Chapter 20
Productive Teacher Noticing: Implications for Improving Teaching

Ban Heng Choy and Jaguthsing Dindyal

Abstract Although there have been calls to focus on teacher-inquiry approaches to teacher professional development, simply putting teachers together in a professional learning team is not sufficient for improving teaching. What matters is what, and how, teachers notice when they learn from their teaching practices. In this chapter, we first give an overview of the crucial role of teacher noticing in professional development by drawing on relevant literature. Next, we explain the notion of productive teacher noticing by highlighting what and how teachers notice as they attempt to enact teaching practices, aimed at enhancing students' reasoning. Following this, we describe two studies on productive teacher noticing in Singapore before we highlight some implications for improving practice and suggest possible future trajectories of research into teacher noticing.

Keywords Learning from teaching · Mathematics teacher noticing · Teacher education · Teacher professional development

20.1 Introduction

There has been a shift towards professional development activities that involve some form of job-embedded collaborative teacher inquiry with teachers learning from their own teaching (Lave 1996; Timperley et al. 2007). Examples of this kind of professional development activities include video clubs where teachers examine and reflect on practices using video recordings of lessons (van Es 2012); study groups where teachers examine classroom artefacts (Goldsmith and Seago 2013) or analyse lesson plans (Santagata 2011); and lesson study (Lewis et al. 2006). However, participating in these activities alone does not necessarily help teachers to change or improve their

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teaching. The contention is that what matters is, not the kind of professional development activities, but what teachers focus on and how they engage with the activities within the contexts of learning communities (Lampert 2009). Although teachers in Singapore generally take an active role in their professional development, what they learn from their participation in learning communities, and how this learning actually helps improve their quality of mathematics instruction have not been well understood (Chua 2009).

Recent literature in mathematics education has positioned mathematics teacher noticing—a component of teaching expertise and a form of professional vision (Goodwin 1994)—to be critical for examining one's teaching practices, and for improving instruction (Barnhart and van Es 2015; Choy 2013; Goodwin 1994; Mason and Davis 2013; Schoenfeld 2011). From the perspective of professional vision, mathematics teacher noticing is conceptualised as the process of attending to students' mathematical ideas, and making sense of the information to make instructional decisions (Jacobs et al. 2010; van Es and Sherin 2008). On the other hand, Mason (2002) views noticing as a set of practices, aimed at raising one's awareness to have a different act in mind. Although all teachers notice instructional details to some extent, not all noticing is productive (Choy et al. 2017). As highlighted by Choy (2015), productive noticing not only empowers teachers to shift their foci of attention (Mason 2002), but more importantly, decide and implement instructional decisions that potentially enhance students' mathematical thinking. But the ability to "notice productively" during mathematics teaching is both difficult to master, and complex to study (Jacobs et al. 2011, p. xxvii). In this chapter, we first give an overview of the critical role of teacher noticing in professional development by drawing on relevant literature. We then describe two studies on teacher noticing in Singapore to provide a characterisation of productive noticing. Following this, we illustrate the notion of productive teacher noticing through a series of snapshots what and how teachers notice as they attempted to enact productive teaching practices. Lastly, we highlight some implications for improving practice and suggest possible future trajectories of research into teacher noticing.

20.2 Learning from Professional Development: The Role of Teacher Noticing

People notice all the time, but they are "sensitised to notice certain things" (Mason 2002, p. xi). This sensitised noticing is closely associated with the idea of "professional vision, which consists of socially organised ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (Goodwin 1994, p. 606). Following Goodwin's (1994) idea of professional vision, teacher noticing has also been understood in terms of three inter-related processes: attending, interpreting, and deciding to respond in teaching contexts (Sherin et al. 2011). But how does noticing enhance teacher development? To this end, Mason

(2002) sees noticing as a discipline and highlights noticing as "the heart of all practice" (p. 1) and offers it as a means by which teachers can "do something about it [teaching] in a practical and disciplined manner" (p. 1). Hence, the practical aim of noticing is to improve teachers' sensitivity to act differently during instructional situations (Mason 2002) and is distinguished from spontaneous or impromptu noticing by adopting a more disciplined approach (Mason 2002, p. 61):

> The idea is simply to work on becoming more sensitive to notice opportunities in the moment; to be methodical without being mechanical. This is the difference between 'finding opportunities' and 'making them'. Instead of being caught up in moment by moment flow of events according to habits and pre-established patterns, the idea is to have the opportunity to respond freshly and creatively yet appropriately, every so often.

As argued by Mason (2002), teacher noticing is critical for improving teaching practices—teachers who notice can learn to respond freshly or have a different act in mind for the future. The potential for changing practices places noticing at the centre of any professional development efforts. But how does noticing support teacher professional development?

With the aim of making opportunities to improve instruction, Mason (2002) highlighted two important ways to enhance noticing: advance preparation and using past experience. According to him, professional learning takes place in the world of personal experience and is supported by one's colleagues while drawing on the world of theories, which informs how noticing can take place. Therefore, noticing does not necessarily occur at an individual level, but rather, the practices of disciplined noticing lie in the "merging" of three worlds of experience (see Fig. 20.1)—"the world of personal experience, the world of one's colleagues' experience and the world of observations, accounts, and theories" (Mason 2002, p. 93).

As seen in Fig. 20.1, the ability to recognise possibilities to act differently lies at the intersection of the three worlds of experience, which underscore the importance of collaborative professional development. By reflecting on their colleagues' and their own experiences systematically, teachers can prepare to notice by developing sensitivity to the common observations, which emerge from their discussion.

Fig. 20.1 Noticing in the three worlds of experience (Mason 2002, p. 94)

They then interpret these observations in light of the theories and observations; and sometimes validate their observations with other people, in order to distinguish and recognise the possibility to act differently. While these actions usually happen retrospectively during post-lesson reflections, the essence of noticing is to bring these "moments of noticing from the retrospective to the spective" (Mason 2002, p. 87) so that the teachers are better prepared to notice in the moment during lessons.

Our perspective of noticing adopted in this chapter sees noticing as a form of professional vision view (Jacobs et al. 2011; van Es 2011) and at the same time, as a discipline or practice (Mason 2002, 2011). First, we examine teachers' noticing in terms of the events or details they attend to, how they interpret these events, and the instructional decisions they make based on their interpretations. Next, we use Mason's (2011) notion of noticing as a shift in attention to investigate what and how teachers notice in terms of the following micro-level structure of their attention: holding wholes; discerning details; recognising relationships, and perceiving properties.

Not all noticing, however, is productive with regard to improving instruction, and it is challenging for teachers to move their "moments of noticing from the retrospective to the spective". Erickson (2011) highlights that teacher noticing is very selective in its focus, and while different teachers can notice various aspects of the classrooms, what they notice may not always be helpful, and at times, direct students' attention away from the mathematical issues. It is common for teachers, for example, to see students' active participation in the tasks, or their enthusiastic raising of hands to answer questions, as indicators of students' understanding (Erickson 2011; Star et al. 2011; Star and Strickland 2008). Furthermore, what teachers notice depends on their knowledge (Kazemi et al. 2011; Schifter 2011) and beliefs or philosophical stance towards teaching (Erickson 2011; Schoenfeld 2011). This diversity of knowledge and orientations has the potential for both "insight" and "misperception" in noticing (Erickson 2011, p. 32; Miller 2011). This begs the question, what makes teacher noticing productive?

20.3 What Makes Teacher Noticing Productive? Findings from Two Studies

The construct of teacher noticing is still relatively new to mathematics teachers and mathematics educators in Singapore. Many of the studies, based overseas, investigated pre-service teachers' development of noticing expertise within the contexts of video clubs, or video-based professional development (Miller 2011; Seidel et al. 2011; Star et al. 2011; van Es 2011). In Singapore, however, most of the studies were situated within the contexts of lesson study discussions (Choy 2014a, b, 2015, 2016; Lee and Choy 2017), or other related professional development activities, where in-service teachers had opportunities to plan, observe, and discuss lessons (Seto and Loh 2015). These studies, together with a few others (Chia 2017; Choy and Dindyal

2017a, b), have begun to expand the contexts of research on teacher noticing beyond video-related professional development activities and pre-service teacher education. In this section, we focus on two studies on teacher noticing situated in Singapore to highlight the notion of productive noticing. In the first study (see Sect. 20.3.1), the notion of productive noticing goes beyond the three inter-related processes—attending, interpreting, and deciding to respond—by including the *foci* for teachers to notice, and extending the notion of noticing beyond the walls of the classrooms to include planning during lesson study. In the second study (see Sect. 20.3.2), we built on Choy's (2015) notion of productive noticing and looked at how teachers planned day-to-day lessons around rapid cycles of simple tasks instead of a single rich task, for example, in the context of lesson study.

20.3.1 Study 1: The FOCUS Framework

The FOCUS Framework was developed from a doctoral study (Choy 2015), which used a design-based research approach (Cobb et al. 2003) to address the twin challenges of theoretical development and practical application (Zawojewski et al. 2008). Data collection for the doctoral study, which consisted of three phases, took place in Singapore over a period of eight months in 2012 and 2013. A total of 36 teachers from three schools, a primary school and two secondary schools, volunteered to participate in the study. All three schools had actively supported learning communities by providing teachers time and space to discuss curriculum-related matters. The teacher participants had used lesson study as a professional development activity, and they were familiar with the lesson study protocol.

The voice recordings of the lesson study discussions and video recordings of the observed lessons formed the bulk of the data collected. The recordings were marked for segments that focused on the key tasks of lesson study and notable episodes involving mathematically significant moments. Segments related to logistics, administrative matters, and other irrelevant incidents were discarded. The selected segments were reviewed and initially classified according to the framework for noticing students' thinking (van Es 2011), shown in Table 20.1. The reviewed segments were then transcribed before they were coded using a "thematic approach" (Bryman 2012, p. 578).

The FOCUS framework, developed from Choy's (2015) study, reflects the following two characteristics of productive mathematical noticing:

1. The focus—what to notice: (a) Specific mathematically significant aspects of learning and teaching, such as the three points; mathematics-learner-teacher milieu; or simply the concept, confusion, and course of action. (b) The alignment between the teaching approach and students' learning difficulties associated with mathematical concepts; and
2. The focusing—how to notice: The central role of sense-making or reasoning as a mediator between seeing and responding. It is the analysis of the observations

Table 20.1 Framework for noticing students' thinking adapted from van Es (2011, p. 139)

	What teachers notice	How teachers notice
Level 1 Baseline	Attend to generic aspects of teaching and learning, e.g. seating arrangement and student behaviour	Provide general descriptive comments with little or no evidence from observations
Level 2 Mixed	Begin to attend to particular instances of students' mathematical thinking and behaviours	Provide mostly evaluative comments with a few references to specific instances or interactions as evidence
Level 3 Focused	Attend to particular students' mathematical thinking	Provide elaborate and interpretive comments by drawing upon specific instances and interactions from observations as evidence
Level 4 Extended	Attend to the relationships between particular students' mathematical thinking, mathematical concepts, and teaching approaches	Provide elaborate and interpretive comments by drawing upon specific instances and interactions from observations as evidence, make connections to principles of teaching and learning and propose alternative pedagogical solutions

that provide the evidence or justification for making an instructional response that promotes student reasoning.

Choy (2015) positions teachers' noticing as productive when teachers' noticing leads to one or more of the following teaching practices for enhancing students' mathematical thinking: designing or planning tasks to reveal students' thinking; listening and responding to students' thinking, and analysing students' thinking. This stance extends the study of teacher noticing from examining teachers' practices during and after lesson to the planning processes before instruction. In particular, Choy (2015) focuses on what and how a teacher anticipates students' responses to a task during planning. This is similar to the practice of anticipation as highlighted by Smith and Stein (2011). Moreover, Choy (2015) also finds it necessary to focus teacher's attention on some specified aspects of teaching and learning so that teachers can cope with the enormous amount of information encountered during real-time teaching.

Building on the work by Yang and Ricks (2012), Choy (2015) highlights that an explicit focus is an essential characteristic of productive noticing, especially in the context of professional development. More specifically, he proposed that teachers could focus on the following focal points to promote productive noticing:

1. Mathematics Concept. The key mathematical ideas, themes, or constructs that are of interest to the lesson, discussion, or teaching episode;
2. Students' Confusion. The mathematical difficulties, cognitive obstacles, errors, misconceptions, or uncertainties demonstrated by students; and
3. Teachers' Course of Action. The instructional decision or response by teachers during the planning, teaching, or reviewing of the lesson.

Besides noticing specific aspects of these three focal points, it is also crucial for teachers to notice the alignment between these three points. That is, whether the teacher's course of action addresses students' confusion when learning the concept. Focusing on the alignment between the three focal points, or more generally, the mathematics-learner-teacher milieu (Brousseau 1997) is related to what researchers like van Es (2011), and Barnhart and van Es (2015), had highlighted about responding with instructional decisions that are based on teachers' observations. However, these researchers were more concerned about the issue of alignment during the responding component of noticing, whereas Choy's (2015) study demonstrates that the alignment of the milieu is crucial even during the attending and making sense stages of noticing.

Seeing the alignment between the three focal points is challenging, even for experienced teachers. For example, it is possible for teachers to give a highly detailed description of what they notice about the three focal points, and not generate a pedagogically productive instructional decision (Choy 2014a, b). In other words, it is possible for teachers to discern the details, but not to recognise relationships between the three points, and thus fail to learn from their teaching practices. Therefore, the second key characteristic of productive noticing is the need for pedagogical reasoning. According to Choy (2015), in order to coordinate their instructional decisions with what they observe, it is necessary for teachers to make sense of their students' difficulties when learning the key mathematical ideas. Based on their interpretation of students' errors, teachers can then make a reasoned decision about a potential approach or strategy targeted at the mistakes observed. This highlights the central role of teacher pedagogical reasoning (Shulman 1987) in aligning what teachers see to their instructional responses.

There are two aspects of this alignment to be considered for productive teacher noticing. Firstly, to see whether there is an alignment between the three points; and secondly, to ensure that a teacher's decision to respond is aligned to what he or she has seen and interpreted with regard to the concept and confusion. For each of these aspects, Choy (2015) argues that teacher reasoning is essential for focusing one's noticing. Although other similar research suggests the importance of analysing or interpreting instructional details during teacher reflection (Barnhart and van Es 2015; Berliner 2001; Timperley et al. 2007), Choy's (2015) study extends their findings by addressing the object of this teacher reasoning process. In a way, this "completes" the micro-level structure of attention—reasoning (Mason 2011). For teachers to notice productively, it is necessary for them to hold the wholes (see the big ideas involved in the concept) and discern the details of the concept and students' confusion. They have to recognise the relationships or connections between the three focal points and perceive the affordances or properties of their instructional strategies before deciding on the course of action based on their pedagogical reasoning.

A theoretical model of noticing was developed from the FOCUS Framework to describe what, and how, a teacher can notice productively when learning from practice (see Choy 2015 p. 178.). It maps a teacher's noticing processes (attending, making sense, and responding) through three stages of learning from practice (planning, teaching, and reviewing) to the three key productive practices for mathematical reasoning (designing lesson to reveal thinking; listening and responding to student thinking, and analysing student thinking). In other words, the model describes a theoretical process of productive noticing, which highlights explicitly the three crucial focal points, and how the alignment between these three points can be achieved. A teacher's noticing can be analysed and then compared against this theoretical model to highlight the similarities and differences, so that specific actions can be pinpointed as part of a teacher's reflection on his/her instruction (Choy 2015, 2016).

To summarise, Choy's (2015) study introduced the notion of productive noticing as a means to distinguish the kind of noticing expertise that sets expert teachers apart from less accomplished ones. Furthermore, noticing expertise is not necessarily a function of experience. A comparison of noticing expertise in pre-service teachers in a US teacher preparatory course and in-service teachers in a Singapore school highlights that both groups of teachers faced the same challenges in noticing the relevant instructional details (Lee and Choy 2017). Since then, several other studies have explored the notion of productive teacher noticing in different contexts. For example, Seto and Loh (2015) examined how a teacher mentor can direct a teacher's noticing to focus on relevant instructional details related to the concept of decimals during mentoring conversations. Their study highlights the importance of using appropriate questions to support teachers in shifting their focus on their own thinking to how students think. Similarly, Choy (2017) details how the knowledgeable other (Watanabe and Wang-Iverson 2005) in lesson study can redirect teachers' attention to mathematically significant details during the kyouzai kenkyuu stage of lesson study. Research on teacher noticing has also begun to explore the use of the construct to reflect on teaching at the micro-level. For instance, Chia (2017) investigates what and how a teacher may notice about her use of multiple representations when teaching percentage from a commognitive perspective (Sfard 2008).

20.3.2 Study 2: Noticing and Orchestrating Learning Experiences

We now turn our attention to the second study on teacher noticing, which is still ongoing at the time of writing. This project extends the investigation of what teachers notice beyond professional development activities such as lesson study, to explore the role of noticing in the context of day-to-day teaching. In this section, we give a preliminary report on an ongoing project, which examines what, and how, three experienced mathematics teachers in Singapore notice as they orchestrate learning experiences in their own classrooms. Learning experiences refer to "the interaction between the learner and the external conditions in the environment to which he can react" (Tyler 1949, p. 63). In this project, we refer to students' learning experiences as their engagement with mathematical tasks selected or designed by the teachers, through which the students develop their mathematical processes. Learning experiences have been incorporated into the current mathematics syllabus and may involve teachers providing "opportunities for students to discover mathematical results on their own", or "work together on a problem and present their ideas using appropriate mathematical language and methods" (Ministry of Education-Singapore 2012). This project is therefore situated within the context of everyday teaching activities and not within the context of a particular professional development activity as in Choy's (2015) study.

This ongoing project adopted a design-based research paradigm (Design-Based Research Collective 2003), similar to Choy (2015), to develop a toolkit to support teachers in noticing and a theory to describe their noticing when orchestrating learning experiences. We engaged in three iterative cycles of theory-driven design, classroom-based field testing, and data-driven revision of the Mathematical Learning Experience Toolkit (MATHLET) to provide a theoretical justification for the analytical frameworks on which the toolkit is based. By engaging with our teacher participants in designing, implementing, and reviewing learning experiences using the MATHLET, we aimed to develop a deeper theoretical understanding of how teachers orchestrate mathematically meaningful learning experiences.

Four experienced mathematics teachers from three secondary schools, with different achievement bands and demographic factors, participated in this study. Each teacher designed and implemented a lesson of their choice during each design-cycle phase using the MATHLET. This would result in 12 design cycles at the end of the project across three different schools. Data were generated through voice recordings of planning discussions, pre-lesson discussions, post-lesson discussions, video recordings of lessons, and lesson artefacts. Findings were then developed using a "thematic approach" (Bryman 2012, p. 578) together with the two characteristics of productive noticing as proposed by the FOCUS Framework (Choy 2015).

In this project, we focus on what the teacher attended in relation to the interactions between students, content, and the task. These interactions can be visualised as a socio-didactical tetrahedron as shown in Fig. 20.2 (Rezat and Sträßer 2012). We follow Rezat and Sträßer (2012) in seeing each face of the tetrahedron as an instantiation of the interactions between task, students, teacher, and mathematics. For example, the task–students–teacher face represents the interactions that occur amongst teacher, students, and the task. Given our emphasis on teacher noticing, we put "teacher" as the apex of the tetrahedron as seen in Fig. 20.2 to reflect our focus on how the teacher managed these interactions.

Preliminary findings from this study go beyond Choy's (2016) work on productive noticing involving a single task and extend the study of noticing into the realm of using a sequence of typical problems to bring about mathematically productive learning experiences for students. Typical problems are examination-type or textbook-type questions, often used by teachers to develop procedural skills. What we have found is that experienced teachers notice the affordances of typical problems and modify them to develop both procedural skills and conceptual understanding. By affordances, we refer to what typical problems have to offer to develop conceptual understanding beyond their usual usage. In particular, we focus on how teachers are able to notice the characteristics of the task in relation to the particular understandings of the related concepts, and see how these problems are deployed in the classrooms.

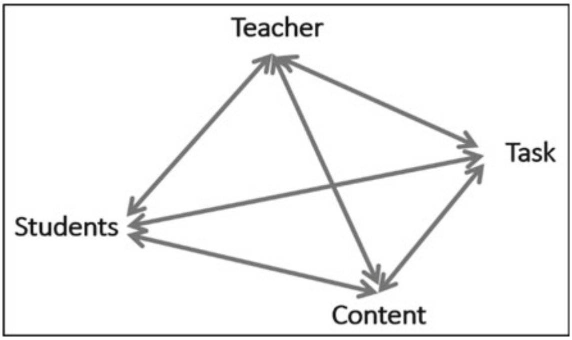

Fig. 20.2 Socio-didactical tetrahedron for using the task to orchestrate learning experiences

Example 1

[Nov 2013] Teresa and Robert attend the same school. They keep a record of the awards they have earned and the points gained. The matrices show the numbers of awards and the points gained for each award.

$$\begin{array}{c} \\ \text{Teresa} \\ \text{Robert} \end{array} \begin{array}{c} \text{Gold Silver Bronze} \\ \begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix} \end{array} \qquad \begin{array}{c} \text{Points} \\ \begin{array}{c} \text{Gold} \\ \text{Silver} \\ \text{Bronze} \end{array} \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} \end{array}$$

(a) Find $\begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$.

(b) Explain what your answer to (a) represents.

Fig. 20.3 An example of a typical problem used by Alice

We see that experienced teachers who harnessed the affordance of typical problems to enhance students' learning experiences do so in two ways. First, they are able to see, within the typical problem, opportunities for developing mathematical ideas. For example, as described in the case of Alice (Choy and Dindyal 2017a, b), she modified a typical matrix calculation problem (see Fig. 20.3) by opening up its solution space, which provided opportunities for students to use different methods to solve the problem. Alice used students' responses to the typical problems to develop relational understanding by connecting their responses to different key mathematical ideas in the same topic.

Another teacher, John, exploited a sequence of typical problems through the idea of *bianshi* (Wong et al. 2013), which is similar to Marton and Pang's idea of variations (see Marton and Pang 2006), by making deliberate modifications to typical problems for broadening and deepening students' understanding of the skills and concepts. He tried to guide students in making connections between the procedural skills and the concepts they had learned. His use of typical problems was characterised by deliberate changes to the structure of the chosen problems to highlight specific aspects of the

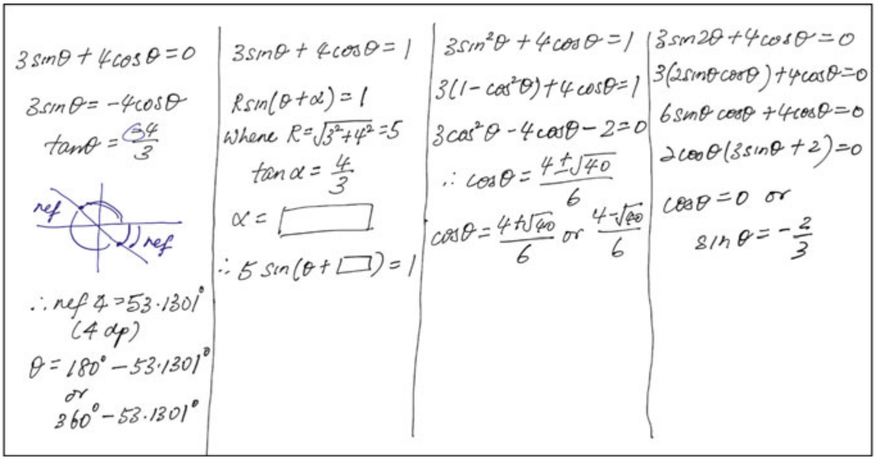

Fig. 20.4 A sequence of four typical problems used by John

concept or skill. Referring to Fig. 20.4, we see that John used four trigonometric equations, which looked similar but are structurally different.

For example, the difference between the first equation ($3\sin\theta + 4\cos\theta = 0$) and the second equation ($3\sin\theta + 4\cos\theta = 1$) lies in the number on the right-hand side. This variation in the number changes the structure and solution method of the first equation. In the first equation, we see that students can divide both sides by $\cos\theta$ to obtain an equation containing only the tangent function. The second equation, however, requires students to transform the equation into the form $R\sin(\theta + \alpha) = 1$. By harnessing variations, John was able to enhance his students' understanding of the solution methods and highlight the key considerations when solving such equations. Therefore, both Alice and John noticed the mathematical opportunities embedded within typical problems and planned how they could be used in the classroom (teacher-mathematics-task face of the tetrahedron, see Fig. 20.3).

Secondly, experienced teachers harnessed typical problems by orchestrating discussions (teacher-student-task face) about such problems to bring out key mathematical ideas (teacher-student-mathematics face). Smith and Stein (2011) highlight the importance of a good task in orchestrating mathematically productive discussions. In their model, they suggest an instructional sequence which centres about a single rich task in which students attempt, present, and discuss the mathematics under the orchestration of a skilful mathematics teacher. However, Alice's lesson differed from that envisioned by Smith and Stein (2011) in the plurality of tasks within the same lesson, punctuated by several more rapid successions of the same discussion moves: monitoring, selecting, sequencing, and connecting. This structure was made feasible by the use of typical problems which generally take a shorter time to complete. Similarly, in John's case, he orchestrated a series of short discussions about the four trigonometric equations.

Both Alice and John's noticing were productive because they were able to enact the productive practices of using tasks to reveal or enhance students' thinking, and at the same time, listened and used students' responses to the tasks to further students' understanding of the concepts. From our interviews with them, we realised that they were able to hold the whole curriculum in their minds, while attending to the details of each task. They recognised the relationships of each task to the topic taught and perceived the potential affordances of these problems. Although more work is needed before we can theorise further about their noticing, these findings suggest these teachers were able to recognise possibilities and have a different act in mind (Mason 2002). This sets up the stage to improve one's practices of using typical problems to enhance students' learning experiences.

20.4 Implications for Improving Teaching

These findings beg the question: whether such noticing is trainable, and if so, how? We believe so. Mason (2002) persuasively argues that the object of noticing is to recognise possibilities and generate alternatives from our habitual responses. This, according to Mason (2002), is achieved through noticing from the three worlds of experiences: one's experiences, other colleagues' experiences, and the world of theory and observations (p. 94). To support teachers in seeing the mathematical connections afforded by typical problems, there is a need to empower teachers to do this work through professional development, which focus on noticing the mathematics envisioned in the curriculum. This has important implications for mathematics educators in how we conceptualise professional development.

20.4.1 Noticing the Mathematics: Going Beyond the Surface

As seen from Alice's case, it is equally important to notice mathematical possibilities during the planning of the lesson, and not just to notice in the moment during the lesson. We realised that it is not so much of learning new content, but rather using teachers' existing knowledge to delve deeper into school mathematics. It is more about supporting teachers to use what they know and guiding them to see new connections between different aspects of the mathematics they are teaching. In a way, it is about guiding them to see the "forest and the trees". Teachers need to have opportunities to zoom in and zoom out of the curriculum, and notice systematically about the details of the curriculum (Mason 2011). In particular, they have to learn how to attend to the whole curriculum (holding wholes); discern the details of the concept; seeing the teaching of this concept in a sequence of lessons; conceptualising a lesson as a sequence of tasks, and encapsulating the mathematics within the tasks, paying careful attention to inter typical problem differences (see Atkinson et al. 2000). Although both Alice and John had thought of using their respective sequence of tasks

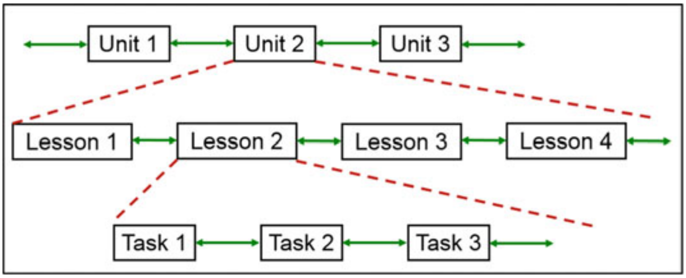

Fig. 20.5 Embeddedness of tasks–lessons–units

in different ways, both teachers highlight the importance of noticing and drawing on the connections between the sequence of tasks embedded within a sequence of lessons, which in turn are embedded within a sequence of units (see Fig. 20.5).

20.4.2 Listening to and not Listening for

Much of the research on noticing focused on what teachers see as a proxy to analyse what teachers attend to. As the two studies have demonstrated, it is crucial for teachers to listen and respond to students' thinking, instead of focusing on what teachers themselves are thinking for noticing to be productive. According to Davis (1997), this stance of listening is the difference between listening to and listening for. In other words, teachers who listen for certain responses in students' answers are less likely to orchestrate mathematically meaningful conversations because the teachers' attention is placed on getting students to utter the "correct responses". Instead, when teachers truly attend to students' thinking by listening to their students in order to make sense of the mathematics embedded in students' responses, they are more likely to move beyond the standard initiate–response–evaluate (IRE) discourse patterns, to a more interactive discourse pattern. The *listening to* stance positions teachers to notice other possibilities in orchestrating discussions, by focusing on what students say, instead of what students ought to say. This stance is a critical shift towards enhancing teachers' formative assessment of their students' understanding.

Moreover, the teachers in our studies demonstrate their competencies in orchestrating discussions by supporting students to make connections between the different ideas presented. For example, in Alice's case, she demonstrated her productive noticing through her attempts to make connections between the ideas presented by different students:

1	Alice:	(Walks around the class and comes to Student S1.) Can you write this for me on the board?
2	S1:	Ok. (Walks to the whiteboard and writes the following: $T = 5 \times 29 + 3 \times 10 + 2 \times 5 = 185$ $R = 5 \times 30 + 3 \times 6 + 2 \times 8 = 184$)
3	Alice:	(Walks around while waiting for Student S1 to finish writing.) Ok. Most of you have written what [Student S1] has written. 5 points for 29 gold, 3 points for 10 silver and 2 points for 5 bronze. Most of you have written in this manner. The last few days, we have been talking about matrices, right? Would you like to convert this to a matrix problem? (Looks at Student S2) Have you written it in matrix form? (Student S2 nods and Alice goes over to look at his answers.) Okay. Can you write your answer on the board?
4	S2:	(Walks to the board and writes the following) $T = \begin{pmatrix} 29 & 10 & 5 \end{pmatrix} \times \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = 185 \text{ and } R = \begin{pmatrix} 30 & 6 & 8 \end{pmatrix} \times \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = 184$
5	Alice:	Any other answers from [Student S2's] answer? (Walks around the class and selects Student S3's answer) Can you write this on the board?
6	S3:	(Walks to the board and writes the following) $\begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 29 \times 5 + 10 \times 3 + 5 \times 2 \\ 30 \times 5 + 6 \times 3 + 8 \times 2 \end{pmatrix} = \begin{pmatrix} 185 \\ 184 \end{pmatrix}$
7	Alice:	Thank you all three of you. [Student S1] has written using an arithmetic method. Most of you have written in this manner. This one comes very naturally to you, ok? [Student S2] has written Robert and Theresa's award separately. He has tried to use the matrix method, (points to Student S1's solution.) Something like this, ok? Let's check whether the order of matrix is correct or not (Alice goes through the method of matrix multiplication and gets the class to check the order of Student S2's matrices) … Ok. Student S3 has written Robert's and Theresa's together so that you only write this matrix once (points to the column matrix [5 3 2]). Don't need to write two times, correct or not? See. Over here. You have to write two times but here, [Student S3] only has to write it once. Let's check the order again…
8	Alice:	(After a short time) I would like to bring this problem a little bit further. Notice that Student S3 presented the information this way. Is there another way to represent the same information? (After some time, Student S4 highlights another possible way)

Here, we see how Alice orchestrated a mathematically productive discussion (Smith and Stein 2011). Alice carefully attended to students' answers before she asked for volunteers during the whole class discussion. However, it can be inferred that she was deliberate in her selection and sequencing of students' responses (see Lines 1 to 6). By beginning with an arithmetic solution, Alice connected Student S1's arithmetic operations to matrix multiplications through the sequencing of Student S2's and Student S3's matrix solutions. The reason for using a single matrix multiplication (Student S3's solution) was also made explicit when Alice moved from Student S2's solution to Student S3's using a matrix approach (Line 7) before she highlighted the different ways to express the given information as matrices (Line 8), which was an important idea for the lesson. Hence, we see how Alice connected the different ideas together during the discourse. This practice of connecting (Smith and Stein 2011) is predicated on Alice's ability to *listen to* her students (Davis 1997). This stance is challenging for teachers to adopt, even experienced ones. Developing and supporting teachers in listening to students during classroom discussion will hence be an important area for investigation.

20.4.3 Support Needed by Teachers

As discussed, teacher noticing is critical but can be a difficult skill to hone. Hence, we propose supporting teachers in (a) thinking about the use of tasks, both mathematically rich tasks and typical problems, through guided exploration of the curriculum materials to see the mathematical potential during planning; (b) empowering teachers to orchestrate discussions through discussion prompts during the lesson, and (c) encouraging them to relook at the mathematical connections from students' work after the lesson. Accordingly, we have developed the MATHLET (toolkit) to address three aspects of incorporating and modifying different tasks for orchestrating learning experiences. First, we provide a protocol for teachers to unpack the big ideas of the unit and specify the learning outcomes in terms of concepts, conventions, techniques, results, and processes for a lesson (Backhouse et al. 1992). Second, we support teachers to select tasks, including typical problems, and suggest how they can modify the structure of such problems to facilitate productive mathematics discussions. Last, we suggest prompts and questions enrich classroom discussions around typical problems that will meet the dual objectives of developing skills and concepts. The MATHLET is still under development at this stage, but we believe that such a toolkit will be useful for teachers as they engage in practices that are aimed at bringing a different act to mind.

20.5 Possible Future Trajectories for Research

Teacher noticing matters. As we have described in this chapter, teachers who notice productively about instructional and curricular details have a higher likelihood of improving their teaching practices. As a critical component of teaching expertise, it is important for mathematics educators to continue their research into this construct. We see at least three potential trajectories for this research:

1. *Unlocking the "black box" of teacher noticing*. As argued by Scheiner (2016), the processes of attending and interpreting instructional events are still unclear

and hidden within a black box. In particular, it is crucial to investigate how and why teachers attend to particular events and other instructional details in the background simultaneously. This may help to answer how and why teachers may attend to something and yet not perceive the object, and vice versa. A promising way is to tap into cognitive sciences and human factor research. This poses several methodological issues, which require more thought. Nevertheless, such investigations may potentially unlock the black box of noticing and offer us insights into the underlying mechanisms of productive teacher noticing;

2. *Overcoming methodological challenges in researching teacher noticing*. Currently, data collected on teacher noticing come from records of teachers' conversations during video clubs (Santagata and Yeh 2013; van Es 2012), lesson study sessions (Choy et al. 2017; Lee and Choy 2017), and other professional development settings (Fernandez et al. 2012; Goldsmith and Seago 2013; Seto and Loh 2015). While there were some attempts to use technology, such as wearable cameras (Sherin et al. 2011), to capture what teachers see, it remains unclear how this information can be connected to what teachers attend to, and how they interpret classroom details. For example, it is possible now to record and analyse gaze plots of teachers using wearables, but the link between the gaze plots and teaching actions remains tenuous at best; and
3. *Developing teachers' expertise in noticing*. We believe that teacher noticing is trainable (Choy and Dindyal 2017b), but it remains to be seen how this can be implemented across teachers of varying experience levels, and across different schooling years. Research into teacher noticing in Singapore is still in its infancy, and developmental research into novel professional development programmes will be a fertile area to look into. While researchers have used video clubs and lesson study sessions to develop teaching expertise, there may be a need to develop programmes to hone teachers' noticing skills in Singapore. This is one key area in our current research and there is definitely room to examine more closely, the features of, and the effectiveness of such a programme.

To conclude, we argue for the critical role that productive noticing plays in teacher education and professional development and highlight how this component of teaching expertise may be developed and honed. More importantly, as Mason and Johnston-Wilder (2006, p. 127) put it: "The heart of teaching is interaction with learners; the rest is preparation to make this interaction useful", it is important for us to see noticing as a deliberate practice, which needs preparation, and not something that is impromptu. Many questions about teacher noticing and challenges in research and development of teachers' noticing expertise remain. Nevertheless, we find it exciting that to see research in this area gaining momentum in Singapore, and we look forward to see how future research trajectories of productive teacher noticing in Singapore can contribute towards a more comprehensive understanding of this important construct in the teaching and learning of mathematics.

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