

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5.1 Introduction

The turn of the twenty-first century saw a global movement to building SMART nations, nations where people are empowered by technology to lead meaningful and fulfilled lives, and technological advancements in Engineering. Responding to this worldwide trend, universities began re-looking at ways of equipping their graduates to meet the expanding demands in the areas of Science, Technology, Engineering and Mathematics (STEM), and hence a re-emphasis on STEM education. As the vehicular language for Science, Technology and Engineering, Mathematics as an academic subject is of central importance, starting from primary, through secondary and pre-university, and culminating at tertiary education. With regards to this recent movement towards STEM education, the Ministry of Education in Singapore (MOE) has stipulated in the official document of the mathematics syllabus at the Advanced Level that 'H2 Mathematics is designed to prepare students for a range of university courses, including *mathematics*, sciences, engineering and related courses, where a good foundation in mathematics is required. It develops mathematical thinking and reasoning skills that are essential for *further learning of mathematics*.' The focus of this present chapter was on this further learning of mathematics—tertiary mathematics education. More precisely, we describe how Singapore is moving *beyond school mathematics*. We expound on this aspect in the ensuing three sections.

A discussion of the changes in the way mathematics have been taught at the tertiary level in Singapore can never begin without tracing the relevant parts of the major changes that occurred in the Singapore mathematics syllabi at the pre-university level, i.e. A-level mathematics taught in junior colleges and centralised institutes. MOE constantly tapped on expert advice from junior college teachers and university professors regarding the reshaping and re-crafting of the mathematics syllabi at the A-level. We begin Sect. 5.2, with a brief chronological recount of the significant education policies introduced at different junctures of time that directly brought about the changes in the A-level Mathematics syllabi. These changes are then analysed through the lens of curriculum orientation—in this case, it represents a shift from scholar academic to social efficiency. This critical analysis provides insight into how Singapore, as a young nation, endeavours to equip the next generation of her citizens with the needful twenty-first century life-skills, amongst which mathematical competencies take central status.

With the chronicle of the changes in the mathematics syllabus landscape at the A-level as the backdrop, in Sect. 5.3, we zoom into the microscopic aspects of the pre-university mathematics education in Singapore that have direct bearings on the way mathematics will be taught and learnt at the tertiary level. Here, we focus on two specific domains. The first domain deals with the H2 Mathematics syllabus implemented by MOE. Further equipping students to study mathematics at a more rigorous level is one key area that MOE has identified at A-level. Students who are mathematically inclined and capable are encouraged to take up Mathematics at Higher 3 level, which aims to develop their advanced mathematical problem-solving skills, mathematical reasoning and communication skills through mastery of precise mathematical language. In Sect. 5.3.1, we examine the implementation of the H3 Mathematics syllabus, following the revamp of the GCE A-level curriculum by MOE in 2006. More specifically, we exhibit the various options available to students taking H3 Mathematics, such as modules offered by the National University of Singapore (NUS), the Nanyang Technological University (NTU) and MOE, and describe the syllabi and content coverage of these modules, as well as major changes in the syllabi by MOE.

The second domain concerns MOE's designation of certain schools that provide specific learning to students in certain niche-areas—with particular emphasis on mathematics and science. In Sect. 5.3.2, we give an example of such a niche-area school—the NUS High School of Mathematics & Science (NHSMS). In particular, we look at how education strategies deployed in NHSMS achieves her vision and mission statement.

Section 5.4, the climax of this chapter, is where we compare and contrast the different ways tertiary mathematics education has evolved in selected local universities. Traditionally, professors delivered mathematics lessons at the tertiary level in lecture-cum-tutorial style. However, in recent years, many mathematics departments worldwide moved from teacher-centric didactics to student-centric pedagogies. We interviewed seven professors from three different mathematics departments of the aforementioned local universities and provide some insight into the new ways of teaching university mathematics with special focus on Singapore context. Finally, in Sect. 5.5, we conclude with what lessons can be gleaned from the narrative we have presented so far with regard to the tertiary education landscape.

5.2 Changes in A-Level Mathematics Syllabi in Singapore

An in-depth exploration of the changes that have taken place in the Singapore tertiary mathematics education so far makes sense only if we understand what changes had taken place in the education experience of students *prior to* their enrolment into university mathematics courses. This is clear since one aim of the A-level H2 Mathematics is to enable students 'to acquire mathematical concepts and skills to prepare for their tertiary studies in *mathematics*...' (MOE 2017a, p. 2).

Through the years, MOE has been conscientious in revising the A-level Mathematics syllabi to keep up with national and global issues, needs and trends. It is important for us to trace this journey to elucidate the major revisions in order to ascertain the landscape of issues and trends affecting the curriculum orientation, i.e. what is valued by the Singaporean society, which underpins the curriculum decisions. Here, curriculum decisions include content selection, high-stake examinations, scope and sequence of scheme of work, and classroom teaching and learning experiences.

5.2.1 *A Brief Chronicle of Major Educational Policies that Shaped A-Level Mathematics Landscape*

Since the 1990s, several significant education-related initiatives have impacted the A-level mathematics curriculum. These initiatives are detailed in Chaps. 2 and 3 of this book. Table 5.1 gives an overview of the initiatives and their impact on the A-level Mathematics syllabi.

5.2.1.1 Thinking Schools Learning Nation (TSLN) and Teach Less Learn More (TLLM)

Responding to the Thinking Schools Learning Nation (TSLN) initiative, the MOE undertook a fundamental review of the school curriculum and assessment system to allow for the development of creative thinking and learning skills required for the future. To achieve this, an important step was taken to reduce the amount of content knowledge that the students needed to learn so that both teachers and students can free up more time to engage in activities that develop the aforementioned skills (e.g. Project Work). An outcome of this content reduction in the A-level Mathematics curriculum was the Mathematics Syllabus 9233 which took its final form in 2001 after its 1999 interim version was phased out.

In 2002, the Junior College (JC)/Upper Secondary Education Review committee was set up to look into a major reshaping of the pre-university education landscape in the junior colleges and centralised institutes. While the aforementioned review was in progress, another independent and major education initiative the 'Teach Less Learn More' (TLLM) was introduced by MOE in 2005.

Independently, but happening in parallel to the JC/Upper Secondary Education Review, the development of the new A-level syllabi took place and was largely completed by the end of 2004. These new A-level syllabi took effect in 2006. In order to broaden the learning experience of the student, the notion of a contrasting subject came into being—it refers to a content-based subject taken outside a student's main area of specialisation. Under the new system, subjects are offered at either H1, H2 or H3 level. H2-level subjects are equivalent to the previous A-level subjects in terms of demand and intellectual challenge but would have their content reduced to free up the curriculum time for contrasting subjects and non-academic pursuits. H1-level subjects can be seen as half of their H2 counterparts in terms of curriculum time. For a subject taken at the H3 level, specific H3 programmes are offered to allow academically exceptional students to pursue that subject or area in which they have passion and aptitude. Unlike the previous Special Papers (commonly known as 'S' Papers), H3 programmes run on a separate syllabi which go beyond the H2 syllabi. We shall elaborate on the various H3 Mathematics programmes in Sect. 5.3.1. Notably, a majority of the A-level students sit for examinations under the H2 Mathematics Syllabus 9740.

Table 5.1 Initiatives since 1990s and their impact on the A-level Mathematics syllabi

Initiative	General description	Syllabus change
TSLN (Thinking Schools, Learning Nation), 1997	Lifelong learning, collective tolerance for change, schools as learning organisations, students develop both lower and higher thinking skills and processes	Mathematics 9205 changed to Mathematics 9233 (Interim), 1999
ICT Masterplan I (mp 1), 1997	Equip schools with ICT hardware, LCD projector in every classroom, whole school networking, ICT use in 30% curriculum time	• Graphing Calculators (GCs) were introduced in A-level mathematics and used in the 2001 Further Mathematics (FM) 9234 (Revised) 'GC-neutral' examination • Mathematics 9233 (Interim) changed to 9233 (Revised), 2001
Review of Junior College (JC)/Upper Secondary Education, 2002	Reviewing at, both the macro and the micro levels, the curricula of all subjects for Junior College	The development of the 2006 A-level curriculum took place parallel to the Junior College (JC)/Upper Secondary Education Review and was largely completed by 2004
ICT Masterplan II (mp 2), 2002	Integrate ICT with curriculum design, student-centred learning environment, evaluation of the use of ICT in education	GCs are used for teaching and learning as well as assessment, i.e. in the H2 Mathematics (9740) examinations
TLLM (Teach Less, Learn More), 2005		New A-level curriculum, 2006 • Broader and more flexible JC curriculum • Subjects offered at H1, H2 or H3 • Contrasting subjects for H1 and H2 • Mathematics 9233 (Revised) changed to H2 Mathematics 9740 • Removal of the subject Further Mathematics 9234
ICT Masterplan III (mp 3), 2009	Transforming the learning environment, a continuum of mp1 and mp 2	
SMART Nation, 2014	Making full use of new technologies to develop sustainable and innovative solutions, not just to run the place better but to make a difference to people's live	A re-emphasis on STEM education

(continued)

Table 5.1 (continued)

Initiative	General description	Syllabus change
ICT Masterplan IV (mp 4), every school a good school, 2012–2015	4 every's in MOE education initiatives: • Every School a Good School • Every Student an Engaged Learner • Every Teacher a Caring Educator • Every Parent a Supportive Partner	• H2 Mathematics 9740 changed to H2 Mathematics 9758 – Twenty-first century competencies (e.g. creative and inventive thinking) – Re-emphasis on STEM education – Focus on 'disciplinarity'; constructivist pedagogies • Reintroduction of the subject Further Mathematics 9649

Aligning with the call for a more broad-based education, Further Mathematics 9234 was removed in 2005. Expert advice from the syllabi review and steering committees was sought to decide which essential topics in Further Mathematics were to be included in the H2 Mathematics Syllabus 9740, and which topics in Mathematics Syllabus 9233 were to be removed (and in some instances, moved to O-level Additional Mathematics) to create sufficient space for the additional topics. Strictly speaking, the change from 9233 to 9740 involved more intricate restructuring of topics so that coherence of the new syllabus was ensured.

5.2.1.2 Information and Communications Technology Masterplans and SMART Nation

The Information and Communications Technology (ICT) Masterplans are described in Chap. 3. There are four altogether. A significant impact ICT Masterplan I (mp 1) had on the A-level mathematics curriculum was the introduction of the use of Graphing Calculators (GCs), i.e. hand-held scientific calculators that have facilities for plotting graphs, computing terms of sequences, solving equations and performing other tasks with variables or simple data structures, e.g. lists. GCs were first allowed in examinations for the Further Mathematics 9234 (Revised) syllabus in 2001, where it was maintained that the question items were set to be 'GC-neutral', meaning that students who used GCs would have no absolute advantage over those who did not. ICT Masterplan II (mp 2) advanced the role of GCs in the teaching and learning of A-level mathematics. Since 2006 GCs became a mode of assessment under the H2 Mathematics Syllabus 9740 in that 'the examination papers will be set with the assumption that candidates will have access to GCs. As a general rule, unsupported answers obtained from GCs are allowed unless the question states otherwise. … For questions where graphs are used to find a solution, candidates should sketch these graphs as part of their answers. … if there is written evidence of using GCs correctly, method marks may be awarded' (MOE 2017c, p. 3).

The SMART Nation initiative in Singapore was launched by her Prime Minister Lee Hsien Loong during the 2014 National Day Rally (Lee 2014). It called for Singaporeans to be empowered by technology to lead meaningful and fulfilled lives through technological advancements in Engineering. This brought about a renewed focus on STEM initiatives. The timely development of ICT Masterplan 4 (mp 4), in 2015, supported the impetus to develop life-skills and competencies relevant to the twenty-first century. In particular, the interconnection among the disciplines of Science, Technology, Engineering and Mathematics (STEM) now takes central stage, and the nation's education system must gear itself towards equipping the next generation with STEM-related knowledge.

5.2.1.3 Every School a Good School

Beginning in 2012, the MOE set a new goal in education to provide every child with the opportunity to develop holistically and maximise his or her potential; and to do that, MOE must ensure that 'every school is a good school'. What then makes a good school? Over four Ministry of Education Work Plan Seminars from 2012 to 2015, MOE rolled out in stages the four 'every's:

(i) *Every school a good school.* A good school cares for its students, studying and knowing the needs, interests and strengths of her students and motivates them to learn and grow. The call for 'every school to be a good school' was first formalised by Minister of Education, Mr. Heng Swee Keat, in 2012 and subsequently expanded into the following three aspects (ii), (iii) and (iv) (Heng 2015).
(ii) *Every student an engaged learner.* A good school ensures all students acquire strong fundamentals of literacy and numeracy and develops them holistically, in character, knowledge and critical competencies. A good school creates a positive school experience for each student, making him a confident and lifelong learner.
(iii) *Every teacher a caring educator.* A good school has caring and competent teachers who are steadfast in their mission to impact lives.
(iv) *Every parent a supportive partner.* A good school has the support of parents and the community, working together to bring out the best in our children.

Bearing in mind the importance of a STEM-based education as well as catering for a wider variety of students' needs and interests, MOE created in 2016 an expanded suite of A-level syllabi including, in particular, Further Mathematics. The H2 Further Mathematics Syllabus 9649 is specifically designed for 'students who are mathematically-inclined and who intend to specialise in mathematics, sciences or engineering or disciplines with higher demand on mathematical skills. It extends and expands on the range of mathematics and statistics topics in H2 Mathematics and provides these students with a head start in learning a wider range of mathematical methods and tools that are useful for solving more complex problems in mathematics and statistics' (MOE 2017a). To better engage learners and to immerse them in authentic and relevant learning, H2 Mathematics and Further Mathematics syllabi included two new components, one is assessed in the 'A'-level examinations and the other to augment the 'examinable'. The part which will be assessed in the 'A'-level examinations appears in the form of *Problem in Real-World Contexts* (PRWC), where real-world situations are mathematised via suitable mathematical models and question items are designed to allow students solve problems pertaining the given real-world situation. The augmenting component targeted to enhance teaching and learning is *Learning Experience*, where students are immersed into meaningful discussions, giving them appropriate platforms in which they actively reason and communicate their understanding of concepts. Learning experiences typically manifest as lessons designed to make connections between ideas in different topics, between abstract mathematics and real-life applications, and between mathematics and other disciplines. An exemplar of learning experience created by and for teachers' use is given in the Appendix; it illustrates the relevant mathematical content knowledge that connects two different topics in the Mathematics 9758 Syllabus. It is hoped that, through learning experiences, students may learn to form their own understanding of concepts independently before these are formally taught to them. The detailed record of all content adjustments that took place in A-level mathematics syllabi from 1997 to 2017 can be found in Ho and Ratnam-Lim (2018).

5.2.1.4 Some Concerns Raised by Junior College Teachers

Having discussed the major education initiatives put forth by the MOE and how these initiatives have shaped the A-level Mathematics curriculum, we now turn to look briefly at what some experienced JC teachers have to say about the curriculum changes that occurred in the period 1997–2017. For the purpose of this chapter, we only focus on the comments that have implications on tertiary mathematics education. Though the sample of JCs teachers is small, they are holding or have held middle-leadership roles (e.g. level heads, head of departments) in JCs during the stated period, and hence, their views on the syllabi changes are definitely representative of the views of most JC teachers. Each participating teacher answered a questionnaire pertaining to the A-level mathematics syllabi changes and their perceived outcomes. We present, in summarised form, the data we obtained:

- *Is there a real need for changes in the A-level mathematics syllabi? If so why?*

All the participants of the interview expressed that there is a genuine need to have timely change in the syllabi. Moreover, they listed some criteria to consider as far as syllabi changes are concerned:

> I think the syllabus should change according to the changing needs. Beauty, suitability, relevance and applicability are some parameters for consideration when deciding on the content of the syllabus.
>
> Yes. To respond to changes occurring in the world such as technological advances.

- *Describe the changes that you see in the learning outcomes of the students over these years of changes in the syllabus.*

Some teachers perceived that the changes in the learning outcomes of their students were generally positive, i.e. they are beneficial to the students' academic development:

> The changes made to the syllabus over the last few years made the learning of mathematics more relevant to the student by relating it to the real-world contexts. The implementation of 'Learning Experience' and the introduction of 'Application Questions' in the examination aim to achieve this goal.

Some teachers held different opinions:

> I do agree that mathematics should be seen as an effective tool in solving real-world problems but I also do not discount the fact that studying mathematics should be an end itself. That is, mathematics should be pursued regardless of whether it has potential for applications. But it almost always turns out that some obscure piece of mathematics holds the key to the answer of some deeper questions in science.
>
> No mathematical rigour. Can be seen from their [students'] work. Poor algebraic skills. Students are mostly performance-driven.

- *Sum up your experience/opinions concerning the changes in A-level mathematics syllabi over the stated period.*

JC teachers are also concerned whether the content reduction in the A-level Mathematics syllabi may result in weakening students' mathematical content knowledge, and thus, whether students will be ready for learning mathematics at the tertiary level. Here are some of their voices:

> I think the H2 Math syllabus should provide opportunities for students to think logically and articulate mathematically. One area where students can develop these good qualities is 'Proofs'. I am actually saddened that Mathematical Induction has been removed from the H2 Math syllabus. Mathematical Induction is an important tool in proving mathematical statements.
>
> Students can also appreciate the beauty of this technique and its logical foundation. It's a beautiful piece of mathematics. I feel that some of the 'proofs' in the current H2 Math syllabus are not rigorous enough. For example, to 'proof' that a function is 1-1, one uses the horizontal line test which is incorrect.
>
> Mechanics is another subject which in my opinion lends itself perfectly to mathematical modelling and applications in real-world contexts. It also has cross-disciplinary interaction with Physics.
>
> I strongly maintain that traditional pure mathematics topics like 'Group Theory' should be brought back to the 'A' level FM syllabus to let students have a taste of handling mathematical proofs and understanding what a mathematical structure is. This idea of structure in mathematics is an important one and permeates almost all branches of mathematics – Group structure, Measure Spaces, Normed Spaces, and Topological Spaces etc. are all mathematical structures.
>
> I remember the Math B and Further Math B syllabus I did as a student have given me a strong foundation to study mathematics in the university. I can't really say the same about our current syllabus.

Experienced JC teachers also showed concern towards the deteriorating standards of mathematical content knowledge of JC teachers in the current state as well as in the future, and this observed phenomenon may have implications on JC teachers' competencies and professional development.

> Removal of FM results in loss of expertise, experience and resources in higher math.
>
> Many of the ex-FM teachers have retired or just teaching H2 Math due to small candidature.
>
> The quality of teachers' knowledge is lacking – content, assessment, etc. There are gaps which the teachers cannot see themselves, and they wonder what is wrong. It seems to be getting worse.
>
> Re-introducing FM – seems to place more emphasis on engineering, and hence engineer-trained teachers are teaching FM.

While the JC teachers' feedback and concerns from their implementation of the various revised A-level Mathematics syllabus are genuine and truthful, we need to remain objective in understanding the current situation of the A-level mathematics syllabi and the readiness of the next generation of Singapore mathematics learners to progress beyond the school mathematics. To achieve this objectivity in our analysis, we must acquaint ourselves with relevant and established curricular theories. The framework we choose is the spectrum of curriculum ideologies as presented by Schiro (2013), and in Sect. 5.2.2, we follow closely his interpretation of the central curricular ideologies. In Sect. 5.2.2, we analyse the shifts of curricular orientation that Singapore A-level Mathematics experienced from 1997 to 2017 so as to better understand how well Singapore students are equipped for learning mathematics at tertiary level.

5.2.2 Curriculum Ideologies

We give a brief introduction of the four main curriculum ideologies below: Scholar Academic, Social Efficiency, Learning Centred and Social Reconstruction. For each ideology, wherever possible, we tease out those syllabi aims of the different 'A'-level Mathematics Syllabi over the past two decades, which will provide evidence for the presence of influence of that ideology.

5.2.2.1 The Scholar Academic Ideology

Scholar academics advocate that the human culture has amassed a body of important knowledge which has been organised into academic disciplines institutionalised in universities. Education, hence, is aimed at inducting young children into the system of acquisition of such knowledge, i.e. the different academic disciplines. An academic discipline is perceived to be a hierarchy of people in search of truth and knowledge. The *top* comprises scholars who discover new truths, i.e. university professors and researchers, the *middle* teachers of the discipline who disseminate the truths discovered by the scholars and the *bottom* school students whose responsibility is to learn the truth so as to become more proficient members of the academic community. Mathematics as a long-established discipline requires of its learners a specific *disciplinarity*—the way a mathematician think and the way a mathematician works—the very characterisation of the practices of a working mathematician. The aim of education in mathematics, according to the Scholar Academic ideology, is to equip young members of the mathematical discipline with the disciplinarity of mathematics, moving them from the bottom (primary mathematics) towards the top (tertiary mathematics) of the aforementioned hierarchy.

Regarding thinking and working, both the 9205 and 9233 Syllabi are explicit about the need for clarity in thinking, and accuracy and carefulness in working in the wording of the syllabus aims:

> [item (c), 9205] encourages *clear thinking* and *accurate working;*
>
> [item (6), 9233] develop their [students'] ability to *think clearly*, *work carefully* and …;

Additionally, both these syllabi highlight logic and coherence as important aspects of the mathematical disciplinarity:

> [item (g), 9205] develops a *logical and coherent view of mathematics;*
>
> [item (4), 9233] appreciate mathematics as a *logical* and *coherent* subject with rich interconnections;

All the Mathematics Syllabi (9205, 9233, 9758) emphasised on the upward movement of youth members of the mathematics discipline from the school level to the university level, and this is evidenced in the explicit statements found in their respective syllabus aims:

> [item (d) & (e), 9205] provides as much as possible of the mathematics necessary for the student's concurrent study at A-level; provides a suitable foundation for *beginning a degree level course in mathematics* or a related discipline;
>
> [item (9), 9233] acquire a suitable foundation for *further study of mathematics* and related disciplines;
>
> [item (a), 9758] acquire mathematical concepts and skills to prepare for their *tertiary studies in mathematics*, sciences, engineering and other related disciplines;

5.2.2.2 The Social Efficiency Ideology

The Social Efficiency Ideology advocates that the purpose of schooling is to efficiently meet the needs of the society by training its youth to function as future mature contributing members of society. Skills and procedures needed at workplace and at home are deemed as of paramount importance to ensure productive lives and to perpetuate the functioning of society.

Through the lens of Social Efficiency Ideology, mathematics learnt in schools must be functional and useful at the workplace. Thus, certain fields of mathematics at the tertiary level that are inclined towards this ideology include financial mathematics, engineering mathematics, econometrics, mathematical biology and operations research methods; all of these have a natural tendency to be interdisciplinary and focus more on real-life applications. As the emphasis shifts towards STEM education, one observes that the changes in the 'A'-level Mathematics Syllabi lean towards the applicability of mathematics at the workplace.

> [item (h), 9205] presents at least one major area of application of mathematics—either particle mechanics or probability and statistics—so that students can see examples of the *usefulness* of mathematics in the *real world*;
>
> [item (8), 9233] appreciate how mathematical ideas can be *applied in everyday world;*

In order that mathematics learnt at school is truly functional at the workplace, the entirety of skills related to applicability to real-life situations must be realised by effective social interactions and output, i.e. collaboration, communication and invention. Traces of this social efficiency aspect can be found in the 'A'-level Mathematics Syllabi:

> [9740] produce imaginative and *creative* work arising from mathematical ideas; develop abilities to reason logically, to *communicate* mathematically, and to learn *cooperatively* and independently; and make effective use of variety of mathematical tools (including information and *communication* technology tools) in the learning and *application* of mathematics;
>
> [9758] develop thinking, reasoning, *communication* and modelling skills through a mathematical approach to problem solving;

5.2.2.3 The Learner Centred Ideology

The Learner Centred Ideology anchors itself on the needs and concerns of the individual learners. Thus, the goal of education is the growth of individuals, each in harmony with his or her own unique intellectual, social, emotional and physical attributes. In the lens of Learner Centered Ideology, the emphasis *shifted* from personal enjoyment to a holistic experiential appreciation of the subject over the recent years.

> [9205] develops further the mathematical knowledge of students in a way that encourages *confidence* and provides understanding and *enjoyment*;
>
> [items (1)-(2), 9233] develop further their understanding of mathematics and mathematical processes in a way that encourages *confidence* and *enjoyment*; develop a *positive attitude* to learning and applying mathematics;
>
> [9740] develop *positive attitudes* towards mathematics;
>
> [9758] *experience* and *appreciate* the nature and beauty of mathematics and its value in life and other disciplines;

Already mentioned in Sect. 5.2.1.4, 'Learning experiences' are explicitly incorporated into the 9758 Mathematics and 9649 Further Mathematics Syllabi to promote such an experiential appreciation of Mathematics in the classroom.

5.2.2.4 The Social Reconstruction Ideology

The Social Reconstruction Ideology comes from a social perspective in that it assumes that the current state of the society is unjust and plagued with certain societal problems such as racial, ethnic, gender or economic inequalities, or some form of threat to the society. To resolve these societal problems, Social Reconstruction Ideologists advocate education is the way to facilitate the construction of a new and more just society that offers maximum satisfaction of its members.

Although Singapore has enjoyed many years of social peace and harmony under the leadership of a corrupt-free and efficiency government, the perpetuation of social justice and stability must never be taken for granted, especially given the current volatile world trends and rampant terrorist threats. Staying vigilant and ready to respond towards potential crisis and threats to Singapore's survival, the education system must be robust and quick enough to respond to changing demands, constantly positioning Singapore at the competitive front. Top-quality education is the only way to safeguard Singapore's regional competitiveness and social stability, and this must be available not just to a few elite schools but every school in Singapore—whence, 'every school a good school' (see Sect. 5.2.1.4). Although no part of the 'A'-level Mathematics Syllabi articulates the young nation's uncompromising stand for social justice or stability, it is the subtle insistence on the virtues of diligence, persistence and resilience to be developed in students during their course of the mathematical training that nurtures these students to be ready in times of crisis in the future.

5.2.3 Analysis of the Changes in the A-Level Mathematics Syllabi Based on Curriculum Orientations

Scanning through the evolutionary history of the 'A'-level Mathematics Syllabi, one observes a gradual but clear change in curriculum orientation, namely from Scholar Academic, through Social Efficiency to Learner Centred. Other than the *statements of the syllabi aims*, the actual changes in the selection and alignment of topics found in each syllabus revision witness this change in curriculum orientation. Here, we focus on three major 'A'-level Mathematics syllabus revision which resulted in three different syllabi: (R1) Revised 2001 Mathematics Syllabus (9233), (R2) 2006 'A'-level Mathematics Syllabus (H2 9740) and (R3) 2016 'A'-level Mathematics Syllabus (H2 9758). Since Further Mathematics was withdrawn as an 'A'-level subject from 2006 to 2016, we choose to focus on the syllabi change occurring for Mathematics (and not Further Mathematics) as our main purpose is to highlight the shift in the syllabi orientation throughout a continuous time interval.

5.2.3.1 Revised 2001 'A'-Level Mathematics Syllabus (9233)

As mentioned earlier, the MOE TSLN initiative pushed forward the reduction in content to provide schools with more time to incorporate more thinking activities and infuse ICT into lessons. This content reduction 'movement' supports the view that MOE began to value the higher-order cognitive processes in teaching and learning across all subjects, departing from the old Scholar Academic stance. The most notable change was a substantial removal of trigonometry from the syllabus (e.g. general solution of trigonometrical equations, etc.). Graphing Calculators (GCs) was introduced to Further Mathematics students, though it was maintained that examination questions in the 'A'-level Further Mathematics paper were 'GC-neutral', i.e. students who used GCs would have no unfair advantage over those who did not. The use of GCs was in line with the ICT Masterplans mentioned in Sect. 5.2.1.2 as well as to expose students to the use of a powerful computational tool. In order to create a greater flexibility of choice of topics for the students, the assessment format in the 'A'-level examination changes: an 'Either-Or' option was available for the last question of Paper 1. This first syllabus revision spells the first step of departure from Scholar Academic, emphasising usability and learner-centredness.

5.2.3.2 2006 'A'-Level Mathematics Syllabus (H2 Mathematics 9740)

Emphasis was placed on solving real-world problems, and activities including communication about the mathematics involved in solving a problem and interpretation of the solution in the context of the problem were encouraged. GCs took up a more significant role in teaching and learning of 'A'-level Mathematics in the classrooms, and crucially GCs had since been officially required in the examinations. In the second wave of content reduction, significantly more topics were removed from the syllabus (see Table 5.2); with regards to assessment, this reduction resulted in no question choice in the examination format. At this stage, the curriculum orientation for Mathematics at 'A'-level was then steering towards the Social Efficiency Ideology, where applicability of mathematics in the real world, e.g. at the workplace, in the community, etc., is emphasised.

Table 5.2 Da Vinci programme structure

Year	Course item	Requirement
1 and 2	Creative thinking	Participate in activities that stimulate creative thinking
3 and 4	Independent Research Studies	Complete a Research Methodology module; encourage to work on a research project under the guidance of a teacher-mentor; have the option to participate in external research programmes at universities
5 and 6	Advanced Research Project in either Mathematics or Science	Completed within 9–18 months, dependent on the nature of the project, including at least two weeks of full-time research to be mentored by professors at leading research institutions, universities or polytechnics; research project must be showcased at the annual Research Congress held in March; possible grades received: distinction, merit, pass or fail

5.2.3.3 2016 'A'-Level Mathematics Syllabus (H2 Mathematics 9758)

An expanded suite of syllabi, with H2 Further Mathematics, was introduced to give students more options to choose from, thus catering better to their diversified needs. An emphasis was placed on mathematical processes such as mathematical reasoning, mathematical modelling and communication. Learning experiences, which are stated in the syllabus, are instituted to positively influence the ways teachers teach and how students learn so that curriculum objectives can be achieved. It is stated explicitly in the revised syllabus that teachers are also encouraged to use pedagogies that are constructive in nature. At this stage, we witness a shift of the curriculum orientation from Social Efficiency to Learner Centred Ideology.

5.3 Preparing for Tertiary Mathematics Education at Schools

The analysis given in the previous section provides insight into how education system is moving towards, the general 'big' direction. Now we are ready to see whether our findings and deductions account for the ways Singapore prepare her students for tertiary education, in our interest area—tertiary mathematics education. In this section, we look at these preparatory processes in two ways: H3 Mathematics syllabus and its implementation, and the set-up of niche-area schools—a case study of NHSMS.

5.3.1 H3 Mathematics Syllabus and Its Implementation

With the revamp of the A-level curriculum in 2006, H3 Mathematics was introduced to provide opportunity for students who have an exceptional aptitude and passion for mathematics to pursue it at a higher level than that of H2 Mathematics. Students offering H3 Mathematics have several options available to them. They can choose to read the mathematics module Linear Algebra I or Numbers and Matrices offered by NUS or NTU, respectively; undertake research projects supervised by academic staff from NUS or NTU; or read H3 Mathematics offered by MOE.

The module Linear Algebra I offered by NUS to pre-university students taking it as a H3 module is also a regular module that is offered to NUS undergraduates, and is taught by NUS lecturers. It is a typical first course in linear algebra which covers systems of linear equations, matrices and matrix algebra, vector spaces and linear transformations. In contrast, the module Numbers and Matrices offered by NTU was specially designed as a H3 module for pre-university students. It covered basic number theory, and basic linear algebra topics such as matrix algebra and vector spaces, and was taught by NTU lecturers.

We shall now focus on the H3 Mathematics offered by MOE. Since its implementation in 2007, the MOE H3 Mathematics syllabi have undergone minor and major changes. Even though the syllabi have changed over the years, the focus of MOE H3 Mathematics has been to develop students' abilities to solve non-routine problems and write mathematical proofs. The students are expected to develop their fluency with mathematical language and notation, and the concepts of proposition and its converse, contrapositive and inverse. They also need to have knowledge of the different methods of proof. In terms of content coverage, there has been a shift from new additional content on top of H2 Mathematics to content that builds on the knowledge acquired in H2 Mathematics. We elaborate below the H3 Mathematics syllabi and the changes since 2007.

5.3.1.1 2007 'A'-Level Mathematics Syllabus (H3 Mathematics 9810)

There were four topics, namely, Differential Equations, Plane Geometry, Graph Theory and Combinatorics. While Differential Equations, Combinatorics built on the topics of differential equations and permutations and combinations in H2 Mathematics, and Plane Geometry expanded on plane geometry topic in 'O'-level Additional Mathematics, Graph Theory was a completely new content that was not related to any topic in H2 Mathematics. From past examination papers, it was apparent that Graph Theory and Plane Geometry were used as medium to develop students' mathematical reasoning and proof-writing skills, while Combinatorics focus on honing students problem-solving skills through applying basic principles of counting to solve a variety of counting problems. On the other hand, the emphasis of Differential Equations was on analytical and numerical methods of solving first-order differential equations, and its applications in modelling population dynamics. The examination paper consisted of two sections: Section A, which contained four questions on Differential Equations totalling 40 marks, and candidates had to answer all the questions; and Section B, with two questions on each of Plane Geometry, Graph Theory and Combinatorics, and candidates were required to answer any four questions, with each question worth 14 marks. There were 4 marks allocated for the clarity of presentation.

5.3.1.2 2010 Revised 'A'-Level Mathematics Syllabus (H3 Mathematics 9810)

A notable change in this revised syllabus was the removal of Plane Geometry. Other than this, there were not much changes in the syllabi of Differential Equations, Graph Theory and Combinatorics, except that digraphs and tournament were removed from Graph Theory. There were no significant changes to the examination format, except that candidates had to answer all questions in Section B, with two questions on each of Graph Theory and Combinatorics, and each question still worth 14 points.

5.3.1.3 2013 'A'-Level Mathematics Syllabus (H3 Mathematics 9824)

This syllabus represented a significant change in content coverage from Syllabus 9810, as Graph Theory was no longer included. In its place were topics from H2 Mathematics: Functions and Graphs, Sequence and Series, and Calculus. The other two topics from Syllabus 9810, namely, Combinatorics and Differential Equations, were still in this syllabus, though the latter had been renamed as Differential Equations as Mathematical Models. The topics in Combinatorics and Differential Equations as Mathematical Models remained largely unchanged from Syllabus 9810, although second-order homogeneous linear differential equations and mathematical models of vibrating springs had been added in the latter. As noted earlier, the content built on knowledge acquired in H2 Mathematics, but in greater depth and breadth. There were also notable changes to the examination format. The examination paper consisted of eight questions of varying lengths and marks, with three questions on each of Functions and Graphs and Differential Equations, and two questions from Combinatorics, and candidates were required to answer all questions. No marks were allocated for clarity of presentation.

5.3.1.4 2017 'A'-Level Mathematics Syllabus (H3 Mathematics 9820) (MOE 2017b)

With the introduction of Further Mathematics, most topics in Differential Equations in the previous H3 Mathematics syllabi are now covered in Further Mathematics. Therefore, Differential Equations no longer features prominently in this syllabus and is subsumed under the broad topic of Functions, which also includes graphs, symmetries, derivatives and integrals. The other broad topics in this syllabus are as follows: Numbers, Sequences and Series, Inequalities and Counting. Comparing with Syllabus 9824, the topic on Numbers is new. Although it is new additional content, the amount of materials it covers is substantially less than that in Graph Theory in Syllabus 9810. Further, Numbers builds on the topics of prime and composite numbers and greatest common divisor in secondary mathematics. Nevertheless, students need to learn the formal definitions and properties of divisibility, prime numbers and greatest common divisors, as well as new concepts on congruence and modular arithmetic. That said, the content of Numbers is elementary and well-suited to serve as a means for students to learn mathematical reasoning and proofs. There are also changes to the examination format. The examination paper will consist of eight to ten questions of different lengths, with each question worth 8–16 marks, and candidates will be expected to answer all questions. The scheme of examination paper does not spell out how many questions there are for each broad topic.

5.3.2 *A Case Study of a Niche-Area School: NUS High School of Mathematics and Science*

The NUS (National University of Singapore) High School of Math and Science (NHSMS, for short) is a specialised independent high school in Singapore offering a six-year Integrated Programme (IP) leading to the NUS High School Diploma. The school offers a highly accelerated mathematics and science curriculum integrated with language, arts, humanities, sports, in a modular system. It is estimated that about 90% of its graduates have pursued Science, Technology, Engineering and Medicine-related courses in University.

5.3.2.1 Academic Curriculum

Although the NUS High School is an Integrated Programme school, which means students need not take the O-levels, it does not offer A-level or International Baccalaureate programmes, unlike other Integrated Programme schools in Singapore. In place of these, an NUS High School Diploma is conferred onto her graduates, and this diploma is recognised by all universities both locally and worldwide by virtue of its high level of academic rigour that is comparable to the above-mentioned qualifications. What makes NHSMS a niche school is its accelerated curriculum for mathematics and science curriculum. Honours courses in the Specialization Stage for mathematical and scientific disciplines are offered to further stretch the academic abilities of able students beyond the already-accelerated curriculum.

The graduation requirement for the NUS High School Diploma mandates that the students take Mathematics and at least two science subjects (including computing studies) at the major (basic) level in the Advancement Stage. Students are given the option to read a fourth subject from any subject group (sciences, humanities and the arts), and take any math/science subject at the honours level. In addition, students must complete an Advanced Research Project under the school's Da Vinci Research Programme. This is a mandatory research curriculum programme that every NHSMS student must go through. The Office of Research, Innovation and Enterprise is the primary body responsible for developing and implementing this research curriculum, with the programme structure given below in Table 5.2. In their senior years, students are encouraged to sit for Advanced Placement and Scholastic Assessment Test (SAT) examinations for credits for admission into foreign universities. Note that these additional sittings of examinations are not part of the graduation requirement.

Talent programmes are a central hallmark of a student's school experience at NUS High School. Apart from the Da Vinci Programme, four other specially featured talent programmes include (IP) Internationalisation Programme (exchange programmes with other math and science schools, Summer Academic Programmes), (E+P) Einstein+Programme (academic mentorship by NUS Professors, Olympiad training programme), (SP) Socrates Programme (for talented students in the humanities) and (AAP) Aesthetic Appreciation Programme.

Table 5.3 Different undergraduate mathematics programmes offered by universities in Singapore

Conferring university	College/department	Programme title	Duration of candidature
Nanyang Technological University (NTU)	National Institute of Education (Mathematics and Mathematics Education Academic Group)	BA/BSc (Ed)	4 years
	School of Physical and Mathematical Sciences (division of mathematical sciences)	B.Sc./B.Sc. (Hons) [Mathematical Sciences]	4 years
National University of Singapore (NUS)	Department of Mathematics	B.Sc. (Hons) with Major in Mathematics (MA)	4 years
Singapore University of Social Sciences (SUSS)	School of Science and Technology	B.Sc. Mathematics	3 years

5.4 Tertiary Mathematics Education in Singapore

At the time of writing this chapter, there are three universities in Singapore offering mathematics at undergraduate level under different programme titles (see Table 5.3). Note that Nanyang Technological University offers two distinct Bachelor of Science degree programmes under the School of Physical and Mathematical Sciences and the National Institute of Education, respectively (Table 5.5).

5.4.1 Programme Structures

5.4.1.1 Nanyang Technological University/National Institute of Education (NIE)

We point the reader to Chap. 15 (Sect. 15.4.1) for the detailed programme structure for B.A/B.Sc. (Ed) offered by NIE. In that section, the reader will also find more information about the *distinctive* quality of the NIE undergraduate programme, i.e. teaching and learning of tertiary mathematics, unlike the other programmes mentioned herein, is guided and shaped by the pedagogical principles as advised by the mathematics educator colleagues of the Mathematics and Mathematics Education Academic Group.

5.4.1.2 Nanyang Technological University/School of Physical and Mathematical Sciences (SPMS)

Based on a social efficiency orientation, the curriculum for the undergraduate mathematics programme in NTU/SPMS is designed with the objective of equipping the graduate with rigorous training needed for the new economy. The approach is also backed up with the belief for continual lifelong learning so that the graduates can be adaptive individuals that can contribute towards the society. Both breadth and depth in knowledge and competencies are emphasised: breadth in knowledge and competency in useful skills such as communication as well as depth in knowledge domains rooted to the discipline of mathematics that is required of a mathematics major. For the Major in Mathematical Sciences (MAS), students will be trained in analytical and reasoning skills, together with problem-solving skills, through the acquisition of rigorous mathematical concepts. Additionally, undergraduate mathematics students are trained in computing, technical communication, and exposed to the interdisciplinary nature of mathematics, especially with other disciplines such as biology, computer science, economics and finance. Deeper investigations in the subject can be taken up by students via special courses, supervised independent study and research projects.

Given the broadness of mathematical sciences, four distinct tracks: (1) Pure Mathematics, (2) Applied Mathematics, (3) Statistics and (4) Business Analytics, which are offered within the Major in Mathematical Science, cater to the varying interests of students. The summarised programme structure of MAS is given in Table 5.4.

For the purpose of comparison, we look more closely at the courses offered for Track (1) Pure Mathematics only (see Table 5.5).

Table 5.4 Summarised Programme Structure for MAS (NTU/SPMS)

Courses	AU	Remarks
MAS Core Courses for all track	48	
MAS Core Courses for a specific track (Pure Math/Applied Math/Statistics)	11	
MAS Prescribed Electives for a specific track, including project (Pure Math/Applied Math/Statistics)	25	A grade of A− or better in the Final Year Project (MH4900, 8 AU) is compulsory for the award of Honours (Highest Distinction)
GER: General Elective Requirement GER Core Courses GER Elective Courses	12 15	
Unrestricted Electives	21	
Total	132	

AU—Academic Unit

Table 5.5 Courses overview for MAS Track (1) Pure Mathematics

Year/courses	AU
Year 1	
Calculus I & II, Linear Algebra I & II, Foundations of Mathematics, Discrete Mathematics, Algorithms and Computing I & II	27
Year 2	
Calculus III, Groups and Symmetries, Algorithms & Computing III, Probability and Introduction to Statistics, Real Analysis I, Ordinary Differential Equations	21
One core course of Track (1)	3–4
Year 3	
Two core courses of Track (1)	7–8
Prescribed electives of Track (1)	See below
Year 4	
Prescribed electives of Track (1)	See below
Note A grade of A- or better in the Final Year Project (MH4900, 8 AU) is compulsory for the award of Honours (Highest Distinction)	
Track in Pure Mathematics (1)	
Courses offered	
Core courses: Complex Analysis, Knots And Surfaces: Introduction To Topology, Abstract Algebra	4/4/3AU
Prescribed electives: List 1: Real Analysis II, Algebraic Topology, Differential Geometry, Continuous Methods List 2: Number Theory, Abstract Algebra II, Set Theory and Logic, Algebraic Methods List 3: Coding Theory, Cryptography, Combinatorics, Discrete Methods, Algorithms and Theory of Computing, Algorithms for the Real World List 4: Final Year Project, Professional Internship	4 AU each 8AU/11AU

AU—Academic Unit

5.4.1.3 National University of Singapore/Department of Mathematics

The B.Sc. (Hons) with Major in Mathematics (MA) is advertised as the flagship major that any leading university of the world is obliged to offer. The objective of the programme is to expose students to all the important areas of mathematical knowledge including algebra, logic, number theory and combinatorics, real and complex analysis, differential equations, geometry and topology with focus on mathematical foundations and fundamental techniques. The prerequisite to the programme is a pass in the 'A'-level H2 Mathematics; a lack of basic background may be made up for by reading a certain 'bridging' module pegged at Module Level 1000.

To graduate with a B.Sc. (respectively, B.Sc. (Hons)) with primary major in Mathematics, a student must complete a total of 120 (respectively, 160) Modular Credits

Table 5.6 NUS B.Sc./B.Sc. (Hons) programme degree requirement

Module level	Major requirements	Cumulative MCs
1000	Fundamental concepts of Mathematics or Discrete Structures Linear Algebra I, Calculus, Programming Methodology	16
2000	Linear Algebra II. Multivariate Calculus, Mathematical Analysis I Algebra I, Probability One additional module from List II, III, IV	40–44
3000	Mathematical Analysis II, Complex Analysis I, Two modules from List MA3 Pass one additional module from List III, IV	60–66
4000	Honours Project in Mathematics Four modules from List MA4 Pass one additional module from List IV	92–98
Lists II, III and IV are not available on the public domain		
List MA3: Algebra II, Set Theory, Mathematical Analysis III, Ordinary Differential Equations, Introduction to Number Theory, Introduction to Fourier Analysis		
List MA4: Galois Theory, Mathematical Logic, Functional Analysis, Partial Differential Equations, Complex Analysis II, Measure and Integration, Topology, Differential Geometry of Curves and Surfaces		

MC—Modular Credit

(MC) of courses, inclusive of 20 MC of university requirements, 4–8 (respectively, 4–12) MC of faculty requirements, 60–66 (92–98) MC of major requirements and 26–36 (respectively, 30–44) MC of free electives. Furthermore, the major requirements in Table 5.6 must be satisfied.

5.4.1.4 Singapore University of Social Science (SUSS)/The School of Science and Technology

The SUSS mathematics programme offers graduates a rigorous and broad foundation in the three main pillars of pure mathematics, applied mathematics and statistics. The programme aims to have her graduates explore in greater depth any of a combination of these three important pillars via a range of elective courses that includes abstract algebra, financial mathematics, mathematical modelling, mathematics in computing, mathematical logic, number theory, probability and statistics.

An interesting and seemingly attractive feature of the programme in SUSS is that all foundational mathematics is re-examined and reviewed within the compulsory core Level 1 mathematics courses. This facilitates anyone who meets the general university entry requirement with an interest in learning mathematics to be eligible for the programme; in particular, an 'A'-level pass in H2 Mathematics is *not* a prerequisite of the BSc Mathematics programme.

To graduate with a basic degree, students are required to complete a total of 130 Credit Units (CU) of courses, inclusive of 10 CU of university core courses. The breakdown of the CU's to be completed for the BSc Mathematics programme is as follows: (1) 70 CU of Compulsory Courses; (2) 50 CU of Elective Courses; and (3) 10 CU of University Core Courses. The curriculum has a three-tier structure. Level 1 courses comprise a basic suite of four courses covering all aspects of foundational mathematics and statistics. Level 2 courses consist of a set of core courses in pure mathematics, applied mathematics and statistics that will prepare students for higher level mathematics courses, together with a number of elective courses in financial mathematics, mathematics in computing and computer programming in C++. Level 3 courses consist of a collection of advanced elective courses such as graph theory, complex analysis, optimisation, logic, number theory and applied probability, where students can choose courses to suit their own interests and abilities.

5.4.2 University Professors' Viewpoints on the Changes in Tertiary Mathematics

It is not our purpose here to compare the different programme structures of the above lists of undergraduate degree programmes since we trust that it is the *substance* of the programme rather than its *structure* which makes the difference in the quality of programme. Here, the 'substance of the programme' is characterised by the manner and quality of the teaching and learning that take place in the undergraduate mathematics courses offered under each programme. Based on this view, we believe it is more meaningful to interview professors, seven in total, who have taught or are currently teaching mathematics courses offered under these programmes in the aforementioned three universities. It is hoped that their responses will offer insights into the way tertiary mathematics is imparted to the mathematics majors by these interviewees. Our ensuing analysis of the interview data uses a qualitative approach. Admittedly, the small number of professors yields data that are far from being representative of the general approach taken at the respective universities. Nevertheless, what we compromise for numbers we make up by the rich teaching experience of these professors—the minimum being 12 years; the maximum 57 years. Table 5.7 summarises the profiles of the seven interviewees, whom we label as Professors A-G.

In the remaining of this subsection, we shall summarise the information gathered from the interview data based on the inputs of the professors we interviewed. The information is categorised accordingly to (a) the major changes in tertiary mathematics education system and their objectives and (b) the future of tertiary mathematics education, with special focus on whether schools students are ready to read mathematics at university level and/or to become mathematicians.

Table 5.7 Profiles of the seven interviewees (Professors A-G)

Professor	Universities Taught/Teaching	Teaching experience (Years)	Undergraduate math courses taught	Teaching Philosophy (P)/Teaching Approach (A)
A	NTU/NIE	19	Graph Theory Number Theory Computational Math	P: Guide students to understand and be infused by the disciplinarity of mathematics—rigour, proof, problem-solving, beauty A: Awareness of the different capabilities of students and adjust accordingly
B	NUS NTU/NIE	57	All undergraduate courses	P: Teach students to have sharp observation and critical analysis A: Mindful of pitching teaching at different levels for different students
C	NTU/SPMS	12	Discrete math Real analysis Abstract algebra	P: Motivate students in thinking and practicing mathematics A: Use teaching methods that target on students' motivation and interest levels
D	NTU/NIE NTU/SPMS	19	Calculus Multivariate Calculus Real Analysis Complex Analysis	P: Teaching and learning is a social activity and involves intellectual exchange, where the teacher and the students have to commit their attention so that meaningful learning takes place A: Work out details in classes; not keen about using power-point
E	NUS NTU/NIE	34	Calculus (Engineer) Analysis Measure Theory Functional Analysis Mathematical Methods	P: Awareness of students' background knowledge; progress from fundamental concepts; go deep rather than broad A: Start with examples and end with examples/counterexamples; use whiteboard to teach small class; power-point for mass lectures; use computer animations

(continued)

Table 5.7 (continued)

Professor	Universities Taught/Teaching	Teaching experience (Years)	Undergraduate math courses taught	Teaching Philosophy (P)/Teaching Approach (A)
F	NUS	21	Fundamental Concepts of Mathematics Calculus (Engineer) Linear Algebra Multivariate Calculus Quantitative Reasoning	P: Help students transit smoothly from A-level to tertiary mathematics; changing students' mindset/attitude towards learning math A: Engaging students intellectually in class; teacher–students and student–student interactions
G	NUS SSU	45	Calculus Advanced Calculus Linear Algebra Modern Algebra Combinatorics Graph Theory Discrete Math	P: Teaching is an interaction between the teacher and his/her audience; this meaningful interaction contributes to the growth of both parties A: Present abstract ideas in concrete or geometrical ways; use historical remarks/picture of mathematicians to motivate topics; talk to students: obtain feedback on teacher's teaching and students' difficulties

5.4.2.1 From Past to Present

Finding out from the various professors what the major changes took place in the respective university mathematics department in terms of undergraduate programme in mathematics is a key step in our current undertaking to understand what goes beyond school mathematics, particularly what is the tertiary mathematics education landscape like. The interview question below was intended to tease out exactly this required information:

> What are the major changes in the tertiary education system over the last 10-20 years in the university stated you are teaching in, in terms of an undergraduate programme in mathematics? Be more specific in terms of the description, e.g., change in course structures, assessment modes, modular system, honors-year thesis, etc., and the estimated year of occurrence.

The interview responses all revealed both changes to the programme structures and to the ways degree programme courses in mathematics were taught, as well as the manner in which students were assessed.

Let us note the changes in programme structure that took place in the National Institute of Education over the last two decades. According to Professor A, three major programmatic changes occurred at NIE in around 1998, 2003, 2008 and most recently in 2017. While these changes might be brought about due to direct impact from new education policies made and perhaps be justified or understood at a programmatic level, additional insight can be obtained by matching the years in which these changes took place (1998, 2003, 2008 and 2014) with the years in which a revision of the mathematics syllabus at 'A'-level took place (1997, 2001, 2006, 2016). With the exception of the 2014 programme revamp, a major structural change in the degree programme at NIE occurred one or two years *after* a major revision in the 'A'-level Mathematics syllabus. This approximate correspondence is an indication that the decision making at the policy level in NIE was *responding just in time* to important educational policy changes that took place at the school level in much the same way as schools responding to the major changes in the educational directives (e.g. TSLN, TLLM and ICT Masterplans) initiated by the government through the MOE. In 1997, a new 'interim' Mathematics Syllabus C (9205) was introduced which was applicable to Singapore 'A'-level candidates only. In this revised syllabus, more emphasis was put on *higher-order thinking (H.O.T.) questions*. Responding to this was the major change in NIE that took place around 1998 which saw more hours pumped into the AS component of the programme to equip students with a more complete coverage of content mathematics at the tertiary level.

In 2001, the new Mathematics Syllabus 9233 was introduced with the main aim of reducing the content, freeing up for more space in thinking and infusing Information Technology into teaching and learning of mathematics. Graphing Calculators made their first appearance in the Further Mathematics Syllabus. Interestingly, this cut in content at the 'A'-level brought about a similar change in the mathematics degree programme at NIE as witnessed by Professor A:

> The most major changes are the *loss of the 5th Honours year* (and the academic exercise) for Maths majors (2008), and … – Prof. A.

We now saw that the 'emphasis more on other aspects of teacher education' was in actuality a systems response to the changing demands in the school education landscape, namely, higher-order thinking and using Information Technology.

The year 2006 saw the removal of the Further Mathematics as an 'A'-level subject, and the H2 Mathematics Syllabus introduced put emphasis on solving real-world problems, i.e. solving a problem and interpreting its solutions. To ensure that the student teachers acquire more content knowledge to meet with the aforementioned change in educational emphasis in schools, the NIE degree programme for mathematics experienced a third major change in 2008 with an increase in the number of Academic Units for AS courses but the Academic Exercise did not return until later:

The fourth change in the NIE degree programme for mathematics was noted by Prof. A as follows:

> … and the 'revival' of the proportionate emphasis on AS1 (51 AUs compared to 39 AUs) with the academic exercise (2014). – Prof. A.

This change unlike other previous changes did not take place after a change in the Mathematics syllabus at 'A'-level. Crucially, this 2014 change brought about a renewed emphasis on content mathematics and the Academic Exercise was reinstated in the fourth year of the degree programme.

In the NUS, degree programme in mathematics has undergone several changes. Notably, in around 2005, a group-work approach to the Honours Year Project started (as opposed to individual work in the past). Concerning the reduction of content, Professor F has the following to say:

> Reduced syllabus - Over the years, there have been a rebalance between the breadth and depth in university education, with a *reduction of the major program requirement*, and an *increase in the general education requirement*. – Prof. F.

In both NTU/SPMS and SUSS, an emphasis was placed on content mathematics through Honours Year Thesis; for instance:

> For honor's projects, we have one-year FYP projects, to make sure that students can have deep understanding of the research area.- Prof. C
>
> Introducing more applied modules & honors-yr thesis – Prof. G.

Apart from the changes in the programme structures of mathematics undergraduate programmes, we also see changes that cater to the need of using technology in teaching and learning. We may interpret such changes as the impact of the ICT Masterplans rolled in the period 1997–2009. Three professors made special mention about the use of technology in their tertiary mathematics teaching and related concerns:

> Use of ICT in teaching - Both top down and ground up; this is a natural trend with the advancement of technology and new generation of learners. – Prof. F.
>
> "We provide chances for students to use Mathematica and Matlab in our teaching of calculus and linear algebra, since year 2005." – Prof C.
>
> I think the main change is the push of using TEL. It aims to use technology to enhance learning. But I believe that some struggles are still necessary. Over reliance on technology to relief the growing pains may end up not growing at all. Google and other search engine also brought forth an important change in how students obtain information. It used to be hard to get information (e.g., proof of a theorem) but now it is readily available. – Prof. D.

Looking for new pedagogies/methods of delivering mathematics at the tertiary level has now received more attention than in the past. Teacher belief in this aspect has also started to change:

> I relied on the use of mathematics software to illustrate concepts especially exploiting computing animations. – Prof. E.
>
> Blended learning/flipped classroom - More traditional lectures are being replaced by recorded video. Students come to classes for hands on, practical, group discussion. – Prof. F.

Changes also took place in the form of assessment; with regard to assessment, many of the interviewees put forth their views (and sometimes quite different within the same institution):

> In Singapore, the major changes took place in 1971 and in the last 20 years. In 1971, it was the introduction of new courses. Recently, it was *assessment*. Roughly, the *change in assessment* is from the British system to the American. – Prof. B.
>
> For my course on Abstract Algebra II, students have presentations on some topics they are interested, and presentation is part of the assessment, since 2014. – Prof. C.
>
> Grade-free modules - Probably something unique in NUS; students in their first year have the option of not counting the grade of the module, but opt for satisfactory/unsatisfactory.
>
> Open book exam – Though this has become more common in university exam to discourage students from memorizing, not many examiners are adopting it. It is more common for students to bring help sheets to math exams. – Prof. F.
>
> less closed-book exams – Prof. G.

Let us summarise what we have heard from the seven mathematics professors. Common to the responses of all the interviewees is the phenomenon of constant change in the tertiary mathematics education landscape. These changes usually took place at the university level as a response to significant initiative changes that occurred at the school level—in particular, about two years after a major revision in the 'A'-level Mathematics syllabus. Such changes in the tertiary mathematics education landscape ranged from structural changes in undergraduate degree programmes to the manner mathematics was taught and assessed at the university. We also see the impact the ICT Masterplans had on university teaching as mathematics professors looked for innovative ways to convey mathematical ideas to students by relying on computer and video technology. For an elaboration on the use of ICT in teaching and learning of mathematics at all levels in Singapore, the reader is pointed to Chap. 12. These changes have their repercussions whether for better or for worse as pointed out by Professor A below:

> Both changes significantly affected the ability of maths majors to get deeper into the disciplinarity, the first adversely [referring to the "C" series] and the latter, positively [referring to the new "A" series]. Time and content for reading and writing mathematics were affected. – Prof. A.

5.4.2.2 From Present to Future

In the interviews, all the professors indicated that content reduction is one of the most significant changes that took place at the university level for undergraduate mathematics degree programmes; generally, the coverage of pure mathematics at the tertiary level changed as a response to the content reduction in the 'A'-level Mathematics syllabus and has been reduced over the years. This then begs the following questions:

(a) What will the future of tertiary mathematics education in Singapore look like?
(b) Are the younger generation (school students) better prepared to read mathematics at the university level?
(c) Are they better prepared to become mathematicians in the future?

Here, we have three camps: the optimistic, the realistic and the less-than-optimistic.

The *optimistic* camp holds the view that the mathematics education students receive at school equips them sufficiently so that they may, if the situation allows, take up mathematics as a career.

> Tertiary mathematics education can provide a platform for students to learn math-related subjects, like finance, business analytics, modelling, etc. Quite a lot of students from our division find their jobs in financial industry. Yes. Students from JC are well-prepared to read math at university level.—Prof. C.

Optimism sometimes comes with bold creativity in that mathematics need to be redefined in order to enlarge the scope of its meaning. By so doing, this allows ones to see that many other skills and knowledge domains need to be imparted to students at school so that they can become a new generation of mathematicians:

> We need to redefine mathematics and mathematician. We no longer need to produce the same kind of mathematicians. I believe the same breeds are equally good. – Prof. B.

Most interviewees recognised that it is only *realistic* that not all people become mathematicians, and so tertiary mathematics education should not be solely aimed at producing mathematicians. Indeed, many careers call for analytical skills, problem-solving skills and logical reasoning which are expected attributes of mathematics graduates. A point to add here is that STEM students, in some of these universities, have the option to read Mathematics as a second major, and so tertiary mathematics education ought to be more inclusive to cater for the needs of this new group of students.

> I think students now are no longer the same as what we had. *Not many reading mathematics intend to be research mathematicians.* Nonetheless, there are some good mathematicians who came from other backgrounds. So I am not too dogmatic about that. – Prof. D.

> The trend seems to be students are becoming more "pragmatic" and choose applied math over pure math. There will still be a small group of students who will go for pure math but majority will choose to do applied math. I am not worried about this. What I hope to see is for more STEM students to do math as their second major (if their first major is science, computing, engineering etc.) to build a stronger foundation for their analytical skill. – Prof. F.

Awareness of the wide difference between making mathematics available for the majority and training the mathematical elite to be researchers in mathematics, the challenge here is how tertiary mathematics education can position itself in middle ground. Professor A proposed a realistic opinion about this:

> The future looks like it will be severely bifurcated – mathematics for applications for the majority and 'hard-core' research publishable mathematics for the elite. I think this is happening in NUS and NTU/SPMS. I hope that it will not happen in NTU/NIE – attempts are being made to review the curriculum to achieve the objective of a mathematics major who can read and write maths, can tackle unfamiliar problems, is exposed to the 'canon' of mathematics, can code, and have a positive attitude towards mathematics. – Prof. A.

Some interviewees held a *less-than-optimistic* view about the future of tertiary mathematics education, as far as the undergraduate degree programme is concerned.

> I don't think the A-level mathematics syllabus prepares student sufficiently for mathematics education at the tertiary level. Not enough rigour. – Prof. E.
>
> I hope with the return of 'A' level further math, students entering university will be better prepared. In general no [answering (b)]. Our education caters to the mass, which aims at equipping students enough math skills for the job market. To be mathematicians, this requires more in depth training, and only selected few with the passion and aptitude will make it. – Prof. F.
>
> No for the second [answering (a)] and third [answering (b)] questions. – Prof. G.

5.4.3 Interpretation of Findings About Tertiary Mathematics Education in Singapore

From the classification of the interview responses by the professors, we see how education initiatives and policies have their impact, through the different school levels ('O'-level and 'A'-level education), on tertiary education—focusing on mathematics as a subject at school level and Mathematics as a discipline at the tertiary level. Shaping the tertiary mathematics took the form of policy-driven changes in undergraduate programme structures as well as the self-directed changes in teachers' beliefs which later translate into various classroom implementation, e.g. alternative pedagogies and the use of technology in teaching and learning tertiary mathematics. We have learnt from the responses of the eight professors that teaching style has slowly moved from chalk-and-talk to more student-centred learning. Putting on our curriculum ideological lens, it is not difficult to see that a trend shift has taken place at the tertiary mathematics education landscape: there is a significant shift from Scholar Academic through Social Efficiency to Learner Centred; following more or less a similar movement as that in schools.

Perhaps this finding is not surprising if one considers the output of schools to be the students who graduated from the school system, having attained the intended level of content knowledge and skills in mathematics. The mathematical competency of these students who have graduated from schools and ready to enter university is a part of the Learned Curriculum (as opposed to the Intended Curriculum articulated in the detailed syllabus). We see that the changes at the tertiary level are ways in which the university, as a system, handle the effects of the changes that took place upstream, i.e. at the school level.

However, this is only one direction of the flow. Now this is where the NIE stands in contrast to the rest of the other local universities as far as tertiary mathematics is concerned. As the sole teacher training institute, NIE is responsible of ensuring a high-quality teaching force is ready to be feedback into the school system. As such, mathematics student teachers must be master of *both* mathematical content knowledge and pedagogical knowledge. In the next subsection, we shall look at a snapshot of how MME makes use of teaching innovations which are backed by sound pedagogical theories to enhance teaching and learning in an undergraduate core course in mathematics.

5.5 Conclusion

In this chapter, we have attempted to paint the two sides of the tertiary mathematics education landscape in Singapore: the preparatory side at the pre-university level and teaching and learning of mathematics at the university level. Through the lens of curriculum ideologies, we have begun to understand the observed trend, i.e. there is an evident shift of the curriculum orientation from Academic Scholar through Social Efficiency to Learner Centred. Changes at the school level and at the university level are a manifestation of systemic response to the changing demands of the society and the world through the seemingly more direct 'top-down' impact of new educational initiatives.

For mathematicians-educators (mathematicians who are passionate about mathematics education) in NIE, it is perhaps time for us to reflect on what we, as teachers at the tertiary level, want and what the society needs insofar as mathematics learning is concerned. Not everyone needs to be a mathematician or even needs to love mathematics. Not every student needs to excel in mathematics, not every student who excels in mathematics needs to major in mathematics, and not every mathematics major needs to end up as a research mathematician. However, it will be in our interest to see that as long as mathematics is taught in the schools, for whatever purpose, it is taught correctly and in the way and spirit that it should be taught.

At a round-table discussion at the ICIAM 2003 (The International Congress on Industrial and Applied Mathematics) in Sydney, Australia, in response to a heated argument on why Australian students coming to the universities were not well prepared mathematically, the late Professor Renfrey Potts (1925–2005) stood up to say, 'It is the duty of *every mathematician* at the universities of this country to help and make sure that mathematics is taught right in the schools.'s We believe tertiary mathematics education has this to take care of, especially at NIE.

Appendix

An exemplar for creating learning experience in H2 Mathematics Syllabus 9758

Complex Numbers

Lesson Objective

Based on an 'old' idea of $C + iS$, this learning experience involves the students to create the imaginary counterpart of a sinusoidal voltage function across a resistor arising from an alternating current source. By so doing, the students reinvent the phasor of the voltage function, which takes advantage of the vector nature of complex numbers, and exploit it to calculate the resultant voltage function that results from adding in series two alternating current sources that are not necessarily in phase. Engineers use this method, called *phasor analysis*, to think and reason about alternating current voltages, and related quantities.

Problem

An alternating current source has the following voltage function:

$$V_1 = 3\cos\left(2t + \frac{\pi}{4}\right),$$

where V_1 is the voltage (V) across a given resistor, and t is the time lapsed (s) since the source was turned on.

Another alternating current source whose voltage function is given by

$$V_2 = 4\cos\left(2t + \frac{\pi}{6}\right)$$

is now placed in series with the above-mentioned source so that the resultant voltage is calculated by their sum:

$$V_1 + V_2.$$

What is the amplitude and the period of the resultant voltage?

Mathematical Content Knowledge

For the first voltage V_1, we create the imaginary sine counterpart of the function $3\sin\left(2t + \frac{\pi}{4}\right)$ and construct the complex voltage function:

$$3\left(\cos\left(2t + \frac{\pi}{4}\right) + i\sin\left(2t + \frac{\pi}{4}\right)\right) = 3e^{i\left(2t+\frac{\pi}{4}\right)}.$$

Similarly, for the second voltage V_2, we have the complex voltage function:

$$4\left(\cos\left(2t + \frac{\pi}{6}\right) + i\sin\left(2t + \frac{\pi}{6}\right)\right) = 4e^{i\left(2t+\frac{\pi}{6}\right)}.$$

Now, we sum these two complex voltages together:

$$3e^{i\left(2t+\frac{\pi}{4}\right)} + 4e^{i\left(2t+\frac{\pi}{6}\right)}.$$

A preliminary investigation using a GCs reveals that the above sum can be reduced to a single trigonometric function.

From the vector geometry of complex numbers, one can show rigorously that $3e^{i\left(2t+\frac{\pi}{4}\right)} + 4e^{i\left(2t+\frac{\pi}{6}\right)} = Re^{i(2t+\alpha)}$, where $R = \sqrt{\left(4\sin\frac{\pi}{6} + 3\sin\frac{\pi}{4}\right)^2 + \left(4\cos\frac{\pi}{6} + 3\cos\frac{\pi}{4}\right)^2}$ and $\tan\alpha = \frac{4\sin\frac{\pi}{6}+3\sin\frac{\pi}{4}}{4\cos\frac{\pi}{6}+3\cos\frac{\pi}{4}}$, which in particular are independent of t.

Further exploration

The phasor addition works because the two voltages are of the same angular frequency. A natural question to ask is how one can tackle the case when the angular frequencies are different. Use a GCs to investigate this situation.

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