Chapter 13
Low Attainers and Learning of Mathematics

Tin Lam Toh and Berinderjeet Kaur

Abstract This chapter describes the main studies which have been carried out in the Singapore mathematics classrooms to identify and address the learning needs of low attainers in mathematics at the primary and secondary levels. This chapter begins with describing two research projects on low attainers: the first is an exploratory study on low attainers at the primary level and the second a survey on teachers' perception of low attainers from the Normal (Technical) stream at the secondary level. These two studies identified the characteristics of low attainers and their content knowledge, teachers' perception about their motivation and competency in mathematics, and provide a preliminary knowledge of how teachers have attempted to facilitate them to learn mathematics better. The chapter further presents three intervention research projects that were conducted by researchers from the Singapore National Institute of Education (NIE) in collaboration with school teachers to facilitate the low attainers in learning mathematics. The first project was an action research proposed by a school to facilitate the mathematics learning of students from the Normal (Academic) stream through the use of cooperative learning strategies. The researchers proposed a framework of cooperative learning that was trialled in their school setting. The second project was another research project initiated by researchers from NIE on using comics and storytelling in teaching mathematics in the Normal (Technical) stream. The study shows that there was an overall positive impact of this approach on students' motivation in learning mathematics and their performance in mathematics achievement test. In the third project, another team of researchers from NIE attempting to use the Concrete–Pictorial–Abstract heuristic to help Normal (Academic) students learn mathematics by assisting them to make abstract algebra meaningful and manageable. This chapter concludes with describing the projected initiated by the Ministry of Education at the national level on building teacher capacity to facilitate learning of mathematics among the low attainers in mathematics.

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13.1 Introduction

Despite the fact that Singapore mathematics education has received much attention from educators throughout the world, there was a concern among the policy makers in the Singapore Ministry of Education (MOE) and the school leaders about the significantly large proportion of Singapore students who are not performing well in mathematics. This concern is not surprising as mathematics is a core subject in the Singapore primary and secondary mathematics curriculum. As Kaur et al. (2012) aptly put it, “failure in mathematics due to factors that may be controlled would be unjust for the [students]” (p. 2).

This chapter reports the various efforts taken by the MOE, mathematics educators and researchers from the Singapore National Institute of Education (NIE) in addressing the learning needs of the low-performing students (or “low attainers”) in mathematics. These efforts are crucial and in fact should begin early in students’ career as data (such as the TIMSS 2003 data for the USA) have shown that students who fail at a lower grade will very likely fail at subsequent grades if no intervention has been taken.

13.2 Who Are the Low Attainers?

We first begin with defining and explicating the choice of the term “low attainer” that is used throughout this paper. Early researchers used the term “low attainers” to describe those students who fall into the bottom 20% of mathematics attainment in their age group in national examinations (e.g. Denvir et al. 1982). Haylock (1991) used the term “low attainers” to define students who attain very much less in mathematics when compared to their contemporaries. In this chapter, we choose the words “low attainer” over several other terms like “slow learners”, “at-risk students”, “special needs students” and so on, to describe this group of students at the various levels. As in Kaur et al. (2012), we adopt the use of the term “low attainer” is a purely descriptive term and does not make any judgement about the reasons for the students’ low attainment in mathematics.

Teaching low attainers has always been a challenging task faced by school teachers. Low attainers generally show little interest in the various academic subjects. They are usually not mentally focused in the classrooms, tend to be restless in classes and have relatively short attention span (Lui et al. 2009).

What are the reasons that low attainers behave in this manner in the classrooms? Many educators have argued that low attainers are what they are because there is a misfit between their needs and the existing educational programmes; they generally need educational programmes which are more skill-based than theory-based. Most teachers, based on their classroom experience and anecdotes, generally associate low attainers with kinaesthetic learners. In Dunne et al. (2007), a senior member of a high school commented that:

> …A lot of the low-ability pupils are very kinaesthetic and they just can't sit still and that's why they are low-ability pupils. They just don't work like this (p. 62)

Educators and researchers also recognized that many of them are kinaesthetic learners rather than visual or audio learners (Amir and Subramaniam 2007; Rayneri et al. 2003). Empirical studies such as that by Shahrill et al. (2013) concluded that the learning style of students of lower mathematical ability tends to be kinaesthetic. However, educational programmes worldwide have evolved into one that is more theory-based than skill-based, and most of the instructional programmes tend to focus on students who are visual and audio learners (Glass 2003). This has put the lower-attaining students at a disadvantage. It is thus not surprising that low attainers generally do not show interest in the academic subjects or are perceived as lack of competence in these subjects.

In the situation of Singapore, a study conducted by a group of researchers from the Singapore NIE on low attainers in primary four mathematics (Chang et al. 2010; Kaur and Ghani 2012) reported that there was a mismatch between how pupils think they best learn mathematics and how teachers teach them mathematics. Most students in the study preferred to be engaged in group activity during mathematics lessons and believed that the use of computers would help them learn mathematics better. However, the instructional method adopted by the teachers was mainly teacher demonstration, seatwork and review of pupil work. The study recommended that teachers provide pupils with opportunities to talk and clarify their thinking as well as motivate pupils to ask questions and freely share with others their thoughts. This could offer a further clue on the lack of good behaviour of low attainers in the classrooms.

13.2.1 Characteristics of Low Attainers of Mathematics

Education literature abounds with elaborate descriptions of the characteristics of low attainers. Much research has been conducted on identifying the characteristics of low attainers in mathematics. Generally, these characteristics can be broadly classified under three broad categories:

1. *Cognitive and metacognitive factors*: The low attainers generally lack metacognitive strategies (Cardlle-Elawar 1995; Kruteskii, 1976; Mercer and Mercer 2005; Verschaffel and De Corte 1995). Most of them suffer "cognitive overload" and tend to have short-lived memory for mathematical procedures (Keijzer and Terwei 2004; Mercer and Mercer 2005). Also, they lack the ability to apply the appropriate heuristics for different situations (Nelissen and Tomic 1998;

Verschaffel and De Corte 1995) or to apply domain-specific knowledge flexibly (Kraemer 2000). Not only that, they lack appropriate background/prerequisite knowledge (Mercer and Mercer 2005). They also have difficulties in using more sophisticated representations such that schemata and models or in considering numbers as formal objects (Karsenty et al. 2007; Kraemer and Janssen 2000).

2. *Affective factors*: Low attainers in mathematics usually show negative attitude towards learning mathematics. Some signs of negative attitude include feelings of fear, stress, anxiety and resentment towards mathematics (Haylock 1991; Karsenty 2010; Mercer and Mercer 2005; Lehr and Harris 1988). In addition, they have low motivation and academic self-concept (Fong et al. 2012; Karsenty 2010; Mercer and Mercer 2005).
3. *Social factor*: Many of the low attainers have social problems and lack social skills. Many of these problems of the low attainers can be attributed to their family background or immediate environment (Haylock 1991; Lehr and Harris 1988). However, this crosses into the boundary of social work and psychology and is beyond the scope of this chapter. We shall not discuss this here.

This chapter next discusses how low attainers are recognized early in the students' career and how they are integrated into the school system. The discussion will be based on the perspective from the objective of the Singapore education system. We shall next discuss the various efforts that were undertaken by the various mathematics educators and researchers in identifying and addressing the learning needs of the low attainers in the Singapore school system, and a nationwide effort to facilitate the learning of the low attainers by the Singapore Ministry of Education.

13.3 The New Education System—An Ability-Driven System

Singapore, with its unique historical and geopolitical factors, has seen several phases of development since its independence in 1965. Without any natural resources of her own since her independence, the nation, under the visionary leadership of the first Prime Minister Mr Lee Kuan Yew and his likeminded cabinet at that time, placed great emphasis on developing her human resource. Thus, the education of the population was the main priority at that point of consideration. Mr. Lee Kuan Yew stressed that the "simple objective" of education is to "educate a child to bring out his greatest potential so that he will grow into a good man and a useful citizen" (Lee 1979).

In the phase of education development in 1979, the Singapore Ministry of Education's Study Team led by the then Minister of Education Dr. Goh Keng Swee (Goh and The Study Team 1979) identified that one of the key weaknesses of the education system at that time was the high education wastage. A high proportion of the population did not receive the minimum number of years of secondary education. This led to the low literacy level in the country (Goh and The Study Team 1979). With the New Education System (NES) introduced in Singapore in 1979, an education that was ability-driven was implemented. Instead of having a common core curriculum for all students, in which students' various capabilities was not taken into consideration, streaming was introduced. This allowed students of various capabilities to progress at a pace that was suitable for them. All students had the opportunity to complete the minimum number of years of schooling in order to acquire basic literacy and numeracy.

At this phase of the NES, ability-based streaming of students was carried out at primary three into Foundation and Normal programme and at the secondary one level into Express/Special and Normal. Subsequently, several fine-tuning of streaming was done. In 1991, the streaming at primary three level was adjusted to primary four. In 1994, the Normal Stream at the secondary level was further divided into Normal (Academic) and Normal (Technical), paying particular attention to students in the Normal Stream who are more inclined to technical education.

As a consequence of this process of streaming, students from the Foundation programme at the primary level and the Normal (Technical) stream at the secondary level tended to be labelled as "low attainers". Readers are cautioned that labelling students from these streams as "low attainers" is incorrect (although it may be true that a significant proportion of students from the Foundation and the Normal (Technical) streams are less academically inclined). For example, some students in the Normal (Technical) stream could be proficient in mathematics but less inclined to most other academic subjects. In this chapter, we recognize that interventions that were carried out to assist low attainers in mathematics were usually divided according to the streams they were assigned.

Readers should note that streaming in the Singapore school system is not static. Recognizing that the stage of student development varies with individual, channel is available for students to be transferred from one stream (especially those who are classified as "low attainers") to another if they prove to be capable to manage the challenges required in the latter stream.

13.3.1 Identification and Support for Low-Attaining Group of Students

Even before the level of streaming at primary four, efforts are made by MOE to identify low-attaining groups of students, so that additional support can be provided to facilitate their learning. In primary one and primary two schools, identify low-attaining students with the help of school-based School Readiness Test (SRT), school-based entry/diagnostic tests and performance of students in semestral assessment. Schools adopt varying strategies to support these students in their learning. Two strategies adopted by schools are banding based on either the students' overall performance or their performance in the core subjects (English and Mathematics) and placement in the Learning Support Programme where teachers work either with

Table 13.1 TIMSS 2007 Grade 4 mathematics mean scores

Country	Rank	TIMSS 2007 Grade 4 mathematics mean scores of pupils up to the *n*th percentiles				
	n	5th	10th	15th	20th	25th
Hong Kong	1	462	488	503	514	524
Singapore	2	408	440	461	477	490
Chinese Taipei	3	427	453	469	481	490
Japan	4	404	432	449	463	474

individual students or a small group of them for the subject they are identified as low attaining.

In the next two sections, we shall report on two studies targeted to obtain a better understanding of low attainers in mathematics: the first is an exploratory study at the primary level, and the second is a survey on the teachers teaching Normal (Technical) mathematics at the secondary level.

13.4 Exploratory Study One: Low Attainers in Primary Mathematics (LAPM)

Although Singapore students in general topped the various international comparative studies in mathematics (TIMSS and PISA), the findings from 2007 TIMSS on the performance of low attainers, as shown in Table 13.1, indicate that the lowest performing students were not performing as well as their counterparts in several other Asian countries.

Table 13.1 shows the mean score in the mathematics section of Grade 4 students from four Asian countries (Hong Kong, Singapore, Chinese Taipei and Japan) in 2007 TIMSS according to the various *n*th percentile levels ($n = 5, 10, 15, 20, 25$). The table shows that the lowest achieving students at the *n*th percentile (5, 10, 15, 20, 25) are not performing as well as several other countries (Hong Kong, Chinese Taipei and Japan), which are on par with Singapore in the overall student performance in 2007 TIMSS.

This concern among the policy makers in the Ministry of Education and the school leaders about the high proportion of low attainers in mathematics at the primary schools led to the conceptualization of a research project involving an exploratory study named "Low Attainers in Primary Mathematics" (LAPM). The project, which was initiated by researchers in the Singapore NIE and supported by the MOE, was conducted on low attainers in mathematics at the primary level.

In particular, LAPM aims to identify (1) how schools and teachers motivate and inspire low attainers in primary mathematics to learn the subject; and (2) how schools and teachers address the diverse learning needs of low attainers in primary mathematics. The project also serves to determine the characteristics of low attainers in mathematics in their relation to (3) their school (e.g. behaviour during class, absenteeism, interactions with peers); (4) their home (e.g. home support, resources, environment) and (5) their experiences of learning mathematics. We shall next present a brief summary of the findings of the LAPM under five main subcategories:

13.4.1 How Teachers Motivate Low Attainers in Mathematics at the Primary Level

The result of LAPM shows that primary school mathematics teachers used a variety of strategies to motivate and inspire low-attaining students in mathematics. The teachers modified the existing teaching packages by chunking up the resources in each teaching unit into small chunks, and to teach the students using their modified package in smaller groups. In addition, teachers also used various forms of activity-based learnings, such as the use of pictures, games, songs and manipulatives, in their mathematics lessons. Technology was also harnessed during lessons to enhance the students' learning. Regarding the general educational psychological principle, teachers also used games and quizzes, words of encouragement and extrinsic rewards to motivate and inspire the low attainers.

13.4.2 How Schools and Teachers Address the Learning Needs of Low Attainers

According to the study, the teachers strongly believed that large class sizes, the nature of the mathematics curriculum, time constraint and the mode of assessment are the main hurdles in addressing the learning needs of low attainers. All the schools addressed the above hurdles by engaging additional staff (the provision of either allied educators or adjunct teachers) and provided supplementary and remedial lessons for their students.

The teachers also gave three main recommendations in assisting them to further address the needs of low attainers:

1. They should be provided with teacher professional development programme which specifically address the learning needs of low attainers, in addition to the general student population.
2. Enrichment programme should be provided to the low attainers.
3. More curriculum time should be allocated for core subjects (in particular, mathematics).

Table 13.2 Performance of 390 low-attaining students in mathematics content tests

Test	Max. score possible	Mean	Standard deviation
Whole numbers (concept)	30	17.1	4.49
Whole numbers (operations)	29	22.1	3.98
Whole numbers (word problems)	9	3.52	2.45
Fractions	29	13.71	6.98
Measurement	21	10.14	3.69
Measurement (word problems)	8	3.20	2.05
Geometry	10	6.34	2.16
Data analysis	10	6.37	2.55

13.4.3 Low Attainers' Mastery of Primary Three Mathematics Content

One key research area of LAPM involves examining the performance of 390 primary four low attainers in a test with various strands in the primary school mathematics curriculum. It shows that their poor performance in mathematics was also interfered by their language ability and the psychological hurdle of perpetual failure in mathematics. Table 13.2 shows the mean and standard deviation of the students in these tests.

A full description of the pupils' performance in the individual items is described in greater detail by Koay et al. (2012), which will not be elaborated here.

It is clear from Table 13.2 that the students performed best in items related to concept and operations involving whole numbers and worst in word problems on whole numbers and measurement. Koay et al. (2012) attributed this to their poor reading ability. Sufficiently, competent language ability is necessary in understanding and solving word problems. They suggested the strategy of using visual imagery to solve the word problems by engaging by the bar model method and classroom instruction to address these deficits among the low attainers.

It was encouraging that Koay et al. (2012) asserted that most low attainers had the capability to learn. However, perpetual failure in mathematics created a major obstacle along their journey of mathematics education. It was recommended that classroom teachers refine their instructional approach and set achievable learning goals and suitable assessment tasks for this group of pupils.

13.4.4 *Characteristics of Low Attainers in Relation to the Schools*

With regard to the general characteristics of low attainers, it was found from LAPM that the majority of the low attainers (70%) were seldom or never attentive in class while the teachers were teaching. Few students (15%) sought help from the teachers when in doubt. As they were not attentive and did not seek help from their teachers, less than half of the students (about 39.6%) were able to submit classwork on time.

In relation to their learning of mathematics, the majority of the pupils appreciated the importance (about 88.9%) and usefulness (65%) of mathematics to their daily life. Although the vast majority of the pupils (about 89.6%) believed that they could perform better in mathematics, approximately half of them (54%) believed that they were not good in mathematics. The learning of mathematics aroused a diverse range of emotions from feeling of happiness, dislike, anger, confusion, stress and anxiety to boredom among this group of low-attaining pupils.

Although approximately half of the pupils had parents with secondary (45.3%) and tertiary (8%) education, about one-third of the pupils (28.8%) reported that their parents did not check their mathematics work at home.

13.5 Exploratory Study Two—Teaching and Learning Mathematics at the Normal (Technical) Stream in the Secondary Level

Students from the Normal (Technical) stream at the secondary level are believed to be relatively less inclined towards mathematics. In an effort to understand the perception of teachers teaching Normal (Technical) mathematics about their students, and the strategies they had adopted to help their students better learn mathematics, an exploratory study was carried out by Toh and Lui in 2013. This took place during a teacher professional development workshop that was concurrent with the book launch of one new secondary mathematics textbook series for the Normal (Technical) students for use from 2014 onwards. The findings of this exploratory study were reported in Toh and Lui (2014).

During this survey, the participating teachers were asked to discuss if it was a challenge to teach Normal (Technical) mathematics and to answer three key questions about their

(1) Perception of the reasons of their students' lack of interest in mathematics;
(2) Perception of the reasons for their students' learning difficulties in mathematics;
(3) Strategies and resources that they had used to help their students learn mathematics.

All the teachers' answers were affirmative to the fact that it was a challenge to teach Normal (Technical) students mathematics. The breakdown of the answers to the above three key questions is presented below.

13.5.1 Teachers' Perception of the Reasons of Their Students' Lack of Interest in Mathematics

The reasons provided by the teachers for this question in the survey can be broadly classified under two main categories: (1) cognitive and (2) affective factors.

(1) Cognitive factors

The main reason that students lacked interest in mathematics was that mathematics was difficult for many of them, and not relevant to daily life.

(2) Affective/psychological factors

Mathematics was boring for these students. Not only that, many of these students had never experienced success in mathematics before, as they rarely passed mathematics when they were in primary school.

13.5.2 Teachers' Perception of the Reasons of Their Students' Difficulty in Mathematics

The reasons provided by the teachers on their perception of the reasons of their students' difficulty in mathematics can be classified under two main categories: (1) cognitive and (2) affective factors.

(1) Cognitive factors

Most of the teachers responded that their students had poor mathematics and language foundation, which led them to being unable to understand the mathematics problems. Not only that, the students were easily confused by mathematics problems which involve multiple steps. In addition, the students' style of learning is through numerous and a wide variety of worked examples in order to acquire various mathematical concepts. Several teachers also reflected that the low attainers experienced much difficulty memorizing the various mathematical formulae. It was also reported in the survey that many of these students lacked the perseverance in solving mathematics problems.

(2) Affective/psychological factors

Two key factors cited were that the students were generally not interested in mathematics, and they generally had short attention span. The students' negative attitude towards the subject, manifested by a lack of interest, and short attention span in the classroom could easily lead to disruptive behaviour during the lesson. This further led them to dislike mathematics even more.

13.5.3 Teachers' Use of Strategies and Resources to Help Their Students Learn Mathematics

The numerous responses provided by the teachers for this item in the survey can be broadly classified under six main categories:

(1) Use of manipulatives (including standard manipulative proposed by MOE and others);
(2) Information and communications technology (online learning platforms, content websites and mathematical tools);
(3) Media (newspaper cuttings, existing video clips from YouTube and other resources);
(4) Modification of standard pedagogical practices (managing the pace of lesson, peer coaching, individual explanation of mathematical concepts, use of appropriate language, relate mathematics to everyday life);
(5) Psychology (building up students' confidence in mathematics by providing them with small opportunities of success in the process of learning mathematics);
(6) Alternative pedagogy (use of storytelling, cartoons with humours, games and quizzes).

This preliminary survey (Toh and Lui 2014) clearly indicates that teachers were already engaging their students using a variety of strategies, both utilizing the teaching material available and adopting and adapting existing material outside the traditional teaching resource. It was encouraging to the researchers that many of these strategies, other than the standard manipulatives and teaching approaches proposed by the MOE, were the creative invention or adaptation made by the teachers in addressing the learning needs of their students.

Teachers were clearly aware of the learning difficulties and the lack of interest in mathematics among the low attainers in mathematics in the Normal (Technical) stream. The teachers had devised their own intervention of addressing their students' learning difficulties and lack of interest in the subject. Their intervention strategies can also be broadly classified broadly under two broad categories: (1) that of addressing students' learning difficulties in the subject and (2) that of addressing students' motivation and self-esteem.

In addition to the exploratory studies in understanding the teaching and learning of mathematics among the low attainers, a variety of intervention studies were also conducted by researchers and mathematics educators to address the teaching needs of teachers and the learning needs of students. These interventions include pedagogical approaches that attempt to build up students' interest in the subject and also approaches that attempt to help students learn mathematics better by unpacking the abstractness of mathematics. The next sections shall introduce several of these intervention programmes.

13.6 Rethinking Cooperative Learning Strategies in the Mathematics Classroom

This section reports on a study that was conducted on the learning of mathematics among low attainers by introducing cooperative learning strategies into the mathematics classroom. Mathematics teachers from one mainstream secondary school, in consultation with a team of researchers from NIE, engaged in an action research project on studying the impact of infusing cooperative learning strategies in the Normal (Academic) and Normal (Technical) classrooms.

Earlier, a team of researchers led by Lui (reported in Lui 2003; Lui et al. 2005) conducted a research in Singapore schools in validating an instrument of measuring students' academic self-concept and motivation in the Singapore schools, using mathematics as a context. This collaboration between NIE and schools sparkled the interest of one mainstream secondary school to embark on the action research on introducing cooperative learning strategies into the mathematics classroom. According to Lui et al. (2009), the teachers were keen in helping their students "who [were] weak in mathematics" by using incorporating cooperative learning strategies in the lower secondary mathematics classrooms. The study was conducted with the objective to empower the low attainers in learning mathematics.

The theoretical background of this action research was Vygotsky's (1982) theory of social constructivism. According to Vygotsky (1982), students learn best in group activities, and greater opportunity for such interactions will widen the zone of proximal development for the students. According to Burns (1990), social interaction is one key factor in learning mathematics. Greater opportunity for interaction of students with their peers, parents and teachers will allow them to have more exposure to a more variety of viewpoints, thereby stimulating them to reflect on their own.

13.6.1 Various Levels of Instructions for Cooperative Learning

The action research project was divided into two phases, and cooperative learning activities were conducted for the various mathematical topics for secondary two in the Normal (Academic) stream.

In the first phase, in consultation with mathematics professors from NIE, the teachers crafted three activities for three mathematical topics, with differing level of instructions on cooperation proposed to the students. The lesson objectives of the three activities were clearly stated. Activity one, which involved work in computer laboratory in solving a mathematics task, did not contain explicit instructions for students to work cooperatively. During the lesson, the teachers encouraged the students to work in pair. Activity two contained explicit instructions for students to work cooperatively. The teachers gave the students explicit instructions on how they could do cooperative learning to complete the task. In addition to clear instruction for cooperative learning in activity three, the proposed group interaction was also structured to ensure the accountability of each group member towards the completion of the task. The content of the three activities crafted for the study was checked by mathematics educators from NIE to ensure that it was mathematically and pedagogically sound. The findings for the first phase were used to better plan and refine cooperative learning activities for the second phase.

Findings from the first phase

In activity one, the students did not exhibit much interaction with their partners even though they were verbally encouraged to do so by their teacher. The students had not learnt from their partners nor taught their partners during the interaction. Not only that, most of the students indicated that they had a low level of confidence in completing the task. Thus, the presence of the mere physical infrastructure (in this case, the classroom arrangement being conducive for group interaction) and teacher encouragement is insufficient to ensure students working collaboratively. In order to encourage cooperative learning among students, teachers must provide a clear set of instructions on how to work cooperatively. In addition, teachers should create a spirit of cooperation among the students and make them accountable for the task to be completed.

In activity two, explicit instructions were given to students to work in groups, and the teachers also briefed the students on how to work cooperatively. In this activity, student cooperation and discussion were observed. Furthermore, the students were able to state at least one thing they had learnt from or taught their partners. There was a higher rating on student confidence in completing the task in this activity. However, it was observed that little learning took place among the pairs who were both mathematically weak. Also, the students lost focus when they continued to work in pairs for a long period of time, as students, in particular, the low attainers, generally had short attention span. This taught the teacher researchers the lesson that in executing cooperative learning, the grouping or pairing of students based on their capacity was crucial in order to ensure that learning took place. The tasks for cooperative learning must be designed in a way that it is engaging, and not an unduly lengthy task that demands too much time in view of students' short attention span.

In activity three, in addition to the explicit instructions and teacher briefing, the students were divided into groups of four. The students were given clear briefing on the importance of cooperation, the tasks to be completed and the allocation of the scores for the activity. This activity was conducted in an open area in school. The students were clearly aware of the task that they had to complete and the demand of the task. However, the teachers faced greater difficulty in managing the students in open area compared to conducting lessons in the usual classroom. Moreover, in groups of four, there was evidence of some students leaving most of the work to their

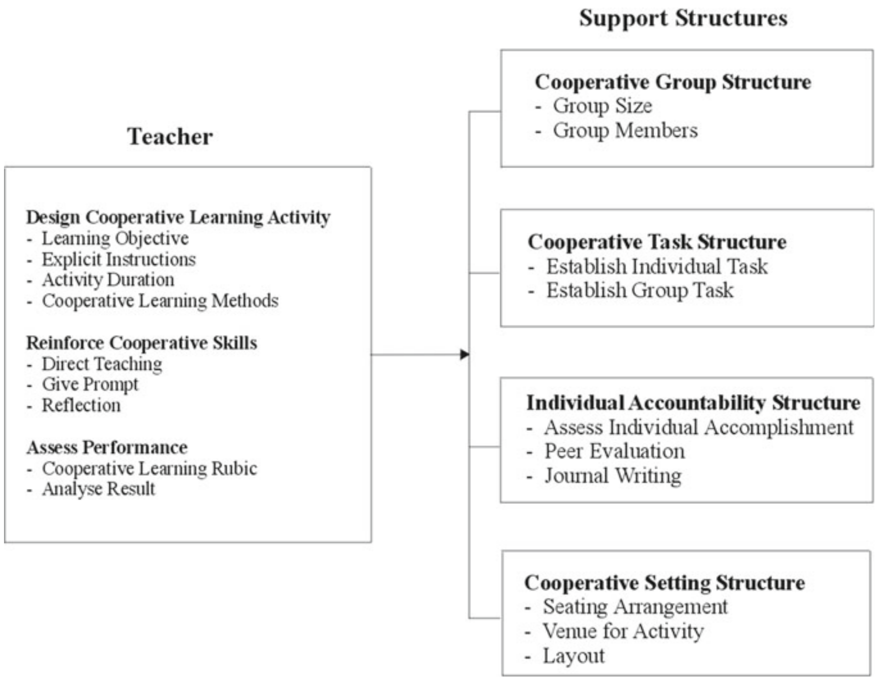

Fig. 13.1 A cooperative learning framework (Lui et al. 2009)

peers and completing minimal work themselves. This appears to point to a higher chance for successful implementation of cooperative learning in pair work rather than in bigger group.

The findings from the first phase provided the researchers with greater understanding of implementing cooperative learning in the second phase.

Implementation of cooperative learning in the second phase

With the learning points in the first phase taken into consideration, the implementation of cooperative learning in second phase incorporated the following arrangement:

- Listing clearly and enforcing class rules;
- Partnering of students of different mathematical abilities;
- Developing a spirit of cooperation in class by constantly encouraging peer tutoring;
- Prompting students to use cooperative skills when offering or asking for help.

The implementation in the second phase involved using cooperative learning strategies for four mathematical activities that were designed by the same team of school teachers, and endorsed by the mathematics professors from the NIE. The activities covered four different subtopics from the secondary two mathematics curriculum.

After the implementation in the second phase, most of the students (about 80%) indicated that they had been able to learn and help one another using cooperative skills. They showed a high degree of positive interdependence, and there was clear evidence of peer tutoring among the students.

Based on the findings of this research project, the researchers proposed a cooperative learning framework that posits the implementation of cooperative learning through the dependence of two main factors: (1) teacher and (2) support structures (Fig. 13.1).

13.7 Contextualizing Mathematics and Comics in Teaching and Learning Mathematics

In the survey conducted by Toh and Lui (2014) described in 12.5, it was seen that there was sporadic effort among the mathematics teachers in using creative teaching strategies to help their students in learning mathematics. For example, they were already using strategies like storytelling and cartoons in teaching mathematics for the low attainers in the Normal (Technical) mathematics classroom. However, it seemed that there was a lack of concerted effort among teachers to explore how these alternative approaches could be used for mathematics instructions. In 2014, a team of researchers from the Singapore NIE initiated a research project with the objective of studying (1) the feasibility of using comics and storytelling for mathematics instruction and (2) the impact on the students' motivation, academic self-concept and performance in mathematics achievement test of using comics and storytelling as a mode of instruction.

Although the idea of using comics in the teaching of mathematics was not entirely new [as the survey in 2013 reported in Toh and Lui (2014)], this marked the beginning of an organized concerted effort headed by researchers in collaboration with teachers from the Singapore school in designing teaching units of secondary one mathematics using the comics and storytelling approach.

Even before the survey in 2013 was carried out by Toh and Lui (2014), Toh (2009) discussed the possibility of using cartoons and comics in teaching mathematics to the less-motivated students. It was argued in the paper that comics not only has the potential to motivate students to yearn to learn mathematics, but it could be useful in teaching abstract concepts in mathematics. Several examples on how comics could be used to expound the various abstract algebra concepts were detailed in the paper. Subsequently, this idea was adopted by a series of textbook for Normal (Technical) stream that was launched in 2014 and adopted in the Singapore schools.

In this research project led by Toh et al. (2017), the researchers designed comics teaching packages for selected secondary one mathematics unit in the Normal (Technical) mathematics curriculum. The package for each unit was complete in both the content coverage and practice questions accompanying the comics. In other words, the package that was developed was used as a "replacement unit" for the existing curriculum resource. The comics package for each unit was offered in both hardcopy versions and online versions for which schools were given the option to select. The characteristics of the two versions and the feedback provided by the teachers after the first cycle of implementation are described below.

Hardcopy version: This version of the comics package was offered in the form of printed worksheets that consisted of both the comic strips and the accompanying practice questions. During each mathematics lesson, the teacher would give up each set of comics relevant to that particular lesson. In each lesson, teacher would go through the comic strips and give students sufficient time to attempt solving the questions. In the feedback provided by the teachers, the hardcopy version had the advantage in that students were provided with some concrete material to rely on and to take notes whenever necessary. However, the comics given to students was in black and white version; the attraction to the comics was reduced compared to the original design which was fully coloured.

Online version: The same set of comic resources was also made available using the online platform (a sample comic strip with the accompanying practice questions can be found in http://math.nie.edu.sg/magical). The differences of the online version from the hardcopy version are that

1. The former is in full-colour form which was designed such that it was compatible with iPad and mobile, and that students were able to swipe the comics across their mobile or iPad in the same way that they read e-comics.
2. The accompanying practice questions in the online version provide immediate feedback to the students' response. Not only that, the package was designed in such a way that the teachers could keep track of their students' responses for any particular practice question.

In addition, the researchers tapped on the affordance of the online comics version and randomize the practice questions each time a different question appeared at different logging in. However, the feedback provided by the teachers was that, as the questions were randomized each time, the teacher was unable to identify a common question to discuss during the classroom instruction, and this led to a messy situation.

The participating teachers covered the entire unit using the comics package. The researchers provided proposed lesson plans that accompanied each set of comics. The teachers executing the lesson were given ideas on the stories to tell at each juncture of the comics and the plausible activities to engage their students during the mathematics lessons. The teachers were also given the liberty to tweak the package according to the needs of the students at various junctures, although they were reminded that the whole research was to study the impact of comics on student motivation and learning of mathematics.

To assist the researchers to better understand what happened in the comics lessons, in particular, how teachers tweaked the package or adapted the proposed lesson plans during the actual lesson implementation, all the comics lessons were video-recorded and analysed by the researchers.

After each cycle of implementation, the researchers discussed with the participating teachers. The latter's feedback of the teaching package provided the researchers ideas on how best to fine-tune the existing package for subsequent implementation within the school. Through this cycle of design, fine-tuning and even redesigning, the comics lesson package that had been designed was customized to the needs of the individual schools.

In the preliminary interview conducted by the Straits Times (Teng 2016, May 30), one teacher from a participating school reported that it was quite challenging to engage his Normal (Technical) students in the mathematics classes. With the use of comics, he could see that there was greater engagement from his students during the lesson. Also, more two-way communication between teacher and students on mathematics started to surface during his mathematics lessons. More importantly, for the unit that he taught using comics, his students performed better in the school assessment compared with the other topics that were taught by the usual textbook resource.

13.7.1 Twenty-First-Century Competencies

Using the twenty-first-century competency framework developed by the MOE, Toh et al. (2017) reported from a preliminary analysis of the video recordings of the comics lessons taught by one team of teachers in one research school. It was interesting to the researchers to notice that, in the process of unpacking the teachers' use of the comics package in teaching mathematics, the teachers (both consciously and subconsciously) attempted to develop in their students other competencies, skills and values in their lesson delivery using the comics package. For example, the teachers rode on the affordance of the comic strips in several instances to facilitate their students to extract information from the visual cues of the comic strips, thereby making sense of the social context provided by the story. This contributed to raising students' social awareness in the lesson delivery of lessons.

In another instance, in a set of comics on the statistics unit for secondary one students, the scene was a typical office setting but the context was not explicitly stated. The teachers made the effort to engage their students to offer a possible interpretation of the context of the comics on the process of carrying out a survey, thereby making the process of data collection and tabulation in statistics sensible to the students. Another point noteworthy to mention is that the teachers went the extra mile to engage their students in communication and collaborative work through the use of comics. There were also instances in which one participating teacher attempted to instil in her students the idea of civic literacy and global awareness (two important attributes of the twenty-first-century competency framework) within the lessons.

13.7.2 Students' Perception of the Comics

Toh et al. (2017) reported on several interviews with the students on their perception of the use of comics in classroom lessons. The students found the comics lessons much more interesting, compared to the regular mathematics lessons. The students described the usual mathematics lessons as "quite boring". Not only that, one student highlighted that the comics lessons had provided them with an opportunity to discuss mathematics in the context of the real world with his friends. In this sense, the comics had also contributed to facilitate the students in communicating their ideas, and this translates to developing them into active contributor who can communicate and work in teams.

Through regularly being engaged in interpreting plausible contexts of the comics, the students became more confident in dealing with mathematics in the real-world context. The students were also engaged in role-play during the comics lessons. They developed a better understanding of different perspectives, and a stronger sense of right and wrong, and being discerning in making judgement.

It was also highlighted in the student interview that the context provided by the comics provided the platforms for them to alter the problems in the original comic strips thereby solving more problems that were created by the students.

Another pleasant surprise to the researchers in the interview with the students was the interview with one student who identified himself as dyslexic. He reflected that he had recognized the importance of mathematics in the real world, although mathematics was a very difficult subject for him. He asserted that he had a positive change in attitude of the student due to the use of comics for mathematics instruction. The student claimed that at first he was neither good nor confident in mathematics. However, the comics got him excited about mathematics. Consequently, he started to read and re-read the comics repeatedly and revised the related mathematics regularly. This constant engagement with comics reading made him develop further interest in mathematics. He claimed that due to being dyslexic, he had difficulty reading mathematics, and each time in attempting to understand a problem, he had to "read a lot of times". As the comics teaching package was presented with colourful pictures and cartoons, it became easier for him to understand the problem and it helped him learn mathematics better.

At the time this chapter was written, the research was still going on in several Singapore secondary schools. The researchers will report more findings on this research sometime in the future.

13.8 Concrete–Pictorial–Abstract Approach in Mathematics Instruction

Algebra presents much difficulty to many students, especially low attainers in mathematics. The difficulty can be attributed to two plausible reasons: (1) the students have poor foundation and might not have met the prerequisite knowledge necessary for the learning of algebra; (2) much of the learning emphasized in schools is based on rote learning rather on facilitating students to make sense of the abstract mathematics (Quek et al. 2016).

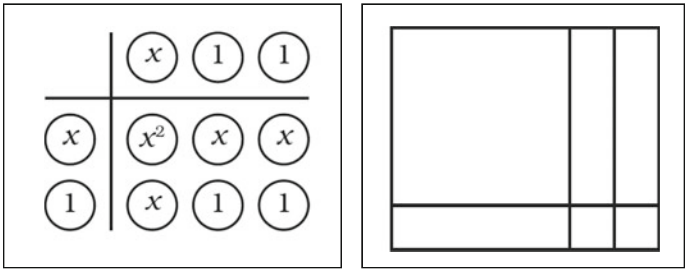

Fig. 13.2 Using AlgeDiscs™ (left) and algebra tiles (right) in factorizing $x^2 + 3x + 2$

The Concrete–Pictorial–Abstract (CPA) approach of teaching mathematics has been highlighted as one of the key instructional strategies in the Singapore mathematics curriculum since its introduction in the 1980s. This is an adaptation of the "enactive–iconic–symbolic" modes of representation first proposed by Bruner in 1966 in his book *Toward a Theory of Instructions*. Leong et al. (2015) provided a survey of the origin of CPA approach used in Singapore in relation to the original proposed idea of Bruner (1966).

According to the Singapore MOE, CPA approach is an activity-based approach of learning mathematics by doing.

> Students engage in activities to explore and learn mathematical concepts and skills …. They could use manipulatives or other resources to construct meanings and understandings. From concrete manipulatives and experiences, students are guided to uncover abstract mathematical concepts or results. … During the activity, students communicate and share their understanding using concrete and pictorial representations. The role of the teacher is that of a facilitator who guides students through the concrete, pictorial and abstract levels of understanding by providing appropriate scaffolding and feedback (MOE 2012, p. 23)

In aligning to CPA as an instructional approach recommended for mathematics instructions, manipulatives have been used in the Singapore mathematics classrooms to make students learn abstract mathematics more sensibly and meaningfully. Two types of manipulatives, the AlgeDisc™ and algebra tiles, were introduced into the Singapore secondary mathematics curriculum as an attempt to help students to "concretise" the otherwise abstract algebra in the teaching and learning processes.

For example, instead of getting students to memorize the procedure of algebraic factorization and completing squares, concrete manipulatives were introduced in the classroom to help students make sense of these and other algebraic processes (Fig. 13.2).

The AlgeDisc™ has the advantage of explicit labelling of the algebraic quantities $(1, -1, x, -x)$ on the discs. The "negative" of this algebraic/numerical quantity can be represented by a "flip" of the disc. On the other hand, the alge-tiles has the added advantage of the geometrical feature of representing each algebraic quantity by the dimensions and the area of a tile.

In adapting the advantages of the algebra discs and algebra tiles, Leong et al. (2010) developed a hybrid form of manipulative that combines the advantages of both the AlgeDisc™ and the algebra tiles which was called the *AlgeCards*.

The AlgeCards involve the use of three types of rectangles (of areas x^2, x and 1), which the length of each rectangle could easily be associated with the algebraic quantity. For example, the rectangle (or square) with area x^2 has dimensions x by x units while the rectangle with area x has dimension x by 1 unit. The flipped side of the rectangle x represents $-x$, and so on. Here, we present two of the studies by Leong et al. (2010), Quek et al. (2016) in their studies on the use of AlgeCards in teaching and learning school algebra.

13.8.1 Study One: Quadratic Factorization

Leong et al. (2010) discusses how the process of factorization of quadratic expressions by the "trial-and-error" method, which has usually been perceived by students as a meaningless arbitrary procedure, can be made meaningful for students. The authors also discuss how the physical use of the manipulative can be meaningfully translated into realistic heuristics that could be applied during the usual paper-and-pencil tests.

Leong et al. (2010) describe in great details in how the AlgeCard, which is used as a scaffolding, can be used to move students from concrete to abstract stage in performing factorization of quadratic expressions during the two lessons on algebraic factorization. In this paper, they described their study on the impact of AlgeCard on the learning of mathematics among Normal (Academic) students from a Singapore mainstream school. They specifically focused on the algebra subtopic on factorization of quadratic expressions.

In the first lesson, the students were introduced to the use of AlgeCard and rectangle diagrams in performing factorization, thereby making sense of the entire otherwise abstract process (factorization of quadratic expressions can be interpreted as forming rectangles with a total fixed area). This parallel use of concrete manipulative and pictorial representation enabled the students to make greater connection of the algebraic process with the pictorial interpretation. Both AlgeCard and rectangle diagrams were used for all the questions in the worksheet in this first lesson.

In the second lesson, the researchers emphasized more on the pictorial representation using rectangle diagrams and downplayed the use of the concrete AlgeCard. In addition, more cumbersome quadratic expressions were introduced in the worksheet to demonstrate the inadequacy of the physical manipulative, although the students were still allowed to use the AlgeCard and the rectangle diagrams if they chose to do so. In the last part of the second lesson on quadratic factorization, quadratic expressions with negative coefficients were introduced in the worksheet. This rendered the use of physical manipulative unnatural.

It was observed that the students were able to respond to this gradual process of scaffolding and most of them eventually moved away from the use of AlgeCard

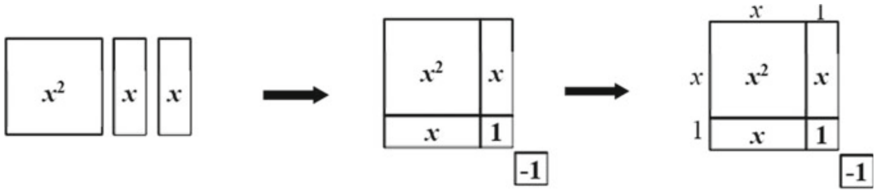

Fig. 13.3 Illustration using AlgeCard on completing squares in $x^2 + 2x$

and resorted to the use of rectangle diagrams and algebraic processes to complete the factorization. What was noteworthy was that this approach increased the level of student engagement among those who were originally uninterested in mathematics.

13.8.2 Study Two: Completing the Squares

Quek et al. (2016) extend the legacy of the AlgeCard experience in Leong et al. (2010) to another Singapore mainstream school to teach the secondary three students from the Normal (Academic) stream the process of completing squares in quadratic expressions. Continuing the line of thought in Leong et al. (2010), the researchers Quek et al. (2016) attempt to assist students to make sense of the process of completing squares and to develop in them the skill to perform typical assessment type items on completing squares. An example provided in Quek et al. (2016) on completing the squares in $x^2 + 2x$ is shown in Fig. 13.3.

They used experiment control approach to test the effect of using the above approach in teaching completing squares compared to the usual procedural emphasis taught by most teachers. The class that was taught using the CPA approach with the use of AlgeCards and another comparable class (that was taught using the usual procedural approach) were required to sit a test, which consisted of typical examination-type items about completing squares in quadratic expressions. The result of the test shows that the experimental group taught using the CPA approach performed significantly better (with higher mean score and smaller standard deviation) than the class that served as control.

13.9 Nationwide Teacher Capacity Building for Teachers Teaching Low Attainers in Mathematics

In recognizing the importance of preparing teachers specifically for low attainers in mathematics, the Singapore Ministry of Education embarked on a project *Improving Confidence And Numeracy* (with the acronym ICAN) to equip her mathematics teachers' proficiency in teaching low attainers in mathematics.

The project ICAN, which started in 2013, targets to assist the low attainers in mathematics (which is identified as the bottom 15% of each cohort) from both primary and secondary schools. The project involves building teachers' capacity to facilitate the learning of mathematics among the low attainers in mathematics. The eight pedagogical principles identified for the teacher building capacity of ICAN are listed below:

1. Establish the classroom norms—getting students ready for the lesson and setting expected behaviour.
2. Check and diagnose students' prerequisite knowledge—to bridge any learning gaps.
3. Create a motivating environment.
4. Focus on fundamentals—during lesson delivery.
5. Giving direct and explicit instruction.
6. Simplify and scaffold.
7. Provide guided practice/communication—oral explanation and reasoning are encouraged.
8. Provide individual practice and review.

It is not difficult to observe that the eight principles of ICAN, which serve to help the low attainers to get the basics right, also serve to address the five dimensions of problem-solving as represented by the five sides of the Singapore school mathematics framework (Fig. 3.1, Chap. 3). Principle 1 relates to setting a conducive environment for learning. Principles 2, 4 and 5 address the importance of skills and concepts dimensions of problem-solving. Principles 7 and 8 address the process of learning mathematics. Principle 3 stresses on the affective dimension of learning—that of attitude. Principle 8, that of providing simplification and scaffolding, addresses the metacognitive dimension of problem-solving.

The support for teachers includes workshops, mentoring, network meetings, pedagogical resources and an annual symposium. To build the capacity at the ground and to sustain ICAN for the longer term, a pool of cluster mathematics mentors from primary schools and secondary schools are supporting the training and mentoring effort of teachers and school mathematics mentors at the cluster level. As the work is still work in progress at the time this chapter is written, we are unable to report the findings of the project at the current stage.

13.10 Conclusion

This chapter presents the various studies that have been conducted in the Singapore education context to provide assistance to low attainers in mathematics at the various levels. In Singapore, the low attainers are identified early in the early years of schooling so that additional assistance can be provided to help them learn better. Not only that, various efforts made by the MOE and the various mathematics education researchers to facilitate the low attainers learn mathematics better are provided at both primary and secondary levels. These additional assistance being offered come in the various types of pedagogical innovation, which include the domain-specific (e.g. the use of mathematics manipulatives in the mathematics lessons), and interpretation and implementation generic pedagogical principles (the use of cooperative learning strategies), and the use of ideas from pop culture (incorporating the use of comics in mathematics instruction).

To ensure that these pedagogical innovations were evidence-based, they were introduced as research projects conducted by the researchers. This chapter provides a brief report of these studies. Readers are encouraged to read up the original research papers if they are keen to have a more detailed understanding of each of the innovative practices to assist the low attainers.

Not only that, these innovative approaches could not have been implemented in classroom if teachers are not well-prepared. Thus, teacher capacity building has always been recognized as an important aspect of better assisting the learning of the low attainers. It is thus not surprising that the MOE embarked on the nationwide effort to build teacher capacity for teachers working with low attainers.

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