Chapter 16
Exemplary Practices of Mathematics Teachers

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Abstract In the first section of this chapter, we review the growing literature on "practices", focusing on the purpose of studying teacher practices in actual classrooms in view of its potential in teacher professional development. Following that, we zoom in to the Singapore situation by reviewing other studies here on mathematics teacher practices. In the second section, we describe an ongoing project and its contribution to research on exemplary practices of Singapore mathematics teachers. In the final section, we discuss the usefulness of this review in relation to the effort of building portraits of Singapore mathematics teacher practices.

Keywords Exemplary teaching · Instructional practices · Mathematics · Singapore

16.1 Introduction

Currently, one of the challenges faced by the Singapore mathematics education community, especially those involved in professional development (PD) work for teachers, is that we do not yet have a coherent portrait of exemplary practices from which to take reference when considering areas of mathematics instruction that can be improved. This can result in teachers having the impression that a myriad of disconnected pedagogical innovations—often introduced simultaneously—are running parallel to each other. As an example, one may advocate improvement in questioning

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T. L. Toh et al. (eds.), *Mathematics Education in Singapore*,
Mathematics Education – An Asian Perspective,
https://doi.org/10.1007/978-981-13-3573-0_16

techniques, another in alternative assessment modes, among others. This can result in the dilution of the effects of PD and ultimately to PD fatigue. In this chapter, we review research on exemplary instructional practices carried out by Singapore mathematics teachers. We begin by drawing from the international literature to clarify the term “practices”.

16.2 Exemplary Practices

“Practices” within the context of education has gained interest as seen from the recent literature (e.g. Lampert 2010). They can be thought of as a set of easily recognizable units of work that mathematics teachers commonly carry out in the classroom. By “practices”, we have in mind the following characteristics—drawn from the international literature (Ball et al. 2009; Hatch and Grossman 2009) and our experiences with mathematics teacher professional development, especially school-based designs of instructional innovations: (1) they are professional units of work that teachers do on a regular basis in school. Seen in this way, “instructional practices” are analogous to “medical practices” or “legal practices”—the work practitioners do as closely identifiable to the image of their respective professions; (2) they are units of work that are sufficiently isolable so as to allow for analysis, rehearsal and honing for improvement. In this sense, “instructional practices” carry the meaning of practices similar to routines—such as in a sports arena (e.g. specific skill drills in football)—that through repeated trials and fine-tuning become increasingly part of the overall work of high-quality teaching.

This leads naturally to the question of the kind of practices that ought to be upheld as exemplary for analysis and learning by teachers. The community is in need of a clear articulation of the standards of exemplary practices that are worth pursuing.

Calls for reforms in mathematics teaching towards exemplary practices are often expressed using contrastive pairs to present the traditional new distinction. Kirshner (2002) observed that, in the USA, “the Learning Principle propounded in Principles and Standards for School Mathematics (NCTM 2000) rehearses the familiar distinction between facts/procedures and understanding as a central guiding principle of teaching reform” (p. 46). Boaler (2002) presented the distinction as one between “skill-oriented” and “reform-oriented” teaching approaches. Other researchers, who avoided association with prescriptive methods, sought rather to describe methods that teachers use in their classroom practices. Some of them have also used contrasting dualistic descriptions, as in “calculationally oriented mathematics teacher” versus “conceptually oriented mathematics teacher” (Thompson et al. 1994) and teaching by “procedural instruction” versus teaching by “inquiry” (Cobb et al. 1998).

There have, however, been calls to move away from this simplistic traditional new dualistic casting of the teaching enterprise. In this alternative perspective, enactment of exemplary practices is not about merely applying a single teaching approach but a variety of instructional methods suited for different contexts and purposes. Quality teaching can be a complex mix and match of different instructional forms whose choice is dependent on various factors and competing priorities. Apart from this eclectic stance in considering exemplary practice, we advocate that a pragmatic dimension is added into the dialectic. In other words, instead of thinking about exemplary practices along universal categories, we should ask the question about exemplary practices for who? Would the images of exemplary practices differ between a mathematics classroom in urban low-resource USA and a mathematics classroom in "neighbourhood" Singapore schools? To deny the need to make these distinctions is to run the risk of divorcing teaching from its context. Teaching is a cultural activity (Stigler and Hiebert 1999). This provides the basis for studying exemplary practices within the cultural context of teaching in Singapore—and we should not be surprised that while there are features that would resonate globally, there would be characteristics distinctive to the Singapore context.

In the next sections of this chapter, we review two studies—one recent and one ongoing—on exemplary practices of Singapore mathematics teachers.

16.3 The Learner's Perspective Study—A Study of Competent Grade 8 Mathematics Teachers

Singapore's participation in the Learner's Perspective Study (LPS) may be marked as the beginning of research with a focus on exemplary practices of mathematics teachers in Singapore schools. The Learner's Perspective Study (LPS) is an international study helmed by David Clarke at the University of Melbourne. It started in 1999 with Australia, Germany, Japan and the USA examining the practices of Grade 8 mathematics classrooms in a more integrated and comprehensive manner than had been attempted in past international studies, in particular the TIMSS Video Studies of 1995 and 1999. The study has several distinguishing features among which are (a) documentation of a sequence of lessons rather than just single lessons, (b) the exploration of learner practices and (c) use of the complementary accounts methodology developed by Clarke (1998) for data collection of classroom practice—an activity where both teacher and students are key participants (Clarke et al. 2006).

Three Grade 8 mathematics teachers, T1, T2 and T3, recognized for their locally defined "teaching competence" and their respective classes of students participated in the study in 2005. These teachers are from a pool of teachers deemed as "experienced and competent", where experience was a measure of the number of years they have taught mathematics in secondary schools and competency was a composite measure of their students' performance at examinations and their performance in class in the eyes of their students. The teachers were nominated by their respective school leaders and the LPS research team in Singapore followed up on the nominations and interviewed the teachers. A strict requirement for participation in the study was that the teacher had to teach the way he/she did all the time; i.e. no special preparation was allowed. Details about the study are reported elsewhere (Kaur 2008, 2009; Kaur and Loh 2009). Data and findings of the study have also been reported in numerous publications (Kaur 2008, 2009, 2010, 2011, 2013, 2014; Kaur and Loh 2009; Kaur et al. 2006; Seah et al. 2006; Mok and Kaur 2006). In the following subsections, some selected data and findings on exemplary practices of three competent Grade 8 mathematics teachers, specifically their instructional patterns, nature of mathematical tasks used and purpose of homework, are presented.

16.3.1 Instructional Approaches

The video records of the 10-lesson sequence for each of the teachers in the study were the main source of the data analysed. On average, there are about six 45-minute lessons allocated to mathematics in the Singapore classrooms per week. For the first phase of the data analysis, a wide-angle lens was adopted. The researchers viewed the video records and located global features related to the patterns of instruction of the three teachers. For the second phase of the data analysis, a close-up lens was used and the grounded theory approach was adopted. An activity segment, "the major division of the lessons", served as an appropriate unit of analysis for examining the structural patterns of lessons since it allowed us "to describe the classroom activity as a whole" (Stodolsky 1988, p. 11).

For the purpose at hand, the activity segments were distinguished mainly by the instructional format that characterized them, although there were other segment properties, such as materials that differed among the various activity segments identified. Six categories of activity segments emerged through reiterative viewing of the video data. These mutually exclusive segments were found to account for most of the 30 lessons, 10 each from T1, T2 and T3. Table 16.1 shows the categories, and Table 16.2 shows the analysis of lesson structure with mathematical content of T2.

Coding of the video data revealed patterns of instructional cycles that consisted mainly of combinations of the three main categories of classroom activity: whole-

Table 16.1 Categories of activity segments

Category	Description
Whole-class demonstration [D]	Whole-class mathematics instruction that aimed to develop students' understanding of mathematical concepts and skills
Seatwork [S]	Students were assigned questions to work on either individually or in groups at their desks
Whole-class review of student work [R]	Teachers' primary focus was to review the work done by students or the task assigned to them
Miscellaneous [M]	A catch-all category during which the class was involved in managerial and administrative activities
Group quiz [Q]	Found in T2's lessons; students solved tasks in groups in a competitive manner
Test [T]	Found only in the lessons of T1 and T3

Table 16.2 Analysis of lesson structure for T2

Lesson No.	Activity segment code	Mathematical content	Instructional objective	Cycle No.
1	[D]	Worked example: $(3x + 2y)^2 - 6x - 4y$	Factorization by grouping	1
	[S]	Practice task: $2x + 4y - 3(x + 2y)^2$		
	[R]	Student wrote answers for practice task on board		
1	[D]	Worked examples: $x^2 - 9$, $y^2 - 1/16$, $9y^2 - 4z^2$	Factorization of expression in the form of difference of two squares	2
	[S]	Practice tasks: $a^2x^2 - 16y^2$, $50x^2 - 2p^2$		
	[R]	Teacher and students worked out practice tasks on board		
	[S]	Practice tasks: $18m^2 - 8n^4$ $(x - 1)^2 - (2x + 3)^2$		
	[R]	Teacher and students worked out practice tasks on board		
	[Q]	Quiz tasks $4x^2 - 25$ $121 - 36x^2$ $49x^2 - 1$ $\pi R^2 - \pi r^2$		
2	[R]	Reviewed solutions of $6p^4 - 24q^2$ $32xy^4 - 2x^2$ $16n^2 + 8ne + e^2$ $49y^2 + 42yz + 9z^2$ $9f^2 + 24fg + 16g^2$	Factorization of expressions by grouping and difference of two squares	1

class demonstration [D], seatwork [S] and whole-class review of student work [R] for the sequences of ten lessons each for T1, T2 and T3. Figure 16.1 shows the segment sequence for the ten lessons each for T1, T2 and T3. Activity segments that served different instructional objectives were separated by a dotted vertical line. In an instructional cycle, the mathematical tasks shared the same instructional objective.

To understand the instructional approaches further, it is necessary to go beyond structural patterns of the lesson sequence. The key features of the classroom talk through which the teachers realized their roles in not just the teaching of mathematics but also in engaging students to learn it are described elsewhere (see Kaur 2009).

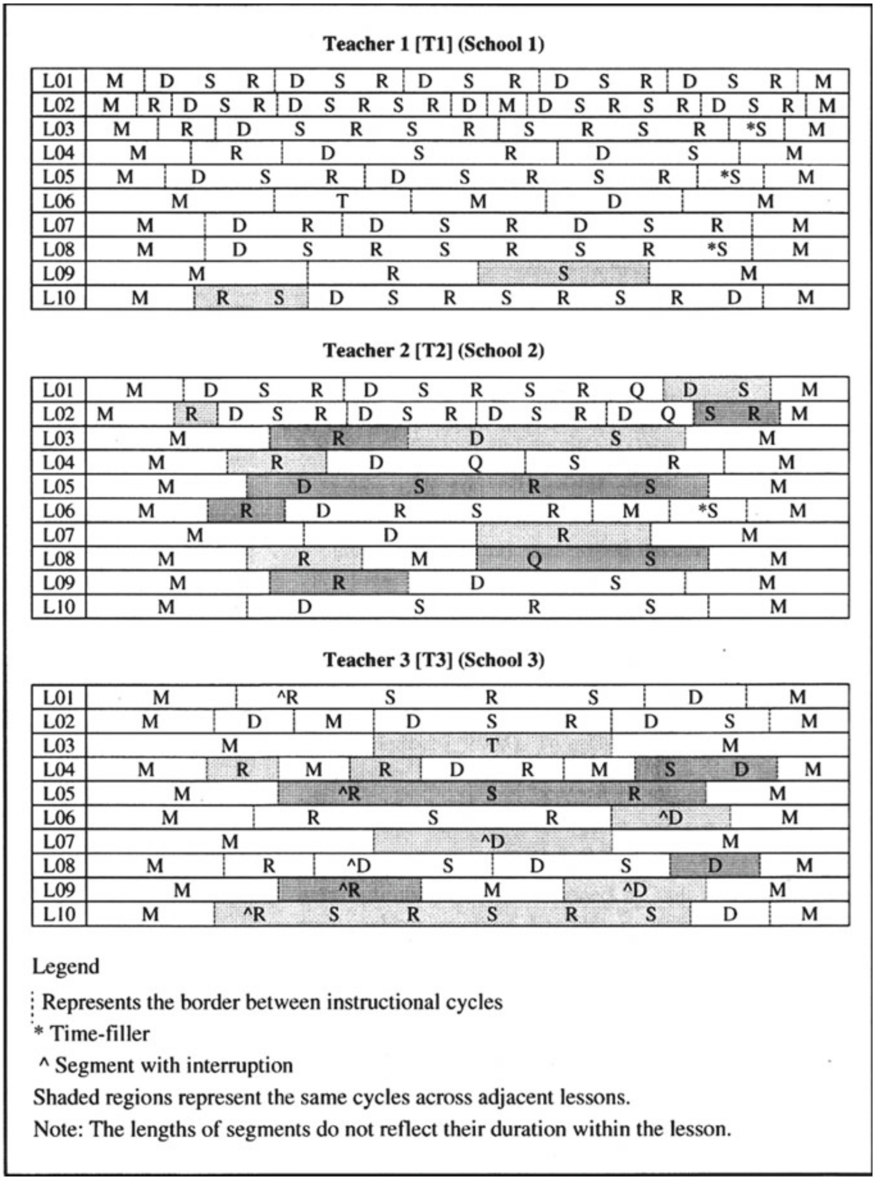

Fig. 16.1 Structural patterns of the lesson sequences of T1, T2 and T3 (Kaur 2009, p. 338)

The wide-angle lens findings show that the pattern of instruction in the Grade 8 classrooms of the three competent teachers was as follows: (1) set the stage for a topic/review past knowledge, (2) present a concept/procedure and show how to work out the solution of a problem, (3) do seatwork, and (4) correct seatwork and assign homework. Lessons were also deemed to be teacher-centred, mainly comprising teacher exposition coupled with student practice. This is often interpreted as "drill and practice" by many who have no other information about the what and the how of the lessons. On the contrary, the close-up lens findings show that lessons consisted of instructional cycles that were highly structured combinations of D, S and R. Specific instructional objectives guided each instructional cycle, with subsequent cycles building on the knowledge. Carefully selected examples that systematically varied in complexity from low to high were used during whole-class demonstrations. There was also active monitoring of student's understanding during seatwork (teachers moved from desk to desk guiding those with difficulties and selecting appropriate student work for subsequent whole-class review and discussion). Most importantly, student understanding of knowledge expounded during whole-class demonstrations was reinforced by detailed review of student work done in class or as homework, and lessons were both teacher- and student-centred.

16.3.2 Nature of Mathematical Tasks

There were three main types of tasks, learning, practice and assessment, used by the teachers. A learning task (Mok 2004) is an example the teacher uses to teach the students a new concept or skill. A review task is a task used by the teacher to review previously learnt concepts and/or skills so as to facilitate the learning of new concepts and skills. Practice tasks are tasks used during the lesson to either illuminate the concept or demonstrate the skill further and tasks the teacher asks students to work through during the lesson either in groups or individually or during out of class time. Assessment tasks are tasks used to assess the performance of the students. Based on these considerations, the tasks used by the teachers, in particular the source of the tasks and aspects of the demands the tasks make on the learners, were studied (see Kaur 2010 for details).

It was found that learning tasks used by the teachers either introduced new concepts and skills, made connections between new and old concepts or skills, or introduced students to knowledge or information that might excite them (Example T3 showing them the history of Pythagoras via the Internet) or explained some of their observations (Example T3 working through the generalized representations of Pythagorean triplets). These tasks were either taken from the textbook or sourced by teachers from their personal resources.

Practice tasks often preceded a learning task, and there was emphasis on "practice makes perfect". They were either taken from the textbook or sourced by teachers from other books. The ones from the textbook were procedural in nature. The textbooks used in the three classrooms adopted the exposition–examples–exercises

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<td>Generate Pythagorean triplets (a, b, c)
Such that $a^2 + b^2 = c^2$</td>
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<tr><td>Task T1-L01-P3</td><td>Task T3-L02-P1</td></tr>
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Fig. 16.2 Practice tasks

model (Love and Pimm 1996), and therefore, the exercises of the textbook for the relevant topic formed the bulk of the practice tasks. These tasks were mainly procedural and algorithmic in nature. Tasks from other books were word problems contextualized in some “real-world” context or like those shown in Fig. 16.2 that provided students with opportunities to engage in thinking skills such as comparing, inductive reasoning and systematic listing.

The assessment tasks were taken from past examination papers. These tasks mainly tested the reproduction of facts or procedures, manipulation of algebraic expressions, computations and application of mathematical concepts and procedures to solve simple and routine problems. Bearing in mind the limitations of pencil–paper tests, these items appeared to largely test for concepts and skills.

16.3.3 Homework

All the three teachers assigned their students’ homework for instructional purposes. An analysis of the nature and source of mathematics homework was carried out. The details are described elsewhere (see Kaur 2011). It was found that the goal of the homework was to engage students in consolidating what they were taught in class and prepare them for upcoming tests and examinations. The homework only involved paper and pencil, was compulsory and often due for submission within a week from being assigned. It was homogenous for the whole class and meant for individual work.

The homework assignments were of only two types, i.e. Type I and Type II. Type I homework was meant to review, practise and drill same-day content, while Type

II was meant to amplify, elaborate and enrich previously learnt information. For all three teachers, the main source of homework assignments was the textbook that the students used for the study of mathematics at school. Teachers also gave their students homework from past examination papers and non-school textbooks so that they would experience a wide range of questions, of varying levels of difficulty, for a particular mathematical topic. All three teachers monitored their students' homework and graded the assignments, giving them feedback. They also helped their students with their homework. When several students faced a common difficulty in their assignments, the teachers convened a focused discussion of the homework task and demonstrated the solution on the whiteboard.

The perspectives of the teachers regarding the role of homework they assigned their students were also explored. Again, the details were described elsewhere (see Kaur 2011). From the perspectives of the teachers, the role of homework they assigned their students was threefold. Firstly, "practice makes perfect" appears to be an underlying belief of all three teachers when rationalizing why they gave their students homework assignments. For all of them, it was important that their students "*hone their skills and comprehend the concepts*" of mathematical knowledge they were taught. Secondly, T2 also gave her students homework with the view that it was an extension of the lesson during which students were engaged in individual seatwork. Thirdly, T1 and T2 also gave their students homework to cultivate a sense of responsibility towards their learning. Certainly, the main underlying belief that "practice makes perfect" resonates with the finding of Macbeath and Turner (1990) about the most important functions of homework according to secondary school teachers, i.e. reinforcement, review and practice of work so that students perform well in tests and examinations. Inferring from the types of homework the teachers assigned their students, it is apparent that the homework was related to ongoing classroom work. T2, specifically, assigned her students challenging tasks as part of homework, and T3 was mindful of the fact that if he gave his students too much homework they were unable to cope with it. These findings resonate with that of Hallam's (2004) about homework being related to ongoing classroom work, be manageable, be challenging but not too difficult and that there be guidance and support to complete the work.

16.3.4 A Juxtaposition of Teachers' Practice and Students' Perception

Findings about how competent teachers teach Grade 8 mathematics reported here as well as students' perceptions about a good mathematics lesson (presented in Chap. 12) are essential for the creation of an image of exemplary instructional practices. This is exactly what the data and nature of analysis adopted in the Singapore LPS allowed the researchers to do. In so doing, the researchers questioned the stereotype of East Asian mathematics teaching (Leung 2001) and have been motivated to delve deeper into their classrooms and create a model of mathematics teaching in Singapore schools.

The next section reports on research that is presently underway to document the enactment of the school mathematics curriculum in secondary schools. This project involves the study of exemplary practices that are carried out by some 30 secondary mathematics teachers that are viewed as competent by the professional community in Singapore. As the project is ongoing, for the purpose of this chapter, we report preliminary findings based on a subset of the data corpus.

16.4 Enactment Project: Exemplary Practices in Relation to the Intended Curriculum

For this section on exemplary practices in relation to the intended curriculum, we examined 21 lessons from four teachers. The design of the study is such that two of these teachers were from School A while the other two were from school B so that any possible inter- and intra-school issues, if they exist at all, may be investigated.

As each enacted lesson was about an hour long, the enacted lessons were segmented into phases to facilitate comparison of the lessons and to examine in detail how the intended curriculum was enacted by these teachers. Since these Singapore teachers are familiar with the segmentation of lessons into the four phases—introduction, development, consolidation and closure (Lee 2009)—these phases were used and operationalized as follows:

- Introduction: Teacher setting the stage for current learning, such as checking for mastery of prerequisite knowledge (linkages to other subjects) and use of motivating stories/contexts.
- Development: Teaching for the attainment of the objective of the current lesson (alignment with other subjects).
- Consolidation: Teacher providing opportunities for students to practise on tasks related directly to the objective of the current lesson. It entails:
  - Students' independent work
  - Teacher selects and explains questions
  - Teacher asks students to explain their work
  - Teacher draws connections between previous lesson's tasks done in class or at home, and goals of the present lesson.
- Closure: Summary of lesson, setting of homework and/or assigning follow-up activity to set the stage for the next lesson.

These four phases of lessons also correspond closely to the phases of learning reflected in the syllabus document (Ministry of Education 2012), as presented in Table 16.3.

To gain further insights into how the intended curriculum was enacted by these teachers, each segment of these lessons was examined from the perspective of each

Table 16.3 Phases of lesson and phases of learning

Phases of lesson	Phases of learning
Introduction	Readiness (R)—In the readiness phase of learning, teachers prepare students so that they are ready to learn. This requires considerations of prior knowledge, motivating contexts and learning environment
Development	Engagement (E)—This is the main phase of learning where teachers use a repertoire of pedagogical approaches to engage students in learning new concepts and skills
Consolidation	Mastery (M)—This is the final phase of learning where teachers help students consolidate and extend their learning. The mastery approaches include motivated practice, reflective review and extended learning
Closure	

of the five interrelated aspects of the School Mathematics Curriculum Framework (SMCF) (Ministry of Education 2012), namely concepts, skills, processes, metacognition and attitude (see Fig. 3.1 in Chap. 3). However, it is observed that the level of enactment of the various aspects of the SMCF was very much dependent on the nature of the lessons. A skill-based lesson, for example, naturally yielded more codes under the skills aspect of the SMCF, while a concept-based lesson correspondingly yielded more codes under the concepts aspect of the SMCF. Consequently, the lessons were classified into the following five types to better reflect the nature of each lesson for further comparison:

- Type 1: Introducing new concepts
- Type 2: Revisiting learnt concepts
- Type 3: Introducing new skills
- Type 4: Revisiting learnt skills
- Type 5: Problem-solving (Barkatsas and Hunting 1996):
  - Type 5A: The application of learnt concepts and skills to solve either complex/non-routine problems (there must be a blockage to the students in general)
  - Type 5B: The application of learnt concepts and skills to solve either complex/non-routine problems (there must be a blockage to the students in general) demonstrated through implicit or explicit enactment of Polya's four-step approach.

The distribution of the types of lesson that were enacted in the 21 lessons by the 4 teachers is shown in Table 16.4.

From Table 16.4, it can be seen that these experienced mathematics teachers enacted a good spread of the different types of lesson. In particular, there is a good mix of addressing conceptual understanding and teaching of procedural skills; while slightly more than half of the occurrences are introducing and revisiting mathematics skills, a fifth of them were on introducing and revisiting learnt concepts. In particular, all these teachers were observed to weave in many short cycles of development and consolidation phases within each and between lessons, i.e. Engagement Mastery

Table 16.4 Distribution of the types of lesson

Type of lesson	Number of occurrences	Percentage
Introducing new concept	6	13.6
Revisiting learnt concept	4	9.1
Introducing new skills	11	25.0
Revisiting learnt skills	14	31.8
Problem-solving	9	20.5
Total	44	100.0

Note The total count is more than 21 as some of the lessons were coded as more than one type of lessons

cycles, to ensure that students reach a reasonable level of mastery in the relevant mathematical skills with a good grasp of the underlying conceptual understanding.

Furthermore, all these teachers were also very selective in their choice of questions to be used for teacher modelling, guided practice and independent practice. There appeared to be generally a good alignment for these questions to ensure that the students have sufficient practice to acquire the relevant mathematical skills to tackle such questions.

All in all, these teachers seemed to focus much on promoting conceptual understanding and fluency in procedural skills.

It is also observed that all these teachers tapped on the affordance of information and communications technology (ICT) to achieve their lesson objectives, though the role of ICT use might differ. There was use of YouTube videos to facilitate flipped classroom teaching, while some animations and videos were used to create motivating teaching materials. There was also use of graphing tool and commercially produced technological resources to exemplify mathematics ideas for proving or promoting of understanding of mathematical concepts. There was also an attempt to use a digital textbook to facilitate the teaching of and to make visible the problem-solving process. These seemed to reflect an impact on these teachers' enactment of the intended curriculum as a result of the four ICT master plans that have been put in place (see also Chap. 3).

From the perspective of the concepts and skills of the SMCF, these teachers seemed to be pedagogically strong in promoting conceptual understanding through the various use of technological aids and learning experiences, and procedural skills were taught alongside an understanding of the underlying principles/concepts.

In addition, there was also a reasonable good emphasis on problem-solving, as can be seen from Table 16.4 that about a fifth of the occurrences of the lesson types are on problem-solving. In other words, these teachers also provided opportunities for students to apply their learnt concepts and skills to solve either complex and/or non-routine problems.

Furthermore, these teachers also made conscious efforts to teach mathematical language explicitly and both written and verbal communication were encouraged. The teachers were also observed to promote reasoning by getting students to explain/justify their work through such mathematical communication. The thinking skills induction and deduction were employed in many of the lessons. The most common heuristics that were observed to be employed in these lessons were drawing a diagram, guess and check, and working backwards.

Thus, from the perspective of the process aspect of the SMCF, these teachers provided rich opportunities for the enactment of these mathematical processes.

In terms of the metacognitive aspect of the SMCF, all the teachers were observed to encourage students not to think impulsively but instead pause to monitor their thoughts. There were also some attempts to make visible the problem-solving process through digital means, as noted earlier. In addition, all the teachers were also observed to encourage offline metacognition, i.e. reflection (see Chap. 10), among the students.

From the attitude aspect of the SMCF, these teachers have certainly established a rather impressive rapport with their students; students were generally observed to be interested and enjoyed the lessons. The teachers were also seen to be consciously structuring their teaching by breaking the learning and doing into smaller chunks to boost students' confidence.

Thus, these experienced mathematics teachers' classroom practices seemed to be very well informed and guided by the intended curriculum.

16.5 Enactment Project: Exemplary Case Studies on Teachers' Use of Instructional Materials

The first is a case of a teacher's use of instructional material in "making things explicit" to his students. [The more detailed version of this case study can be found in Leong et al. (in press). We provide a summary here.] In searching the international literature, it is found that a popular conception of "explicit" is found in "explicit instruction" and it is seen as closely associated with other methods of instruction such as "teacher-directed instruction" (Doabler et al. 2015) and "direct instruction" (Gersten and Carnine 1984). The former highlights the primary role of the teacher in structuring lesson sequences; the latter focusses on the direct manner in which procedural steps "pass from" teacher to students. But in the case of "making things explicit" that we studied in the project, we began with a different starting point: we were not limiting "explicit" to these forms of instruction; but we started with the teacher's conception of explicitness; in particular, we examined his use of instructional materials as an instrument for making things explicit.

Our findings revealed that the teacher's attempt at using instructional materials for making things explicit can be summarized along these lines: explicit–from base; explicit–within materials and explicit–to instruction. These three conceptions correspond roughly to the three arrows shown in Fig. 16.3.

Explicit–from base. The teacher referred extensively from the school-subscribed textbook as his base curricular material. However, the transference from textbook to the instructional materials he used was not merely one of the direct lifting nor minor

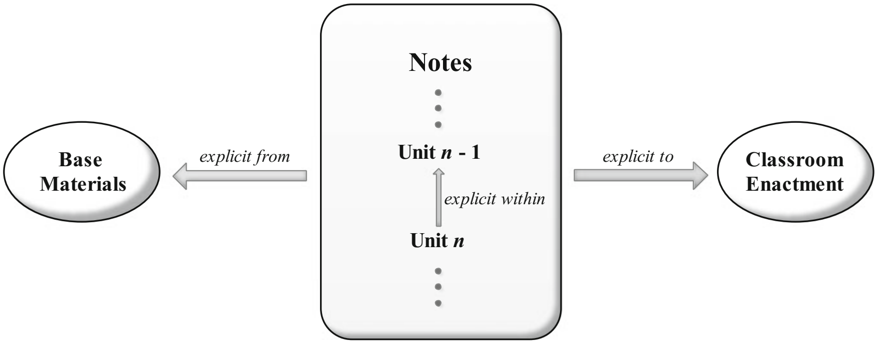

Fig. 16.3 Illustration of how the notes were used for explicit–from, explicit–within and explicit–to

adaptations. He saw the move between these material domains as primarily one of "making explicit". This explicitation can be further categorized into: filling gaps in the textbook content, making links between representations given in the textbook and highlighting critical ideas—without which students may inadvertently develop misconceptions—not emphasized in the textbook.

Explicit–within materials. The teacher used each unit within the notes he prepared to focus on one main concept. As is usually the case in mathematics, the focused concept is tightly linked to other related ideas. Instead of highlighting all the ideas in one-go within a unit, he used the strategy of foregrounding a particular idea while holding the other related ones as "supporting cast" at the background. This inter-unit implicit-to-explicit strategy reveals a level of sophistication in the crafting of instructional materials that we had not previously studied. The common anecdotal portrayal of Singapore mathematics teachers' use of materials is one of the numerous similar routine exercise items for students to repetitively practise the same skill to gain fluency. In the case of this teacher's notes, it was not pure repetitive practice that was in play; rather, students were given the opportunity to revisit similar tasks and representations but with added richness of perspective each time. In other words, each revisit allowed students to reinforce previously introduced ideas and to connect to new ones.

Explicit–to instruction. The teachers recognized the limitations to the extent in which the notes by itself can help make things explicit to the students. The explicitation strategy went beyond the contents contained in the notes. In particular, he used the notes as a springboard to connect to further examples and explanations he would provide during in-class instruction. He drew students' attention to questions spelt out in the notes, created opportunities for students to formulate initial thoughts and used these preparatory moves to link to the explicit content he subsequently covered in class.

From the point of view of students' learning experience, the chronology of first prompting their thoughts followed by the teacher's explicitation inverts the more traditional order of teacher–teach proceeded by student–practice. While the latter

Example 3	Solve the inequality $2x^2 - 7x + 6 < 0$.
Example 4	Solve the following inequality using a graphical approach:
	(a) $x^2 - 4x + 3 > 0$
	(b) $3x^2 - 4x - 7 < 0$
	(c) $4 - x^2 < 0$

Fig. 16.4 Examples in the instructional materials

tends to foster a passive adherence to teacher-demonstrated steps, the former allows students to carry out their first-cut thought experiments before the teacher points out the salient ideas or demonstrate some canonical methods. This sequence provides students the opportunity to contrast their more naive preliminary ideas against the explicit treatment provided by the teacher and thus learn to better appreciate the mathematical explicitation.

The second case features the principles a teacher used to sequence examples in his notes in such a way as to support mathematical reasoning. This is a significant study both in terms of the place that "reasoning" holds in the Singapore mathematics curriculum (see Fig. 3.1 in Chap. 3) and also in the ongoing interest in "reasoning" within the international mathematics education community (e.g. Jeanotte and Kieran 2017; Lampert 1990).

It is less surprising to find that the teachers sequenced the examples to "advance a method" (the teacher's own words) that he had demonstrated to the students. Figure 16.4 provides an illustration of a sequence of examples he gave within the topic of solving quadratic inequalities.

The method that was demonstrated to the students—for Example 3—was a series of steps that involved quadratic factorization followed by the use of graphical representation to show that the solution to the quadratic inequality corresponded to the x-values of the portion of the graph that is below the x-axis. This method was "advanced" as the subsequent examples retained the main thrust of the method but with refinements to deal with tweaks—such as the switch to ">" in Example 4(a), to non-strict inequality in Example 4(b) and to an inequality with zero coefficient for the x-term. The advancing of method principle is further reinforced as he proceeded with subsequent examples (see Fig. 16.5) as he modified the method to handle quadratic expressions that are not factorizable over the rationals.

Through the post-interview and classroom videos, it became also clear that "advance the method" was not the teacher's only goal in his use of this sequence of examples. The teacher expanded the examples systematically to a whole suite of what he called non-standard cases in Examples 5 and 6.

Analysis of the teacher's progression from Example 6(a) to 6(b) in his lessons showed that while he demonstrated how the same method applied, he also advocated an alternative method as he advocated that students "think flexibly". In other words, he wanted students not merely to follow strictly to the method he demonstrated but to constantly exercise reasoning behind the method and the procedural steps.

Example 5	Solve the inequality $2x^2 + x - 4 > 0$. Solution: We observe (or check) that the expression $2x^2 + x - 4$ is not easily factorized. In this case, we have to find the x-intercepts using the quadratic formula. We present our working in this way:
Example 6	Solve the following inequalities, giving exact answers: (a) $x^2 + 4x - 7 > 0$ (b) $2x^2 < 5$ (c) $x^2 + 2x + 11 > 0$ (d) $3x^2 - 30x + 75 < 0$

Fig. 16.5 Examples 5 and 6 in the instructional materials

This goal to encourage students' habitual reasoning is more obvious in Example 6(c). In this case, the solution of the associated equations is "no real roots". The students were unable to simply apply the method used in previous examples. They were thus "forced" to reason their way out of the quandary. That reasoning was inbuilt into the design of the examples was attested by the teacher during the interview: "today the focus is on the non-standard examples [Examples 5 and 6] … . So here is to promote reasoning in general, because here the basic idea is … to get the sketch of the graph, [then] use the graph to deduce a solution … . This way we make sure that they know the thinking behind the particular graphical method, and we put in all these parts to make sure that they are actually applying the reasoning behind the graphical method" (emphases added). The teacher was not merely using the sequence of examples to advance a method; he also wanted students to attend to the mathematical reasoning behind the (advancement of the) method. In other words, the advancement of the method "pulled along" the underlying mathematical reasoning.

The two cases described enabled us to uncover complex design considerations behind what may look to a casual observer as "simply drill-and-practice" instructional materials. In the enactment project, we are just beginning to examine these exemplary practices that are helpful in developing portraits of high-quality teaching in Singapore mathematics classrooms.

16.6 Discussion

This chapter reports on two significant mathematics education research projects that have been conducted in the Singapore mathematics classroom in identifying pedagogical approaches and exemplary practices exhibited by mathematics teachers in their enactment of the school mathematics curriculum. The researchers have moved away from the traditionally prescriptive approach in identifying classroom practices or using contrasting dualistic lens in the study of the mathematics classrooms. The researchers recognize that classroom teaching, being culture- and context-bound, is a much more complex process than it has traditionally been perceived by researchers. In the study of the Singapore mathematics classrooms reported in this chapter, the researchers have also questioned the stereotyped "East Asian pedagogy" (Leung 2001) in favour of delving deeper into the authentic mathematics classroom.

The message of the various studies reported in this chapter can be summarized in the following key points. To identify teachers' instructional approaches and exemplary practices, it is essential to

- Transcend the superficial patterns of the lesson sequence, but to take into consideration the totality of teacher instruction and role in engaging students in the entire process of learning during the class;
- Take into account the local factors in the educational landscape. In particular, it is crucial to study the classroom lessons or lesson segments using the lens of the underlying reasons and principles of the intended school curriculum, which is one of the key factors that drives the way lessons are conducted in the classrooms (in the researchers' experience with the Enactment Project described above, the lessons that were examined using the mathematical problem-solving framework in the Singapore mathematics curriculum document); and
- examine the instructional materials that are used by the teachers. As described in the preceding sections, teachers did not use the existing teaching resource wholesale in delivering a lesson. The teachers made many careful considerations in adapting or developing the instructional resource for lesson delivery. This aspect, though not directly visible in classroom observations, contributes to an extremely important component in the study and identification of teachers' exemplary practices.

At the time that this chapter is written, the enactment project is still work in progress. After identifying the exemplary practices of this relatively small sample of experienced mathematics teachers, the next step for the researchers is to identify how widespread these exemplary practices are among the mathematics teachers in the Singapore education system in general. This will allow the researchers to have a fuller picture of the overall mathematics classrooms in Singapore. A study of how these exemplary practices among mathematics teachers impact on students' learning (cognitive, metacognitive and affective dimensions) of the subject is another area which will likely attract international attention on Singapore mathematics.

The researchers of the enactment project used a coding scheme that attempted to explain in great depth the intent of the teacher. Besides the Singapore mathematics curriculum document, Schoenfeld's Teaching for Robust Understanding (TRU) framework is one of the theoretical frameworks that was used at least in the initial phase in designing the coding scheme (Kaur et al. 2018). As teaching has been recognized to be cultural and it is very much context-dependent, perhaps what we need next is to develop a local Singapore teaching framework. Although we would not go so far as to suggest to develop a prescriptive list of "exemplary" practices, such a local teaching framework would be useful for researchers in understanding the specific pedagogical approaches of the teacher which could be unique to Singapore in recognition of its unique social–cultural factors.

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