Hydroelasticity Analysis of a Container Ship Using a Semi-analytic Approach and Direct Coupling Method in Time Domain



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Abstract A three-dimensional semi-analytic method in time domain is used to predict the hydroelastic effects due to wave-induced loads on a container ship. A container ship with zero forward speed has been taken to perform the hydroelastic analysis. The pertinence of the proposed method is verified with the results obtained from the direct coupling between FEM and BEM in time domain. In both the approaches, the proposed structure has been modelled as an Euler-Bernoulli beam. However, in case of the semi-analytic approach, the container ship has been assumed as an equivalent rectangular barge with uniformly distributed mass. The hydrodynamic forces are obtained in time domain through impulse response function. The Duhamel integral is employed in order to get the structural deflections, velocity, etc. The hydrodynamic and structural part is then fully coupled in time domain through modal analysis to capture the proper phenomena. On the other hand, in case of the direct coupling, a 3D time domain lower order panel method is used for the solution of the hydrodynamic problem. Structural responses, shear forces and bending moments are calculated at different sections of the ship. The validation of the computed results is confirmed as satisfactory agreement is found between these two methods. It may be noted that the present semi-analytic technique appears to be time efficient, robust and could be a very useful tool in predicting the hydroelastic effects on a container ship in terms of shear force, bending moment, structural deflection at initial design stage.

Keywords Hydroelasticity \cdot Container ship \cdot Impulse response function Duhamel integral \cdot BEM–FEM

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1 Introduction

The study of fluid-structure interaction problem, especially for large floating structures, has been of great importance since past few decades. Nowadays, shipbuilding and offshore industries are constructing larger structures to carry more cargo/passenger and to achieve more production benefit. Large offshore structures and ships are relatively more flexible and their natural frequencies can fall into the range of the encounter frequencies of commonly occurring sea spectrum. Hence, in this context the study of hydroelastic behaviour of ship like structures has been a key topic for research and development.

Since the progress of the classical hydroelastic tool to explore the fluid-structure interaction problem [1], scientific investigators have been contributed to the several aspects of this domain. Several numerical techniques have been proposed to investigate the hydroelastic analysis of floating bodies considering various aspects and challenges of this domain. The following are the few such examples of hydroelastic analysis of simplistic structures; the interaction of monochromatic incident waves with a horizontal porous flexible membrane has been investigated in the context of two-dimensional linear hydroelastic theory [2], a contemporary hydroelastic theory has been developed to describe how sea ice responds to an ocean wave field and those that relate to a very large floating structure (VLFS) experiencing comparable forcing [3], three methods have been proposed to determine the motion of a two-dimensional finite elastic plate floating on the water surface, allowing evolving freely [4].

However, it is seen from the literature that there is a need to develop a robust methodology to assess the hydroelastic analysis for complicated ship like structures more precisely. Few initial attempts were made to study the fluid-structure interaction problem for real ship shaped body [1]. Later, other noteworthy contributions in this area are as follows; a two- and three-dimensional hydroelasticity theory is developed to predict and compare the dynamic behaviour of a bulk carrier hull in waves [5], a hydroelasticity theory is used to analyse the response of barge and large container ship in waves [6], a fully coupled time domain ship hydroelasticity problem is investigated focusing on a springing phenomenon using a hybrid BEM–FEM scheme adopting a higher-order B-spline Rankine panel method [7] etc.

From the above discussion, it is interesting to note that either an analytical method is introduced to study the hydroelastic behaviour of very simplified two- or threedimensional structures [8, 9] or rigorous numerical technique has been developed in order to deal with the fluid-structure interaction problem for ship shaped bodies [10, 11]. In this context, the present study is an attempt to inspect the hydroelastic behaviour of a container ship with zero speed and to evaluate the response assuming the ship as an equivalent rectangular barge with uniformly distributed mass. A robust and efficient semi-analytical approach in time domain [12] is used to obtain the structural deformation, bending moment, shear force, etc. which will be useful at the preliminary design stage. The hydrodynamic problem is solved using impulse response function [13, 14]. The structural part of the hydroelastic analysis has been solved using modal superposition approach of an Euler–Bernoulli beam. Finally, the response of the structure is calculated semi-analytically by means of Duhamel integral. To check the robustness and efficiency of the present method, the obtained results for bending moments and shear forces are compared with a direct coupled BEM–FEM method in time domain and also with available published experimental results [15]. It is found out that the results obtained from the present study are showing satisfactory agreement with published results as well as the direct BEM–FEM coupling method. The main idea of the paper is to model a complicated ship like structure in a simplified way and to get the results good enough to predict the response accurately at initial design stage which can be further compared with sophisticated and computationally expensive methods.

2 Mathematical Formulation

In the present study, the fluid-structure interaction problem is solved in time domain. A one-dimensional beam model is taken into account to study the 3D barge structure. The structural deflection has been obtained using Duhamel integral, whereas the radiation forces are calculated using IRF in frequency domain. The coupling phenomena between the structural part and the hydrodynamic part are then fully investigated in order to execute the hydroelastic analysis more accurately.

2.1 Governing Equation

The deflection of the floating body is based on the foundation of an Euler–Bernoulli beam equation in the present study. The beam is free at both ends which means bending moment and shear force are zero at both the ends. The floating barge is considered as an equivalent beam on elastic foundation, and hence, the governing equations are obtained as,

$$\frac{\partial^2}{\partial x^2} \left(EI \frac{\partial^2 w}{\partial x^2} \right) + \rho A \frac{\partial^2 w}{\partial t^2} + k_f w = f(x, t) \tag{1}$$

where *E* is Young's modulus, *I* is the vertical moment of inertia of the cross section, *w* is the elastic deflection of the body, ρ is the density of the material, *A* is the cross section area of the beam, k_f is the hydrostatic restoring coefficient and f(x, t) is the external wave force applied on the beam.

The above-mentioned differential equation is solved using modal superposition technique, i.e. the elastic deformation w(x, t) is assumed to be the summation of modeshape function W(x) and modal displacement q(t) as follows,

$$w(x,t) = \sum_{n=1}^{\infty} W_n(x)q_n(t)$$
(2)

As the beam is free-free at both the ends, the boundary conditions are as follows,

$$\frac{d^2 W(x)}{dx^2} = 0, \ \frac{d^3 W(x)}{dx^3} = 0 \ \text{at} \ x = 0 \text{ and } x = L$$
(3)

2.2 Hydrodynamic Solution

The hydrodynamic problem is solved using potential flow theory. The basic formulation has not been discussed here rigorously. Primary focus is given in the modelling of the hydrodynamic forces. The hydrodynamic forces comprise of wave excitation force F^{exc} and radiation force F^{R} as,

$$f(x,t) = F^{\mathrm{R}}(x,t) + F^{\mathrm{exc}}(x,t)$$
(4)

The radiation force is proportional to the velocity and acceleration of the body which not only varies over time but also with the length of the vessel. Hence, proper fluid-structure model is necessary to estimate the added mass and damping coefficient for elastic modes. The radiation forces are calculated using impulse response function theory in time domain. On the other hand, the wave exciting force is assumed as a periodic impulsive point load acting at the centre of gravity of the structure. The detailed modelling and the corresponding formulations of these forces can be found in [12].

2.3 Mode Functions

Deflection of the floating body has been represented by free-free beam modes. Six elastic modes have been taken for the convergence. Orthogonal property of the mode-shapes has been used in order to get the functions. Then, after few steps, the mode-shape functions for the Euler–Bernoulli beam are obtained as,

$$W_n(x) = (\cos(\alpha_n x) + \cosh(\alpha_n x)) - K_n(\sin(\alpha_n x) + \sinh(\alpha_n x))$$
(5)

2.4 Coupling of Fluid-Structure

From the available literature [10], it is found out that the coupling phenomena are independent of the chosen hydrodynamic model. The coupling has been studied for the zero speed case of the container ship under head waves. While solving Eq. (1) by modal superposition approach, the following equation is obtained to get the corresponding modal deflection,

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$$\ddot{q}_n(t) + \omega_n^2 q_n(t) = \frac{1}{\rho A} P_n(t) \tag{6}$$

The above differential equation has been solved by Duhamel integral technique as the generalized force, $P_n(t) = \int f(x, t) W_n(x) dx$ does not fit to any known mathematical form. The deflection $q_n(t)$ is thus obtained as,

$$q_n(t) = \frac{1}{\rho A \omega_n} \int_0^t P_n(\tau) \sin \omega_n(t-\tau) \,\mathrm{d}\tau \tag{7}$$

Differentiating the deflection gives the normal velocity $\dot{q}_n(t)$ of the concerned modeshape. Once the amplitude and velocity are obtained, the corresponding radiation force is also calculated for the next time step and thus the fluid-structure coupling is made in time domain.

2.5 Direct Coupling of BEM and FEM

The results in this paper are compared with numerical method based on direct coupling between BEM and FEM in time domain. Bending moment and shear force results for the container ship have been computed by this method. For the completeness of the paper, the method has been briefly discussed below.

2.5.1 Formulation of the Structural Problem

The ship hull is modelled as an Euler–Bernoulli beam floating in water. The governing differential equation of this beam is given by,

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + gBw = f \tag{8}$$

here *E* is the modulus of elasticity of the material, *I* is the moment of inertia, is the displacement along *z*-axis, ρ is the density of the material, *A* is the cross-sectional area of the beam, *B* is the width of the hull at the water plane, *g* is the acceleration due to gravity and *f* is the external force per unit length to the hull at *x*. The external force per unit length at *x* is obtained by integrating the total pressure on the hull surface at *x*.

The hull girder idealized as an Euler–Bernoulli beam has been discretized into a set of N finite elements. Each element has two nodes, each node having two degrees of freedom w and w_x . The subscript x denotes derivation with respect to x. In element e, w at any point can be written in terms of the nodal degrees of freedom as,

$$w = \{N\}^{eT} \{d\}^{e}$$
(9)

here $\{N\}^e = \{N_1^e N_2^e N_3^e N_4^e\}$ where $N_i^e(i = 1, 4)$ are the Hermite shape functions and $\{d^e\} = \{w_1^e w_{x1}^e, w_2^e w_{x2}^e,\}$ is a vector of the degrees of freedom at the two nodes of element *e*.

Applying the Galerkin technique to Eq. (8) and using Eq. (9), the elemental finite element equation for element *e* can be written as,

$$[M^{e}]\{\ddot{d}^{e}\} + [K^{e}]\{d^{e}\} = \{F^{e}\}$$
(10)

Value of f comes from the hydrodynamic analysis explained in Subsect. 2.5.2. Equation (10) is a system of four linear simultaneous ordinary differential equations. Combining these elemental equations with consideration to the nodal connectivity of different elements, we get the global finite element equation. This is a set of $2(N_e + 1)$ linear simultaneous ordinary differential equations. Adding a Rayleigh damping term, we get the following system of global finite element equations,

$$[M]\{\dot{d}\} + [C]\{\dot{d}\} + [K]\{d\} = \{F\}$$
(11)

2.5.2 Hydrodynamic Analysis

The hydrodynamic problem is solved using 3D linear time domain panel code. The boundary value problem is solved using 3D transient free surface Green's function as can be seen in [16]. Detailed analysis of the numerical scheme used here is discussed in many sources [17–19] and hence not repeated here.

2.5.3 Numerical Integration

The global finite element Eq. (11) and the boundary element and pressure equations have been solved with the help of Newmark's time integration method. Solution of Eq. (11) gives the values of displacements, velocities and accelerations at the nodes. However, to solve the Euler–Bernoulli beam Eq. (11), force per unit length is required. The values of *f* in Eq. (11) at different locations can be obtained by solving the hydrodynamic problem.

According to Newmark's method, nodal displacement vector at any time instant t can be related to $\{d\}$ at the previous time instant by the following equation,

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$$\left(\frac{[M]}{\alpha\Delta t^{2}} + \frac{\delta}{\alpha\Delta t}[C] + [K]\right)\{d\}_{t}$$

$$= \{F\}_{t} + [M]\left[\frac{1}{\alpha\Delta t^{2}}\{d\}_{t-\Delta t} + \frac{1}{\alpha\Delta t}\{\dot{d}\}_{t-\Delta t} + \left(\frac{1}{2\alpha} - 1\right)\{\ddot{d}\}_{t-\Delta t}\right]$$

$$+ [C]\left[\frac{1}{\alpha\Delta t}\{d\}_{t-\Delta t} + \left(\frac{\delta}{\alpha} - 1\right)\{\dot{d}\}_{t-\Delta t} + \frac{\Delta t}{2}\left(\frac{\delta}{\alpha} - 2\right)\{\ddot{d}\}_{t-\Delta t}\right] \quad (12)$$

After developing $\{d\}$, the velocities and acceleration vectors can be calculated using the following equations,

$$\{\ddot{d}\}_{t} = \frac{1}{\alpha \Delta t^{2}} (\{d\}_{t} - \{d\}_{t-\Delta t}) - \frac{1}{\alpha \Delta t} \{\dot{d}\}_{t-\Delta t} - \left(\frac{1}{2\alpha} - 1\right) \{\ddot{d}\}_{t-\Delta t}$$
(13)

$$\{\dot{d}\}_t = \{\dot{d}\}_{t-\Delta t} + \Delta t \left[(1-\delta)\{\ddot{d}\}_{t-\Delta t} + \delta\{\ddot{d}\}_t \right]$$
(14)

Using the velocities obtained from Eq. (14), force for the next time step can be calculated. Using this force and with the help of Eqs. (12)–(14), displacement, velocity and force vectors can be obtained for the next time step. In this way, the time marching is continued. The force on the hull is calculated from the boundary element equations, and the displacement, velocity and the acceleration vectors are calculated using the finite element equations at different time steps.

3 Results and Discussions

In the present study, a semi-analytic method in time domain is used to predict the hydroelastic response of a container ship modelled as a rectangular barge with uniformly distributed mass. The length of the barge is taken as 286.6 m which is same as of the container ship. The maximum breadth of the ship has been taken as the beam of the barge which is 40 m. And, the draft-to-length ratio is taken as 0.02. For the sake of simplicity, zero forward speed case of the ship has been considered for the present hydroelastic analysis.

Figure 1 demonstrates the geometry of the container ship and Fig. 2 shows the first four deformable modes of that ship which has been used in the coupled BEM–FEM method to get the wave-induced responses.

Six deformable modes have been considered to represent the elastic deflection of the floating body while analysing the fluid-structure interaction problem through the IRF method. Figure 3 shows the first four deformable modes of the vessel. The deflections are unscaled and only to demonstrate how the elastic deformation varies over the length at any instant t. The modes are numbered from 7 to 10 as the first six modes are the usual rigid body modes.

Figure 4 represents the radiation impulse response function obtained for the first six deformable modes. It can be seen from the figure that as the mode number increases the value of response function also gets decreased which indicates the fact



Fig. 1 Geometry of discretized container ship



Fig. 2 Deflection of the container ship for first four flexible modes

that higher modes have lesser contribution in total displacement and other responses due to wave induces loads.

For establishing the idea to estimate hydroelastic response of a container ship with a slender rectangular barge, the results for bending moments and shear forces have been compared with numerically enriched coupled BEM–FEM method. Results from the present methodology are termed as "IRF", whereas the results obtained from the time domain panel method code are termed as "TD". For better understanding and authenticity of the present formulation, the transfer function for bending moment is further compared with published experimental result [15]. In Fig. 5, the three results for bending moment RAOs have been plotted and compared with each other. The bending moments are non-dimensionalized by $1/(\rho g A L^2 B)$ and have been plotted against non-dimensional wavelength λ/L . It is interesting to note that the result obtained from the present IRF method agrees well with the other two numerically expensive methodologies. Although there are slight differences between the bending moment values over all the frequency regions; however, it can be justified as the



Fig. 3 Deflection of the barge for first four flexible modes



Fig. 4 Radiation impulse response function for first six deformable modes

present IRF method is a semi-analytic approach and the results are obtained considering the container ship to be a rectangular barge with uniformly distributed mass. Hence, it is practically not possible to match the data fully. However, the error is within 5% range of accuracy when compared to the other two methods and thus, could be useful at the initial design stage of such a container ship. Henceforth, it can be concluded that the idea which has been proposed as an alternative approach to estimate the response of a container ship is established. It also proves the robustness and efficiency of the present methodology.



Fig. 5 Comparison of bending moment RAOs between IRF, coupled BEM–FEM and experimental results from Rajendran et al.



Fig. 6 Comparison of time signals of bending moments between coupled BEM-FEM and the present IRF code

After establishing the authenticity of the present formulation, few more comparisons regarding shear force and bending moment results have been discussed for the sake of completeness of the present hydroelastic study. In Fig. 6, the time signal for bending moment at midship for $\lambda/L = 1$, i.e. when wavelength is close to the structural length, obtained from "IRF" and coupled BEM–FEM method have been compared. Figure 7 shows the comparison between bending moments along the length of the vessel obtained from these two methods. It is seen that the results match very well and thus it confirms the efficiency of the present analysis.

Shear force results have also been compared with the coupled BEM–FEM method for the completeness of the paper. In all the remaining figures, shear forces are non-dimensionalized by $1/(\rho gALB)$ and have been plotted against non-dimensional



Fig. 7 Comparison of bending moments along the length of the vessel between coupled BEM–FEM and the present IRF code



Fig. 8 Comparison of shear forces between coupled BEM-FEM and the present IRF code

wavelength λ/L . Figure 8 shows the comparison between the shear force RAOs between coupled BEM–FEM and the present IRF code calculated at -L/4 position of the vessel. From the figure, excellent agreement can be seen between these two methodologies. Keeping in mind the simplistic modelling of the structural problem, the slight differences over the lower frequency zone are justified in the range of engineering accuracy.

The study has been further progressed by showing the comparison between time signal of shear forces and shear forces along the length of the vessel between these two methods. In Fig. 9, the time signals have been plotted for $\lambda/L = 1$, i.e. when wavelength is equal to body length at -L/4 positions. And Fig. 10 shows the comparison between shear forces along the length of the vessel obtained from these two methodologies. Excellent agreement can be seen from these two figures as well.



Fig. 9 Comparison of time signals of shear forces between coupled BEM–FEM and the present IRF code



Fig. 10 Comparison of shear forces along the length of the vessel between coupled BEM–FEM and the present IRF code

Therefore, as a conclusion, one can say that both the methods are representing the same phenomena although numerical approach and the structural modelling are different.

4 Conclusions and Future Scope

In the present paper, a semi-analytic three-dimensional time domain methodology has been delivered to estimate the hydroelastic response of a container ship with zero forward speed exposed to waves. An Euler–Bernoulli beam has been considered to model the rectangular barge with analytically defined modeshapes. The analysis has been carried out through impulse response function. Duhamel integral is employed in order to get the structural response, bending moments and shear forces. The structural and hydrodynamic parts are then fully coupled in time domain for the solution of the hydroelastic problem in the present solution scheme.

Main focus is given in developing the idea of estimating the response of a container ship due to wave-induced loads with the help of a rectangular barge with uniformly distributed mass. The results for shear forces and bending moments RAOs obtained from this IRF method have been compared with the results obtained from the direct coupled BEM–FEM method. Time signals and along the length properties are also compared. Satisfactory agreement has been found between these two approaches which proves the robustness of the present concept. To establish the authenticity of the present work, the result for bending moment RAO at midship has been compared with the experimental results obtained from Rajendran et al. which once again proves the efficiency of the proposed state of the art.

For simplicity, a barge with uniformly distributed mass is studied in the present hydroelastic analysis. It is found out from the comparison results that despite simple assumptions, the results are good enough to fit in the range of engineering accuracy. The present methodology is more time efficient compared to the other two methods. Hence, the present concept can be used as an alternative procedure to guess the shear force, bending moment of a container ship at initial design stage.

As a future scope, the hydroelastic analysis can be carried forward by considering the coupled flexural and torsional vibrations. This methodology can be further extended to investigate the hydroelastic analysis of a container ship with forward speed where some nonlinearities such as nonlinear Froude–Krylov force, nonlinear hydrostatics, etc. can be incorporated. Horizontal bending of the ship can further be investigated to portray the fluid-structure interaction problem more accurately. This methodology is also able to study the response of a container ship arising due to local phenomena such as green water loading, slamming, etc. These forces could be obtained from rigorous CFD solver and then coupled with the present IRF method.

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