Chapter 3
The Intended School Mathematics Curriculum

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Abstract This chapter examines the changes to the intended Singapore School Mathematics Curriculum since 1990 to the present that resulted from reviews carried out periodically. Special features and key approaches are identified to gain better insights of the curriculum. The curriculum is also examined from the perspective of the three educational initiatives that were implemented in 1997: The Critical and Creative Thinking (CCT) Initiative, the National Education (NE) Initiative, and the Information and Communications Technology (ICT) Initiative. A short discussion on textbooks is also included as they contain and communicate the intended School Mathematics Curriculum. The chapter concludes with an examination of the intended School Mathematics Curriculum from two levels: national versus school. This discussion is taken from the perspective of the process of curriculum development.

Keywords Singapore School Mathematics Curriculum · School Mathematics Curriculum Framework · Intended School Mathematics Curriculum · Nation-building initiatives and School Mathematics Curriculum · 21CC and mathematics education · ICT in mathematics education · Textbook in mathematics education · Mathematics curriculum development

3.1 The Problem-Solving Mathematics Curriculum

As noted by Kaur in Chap. 2, the Singapore Ministry of Education's (MOE) goal of setting up of the mathematics syllabus review committee to review and revise the mathematical syllabuses in use since 1981 was to study the adequacy of the

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syllabuses in meeting the needs of the students as well as to reflect relevant newer trends in mathematics education. One major outcome of this effort was the positioning of developing students' ability in mathematical problem-solving as the primary aim of the Singapore School Mathematics Curriculum (MOE 1990a, p. 3), reflecting the impact of the then considerable amount of research on mathematical problem-solving (Lester 1994) on the Singapore Mathematics Curriculum. This Problem-Solving Mathematics Curriculum of Singapore was first implemented in 1992, and though it has undergone several rounds of review of revision (MOE 1990a, b, 2000a, b, 2006a, b, 2012a, b, c), problem-solving remains central to the Singapore Mathematics Curriculum.

Lee (2016) in an analysis of the Singapore School Mathematics Curriculum (MOE 2012a, b, c), identified the key approaches and key features of it that exemplify a connected curriculum (MOE 2012a, p. 11). In the sections, that follow these are elaborated, and henceforth in this chapter, the Singapore School Mathematics Curriculum would also be referred to as the School Mathematics Curriculum.

3.1.1 Key Approaches

Lee (2016) identified two key approaches in the School Mathematics Curriculum, namely the curriculum development approach and the pedagogical approach.

3.1.1.1 Curriculum Development Approach—Spiral Curriculum Development Approach

The School Mathematics Curriculum recognizes the 'hierarchical' nature of mathematics and adopts a 'spiral approach' to the design of the curriculum (MOE 2012a, p. 11). Each topic is revisited and introduced in increasing depth from one level to the next to enable students to consolidate the concepts and skills learned and to develop these concepts and skills further. This is basically aligned with Bruner's (1960) idea of readiness for learning wherein he believed that a spiral curriculum can foster or scaffold that readiness by 'deepening the child's powers where you find him here and now'. An example of how the spiral curriculum is exemplified in the teaching of addition and subtraction of fractions at the primary levels is shown in Table 3.1. The table illustrates how clearly and refined the spiralling of the content is specified in the curriculum document (MOE 2012a).

Garland (2013) reported that based on a 2012 study by William Schmidt and Richard Houang, it is found that the (USA) Common Core Math Standards were highly correlated with those of high-performing countries, including Singapore. In fact, she noted that '[A]n analysis by Achieve, a nonprofit organization that has supported the Common Core, found that Singapore's math curriculum was similar to Common Core, but that in Singapore, students more quickly reach a higher level of

Table 3.1 Spiralling of the teaching of addition and subtraction of fractions at the primary level (MOE 2012a)

Level	Topic
Primary 2	Addition and subtraction of fractions: Adding and subtracting like fractions within one whole with denominators of given fractions not exceeding 12
Primary 3	Addition and subtraction of fractions: Adding and subtracting two related fractions within one whole with denominators of given fractions not exceeding 12
Primary 4	Addition and subtraction of fractions: • Adding and subtracting fractions with denominators of given fractions not exceeding 12 and not more than two different denominators • Solving up to two-step word problems involving addition and subtraction
Primary 5	Addition and subtraction of mixed numbers

math proficiency', reflecting on the efficiency of Singapore's spiral approach towards curriculum development.

3.1.1.2 Pedagogical Approach—The Concrete–Pictorial–Abstract (C-P-A) Approach

The School Mathematics Curriculum also recognizes the need for 'age-appropriate strategies' such as through 'the use of concrete manipulatives and pictorial representations to scaffold the learning and for sense making' (MOE 2012a, p. 33). Consequently, the key pedagogical approach advocated by the curriculum document is the 'concrete–pictorial–abstract' (C-P-A) approach, particularly for the teaching of the number and algebra strand.

As was observed by Leong et al. (2015), this approach is an adaptation of Bruner's conception of the 'enactive-iconic-symbolic' modes of representation (Bruner 1966). They also argued that Bruner is interested in the external representations of knowledge when putting forth these three modes. Ng (2009), in advocating the use of the C-P-A development of concepts, advised teachers to 'structure' the external representations in the learning environment, wherever possible, to enable students to progress from 'concrete and pictorial levels to abstract representation'. Lee and Tan (2014) observed that in fact it is a common practice for teachers adopting the C-P-A approach not only to present mathematical ideas in concrete, pictorial and abstract representations, but also encourage students to establish linkages among these external representations to aid students in their development of their internal representation system of an abstract mathematical idea.

While the key curriculum approach—the spiral curriculum approach—promotes connecting to extend existing knowledge and skills, i.e. inter-conceptual connection, the key pedagogical approach, the C-P-A approach encourages connecting to make sense of learning through multiple representations, i.e. intra-conceptual connections.

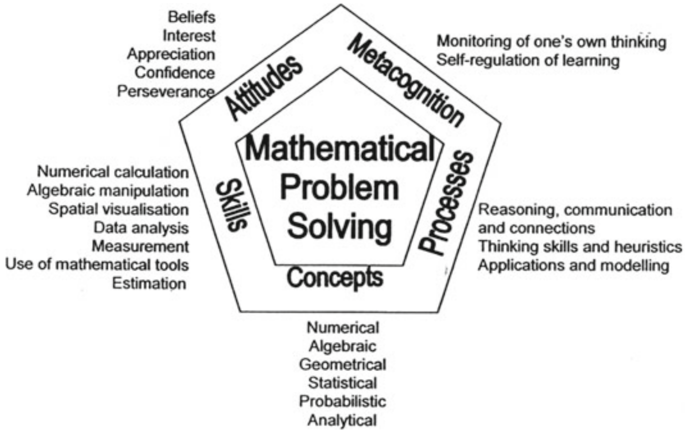

Fig. 3.1 School Mathematics Curriculum Framework (MOE 2012a, p. 16)

3.1.2 Key Features

Lee (2016) also identified two key features of the School Mathematics Curriculum, namely the School Mathematics Curriculum Framework (SMCF) and the pedagogical tool—the Model Method.

3.1.2.1 The School Mathematics Curriculum Framework (SMCF)

The SMCF, shown in Fig. 2.6 in Chap. 2, has problem-solving as its 'central focus'. The framework 'stresses conceptual understanding, skills proficiency and mathematical processes and gives due emphasis to attitudes and metacognition', with these five components being viewed as 'inter-related'. As observed by Wong (1991), the framework describes 'the philosophy' of the curriculum and integrates the 'aspects about mathematics learning and teaching'.

As pointed out by Kaur in Chap. 2, these five components of the framework, concepts, skills, attitudes, metacognition and processes have remained 'steadfast although some refinements have been made of their attributes at periodic subsequent revisions' of the curriculum. In the following, we detail each of these components and trace the refinements made during the periodic subsequent revisions of the curriculum. Figure 3.1 shows the present version of the SMCF (MOE 2012a, p. 16).

Concepts: To encourage the development of deep understanding of mathematical concepts, which forms the foundation of the SMCF, the syllabus document advocate teaching that 'make sense of various mathematical ideas as well as their connections and applications' so as to help students to 'relate abstract mathematical concepts with concrete experiences' (MOE 2012a, p. 17). The two key approaches mentioned reflects the emphasis on promoting conceptual understanding through conceptual inter- and conceptual intra-connectedness. The original SMCF (MOE 1990a, b) and that follows from the immediate revision of the curriculum for implementation in 2001 (MOE 2000a, b) included only numerical, geometrical, algebraic and statistical concepts. However, in subsequent revisions (MOE 2006a, b, 2012a, b, c), probabilistic and analytical concepts were also included. The inclusion of these two groups of concepts also marks the move away from relying purely on examination syllabuses for upper secondary and pre-university levels, which are tied closely to the emphasis placed on the national examinations at these levels then. For the first time, the SMCF articulated the philosophy of the curriculum from primary to pre-university levels when the revised syllabus was implemented in 2007 (MOE 2006a, b). In fact, the latest teaching and learning curriculum documents articulate clearly not only the generic philosophy of the curriculum but also interpretations of the curriculum approaches and features in relation to the respective year levels and courses of study (MOE 2012a, b, c, 2015).

Skills: As reflected in the curriculum document (MOE 2012a, p. 17), these are skills 'specific to mathematics and are important in the learning and application of mathematics. The set of skills have not changed much over the various rounds of revision of the curriculum, except for the way these skills are grouped. Numerical calculation, for example, now encompasses mental calculation and arithmetic manipulations (elaborated in the earlier versions of the SMCF—MOE 1990a, b, 2001a, b). Other skill sets included algebraic manipulation, spatial visualization, data analysis, measurement, use of mathematics tools and estimation. Though many of these skills are procedural in nature, the curriculum places an emphasis for these to be taught with 'an understanding of the underlying mathematical principles'. In other words, the curriculum advocates the promotion of relations understanding, as purported by Skemp (1976), thus encouraging the address of conceptual-procedural connections.

Processes: From the perspective of the curriculum document, processes refer to the 'process skills involved in the process of acquiring and applying mathematical knowledge' (MOE 2012a, p. 17). This aspect of the SMCF has undergone the greatest refinement over the years. In the first version (MOE 1990a, b), this aspect only included heuristics and deductive and inductive reasoning—the two most common types of reasoning involved in the learning and doing of mathematics. In the subsequent revision of the curriculum (MOE 2000a, b), heuristics remained but deductive and inductive reasoning were then subsumed under the generic group of thinking skills. The twelve heuristics listed in the original curriculum were reduced to eleven with 'use of tabulation' and 'make a systematic list' combined as 'make a systematic list'. At the same time, a list of eight core thinking skills was listed under the more generic term 'thinking skills', and the eight included both induction and deduction, reflecting then the impact of the vision for 'Thinking School Learning Nation' (see Chap. 2). In the next revision of the curriculum (MOE 2006a, b), the process aspect continued to undergo further refinement. Firstly, communication, both written and verbal, was reclassified from being a mathematical skill to a mathematical process skill, signifying an increased emphasis of the role classroom discourse have in the teaching and learning of mathematics. Furthermore, the distinction was made between the microthinking skills versus reasoning as a process. Emphasis was also placed to explicitly address the issue of encouraging students to see and make connections ‘among mathematical ideas’ as well as ‘between mathematics and other subjects, and between mathematics and everyday life’ (MOE 2006a, p. 17). In the latest revision of the curriculum (MOE 2012a, b, c), this emphasis on getting students to make sense of what they learn in mathematics and relate to real life is given a further boost with the explicit inclusion of applications and modelling to the process aspect of the SMCF. The continued refinement of the process aspect of the SMCF with emphasis both on the necessary cognitive skills—thinking skills, heuristics, communication, reasoning, connections, as well as the actual process of addressing real-life problems using mathematics—solving problems in real-world contexts and mathematical modelling (see Chap. 8) reflects the importance of real-life connections in the SMCF. In fact, the importance of the process aspect of the curriculum has been further elevated by the recognition of this aspect as one of the strands, others being the content strands—number and algebra, measurement and geometry, and statistics—that cut across the three content strands (MOE 2012a, p. 32).

Metacognition: Despite the fact that metacognition was a term that is coined by Flavell only in 1976, metacognition was featured as one of the five aspects of the original version of the SMCF (MOE 1990a, b), which was developed in the 1980s. This reflects that the Singapore Mathematics Curriculum is not only a forward-looking curriculum but also one that is informed by theory and research, of which the impact of Bruner’s theories (Bruner 1960, 1966) has already been discussed earlier. Though there were no major changes made to this aspect of the SMCF, there was a conscious effort to refine and operationalize the construct, reflecting the continuous work on addressing this aspect of learning and doing mathematics (see Chap. 11). In the first version and subsequent first revision of the SMCF, metacognition was explained to be ‘the ability to control one’s own thinking processes’, and it includes the ‘constant (and conscious) monitoring of the strategies (and thinking processes)’ (MOE 1990a, p. 4, 2000a, p. 11). In the subsequent revisions, metacognition was further refined and elaborated as ‘thinking about thinking’ that involves ‘awareness of’, ‘monitoring of’ and ‘regulation of’ one’s thinking and learning. Such an emphasis on the executive processes over cognition (Tarricone 2011, p. 147) points towards the SMCF’s promotion of the executive control connections.

Attitude: As with the other aspects of the SMCF, the attitudes aspect, which refers to ‘the affective aspects of mathematics learning’ (MOE 2012a, p. 19), underwent constant refinement over the various rounds of curriculum reviews. In the first version of the SMCF (MOE 1990a, p. 4), attitudes encompassed three affective aspects, namely (interest in and) enjoying doing mathematics, appreciating the beauty and power of mathematics, and showing confidence in using mathematics. In the following round of revision, persevering in solving a problem was added (MOE 2001a, p. 11), and with the next revision that follows, beliefs about mathematics and its usefulness were added (MOE 2006a, p. 15). The most significant address to this aspect of the SMCF occurs in the latest revised School Mathematics Curriculum—the introduction of learning experiences in the curriculum documents (MOE 2012a, b, c, 2015). Learning experiences are stated explicitly in the curriculum documents ‘to influence the ways teachers teach and students learn so that curriculum objectives can be achieved' (MOE 2012a, p. 22). Though the learning experiences stipulated in the curriculum documents are not meant to be exhaustive, a conscious effort was made to include them as the key learning experiences in each of the topics addressed in the curriculum. Textbooks endorsed by MOE for use in schools, for example, are required to reflect these so that these learning experiences are addressed in all mathematics classes in Singapore. The idea is to 'remind teachers of the student-centric nature' (MOE 2012a, p. 22) of learning mathematics. The five attitudinal components of the attitudes aspect of the SMCF do provide guidance in the choice and design of learning experiences to help students develop a more positive attitude towards and in the process of learning mathematics. However, with the explicit and deliberate address of these learning experiences in MOE sanction instructional materials, there appears to be a conscious effort to level up the learning experiences resulting from possible differences in teachers' level of expertise. This is aligned with the philosophy held by MOE—that 'Every School a Good School', a slogan popularized by then Minister of Education, Mr. Heng Swee Kiat, when he spoke during the 2014 Committee of Supply debates on 7 March 2014. Minister Heng elaborated that 'Every School a Good School does not mean Every School the Same School, but it does mean Every School Good in its Own Way, seeking to bring out the Best in Every Child'. The various pathways and possible lateral transfers among the course of study shown in Fig. 2.3 in Chap. 2 are another reflection of MOE's student-centric approach towards learning of mathematics. MOE regularly updates and conducts workshops for teachers who may be involved in the teaching of students of different courses of study so as to ensure that students continue to be well supported and provided with positive learning experiences even when they switch course of study. A secondary one mathematics teacher teaching secondary 1 normal (academic) mathematics, for example, may have students who studied mathematics as well as those who studied foundation mathematics. Such teachers are not only informed of the differential entry knowledge of these students, but also invited to attend workshops to help them level up the two different groups of students in their classes. All in all, the attitude aspect of the SMCF seeks to address the affective aspects of learning so as to achieve a more holistic learning experience in the mathematics classroom. In other words, the attitudes aspect of the SMCF promotes holistic learning connections.

3.1.2.2 The Model Method

The Model Method refers to the use of rectangular drawings to represent a problem situation and to visualize and explore relationships among the quantities related to the problem situation. The introduction of the Model Method is 'an essential element of the concrete–pictorial–abstract approach', students progress from 'use of concrete objects', to 'drawing of rectangular bars as pictorial representations of the models', to using the models 'solve abstract mathematics word problems' (MOE 2009, p. 15). A more detailed treatment of the Model Method can be found in Chap. 8. Here, we present the role and benefits of the Model Method in the intended Singapore Mathematics Curriculum.

As explicated in MOE (2009), the Model Method serves to:

- Exemplify and make visible the part-whole thinking that is key to the learning of, particularly primary, mathematics (see Chaps. 3 and 4 in MOE 2009).
- Provides pupils with an efficient and effective problem-solving heuristic (see Chap. 5 in MOE 2009).
- Expose pupils at the primary levels to informal algebra by promoting algebraic thinking years before they are ready for formal algebra (see Chap. 6 in MOE 2009).

It is thus not surprising that teachers generally find the Model Method to be beneficial in the following ways:

- The model is a simplifying tool; many constraints can be handled simultaneously. Fraction problems, for example, can be solved without cumbersome computations involving fractions.
- Students are able to solve challenging problems without the use of form algebra.
- Students are able to engage in algebraic thinking years before they are ready for formal algebra. It can subsequently help students to make sense of formal algebra.

The Model Method that is highly emphasized in the primary school mathematics classrooms appears to lend well as a strong connection between primary and secondary mathematics learning. This key feature of the School Mathematics Curriculum points towards an address of transitional connections.

3.1.3 The Connected School Mathematics Curriculum

In the above analysis of the School Mathematics Curriculum through an examination of the key approaches and key features of the curriculum, it is also apparent that the intended School Mathematics Curriculum is a multidimensional connected curriculum that promotes:

- Intra-conceptual connections
- Inter-conceptual connections
- Conceptual-procedural connections
- Real-life connections
- Executive control connections
- Holistic learning connections
- Transitional learning connections.

This multidimensional approach towards a connected curriculum is similar to the one proposed by Kaur and Toh (2012):

> Teachers must provide students with opportunities to experience connection in the mathematics they learn. This is possible through links between the conceptual and procedural knowledge, connections among mathematics topics and equivalent representations of the same concept. Similarly, teachers must also provide students with opportunities to experience connections between mathematics and other disciplines of the school curriculum and daily life needs. (pp. 6–7)

In fact, Perkins has as early as 1993 argued for the need of a connected curriculum:

> A good deal of the typical curriculum does not connect – not to practical applications, nor to personal insights, nor to much of anything else. It's not the kind of knowledge that would connect. Or its not taught in a way that would help learners to make connections … What is needed is a connected rather than a disconnected curriculum – one full of knowledge of the right kind, one taught in a way to connect richly to future insights and applications. (p. 91)

3.2 The Impact of Nation-Building Initiatives on the Intended Mathematics Curriculum

To better appreciate the modifications and refinement to the School Mathematics Curriculum over the years that was first implemented in 1992 (MOE 1990a, b), there is a need to examine these from the perspective of the three education initiatives (see Chap. 2) that were introduced in 1997:

1. Critical and Creative Thinking (CCT)
2. National Education (NE)
3. Information and Communications Technology (ICT).

These three initiatives have a major and significant impact on the school curriculum as they were nation-building initiatives based on the concerns that plagued the nation then (Lee 2008). All school subjects, including mathematics, were required to respond to the initiatives accordingly.

3.2.1 Impact of the CCT Initiative

As was noted by Lee (2008), one of the approaches taken to respond to the CCT Initiative was the infusion of teaching of thinking skills into the core subjects, English, science, mathematics, geography and history. About 30% of curriculum time for these subjects consisted of such infusion lessons. Thinking skills and teaching strategies that promoted thinking were integrated into content instruction. To accommodate for the extra time needed to cope with such an approach, these subjects, including mathematics, underwent a content reduction ranging from 10 to 30% and reduced content syllabuses became effective in 1999 (see Chap. 2).

In the meantime, a more systematic review of the mathematics curriculum was carried out in 1998 to take into consideration both the content reduction that occurred in the interim curriculum implemented in 1999 and the teaching of thinking. This resulted in the refinement of the process aspect of the SMCF for the version of the curriculum to be implemented in 2001 (MOE 2000a, b) as mentioned previously. Instead of deductive reasoning and inductive reasoning for the process aspect of the SMCF, a list of eight core thinking skills were listed under the more generic term 'thinking skills'. The eight core thinking skills, which are not meant to be exhaustive, are:

1. Classifying
2. Comparing
3. Sequencing
4. Analysing parts and whole
5. Identifying patterns and relationships
6. Induction
7. Deduction
8. Spatial Visualization.

This is aligned with the intention of the CCT Initiative to get teachers more deliberate in the address of the teaching of thinking in the mathematics classrooms. To establish a more common understanding of what these thinking skills are, an operationalization of these thinking skills was provided in an appendix of the curriculum documents (MOE 2000a, p. 131; 2000b, p. 87). Thus, the School Mathematics Curriculum was refined, not displaced, with minor refinements in response to the CCT Initiative. The list of thinking skills continued to be refined, and the eight thinking skills that are reflected in the latest curriculum documents (2012a, b, c) are:

1. Classifying
2. Comparing
3. Sequencing
4. Generalizing
5. Induction
6. Deduction
7. Analysing (from whole to parts)
8. Synthesizing (from parts to whole).

The refined list is essentially the same as the original list except for 'spatial visualization' missing. In fact, 'spatial visualization' is not missing from the curriculum document, it was removed from the list of generic thinking skills but explicitly mentioned and addressed in the teaching of measurement and geometry, in view of its relevance in the teaching of this content strand. In other words, the CCT Initiative has a lasting impact on the intended mathematics curriculum till today.

3.2.2 Impact of the NE Initiative

Lee (2008) observed that the NE Initiative, in its original form, only requires its infusion across a core group of subjects, namely social studies, history and geography, where the NE values have been identified as being especially suited for infusion into. Though mathematics does not belong to this group of subjects, mathematics teachers were still encouraged to incorporate NE in their teaching (MOE 2000a):

> National Education is part of Total Education; therefore every teacher has a role to play. In the context of mathematics, National Education can be integrated into instruction by drawing examples from the prevailing national and current issues during mathematics lessons. These examples can be expressed in the problem context during problem solving or incorporated into practical work. (p. 18)

The call for application of mathematics to problems in real-world contexts seems to have its roots in the NE Initiative.

As was noted in Chap. 2, in facing a more globalized world in the twenty-first century, MOE introduced the 21CC framework in 2010 (MOE n.d.a), consisting of a circle at the centre surrounded by two concentric rings (see Fig. 2.1 in Chap. 2). At the centre of this framework is a circle that captures the core of 21CC. The first ring that encircles this core represents the social and emotional competencies, namely self-awareness, self-management, social awareness, relationship management and responsible decision-making. The outer ring that goes round the first ring encompasses the three main clusters of emerging 21CC:

1. Civic literacy, global awareness and cross-cultural skills
2. Critical and inventive thinking
3. Communication, collaboration and information Skills.

Clearly, both the CCT Initiative and NE Initiative, in fact even the ICT Initiative (to be elaborated in this chapter), are encapsulated in the 21CC framework. The explicit inclusion of learning experiences in the latest curriculum documents is not just, as mentioned above, to address the affective aspect of learning mathematics; these are also carefully chosen and designed to ensure that mathematics classrooms are rich with opportunities 'to provide the platform for students to develop these twenty first century competencies' (MOE 2012a, p. 22), including values related to the NE Initiative and skills related to the CCT Initiative (and ICT Initiative). The explicit inclusion of mathematical modelling and applications to problems in real-world contexts within the process aspect of the SMCF (MOE 2012a, p. 16) allow the address the contexts not only related to the NE Initiative, but now expanded to that face by globalization.

3.2.3 Impact of the ICT Initiative

The ICT Initiative in the education system of Singapore has evolved over the years, from the development of masterplan 1 for ICT in education (officially abbreviated as mp1) to mp2 and then mp3 (which are, respectively, the abbreviations for the second and third ICT masterplans in education) and in 2015, the fourth masterplan for ICT in education—mp4. The goal of mp4 is to put 'quality learning in the hands of every learner empowered with technology' (MOE n.d.b).

While mp1, implemented from 1997 to 2002, laid a strong foundation for schools to harness ICT, mp2 implemented from 2003 to 2008 built on mp1 to strive for effective and pervasive use of ICT in schools. mp3, implemented from 2009 to 2014, harnessed the developments of mp1 and mp2 and enriched and transformed the learning environments of students equipping them with critical competencies and dispositions to succeed in a knowledge economy (MOE n.d.b).

When MOE rolled out mp1 in 1997, the ICT masterplan was guided by the principle of 'appropriate and judicious use of technology in teaching and learning' (MOE n.d.b). The initiative laid strong foundations for schools to embrace ICT in their respective curriculums, particularly in the provision of basic ICT infrastructure and equipping teachers with a basic level of ICT integration competency. In other words, the foundations were laid to harness ICT through building the infrastructure and developing resources including ICT competency for teachers.

As outlined by Koh and Koh (2006), at the end of mp1, effective use of ICT tools in the mathematics curriculum in Singapore could be classified as follows:

(1) Productivity tool to help teachers and students to manage and speed up administrative tasks associated with teaching and learning mathematics;
(2) Informational tool to facilitate students' access to information on mathematics;
(3) Instructional or assessment tool to assist teachers to automate aspects of teaching mathematics and assessing learning;
(4) Visualization or simulation tool to facilitate learners in recognizing patterns, trends or relationships and in visualizing or simulating abstract mathematical phenomena;
5) Connection tool to allow teachers and students to engage one another on mathematical learning anytime and anywhere; and
6) Reconstruction tool to provide students with an integrated learning environment that is equipped with a suite of ICT-based tools for the reconstruction and experience of some subdomain of mathematics.

The second ICT masterplan, mp2, launched in 2003, built on the foundation laid by mp1 to establish baseline ICT standards for students and seeding innovative use of ICT among schools. Indeed, as part of MOE's continual effort to level up the ICT competency, mp2 focused on the pervasiveness of ICT in the classroom through the amalgamation with the educational curriculums. The charting of directions of the first two masterplans was primarily influenced by Singapore's economic development, from a survival-driven industrialization phase to the current knowledge and ability-based phase (see also Chap. 2), working towards an innovation and values-driven future (MOE n.d.b). According to Ng and Leong (2009) during the progression from mp1 to mp2, the use of ICT in the mathematics classroom could be classified as follows:

(1) ICT-use as a better way for teaching mathematics;
(2) ICT-use as a better way for learning mathematics; and
(3) ICT-use in relation to other factors in the instructional environment.

The third ICT masterplan, mp3, launched in 2009, was a continuum of the vision of mp1 and mp2, which is to enrich and transform the learning environments of the students and equip them with the critical competencies and dispositions to succeed in a knowledge economy. It focused on promoting self-directed learning and collaborative learning for learners through strengthening and scaling the potential of individuals to leverage on technology effectively, with the intention of such ICT-enabled learning being delivered anytime and anywhere. The initiative also empowered and supported teachers to have the capacity to plan and deliver ICT-enriched lessons. Students were able to use ICT extensively for school work, and teachers were able to adapt a wide variety of ICT tools.

As part of the goals and objectives of the mathematics curriculum, students are expected to 'use technology to present and communicate mathematical ideas' (MOE 2012a, b, c) and undergo specific learning experiences through the use of ICT tools so as to enhance conceptual understanding. The presence of such instructions on students' learning experiences across the syllabi for all levels of mathematics is the culmination of the development of mp3.

At the primary level, teachers are expected to use digital manipulatives, in addition to other learning tools, to illustrate the various algorithms for the four operations on whole numbers and fractions so that students can better make connections between the operations for whole numbers and those for fractions. In addition, teachers could include activities in which pupils construct bar charts, pie charts and line graphs using a spreadsheet software and make connections among the different graphical representations (MOE 2012a).

Virtual manipulatives could be used as a visual image like a static picture, manipulated like a concrete manipulative, or linked with verbal and symbolic notations (Goldin and Shteingold 2001). Virtual manipulatives, being capable of embodying several representations, thus lend itself to supporting the learner in connecting different mathematical concepts and ideas. In addition, virtual manipulative, if used appropriately, could be a powerful cognitive tool for learners (Moyer-Packenham et al. 2008) because learners would need to remain focussed within a virtual mathematical environment and constantly interact with the visual, verbal and/or symbolic feedback in relation to their actions on the virtual manipulative.

As discussed above, the C-P-A approach has helped learners to relate their concrete experiences with the abstract mathematical ideas, thus closing the cognitive gap between the two representations. According to Lee (2014), virtual manipulatives could help to further narrow the cognitive gap between the concrete and pictorial representations. However, as noted by Lee and Tan (2014), to incorporate the use of virtual manipulatives in the C-P-A approach, it would be unwise to simply replace the 'C' with 'V' (where V refers to external representation arising from the use of virtual manipulatives) or add 'V' into the original approach. As such, they proposed a revision of the C-P-A approach into a two-part approach: C-V and V-P-A approach. The authors elaborated that the advantages of using the aforesaid two-part revised model of the C-P-A approach include helping teachers to better understand the role of virtual manipulatives as a technological tool within the context of the commonly used C-P-A approach. Furthermore, using an integrative rather than additive approach to revising the C-P-A approach not only would increase receptivity of the revised model among teachers, but also improve the effectiveness and efficiency of lesson delivery in applying the revised model which in turn aid learners in developing conceptual understanding.

For mathematics at the secondary level, examples of ICT opportunities students are expected to receive include use of spreadsheets (e.g. Microsoft Excel) to explore the concept of variables and evaluate algebraic expressions, compare and examine the differences between pairs of expressions such as $2n$ and $2 + n$, n^2 and $2n$, $2n^2$ and $(2n)^2$ and study how the graph of $y = ax + b$ changes when either a or b varies or how the graph of $y = ax^2 + bx + c$ changes when either a, b or c varies. In addition, teachers are expected to use the AlgeDisc™ application in AlgeTools™ to help students make sense of addition, subtraction and multiplication involving negative integers and develop proficiency in the four operations of integers, make sense of and interpret linear expressions with integral coefficients such as $4x - 3y$ and $-3(x - 2)$, construct and simplify linear expressions with integral coefficients and factorize a quadratic expression of the form $ax^2 + bx + c$ into two linear factors where a, b and c are integers. Teachers could also explore the use of other ICT tools in helping students develop understanding mathematical concepts. For instance, the AlgeBar™ application in AlgeTools™ could be used to formulate linear equations to solve problems; Graphmatica, applets or other graphing software could be used to explore the characteristics of various graph functions, draw the graph of $ax + by = c$, check that the coordinates of a point on the straight line satisfy the equation and explain why the solution of a pair of simultaneous linear equations is the point of intersection of two straight lines. Furthermore, computer simulations could be used to compare and discuss the experimental and theoretical values of probability (MOE 2012b).

At the pre-university level, the use of graphing calculators, which has been integrated into the advanced level mathematics curriculum since 2006, has impacted the teaching and learning mathematics in various ways. In particular, in examinations, students are expected to use graphing calculators to graph a given function, solve an equation exactly or approximately, solve a system of linear equations, find the approximate value of a definite integral, locate maximum and minimum points and find the approximate value of a derivative at a given point (MOE 2007). More details regarding the effects of the graphing calculator are discussed in Chap. 14.

The above examples illustrated how the ICT Initiative has widened the choices of tools and platforms that mathematics teachers may employ to better achieve conceptual understanding and procedural skills fluency. Thus, the initiative enriched the pathways to better realize both the concepts and skills aspects of the SMCF.

Unveiled in 2015, the fourth ICT masterplan mp4 aims to nurture ‘future-ready and responsible digital learners’ with the productive and efficient use of ICT in support of the total curriculum in order to deepen subject mastery and develop the twenty-first-century competencies (MOE n.d.b). Its focus is on deepening learning through quality ICT-enabled learning and design, addressing cyber-wellness issues, developing new media literacies and sharpening the use of ICT in teaching practices. It serves a greater mission to prepare our nation’s only natural resource—people, to be ICT-savvy besides having subject-specific knowledge. This helps to further realize the development of the 21CC within the mathematics classrooms, providing further impetus to the realization of the attitudes, process and metacognition aspects of the SMCF.

The four ICT masterplans have collectively set the direction for schools to plan, design, implement and evaluate ICT-integrated mathematics curriculum.

3.2.4 Overall Impact of the Three Education Initiatives

In the above examination of the impact of the three education initiatives on the School Mathematics Curriculum, it is clear that the curriculum was modified and refined but not displaced. In fact, the CCT Initiative and the NE Initiative appear to provide the necessary contexts for the refinement and clarifications, while the ICT Initiatives expanded and enriched the pathways towards realizing the curriculum. In fact, Lee (2008, 2015) observed that the SMCF, developed in the 1980s, has remained steadfast, undergoing only minor changes resulting from the numerous curriculum reviews undertaken to date. This is in part due to the rigour and robustness of the philosophy and principles underlying the decisions made about what mathematics education should equip students within the SCMF.

3.3 Textbooks and the Intended Mathematics Curriculum

A chapter on the intended mathematics curriculum would be incomplete without a discussion on the role of textbooks as they contain and communicate the intended School Mathematics Curriculum (Schmidt et al. 1997). In fact, Ang (2008) used the word ‘textbook’ and ‘curriculum’ interchangeably as reflected by the high occurrence of ‘textbook (curriculum)’ in the article. It seems that she saw textbook as equivalent to curriculum. She even further elaborated with an example in primary mathematics:

> When the new ‘part-whole’ model method was introduced in the syllabus, textbooks were specifically designed to incorporate this and its associated teaching approaches and strategies. (p. 81)

This is not surprising as textbooks in Singapore must be formally approved by MOE before they could be adopted by schools.

When the Problem-Solving Mathematics Curriculum was first implemented in 1992, primary mathematics textbooks continued to be produced by MOE based on the materials developed by CDIS in the 1980s (see Chap. 2). However, mathematics textbooks for secondary schools were published by commercial publishers. As pointed out in Chap. 2, for the mathematics curriculum that was implemented in 2001, both the primary and secondary mathematics textbooks were all produced by commercial publishers. Despite the involvement of the commercial publishers in the production of the mathematics textbooks, MOE continues its rigorous process of vetting these books for alignment with the intended curriculum, involving both mathematics teachers and curriculum specialists in the process.

As Singapore has consistently performed well in TIMSS since 1995, Singapore mathematics textbooks have also been of interest to researchers around the world. Oates' (2014) policy paper reported that Singapore mathematics textbooks clearly conveyed key concepts, provided systematic learning progression, included a variety of examples and applications and encouraged learner reflection. The paper also opined that while textbooks in Singapore had to be approved MOE, the textbooks did not dictate teachers' teaching styles. Instead, teachers used textbooks in different ways: teachers might ask their pupils to read the text in class or at home and then discuss the main concepts as a whole class. Some teachers used the textbooks as a guide when structuring their lessons and others selected assessment items from the textbooks for the pupils to attempt. The policy paper also reported that 70% of students in Singapore had mathematics teachers who used textbooks as a basis for instruction, as evidenced from TIMSS 2011.

From the perspective of the SMCF, Low (2011), as part of his master's study, investigated the extent that the framework is represented in secondary school textbooks. In the study, Low and two other coders analysed chapters categorized under the topic algebra in a Secondary Three Mathematics textbook used in Singapore. They coded sections of the chapters according to the five aspects of the SMCF. Of the contents coded, 31.5% were classified as concepts, 44.4% as skills, 11.7% as processes, 3.1% as attitudes, 7.4% as metacognition. Although there are limitations to the study, especially when it only examined the teaching of algebra, the study did show that all the five aspects of the framework were represented to a certain extent in the textbook selected. However, the glean of the distribution of the coded content across the five aspects of the SMCF raise another pertinent issue on the intended curriculum—Is it important, reasonable or even sensible to discuss about what is the ideal distribution of such codes across the five aspects of the SMCF from the perspective of the intended curriculum?

3.4 Conclusion: National Versus School Intended Mathematics Curriculum

In this chapter, we have presented the intended School Mathematics Curriculum from the perspective of the national curriculum. Singapore has a national School Mathematics Curriculum, and the philosophy, principles, goals and objectives are articulated through the curriculum documents which were produced by and disseminated by MOE and that this chapter has made reference to (MOE 1990a, b, 2000a, b, 2006a, b, 2012a, b, c, 2015).

Olivia (2013) observed that models of curriculum development are generally deductive or inductive. Deductive models of curriculum development proceed 'from the general (e.g. examining the needs of society) to the specific (e.g. specifying instructional objectives)', as pointed out by Lunenburg (2011a). Tyler's (1949) classic work is an excellent example of a deductive model of curriculum development. The way nation building in Singapore has impacted the national School Mathematics Curriculum, as presented in this chapter, also shows that it follows the deductive model of development. In fact, Lunenburg (2011a) also noted that most curricular makers adhere to the deductive approach of curriculum development as it allows the broader needs of society to be addressed. With Singapore being a young nation, a deductive approach would help ensure necessary changes to the education system are effected to meet nation-building needs (Lee 2008).

However, as Singapore enters into an ability-based, aspiration-driven phase (1997–2011) (see Chap. 2), there is a greater focus on the development of the individual student. In fact, in response to the 'Teach Less Learn More' (TLLM) Initiative mentioned in Chap. 2, Lee (2014) reported that many school teachers have embarked on a number of interesting school-based curriculum innovations, with generous support from MOE, to cater to the specific needs of the students in their respective schools. Furthermore, under the earlier mentioned Minister Heng's vision of 'Every School A Good School' for the values-based, student-centric phase of the Singapore education system (see Chap. 2), 'schools have been resourced to offer customized programmes … Different schools also offer a variety of programmes to develop the varied interests and abilities of their students' (MOE n.d.c). All these school-based curriculum innovations and programmes appear to be more aligned with an inductive approach, where the process starts with the actual 'development of curriculum materials and leading to generalization' (Lunenburg 2011b). As Lunenburg (2011b) noted, such an approach has incorporated 'a postmodern view of curriculum, because they are temporal and naturalistic'.

Lee (2014) observed that the centrally controlled national mathematics curriculum coupled with school-based mathematics curriculum innovations and programmes have created a new mathematics curriculum that is evolving in Singapore schools. This new intended School Mathematics Curriculum 'starts with the actual development of curriculum materials to target the specific needs of the pupils from the respective schools, but that is also aligned with the national mathematics curriculum' (Lee 2014). The approach taken to the development of such a mathematics curriculum appears, as Lee (2014) proposed, to be a mixed model one—one containing the elements of both the deductive and inductive models.

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