Chapter 21
Reviewing the Past, Striving in the Present and Moving Towards a Future-Ready Mathematics Education

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Abstract This book serves the purpose of discussing the state of mathematics education in Singapore at the time it is written, and an update of a previous edition of the book on Singapore mathematics education published about a decade ago. This chapter identifies the central message that runs across the chapters in this book: Singapore is striving for excellence in mathematics education while addressing her imperfections; Singapore is also learning from good practices of other countries and at the same time bases her practices on sound educational theory. In addition, Singapore places great emphasis on teacher professional development. Further, this chapter discusses the future trends of mathematics education in Singapore: an increasing emphasis on big ideas, big data and computational thinking.

Keywords Mathematics education · Big ideas · Big data · Computational thinking · Early childhood · STEM

21.1 High-Quality Education in Singapore

According to the latest report (Save The Children 2018) released on 31 May 2018 by the non-governmental organization Save The Children, Singapore is ranked first among all the countries in the world for children to grow up. Among the many reasons, equitable high-quality education is one of the factors.

> Singapore is a great place for children to grow up with *good access* to *high quality education* [emphasis added] and medical care services, while also being one of the safest countries in the world… (The Straits Times, 1 Jun 2018)

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Being internationally recognized for her "high-quality education" accessible by all children, it is indeed not surprising that Singapore has performed well in mathematics, science and language in various international comparative studies as mentioned in Chap. 1.

In one of his recent visits to NIE, the Deputy Prime Minister of Singapore, Mr. Teo Chee Hean, stressed the importance of Singapore having a high level of quality education. In his closing address at the visit, Mr. Teo also highlighted the importance for Singapore to understand the reasons why the country had performed well in the various areas of education. Understanding the reasons why Singapore was performing well would enable us to better adapt to the constantly changing world, thereby allowing us to sustain the good performance in the future. He also emphasized on three aspects of education that Singapore would be focusing on—mathematics, science and language.

This concluding chapter summarizes the earlier chapters of this book in taking stock of what Singapore has been doing in mathematics education and the future directions that it will be taking.

21.2 A Sense of Singapore Mathematics Education from the Earlier Chapters in This Book

What are some key impressions that the various chapters of this book coherently convey? It is not difficult for the reader, as he or she reads through all the chapters in this book, to get a sense that Singapore is not resting on her laurels despite having been recognized as a country having one of the best mathematics education systems in the world. As the various chapters unfold, the Singapore mathematics educators are self-critical in their approach to mathematics education. On the one hand, the educators are evidently striving for excellence in the various fields of mathematics education of their specialization. On the other hand, they are continuing to tease out the unresolved imperfections in order to approach excellence in mathematics education.

A good illustration of Singapore mathematics educators' effort in regularly attempting to address the imperfections in the midst of good general performance is in Kaur's (2009) paper on international comparative studies. It was identified that Singaporean students were still relatively unfamiliar with solving unseen problems, amidst the relatively good performance of Singapore students in the international comparative studies. This paper led researchers to deliberate on the true spirit of mathematical problem-solving beyond the mere performance in the usual paper-and-pencil test (Chap. 7). Metacognition, one of the five important dimensions of problem-solving in the Singapore mathematics curriculum, continues to receive the attention of mathematics educators and researchers (Chap. 8), as metacognition is recognized as one of the five attributes of mathematical problem-solving in the Singapore mathematics curriculum (Chap. 3).

We also recognize that Singapore mathematics education is not flawless. For example, Chap. 13 reports that Singapore has a significant number of low-attaining students who are not performing well in comparison with the low-attaining students in several other high-performing East Asian countries. As a result, efforts from a spectrum of concerned educators that range across researchers in NIE, education and policy officers from the Ministry of Education and school teachers were taken to address the learning needs of the low-attaining students in the education system. In the process, there was much collaborative effort among the educators in addressing the imperfection in the system.

Any reader familiar with Singapore history will know that the Singapore education system evolved from the inherited colonial British education system. However, the various chapters in this book also show that Singapore has learnt from various other countries, such as the USA, Australia, New Zealand, and even non-English speaking countries such as Finland and Japan (Wong et al. 2009). Singapore has boarded the bandwagon of the latest trends in mathematics education, but with caution. To cite two examples of Singapore learning from other countries and testing them out in our local education system: (1) information and communications technology (ICT) is being used in Singapore schools in teaching and learning mathematics in various degrees across all levels (Chap. 14); (2) mathematical modelling and problem-solving in the real-world context are being implemented in the Singapore schools at the primary and secondary levels (Chap. 9).

It is also important to note that Singapore does not take ideas wholesale from overseas. Rather, Singapore adapted these ideas to fit the unique Singapore context and to reinterpret and reconfigure rather than replace the existing mathematics curriculum with the new trends. Corresponding to the two examples cited in the preceding paragraph: (1) technology is introduced in the Singapore schools not to replace students' computation, but to stretch students' higher-order thinking skills in order to enhance their problem-solving ability. (2) The introduction of mathematical modelling and problems in the real-world context into the Singapore mathematics curriculum was not meant to displace the centrality of mathematical problem-solving in the mathematics curriculum. Instead, mathematical modelling is used to enrich the problem-solving emphasis of the curriculum: modelling and application of mathematics (in real-world context) have now become important *processes* of mathematical problem-solving, one of the five important attributes of problem-solving.

Singapore builds from established international education theories found in education literature as the basis to address the local education needs in order to solve local problems or to stretch for excellence. As an example, algebra learning has always been difficult for students worldwide. Piagetian theory suggests the futility of teaching algebra to young children who are not yet ready cognitively to acquire abstract mathematical concepts. The well-known Singapore model method in solving word problems (or more affectionately known as "Singapore Math" in the west), and the Algebra Manipulatives (AlgeDiscs™ and AlgeCards described in Chaps. 8 and 13), are founded on Bruner's theory of three modes of representation and subsequently the more recently developed Concrete-Pictorial-Abstract (CPA) theory. These practices used in the Singapore schools were tested in some Singapore classrooms to ensure its efficacy before its large-scale implementation in the classrooms through the national mathematics curriculum document. As a side note, in the chapters in which all the various teaching practices are discussed, one can easily find a long list of references on the studies carried out to test these practices in Singapore schools. Perhaps, this could be the reason why most practicing Singapore mathematics teachers will wholeheartedly use these innovative practices in their classrooms, as confidence has been built in the teachers that these tested practices will improve their students' learning.

As one reads through the various chapters of Part III of this book, it is evident that Singapore places much emphasis on teacher professional development, as we believe that the most important single factor for the quality of education is the quality of the teachers' training as discussed in Chap. 15. In mathematical jargon, we believe that the quality of the teachers is bounded by the quality of the teacher education programme. It is also clear that teacher professional development in Singapore has transcended the traditional workshop deficit model of professional development. Teacher professional development has moved to the strong partnership between the practicing teachers and the researchers in the Singapore National Institute of Education (NIE), and/or education and policy officers from the Singapore Ministry of Education over a sustained period of time. This close partnership ensures the validity and sustainability of the professional development, which is both practice-oriented and theory-based.

The reader will also get a sense from reading this book that teaching practice, theory and research are strongly intertwined in the Singapore mathematics education landscape. In each of the three parts of this book (curriculum, practice and professional development), the theme discussed in each chapter revolves around practices which are based on sound education theory and informed by research carried out locally and internationally. In addition to these practices, how Singapore teachers are prepared are also discussed in detail.

21.3 Some Other Areas of Mathematics Education

Are there other aspects of mathematics education that deserve our attention? Wong et al. (2009) asserted that there is a "huge gap in our knowledge about how best to help young children make the transition from less-structured acquisition of mathematics at kindergartens to more formal instruction at Primary 1…" (p. 526). They lamented that there was only one chapter on early childhood education in their book (Wong et al. 2009). We find the same situation in this book that is published about one decade later. There is little mention of research or teaching practice in early childhood years.

Following the recent announcement by the Singapore Ministry of Education placing its emphasis on early childhood education and channelling much resource into this area of education, more mathematics education research will be carried out at the early childhood level. Studies on early numeracy and other aspects of mathematics education on early childhood are in the pipeline with this new initiative. We resonate with the government's decision in focusing on early childhood education, as this is the first step of education in an individual. We believe that there will be more chapters on early childhood mathematics education if there is a next volume on Singapore mathematics education published several years after the publication of this book.

Singapore has been perceived by the world to have provided her students with a strong STEM education. STEM education has received the world's attention in recent years. It is noted that countries such as Thailand and South Korea have looked towards Singapore, and the NIE in particular, for STEM education. There seems to be a lack of attention on STEM education (only technology is mentioned in Chap. 14, not as an integrated interdisciplinary approach). This might not be too surprising since there is no isolated STEM subject in the Singapore curriculum either in primary or secondary level. Singapore does not offer a unique STEM subject in their school curriculum, a subject with explicit integration across all the four disciplines. In the context of Singapore, the concept of STEM is perhaps well integrated into the system implicitly through the connections and applications taught in the individual subjects of science and mathematics. With the increasing emphasis on STEM education, the prominent role of mathematics in an interdisciplinary context might spur some interdisciplinary research related to STEM—mathematics will no longer be perceived as playing merely a supporting role, but as leading in other systems of thought such as design thinking and scientific thought.

21.4 Looking Forward in the Journey of Mathematics Education

In addition to striving towards excellence from what Singapore has achieved, we see at least two emerging trends in Singapore that we shall discuss below.

21.4.1 Big Ideas in Mathematics

We see that the next leap in mathematics education in Singapore is the incorporating of big ideas into the mainstream discussion in the design of the school mathematics curriculum. As early as 2000, the notion of big ideas has been discussed in the NCTM document.

> Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise. (NCTM 2000, p. 17)

The definition of a big idea was not made explicit then, although it was briefly discussed in the NCTM document. It seems that it was only after the seminal paper by Charles (2005) that the notion of big ideas began to attract much attention internationally and was subsequently introduced into the "conversations about mathematics standards, curriculum, teaching, learning, and assessment" (p. 9). According to Charles (2005), a big idea is "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p. 10).

Understanding big ideas allows an individual to develop a deep and robust understanding of mathematics. After all, the notion of big ideas shows the connection across the many topics of mathematics. The Singapore Ministry of Education (MOE) has since the latest curriculum review incorporated the use of big ideas in the new school mathematics curriculum. This development of the school mathematics curriculum was supported by the new series of textbooks that support the approach of big ideas in the curriculum. To support the effort of MOE in her implementation of big ideas in the mathematics curriculum, the theme of the annual Mathematics Teachers' Conference 2018 held in Singapore was "Big Ideas in Mathematics". This platform gathered a group of international and local mathematics educators and researchers to share their knowledge and experience on the theory and practice of big ideas in mathematics classrooms. Subsequently, it raised an awareness among Singapore mathematics teachers on the concept of big ideas.

It seems that research projects on big ideas are in the pipeline with the rising awareness among educators and teachers. The next big step of mathematics education in Singapore taken in the latest school mathematics curriculum revision is a re-examination of the mathematics curriculum with a focus on big ideas in mathematics using a threefold approach: (1) incorporation in the mathematics curriculum; (2) alignment with the school textbooks and (3) increasing awareness among the Singapore mathematics teachers and conducting teacher professional development on big ideas.

21.4.2 Big Data and Computational Thinking

With the exponential increase in the amount of information every moment, it is not surprising that now we are faced with extremely large volumes of data that are almost impossible to process using the traditional data processing techniques. The advent of highly sophisticated technology and powerful computers today makes it possible to analyse large data in order to identify patterns and trends which could be crucial information for day-to-day business. This challenges the traditional method of analysing small sample data sets in order to obtain information about the whole population. As mathematics is closely linked to computer science due to the nature of its logical deductive reasoning approach, it will indeed not be surprising that mathematics education in Singapore could be paying much attention to big data in preparing our students to be future-ready.

In order to be program computers act on large amounts of information, the notion of computational thinking could become a new trend in education and, in particular, mathematics education. The term *computational thinking* was first used by Papert (1980). According to Papert (1980, 1996), *computational thinking* is defined as the thought processes in formulating a problem and expressing its solution in a way that computers (and human beings) can work on it. This is reminiscent of mathematical problem-solving as it shares the similarity in formulating the problem and looking for solutions to the problems. Indeed, computational thinking shares similar thought processes with problem-solving, and in fact, the possibility of formulating a problem is expressing a problem into its equivalent form—one big idea in mathematics (Charles 2005).

21.5 Conclusion

This book reports the current state of mathematics education in Singapore in 2018, and it is a timely update of the book by Wong et al. (2009). The issues discussed in this book include a broad spectrum of issues related to mathematics education faced in Singapore. To sum up the state of the art of mathematics education in Singapore, we would say it is encapsulated in the phrase "*Reviewing the past, striving in the present and moving towards a future-ready mathematics education*".

Interested readers may consider examining many of the ideas presented in this book, which is written in the Singapore context, bearing in mind the contexts of their own country. We sincerely hope that this book will be of use to readers who are interested in mathematics education. Hopefully, the ideas expressed here can be transferred to another context.

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