# Improved Conditional Value-at-Risk (CVaR) Based Method for Diversified Bond Portfolio Optimization

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**Abstract.** In this study, an improved CVaR-based Portfolio Optimization Method is presented. The method was used to test the performance of a diversified bond portfolio in providing low expected loss and optimal CVaR. A hypothetical diversified bond portfolio, which is a combination of Islamic bond or *Sukuk* and conventional bond, was constructed using bonds issued by four banking institutions. The performance of the improved method is determined by comparing the generated returns of the method against the existing CVaR-based Portfolio Optimization Method. The simulation of the optimization process of both methods was carried out by using the Geometric Brownian Motion-based Monte Carlo Simulation method. The results of the improved CVaR portfolio optimization method show that by restricting the upper and lower bounds with certain floor and ceiling bond weights using volatility weighting schemes, the expected loss can be reduced and an optimal CVaR can be achieved. Thus, this study shows that the improved CVaR-based Portfolio Optimization Method is able to provide a better optimization of a diversified bond portfolio in terms of reducing the expected loss, and hence maximizes the returns.

**Keywords:** Value-at-Risk (VaR) · Conditional Value-at-Risk (CVaR)
CVaR optimization · Bond · *Sukuk*

## 1 Introduction

Capital markets are markets where securities such as equities and bonds are issued and traded in raising medium to long-terms funds [1]. Securities are important components in a financial system, which are issued by public or private companies and entities including governments. Islamic capital markets carry the same definition as the conventional capital markets, except that all transaction activities are *Shariah* compliant.


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Bond is a type of debt investment, which is basically a transaction of loan that involves a lender (investor) and a borrower (issuer). There are two types of bonds which are conventional bond and Islamic bond or *Sukuk*. In the capital markets the *Sukuk* has been established as an alternative financial instrument to the conventional bond. The *Sukuk* differs from the conventional bond in the sense that *Sukuk* must comply with the *Shariah* principles, while the conventional bond involves debt upon sale which is prohibited in Islam.

From the bond issuance perspective, the issuer will either issue a conventional bond or *Sukuk* to the investor in order to finance their project(s). Based on the agreement that has been agreed upon by both parties, the issuer will make regular interest payments to the investor at a specified rate on the amount that have been borrowed before or until a specified date. As with any investment, both conventional bonds and *Sukuk* carry risks such as market and credit risks. A known technique to manage risk is diversification. Diversification is a risk management technique that is designed to reduce the risk level by combining a variety of investment instruments which are unlikely to move in the same direction within a portfolio [2]. To move in different directions here means that the financial instruments involved in a diversified portfolio are negatively correlated and have different price behaviours between them. Hence, investing in a diversified portfolio affords the possibility of reducing the risks as compared to investing in an undiversified portfolio.

Value-at-Risk (VaR) is an established method for measuring financial risk. However, VaR has undesirable mathematical characteristics such as lack of sub-additivity and convexity [3]. The lack of sub-additivity means that the measurement of a portfolio VaR might be greater than the sum of its assets [4]. While, convexity is the characteristics of a set of points in which, for any two points in the set, the points on the curve joining the two points are also in the set [5]. [6, 7] have shown that VaR can exhibit multiple local extrema, and hence does not behave well as a function of portfolio positions in determining an optimal mix of positions. Due to its disadvantages, VaR is considered a non-coherent risk measure.

As an alternative, [3] proved that CVaR has better properties than VaR since it fulfils all the properties (axioms) of a coherent risk measure and it is convex [8]. By using the CVaR approach, investors can estimate and examine the probability of the average losses when investing in certain transactions [9]. Although it has yet to be a standard in the finance industry, CVaR appears to play a major role in the insurance industry. CVaR can be optimized using linear programming (LP) and non-smooth optimization algorithm [4], due to its advantages over VaR.

The intention of this study was to improve the CVaR-based portfolio optimization method presented in [4]. In this paper, the improved CVaR portfolio optimization method is introduced in Sect. 2. The method finds the optimal allocation (weight) of various assets or financial instruments in a portfolio when the expected loss is minimized, thus maximizing the expected returns. The results of the implementation of the existing CVaR-based method in [4] and the improved CVaR-based method of this study are presented and discussed in Sect. 3 and concluded in Sect. 4.

## 2 Conditional Value-at-Risk (CVaR) - Based Portfolio Optimization Method for Diversified Bond Portfolio

Diversification has been established as an effective approach in reducing investment risk [2]. Portfolio optimization is considered a useful solution in investment diversification decision making where the investors will be able to allocate their funds in many assets (portfolios) with minimum loss at a certain risk level. Hence, the CVaR-based Portfolio Optimization Method has been developed in [4] to find the optimum portfolio allocation with the lowest loss at a certain risk level.

### 2.1 CVaR-Based Portfolio Optimization Method

In this study, the portfolio optimization problem using the CVaR-based Portfolio Optimization Method in [4] is solved by applying the approach presented in [2], which uses linear programming. The optimization problem is described as follows:

$$\min \quad -w^T\bar{y}$$

subject to

$$w \in W, \varphi \in \Re$$

$$\varphi + \frac{1}{J(1-\beta)}\sum_{j=1}^{J} s_j \leq \delta \tag{1}$$

$$s_j \geq 0, \qquad j = 1,\ldots,J$$

$$w^T r_j + \varphi + s_j \geq 0, \qquad j = 1,\ldots,J$$

where $w$ represents the weight, $\bar{y}$ is the expected outcome of $r$, $r_j$ is the vector representing returns, $\varphi$ is the value-at-risk (VaR), $\delta$ is the conditional value-at-risk (CVaR) limit, $\beta$ is the level of confidence, $J$ is the number of simulations and $s$ is the auxiliary variable. The computation for the optimization of (1) to find the portfolio allocation when loss is minimized (or return is maximized) within a certain CVaR (risk) limit is implemented using the MATLAB `fmincon` function. The `fmincon` function is a general constraint optimization routine that finds the minimum of a constrained multivariable function and has the form

$$[w\,, \mathit{fval}] \;=\; \texttt{fmincon}\;(\mathit{objfun}\,,\; w_0\,,\; A\,,\; b\,,\; \mathit{Aeq}\,,\; \mathit{beq}\,,\; \mathit{LB}\,,\; \mathit{UB}\,,\; [\,]\,,\; \mathit{options}),$$

where the return value *fval* is the expected return under the corresponding constraints.

To use the `fmincon` function, several parameters of the linear programming formulation of (1) need to be set up which are described as follows:

i. Objective Function
The aim of the formulation is to minimize the loss $-w^T\bar{y}$ in order to maximize the expected returns.

ii. Decision variables
The decision variables of this formulation are $w_1, w_2, \ldots, w_N$ which represent the weights for $N$ assets of the optimal portfolio.

iii. Constraints
(a) Inequality Constraints
The linear inequality of this formulation takes the form of $Aw \leq b$, where $w$ is the weight vector. Matrix $A$ represents the constraint coefficient which consists of the asset weights $(w_1, w_2, \ldots, w_N)$, VaR $(\varphi)$ and the auxiliary variables $(s_1, s_2, \ldots, s_j)$ as expressed in (1). Matrix $b$ describes the constraints level. Following (1), matrix $A$ and $b$ can be expressed as follows:

$$A = \begin{pmatrix} & w_1 & w_2 & \cdots & w_N & \varphi & s_1 & s_2 & \cdots & s_j \\ & 0 & 0 & \cdots & 0 & 1 & \frac{1}{J*(1-\beta)} & \frac{1}{J*(1-\beta)} & \cdots & \frac{1}{J*(1-\beta)} \\ & -r_{11} & -r_{12} & \cdots & -r_{1N} & -1 & -1 & 0 & \cdots & 0 \\ & -r_{21} & -r_{22} & \cdots & -r_{2N} & -1 & 0 & -1 & \cdots & 0 \\ & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ & -r_{j1} & -r_{j2} & \cdots & -r_{jN} & -1 & 0 & 0 & \cdots & -1 \end{pmatrix}$$

$$b = \begin{pmatrix} -\delta \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

The first row in matrix $A$ and $b$ represents the condition $\varphi + \frac{1}{J(1-\beta)}\sum_{j=1}^{J} s_j \leq \delta$ in (1), while the remaining rows represent the condition $w^T r_j + \varphi + s_j \geq 0$. Since the objective of the formulation is to minimize the loss, then the returns must be multiplied by $-1$. $N$ and $J$ in matrix $A$ represents the number of bonds in a portfolio and the number of simulations respectively.

(b) Equality Constraints
The equality constraints in this formulation are of the form $Aeq * w = beq$. The equality matrices $Aeq$ and $beq$ are used to define

$$\sum_{i=1}^{N} w_i = 1,$$

which means that the sum of all the asset weights is equal to 1 or 100%. The equality matrices can be represented in the following matrix form:

$$Aeq = \begin{pmatrix} 1 & 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}.$$

$$beq = (1).$$

iv. Lower and Upper Bounds

The lower and upper bounds in this formulation follow the formulation in [2] and are not restricted to the condition that any asset in a portfolio can have a maximum of 100% of the portfolio weight and must be greater than 0. Matrices *UB* (upper bound) and *LB* (lower bound) can be in the form of:

$$\begin{array}{lllllllllll} & w_1 & w_2 & \cdots & w_N & \varphi & s_1 & s_2 & \cdots & s_j \\ UB = & (UB_1 & UB_2 & \cdots & UB_N & \text{inf} & \text{inf} & \text{inf} & \cdots & \text{inf}). \\ LB = & (LB_1 & LB_2 & \cdots & LB_N & 0 & 0 & 0 & \cdots & 0). \end{array}$$

The constraint is defined as $s_j \geq 0$, where $j = 1, \ldots, J$ and $s_1, s_2, \ldots, s_j = 0$ in LB.

v. Initial Parameter

The initial parameter for the fmincon needs to be set up first before it is used by the optimizer. The initial parameter is the vector $w_0$, consists of the values $w_1, w_2, \ldots, w_N$ that are initialized by $\frac{1}{N}$, the initial values of $s_1, s_2, \ldots, s_j$, which are all zeros and the initial value for φ, which is the quantile of the equally weighted portfolio returns, namely $VaR_0$. Given these initial value $w_0$ can be described as

$$w_0 = \begin{pmatrix} \frac{1}{N} & \frac{1}{N} & \cdots & \frac{1}{N} & VaR_0 & 0 & 0 & \cdots & 0 \end{pmatrix}.$$

Various CVaR limits (δ) were used to see the changes in the returns. The optimization computations the weight vector *w* of the optimal portfolio where $w_1, w_2, \ldots, w_N$ are the corresponding weights of *N* assets. Meanwhile, $w_{N+1}$ is the corresponding VaR and *fval* is the expected return.

## 2.2 Improved CVaR-Based Portfolio Optimization Method

Asset allocation of a portfolio is one of the important key strategies in minimizing risk and maximizing gains. Since the asset allocation in a portfolio is very important [10], thus, an improvement of the existing CVaR-based Portfolio Optimization Method is proposed in this paper. The improved CVaR-based Portfolio Optimization Method focused on determining the upper and lower limits of the bond weight in a diversified portfolio. In estimating the upper and lower limits of each bond weight, the volatility weighting schemes have been used in this study due to the close relationship between volatility and risk. Bond portfolio weight can be obtained by applying the formula in [11] as follows:

$$w_i = k_i \sigma_i^{-1} \tag{2}$$

where

$w_i$ = weight of bond $i$,

$\sigma_i$ = volatility of returns of bond $i$,

$k_i$ = variable that controls the amount of leverage of the volatility weighting such that

$$k_i = \frac{1}{\sum_{i=1}^{n} \sigma_i^{-1}} \tag{3}$$

in a diversified portfolio for $i = 1, 2, \ldots, n$. The weight of each bond in the diversified portfolio in (2) is used as an indication in setting the upper and lower limits by setting the respective floor and ceiling values as follows:

$$\lfloor w_i \rfloor \leq w_i \leq \lceil w_i \rceil. \tag{4}$$

The floor and ceiling values of $w_i$ are rounded to the nearest tenth due the values of $w_i$ being in percentage form, which have been evaluated using Microsoft Excel. Thus, the improved CVaR-based Portfolio Optimization Method can be presented as follows:

$$\min - w^T \bar{y}$$

subject to

$$w \in W, \varphi \in \Re \tag{5}$$

$$\varphi + \frac{1}{J(1-\beta)} \sum_{j=1}^{J} s_j \leq \delta$$

$$s_j \geq 0, \qquad j = 1, \ldots, J$$

$$w^T r_j + \varphi + s_j \geq 0, \qquad j = 1, \ldots, J$$

$$\lfloor w_i \rfloor \leq w_i \leq \lceil w_i \rceil$$

## 2.3 Simulation of Existing and Improved CVaR-Based Portfolio Optimization Methods

The simulation of the optimization process of both the existing CVaR-based and the improved CVaR-based Portfolio Optimization Methods in generating the returns were carried out using the Monte Carlo Simulation method. The Geometric Brownian Motion (GBM), or the stochastic pricing model of bonds, was used in the simulation to generate future price of bond. GBM, which is also known as Exponential Brownian Motion, is a continuous-time stochastic process that follows the Wiener Process, and is defined as the logarithm of the random varying quantity.

The diversified or multiple asset bond portfolios of this study comprises of bonds issued by four banking institutions namely the Export-Import Bank of Malaysia Berhad (EXIM), Commerce International Merchant Bankers (CIMB) Malaysia, European Investment bank (EIB) and Emirates National Bank of Dubai (Emirates NBD). EXIM and EIB issued the *Sukuk* while CIMB and Emirates NBD issued the conventional bonds. Each bond price evolves according to the Brownian motions that are described in (6):

$$\begin{aligned} S(\Delta t)_1 &= S(0)_1 \exp\left[\left(\mu_1 - \tfrac{\sigma_1^2}{2}\right)\Delta t + \left(\sigma_1\sqrt{\Delta t}\right)\varepsilon_1\right] \\ S(\Delta t)_2 &= S(0)_2 \exp\left[\left(\mu_2 - \tfrac{\sigma_2^2}{2}\right)\Delta t + \left(\sigma_2\sqrt{\Delta t}\right)\varepsilon_2\right] \\ &\vdots \\ S(\Delta t)_i &= S(0)_i \exp\left[\left(\mu_i - \tfrac{\sigma_i^2}{2}\right)\Delta t + \left(\sigma_i\sqrt{\Delta t}\right)\varepsilon_i\right] \\ &\vdots \\ S(\Delta t)_N &= S(0)_N \exp\left[\left(\mu_N - \tfrac{\sigma_N^2}{2}\right)\Delta t + \left(\sigma_N\sqrt{\Delta t}\right)\varepsilon_N\right] \end{aligned} \tag{6}$$

for $i = 1, 2, \ldots, N$, where

$S(\Delta t)_i$ = Simulated bond price for bond $i$.
$S(0)_i$ = Initial bond price for bond $i$.
$\mu_i$ = Drift rate of returns over a holding period for bond $i$.
$\sigma_i$ = Volatility of returns over a holding period for bond $i$.
$\Delta t$ = Time step for a week.

The random numbers $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N$ are correlated, whereby their correlation patterns depend on the correlation patterns of bonds returns [12]. By using Cholesky factorization of variance-covariance matrix, the correlated asset paths are generated from the given correlation matrix. The Cholesky factorization can be described as follows:

$$C = U^T U. \tag{7}$$

Correlated random numbers are generated with the help of the upper triangular matrix (with positive diagonal elements) $U$ as follows:

$$R_{r,c} = W_{r,c} * U_{c,c}. \tag{8}$$

Before (8) can be applied, the uncorrelated random numbers $W$ need to be generated first, followed by the construction of bond prices paths using (6) for all bonds. The Cholesky factorization procedure is available in many statistical and computational software packages such as ScaLAPACK [13] and MATLAB. In this study, Cholesky factorization was evaluated by repeating the procedure 3000, 5000, 10000, 20000 times to obtain a distribution of the next period's portfolio price. The simulation for the correlated bond prices based on the existing CVaR-based and the improved CVaR-based Profolio Optimization Methods were generated in MATLAB using a source code modified from [2] (Refer Appendix A).

The results of the simulated bond prices were presented in the form of T-by-N-by-J dimensional matrix where each row represent a holding period $(t_1, t_2, \ldots, t_T)$, each column represents a different bond $(a_1, a_2, \ldots, a_N)$ and each slice in the third dimension represents the number of simulations $(S_1, S_2, \ldots, S_N)$. The returns from the simulated prices were calculated using the log-normal formula which is expressed as follows:

$$R_i = \ln\left|\frac{P_i}{P_{i-1}}\right|, \tag{9}$$

where

$R_i$ = Bond returns at week $i$.
$P_i$ = Bond price at week $i$.
$P_{i-1}$ = Bond price at week $i - 1$.

## 3 Results

The performance of the existing and the improved CVaR-based Portfolio Optimization Methods in optimizing the diversified bond portfolio of this study were compared in order to determine which of the two methods provides a better optimization. The existing and the improved CVaR-based Portfolio Optimization Methods are summarized in Table 1.

**Table 1.** CVaR portfolio optimization method and the improved CVaR portfolio optimization method

| | Existing CVaR portfolio optimization by Rockafellar and Uryasev [4] | Improved CVaR portfolio optimization |
|---|---|---|
| Method | $\min \ -w^T\bar{y}$<br>subject to<br>$w \in W, \varphi \in \Re$<br>$\varphi + \frac{1}{J(1-\beta)}\sum_{j=1}^{J} s_j \leq \delta$<br>$s_j \geq 0, \quad j = 1, \ldots, J$<br>$w^T r_j + \varphi + s_j \geq 0, \quad j = 1, \ldots, J$ | $\min \ -w^T\bar{y}$<br>subject to<br>$w \in W, \varphi \in \Re$<br>$\varphi + \frac{1}{J(1-\beta)}\sum_{j=1}^{J} s_j \leq \delta$<br>$s_j \geq 0, \quad j = 1, \ldots, J$<br>$w^T r_j + \varphi + s_j \geq 0, \quad j = 1, \ldots, J$<br>$\lfloor w_i \rfloor \leq w_i \leq \lceil w_i \rceil$ |

Table 2 shows that the results of the optimal CVaR and the expected loss generated using the improved method, which has restricted condition for the upper and lower bounds, are lower than that of the existing method in [4], which has no restricted conditions. The correct choice of maximum and minimum bond weight when performing the optimization process can help reduce the portfolio's VaR and CVaR along with the expected loss.

As demonstrated by the results in Table 3, the inclusion of the upper and lower bounds for each bond in the diversified portfolio shows that each bond plays a significant role in reducing the expected loss resulting in a more balanced portfolio as compared to the optimization using the existing method. However, the Sukuk appears to provide more benefits to investors and issuers in producing a balanced diversified portfolio due to the reduced CVaR. The results obtained from the existing CVaR-based Portfolio Optimization Method show unbalanced bond weight allocations of the diversified portfolio leading to a bias towards the positive drift rate.

**Table 2.** Results generated by existing CVaR portfolio optimization method and the improved CVaR portfolio optimization method

| | | Existing CVaR portfolio optimization method | Improved CVaR portfolio optimization method |
|---|---|---|---|
| Results | Risk limit | −2.50 | |
| | Confidence level | 99.9 | |
| | Expected loss | −0.0264 | −0.0194 |
| | VaR portfolio | −0.0125 | −0.0125 |
| | CVaR portfolio | −0.0148 | −0.013 |

**Table 3.** Assets weights generated by existing CVaR portfolio optimization method and the improved CVaR portfolio optimization method in the diversified portfolio

| | | Generated assets weights | |
|---|---|---|---|
| | | Existing CVaR portfolio optimization method | Improved CVaR portfolio optimization method |
| Results | EXIM *Sukuk* (%) | 0.013 | 19.49 |
| | EIB *Sukuk* (%) | 0.1777 | 29.96 |
| | CIMB (%) | 99.789 | 39.97 |
| | Emirates NBD (%) | 0.0193 | 10.58 |

## 4 Conclusion

In conclusion, this study has successfully improved the existing CVaR-based method for optimizing a diversified portfolio presented in [4] by using the approach presented in [2]. The need to improve the existing method is due to the possibility of the method resulting in an unbalanced bond weight allocation for a diversified portfolio. The improved method proposed in this study appears to overcome this problem. The method is found to be more helpful in allocating the optimal weight of bonds in a diversified portfolio in order to minimize the loss for a certain risk level. The improved CVaR-based Optimization Method minimizes the loss by introducing new constraint level on the upper and lower limit of the bond weight. The constraint is based on the volatility weighting scheme for the optimization formulation since there is a strong relationship between volatility and risk. Given the results, it can be concluded that the improved CVaR-based Optimization Method is able to provide positive results in terms of lower expected loss and optimal CVaR.

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# APPENDIX A

## Source Code

### A.1: Simulated Price and Return to Run A.2

```
function[S,r] = Simulated(S0,drift,vol,corr,steps,nsims)

    nAssets = length(S0);
    dt=1/52;%time steps for one week%
     %to stimulate correlated asset path (bond prices) based on MCS%
    R = chol(corr);%cholesky factorization%
    S = nan(steps+1,nsims,nAssets);
    for irand = 1:nsims
       x = randn(steps,size(corr,2));
       ep = x*R;
       S(:,irand,:) = [ones(1,nAssets); ...
     cumprod(exp(repmat(drift*dt,steps,1)+ep*diag(vol)*sqrt(dt)))]*
      diag(S0);
    end

    nAssets=size(S,3);
    nsims=size(S,2);
    %to generate return from simulated prices%
    r=nan(nsims,nAssets);
    for iSim = 1: nAssets
       k = squeeze (S(:,:, iSim ));
       rSim = log(k(end,:)./k(1 ,:));
       r(:,iSim) = rSim;
    end

end
```

### A.2: CVaR Portfolio Optimization

```
function Optimization_CVaR (r,beta,CVaRLimit,UB,LB)
    % Sizes
    [nsims,nAssets]=size(r);
    % Inequality constraints
   A1=[zeros(1,nAssets) 1 1/(1-beta)*1/nsims*
   ones(1,nsims)];
    A2=-r;
    A3=-ones(nsims,1);
    A4=-eye(nsims,nsims);
    A=[A2 A3 A4];
    A=[A1;A];
    b=[-CVaRLimit zeros(1,nsims)];
    b=b';
    % Equality constraints --> sum of weights has to be
  100%
    Aeq = [ones(1,nAssets) zeros(1,nsims+1)];
    beq = [1];
    % Upper and lower bounds
    if UB==1
       UB=[repmat(UB,1,nAssets) +Inf*ones(1,nsims+1)];
    else
       UB=[UB +Inf*ones(1,nsims+1)];
    End

    if LB==0
       LB = [repmat(LB,1,nAssets) zeros(1,nsims+1)];
    else
       LB = [LB zeros(1,nsims+1)];
    end
    % Initial weights and initial VaR
    w0=[(1/nAssets)*ones(1,nAssets)];
    VaR0=quantile(r*w0',beta);
    w0=[w0 VaR0 zeros(1,nsims)];
    % Objective function
    objfun = @(w) -mean(r(:,1:nAssets))*w(1: nAssets)';
    options = optimoptions(@fmincon,'Algorithm',
  'interior-point');
    options = optimoptions(options,'MaxFunEvals',
  100000);
    % Optimization %
    [w,fval,exitflag,output]=fmincon(objfun,w0,A,b,Aeq,
  beq,LB,UB,[],options);
    history = [];

    wopt=w(1:nAssets)'; % Optimal weight%
    Asset_Weight_Optimal=wopt*100
    ExpectedReturn=-fval % Expected Return %
    ropt=r*wopt; % Optimal Return %
    VaR_OptPort=-w(nAssets+1)/100 %Optimal VaR%
    p=sort(ropt,'descend');
    CVaR_OptPort= mean(ropt(ropt<VaR_OptPort));
     % Optimal CVaR%
    display (CVaR_OptPort);
end
```

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