Chapter 8
Neutrosophic Soft Rough Graphs

Neutrosophic soft rough set model is a hybrid model by combining neutrosophic soft sets with rough sets. We apply neutrosophic soft rough sets to graphs. We present the concept of neutrosophic soft rough graphs and describe different methods of their construction. We develop an efficient algorithm of our method to solve decision-making problems. This chapter is due to [17].

8.1 Introduction

Pawlak [142] introduced the concept of rough set. He was a Polish mathematician (citizen of Poland) and computer scientist. Rough means approximate or inexact. Rough set theory expresses vagueness in terms of a boundary region of a set not in terms of membership function as in fuzzy set. The idea of rough set theory is a generalization of classical set theory to study the intelligence systems containing inexact, uncertain or incomplete information. It is an effective drive for bestowal with uncertain or incomplete information. Rough set theory is a novel mathematical approach to imprecise knowledge. Rough set theory expresses vagueness by means of a boundary region of a set. The emptiness of boundary region of a set shows that this is a crisp set, and nonemptiness shows that this is a rough set. Nonemptiness of boundary region also describes the deficiency of our knowledge about a set. A subset of a universe in rough set theory is expressed by two approximations which are known as lower and upper approximations. Equivalence classes are the basic building blocks in rough set theory, for upper and lower approximations. Dubois and Prade [74] investigated rough sets and fuzzy sets and concluded that these two theories are different approaches to handle vagueness. They reported that these are not opposite theories and to obtain beneficial results, both theories can be combined. Following this idea, Broumi et al. [61] introduced the concept of rough neutrosophic sets. Yang et al. [177] proposed single-valued neutrosophic rough sets by combining single-valued neutrosophic sets and rough sets, and established an algorithm for decision-making problem based on single-valued neutrosophic rough sets on two

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Table 8.1 List of notations

Symbols	Stand for
X	Universal set
P	Parameter set
M	Subset of parameter set
$\mathbb{R}$	Neutrosophic soft relation on X
(F, A)	Neutrosophic soft set
A	Neutrosophic set on M
$\mathbb{R}A$	Neutrosophic soft rough set on X
$\underline{\mathbb{R}}(A)$	Lower neutrosophic soft rough approximation on X
$\overline{\mathbb{R}}(A)$	Upper neutrosophic soft rough approximation on X
$\acute{X}$	$X \times X$
E	Subset of $\acute{X}$
$\acute{M}$	$M \times M$
L	Subset of $\acute{M}$
$\mathbb{S}$	Neutrosophic soft relation on E
B	Neutrosophic set on L
$\mathbb{S}B$	Neutrosophic soft rough relation on $\acute{X}$
$\underline{\mathbb{S}}(B)$	Lower neutrosophic soft rough approximation on E
$\overline{\mathbb{S}}(B)$	Upper neutrosophic soft rough approximation on E
α	The sum of upper neutrosophic soft rough set and lower neutrosophic soft rough set
β	The sum of upper neutrosophic soft rough relation and lower neutrosophic soft rough relation
γ	The score function

universes. Zhang et al. [203] presented the notion of intuitionistic fuzzy rough sets. The notions of soft rough neutrosophic sets and neutrosophic soft rough sets as hybrid models are described in [26]. We give a list of notations in Table 8.1.

Definition 8.1 Let X be an initial universal set, P a universal set of parameters and $M \subseteq P$. For an arbitrary neutrosophic soft relation $\mathbb{R}$ over $X \times M$, $(X, M, \mathbb{R})$ is called neutrosophic soft approximation space.

For any neutrosophic set $A \in \mathcal{N}(M)$, we define the upper neutrosophic soft rough approximation and the lower neutrosophic soft rough approximation operators of A with respect to $(X, M, \mathbb{R})$ denoted by $\overline{\mathbb{R}}(A)$ and $\underline{\mathbb{R}}(A)$, respectively, as follows:

$$\overline{\mathbb{R}}(A) = \{(x, T_{\overline{\mathbb{R}}(A)}(x), I_{\overline{\mathbb{R}}(A)}(x), F_{\overline{\mathbb{R}}(A)}(x)) \mid x \in X\},$$
$$\underline{\mathbb{R}}(A) = \{(x, T_{\underline{\mathbb{R}}(A)}(x), I_{\underline{\mathbb{R}}(A)}(x), F_{\underline{\mathbb{R}}(A)}(x)) \mid x \in X\},$$

where

Table 8.2 Neutrosophic soft relation $\mathbb{R}$

$\mathbb{R}$	x_1	x_2	x_3	x_4
m_1	(0.3, 0.4, 0.5)	(0.4, 0.2, 0.3)	(0.1, 0.5, 0.4)	(0.2, 0.3, 0.4)
m_2	(0.1, 0.5, 0.4)	(0.3, 0.4, 0.6)	(0.4, 0.4, 0.3)	(0.5, 0.3, 0.8)
m_3	(0.3, 0.4, 0.4)	(0.4, 0.6, 0.7)	(0.3, 0.5, 0.4)	(0.5, 0.4, 0.6)

$$T_{\overline{\mathbb{R}}(A)}(x) = \bigvee_{m\in M} \big(T_{\mathbb{R}(A)}(x,m) \wedge T_A(m)\big), \quad I_{\overline{\mathbb{R}}(A)}(x) = \bigwedge_{m\in M} \big(I_{\mathbb{R}(A)}(x,m) \vee I_A(m)\big),$$
$$F_{\overline{\mathbb{R}}(A)}(x) = \bigwedge_{m\in M} \big(F_{\mathbb{R}(A)}(x,m) \vee F_A(m)\big); \quad T_{\underline{\mathbb{R}}(A)}(x) = \bigwedge_{m\in M} \big(F_{\mathbb{R}(A)}(x,m) \vee T_A(m)\big),$$
$$I_{\underline{\mathbb{R}}(A)}(x) = \bigvee_{m\in M} \big((1 - I_{\mathbb{R}(A)}(x,m)) \wedge I_A(m)\big), \; F_{\underline{\mathbb{R}}(A)}(x) = \bigvee_{m\in M} \big(T_{\mathbb{R}(A)}(x,m) \wedge F_A(m)\big).$$

The pair $(\underline{\mathbb{R}}(A), \overline{\mathbb{R}}(A))$ is called *neutrosophic soft rough set* of A w.r.t. $(X, M, \mathbb{R})$, *and* $\underline{\mathbb{R}}$ and $\overline{\mathbb{R}}$ are referred to as the lower neutrosophic soft rough approximation and the upper neutrosophic soft rough approximation operators, respectively.

Example 8.1 Suppose that $X = \{x_1, x_2, x_3, x_4\}$ is the set of careers under consideration, Mr. X wants to select best suitable career. $M = \{m_1, m_2, m_3\}$ be a set of decision parameters. Mr. X describe the "most suitable career" by defining a neutrosophic soft set $\mathbb{R} = (F, M)$ on X which is a neutrosophic relation from X to M as shown in Table 8.2.

Now, Mr. X gives the most favourable decision object A which is a neutrosophic set on M defined as follows: A=$\{(m_1$,0.5,0.2,0.4),(m_2,0.2,0.3,0.1),(m_3,0.2,0.4,0.6)$\}$. By Definition 8.1, we have

$$T_{\overline{\mathbb{R}}(A)}(x_1) = 0.3, \quad I_{\overline{\mathbb{R}}(A)}(x_1) = 0.4, \quad F_{\overline{\mathbb{R}}(A)}(x_1) = 0.4,$$
$$T_{\overline{\mathbb{R}}(A)}(x_2) = 0.4, \quad I_{\overline{\mathbb{R}}(A)}(x_2) = 0.2, \quad F_{\overline{\mathbb{R}}(A)}(x_2) = 0.4,$$
$$T_{\overline{\mathbb{R}}(A)}(x_3) = 0.2, \quad I_{\overline{\mathbb{R}}(A)}(x_3) = 0.4, \quad F_{\overline{\mathbb{R}}(A)}(x_3) = 0.3,$$
$$T_{\overline{\mathbb{R}}(A)}(x_4) = 0.2, \quad I_{\overline{\mathbb{R}}(A)}(x_4) = 0.3, \quad F_{\overline{\mathbb{R}}(A)}(x_4) = 0.4.$$

Similarly,

$$T_{\underline{\mathbb{R}}(A)}(x_1) = 0.4, \quad I_{\underline{\mathbb{R}}(A)}(x_1) = 0.4, \quad F_{\underline{\mathbb{R}}(A)}(x_1) = 0.3,$$
$$T_{\underline{\mathbb{R}}(A)}(x_2) = 0.5, \quad I_{\underline{\mathbb{R}}(A)}(x_2) = 0.4, \quad F_{\underline{\mathbb{R}}(A)}(x_2) = 0.4,$$
$$T_{\underline{\mathbb{R}}(A)}(x_3) = 0.4, \quad I_{\underline{\mathbb{R}}(A)}(x_3) = 0.4, \quad F_{\underline{\mathbb{R}}(A)}(x_3) = 0.3,$$
$$T_{\underline{\mathbb{R}}(A)}(x_4) = 0.5, \quad I_{\underline{\mathbb{R}}(A)}(x_4) = 0.4, \quad F_{\underline{\mathbb{R}}(A)}(x_4) = 0.5.$$

Thus, we obtain

$$\overline{\mathbb{R}}(A) = \{(x_1, 0.3, 0.4, 0.4), (x_2, 0.4, 0.2, 0.4), (x_3, 0.2, 0.4, 0.3), (x_4, 0.2, 0.3, 0.4)\},$$
$$\underline{\mathbb{R}}(A) = \{(x_1, 0.4, 0.4, 0.3), (x_2, 0.5, 0.4, 0.4), (x_3, 0.4, 0.4, 0.3), (x_4, 0.5, 0.4, 0.5)\}.$$

Hence $(\underline{\mathbb{R}}(A), \overline{\mathbb{R}}(A))$ is a neutrosophic soft rough set of A.

The conventional neutrosophic soft set is a mapping from a parameter to the neutrosophic subset of universe, and let $\mathbb{R}=(F, M)$ be neutrosophic soft set. Now, we present the constructive definition of neutrosophic soft rough relation by using a neutrosophic soft relation $\mathbb{S}$ from $M \times M = \acute{M}$ to $\mathcal{N}(X \times X = \acute{X})$, where X be a universal set and M be a set of parameter.

Definition 8.2 A *neutrosophic soft rough relation* $(\underline{\mathbb{S}}(B), \overline{\mathbb{S}}(B))$ on X is a neutrosophic soft rough set, and $\mathbb{S} : \acute{M} \to \mathcal{N}(\acute{X})$ is a neutrosophic soft relation on X defined by $\mathbb{S}(m_i m_j) = \{x_i x_j \mid \exists x_i \in \mathbb{R}(m_i), x_j \in \mathbb{R}(m_j)\}$, $x_i x_j \in \acute{X}$, such that

$$\begin{aligned}
T_{\mathbb{S}}(x_i x_j, m_i m_j) &\le \min\{T_{\mathbb{R}}(x_i, m_i), T_{\mathbb{R}}(x_j, m_j)\}\\
I_{\mathbb{S}}(x_i x_j, m_i m_j) &\le \max\{I_{\mathbb{R}}(x_i, m_i), I_{\mathbb{R}}(x_j, m_j)\}\\
F_{\mathbb{S}}(x_i x_j, m_i m_j) &\le \max\{F_{\mathbb{R}}(x_i, m_i), F_{\mathbb{R}}(x_j, m_j)\}.
\end{aligned}$$

For any $B \in \mathcal{N}(\acute{M})$, $B = \{\big(m_i m_j, T_B(m_i m_j), I_B(m_i m_j), F_B(m_i m_j)\big)\ m_i m_j \in \acute{M}\}$,

$$\begin{aligned}
T_B(m_i m_j) &\le \min\{T_A(m_i), T_A(m_j)\},\\
I_B(m_i m_j) &\le \max\{I_A(m_i), I_A(m_j)\},\\
F_B(m_i m_j) &\le \max\{F_A(m_i), F_A(m_j)\}.
\end{aligned}$$

The upper neutrosophic soft approximation and the lower neutrosophic soft approximation of B *w.r.t.* $(\acute{X}, \acute{M}, \mathbb{S})$ are defined as follows:

$$\overline{\mathbb{S}}(B) = \{(x_i x_j, T_{\overline{\mathbb{S}}(B)}(x_i x_j), I_{\overline{\mathbb{S}}(B)}(x_i x_j), F_{\overline{\mathbb{S}}(B)}(x_i x_j)) \mid x_i x_j \in \acute{X}\},$$

$$\underline{\mathbb{S}}(B) = \{(x_i x_j, T_{\underline{\mathbb{S}}(B)}(x_i x_j), I_{\underline{\mathbb{S}}(B)}(x_i x_j), F_{\underline{\mathbb{S}}(B)}(x_i x_j)) \mid x_i x_j \in \acute{X}\},$$

where

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B)}(x_i x_j) &= \bigvee_{m_i m_j \in \acute{M}} \big(T_{\mathbb{S}}(x_i x_j, m_i m_j) \wedge T_B(m_i m_j)\big),\\
I_{\overline{\mathbb{S}}(B)}(x_i x_j) &= \bigwedge_{m_i m_j \in \acute{M}} \big(I_{\mathbb{S}}(x_i x_j, m_i m_j) \vee I_B(m_i m_j)\big),\\
F_{\overline{\mathbb{S}}(B)}(x_i x_j) &= \bigwedge_{m_i m_j \in \acute{M}} \big(F_{\mathbb{S}}(x_i x_j, m_i m_j) \vee F_B(m_i m_j)\big),
\end{aligned}$$

$$T_{\underline{\mathbb{S}}(B)}(x_ix_j) = \bigwedge_{m_im_j \in \acute{M}} \big(F_{\mathbb{S}}(x_ix_j, m_im_j) \vee T_B(m_im_j)\big),$$
$$I_{\underline{\mathbb{S}}(B)}(x_ix_j) = \bigvee_{m_im_j \in \acute{M}} \big((1 - I_{\mathbb{S}}(x_ix_j, m_im_j)) \wedge I_B(m_im_j)\big),$$
$$F_{\underline{\mathbb{S}}(B)}(x_ix_j) = \bigvee_{m_im_j \in \acute{M}} \big(T_{\mathbb{S}}(x_ix_j, m_im_j) \wedge F_B(m_im_j)\big).$$

The pair $(\underline{\mathbb{S}}(B), \overline{\mathbb{S}}(B))$ is called *neutrosophic soft rough relation*, and $\underline{\mathbb{S}}, \overline{\mathbb{S}} : \mathcal{N}(\acute{M}) \to \mathcal{N}(\acute{X})$ are called the *lower neutrosophic soft rough approximation* and the *upper neutrosophic soft rough approximation* operators, respectively.

Remark 8.1 Consider a neutrosophic set B on $\acute{M}$ and a neutrosophic set A on M; according to the definition of neutrosophic soft rough relation, we get

$$T_{\overline{\mathbb{S}}(B)}(x_ix_j) \le \min\{T_{\overline{\mathbb{R}}(A)}(x_i), T_{\overline{\mathbb{R}}(A)}(x_j)\},$$
$$I_{\overline{\mathbb{S}}(B)}(x_ix_j) \le \max\{I_{\overline{\mathbb{R}}(A)}(x_i), I_{\overline{\mathbb{R}}(A)}(x_j)\},$$
$$F_{\overline{\mathbb{S}}(B)}(x_ix_j) \le \max\{F_{\overline{\mathbb{R}}(A)}(x_i), F_{\overline{\mathbb{R}}(A)}(x_j)\}.$$

Similarly, for lower neutrosophic soft rough approximation operator $\underline{\mathbb{S}}(B)$,

$$T_{\underline{\mathbb{S}}(B)}(x_ix_j) \le \min\{T_{\underline{\mathbb{R}}(A)}(x_i), T_{\underline{\mathbb{R}}(A)}(x_j)\},$$
$$I_{\underline{\mathbb{S}}(B)}(x_ix_j) \le \max\{I_{\underline{\mathbb{R}}(A)}(x_i), I_{\underline{\mathbb{R}}(A)}(x_j)\},$$
$$F_{\underline{\mathbb{S}}(B)}(x_ix_j) \le \max\{F_{\underline{\mathbb{R}}(A)}(x_i), F_{\underline{\mathbb{R}}(A)}(x_j)\}.$$

Example 8.2 Let $X = \{x_1, x_2, x_3\}$ be a universal set and $M = \{m_1, m_2, m_3\}$ a set of parameters. A neutrosophic soft set $\mathbb{R} = (F, M)$ on X can be defined in Table 8.3 as follows.

Let $E = \{x_1x_2, x_2x_3, x_2x_2, x_3x_2\} \subseteq \acute{X}$ and $L = \{m_1m_3, m_2m_1, m_3m_2\} \subseteq \acute{M}$. Then a soft relation $\mathbb{S}$ on E (from L to E) can be defined in Table 8.4 as follows. Let $A = \{(m_1, 0.2, 0.4, 0.6), (m_2, 0.4, 0.5, 0.2), (m_3, 0.1, 0.2, 0.4)\}$ be a neutrosophic set on M, then
$\overline{\mathbb{R}}(A) = \{(x_1, 0.4, 0.2, 0.4), (x_2, 0.3, 0.4, 0.3), (x_3, 0.4, 0.2, 0.3)\}$
$\underline{\mathbb{R}}(A) = \{(x_1, 0.3, 0.5, 0.4), (x_2, 0.2, 0.5, 0.6), (x_3, 0.4, 0.5, 0.6)\}$.
Let $B = \{(m_1m_3, 0.1, 0.3, 0.5), (m_2m_1, 0.2, 0.4, 0.3), (m_3m_2, 0.1, 0.2, 0.3)\}$ be a neutrosophic set on L, then

Table 8.3 Neutrosophic soft set $\mathbb{R} = (F, M)$

$\mathbb{R}$	x_1	x_2	x_3
m_1	(0.4, 0.5, 0.6)	(0.7, 0.3, 0.2)	(0.6, 0.3, 0.4)
m_2	(0.5, 0.3, 0.6)	(0.3, 0.4, 0.3)	(0.7, 0.2, 0.3)
m_3	(0.7, 0.2, 0.3)	(0.6, 0.5, 0.4)	(0.7, 0.2, 0.4)

Table 8.4 Neutrosophic soft relation $\mathbb{S}$

$\mathbb{S}$	x_1x_2	x_2x_3	x_2x_2	x_3x_2
m_1m_3	(0.4, 0.4, 0.5)	(0.6, 0.3, 0.4)	(0.5, 0.4, 0.2)	(0.5, 0.4, 0.3)
m_2m_1	(0.3, 0.3, 0.4)	(0.3, 0.2, 0.3)	(0.2, 0.3, 0.3)	(0.7, 0.2, 0.2)
m_3m_2	(0.3, 0.3, 0.2)	(0.5, 0.3, 0.2)	(0.2, 0.4, 0.4)	(0.3, 0.4, 0.4)

$$\overline{\mathbb{S}}(B) = \{(x_1x_2, 0.2, 0.3, 0.3), (x_2x_3, 0.2, 0.3, 0.3), (x_2x_2, 0.2, 0.4, 0.3), (x_3x_2, 0.2, 0.4, 0.3)\}$$
$$\underline{\mathbb{S}}(B) = \{(x_1x_2, 0.2, 0.4, 0.4), (x_2x_3, 0.2, 0.4, 0.5), (x_2x_2, 0.3, 0.4, 0.5), (x_3x_2, 0.2, 0.4, 0.5)\}$$

Hence $\mathbb{S}B = (\underline{\mathbb{S}}(B), \overline{\mathbb{S}}(B))$ is neutrosophic soft rough relation.

8.2 Neutrosophic Soft Rough Information

Definition 8.3 A *neutrosophic soft rough graph* on a nonempty X is a four-ordered tuple $(X, M, \mathbb{R}A, \mathbb{S}B)$ such that

(i) M is a set of parameters.
(ii) $\mathbb{R}$ is an arbitrary neutrosophic soft relation over $X \times M$.
(iii) $\mathbb{S}$ is an arbitrary neutrosophic soft relation over $\acute{X} \times \acute{M}$.
(vi) $\mathbb{R}A = (\underline{\mathbb{R}}(A), \overline{\mathbb{R}}(A))$ is a neutrosophic soft rough set of X.
(v) $\mathbb{S}B = (\underline{\mathbb{S}}(B), \overline{\mathbb{S}}(B))$ is a neutrosophic soft rough relation on $\acute{X} \subseteq X \times X$.

$G = (\mathbb{R}A, \mathbb{S}B)$ is a neutrosophic soft rough graph, where $\underline{G} = (\underline{\mathbb{R}}(A), \underline{\mathbb{S}}(B))$ and $\overline{G} = (\overline{\mathbb{R}}(A), \overline{\mathbb{S}}(B))$ are lower neutrosophic approximate graph and upper neutrosophic approximate graph, respectively, of neutrosophic soft rough graph $G = (\mathbb{R}A, \mathbb{S}B)$.

Example 8.3 Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be a vertex set and $M = \{m_1, m_2, m_3\}$ a set of parameters. A neutrosophic soft relation over $X \times M$ can be defined in Table 8.5 as follows.
Let $A = \{(m_1, 0.5, 0.4, 0.6), (m_2, 0.7, 0.4, 0.5), (m_3, 0.6, 0.2, 0.5)\}$ be a neutrosophic set on M, then

Table 8.5 Neutrosophic soft relation $\mathbb{R}$

$\mathbb{R}$	x_1	x_2	x_3	x_4	x_5	x_6
m_1	(0.4, 0.5, 0.6)	(0.7, 0.3, 0.5)	(0.6, 0.2, 0.3)	(0.4, 0.4, 0.2)	(0.5, 0.5, 0.6)	(0.4, 0.5, 0.6)
m_2	(0.5, 0.4, 0.2)	(0.6, 0.4, 0.5)	(0.7, 0.3, 0.4)	(0.5, 0.3, 0.2)	(0.4, 0.5, 0.4)	(0.6, 0.5, 0.4)
m_3	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.2)	(0.5, 0.4, 0.3)	(0.6, 0.2, 0.3)	(0.5, 0.4, 0.4)	(0.7, 0.3, 0.5)

$$\overline{\mathbb{R}}(A) = \{(x_1, 0.5, 0.4, 0.5), (x_2, 0.6, 0.3, 0.5), (x_3, 0.7, 0.4, 0.5), (x_4, 0.6, 0.2, 0.5), (x_5, 0.5, 0.4, 0.5), (x_6, 0.6, 0.3, 0.5)\},$$

$$\underline{\mathbb{R}}(A) = \{(x_1, 0.6, 0.4, 0.5), (x_2, 0.5, 0.4, 0.6), (x_3, 0.5, 0.4, 0.6), (x_4, 0.5, 0.4, 0.5), (x_5, 0.6, 0.4, 0.5), (x_6, 0.6, 0.4, 0.5)\}.$$

Let $E = \{x_1x_1, x_1x_2, x_2x_1, x_2x_3, x_4x_5, x_3x_4, x_5x_2, x_5x_6\} \subseteq \acute{X}$ and $L = \{m_1m_3, m_2m_1, m_3m_2\} \subseteq \acute{M}$. Then a neutrosophic soft relation $\mathbb{S}$ on E (from L to E) can be defined in Tables 8.6 and 8.7 as follows.
Let $B = \{(m_1m_2, 0.4, 0.4, 0.5), (m_2m_3, 0.5, 0.4, 0.5), (m_1m_3, 0.5, 0.2, 0.5)\}$ be a neutrosophic set on L, then

$$\overline{\mathbb{S}}(B) = \{(x_1x_1, 0.5, 0.4, 0.5), (x_1x_2, 0.4, 0.2, 0.5), (x_2x_1, 0.4, 0.2, 0.5), (x_2x_3, 0.5, 0.3, 0.5), (x_3x_4, 0.5, 0.2, 0.5), (x_4x_5, 0.4, 0.3, 0.5), (x_5x_2, 0.5, 0.3, 0.5), (x_5x_6, 0.5, 0.3, 0.5)\},$$

$$\underline{\mathbb{S}}(B) = \{(x_1x_1, 0.4, 0.4, 0.5)(x_1x_2, 0.5, 0.4, 0.4), (x_2x_1, 0.5, 0.4, 0.4), (x_2x_3, 0.4, 0.4, 0.5), (x_3x_4, 0.4, 0.4, 0.5), (x_4x_5, 0.4, 0.4, 0.4), (x_5x_2, 0.4, 0.4, 0.5), (x_5x_6, 0.4, 0.4, 0.5)\}.$$

Hence $\mathbb{S}B = (\underline{\mathbb{S}}(B), \overline{\mathbb{S}}(B))$ is neutrosophic soft rough relation on $\acute{X}$. Thus, $\underline{G} = (\underline{\mathbb{R}}(A), \underline{\mathbb{S}}(B))$ and $\overline{G} = (\overline{\mathbb{R}}(A), \overline{\mathbb{S}}(B))$ are lower neutrosophic approximate graph and upper neutrosophic approximate graph, respectively, as shown in Fig. 8.1. Hence, $G = (\underline{G}, \overline{G})$ is neutrosophic soft rough graph.

Definition 8.4 Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic soft rough graphs on X. The *union* of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1 \cup G_2 = (\underline{G}_1 \cup \underline{G}_2, \overline{G}_1 \cup \overline{G}_2)$, where $\underline{G}_1 \cup \underline{G}_2 = (\underline{\mathbb{R}}(A_1) \cup \underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_1) \cup \underline{\mathbb{S}}(B_2))$ and $\overline{G}_1 \cup \overline{G}_2 = (\overline{\mathbb{R}}(A_1) \cup \overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_1) \cup \overline{\mathbb{S}}(B_2))$ are neutrosophic graphs, such that

(i) $\forall x \in \mathbb{R}A_1$ but $x \notin \mathbb{R}A_2$.

Table 8.6 Neutrosophic soft relation $\mathbb{S}$

$\mathbb{S}$	x_1x_1	x_1x_2	x_2x_1	x_2x_3
m_1m_2	(0.4, 0.4, 0.2)	(0.4, 0.4, 0.5)	(0.4, 0.4, 0.5)	(0.6, 0.3, 0.4)
m_2m_3	(0.5, 0.4, 0.1)	(0.4, 0.3, 0.2)	(0.4, 0.3, 0.2)	(0.5, 0.3, 0.2)
m_1m_3	(0.4, 0.4, 0.1)	(0.4, 0.2, 0.2)	(0.4, 0.2, 0.2)	(0.5, 0.3, 0.3)

Table 8.7 Neutrosophic soft relation $\mathbb{S}$

$\mathbb{S}$	x_3x_4	x_4x_5	x_5x_2	x_5x_6
m_1m_2	(0.4, 0.2, 0.2)	(0.4, 0.4, 0.2)	(0.4, 0.3, 0.4)	(0.3, 0.2, 0.3)
m_2m_3	(0.6, 0.2, 0.4)	(0.3, 0.2, 0.1)	(0.4, 0.3, 0.2)	(0.4, 0.3, 0.4)
m_1m_3	(0.4, 0.2, 0.3)	(0.4, 0.3, 0.1)	(0.5, 0.3, 0.2)	(0.5, 0.3, 0.5)

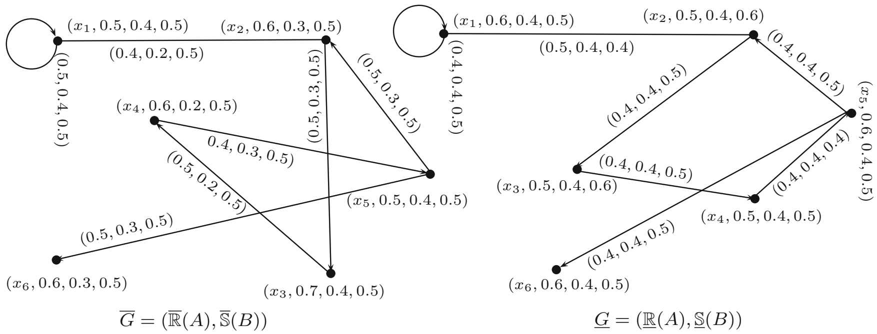


Fig. 8.1 Neutrosophic soft rough graph $G = (\underline{G}, \overline{G})$

$$
\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)\cup\overline{\mathbb{R}}(A_2)}(x) &= T_{\overline{\mathbb{R}}(A_1)}(x),\ T_{\underline{\mathbb{R}}(A_1)\cup\underline{\mathbb{R}}(A_2)}(x) = T_{\underline{\mathbb{R}}(A_1)}(x),\\
I_{\overline{\mathbb{R}}(A_1)\cup\overline{\mathbb{R}}(A_2)}(x) &= I_{\overline{\mathbb{R}}(A_1)}(x),\ \ I_{\underline{\mathbb{R}}(A_1)\cup\underline{\mathbb{R}}(A_2)}(x) = I_{\underline{\mathbb{R}}(A_1)}(x),\\
F_{\overline{\mathbb{R}}(A_1)\cup\overline{\mathbb{R}}(A_2)}(x) &= F_{\overline{\mathbb{R}}(A_1)}(x),\ F_{\underline{\mathbb{R}}(A_1)\cup\underline{\mathbb{R}}(A_2)}(x) = F_{\underline{\mathbb{R}}(A_1)}(x).
\end{aligned}
$$

(ii) $\forall x \notin \mathbb{R}A_1$ but $x \in \mathbb{R}A_2$.

$$
\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)\cup\overline{\mathbb{R}}(A_2)}(x) &= T_{\overline{\mathbb{R}}(A_2)}(x),\ T_{\underline{\mathbb{R}}A_1\cup\underline{\mathbb{R}}(A_2)}(x) = T_{\underline{\mathbb{R}}(A_2)}(x),\\
I_{\overline{\mathbb{R}}(A_1)\cup\overline{\mathbb{R}}(A_2)}(x) &= I_{\overline{\mathbb{R}}(A_2)}(x),\ \ I_{\underline{\mathbb{R}}(A_1)\cup\underline{\mathbb{R}}(A_2)}(x) = I_{\underline{\mathbb{R}}(A_2)}(x),\\
F_{\overline{\mathbb{R}}(A_1)\cup\overline{\mathbb{R}}(A_2)}(x) &= F_{\overline{\mathbb{R}}(A_2)}(x),\ F_{\underline{\mathbb{R}}(A_1)\cup\underline{\mathbb{R}}(A_2)}(x) = F_{\underline{\mathbb{R}}(A_2)}(x).
\end{aligned}
$$

(iii) $\forall x \in \mathbb{R}A_1 \cap \mathbb{R}A_2$

$$
\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)\cup\overline{\mathbb{R}}(A_2)}(x) &= \max\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{R}}(A_2)}(x)\},\\
T_{\underline{\mathbb{R}}(A_1)\cup\underline{\mathbb{R}}(A_2)}(x) &= \max\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{R}}(A_2)}(x)\},\\
I_{\overline{\mathbb{R}}(A_1)\cup\overline{\mathbb{R}}(A_2)}(x) &= \min\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{R}}(A_2)}(x)\},\\
I_{\underline{\mathbb{R}}(A_1)\cup\underline{\mathbb{R}}(A_2)}(x) &= \min\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{R}}(A_2)}(x)\},\\
F_{\overline{\mathbb{R}}(A_1)\cup\overline{\mathbb{R}}(A_2)}(x) &= \min\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{R}}(A_2)}(x)\},\\
F_{\underline{\mathbb{R}}(A_1)\cup\underline{\mathbb{R}}(A_2)}(x) &= \min\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{R}}(A_2)}(x)\}.
\end{aligned}
$$

(iv) $\forall xy \in \mathbb{S}B_1$ but $xy \notin \mathbb{S}B_2$.

$$
\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\cup\overline{\mathbb{S}}(B_2)}(xy) &= T_{\overline{\mathbb{S}}(B_1)}(xy),\ T_{\underline{\mathbb{S}}(B_1)\cup\underline{\mathbb{S}}(B_2)}(xy) = T_{\underline{\mathbb{S}}(B_1)}(xy),\\
I_{\overline{\mathbb{S}}(B_1)\cup\overline{\mathbb{S}}(B_2)}(xy) &= I_{\overline{\mathbb{S}}(B_1)}(xy),\ \ I_{\underline{\mathbb{S}}(B_1)\cup\underline{\mathbb{S}}(B_2)}(xy) = I_{\underline{\mathbb{S}}(B_1)}(xy),\\
F_{\overline{\mathbb{S}}(B_1)\cup\overline{\mathbb{S}}(B_2)}(xy) &= F_{\overline{\mathbb{S}}(B_1)}(xy),\ F_{\underline{\mathbb{S}}(B_1)\cup\underline{\mathbb{S}}(B_2)}(xy) = F_{\underline{\mathbb{S}}(B_1)}(xy).
\end{aligned}
$$

(v) $\forall xy \notin \mathbb{S}B_1$ but $xy \in \mathbb{S}B_2$

Table 8.8 Neutrosophic soft relation $\mathbb{R}$

$\mathbb{R}$	x_1	x_2	x_3	x_4
m_1	(0.5, 0.4, 0.3)	(0.7, 0.6, 0.5)	(0.7, 0.6, 0.4)	(0.5, 0.7, 0.4)
m_2	(0.3, 0.5, 0.6)	(0.4, 0.5, 0.1)	(0.3, 0.6, 0.5)	(0.4, 0.8, 0.2)
m_3	(0.7, 0.5, 0.8)	(0.2, 0.3, 0.8)	(0.7, 0.3, 0.5)	(0.6, 0.4, 0.3)

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\cup\overline{\mathbb{S}}(B_2)}(xy) =& T_{\overline{\mathbb{S}}(B_2)}(xy),\ T_{\underline{\mathbb{S}}(B_1)\cup\underline{\mathbb{S}}(B_2)}(xy) = T_{\underline{\mathbb{S}}(B_2)}(xy),\\
I_{\overline{\mathbb{S}}(B_1)\cup\overline{\mathbb{S}}(B_2)}(xy) =& I_{\overline{\mathbb{S}}(B_2)}(xy),\ \ I_{\underline{\mathbb{S}}(B_1)\cup\underline{\mathbb{S}}(B_2)}(xy) = I_{\underline{\mathbb{S}}(B_2)}(xy),\\
F_{\overline{\mathbb{S}}(B_1)\cup\overline{\mathbb{S}}(B_2)}(xy) =& F_{\overline{\mathbb{S}}(B_2)}(xy),\ F_{\underline{\mathbb{S}}(B_1)\cup\underline{\mathbb{S}}(B_2)}(xy) = F_{\underline{\mathbb{S}}(B_2)}(xy).
\end{aligned}$$

(vi) $\forall xy \in \mathbb{S}B_1 \cap \mathbb{S}(B_2)$

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\cup\overline{\mathbb{S}}(B_2)}(xy) =& \max\{T_{\overline{\mathbb{S}}(B_1)}(xy),\ T_{\overline{\mathbb{S}}(B_2)}(xy)\},\\
T_{\underline{\mathbb{S}}(B_1)\cup\underline{\mathbb{S}}(B_2)}(xy) =& \max\{T_{\underline{\mathbb{S}}(B_1)}(xy),\ T_{\underline{\mathbb{S}}(B_2)}(xy)\},\\
I_{\overline{\mathbb{S}}(B_1)\cup\overline{\mathbb{S}}(B_2)}(xy) =& \min\{I_{\overline{\mathbb{S}}(B_1)}(xy),\ I_{\overline{\mathbb{S}}(B_2)}(xy)\},\\
I_{\underline{\mathbb{S}}(B_1)\cup\underline{\mathbb{S}}(B_2)}(xy) =& \min\{I_{\underline{\mathbb{S}}(B_1)}(xy),\ I_{\underline{\mathbb{S}}(B_2)}(xy)\},\\
F_{\overline{\mathbb{S}}(B_1)\cup\overline{\mathbb{S}}(B_2)}(xy) =& \min\{F_{\overline{\mathbb{S}}(B_1)}(xy),\ F_{\overline{\mathbb{S}}(B_2)}(xy)\},\\
F_{\underline{\mathbb{S}}(B_1)\cup\underline{\mathbb{S}}(B_2)}(xy) =& \min\{F_{\underline{\mathbb{S}}(B_1)}(xy),\ F_{\underline{\mathbb{S}}(B_2)}(xy)\}.
\end{aligned}$$

Example 8.4 Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of universe and $M = \{m_1, m_2, m_3\}$ a set of parameters. Then a neutrosophic soft relation over $X \times \mathbb{M}$ can be written as in Table 8.8.

Let $A_1 = \{(m_1, 0.5, 0.7, 0.8), (m_2, 0.7, 0.5, 0.3), (m_3, 0.4, 0.5, 0.3)\}$, and $A_2 = \{(m_1, 0.6, 0.3, 0.5), (m_2, 0.5, 0.8, 0.2), (m_3, 0.5, 0.7, 0.2)\}$ be two neutrosophic sets on M, Then $\mathbb{R}A_1 = (\underline{\mathbb{R}}(A_1), \overline{\mathbb{R}}(A_1))$ and $\mathbb{R}A_2 = (\underline{\mathbb{R}}(A_2), \overline{\mathbb{R}}(A_2))$ are neutrosophic soft rough sets, where

$$\begin{aligned}
\underline{\mathbb{R}}(A_1) =& \{(x_1, 0.5, 0.6, 0.5), (x_2, 0.5, 0.5, 0.7)(x_3, 0.5, 0.5, 0.7), (x_4 0.4, 0.5, 0.5)\},\\
\overline{\mathbb{R}}(A_1) =& \{(x_1, 0.5, 0.5, 0.6), (x_2, 0.5, 0.5, 0.3), (x_3, 0.5, 0.5, 0.5), (x_4 0.5, 0.5, 0.3)\};\\
\underline{\mathbb{R}}(A_2) =& \{(x_1, 0.6, 0.5, 0.5), (x_2, 0.5, 0.7, 0.5), (x_3, 0.5, 0.7, 0.5), (x_4, 0.5, 0.6, 0.5)\},\\
\overline{\mathbb{R}}(A_2) =& \{(x_1, 0.5, 0.4, 0.5), (x_2, 0.6, 0.6, 0.2), (x_3, 0.6, 0.6, 0.5), (x_4, 0.5, 0.7, 0.2)\}.
\end{aligned}$$

Let $E = \{x_1x_2, x_1x_4, x_2x_2, x_2x_3, x_3x_3, x_3x_4\} \subseteq X \times X$, and $L = \{m_1m_2, m_1m_3, m_2m_3\} \subset \acute{\mathbb{M}}$. Then a neutrosophic soft relation on E can be written as in Table 8.9

Let $B_1 = \{(m_1m_2, 0.5, 0.4, 0.5), (m_1m_3, 0.3, 0.4, 0.5), (m_2m_3, 0.4, 0.4, 0.3)\}$, and $B_2 = \{(m_1m_2, 0.5, 0.3, 0.2), (m_1m_3, 0.4, 0.3, 0.3), (m_2m_3, 0.4, 0.6, 0.2)\}$ be two neutrosophic sets on L. Then $\mathbb{S}B_1 = (\underline{\mathbb{S}}(B_1), \overline{\mathbb{S}}(B_1))$ and $\mathbb{S}B_2 = (\underline{\mathbb{S}}(B_2), \overline{\mathbb{S}}(B_2))$ are neutrosophic soft rough relations, where

Table 8.9 Neutrosophic soft relation $\mathbb{S}$

$\mathbb{S}$	x_1x_2	x_1x_4	x_2x_2	x_2x_3	x_3x_3	x_3x_4
m_1m_2	(0.3, 0.4, 0.1)	(0.4, 0.4, 0.2)	(0.4, 0.5, 0.1)	(0.3, 0.5, 0.4)	(0.3, 0.4, 0.4)	(0.4, 0.5, 0.2)
m_1m_3	(0.2, 0.3, 0.3)	(0.4, 0.3, 0.2)	(0.2, 0.3, 0.5)	(0.4, 0.3, 0.3)	(0.5, 0.3, 0.3)	(0.5, 0.4, 0.3)
m_2m_3	(0.2, 0.3, 0.5)	(0.3, 0.3, 0.3)	(0.2, 0.3, 0.1)	(0.4, 0.3, 0.1)	(0.3, 0.3, 0.5)	(0.3, 0.4, 0.3)

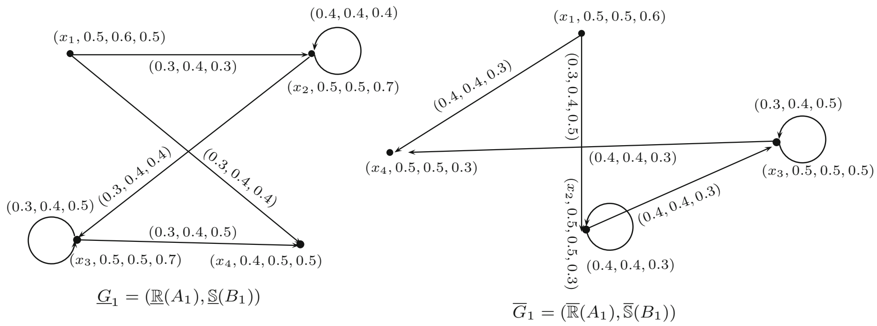


Fig. 8.2 Neutrosophic soft rough graph $G_1 = (\underline{G}_1, \overline{G}_1)$

$$
\begin{aligned}
\underline{\mathbb{S}}(B_1) =& \{(x_1x_2, 0.3, 0.4, 0.3), (x_1x_4, 0.3, 0.4, 0.4), (x_2x_2, 0.4, 0.4, 0.4), (x_2x_3, 0.3, 0.4, 0.4),\\
& (x_3x_3, 0.3, 0.4, 0.5), (x_3x_4, 0.3, 0.4, 0.5)\},\\
\overline{\mathbb{S}}(B_1) =& \{(x_1x_2, 0.3, 0.4, 0.5), (x_1x_4, 0.4, 0.4, 0.3), (x_2x_2, 0.4, 0.4, 0.3), (x_2x_3, 0.4, 0.4, 0.3),\\
& (x_3x_3, 0.3, 0.4, 0.5), (x_3x_4, 0.4, 0.4, 0.3)\};\\
\underline{\mathbb{S}}(B_2) =& \{(x_1x_2, 0.4, 0.6, 0.2), (x_1x_4, 0.4, 0.6, 0.3), (x_2x_2, 0.4, 0.6, 0.2), (x_2x_3, 0.4, 0.6, 0.3),\\
& (x_3x_3, 0.4, 0.6, 0.3), (x_3x_4, 0.4, 0.6, 0.3)\},\\
\overline{\mathbb{S}}(B_2) =& \{(x_1x_2, 0.3, 0.3, 0.2), (x_1x_4, 0.4, 0.3, 0.2), (x_2x_2, 0.4, 0.3, 0.2), (x_2x_3, 0.4, 0.3, 0.2),\\
& (x_3x_3, 0.4, 0.3, 0.3), (x_3x_4, 0.4, 0.4, 0.2)\}.
\end{aligned}
$$

Thus $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ are neutrosophic soft rough graphs, where $\underline{G}_1 = (\underline{\mathbb{R}}(A_1), \underline{\mathbb{S}}(B_1))$, $\overline{G}_1 = (\overline{\mathbb{R}}(A_1), \overline{\mathbb{S}}(B_1))$ as shown in Fig. 8.2

$\underline{G}_2 = (\underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_2))$, $\overline{G}_2 = (\overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_2))$ as shown in Fig. 8.3.

The union of $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ is neutrosophic soft rough graph $G = G_1 \cup G_2 = (\underline{G}_1 \cup \underline{G}_2, \overline{G}_1 \cup \overline{G}_2)$ as shown in Fig. 8.4.

Definition 8.5 Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic soft rough graphs on X. The *intersection* of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1 \cap G_2 = (\underline{G}_1 \cap \underline{G}_2, \overline{G}_1 \cap \overline{G}_2)$, where $\underline{G}_1 \cap \underline{G}_2 = (\underline{\mathbb{R}}(A_1) \cap \underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_1) \cap \underline{\mathbb{S}}(B_2))$ and $\overline{G}_1 \cap \overline{G}_2 = (\overline{\mathbb{R}}(A_1) \cap \overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_1) \cap \overline{\mathbb{S}}(B_2))$ are neutrosophic graphs, respectively, such that

(i) $\forall x \in \mathbb{R}A_1$ but $x \notin \mathbb{R}A_2$.

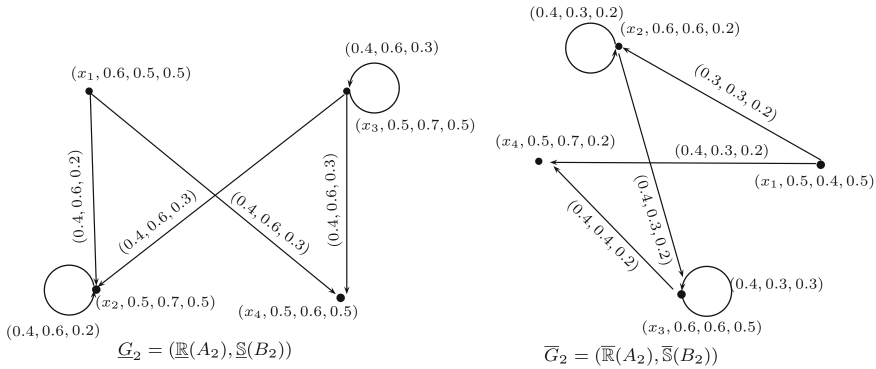


Fig. 8.3 Neutrosophic soft rough graph $G_2 = (\underline{G}_2, \overline{G}_2)$

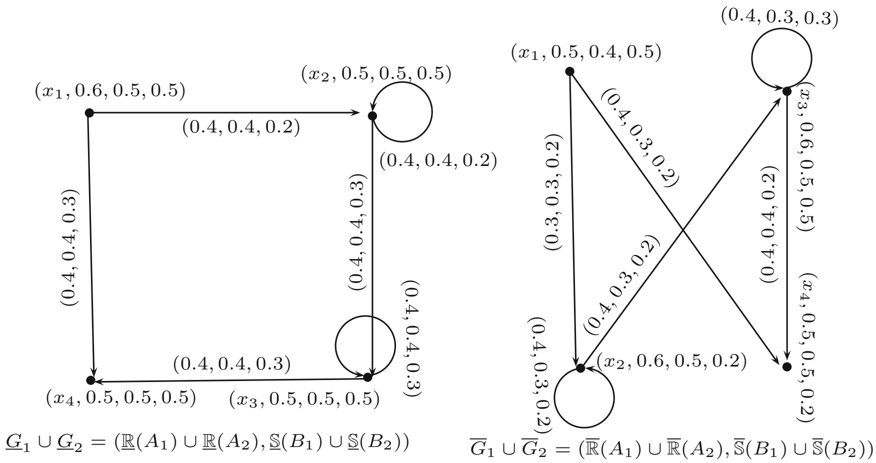


Fig. 8.4 Neutrosophic soft rough graph $G_1 \cup G_2 = (\underline{G}_1 \cup \underline{G}_2, \overline{G}_1 \cup \overline{G}_2)$

$$\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)\cap\overline{\mathbb{R}}(A_2)}(x) &= T_{\overline{\mathbb{R}}(A_1)}(x),\ T_{\underline{\mathbb{R}}(A_1)\cap\underline{\mathbb{R}}(A_2)}(x) = T_{\underline{\mathbb{R}}(A_1)}(x),\\
I_{\overline{\mathbb{R}}(A_1)\cap\overline{\mathbb{R}}(A_2)}(x) &= I_{\overline{\mathbb{R}}(A_1)}(x),\ \ I_{\underline{\mathbb{R}}(A_1)\cap\underline{\mathbb{R}}(A_2)}(x) = I_{\underline{\mathbb{R}}(A_1)}(x),\\
F_{\overline{\mathbb{R}}(A_1)\cap\overline{\mathbb{R}}(A_2)}(x) &= F_{\overline{\mathbb{R}}(A_1)}(x),\ F_{\underline{\mathbb{R}}(A_1)\cap\underline{\mathbb{R}}(A_2)}(x) = F_{\underline{\mathbb{R}}(A_1)}(x).
\end{aligned}$$

(ii) $\forall x \notin \mathbb{R}A_1$ but $x \in \mathbb{R}A_2$.

$$\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)\cap\overline{\mathbb{R}}(A_2)}(x) &= T_{\overline{\mathbb{R}}(A_2)}(x),\ T_{\underline{\mathbb{R}}(A_1)\cap\underline{\mathbb{R}}(A_2)}(x) = T_{\underline{\mathbb{R}}(A_2)}(x),\\
I_{\overline{\mathbb{R}}(A_1)\cap\overline{\mathbb{R}}(A_2)}(x) &= I_{\overline{\mathbb{R}}(A_2)}(x),\ \ I_{\underline{\mathbb{R}}(A_1)\cap\underline{\mathbb{R}}(A_2)}(x) = I_{\underline{\mathbb{R}}(A_2)}(x),\\
F_{\overline{\mathbb{R}}(A_1)\cap\overline{\mathbb{R}}(A_2)}(x) &= F_{\overline{\mathbb{R}}(A_2)}(x),\ F_{\underline{\mathbb{R}}(A_1)\cap\underline{\mathbb{R}}(A_2)}(x) = F_{\underline{\mathbb{R}}(A_2)}(x).
\end{aligned}$$

(iii) $\forall x \in \mathbb{R}A_1 \cap \mathbb{R}A_2$

$$\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)\cap\overline{\mathbb{R}}(A_2)}(x) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{R}}(A_2)}(x)\},\\
T_{\underline{\mathbb{R}}(A_1)\cap\underline{\mathbb{R}}(A_2)}(x) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{R}}(A_2)}(x)\},\\
I_{\overline{\mathbb{R}}(A_1)\cap\overline{\mathbb{R}}(A_2)}(x) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{R}}(A_2)}(x)\},\\
I_{\underline{\mathbb{R}}(A_1)\cap\underline{\mathbb{R}}(A_2)}(v) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{R}}(A_2)}(x)\},\\
F_{\overline{\mathbb{R}}(A_1)\cap\overline{\mathbb{R}}(A_2)}(x) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{R}}(A_2)}(x)\},\\
F_{\underline{\mathbb{R}}(A_1)\cap\underline{\mathbb{R}}(A_2)}(v) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{R}}(A_2)}(x)\}.
\end{aligned}$$

(iv) $\forall xy \in \mathbb{S}B_1$ but $xy \notin \mathbb{S}B_2$.

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\cap\overline{\mathbb{S}}(B_2)}(xy) &= T_{\overline{\mathbb{S}}(B_1)}(xy),\ T_{\underline{\mathbb{S}}(B_1)\cap\underline{\mathbb{S}}(B_2)}(xy) = T_{\underline{\mathbb{S}}(B_1)}(xy),\\
I_{\overline{\mathbb{S}}(B_1)\cap\overline{\mathbb{S}}(B_2)}(xy) &= I_{\overline{\mathbb{S}}(B_1)}(xy),\ \ I_{\underline{\mathbb{S}}(B_1)\cap\underline{\mathbb{S}}(B_2)}(xy) = I_{\underline{\mathbb{S}}(B_1)}(xy),\\
F_{\overline{\mathbb{S}}(B_1)\cap\overline{\mathbb{S}}(B_2)}(xy) &= F_{\overline{\mathbb{S}}(B_1)}(xy),\ F_{\underline{\mathbb{S}}(B_1)\cap\underline{\mathbb{S}}(B_2)}(xy) = F_{\underline{\mathbb{S}}(B_1)}(xy).
\end{aligned}$$

(v) $\forall xy \notin \mathbb{S}B_1$ but $xy \in \mathbb{S}B_2$

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\cap\overline{\mathbb{S}}(B_2)}(xy) &= T_{\overline{\mathbb{S}}(B_2)}(xy),\ T_{\underline{\mathbb{S}}(B_1)\cap\underline{\mathbb{S}}(B_2)}(xy) = T_{\underline{\mathbb{S}}(B_2)}(xy),\\
I_{\overline{\mathbb{S}}(B_1)\cap\overline{\mathbb{S}}(B_2)}(xy) &= I_{\overline{\mathbb{S}}(B_2)}(xy),\ \ I_{\underline{\mathbb{S}}(B_1)\cap\underline{\mathbb{S}}(B_2)}(xy) = I_{\underline{\mathbb{S}}(B_2)}(xy),\\
F_{\overline{\mathbb{S}}(B_1)\cap\overline{\mathbb{S}}(B_2)}(xy) &= F_{\overline{\mathbb{S}}(B_2)}(xy),\ F_{\underline{\mathbb{S}}(B_1)\cap\underline{\mathbb{S}}(B_2)}(xy) = F_{\underline{\mathbb{S}}(B_2)}(xy).
\end{aligned}$$

(vi) $\forall xy \in \mathbb{S}B_1 \cap \mathbb{S}(B_2)$

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\cap\overline{\mathbb{S}}(B_2)}(xy) &= \min\{T_{\overline{\mathbb{S}}(B_1)}(xy), T_{\overline{\mathbb{S}}(B_2)}(xy)\},\\
T_{\underline{\mathbb{S}}(B_1)\cap\underline{\mathbb{S}}(B_2)}(xy) &= \min\{T_{\underline{\mathbb{S}}(B_1)}(xy), T_{\underline{\mathbb{S}}(B_2)}(xy)\},\\
I_{\overline{\mathbb{S}}(B_1)\cap\overline{\mathbb{S}}(B_2)}(xy) &= \max\{I_{\overline{\mathbb{S}}(B_1)}(xy), I_{\overline{\mathbb{S}}(B_2)}(xy)\},\\
I_{\underline{\mathbb{S}}(B_1)\cap\underline{\mathbb{S}}(B_2)}(xy) &= \max\{I_{\underline{\mathbb{S}}(B_1)}(xy), I_{\underline{\mathbb{S}}(B_2)}(xy)\},\\
F_{\overline{\mathbb{S}}(B_1)\cap\overline{\mathbb{S}}(B_2)}(xy) &= \max\{F_{\overline{\mathbb{S}}(B_1)}(xy), F_{\overline{\mathbb{S}}(B_2)}(xy)\},\\
F_{\underline{\mathbb{S}}(B_1)\cap\underline{\mathbb{S}}(B_2)}(xy) &= \max\{F_{\underline{\mathbb{S}}(B_1)}(xy), F_{\underline{\mathbb{S}}(B_2)}(xy)\}.
\end{aligned}$$

Definition 8.6 Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic soft rough graphs on X. The *join* of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1 + G_2 = (\underline{G}_1 + \underline{G}_2, \overline{G}_1 + \overline{G}_2)$, where $\underline{G}_1 + \underline{G}_2 = (\underline{\mathbb{R}}(A_1) + \underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_1) + \underline{\mathbb{S}}(B_2))$ and $\overline{G}_1 + \overline{G}_2 = (\overline{\mathbb{R}}(A_1) + \overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_1) + \overline{\mathbb{S}}(B_2))$ are neutrosophic graphs, respectively, such that

(i) $\forall x \in \mathbb{R}A_1$ but $x \notin \mathbb{R}A_2$.

$$\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)+\overline{\mathbb{R}}(A_2)}(x) &= T_{\overline{\mathbb{R}}(A_1)}(x),\ T_{\underline{\mathbb{R}}(A_1)+\underline{\mathbb{R}}(A_2)}(x) = T_{\underline{\mathbb{R}}(A_1)}(x),\\
I_{\overline{\mathbb{R}}(A_1)+\overline{\mathbb{R}}(A_2)}(x) &= I_{\overline{\mathbb{R}}(A_1)}(x),\ \ I_{\underline{\mathbb{R}}(A_1)+\underline{\mathbb{R}}(A_2)}(x) = I_{\underline{\mathbb{R}}(A_1)}(x),\\
F_{\overline{\mathbb{R}}(A_1)+\overline{\mathbb{R}}(A_2)}(x) &= F_{\overline{\mathbb{R}}(A_1)}(x),\ F_{\underline{\mathbb{R}}(A_1)+\underline{\mathbb{R}}(A_2)}(x) = F_{\underline{\mathbb{R}}(A_1)}(x).
\end{aligned}$$

(ii) $\forall x \notin \mathbb{R}A_1$ but $x \in \mathbb{R}A_2$.

$$T_{\overline{\mathbb{R}}(A_1)+\overline{\mathbb{R}}(A_2)}(x) = T_{\overline{\mathbb{R}}(A_2)}(x),\ T_{\underline{\mathbb{R}}(A_1)+\underline{\mathbb{R}}(A_2)}(x) = T_{\underline{\mathbb{R}}(A_2)}(x),$$
$$I_{\overline{\mathbb{R}}(A_1)+\overline{\mathbb{R}}(A_2)}(x) = I_{\overline{\mathbb{R}}(A_2)}(x),\ I_{\underline{\mathbb{R}}(A_1)+\underline{\mathbb{R}}(A_2)}(x) = I_{\underline{\mathbb{R}}(A_2)}(x),$$
$$F_{\overline{\mathbb{R}}(A_1)+\overline{\mathbb{R}}(A_2)}(x) = F_{\overline{\mathbb{R}}(A_2)}(x),\ F_{\underline{\mathbb{R}}(A_1)+\underline{\mathbb{R}}(A_2)}(x) = F_{\underline{\mathbb{R}}(A_2)}(x).$$

(iii) $\forall x \in \mathbb{R}A_1 \cap \mathbb{R}A_2$

$$T_{\overline{\mathbb{R}}(A_1)+\overline{\mathbb{R}}(A_2)}(x) = \max\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{R}}(A_2)}(x)\},$$
$$T_{\underline{\mathbb{R}}(A_1)+\underline{\mathbb{R}}(A_2)}(x) = \max\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{R}}(A_2)}(x)\},$$
$$I_{\overline{\mathbb{R}}(A_1)+\overline{\mathbb{R}}(A_2)}(x) = \min\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{R}}(A_2)}(x)\},$$
$$I_{\underline{\mathbb{R}}(A_1)+\underline{\mathbb{R}}(A_2)}(x) = \min\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{R}}(A_2)}(x)\},$$
$$F_{\overline{\mathbb{R}}(A_1)+\overline{\mathbb{R}}(A_2)}(x) = \min\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{R}}(A_2)}(x)\},$$
$$F_{\underline{\mathbb{R}}(A_1)+\underline{\mathbb{R}}(A_2)}(x) = \min\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{R}}(A_2)}(x)\}.$$

(iv) $\forall xy \in \mathbb{S}B_1$ but $xy \notin \mathbb{S}B_2$.

$$T_{\overline{\mathbb{S}}(B_1)+\overline{\mathbb{S}}(B_2)}(xy) = T_{\overline{\mathbb{S}}(B_1)}(xy),\ T_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) = T_{\underline{\mathbb{S}}(B_1)}(xy),$$
$$I_{\overline{\mathbb{S}}(B_1)+\overline{\mathbb{S}}(B_2)}(xy) = I_{\overline{\mathbb{S}}(B_1)}(xy),\ I_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) = I_{\underline{\mathbb{S}}(B_1)}(xy),$$
$$F_{\overline{\mathbb{S}}(B_1)+\overline{\mathbb{S}}(B_2)}(xy) = F_{\overline{\mathbb{S}}(B_1)}(xy),\ F_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) = F_{\underline{\mathbb{S}}(B_1)}(xy).$$

(v) $\forall xy \notin \mathbb{S}B_1$ but $xy \in \mathbb{S}B_2$

$$T_{\overline{\mathbb{S}}(B_1)+\overline{\mathbb{S}}(B_2)}(xy) = T_{\overline{\mathbb{S}}(B_2)}(xy),\ T_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) = T_{\underline{\mathbb{S}}(B_2)}(xy),$$
$$I_{\overline{\mathbb{S}}(B_1)+\overline{\mathbb{S}}(B_2)}(xy) = I_{\overline{\mathbb{S}}(B_2)}(xy),\ I_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) = I_{\underline{\mathbb{S}}(B_2)}(xy),$$
$$F_{\overline{\mathbb{S}}(B_1)+\overline{\mathbb{S}}(B_2)}(xy) = F_{\overline{\mathbb{S}}(B_2)}(xy),\ F_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) = F_{\underline{\mathbb{S}}(B_2)}(xy).$$

(vi) $\forall xy \in \mathbb{S}B_1 \cap \mathbb{S}(B_2)$

$$T_{\overline{\mathbb{S}}(B_1)+\overline{\mathbb{S}}(B_2)}(xy) = \max\{T_{\overline{\mathbb{S}}(B_1)}(xy), T_{\overline{\mathbb{S}}(B_2)}(xy)\},$$
$$T_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) = \max\{T_{\underline{\mathbb{S}}(B_1)}(xy), T_{\underline{\mathbb{S}}(B_2)}(xy)\},$$
$$I_{\overline{\mathbb{S}}(B_1)+\overline{\mathbb{S}}(B_2)}(xy) = \min\{I_{\overline{\mathbb{S}}(B_1)}(xy), I_{\overline{\mathbb{S}}(B_2)}(xy)\},$$
$$I_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) = \min\{I_{\underline{\mathbb{S}}(B_1)}(xy), I_{\underline{\mathbb{S}}(B_2)}(xy)\},$$
$$F_{\overline{\mathbb{S}}(B_1)+\overline{\mathbb{S}}(B_2)}(xy) = \min\{F_{\overline{\mathbb{S}}(B_1)}(xy), F_{\overline{\mathbb{S}}(B_2)}(xy)\},$$
$$F_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) = \min\{F_{\underline{\mathbb{S}}(B_1)}(xy), F_{\underline{\mathbb{S}}(B_2)}(xy)\}.$$

(vii) $\forall xy \in \tilde{E}$, where $\tilde{E}$ is the set of edges joining vertices of $\mathbb{R}A_1$ and $\mathbb{R}A_2$.

$$
\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)+\overline{\mathbb{S}}(B_2)}(xy) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{R}}(A_2)}(y)\},\\
T_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{R}}(A_2)}(y)\},\\
I_{\overline{\mathbb{S}}(B_1)\text{`}+\overline{\mathbb{S}}(B_2)}(xy) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{R}}(A_2)}(y)\},\\
I_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{R}}(A_2)}(y)\},\\
F_{\overline{\mathbb{S}}(B_1)+\overline{\mathbb{S}}(B_2)}(xy) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{R}}(A_2)}(y)\},\\
F_{\underline{\mathbb{S}}(B_1)+\underline{\mathbb{S}}(B_2)}(xy) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{R}}(A_2)}(y)\}.
\end{aligned}
$$

Definition 8.7 The *Cartesian product* of G_1 and G_2 is a $G = G_1 \ltimes G_2 = (\underline{G}_1 \ltimes \underline{G}_2, \overline{G}_1 \ltimes \overline{G}_2)$, where $\underline{G}_1 \ltimes \underline{G}_2 = (\underline{\mathbb{R}}(A_1) \ltimes \underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_1) \ltimes \underline{\mathbb{S}}(B_2))$ and $\overline{G}_1 \ltimes \overline{G}_2 = (\overline{\mathbb{R}}(A_1) \ltimes \overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_1) \ltimes \overline{\mathbb{S}}(B_2))$ are neutrosophic graphs, such that

(i) $\forall\ (x, y) \in \mathbb{R}A_1 \times \mathbb{R}A_2$.

$$
\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)\ltimes\overline{\mathbb{R}}(A_2)}(x, y) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{R}}(A_2)}(x)\},\\
T_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x, y) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{R}}(A_2)}(x)\},\\
I_{\overline{\mathbb{R}}(A_1)\ltimes\overline{\mathbb{R}}(A_2)}(x, y) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{R}}(A_2)}(x)\},\\
I_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x, y) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{R}}(A_2)}(x)\},\\
F_{\overline{\mathbb{R}}(A_1)\ltimes\overline{\mathbb{R}}(A_2)}(x, y) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{R}}(A_2)}(x)\},\\
F_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x, y) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{R}}(A_2)}(x_1)\}.
\end{aligned}
$$

(ii) $\forall y_1y_2 \in \mathbb{S}B_2, x \in \mathbb{R}A_1$.

$$
\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\ltimes\overline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\overline{\mathbb{S}}(B_1)\ltimes\overline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\overline{\mathbb{S}}(B_1)\ltimes\overline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\}.
\end{aligned}
$$

(iii) $\forall x_1x_2 \in \mathbb{S}B_1, y \in \mathbb{R}A_2$.

$$
\begin{aligned}
T_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \min\{T_{\underline{\mathbb{S}}(B_1)}(x_1x_2), T_{\underline{\mathbb{R}}(A_2)}(y)\},\\
T_{\overline{\mathbb{S}}(B_1)\ltimes\overline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \min\{T_{\overline{\mathbb{S}}(B_1)}(x_1x_2), T_{\overline{\mathbb{R}}(A_2)}(y)\},\\
I_{\overline{\mathbb{S}}(B_1)\ltimes\overline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \max\{I_{\overline{\mathbb{S}}(B_1)}(x_1x_2), I_{\overline{\mathbb{R}}(A_2)}(y)\},\\
I_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \max\{I_{\underline{\mathbb{S}}(B_1)}(x_1x_2), I_{\underline{\mathbb{R}}(A_2)}(y)\},\\
F_{\overline{\mathbb{S}}(B_1)\ltimes\overline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \max\{F_{\overline{\mathbb{S}}(B_1)}(x_1x_2), F_{\overline{\mathbb{R}}(A_2)}(y)\},\\
F_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \max\{F_{\underline{\mathbb{S}}(B_1)}(x_1x_2), F_{\underline{\mathbb{R}}(A_2)}(y)\}.
\end{aligned}
$$

Definition 8.8 The *cross product* of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1 \circledcirc G_2 = (\underline{G}_1 \circledcirc \underline{G}_2, \overline{G}_1 \circledcirc \overline{G}_2)$, where $\underline{G}_1 \circledcirc \underline{G}_2 = (\underline{\mathbb{R}}(A_1) \circledcirc \underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_1) \circledcirc \underline{\mathbb{S}}(B_2))$ and $\overline{G}_1 \circledcirc \overline{G}_2 = (\overline{\mathbb{R}}(A_1) \circledcirc \overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_1) \circledcirc \overline{\mathbb{S}}(B_2))$ are neutrosophic graphs, respectively, such that

(i) $\forall\ (x, y) \in \mathbb{R}A_1 \times \mathbb{R}A_2$.

$$\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)\circledcirc\overline{\mathbb{R}}(A_2)}(x, y) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{R}}(A_2)}(x)\},\\
T_{\underline{\mathbb{R}}(A_1)\circledcirc\underline{\mathbb{R}}(A_2)}(x, y) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{R}}(A_2)}(x)\},\\
I_{\overline{\mathbb{R}}(A_1)\circledcirc\overline{\mathbb{R}}(A_2)}(x, y) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{R}}(A_2)}(x)\},\\
I_{\underline{\mathbb{R}}(A_1)\circledcirc\underline{\mathbb{R}}(A_2)}(x, y) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{R}}(A_2)}(x)\},\\
F_{\overline{\mathbb{R}}(A_1)\circledcirc\overline{\mathbb{R}}(A_2)}(x, y) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{R}}(A_2)}(x)\},\\
F_{\underline{\mathbb{R}}(A_1)\circledcirc\underline{\mathbb{R}}(A_2)}(x, y) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{R}}(A_2)}(x)\}.
\end{aligned}$$

(ii) $\forall x_1x_2 \in \mathbb{S}B_1,\ y_1y_2 \in \mathbb{S}B_2$.

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\circledcirc\overline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \min\{T_{\overline{\mathbb{S}}(B_1)}(x_1x_2), T_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)\circledcirc\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \min\{T_{\underline{\mathbb{S}}(B_1)}(x_1x_2), T_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\overline{\mathbb{S}}(B_1)\circledcirc\overline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{I_{\overline{\mathbb{S}}(B_1)}(x_1x_2), I_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\underline{\mathbb{S}}(B_1)\circledcirc\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{I_{\underline{\mathbb{S}}(B_1)}(x_1x_2), I_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\overline{\mathbb{S}}(B_1)\circledcirc\overline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{F_{\overline{\mathbb{S}}(B_1)}(x_1x_2), F_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\underline{\mathbb{S}}(B_1)\circledcirc\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{F_{\underline{\mathbb{S}}(B_1)}(x_1x_2), F_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\}.
\end{aligned}$$

Definition 8.9 The *rejection* of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1|G_2 = (\underline{G}_1|\underline{G}_2, \overline{G}_1|\overline{G}_2)$, where $\underline{G}_1|\underline{G}_2 = (\underline{\mathbb{S}}A_1|\underline{\mathbb{S}}A_2, \underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2))$ and $\overline{G}_1|\overline{G}_2 = (\overline{\mathbb{S}}A_1|\overline{\mathbb{S}}A_2, \overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2))$ are neutrosophic graphs such that

(i) $\forall\ (x, y) \in \mathbb{R}A_1 \times \mathbb{R}A_2$.

$$\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)|\overline{\mathbb{R}}(A_2)}(x, y) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{R}}(A_2)}(y)\},\\
T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x, y) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{R}}(A_2)}(y)\},\\
I_{\overline{\mathbb{R}}(A_1)|\overline{\mathbb{R}}(A_2)}(x, y) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{R}}(A_2)}(y)\},\\
I_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x, y) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{R}}(A_2)}(y)\},\\
F_{\overline{\mathbb{R}}(A_1)|\overline{\mathbb{R}}(A_2)}(x, y) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{R}}(A_2)}(y)\},\\
F_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x, y) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{R}}(A_2)}(y)\}.
\end{aligned}$$

(ii) $\forall y_1y_2 \notin \mathbb{S}B_2,\ \ x \in \mathbb{R}A_1$.

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{R}}(A_2)}(y_1), T_{\overline{\mathbb{R}}(A_2)}(y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{R}}B_2)}\big((x, y_1)(x, y_2)\big) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{R}}(A_2)}(y_1), T_{\underline{\mathbb{R}}(A_2)}(y_2)\},
\end{aligned}$$

$$(I_{\overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) = \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{R}}(A_2)}(y_1), I_{\overline{\mathbb{R}}(A_2)}(y_2)\},$$
$$(I_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) = \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{R}}(A_2)}(y_1), I_{\underline{\mathbb{R}}(A_2)}(y_2)\},$$
$$(F_{\overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) = \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{R}}(A_2)}(y_1), F_{\overline{\mathbb{R}}(A_2)}(y_2)\},$$
$$(F_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) = \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{R}}(A_2)}(y_1), F_{\underline{\mathbb{R}}(A_2)}(y_2)\}.$$

(iii) $\forall x_1x_2 \notin \mathbb{S}B_1,\ y \in \mathbb{R}A_2,$

$$T_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) = \min\{T_{\underline{\mathbb{R}}(A_1)}(x_1), T_{\underline{\mathbb{R}}(A)_1}(x_2), T_{\underline{\mathbb{R}}(A_2)}(y)\},$$
$$I_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) = \max\{I_{\underline{\mathbb{R}}(A_1)}(x_1), I_{\underline{\mathbb{R}}(A)_1}(x_2), I_{\underline{\mathbb{R}}(A_2)}(y)\},$$
$$F_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) = \max\{F_{\underline{\mathbb{R}}(A_1)}(x_1), F_{\underline{\mathbb{R}}(A)_1}(x_2), F_{\underline{\mathbb{R}}(A_2)}(y)\},$$
$$T_{\overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) = \min\{T_{\overline{\mathbb{R}}(A_1)}(x_1), T_{\overline{\mathbb{R}}(A)_1}(x_2), T_{\overline{\mathbb{R}}(A_2)}(y)\},$$
$$I_{\overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) = \max\{I_{\overline{\mathbb{R}}(A_1)}(x_1), I_{\overline{\mathbb{R}}(A)_1}(x_2), I_{\overline{\mathbb{R}}(A_2)}(y)\},$$
$$F_{\overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) = \max\{F_{\overline{\mathbb{R}}(A_1)}(x_1), F_{\overline{\mathbb{R}}(A)_1}(x_2), F_{\overline{\mathbb{R}}(A_2)}(y)\}.$$

(iv) $\forall x_1x_2 \notin \mathbb{S}B_1,\ y_1y_2 \notin \mathbb{S}B_2,\ x_1 \neq x_2, y_1 \neq y_2.$

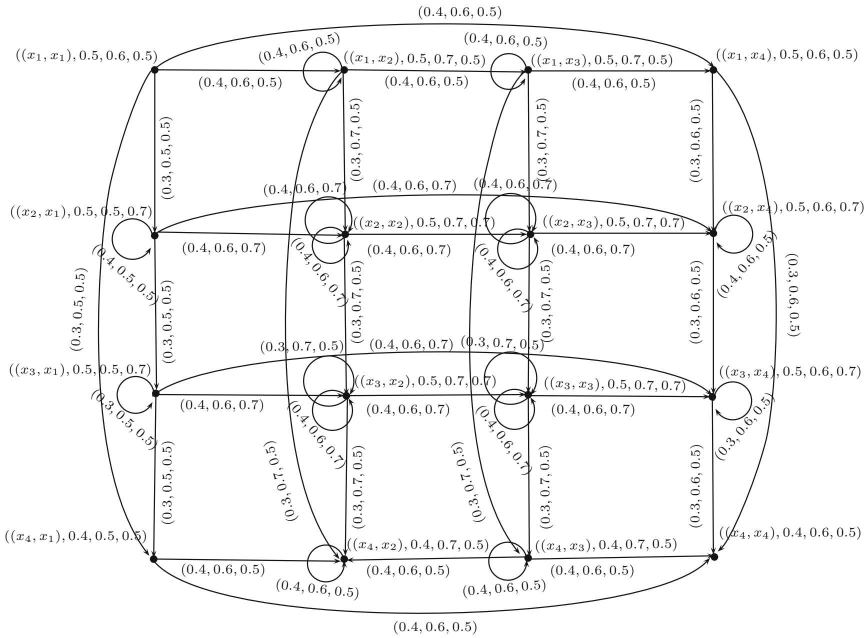
$$T_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) = \min\{T_{\underline{\mathbb{R}}(A_1)}(x_1), T_{\underline{\mathbb{R}}(A)_1}(x_2), T_{\underline{\mathbb{R}}(A_2)}(y_1), T_{\underline{\mathbb{R}}(A_2)}(y_2)\},$$
$$I_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) = \max\{I_{\underline{\mathbb{R}}(A_1)}(x_1), I_{\underline{\mathbb{R}}(A)_1}(x_2), I_{\underline{\mathbb{R}}(A_2)}(y_1), I_{\underline{\mathbb{R}}(A_2)}(y_2)\},$$
$$F_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) = \max\{F_{\underline{\mathbb{R}}(A_1)}(x_1), F_{\underline{\mathbb{R}}(A)_1}(x_2), F_{\underline{\mathbb{R}}(A_2)}(y_1), F_{\underline{\mathbb{R}}(A_2)}(y_2)\},$$
$$T_{\overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) = \min\{T_{\overline{\mathbb{R}}(A_1)}(x_1), T_{\overline{\mathbb{R}}(A)_1}(x_2), T_{\overline{\mathbb{R}}(A_2)}(y_1), T_{\overline{\mathbb{R}}(A_2)}(y_2)\},$$
$$I_{\overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) = \max\{I_{\overline{\mathbb{R}}(A_1)}(x_1), I_{\overline{\mathbb{R}}(A)_1}(x_2), I_{\overline{\mathbb{R}}(A_2)}(y_1), I_{\overline{\mathbb{R}}(A_2)}(y_2)\},$$
$$F_{\overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) = \max\{F_{\overline{\mathbb{R}}(A_1)}(x_1), F_{\overline{\mathbb{R}}(A)_1}(x_2), F_{\overline{\mathbb{R}}(A_2)}(y_1), F_{\overline{\mathbb{R}}(A_2)}(y_2)\},$$

Example 8.5 Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic soft rough graphs on X, where $\underline{G}_1 = (\underline{\mathbb{R}}(A_1), \underline{\mathbb{S}}(B_1))$ and $\overline{G}_1 = (\overline{\mathbb{R}}(A_1), \overline{\mathbb{S}}(B_1))$ are neutrosophic graphs as shown in Fig. 8.2 and $\underline{G}_2 = (\underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_2))$ and $\overline{G}_2 = (\overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_2))$ are neutrosophic graphs as shown in Fig. 8.3. The Cartesian product of $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ is neutrosophic soft rough graph $G = G_1 \times G_2 = (\underline{G}_1 \times \underline{G}_2, \overline{G}_1 \times \overline{G}_2)$ as shown in Fig. 8.5.

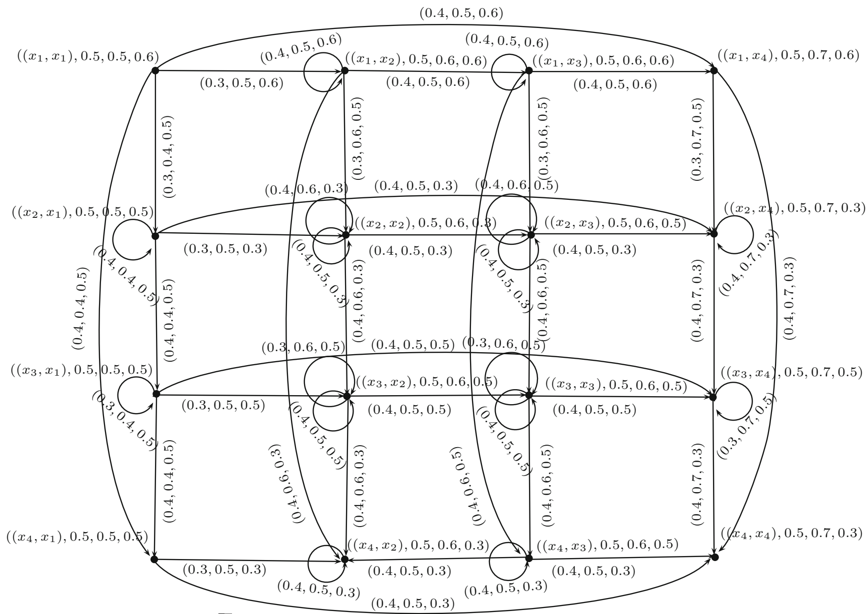
Definition 8.10 The *symmetric difference* of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1 \oplus G_2 = (\underline{G}_1 \oplus \underline{G}_2, \overline{G}_1 \oplus \overline{G}_2)$, where $\underline{G}_1 \oplus \underline{G}_2 = (\underline{\mathbb{R}}(A_1) \oplus \underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_1) \oplus \underline{\mathbb{S}}(B_2))$ and $\overline{G}_1 \oplus \overline{G}_2 = (\overline{\mathbb{R}}(A_1) \oplus \overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_1) \oplus \overline{\mathbb{S}}(B_2))$ are neutrosophic graphs, respectively, such that

(i) $\forall\ (x, y) \in \mathbb{R}A_1 \times \mathbb{R}A_2.$

$$T_{\overline{\mathbb{R}}(A_1)\oplus\overline{\mathbb{R}}(A_2)}(x,y) = \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{R}}(A_2)}(y)\},$$
$$T_{\underline{\mathbb{R}}(A_1)\oplus\underline{\mathbb{R}}(A_2)}(x,y) = \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{R}}(A_2)}(y)\},$$



$\underline{G}_1 \times \underline{G}_2 = (\underline{\mathbb{S}}A_1 \times \underline{\mathbb{S}}A_2, \underline{\mathbb{S}}(B_1) \times \underline{\mathbb{S}}(B_2))$



$\overline{G}_1 \times \overline{G}_2 = (\overline{\mathbb{S}}A_1 \times \overline{\mathbb{S}}A_2, \overline{\mathbb{S}}(B_1) \times \overline{\mathbb{S}}(B_2))$

Fig. 8.5 Cartesian product of two neutrosophic soft rough graphs $G_1 \times G_2$

$$\begin{aligned}
I_{\overline{\mathbb{R}}(A_1)\oplus\overline{\mathbb{R}}(A_2)}(x,y) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{R}}(A_2)}(y)\},\\
I_{\underline{\mathbb{R}}(A_1)\oplus\underline{\mathbb{R}}(A_2)}(x,y) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{R}}(A_2)}(y)\},\\
F_{\overline{\mathbb{R}}(A_1)\oplus\overline{\mathbb{R}}(A_2)}(x,y) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{R}}(A_2)}(y)\},\\
F_{\underline{\mathbb{R}}(A_1)\oplus\underline{\mathbb{R}}(A_2)}(x,y) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{R}}(A_2)}(y)\}.
\end{aligned}$$

(ii) $\forall y_1y_2 \in \mathbb{S}B_2,\ x \in \mathbb{R}A_1.$

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\}.
\end{aligned}$$

(iii) $\forall x_1x_2 \in \mathbb{S}B_1,\ y \in \mathbb{R}A_2.$

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &= \min\{T_{\overline{\mathbb{S}}(B_1)}(x_1x_2), T_{\overline{\mathbb{R}}(A_2)}(y)\},\\
T_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &= \min\{T_{\underline{\mathbb{S}}(B_1)}(x_1x_2), T_{\underline{\mathbb{R}}(A_2)}(y)\},\\
I_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &= \max\{I_{\overline{\mathbb{S}}(B_1)}(x_1x_2), I_{\overline{\mathbb{R}}(A_2)}(y)\},\\
I_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &= \max\{I_{\underline{\mathbb{S}}(B_1)}(x_1x_2), I_{\underline{\mathbb{R}}(A_2)}(y)\},\\
F_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &= \max\{F_{\overline{\mathbb{S}}(B_1)}(x_1x_2), F_{\overline{\mathbb{R}}(A_2)}(y)\},\\
F_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &= \max\{F_{\underline{\mathbb{S}}(B_1)}(x_1x_2), F_{\underline{\mathbb{R}}(A_2)}(y)\}.
\end{aligned}$$

(iv) $\forall x_1x_2 \in \mathbb{S}B_1,\ y_1y_2 \notin \mathbb{S}B_2.$

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &= \min\{T_{\overline{\mathbb{S}}(B_1)}(x_1x_2), T_{\overline{\mathbb{R}}(A_2)}(y_1), T_{\overline{\mathbb{R}}(A_2)}(y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &= \min\{T_{\underline{\mathbb{S}}(B_1)}(x_1x_2), T_{\underline{\mathbb{R}}(A_2)}(y_1), T_{\underline{\mathbb{R}}(A_2)}(y_2)\},\\
I_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &= \max\{I_{\overline{\mathbb{S}}(B_1)}(x_1x_2), I_{\overline{\mathbb{R}}(A_2)}(y_1), I_{\overline{\mathbb{R}}(A_2)}(y_2)\},\\
I_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &= \max\{I_{\underline{\mathbb{S}}(B_1)}(x_1x_2), I_{\underline{\mathbb{R}}(A_2)}(y_1), I_{\underline{\mathbb{R}}(A_2)}(y_2)\},\\
F_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &= \max\{F_{\overline{\mathbb{S}}(B_1)}(x_1x_2), F_{\overline{\mathbb{R}}(A_2)}(y_1), F_{\overline{\mathbb{R}}(A_2)}(y_2)\},\\
F_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &= \max\{F_{\underline{\mathbb{S}}(B_1)}(x_1x_2), F_{\underline{\mathbb{R}}(A_2)}(y_1), F_{\underline{\mathbb{R}}(A_2)}(y_2)\}.
\end{aligned}$$

(v) $\forall x_1x_2 \notin \mathbb{S}B_1,\ y_1y_2 \in \mathbb{S}B_2.$

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x_1), T_{\overline{\mathbb{R}}(A_1)}(x_2), T_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x_1), T_{\underline{\mathbb{R}}(A_1)}(x_2), T_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x_1), I_{\overline{\mathbb{R}}(A_1)}(x_2), I_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},
\end{aligned}$$

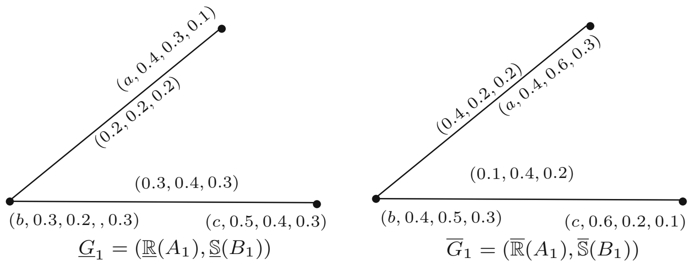


Fig. 8.6 Neutrosophic soft rough graph $G_1 = (\underline{G}_1, \overline{G}_1)$

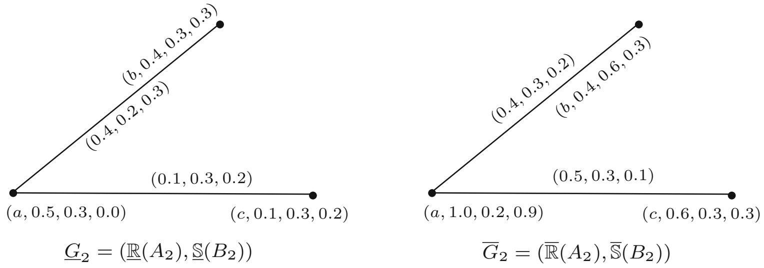


Fig. 8.7 Neutrosophic soft rough graph $G_2 = (\underline{G}_2, \overline{G}_2)$

$$
\begin{aligned}
I_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x_1), I_{\underline{\mathbb{R}}(A_1)}(x_2), I_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\overline{\mathbb{S}}(B_1)\oplus\overline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x_1), F_{\overline{\mathbb{R}}(A_1)}(x_2), F_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\underline{\mathbb{S}}(B_1)\oplus\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x_1), F_{\underline{\mathbb{R}}(A_1)}(x_2), F_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\}.
\end{aligned}
$$

Example 8.6 Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic soft rough graphs on X, where $\underline{G}_1 = (\underline{\mathbb{R}}(A_1), \underline{\mathbb{S}}(B_1))$ and $\overline{G}_1 = (\overline{\mathbb{R}}(A_1), \overline{\mathbb{S}}(B_1))$ are neutrosophic graphs as shown in Fig. 8.6 and $\underline{G}_2 = (\underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_2))$ and $\overline{G}_2 = (\overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_2))$ are neutrosophic graphs as shown in Fig. 8.7

The symmetric difference of G_1 and G_2 is $G = G_1 \oplus G_2 = (\underline{G}_1 \oplus \underline{G}_2, \overline{G}_1 \oplus \overline{G}_2)$, where $\underline{G}_1 \oplus \underline{G}_2 = (\underline{\mathbb{R}}(A_1) \oplus \underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_1) \oplus \underline{\mathbb{S}}(B_2))$ and $\overline{G}_1 \oplus \overline{G}_2 = (\overline{\mathbb{R}}(A_1) \oplus \overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_1) \oplus \overline{\mathbb{S}}(B_2))$ are neutrosophic graphs as shown in Fig. 8.8.

Definition 8.11 The *lexicographic product* of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1 \odot G_2 = (G_{1*} \odot G_{2*}, G_1^* \odot G_2^*)$, where $G_{1*} \odot G_{2*} = (\underline{\mathbb{R}}A_1 \odot \underline{\mathbb{R}}A_2, \underline{\mathbb{S}}B_1 \odot \underline{\mathbb{S}}B_2)$ and $G_1^* \odot G_2^* = (\overline{\mathbb{R}}A_1 \odot \overline{\mathbb{R}}A_2, \overline{\mathbb{S}}B_1 \odot \overline{\mathbb{S}}B_2)$ are neutrosophic graphs, respectively, such that

(i) $\forall\, (x, y) \in \mathbb{R}A_1 \times \mathbb{R}A_2$.

$$
\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)\odot\overline{\mathbb{R}}(A_2)}(x, y) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{R}}(A_2)}(y)\},\\
T_{\underline{\mathbb{R}}(A_1)\odot\underline{\mathbb{R}}(A_2)}(x, y) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{R}}(A_2)}(y)\},
\end{aligned}
$$

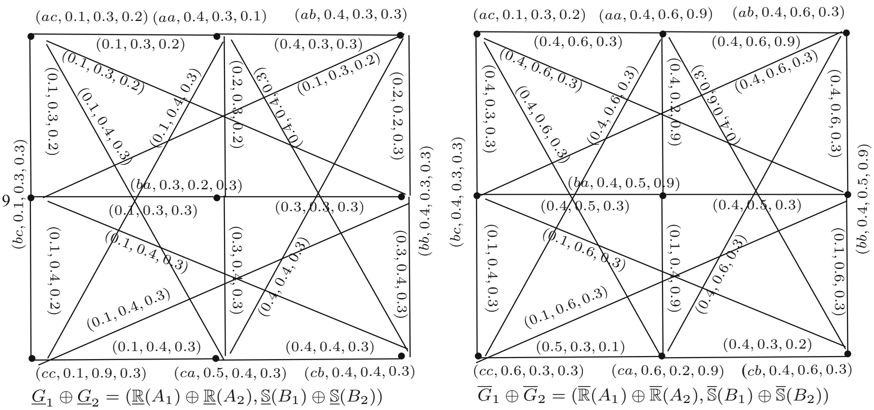


Fig. 8.8 Neutrosophic soft rough graph $G_1 \oplus G_2 = (\underline{G}_1 \oplus G_2, \overline{G}_1 \oplus \overline{G}_2)$

$$\begin{aligned}
I_{\overline{\mathbb{R}}(A_1)\odot\overline{\mathbb{R}}(A_2)}(x, y) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{R}}(A_2)}(y)\},\\
I_{\underline{\mathbb{R}}(A_1)\odot\underline{\mathbb{R}}(A_2)}(x, y) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{R}}(A_2)}(y)\},\\
F_{\overline{\mathbb{R}}(A_1)\odot\overline{\mathbb{R}}(A_2)}(x, y) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{R}}(A_2)}(y)\},\\
F_{\underline{\mathbb{R}}(A_1)\odot\underline{\mathbb{R}}(A_2)}(x, y) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{R}}(A_2)}(y)\}.
\end{aligned}$$

(ii) $\forall y_1 y_2 \in \mathbb{S}B_2,\ x \in \mathbb{R}A_1.$

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\odot\overline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{S}}(B_2)}(y_1 y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)\odot\underline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{S}}(B_2)}(y_1 y_2)\},\\
I_{\overline{\mathbb{S}}(B_1)\odot\overline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{S}}(B_2)}(y_1 y_2)\},\\
I_{\underline{\mathbb{S}}(B_1)\odot\underline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{S}}(B_2)}(y_1 y_2)\},\\
F_{\overline{\mathbb{S}}(B_1)\odot\overline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{S}}(B_2)}(y_1 y_2)\},\\
F_{\underline{\mathbb{S}}(B_1)\odot\underline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{S}}(B_2)}(y_1 y_2)\}.
\end{aligned}$$

(iii) $\forall x_1 x_2 \in \mathbb{S}B_1,\ y_1 y_2 \in \mathbb{S}B_2.$

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\odot\overline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \min\{T_{\overline{\mathbb{S}}(B_1)}(x_1 x_2), T_{\overline{\mathbb{S}}(B_2)}(y_1 y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)\odot\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \min\{T_{\underline{\mathbb{S}}(B_1)}(x_1 x_2), T_{\underline{\mathbb{S}}(B_2)}(y_1 y_2)\},\\
I_{\overline{\mathbb{S}}(B_1)\odot\overline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{I_{\overline{\mathbb{S}}(B_1)}(x_1 x_2), I_{\overline{\mathbb{S}}(B_2)}(y_1 y_2)\},\\
I_{\underline{\mathbb{S}}(B_1)\odot\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{I_{\underline{\mathbb{S}}(B_1)}(x_1 x_2), I_{\underline{\mathbb{S}}(B_2)}(y_1 y_2)\},\\
F_{\overline{\mathbb{S}}(B_1)\odot\overline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{F_{\overline{\mathbb{S}}(B_1)}(x_1 x_2), F_{\overline{\mathbb{S}}(B_2)}(y_1 y_2)\},\\
F_{\underline{\mathbb{S}}(B_1)\odot\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{F_{\underline{\mathbb{S}}(B_1)}(x_1 x_2), F_{\underline{\mathbb{S}}(B_2)}(y_1 y_2)\}.
\end{aligned}$$

Definition 8.12 The *strong product* of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1 \otimes G_2 = (G_{1*} \otimes G_{2*}, G_1^* \otimes G_2^*)$, where $G_{1*} \otimes G_{2*} = (\underline{\mathbb{R}}A_1 \otimes \underline{\mathbb{R}}A_2, \underline{\mathbb{S}}B_1 \otimes \underline{\mathbb{S}}B_2)$ and $G_1^* \otimes G_2^* = (\overline{\mathbb{R}}A_1 \otimes \overline{\mathbb{R}}A_2, \overline{\mathbb{S}}B_1 \otimes \overline{\mathbb{S}}B_2)$ are neutrosophic graphs, respectively, such that

(i) $\forall\ (x, y) \in \mathbb{R}A_1 \times \mathbb{R}A_2$.

$$\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)\otimes\overline{\mathbb{R}}(A_2)}(x, y) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{R}}(A_2)}(y)\},\\
T_{\underline{\mathbb{R}}(A_1)\otimes\underline{\mathbb{R}}(A_2)}(x, y) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{R}}(A_2)}(y)\},\\
I_{\overline{\mathbb{R}}(A_1)\otimes\overline{\mathbb{R}}(A_2)}(x, y) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{R}}(A_2)}(y)\},\\
I_{\underline{\mathbb{R}}(A_1)\otimes\underline{\mathbb{R}}(A_2)}(x, y) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{R}}(A_2)}(y)\},\\
F_{\overline{\mathbb{R}}(A_1)\otimes\overline{\mathbb{R}}(A_2)}(x, y) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{R}}(A_2)}(y)\},\\
F_{\underline{\mathbb{R}}(A_1)\otimes\underline{\mathbb{R}}(A_2)}(x, y) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{R}}(A_2)}(y)\}.
\end{aligned}$$

(ii) $\forall y_1y_2 \in \mathbb{S}B_2,\ x \in \mathbb{R}A_1$.

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\otimes\overline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \min\{T_{\overline{\mathbb{R}}(A_1)}(x), T_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)\otimes\underline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \min\{T_{\underline{\mathbb{R}}(A_1)}(x), T_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\overline{\mathbb{S}}(B_1)\otimes\overline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{I_{\overline{\mathbb{R}}(A_1)}(x), I_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\underline{\mathbb{S}}(B_1)\otimes\underline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{I_{\underline{\mathbb{R}}(A_1)}(x), I_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\overline{\mathbb{S}}(B_1)\otimes\overline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{F_{\overline{\mathbb{R}}(A_1)}(x), F_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\underline{\mathbb{S}}(B_1)\otimes\underline{\mathbb{S}}(B_2)}\big((x, y_1)(x, y_2)\big) &= \max\{F_{\underline{\mathbb{R}}(A_1)}(x), F_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\}.
\end{aligned}$$

(iii) $\forall x_1x_2 \in \mathbb{S}B_1,\ y \in \mathbb{R}A_2$.

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\otimes\overline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \min\{T_{\overline{\mathbb{S}}(B_1)}(x_1x_2), T_{\overline{\mathbb{R}}(A_2)}(y)\},\\
T_{\underline{\mathbb{S}}(B_1)\otimes\underline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \min\{T_{\underline{\mathbb{S}}(B_1)}(x_1x_2), T_{\underline{\mathbb{R}}(A_2)}(y)\},\\
I_{\overline{\mathbb{S}}(B_1)\otimes\overline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \max\{I_{\overline{\mathbb{S}}(B_1)}(x_1x_2), I_{\overline{\mathbb{R}}(A_2)}(y)\},\\
I_{\underline{\mathbb{S}}(B_1)\otimes\underline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \max\{I_{\underline{\mathbb{S}}(B_1)}(x_1x_2), I_{\underline{\mathbb{R}}(A_2)}(y)\},\\
F_{\overline{\mathbb{S}}(B_1)\otimes\overline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \max\{F_{\overline{\mathbb{S}}(B_1)}(x_1x_2), F_{\overline{\mathbb{R}}(A_2)}(y)\},\\
F_{\underline{\mathbb{S}}(B_1)\otimes\underline{\mathbb{S}}(B_2)}\big((x_1, y)(x_2, y)\big) &= \max\{F_{\underline{\mathbb{S}}(B_1)}(x_1x_2), F_{\underline{\mathbb{R}}(A_2)}(y)\}.
\end{aligned}$$

(iv) $\forall x_1x_2 \in \mathbb{S}B_1, y_1y_2 \in \mathbb{S}B_2$.

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\otimes\overline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \min\{T_{\overline{\mathbb{S}}(B_1)}(x_1x_2), T_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)\otimes\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \min\{T_{\underline{\mathbb{S}}(B_1)}(x_1x_2), T_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\overline{\mathbb{S}}(B_1)\otimes\overline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{I_{\overline{\mathbb{S}}(B_1)}(x_1x_2), I_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\underline{\mathbb{S}}(B_1)\otimes\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{I_{\underline{\mathbb{S}}(B_1)}(x_1x_2), I_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\overline{\mathbb{S}}(B_1)\otimes\overline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_1, y_2)\big) &= \max\{F_{\overline{\mathbb{S}}(B_1)}(x_1x_2), F_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},
\end{aligned}$$

$$F_{\underline{\mathbb{S}}(B_1)\otimes\underline{\mathbb{S}}(B_2)}\big((x_1,x_1)(x_1,x_2)\big)=\max\{F_{\underline{\mathbb{S}}(B_1)}(x_1x_2),\ F_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\}.$$

Definition 8.13 The *composition* of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1[G_2] = (\underline{G_{1*}}[\underline{G_{2*}}], \overline{G_1^*}[\overline{G_2^*}])$, where $G_{1*}[G_{2*}] = (\underline{\mathbb{R}}A_1[\underline{\mathbb{R}}A_2], \underline{\mathbb{S}}B_1[\underline{\mathbb{S}}B_2])]$ and $G_1^*[G_2^*] = (\overline{\mathbb{R}}A_1[\overline{\mathbb{R}}A_2], \overline{\mathbb{S}}B_1[\overline{\mathbb{S}}B_2])$ are neutrosophic graphs, respectively, such that

(i) $\forall (x,y) \in \mathbb{R}A_1 \times \mathbb{R}A_2$.

$$\begin{aligned}
T_{\overline{\mathbb{R}}(A_1)\times\overline{\mathbb{R}}(A_2)}(x,y) &=\min\{T_{\overline{\mathbb{R}}(A_1)}(x),\ T_{\overline{\mathbb{R}}(A_2)}(y)\},\\
T_{\underline{\mathbb{R}}(A_1)\times\underline{\mathbb{R}}(A_2)}(x,y) &=\min\{T_{\underline{\mathbb{R}}(A_1)}(x),\ T_{\underline{\mathbb{R}}(A_2)}(y)\},\\
I_{\overline{\mathbb{R}}(A_1)\times\overline{\mathbb{R}}(A_2)}(x,y) &=\max\{I_{\overline{\mathbb{R}}(A_1)}(x),\ I_{\overline{\mathbb{R}}(A_2)}(y)\},\\
I_{\underline{\mathbb{R}}(A_1)\times\underline{\mathbb{R}}(A_2)}(x,y) &=\max\{I_{\underline{\mathbb{R}}(A_1)}(x),\ I_{\underline{\mathbb{R}}(A_2)}(y)\},\\
F_{\overline{\mathbb{R}}(A_1)\times\overline{\mathbb{R}}(A_2)}(x,y) &=\max\{F_{\overline{\mathbb{R}}(A_1)}(x),\ F_{\overline{\mathbb{R}}(A_2)}(y)\},\\
F_{\underline{\mathbb{R}}(A_1)\times\underline{\mathbb{R}}(A_2)}(x,y) &=\max\{F_{\underline{\mathbb{R}}(A_1)}(x),\ F_{\underline{\mathbb{R}}(A_2)}(y)\}.
\end{aligned}$$

(ii) $\forall y_1y_2 \in \mathbb{S}B_2,\ x \in \mathbb{R}A_1$.

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\times\overline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &=\min\{T_{\overline{\mathbb{R}}(A_1)}(x),\ T_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)\times\underline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &=\min\{T_{\underline{\mathbb{R}}(A_1)}(x),\ T_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\overline{\mathbb{S}}(B_1)\times\overline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &=\max\{I_{\overline{\mathbb{R}}(A_1)}(x),\ I_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
I_{\underline{\mathbb{S}}(B_1)\times\underline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &=\max\{I_{\underline{\mathbb{R}}(A_1)}(x),\ I_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\overline{\mathbb{S}}(B_1)\times\overline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &=\max\{F_{\overline{\mathbb{R}}(A_1)}(x),\ F_{\overline{\mathbb{S}}(B_2)}(y_1y_2)\},\\
F_{\underline{\mathbb{S}}(B_1)\times\underline{\mathbb{S}}(B_2)}\big((x,y_1)(x,y_2)\big) &=\max\{F_{\underline{\mathbb{R}}(A_1)}(x),\ F_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\}.
\end{aligned}$$

(iii) $\forall x_1x_2 \in \mathbb{S}B_1,\ y \in \mathbb{R}A_2$.

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\times\overline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &=\min\{T_{\overline{\mathbb{S}}(B_1)}(x_1x_2),\ T_{\overline{\mathbb{R}}(A_2)}(y)\},\\
T_{\underline{\mathbb{S}}(B_1)\times\underline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &=\min\{T_{\underline{\mathbb{S}}(B_1)}(x_1x_2),\ T_{\underline{\mathbb{R}}(A_2)}(y)\},\\
I_{\overline{\mathbb{S}}(B_1)\times\overline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &=\max\{I_{\overline{\mathbb{S}}(B_1)}(x_1x_2),\ I_{\overline{\mathbb{R}}(A_2)}(y)\},\\
I_{\underline{\mathbb{S}}(B_1)\times\underline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &=\max\{I_{\underline{\mathbb{S}}(B_1)}(x_1x_2),\ I_{\underline{\mathbb{R}}(A_2)}(y)\},\\
F_{\overline{\mathbb{S}}(B_1)\times\overline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &=\max\{F_{\overline{\mathbb{S}}(B_1)}(x_1x_2),\ F_{\overline{\mathbb{R}}(A_2)}(y)\},\\
F_{\underline{\mathbb{S}}(B_1)\times\underline{\mathbb{S}}(B_2)}\big((x_1,y)(x_2,y)\big) &=\max\{F_{\underline{\mathbb{S}}(B_1)}(x_1x_2),\ F_{\underline{\mathbb{R}}(A_2)}(y)\}.
\end{aligned}$$

(iv) $\forall x_1x_2 \in \mathbb{S}B_1,\ y_1 \neq y_2 \in \mathbb{R}A_2$.

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B_1)\times\overline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &=\min\{T_{\overline{\mathbb{S}}(B_1)}(x_1x_1),\ T_{\overline{\mathbb{R}}(A_2)}(y_1),\ T_{\overline{\mathbb{R}}(A_2)}(y_2)\},\\
T_{\underline{\mathbb{S}}(B_1)\times\underline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &=\min\{T_{\underline{\mathbb{S}}(B_1)}(x_1x_1),\ T_{\underline{\mathbb{R}}(A_2)}(y_1),\ T_{\underline{\mathbb{R}}(A_2)}(y_2)\},\\
I_{\overline{\mathbb{S}}(B_1)\times\overline{\mathbb{S}}(B_2)}\big((x_1,y_1)(x_2,y_2)\big) &=\max\{I_{\overline{\mathbb{S}}(B_1)}(x_1x_1),\ I_{\overline{\mathbb{R}}(A_2)}(y_1),\ I_{\overline{\mathbb{R}}(A_2)}(y_2)\},
\end{aligned}$$

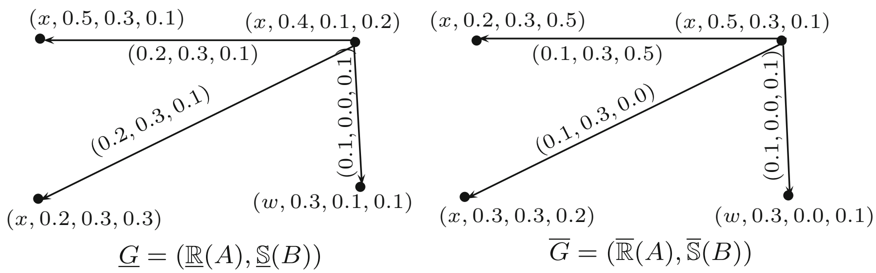


Fig. 8.9 Neutrosophic soft rough graph $G = (\underline{G}, \overline{G})$

$$
\begin{aligned}
I_{\underline{\mathbb{S}}(B_1)\times\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{I_{\underline{\mathbb{S}}(B_1)}(x_1x_1), I_{\underline{\mathbb{R}}(A_2)}(y_1), I_{\underline{\mathbb{R}}(A_2)}(y_2)\},\\
F_{\overline{\mathbb{S}}(B_1)\times\overline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{F_{\overline{\mathbb{S}}(B_1)}(x_1x_1), F_{\overline{\mathbb{R}}(A_2)}(y_1), F_{\overline{\mathbb{R}}(A_2)}(y_2)\},\\
F_{\underline{\mathbb{S}}(B_1)\times\underline{\mathbb{S}}(B_2)}\big((x_1, y_1)(x_2, y_2)\big) &= \max\{F_{\underline{\mathbb{S}}(B_1)}(x_1x_1), F_{\underline{\mathbb{R}}(A_2)}(y_1), F_{\underline{\mathbb{R}}(A_2)}(y_2)\}.
\end{aligned}
$$

Definition 8.14 Let $G = (\underline{G}, \overline{G})$ be a neutrosophic soft rough graph. The *complement* of G, denoted by $\underline{\acute{G}} = (\underline{\acute{G}}, \overline{\acute{G}})$, is a neutrosophic soft rough graph, where $\underline{\acute{G}} = (\underline{\mathbb{R}}(\acute{A}), \underline{\mathbb{S}}(\acute{B}))$ and $\overline{\acute{G}} = (\overline{\mathbb{R}}(\acute{A}), \overline{\mathbb{S}}(\acute{B}))$ are neutrosophic graphs such that

(i) $\forall x \in \mathbb{R}A.$

$$
\begin{aligned}
T'_{\overline{\mathbb{R}}(A)}(x) &= T_{\overline{\mathbb{R}}(A)(x)}, \quad I'_{\overline{\mathbb{R}}(A)}(x) = I_{\overline{\mathbb{R}}(A)(x)}, \quad F'_{\overline{\mathbb{R}}(A)}(x) = F_{\overline{\mathbb{R}}(A)(x)},\\
T'_{\underline{\mathbb{R}}(A)}(x) &= T_{\underline{\mathbb{R}}(A)(x)}, \quad I'_{\underline{\mathbb{R}}(A)}(x) = I_{\underline{\mathbb{R}}(A)(x)}, \quad F'_{\underline{\mathbb{R}}(A)}(x) = F_{\underline{\mathbb{R}}(A)(x)}.
\end{aligned}
$$

(ii) $\forall\ v, u \in \mathbb{R}A.$

$$
\begin{aligned}
T'_{\overline{\mathbb{S}}(B)}(xy) &= \min\{T_{\overline{\mathbb{R}}(A)}(x), T_{\overline{\mathbb{R}}(A)}(y)\} - T_{\overline{\mathbb{S}}(B)}(xy),\\
I'_{\overline{\mathbb{S}}(B)}(xy) &= \max\{I_{\overline{\mathbb{R}}(A)}(x), I_{\overline{\mathbb{R}}(A)}(y)\} - I_{\overline{\mathbb{S}}(B)}(xy),\\
F'_{\overline{\mathbb{S}}(B)}(xy) &= \max\{F_{\overline{\mathbb{R}}(A)}(x), F_{\overline{\mathbb{R}}(A)}(y)\} - F_{\overline{\mathbb{S}}(B)}(xy),\\
T'_{\underline{\mathbb{S}}(B)}(xy) &= \min\{T_{\underline{\mathbb{R}}(A)}(x), T_{\underline{\mathbb{R}}(A)}(y)\} - T_{\underline{\mathbb{S}}(B)}(xy),\\
I'_{\underline{\mathbb{S}}(B)}(xy) &= \max\{I_{\underline{\mathbb{R}}(A)}(x), I_{\underline{\mathbb{R}}(A)}(y)\} - I_{\overline{\mathbb{S}}(B)}(xy),\\
F'_{\underline{\mathbb{S}}(B)}(xy) &= \max\{F_{\underline{\mathbb{R}}(A)}(x), F_{\underline{\mathbb{R}}(A)}(y)\} - F_{\underline{\mathbb{S}}(B)}(xy).
\end{aligned}
$$

Example 8.7 Consider a neutrosophic soft rough graphs G as shown in Fig. 8.9. The complement of G is $\acute{G} = (\underline{\acute{G}}, \overline{\acute{G}})$ obtained by using Definition 8.14, where $\underline{\acute{G}} = (\underline{\mathbb{R}}(\acute{A}), \underline{\mathbb{S}}(\acute{B}))$ and $\overline{\acute{G}} = (\overline{\mathbb{R}}(\acute{A}), \overline{\mathbb{S}}(\acute{B}))$ are neutrosophic graphs as shown in Fig. 8.10.

Definition 8.15 A graph G is called self-complement if $G = \acute{G}$, i.e.

(i) $\forall x \in \mathbb{R}A.$

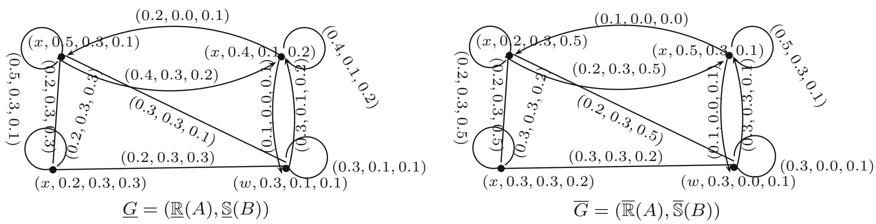


Fig. 8.10 Neutrosophic soft rough graph $\acute{G} = (\acute{\underline{G}}, \acute{\overline{G}})$

Table 8.10 Neutrosophic soft rough set on X

X	$\overline{\mathbb{R}}(A)$	$\underline{\mathbb{R}}(A)$
u	(0.8, 0.5, 0.2)	(0.7, 0.5, 0.2)
v	(0.9, 0.5, 0.1)	(0.7, 0.5, 0.2)
w	(0.7, 0.5, 0.1)	(0.7, 0.5, 0.2)

$$T'_{\overline{\mathbb{R}}(A)}(x) = T_{\overline{\mathbb{R}}(A)(x)},\ \ I'_{\overline{\mathbb{R}}(A)}(x) = I_{\overline{\mathbb{R}}(A)(x)},\ F'_{\overline{\mathbb{R}}(A)}(x) = F_{\overline{\mathbb{R}}(A)(x)},$$
$$T'_{\underline{\mathbb{R}}(A)}(x) = T_{\underline{\mathbb{R}}(A)(x)},\ \ I'_{\underline{\mathbb{R}}(A)}(x) = I_{\underline{\mathbb{R}}(A)(x)},\ \ F'_{\underline{\mathbb{R}}(A)}(x) = F_{\underline{\mathbb{R}}(A)(x)}.$$

(ii) $\forall\, x, y \in \mathbb{R}A$.

$$T'_{\overline{\mathbb{S}}(B)}(xy) = T_{\overline{\mathbb{S}}(B)}(xy),\ \ I'_{\overline{\mathbb{S}}(B)}(xy) = I_{\overline{\mathbb{S}}(B)}(xy),\ \ F'_{\overline{\mathbb{S}}(B)}(xy) = F_{\overline{\mathbb{S}}(B)}(xy),$$
$$T'_{\underline{\mathbb{S}}(B)}(xy) = T_{\underline{\mathbb{S}}(B)}(xy),\ \ I'_{\underline{\mathbb{S}}(B)}(xy) = I_{\underline{\mathbb{S}}(B)}(xy),\ \ F'_{\underline{\mathbb{S}}(B)}(xy) = F_{\underline{\mathbb{S}}(B)}(xy).$$

Definition 8.16 A neutrosophic soft rough graph G is called *strong neutrosophic soft rough graph* if $\forall xy \in \mathbb{S}B$,

$$\begin{aligned}
T_{\overline{\mathbb{S}}(B)}(xy) &= \min\{T_{\overline{\mathbb{R}}(A)}(x), T_{\overline{\mathbb{R}}(A)}(y)\},\\
I_{\overline{\mathbb{S}}(B)}(xy) &= \max\{I_{\overline{\mathbb{R}}(A)}(x), I_{\overline{\mathbb{R}}(A)}(y)\}),\\
F_{\overline{\mathbb{S}}(B)}(xy) &= \max\{F_{\overline{\mathbb{R}}(A)}(x), F_{\overline{\mathbb{R}}(A)}(y)\},\\
T_{\underline{\mathbb{S}}(B)}(xy) &= \min\{T_{\underline{\mathbb{R}}(A)}(x), T_{\underline{\mathbb{R}}(A)}(y)\},\\
I_{\underline{\mathbb{S}}(B)}(xy) &= \max\{I_{\underline{\mathbb{R}}(A)}(x), I_{\underline{\mathbb{R}}(A)}(y)\},\\
F_{\underline{\mathbb{S}}(B)}(xy) &= \max\{F_{\underline{\mathbb{R}}(A)}(x), F_{\underline{\mathbb{R}}(A)}(y)\}.
\end{aligned}$$

Example 8.8 Consider a graph G such that $X = \{u, v, w\}$ and $E = \{uv, vw, wu\}$. Let $\mathbb{R}A$ be a neutrosophic soft rough set of X, and let $\mathbb{S}B$ be a neutrosophic soft rough set of E defined in Tables 8.10 and 8.11, respectively.

Hence, $G = (\mathbb{R}A, \mathbb{S}B)$ is a strong neutrosophic soft rough graph as shown in Fig. 8.11.

Definition 8.17 A neutrosophic soft rough graph G is called *complete neutrosophic soft rough graph* if $\forall\, x, y \in X$,

Table 8.11 Neutrosophic soft rough set on E

E	$\overline{\mathbb{S}}(B)$	$\underline{\mathbb{S}}(B)$
uv	(0.8, 0.5, 0.2)	(0.7, 0.5, 0.2)
vw	(0.7, 0.5, 0.1)	(0.7, 0.5, 0.2)
wu	(0.7, 0.5, 0.2)	(0.7, 0.5, 0.2)

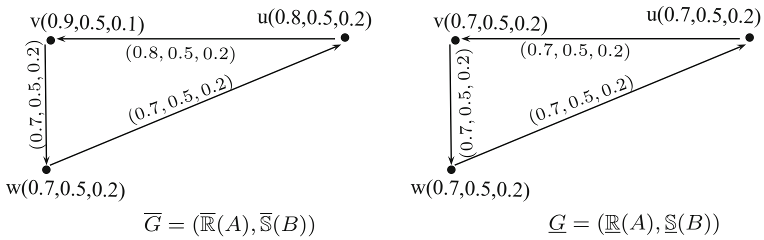


Fig. 8.11 Strong neutrosophic soft rough graph $G = (\mathbb{R}A, \mathbb{S}B)$

$$
\begin{aligned}
T_{\overline{\mathbb{S}}(B)}(xy) &= \min\{T_{\overline{\mathbb{R}}(A)}(x), T_{\overline{\mathbb{R}}(A)}(y)\},\\
I_{\overline{\mathbb{S}}(B)}(xy) &= \max\{I_{\overline{\mathbb{R}}(A)}(x), I_{\overline{\mathbb{R}}(A)}(y)\},\\
F_{\overline{\mathbb{S}}(B)}(xy) &= \max\{F_{\overline{\mathbb{R}}(A)}(x), F_{\overline{\mathbb{R}}(A)}(y)\},\\
T_{\underline{\mathbb{S}}(B)}(xy) &= \min\{T_{\underline{\mathbb{R}}(A)}(x), T_{\underline{\mathbb{R}}(A)}(y)\},\\
I_{\underline{\mathbb{S}}(B)}(xy) &= \max\{I_{\underline{\mathbb{R}}(A)}(x), I_{\underline{\mathbb{R}}(A)}(y)\},\\
F_{\underline{\mathbb{S}}(B)}(xy) &= \max\{F_{\underline{\mathbb{R}}(A)}(x), F_{\underline{\mathbb{R}}(A)}(y)\}.
\end{aligned}
$$

Remark 8.2 Every complete neutrosophic soft rough graph is a strong neutrosophic soft rough graph. But the converse is not true.

Definition 8.18 A neutrosophic soft rough graph G is *isolated* if $\forall x, y \in X$.

$$T_{\underline{\mathbb{S}}(B)}(xy) = 0,\ I_{\underline{\mathbb{S}}(B)}(xy) = 0,\ F_{\underline{\mathbb{S}}(B)}(xy) = 0,\ T_{\overline{\mathbb{S}}(B)}(xy) = 0,\ I_{\overline{\mathbb{S}}(B)}(xy) = 0,\ F_{\overline{\mathbb{S}}(B)}(xy) = 0,$$

Theorem 8.1 *The rejection of two neutrosophic soft rough graphs is a neutrosophic soft rough graph.*

Proof Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic soft rough graphs. Let $G = G_1|G_2 = (\underline{G}_1|\underline{G}_2, \overline{G}_1|\overline{G}_2)$ be the rejection of G_1 and G_2, where $\underline{G}_1|\underline{G}_2 = (\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2))$ and $\overline{G}_1|\overline{G}_2 = (\overline{\mathbb{R}}(A_1)|\overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2))$. We claim that $G = G_1|G_2$ is an neutrosophic soft rough graph. It is enough to show that $\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)$ and $\overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2)$ are neutrosophic relations on $\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)$ and $\overline{\mathbb{R}}(A_1)|\overline{\mathbb{R}}(A_2)$, respectively. First, we show that $\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)$ is a neutrosophic relation on $\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)$.
If $x \in \underline{\mathbb{R}}(A_1)$, $y_1y_2 \notin \underline{\mathbb{S}}(B_2)$, then

$$\begin{aligned}
T_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x,y_1)(x,y_2)) &= (T_{\underline{\mathbb{R}}(A_1)}(x) \wedge (T_{\underline{\mathbb{R}}(A_2)}(y_2) \wedge T_{\underline{\mathbb{R}}(A_2)}(y_2))) \\
&= (T_{\underline{\mathbb{R}}(A_1)}(x) \wedge T_{\underline{\mathbb{R}}(A_2)}(y_2)) \wedge (T_{\underline{\mathbb{R}}(A_1)}(x) \wedge T_{\underline{\mathbb{R}}(A_2)}(y_2)) \\
&= T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x,y_1) \wedge T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x,y_2) \\
T_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x,y_1)(x,y_2)) &= T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x,y_1) \wedge T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x,y_2) \\
\text{Similarly, } I_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x,y_1)(x,y_2)) &= I_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x,y_1) \vee I_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x,y_2) \\
F_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x,y_1)(x,y_2)) &= F_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x,y_1) \vee F_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x,y_2)
\end{aligned}$$

If $x_1x_2 \notin \underline{\mathbb{S}}(B_1)$, $y \in \underline{\mathbb{R}}(A_2)$, then

$$\begin{aligned}
T_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x_1,y)(x_2,y)) &= ((T_{\underline{\mathbb{R}}(A_1)}(x_1) \wedge T_{\underline{\mathbb{R}}(A_1)}(x_2)) \wedge T_{\underline{\mathbb{R}}(A_2)}(y)) \\
&= ((T_{\underline{\mathbb{R}}(A_1)}(x_1) \wedge T_{\underline{\mathbb{R}}(A_2)}(y)) \wedge (T_{\underline{\mathbb{R}}(A_1)}(x_2) \wedge T_{\underline{\mathbb{R}}(A_2)}(y))) \\
&= T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_1,y) \wedge T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_2,y) \\
T_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x_1,y)(x_2,y)) &= T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_1,y) \wedge T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_2,y) \\
\text{Similarly, } I_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x_1,y)(x_2,y)) &= I_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_1,y) \vee I_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_2,y) \\
F_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x_1,y)(x_2,y)) &= F_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_1,y) \vee F_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_2,y)
\end{aligned}$$

If $x_1x_2 \notin \underline{\mathbb{S}}(B_1)$, $y_1, y_2 \notin \underline{\mathbb{S}}(B_2)$, then

$$\begin{aligned}
T_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x_1,y_1)(x_2,y_2)) &= ((T_{\underline{\mathbb{R}}(A_1)}(x_1) \wedge T_{\underline{\mathbb{R}}(A_1)}(x_2)) \wedge (T_{\underline{\mathbb{R}}(A_2)}(y_1) \wedge T_{\underline{\mathbb{R}}(A_2)}(y_2))) \\
&= (T_{\underline{\mathbb{R}}(A_1)}(x_1) \wedge T_{\underline{\mathbb{R}}(A_2)}(y_1)) \wedge (T_{\underline{\mathbb{R}}(A_1)}(x_2) \wedge T_{\underline{\mathbb{R}}(A_2)}(y_2)) \\
&= T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_1,y_1) \wedge T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_2,y_2) \\
T_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x_1,y_1)(x_2,y_2)) &= T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_1,y_1) \wedge T_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_2,y_2) \\
\text{Similarly, } I_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x_1,y_1)(x_2,y_2)) &= I_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_1,y_1) \vee I_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(= y_1,y_2) \\
F_{\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)}((x_1,y_1)(x_2,y_2)) &= F_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_1,y_1) \vee F_{\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)}(x_2,y_2)
\end{aligned}$$

Thus, $\underline{\mathbb{S}}(B_1)|\underline{\mathbb{S}}(B_2)$ is a neutrosophic relation on $\underline{\mathbb{R}}(A_1)|\underline{\mathbb{R}}(A_2)$. Similarly, we can show that $\overline{\mathbb{S}}(B_1)|\overline{\mathbb{S}}(B_2)$ is a neutrosophic relation on $\overline{\mathbb{R}}(A_1)|\overline{\mathbb{R}}(A_2)$. Hence, G is a neutrosophic soft rough graph.

Theorem 8.2 *The Cartesian product of two neutrosophic soft rough graphs is a neutrosophic soft rough graph.*

Proof Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic soft rough graphs. Let $G = G_1 \ltimes G_2 = (\underline{G}_1 \ltimes \underline{G}_2, \overline{G}_1 \ltimes \overline{G}_2)$ be the Cartesian product of G_1 and G_2, where $\underline{G}_1 \ltimes \underline{G}_2 = (\underline{\mathbb{R}}(A_1) \ltimes \underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_1) \ltimes \underline{\mathbb{S}}(B_2))$ and $\overline{G}_1 \ltimes \overline{G}_2 = (\overline{\mathbb{R}}(A_1) \ltimes \overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_1) \ltimes \underline{\mathbb{S}}(B_2))$. We claim that $G = G_1 \ltimes G_2$ is a neutrosophic soft rough graph. It is enough to show that $\underline{\mathbb{S}}(B_1) \ltimes \underline{\mathbb{S}}(B_2)$ and $\overline{\mathbb{S}}(B_1) \ltimes \overline{\mathbb{S}}(B_2)$ are neutrosophic relations on $\underline{\mathbb{R}}(A_1) \ltimes \underline{\mathbb{R}}(A_2)$ and $\overline{\mathbb{R}}(A_1) \ltimes \overline{\mathbb{R}}(A_2)$, respectively. We have to show that $\underline{\mathbb{S}}(B_1) \ltimes \underline{\mathbb{S}}(B_2)$ is a neutrosophic relation on $\underline{\mathbb{R}}(A_1) \ltimes \underline{\mathbb{R}}(A_2)$.
If $x \in \underline{\mathbb{R}}(A_1)$, $y_1y_2 \in \underline{\mathbb{S}}(B_2)$, then

$$\begin{aligned}
T_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}((x,y_1)(x,y_2)) &= T_{\underline{\mathbb{R}}(A_1))}(x) \wedge T_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\\
&\leq T_{\underline{\mathbb{R}}(A_1))}(x) \wedge (T_{\underline{\mathbb{R}}(A_2)}(y_1) \wedge T_{\underline{\mathbb{R}}(A_2)}(y_2))\\
&= (T_{\underline{\mathbb{R}}(A_1))}(x) \wedge T_{\underline{\mathbb{R}}(A_2)}(y_1)) \wedge (T_{\underline{\mathbb{R}}(A_1))}(x) \wedge T_{\underline{\mathbb{R}}(A_2)}(y_2))\\
&= T_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x,y_1) \wedge T_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x,y_2)\\
T_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}((x,y_1)(x,y_2)) &\leq T_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x,y_1) \wedge T_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x,y_2)\\
\text{Similarly, } I_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}((x,y_1)(x,y_2)) &\leq I_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x,y_1) \vee I_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x,y_2)\\
F_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}((x,y_1)(x,y_2)) &\leq F_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x,y_1) \vee F_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x,y_2)
\end{aligned}$$

If $x_1x_2 \in \underline{\mathbb{S}}(B_1)$, $z \in \underline{\mathbb{R}}(A_2)$, then

$$\begin{aligned}
T_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}((x_1,z)(x_2,z)) &= T_{\underline{\mathbb{S}}(B_1))}(x_1x_2) \wedge T_{\underline{\mathbb{R}}(A_2)}(z)\\
&\leq (T_{\underline{\mathbb{R}}(A_1))(x_1)\wedge\underline{\mathbb{R}}(A_1))}(x_2)) \wedge T_{\underline{\mathbb{R}}(A_2)}(z)\\
&= T_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x_1,z) \wedge T_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x_2,z)\\
T_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}((x_1,z)(x_2,z)) &\leq T_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x_1,z) \wedge T_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x_2,z)\\
\text{Similarly, } I_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}((x_1,z)(x_2,z)) &\leq I_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x_1,z) \vee I_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x_2,z)\\
F_{\underline{\mathbb{S}}(B_1)\ltimes\underline{\mathbb{S}}(B_2)}((x_1,z)(x_2,z)) &\leq F_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x_1,z) \vee F_{\underline{\mathbb{R}}(A_1)\ltimes\underline{\mathbb{R}}(A_2)}(x_2,z)
\end{aligned}$$

Therefore, $\underline{\mathbb{S}}(B_1) \ltimes \underline{\mathbb{S}}(B_2)$ is a neutrosophic relation on $\underline{\mathbb{R}}(A_1) \ltimes \underline{\mathbb{R}}(A_2)$. Similarly, $\overline{\mathbb{S}}(B_1) \ltimes \overline{\mathbb{S}}(B_2)$ is a neutrosophic relation on $\overline{\mathbb{R}}(A_1) \ltimes \overline{\mathbb{R}}(A_2)$. Hence, G is a neutrosophic rough graph.

Theorem 8.3 *The cross product of two neutrosophic soft rough graphs is a neutrosophic soft rough graph.*

Proof Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic soft rough graphs. Let $G = G_1 \circledcirc G_2 = (\underline{G}_1 \circledcirc \underline{G}_2, \overline{G}_1 \circledcirc \overline{G}_2)$ be the cross product of G_1 and G_2, where $\underline{G}_1 \circledcirc \underline{G}_2 = (\underline{\mathbb{R}}(A_1) \circledcirc \underline{\mathbb{R}}(A_2), \underline{\mathbb{S}}(B_1) \circledcirc \underline{\mathbb{S}}(B_2))$ and $\overline{G}_1 \circledcirc \overline{G}_2 = (\overline{\mathbb{R}}(A_1) \circledcirc \overline{\mathbb{R}}(A_2), \overline{\mathbb{S}}(B_1) \circledcirc \underline{\mathbb{S}}(B_2))$. We claim that $G = G_1 \circledcirc G_2$ is a neutrosophic soft rough graph. It is enough to show that $\underline{\mathbb{S}}(B_1) \circledcirc \underline{\mathbb{S}}(B_2)$ and $\overline{\mathbb{S}}(B_1) \circledcirc \overline{\mathbb{S}}(B_2)$ are neutrosophic relations on $\underline{\mathbb{R}}(A_1) \circledcirc \underline{\mathbb{R}}(A_2)$ and $\overline{\mathbb{R}}(A_1) \circledcirc \overline{\mathbb{R}}(A_2)$, respectively. First, we show that $\underline{\mathbb{S}}(B_1) \circledcirc \underline{\mathbb{S}}(B_2)$ is a neutrosophic relation on $\underline{\mathbb{R}}(A_1) \circledcirc \underline{\mathbb{R}}(A_2)$.
If $x_1x_2 \in \underline{\mathbb{S}}(B_1)$, $y_1y_2 \in \underline{\mathbb{S}}(B_2)$, then

$$\begin{aligned}
T_{\underline{\mathbb{S}}(B_1)\circledcirc\underline{\mathbb{S}}(B_2)}((x_1,y_1)(x_2,y_2)) &= T_{\underline{\mathbb{S}}(B_1))}(x_1x_2) \wedge T_{\underline{\mathbb{S}}(B_2)}(y_1y_2)\\
&\leq (T_{\underline{\mathbb{R}}(A_1))}(x_1) \wedge T_{\underline{\mathbb{R}}(A_1))}(x_2) \wedge (T_{\underline{\mathbb{R}}(A_2)}(y_1) \wedge T_{\underline{\mathbb{R}}(A_2)}(y_2))\\
&= (T_{\underline{\mathbb{R}}(A_1))}(x_1) \wedge T_{\underline{\mathbb{R}}(A_2)}(x_2)) \wedge (T_{\underline{\mathbb{R}}(A_1))}(y_1) \wedge T_{\underline{\mathbb{R}}(A_2)}(y_2))\\
&= T_{\underline{\mathbb{R}}(A_1)\circledcirc\underline{\mathbb{R}}(A_2)}(x_1,x_2) \wedge T_{\underline{\mathbb{R}}(A_1)\circledcirc\underline{\mathbb{R}}(A_2)}(y_1,y_2)\\
T_{\underline{\mathbb{S}}(B_1)\circledcirc\underline{\mathbb{S}}(B_2)}((x_1,x_2)(y_1,y_2)) &\leq T_{\underline{\mathbb{R}}(A_1)\circledcirc\underline{\mathbb{R}}(A_2)}(x_1,y_1) \wedge T_{\underline{\mathbb{R}}(A_1)\circledcirc\underline{\mathbb{R}}(A_2)}(x_2,y_2)\\
\text{Similarly, } I_{\underline{\mathbb{S}}(B_1)\circledcirc\underline{\mathbb{S}}(B_2)}((x_1,y_1)(x_2,y_2)) &\leq I_{\underline{\mathbb{R}}(A_1)\circledcirc\underline{\mathbb{R}}(A_2)}(x_1,y_1) \vee I_{\underline{\mathbb{R}}(A_1)\circledcirc\underline{\mathbb{R}}(A_2)}(x_2,y_2)\\
F_{\underline{\mathbb{S}}(B_1)\circledcirc\underline{\mathbb{S}}(B_2)}((x_1,y_1)(x_2,y_2)) &\leq F_{\underline{\mathbb{R}}(A_1)\circledcirc\underline{\mathbb{R}}(A_2)}(x_1,y_1) \vee F_{\underline{\mathbb{R}}(A_1)\circledcirc\underline{\mathbb{R}}(A_2)}(x_2,y_2)
\end{aligned}$$

Thus, $\underline{\mathbb{S}}(B_1) \circledcirc \underline{\mathbb{S}}(B_2)$ is a neutrosophic relation on $\underline{\mathbb{R}}(A_1) \circledcirc \underline{\mathbb{R}}(A_2)$. Similarly, we can show that $\overline{\mathbb{S}}(B_1) \circledcirc \overline{\mathbb{S}}(B_2)$ is a neutrosophic relation on $\overline{\mathbb{R}}(A_1) \circledcirc \overline{\mathbb{R}}(A_2)$. Hence, G is a neutrosophic soft rough graph.

8.3 Application of Neutrosophic Soft Rough Graphs

In this section, we apply the concept of neutrosophic soft rough sets to a decision-making problem. In recent times, the object recognition problem has gained considerable importance. The object recognition problem can be considered as a decision-making problem, in which final identification of object is founded on given set of information. A detailed description of the algorithm for the selection of most suitable object based on available set of alternatives is given, and purposed decision-making method can be used to calculate lower and upper approximation operators to progress deep concerns of the problem. The presented algorithms can be applied to avoid lengthy calculations when dealing with a large number of objects. This method can be applied in various domains for multicriteria selection of objects.

Selection of Most Suitable Generic Version of Brand Name Medicine

In pharmaceutical industry, different pharmaceutical companies develop, produce and discover pharmaceutical medicines (drugs) for use as medication. These pharmaceutical companies deals with "brand name medicine" and "generic medicine". Brand name medicine and generic medicine are bioequivalent, generic medicine rate and element of absorption. Brand name medicine and generic medicine have the same active ingredients, and the inactive ingredients may differ. The most important difference is cost. Generic medicine is less expensive as compared to brand name comparators. Usually generic drug manufacturers have competition to produce cost less products. The product may possibly be slightly dissimilar in colour, shape or markings. The major difference is cost. We consider a brand name drug "u = Loratadine" used for seasonal allergies medication. Consider

$$\begin{aligned} X = \{&x_1 = \text{Triamcinolone}, x_2 = \text{Cetirizine/Pseudoephedrine}, \\ &x_3 = \text{Pseudoephedrine}, x_4 = \text{loratadine/pseudoephedrine}, \\ &x_5 = \text{Fluticasone}\} \end{aligned}$$

is a set of generic versions of "Loratadine". We want to select the most suitable generic version of Loratadine on the basis of parameters e_1 = highly soluble, e_2 = highly permeable, e_3 = rapidly dissolving. $M = \{e_1, e_2, e_3\}$ be a set of paraments. Let $\mathbb{R}$ be a neutrosophic soft relation from X to parameter set M, describes truth-membership, indeterminacy-membership and false-membership degrees of generic version medicines corresponding to the parameters as shown in Table 8.12.

Suppose $A = \{(e_1, 0.2, 0.4, 0.5), (e_2, 0.5, 0.6, 0.4), (e_3, 0.7, 0.5, 0.4)\}$ is most favourable object which is a neutrosophic set on the parameter set M under con-

Table 8.12 Neutrosophic soft set $\mathbb{R} = (F, M)$

$\mathbb{R}$	x_1	x_2	x_3	x_4	x_5
e_1	(0.4, 0.5, 0.6)	(0.5, 0.3, 0.6)	(0.7, 0.2, 0.3)	(0.5, 0.7, 0.5)	(0.6, 0.5, 0.4)
e_2	(0.7, 0.3, 0.2)	(0.3, 0.4, 0.3)	(0.6, 0.5, 0.4)	(0.8, 0.4, 0.6)	(0.7, 0.8, 0.5)
e_3	(0.6, 0.3, 0.4)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.4)	(0.8, 0.7, 0.6)	(0.7, 0.3, 0.5)

Table 8.13 Neutrosophic soft relation S

S	x_1x_2	x_1x_3	x_4x_1	x_2x_3	x_5x_3	x_2x_4	x_2x_5
e_1e_2	(0.3, 0.4, 0.2)	(0.4, 0.4, 0.5)	(0.4, 0.4, 0.5)	(0.6, 0.3, 0.4)	(0.4, 0.2, 0.2)	(0.4, 0.4, 0.2)	(0.4, 0.3, 0.4)
e_2e_3	(0.5, 0.4, 0.1)	(0.4, 0.3, 0.2)	(0.4, 0.3, 0.2)	(0.3, 0.3, 0.2)	(0.6, 0.2, 0.4)	(0.3, 0.2, 0.1)	(0.3, 0.3, 0.2)
e_1e_3	(0.4, 0.4, 0.1)	(0.4, 0.2, 0.2)	(0.4, 0.2, 0.2)	(0.5, 0.3, 0.3)	(0.4, 0.2, 0.3)	(0.4, 0.3, 0.1)	(0.5, 0.3, 0.2)

sideration. Then $(\underline{\mathbb{R}}(A), \overline{\mathbb{R}}(A))$ is a neutrosophic soft rough set in neutrosophic soft approximation space $(X, M, \mathbb{R})$, where

$$\overline{\mathbb{R}}(A) = \{(x_1, 0.6, 0.5, 0.4), (x_2, 0.7, 0.4, 0.4), (x_3, 0.7, 0.4, 0.4), (x_4, 0.7, 0.6, 0.5), (x_5, 0.7, 0.5, 0.5)\},$$
$$\underline{\mathbb{R}}(A) = \{(x_1, 0.5, 0.6, 0.4), (x_2, 0.5, 0.6, 0.5), (x_3, 0.3, 0.3, 0.5), (x_4, 0.5, 0.6, 0.5), (x_5, 0.4, 0.5, 0.5)\}.$$

Let $E = \{x_1x_2, x_1x_3, x_4x_1, x_2x_3, x_5x_3, x_2x_4, x_2x_5\} \subseteq \acute{X}$ and $L = \{e_1e_3, e_2e_1, e_3e_2\} \subseteq \acute{M}$.
Then a neutrosophic soft relation S on E (from L to E) can be defined in Table 8.13 as follows:

Let $B = \{(e_1e_2, 0.2, 0.4, 0.5), (e_2e_3, 0.5, 0.4, 0.4), (e_1e_3, 0.5, 0.2, 0.5)\}$ be a neutrosophic set on L which describes some relationship between the parameters under consideration, then $SB = (\underline{S}(B), \overline{S}(B))$ is a neutrosophic soft rough relation, where

$$\overline{\mathbb{S}}(B) = \{(x_1x_2, 0.5, 0.4, 0.4), (x_1x_3, 0.4, 0.2, 0.4), (x_4x_1, 0.4, 0.2, 0.4), (x_2x_3, 0.5, 0.3, 0.4), (x_5x_3, 0.5, 0.2, 0.4), (x_2x_4, 0.4, 0.3, 0.4), (x_2x_5, 0.5, 0.3, 0.4)\},$$
$$\underline{\mathbb{S}}(B) = \{(x_1x_2, 0.2, 0.4, 0.4)(x_1x_3, 0.5, 0.4, 0.4), (x_4x_1, 0.5, 0.4, 0.4), (x_2x_3, 0.4, 0.4, 0.5), (x_5x_3, 0.2, 0.4, 0.4), (x_2x_4, 0.2, 0.4, 0.4), (x_2x_5, 0.4, 0.4, 0.5)\}.$$

Thus, $G = (\underline{G}, \overline{G})$ is a neutrosophic soft rough graph as shown in Fig. 8.12.

The sum of two neutrosophic numbers is defined as follows.

Definition 8.19 Let C and D be two single-valued neutrosophic numbers, and the sum of two single-valued neutrosophic numbers is defined as follows:

$$C \oplus D =< T_C + T_D - T_C \times T_D, I_C \times I_D, F_C \times F_D > . \tag{8.1}$$

The sum of upper neutrosophic soft rough set $\overline{\mathbb{R}}A$ and the lower neutrosophic soft rough set $\underline{\mathbb{R}}A$ and sum of lower neutrosophic soft rough relation $\underline{\mathbb{S}}B$ and the upper

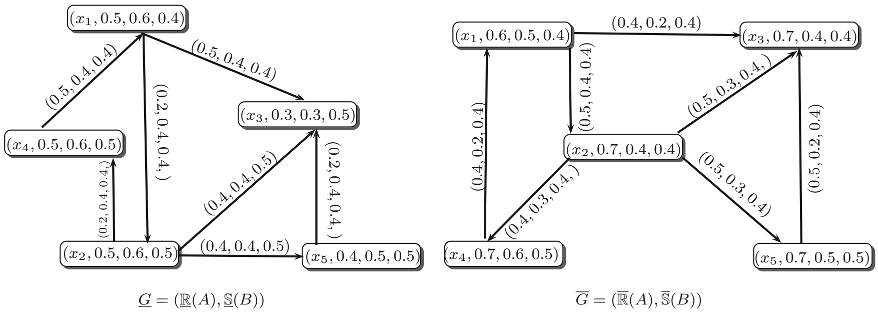


Fig. 8.12 Neutrosophic soft rough graph $G = (\underline{G}, \overline{G})$

neutrosophic soft rough relation $\overline{\mathbb{S}}B$ are neutrosophic sets $\overline{\mathbb{R}}A \oplus \underline{\mathbb{R}}A$ and $\overline{\mathbb{S}}B \oplus \underline{\mathbb{S}}B$, respectively, defined by

$$\alpha = \overline{\mathbb{R}}A \oplus \underline{\mathbb{R}}A = \{(x_1, 0.8, 0.3, 0.16), (x_2, 0.85, 0.24, 0.2), (x_3, 0.79, 0.2, 0.2), (x_4, 0.85, 0.36, 0.25), (x_5, 0.82, 0.25, 0.25)\},$$

$$\beta = \overline{\mathbb{S}}B \oplus \underline{\mathbb{S}}B = \{(x_1x_2, 0.6, 0.16, 0.16), (x_1x_3, 0.7, 0.8, 0.16), (x_4x_1, 0.7, 0.8, 0.16), (x_2x_3, 0.7, 0.12, 0.2), (x_5x_3, 0.6, 0.08, 0.16), (x_2x_4, 0.52, 0.12, 0.16), (x_2x_5, 0.7, 0.12, 0.2), \}.$$

The score function $\gamma(x_k)$ defines for each generic version medicine $x_i \in X$,

$$\gamma(x_i) = \sum_{x_i x_j \in E} \frac{T_\alpha(x_j) + I_\alpha(x_j) - F_\alpha(x_j)}{3 - (T_\beta(x_i x_j) + I_\beta(x_i x_j) - F_\beta(x_i x_j))} \tag{8.2}$$

and x_k with the larger score value $x_k = \max_i \gamma(x_i)$ is the most suitable generic version medicine. By calculations, we have

$$\gamma(x_1) = 0.88, \gamma(x_2) = 0.69, \ \gamma(x_3) = 0.26 \ \gamma(x_4) = 0.57, \ \text{ and } \ \gamma(x_5) = 0.33. \tag{8.3}$$

Here, x_1 is the optimal decision, and the most suitable generic version of "Loratadine" is "Triamcinolone". We have used software MATLAB for calculating the required results in the application. The algorithm is given in Algorithm 8.3.1.

Algorithm 8.3.1 *Algorithm for selection of most suitable objects*

1. Input the number of elements in vertex set $X = \{x_1, x_2, \ldots, x_n\}$.

2. Input the number of elements in parameter set $M = \{e_1, e_2, \ldots, e_m\}$.

3. Input a neutrosophic soft relation $\mathbb{R}$ from X to M.

4. Input a neutrosophic set A on M.
5. Compute neutrosophic soft rough vertex set $\mathbb{R}A = (\underline{\mathbb{R}}(A), \overline{\mathbb{R}}(A))$.
6. Input the number of elements in edge set $E = \{x_1x_1, x_1x_2, \ldots, x_kx_1\}$.
7. Input the number of elements in parameter set $\acute{M} = \{e_1e_1, e_1e_2, \ldots, e_le_1\}$.
8. Input a neutrosophic soft relation $\mathbb{S}$ from $\acute{X}$ to $\acute{M}$.
9. Input a neutrosophic set B on $\acute{M}$.
10. Compute neutrosophic soft rough edge set $\mathbb{S}B = (\underline{\mathbb{S}}(B), \overline{\mathbb{S}}(B))$.
11. Compute neutrosophic set $\alpha = (T_\alpha(x_i), I_\alpha(x_i), F_\alpha(x_i))$, where

$$\begin{aligned} T_\alpha(x_i) &= T_{\overline{\mathbb{R}}(A)}(x_i) + T_{\underline{\mathbb{R}}(A)}(x_i) - T_{\overline{\mathbb{R}}(A)}(x_i) \times T_{\underline{\mathbb{R}}(A)}(x_i), \\ I_\alpha(x_i) &= T_{\overline{\mathbb{R}}(A)}(x_i) \times T_{\underline{\mathbb{R}}(A)}(x_i), \\ F_\alpha(x_i) &= F_{\overline{\mathbb{R}}(A)}(x_i) \times F_{\underline{\mathbb{R}}(A)}(x_i); \end{aligned}$$

12. Compute neutrosophic set $\beta = (T_\beta(x_ix_i), I_\beta(x_ix_j), F_\beta(x_ix_j))$, where

$$\begin{aligned} T_\beta(x_ix_j) &= T_{\overline{\mathbb{S}}(B)}(x_ix_j) + T_{\underline{\mathbb{S}}(B)}(x_ix_j) - T_{\overline{\mathbb{S}}(B)}(x_ix_j) \times T_{\underline{\mathbb{S}}(B)}(x_ix_j), \\ I_\beta(x_ix_j) &= T_{\overline{\mathbb{S}}(B)}(x_ix_j) \times T_{\underline{\mathbb{S}}(B)}(x_ix_j), \\ F_\beta(x_ix_j) &= F_{\overline{\mathbb{S}}(B)}(x_ix_j) \times F_{\underline{\mathbb{S}}(B)}(x_ix_j); \end{aligned}$$

13. Calculate the score values of each object x_i, and score function is defined as follows:

$$\gamma(x_i) = \sum_{x_ix_j \in E} \frac{T_\alpha(x_j) + I_\alpha(x_j) - F_\alpha(x_j)}{3 - (T_\beta(x_ix_j) + I_\beta(x_ix_j) - F_\beta(x_ix_j))};$$

14. The decision is x_i if $\gamma_i = \max_{i=1}^{n} \gamma_i$.
15. If i has more than one value, then any one of x_i may be chosen.