Chapter 17
Continuing from Pre-service: Towards a Professional Development Framework for Mathematics Teachers in the Twenty-First Century

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Abstract Quality teachers have long been recognised as key to preparing the future generation for the nation. Hence, a robust teacher education system with collaborative support from major stakeholders is crucial. Singapore has adopted a complex yet integrated approach in teacher education thus far. This chapter moves on from an earlier report presented in 2009 on the Singapore teacher education system. To pave the way forward, the chapter introduces the visions of key stakeholders in the professional development landscape for teacher education in the twenty-first century. Various factors of influence are analysed before presenting the structure of professional development for mathematics teachers at the National Institute of Education. Current mathematics professional development courses are classified according to aspects of teacher knowledge derived from research so as to gain insights into the content, pedagogical and assessment focuses. Finally, a proposed conceptual framework amidst the multidimensional and multifaceted teacher education landscape is outlined to describe mathematics teacher professional development for the twenty-first century.

Keywords Twenty-first-century competencies · Mathematics pedagogical content knowledge · Mathematics education · Professional development framework · Teacher education · Teacher growth model · Teacher professional development

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17.1 Introduction

Quality teachers have long been recognised as key to preparing the future generation for the nation. Indeed, the influential McKinsey report on how the world's best-performing school systems come on top articulated that "the quality of an education system cannot exceed the quality of its teachers" (Barber and Mourshed 2007, p. 16). Well-qualified teachers not only exert a critical impact on student learning (Gopinathan et al. 2008) but also an enduring influence in the lives of future generations. Hence, it is critical that education ministries, policymakers, and teacher educators work together on several fronts to draw interested individuals into teaching and retain them in education service: (a) provide robust teacher education programmes at pre- and in-service levels at various junctures of the teaching career, (b) maintain rigour in teaching certification, (c) allow for career progression in teaching, and (d) support the teaching fraternity to chart their own professional growth. Collaborations between teacher educators and schools in teaching-research projects (see Ng et al. 2015), formations of professional learning communities among teachers (see Hairon and Dimmock 2012) and establishments of sharing platforms (e.g. conferences, seminars) among key stakeholders in education (i.e. schools, teacher educators, curriculum planners, policymakers) are some infrastructure put in place in many education systems around the world.

Yet, as Tan and her colleagues (2017b) put it, educational systems all over the world face at least two challenges in the twenty-first century. Age old constructs or concepts such as "creativity, critical thinking, collaboration, communication, socio-emotional and lifelong learning aptitudes" are now recognised as "new knowledge economy competencies" and they have been given a renewed lease of attention in view of a technologically dominated globally connected world in the twenty-first century (Tan et al. 2017b). Learning is not confined to traditional forms of delivery. Neither is it confined to individual experts. These have implications on the teacher education system. Firstly, how should schools and educators scaffold and assess students' new knowledge economy competencies individually and collectively? Secondly, how would we activate and sustain a cultural and pedagogical shift from traditional modes of education and perception on achievement to a more inclusive, varied form of education? Some answers may lie with a progressive teacher education framework which is aligned with the learning needs of twenty-first-century teachers. Views on what quality teachers in the twenty-first century are may morph from the complex, multidimensional and multifaceted discussions that follow.

17.2 Teacher Education in Singapore: A Brief Understanding of the Current Landscape

In this chapter, *Teacher Education* refers to a broader concept which encompasses the desired outcomes (e.g. philosophical, theoretical, political and economic) to be

integrated and balanced among major stakeholders such as policymakers, government bodies, education administrators, universities and funding agencies. Ideally, teacher education should be in a continuum of three seamless stages according to the teaching career progression: (a) initial teacher preparation (referred to as "pre-service" in this chapter where student–teachers are in the process of being accredited), (b) induction (referred to as "beginning teachers" where accredited teachers may still work with school mentors during the first few years of teaching experience) and (c) *Teacher Professional Development* (PD) (referred to as "in-service" where experienced teachers may chart their own growth in teaching repertoire). In this chapter, teacher PD includes those conducted in formal delivery-style settings (e.g. workshops, training sessions, talks, seminars, and conferences) as well as those which involve targeted group-based discussions or sharing sessions (e.g. professional learning communities). This is in line with the Teacher Growth Model for twenty-first-century teachers articulated by the Academy of Singapore Teachers (AST) (see Sect. 17.2.3). In addition, we also recognise that beginning teachers can also participate in PD courses alongside with experienced teachers.

We begin with a discussion of the teacher education landscape in Singapore in this chapter and subsequently focus on teacher PD in Singapore, particularly analysing the structure of PD for mathematics teachers. Lim-Teo (2009) provided an in-depth discussion of the context of teacher education in Singapore and factors of influence prior to 2009. She articulated the synergistic effect between the National Institute of Education (NIE) and the Singapore Ministry of Education (MOE) which alleviated teacher education to higher levels globally. This chapter moves on from Lim-Teo's report after 2009, presenting the complex yet integrated approach Singapore has taken and introducing more factors of influence in its teacher education journey thus far. The next section summarises how teacher PD is offered through four major collaborative avenues in Singapore; namely the NIE, MOE, AST, and professional associations. Following which, we outline the current situation of PD for mathematics teachers at the NIE, analysing the factors of influences and their impact on curriculum, and hence PD. Then, some constraints, issues and challenges to mathematics PD faced by teacher educators at NIE are highlighted. A proposed PD conceptual framework tailored for mathematics teachers in the twenty-first century after an analysis of the broader landscape of teacher education in Singapore and beyond is presented. Finally, future directions for mathematics PD and related research in Singapore are discussed.

17.2.1 Teacher Education at the National Institute of Education: The Journey After 2009

Singapore is progressing towards a transformative system that produces quality teachers equipped to raise a new generation of twenty-first-century learners. In this system, teachers are expected to prepare students for a knowledge-driven global economy by

helping them develop higher-order competencies and new technology-based skills, while simultaneously building character and grounding values so that Singapore students still remain rooted in their national identity against the backdrop of a multicultural, globalised world (NIE 2012).

The NIE is Singapore's sole and national teacher education institute (NIE 2017a). As the premier teacher education institute in Singapore and an autonomous institute of the Nanyang Technological University (NTU), Singapore, NIE not only provides teacher accreditation during initial teacher preparation but also designs programmes, courses and workshops to empower accredited teachers in their professional development journey for enhancement of competence and knowledge as they progress in their teaching career. NIE embarked on a Programme Review and Enhancement initiative in 2008 as an institute-wide strategic effort to review and enhance NIE's model of teacher education (NIE 2012). *The Teacher Education 21 Model (TE21)* was inaugurated by NIE in 2009 to cultivate the "thinking teacher" while maintaining "strong partnerships with key stakeholders and the schools to ensure strong clinical practice and to inject the reality of professionalism in teacher development" (NIE 2009). Recommendations in the TE21 Model address the entire initial teacher preparation (i.e. pre-service) to teacher PD (i.e. in-service) continuum (NIE 2017a). It is NIE's mission to provide a curriculum that is "cognizant of nationwide policies and initiatives implemented by the Ministry of Education" (NIE 2017b). TE21 puts major emphasis on teachers' values because values are the "anchor of stability, consistency and centredness in a changing vortex" (Tan 2012, p. 39) in the midst of the rapid changes in curriculum and policy brought about by challenges in the twenty-first century. Three key values are identified: learner-centred values, teacher identity values and the values of service.

Initiatives by the Singapore MOE are put forth to foster students' *new knowledge economy competencies* (see Sect. 17.1) required in the twenty-first century. A crucial recommendation of TE21 is a robust theory-practice nexus in developing teachers' own proficiencies to scaffold students' twenty-first-century competencies, building upon the content and pedagogy associated with different subject disciplines. Hence, NIE works closely with the Singapore MOE, the AST and schools so that policies and initiatives are not only integrated into the pre- and in-service teacher education programmes and courses but also realised in practice among prospective and experienced teachers.

Being in a unique and privileged position, NIE offers a robust system of teacher accreditation at primary, secondary and pre-university levels across various subject-discipline areas in Singapore through various programmes such as the NTU-NIE Teaching Scholars Programme (TSP), the Postgraduate Diploma in Education programme, undergraduate programmes (i.e. Bachelor of Arts (Education), Bachelor of Science (Education) and Diploma programmes. Lim-Teo (2009) summarised the model of pre-service preparation of teachers in Singapore in Fig. 17.1. The model is still applicable to date. Since the implementation of TE21 in 2009, pre-service programmes have been reviewed to address the emphases of TE21. For example, the NTU-NIE TSP is a new undergraduate programme that was launched in August 2014. This programme offers final-year scholars individually supervised research

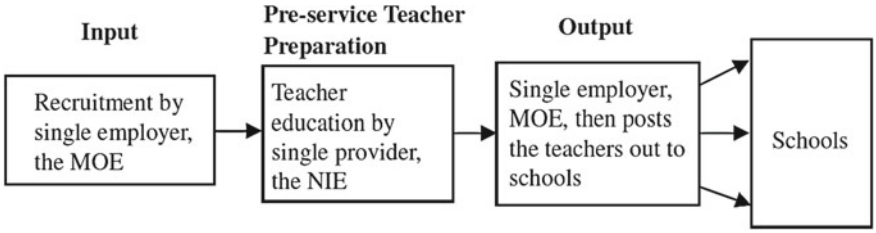

Fig. 17.1 Model of pre-service preparation of teachers in Singapore (Lim-Teo 2009, p. 50)

opportunities with eminent research mentors at NTU or NIE where the scholars and their mentors pursue areas of interest together, immersing in extended content knowledge academic exchanges over a 11-month period. In addition, TSP scholars also undertake an educational research project in their third year of study to build their capacity in the theory-practice nexus alongside their classes in curriculum studies with pedagogical focuses for various subject disciplines (NIE 2017c).

Helmed by the Office of Graduate Studies and Professional Learning at NIE, in-service programmes are crafted for experienced teachers, education officers with the MOE and interested individuals from other educational institutions in Singapore and overseas. Singapore teachers with the MOE are placed in three career tracks: Teaching Track, Leadership Track, Senior Specialist Track (MOE 2017a). NIE in-service and higher degree programmes cater to teachers at various junctures of their careers in these tracks. For example, besides considering a comprehensive list of stand-alone PD courses, primary mathematics teachers can choose to enrol in the Advanced Diploma in Primary Mathematics Education programme should they like to develop their pedagogical content knowledge further and have a deeper conceptual understanding of mathematics content knowledge in the horizon to make connections between primary mathematics topics and beyond (NIE 2017d). In addition, NIE also offers a highly anticipated Teacher Leaders Programme which develops "leaders on the Teaching Track" (i.e. Senior Teachers, Lead Teachers and Master Teachers) through an intricate progression of learning journeys aligned with the *Teacher Growth Model (TGM)* (MOE 2012a; see below for details). The Teacher Leaders Programme aims to "nurture teachers as ethical educators, competent professionals, collaborative learners, transformational leaders and community builders" (NIE 2017a). Separately, working with MOE to develop education officers on the Leadership Track (i.e. school leaders such as Heads of Departments, Vice-Principals, Principals and Cluster Superintendents), NIE provides at least three programmes: Leaders in Education Programme, Management and Leadership in Schools Programme, and Building Educational Bridges—Innovation for School Leaders. MOE education officers who are keen to progress on the Senior Specialist Track can enrol in a myriad of higher degree programmes (e.g. Masters in Education) to enhance their theory-practice nexus for curriculum development. Since 2005, NIE has implemented the *Professional Development Continuum Model (PDCM)* scheme to provide graduate teachers of Singapore MOE with alternative pathways to higher degree certification

(NIE 2017e). Most of the Master's degree programmes at NIE are available under this scheme. A step further into TE21, with effect from August 2012, the *enhanced PDCM* scheme was put in place to allow for more flexibility in structure for candidates with different interests and work commitments. 2014 saw the cross-listing of selected courses in NIE Master's degree programmes with PD. This allows for teachers who meet the entry requirements of the Master's degree programmes to take higher degree courses as PD prior to their admission to the programme under clear accreditation and time frame conditions (NIE 2017f). In this way, teachers can experience the rigour and depth of NIE higher degree courses with a slightly more theoretical and research stance compared to other more practice-orientated PD courses. This also creates opportunities for teachers to design integrated research and practise projects within their own capacities under the tutelage of faculty members, spear-heading innovative pedagogies and curricula approaches in schools, grounded in sound theoretical underpinnings and empirical findings; achieving another TE21 recommendation.

17.2.2 Envisioning Teacher Professional Development for the twenty-first century in Singapore: Ministry of Education

Launched in 2012 by Mr. Heng Swee Keat, then Singapore Minister of Education, the Teacher Growth Model (TGM) is a "professional development model which encourages Singapore teachers to engage in continual learning and become student-centric professionals who take ownership of their growth" (MOE 2012a). Developed by the AST under MOE, the TGM (for in-service teachers) was a result of a collaborative conceptualisation effort among educators of diverse profiles across MOE to construct the learning needs of the twenty-first-century Singapore teacher. Figure 17.2 illustrates the five desired outcomes of the twenty-first-century Singapore teacher in the TGM (The Ethical Educator, The Competent Professional, The Collaborative Learner, The Transformational Leader and The Community Builder). Although the TGM Learning Continuum suggests learning focuses for teachers at various stages of their careers, teachers have the autonomy to plan for their PD based on their needs and interests, bearing in mind alignment to the knowledge and skills needed to nurture students in twenty-first-century competencies (MOE 2012a). Recognising that teachers also have diverse learning needs, the TGM encourages teachers to pursue PD through multiple modes of learning (e.g. face-to-face, ICT-enabled, conferences, mentoring and research-based practice, networked learning, reflective practice and experiential learning). There are seven learning dimensions associated with the TGM and all NIE PD courses for in-service teachers (including those cross-listed with higher degree) are mapped to these learning dimensions (NIE 2017g).

Within the TGM framework, various departments at MOE also conduct PD workshops or sharing sessions for teachers. Curriculum specialists with the curriculum

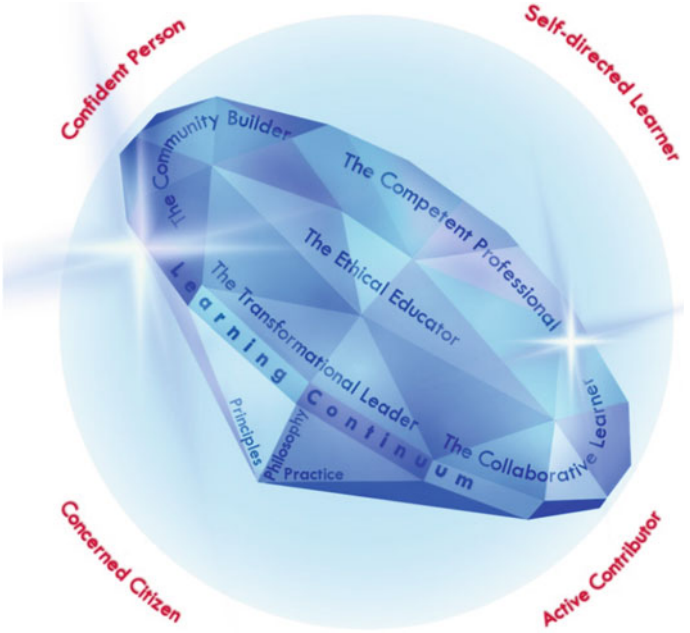

Fig. 17.2 Teacher growth model (Ministry of Education 2012a)

planning and development division at MOE conduct nationwide PD workshops periodically for schools and clusters to communicate and help kick-start the implementation of curriculum initiatives. Senior management in schools send representatives to attend such PD workshops so that they can take leadership in incorporating the initiatives in school-wide programmes. An example is "Fostering Mathematical Reasoning in Classrooms" workshops conducted by the curriculum specialists for secondary school teachers in recent years. Often, curriculum specialists work with teacher educators at NIE to plan complementary workshops for teachers so that the teachers are aware of the background and key messages associated with new curriculum focuses, and have opportunities to draw connections between the curriculum focuses with appropriate pedagogical approaches grounded from theory and research. For instance, mathematics curriculum specialists have conducted introductory workshops on mathematical modelling since 2009 where they show examples of mathematical modelling activities secondary mathematics teachers can use in their classrooms and discuss elements of the modelling cycle (see Balakrishnan et al. 2010; MOE 2012b). However, teachers who are leading mathematical modelling activities in their schools or are interested to have a more in-depth understanding of the design, facilitation and assessment of mathematical outcomes during the full cycle of mathematical modelling are directed to attend NIE PD workshops for a comprehensive hands-on experience (see Ng 2017).

In addition, MOE holds regular meetings and briefing sessions with heads of departments or pedagogical leaders in schools to communicate curriculum and assessment changes. Teachers often attend conferences (e.g. Kwek and Ko 2011), seminars and workshops based on interest or upon the encouragement of school management. Again, within the TGM, many interest-driven professional learning communities (see Hairon and Dimmock 2012) are set up within schools (e.g. among teachers teaching the same level) or within school clusters. These professional learning communities can be steered by teachers, school leaders, mathematics educators, MOE education officers and Master Teachers from the AST. Each professional learning community is set up to achieve explicit objectives and they typically outline PD sessions that are aligned with their progressive implementation of school-based projects or initiatives. Subsequently, the professional learning communities in schools may organise their own in-house PD sessions conducted by the pedagogical leaders in the schools or by invited instructors. For example, between 2014 and 2016, the mathematics department head of one primary school in Singapore held a series of workshops on Talk Moves (see Michaels and O'Connor 2015) with the teachers in the school. There was meticulous planning and mentoring by the mathematics department head to help the mathematics teachers in the school implement what was shared in the workshops in progressive steps, the first of which was lesson observations of the head of department in action with her mathematics class where she used Talk Moves to generate more mathematical productive discourse and encourage students to share their mathematical reasoning. The school finally took on this initiative as a school-wide approach after successful implementation and good reviews from the mathematics department and students (Lee et al. 2016). One advantage of such carefully planned professional learning communities within the school as illustrated above is the close links between theory and practice where teachers engage in iterative cycles of reflective practice among like-minded peers under full support of the school management.

MOE is cognisant of the importance of providing opportunities for serving teachers to extend their professional repertoire in order to help them achieve their academic and professional aspirations. Funding and professional development leave infrastructure are put in place to encourage teachers to engage in lifelong learning. Every teacher in the school system is eligible to 100 h of PD a year, fully funded by MOE directly or through MOE-administered school or cluster budgets (Lim-Teo 2009, p. 66). Teachers can choose from various professional development packages and leave schemes (MOE 2016, 2017a) to participate in PD and higher degree work.

17.2.3 The Role of the Academy of Singapore Teachers in Teacher Professional Development

The Academy of Singapore Teachers (AST) was set up in September 2010 to look into "the development of a teaching fraternity that is characterised by a shared ethos,

strong pedagogical expertise and ownership of professional development" (MOE 2012a). AST serves four functions: to (a) champion the ethos of the profession, (b) foster a teacher-led culture of collaborative professionalism, (c) build a culture of continuous learning and improvement and (d) strengthen enablers of professional development (MOE 2017b). Crafted by AST, the TGM guides the planning and implementation of PD activities at AST. AST articulated the TGM as a "representation of a coherent whole of core learning areas of holistic professional growth and development for Singapore teachers" which "facilitates teachers taking ownership of their professional growth to nurture in students the competencies required for the twenty first century" (MOE 2017c).

To date, AST has organised an array of PD opportunities for in-service education officers (i.e. teachers, school leaders, including those seconded to MOE headquarters), executive and administrative staff working in schools, and allied educators (i.e. teaching assistants). Besides workshops, seminars and talks, PD opportunities from AST also come in the form of focused-group discussions and sharing sessions during subject chapter meetings. Materials from these subject chapter meetings and those from follow-up sessions are typically shared on a private portal with exclusive access rights given to MOE staff.

Master Teachers are identified by MOE as "role models of teaching excellence" based on their track records of "strong pedagogical knowledge" demonstrated in schools over many years. Master Teachers are at the pinnacle of the Teaching Track. The main role of a Master Teacher is to "develop and enhance the capacity of teachers through mentoring and demonstrating good teaching practice" (Ng and Foo 2009, p. 150) through working with schools, cluster schools and beyond (e.g. school-based research projects, curriculum reviews). PD opportunities organised by AST are usually conducted by Master Teachers although they too get invited to be course instructors for school- or cluster-led PD sessions for professional learning communities.

While PD courses conducted by MOE curriculum specialists are mainly to communicate curriculum initiatives, those by AST Master Teachers have predominantly pedagogical focuses with a clear practice-oriented stance. On the other hand, NIE PD courses provide theory-practice nexus where participants learn, experience and reflect on research-based theoretically informed pedagogical practices. NIE collaborates with MOE and AST to plan in-service teacher PD stand-alone courses and programmes offered by NIE every year so that teachers receive a wide selection of complementary PD offers meeting different needs and interests. Funding for most NIE teacher PD courses and programmes comes from annual MOE budgets. In essence, there is a synergistic tripartite collaboration between NIE, MOE and AST for a holistic teacher PD in Singapore.

17.2.4 The Role of the Professional Associations in Teacher Professional Development

Local professional bodies such as the Association of Mathematics Educators (AME) and the Singapore Mathematical Society (SMS) also hold conferences and seminars regularly for mathematics teachers to learn from both foreign and local experts. A case in point is the annual Mathematics Teachers Conference co-organised by AME and the Mathematics and Mathematics Education Academic Group at NIE with support from SMS. This one-day programme includes plenary lectures and PD workshops by invited foreign and local experts for primary school, secondary school and pre-university mathematics teachers, as well as sharing sessions by local academics and mathematics educators to showcase their research studies and share findings with implications drawn for teaching and learning. The Mathematics Teachers Conference has been held for over a decade with a different theme each year articulating current educational focuses in Singapore and globally. Participation in each Mathematics Teachers Conference has been enthusiastic throughout the years with numbers ranging between 500 and 800. In addition, AME also produces a newsletter and an academic journal entitled "The Mathematics Educator" for the mathematics teaching and research fraternity.

17.3 Mathematics Teacher Professional Development at the National Institute of Education

PD courses and programmes for mathematics teachers at NIE are mainly offered by educators from the Mathematics and Mathematics Education Academic Group. Nested within larger global expectations of quality teachers in the twenty-first century, several other factors of influence have impact on current and future PD for mathematics teachers in particular.

17.3.1 Factors of Influence

One key factor of influence on the nature and format of PD for mathematics teachers at NIE is research on mathematics teacher education.

17.3.1.1 Research in Mathematics Teacher Education

Teacher education research gained momentum since Shulman (1986) called for the spotlight to be shone on a teacher's knowledge base on teaching. Grossman (1990) proposed four key components of teacher knowledge: general pedagogical

knowledge, subject matter knowledge, pedagogical content knowledge and knowledge of context. The foundational concept of pedagogical content knowledge was first coined by Shulman, who in a later publication defines it as "that special amalgam of content and pedagogy which is uniquely the province of the teacher" (1987, p. 8). Researchers recognise the need for teacher knowledge base to be discussed in terms of subject-specific disciplines because teaching requires a professional integration of various components of teacher knowledge with respect to the rigour of the discipline. A domain map of mathematical knowledge for teaching was outlined by Hill et al. (2008) and this unpacks Pedagogical Content Knowledge further and distinguishes it from Subject Matter Knowledge. Pedagogical Content Knowledge for mathematics teachers includes "Knowledge of Content and Students, Knowledge of Content and Teacher, and Knowledge of Curriculum". On the other hand, Subject Matter Knowledge refers to "Specialised Content Knowledge, Common Content Knowledge, and Knowledge at the Mathematical Horizon" (p. 377).

Research on Mathematics Pedagogical Content Knowledge (MPCK) of Teachers began to take root after initial efforts by Ball and her colleagues (Ball 1991; Ball et al. 2001, 2008). In Singapore, an inaugural project on MPCK was conducted by an NIE team of mathematics educators to investigate the development of MPCK in primary school beginning teachers. One main goal of this project was to evaluate the impact of a mathematics methods programme on prospective teachers' MPCK at NIE. As part of this large-scale longitudinal project, the team administered a 16-item instrument to measure the performance of pre-service teachers in the Diploma in Education programme before they started their initial teacher preparation journey at NIE and after they completed the programme (Lim-Teo et al. 2007). The instrument used took the form of a MPCK test where items assessed the teachers' (a) own knowledge of mathematical structure and connections, (b) representations (multiple or alternative) of concepts for the purpose of explanations, (c) perceptions of the cognitive demands of the mathematical tasks on learners and (d) identification of the difficulties faced by learners and learners' misconceptions along with teachers' choice of follow-up actions (p. 257). Quantitative pre- and post-test results suggested the pre-teachers with the Diploma programme had generally made some improvements across (a) to (d) at the end of the programme. However, qualitative analysis of the responses revealed that these pre-service teachers were rather weak in mathematical communication; especially in explaining and developing mathematical ideas alongside their logical reasoning using precise mathematical terms and language (p. 251). The researchers also surfaced challenges faced by the pre-service teachers in composing word problems to illustrate mathematical concepts (e.g. quotitive division) (p. 252). Implications were drawn from these findings on reviewing pre-service mathematics methods courses at NIE. In addition, teacher educators can draw upon the research by Lim-Teo and her colleagues when planning PD focusing on MPCK for mathematics in-service teachers so as to further deepen teachers' understanding of subject matter knowledge and introduce innovative pedagogical approaches to help students overcome mathematical learning difficulties.

Another large-scale project measuring teachers' MPCK came from the international Teacher Education and Development Study in Mathematics (TEDS-M) survey.

The performance of NIE pre-service teachers in Mathematics Content Knowledge and MPCK as assessed by the TEDS-M survey was reported in Wong et al. (2010). TEDS-M Mathematics Content Knowledge framework covers four domains (Number, Geometry, Algebra and Data) and three cognitive domains (Knowing, Applying and Reasoning). TEDS-M MPCK framework includes: mathematical curricular knowledge, knowledge of planning for mathematics teaching and learning, and enacting mathematics for teaching and learning (Tatto et al. 2012). Although pre-service primary mathematics teachers at NIE who participated captured top spots in the TEDS-M survey in terms of Mathematics Content Knowledge and MPCK compared to the other participating countries (Wong et al. 2010, p. 300), gaps were identified. There is a need to provide more opportunities for prospective primary mathematics teachers (and even those teaching mathematics at higher levels) to learn different approaches to rectify students' misconceptions in mathematics. Wong et al. also suggested that teacher educators could use publicly released TEDS-M to "explore strategies to remedy misconceptions, design classroom activities that mirror the scenarios described in the TEDS-M items" so as to work towards "assessment for teacher training" (p. 304) with formative purposes. Interested readers may like to refer to Chap. 6 for more in-depth discussions on the results from Singapore's participation in the TEDS-M study.

17.3.1.2 Research in Professional Development Models for Mathematics Teachers

Research projects by Singapore teacher educators on PD models or structures with respect to different fields in mathematics education research can also have an impact on PD for mathematics teachers provided at NIE. These will be summarised briefly in this section.

On mathematical modelling, Tan and Ang (2015) designed a school-based PD programme using Ang's (2015) framework which scaffolds mathematics teachers in secondary schools through progressive stages of modelling task design. The school-based PD programme consists of three phases where teacher reflections from earlier phases provided inputs for subsequent phases. At the end of the programme, participating teachers would have designed mathematical modelling tasks for their schools, facilitated students through the tasks and reflected on their learning about the mathematical modelling process. In another project on mathematical modelling but at the primary level, Ng and her colleagues (see Chan et al. 2012; Ng et al. 2012, 2015) incorporated a multi-tiered teaching experiment (Lesh and Kelly 2000) with adapted design research methods (Dolk et al. 2010) in their PD structure to scaffold the incorporation of mathematical modelling in primary schools.

On design of learning tasks to engage students in reasoning and communication, Kaur (2012) investigated the impact of a hybrid model of PD that integrates the PD training model from Matos et al. (2009) with "sustained support for teachers to integrate knowledge gained from the PD into their classroom practice" (Kaur 2012, p. 5148). This hybrid model of PD advocated three phases: (a) teachers attending

training workshops, (b) teachers enacting what is learnt at the workshops in their schools guided by the PD providers and (c) teachers sustaining what they have learnt from the previous phases through school-based self-directed activities. This PD model took two years to realise, a significantly longer duration compared to other PDs which are constrained by MOE timelines. There are other PD designs implemented for mathematics teachers at NIE. Detailed discussions of another PD design from research involving Replacement Units can be found in Chap. 19 of this book.

17.3.1.3 Curriculum Focuses

Given the widespread implementation across the world, there has been global impact from the results of the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) on policymaking as well as mathematics curriculum planning and review (OECD 2017; Stacey 2012). In Singapore, the mathematics curriculum framework articulates the need for students to solve a variety of problems, including open-ended and real-world problems (MOE 2012c, p. 15). Mathematical modelling has been incorporated into the curriculum framework since 2007 (MOE 2012b) and students' ability to solve "Problems in Real-World Contexts" (similar to applications problems) is assessed formally at the high-stakes GCE "O" Levels mathematics examination since 2016 (MOE 2015). Maintaining the rigour and depth of mathematical content and skills in the mathematics syllabi at the various school levels, but also in response to the global focus on students' competencies to solve non-routine, open-ended real-world problems, Singapore teachers are encouraged to develop their pedagogical content knowledge in support of more student-centric approaches. Such approaches require teachers to scaffold student-directed learning, critical thinking, as well as appropriate mathematical communication and reasoning during group collaborative problem-solving. There is also a need for teachers to be confident facilitators during problem-solving while discussing possibilities of alternative solution pathways in view of real-world constraints stipulated in the context of the problem. Contents of Mathematics PD courses at NIE not only address the mathematics curriculum framework, but also bring in the larger global picture, drawing upon research to provide sound theory-practice nexus during the courses.

17.3.2 Professional Development for Mathematics Teachers at the National Institute of Education

There is a comprehensive array of PD courses for mathematics teachers at NIE across four domains: subject matter knowledge, pedagogical content knowledge, school-based curriculum planning and assessment practices. The PD courses can

be offered as standalones, as a series of progressive courses for the same topic, or as part of a collection of courses during a programme. Stand-alone PD courses are typically short-term hands-on practice-oriented courses which can range from a three-hour workshop to several sessions of three-hour blocks either taken in a school-day afternoon or in a full day during school holidays. In contrast, PD programmes have stipulated entry requirements and time frame for completion of academic units through a number of courses and a subsequent accreditation process. Teachers can register for these programmes as part-time or full-time participants depending on their commitments. Currently, there are two PD programmes focused on mathematics teaching and learning; both for primary school teachers: the Advanced Diploma in Primary Mathematics Education programme and the Certificate for Primary Mathematics Education programme. Primary school teachers are accredited to teach more than one subject. Many also enrol in other NIE-accredited programmes or PD courses which may take a more generic stance (i.e. non-subject-specific) and apply what they learn from these programmes in the various subject disciplines they teach. Courses in PD programmes are at times offered as standalones should participants prefer taking up selected courses within the programme on an ad hoc basis.

17.3.2.1 Professional Development Through MOE-Commissioned Courses

Mathematics teachers in schools can enrol in PD stand-alone courses or programmes through three main avenues, each comprising of complementary PD lists. Firstly, majority of PD courses and programmes for mathematics teachers are MOE-commissioned. Teachers from MOE schools enrol in these through an online system called "TRAISI" (Training Administration System on Internet) using their MOE-registered email and password. MOE, AST and NIE representatives from various subject disciplines engage in annual discussions of course offers by NIE for the following year. Decisions are made based on needs assessment of teachers for further PD on curriculum initiatives and focuses. This is balanced with the overall allocated MOE budget for PD. Table 17.1 summarises the types of TRAISI stand-alone courses offered by NIE mathematics educators between 2014 and 2019. Higher degree courses (i.e. Master's) which are cross-listed with in-service and offered under TRAISI are not reflected in Table 17.1. The types of PD courses are classified according to Hill et al.'s (2008) domain map of mathematical knowledge for teaching (see Sect. 17.3.1.1). An example of a course under Pedagogical Content Knowledge with a focus on Knowledge of Curriculum is that of "Promoting Metacognition in Primary School Children" where mathematics teachers learn how to foster student's use of metacognitive strategies for problem-solving. A Subject Matter Knowledge course can be illustrated with "Algebra in Secondary Additional Mathematics" where teachers learn algebraic concepts that are Specialised Content Knowledge needed for the additional mathematics syllabus. Tan et al. (2017a) called for the incorporation of teachers' assessment literacy in examining teacher knowledge because assessment is a crucial part of curriculum, teaching and learning. Hence, a third classification,

Table 17.1 Types of TRAISI stand-alone courses offered by NIE mathematics educators

Year	Pedagogical content knowledge			Subject matter knowledge			Knowledge of assessment		
	Pri	Sec	Pre-U	Pri	Sec	Pre-U	Pri	Sec	Pre-U
2014	6	7	1	0	7	0	2	0	0
2015	6	6	8	0	7	0	2	0	0
2016	7	2	8	0	4	0	2	1	0
2017	8	3	8	0	3	0	2	1	0
2018	9	3	7	2	3	1	2	0	0
2019	5	3	3	0	4	0	1	1	1

Knowledge of Assessment, is added to provide a more comprehensive representation of the available courses. One example of a course under this classification is "Problems in Real-World Contexts: Design, Implementation and Assessment" where secondary mathematics teachers learn how to design problems situated in real-world contexts which require students to select and apply appropriate mathematics content and skills, similar in format to those assessed in GCE "O" level mathematics examination.

Data shown in Table 17.1 reveal that most TRAISI stand-alone courses are MPCK-related across primary, secondary and pre-university levels. However, at least two gaps in PD can be noted. The first gap refers to Subject Matter Knowledge. Such PD courses are not offered at primary and pre-university levels. Many educators may agree that it is not easy to untangle MPCK and Subject Matter Knowledge in a PD course because competent teachers are often able to integrate both seamlessly to achieve their lesson objectives. Nonetheless, it may be crucial for experienced primary mathematics teachers to attend PD courses on Subject Matter Knowledge because at least a large majority of them are essentially generalist in training and do not have a mathematics degree. Interestingly, graduates who are on the enhanced Postgraduate Diploma in Primary Education programme for pre-service teacher education since its inception in December 2016 have been attending Subject Matter Knowledge courses. On the other hand, it is understandable why teachers teaching pre-university level mathematics are not provided with Subject Matter Knowledge PD courses. Many of them already have honours with their mathematics degrees or even higher degree certification in mathematics. A second gap in PD shown from Table 17.1 is that of Knowledge of Assessment, particularly for mathematics teachers teaching secondary and pre-university levels. The philosophy, types, purposes of assessment, as well as different modes of assessment are taught at pre-service teacher education programmes in NIE. However, MOE dipstick surveys in schools discovered that experienced teachers in schools need refresher PD courses on assessment literacy or other assessment-related courses in view of the changing GCE "O" and "A" levels mathematics examination question types. One such example is the

recent focus on Problems in Real-World Contexts at secondary and pre-university mathematics examinations.

17.3.2.2 Professional Development Through Customisation

Secondly, another PD avenue for mathematics teachers is through customised courses or school-based PD. TRAISI course registration is limited to two teachers from the same school. Customised courses address a need in schools for tailor-made courses by experts to help springboard from the entry levels of specific groups of teachers in a school or cluster. Such courses are particularly popular with professional learning communities and range from a three-hour workshop to a series of consecutive workshops. There are several advantages to a customised approach to PD. As most of the customised courses are conducted in schools, teachers are in their "home ground" working with familiar colleagues, and are hence more willing to engage in an open discussion because of the natural conducive environment. Unlike TRAISI courses which run on standardised timeslots, customised courses can be conducted during periods of time convenient to both the instructor and the participants. Moreover, the customised approach to PD is not bound by the formality of institute-based PD programmes such as tests, examinations, assignments and projects. Generally, there is no prescribed syllabus from MOE for customised courses. These courses are often crafted out of school-based needs analysis where the teacher's voice is heard. In some cases, mathematics educators may offer customised courses to schools in line with their research focuses. In other cases, TRAISI courses can be re-modelled to a customised version should there be a need. Majority of the customised courses for mathematics teachers offered in 2017 are those for pedagogical content knowledge (17 for Primary, 6 for Secondary and 1 for Pre-U). Only one and two courses were offered for subject matter knowledge for Secondary and Pre-U levels respectively in 2017. There were no customised courses on knowledge of assessment in 2017. Similar to what was observed for TRAISI PD courses, it appears that most customised courses centred on MPCK. The two gaps still remain.

17.3.2.3 Research-Based Professional Development

A third PD avenue for mathematics teachers come from research projects. Kaur (2012), Tan and Ang (2015), and Ng et al. (2015) are examples of this. The structure and duration of research-based PD typically follows what is required in the research methodology, not constrained by standardised time frames like TRAISI PD courses. Findings from research-based PD could be used for related TRAISI or customised courses during or after the research project. For example, the contents of a TRAISI course on mathematics modelling by Ng (2017) were reviewed as a result of findings from a research-based PD.

17.4 Some Constraints, Issues and Challenges in Mathematics Teacher Professional Development

The analysis of the TRAISI and customised mathematics PD courses from NIE above seems to show a lack of Subject Matter Knowledge and Knowledge of Assessment PD courses for mathematics teachers. However, a brief scan into the PD course list by MOE and AST reveal that other PD courses on assessments conducted by MOE assessment curriculum specialist are available for teachers. In addition, schools have been known to engage consultants to work with them on their assessment practices. Nevertheless, the case is not the same for Subject Matter courses. PD administrators face constraints and challenges when trying to address this situation. As cautioned by Lim-Teo (2009), teachers prefer to sign up for generic or MPCK courses based on their interests rather than courses on Subject Matter Knowledge which may address their areas of weakness (p. 72). Thus, even when courses on Subject Matter Knowledge are offered, the enrolment for the course may not be sufficient to warrant running it due to high overheads costs. Although such courses have been offered for secondary mathematics teachers, enrolment has declined over the years. In some instances, the courses did not run despite being offered.

The Advanced Diploma in Primary Mathematics Education programme has tried to incorporate courses on Subject Matter Knowledge, MPCK and Knowledge of Assessment. However, this programme has seen a decline in enrolment since 2012. One main reason for this is that the programme was offered on a 13-week full-time immersion basis at NIE during a teaching semester. Because participants needed time off from work to attend the programme, recruitment for the programme was done through top management in schools. There had been challenges mediating between school staff deployment needs and ensuring there were enough minimum cohort size for the programme to be activated. Although teachers have expressed their interest to become full-time participants in the programme so as to focus on their learning journey, it has been very difficult for them to apply for staff development leave to attend this programme because of certain stipulated time frames for leave in order to minimise disruptions to school functioning needs. In response to these challenges, PD administrators presented a revised dual pathway for the programme in 2017 where interested teachers can enrol in the programme on part-time or full-time pathways in a modular stackable structure. Nonetheless, it was a dismay to many that enrolment was still insufficient to meet the minimum class size.

Last but not least, Singapore teachers are offered a wide array of PD courses from NIE, MOE, AST and other organisations; not to mention those from private vendors, professional learning communities and conferences. An abundance of courses to choose from would ensure that the 100-h of PD encouraged by MOE is well-spent or beyond, albeit physically and mentally exhaustive for some teachers who might not be able to make choices as to which courses to attend. Although there have been attempts to streamline course offers from NIE, MOE and AST, more could be done to work out long-term PD plans for teachers, schools and clusters where specific

needs are met with discerning choices of PD. This may ensure a greater and perhaps a more sustainable impact from the PD courses.

17.5 A Conceptual Framework for Mathematics Teacher Professional Development for the Twenty-First Century

An analysis of the mathematics PD climate surfaces the need for a conceptual framework to describe the rationale and progression of PD courses. Such a framework presents a strategic overview of the role of PD within the continuum of teacher education and beyond, incorporating factors of influence, and the emphases of NIE TE21 and AST TGM. The framework will also assist in reflections on possible connections or deliberate overlaps between PD courses and the purposes they serve with respect to the domain map of mathematical knowledge for teaching proposed by Hill et al. (2008). Lastly, this conceptual framework can serve as a point of reference during a comprehensive review of PD courses and programmes for mathematics teachers in time to come, so as to streamline efforts in planning future research-practice nexus.

Figure 17.3 illustrates this conceptual framework. NIE TE21 recommendations underpin the conceptual framework. Mathematics PD courses and programmes can provide platforms for teachers to move on to higher degree or lead to further research by mathematics educators at NIE (as shown by the thick arrows representing pathways of further opportunities). Factors of influence have impact on PD designs and focuses which in turn, have impact on Pedagogical Content Knowledge, Subject Matter Knowledge and Knowledge of Assessment (as shown by the thinner arrows). There is mutual impact between PD and TGM goals from AST (represented by the double arrow).

17.6 Conclusion and Future Directions

This chapter began with a discussion of the key stakeholders in the PD landscape of Singapore and highlighted their vision of teacher education in the twenty-first century. There is a synergistic tripartite collaboration between NIE, MOE and AST in providing a holistic teacher PD in Singapore. The chapter further analysed the various factors of influence (i.e. international comparative studies, research) which bring about curriculum initiatives and thereby have impact on mathematics PD. The structure of NIE mathematics PD was outlined in view of TE21 and TGM. Mathematics PD courses were then classified according to aspects of teacher knowledge derived from research and some insights into the content, pedagogical and assessment focuses of current PD offerings were gleaned. Finally, a case is built for a proposed

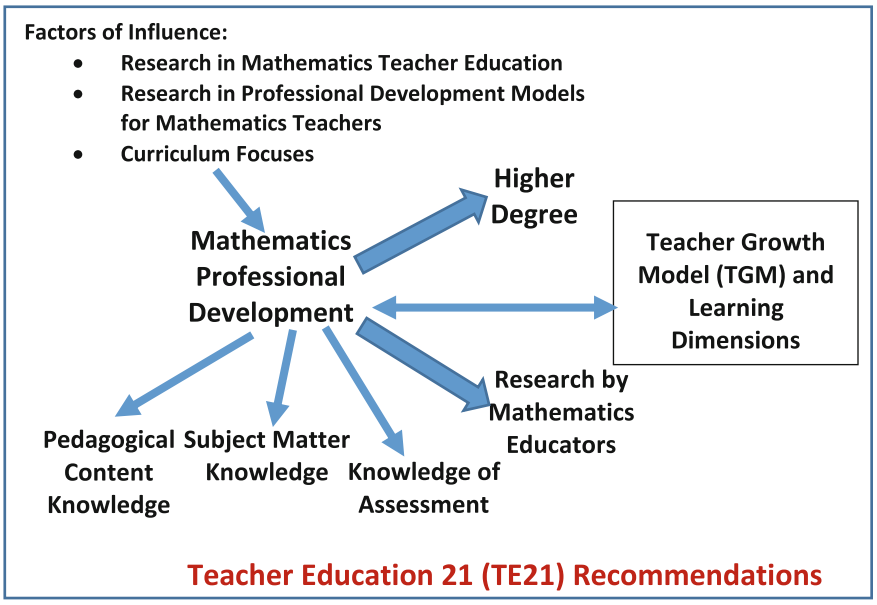

Fig. 17.3 A conceptual framework for mathematics professional development

conceptual framework to describe mathematics teacher professional development for the twenty-first century.

Some implications could be drawn from the conceptual framework and the analysis presented in this chapter. Firstly, a comprehensive review of mathematics PD could follow from this chapter. This review could examine the role of existing mathematics PD within the continuum of teacher education and propose efforts to streamline TRAISI and customised PD courses in view of pre-service teacher programmes. In line with TGM, new and revised PD courses which are connected could be planned in the form of progressive series of PD tailored for schools and clusters.

Secondly, there could be coordinated research into the impact of mathematics PD in schools as well as the sustainability and application of knowledge gleaned from PD. Though ambitious, longitudinal studies could be done to track a cohort of teachers as they advance from pre-service to experienced teachers on their teacher knowledge base expansion pertaining to specific mathematics content topics.

Thirdly, there has been a dearth of research on MPCK since those reported in the chapter. Recruitment requirements of pre-service teachers at NIE have changed much after the time frame of Lim-Teo et al.'s (2009) and Wong et al.'s (2010) research. The time is ripe for more current insights into teacher knowledge base in mathematics from robust research that would be sure to contribute to pre-service and PD course designs.

Lastly, research on effective PD models in the context of Singapore could continue, developing ways to provide impactful PD within the constraints. There could also be

more dialogue or collaborations among mathematics educators to share ideas about various PD models. Professional learning communities spearheaded by like-minded mathematics educators working together could be formed with teacher participants from various PD with connected contents, perhaps further extending the impact and sustainability of PD.

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