Obituary for a Flea

Jasper van Heugten and Sander Wolters

Abstract The Landsman–Reuvers proposal to solve the measurement problem from within quantum theory is extensively analysed. In favor of proposals of this kind, it is shown that the standard reasoning behind objections to solving the measurement problem from within quantum theory rely on counterfactual reasoning or mathematical idealisations. Subsequently, a list of objections/challenges to the proposal are made. Part of these objections are equally important for all attempts at solving the measurement problem, such as the problem of interpreting small numbers in the density matrix, the problem of reproducing the Born rule, the use of pure states as a tool to alleviate the interpretational issues of quantum states, and the necessity of introducing classical certainties which are not strictly present in quantum theory. The additional objections that are particular to the proposal, such as the physical interpretation/origin of the flea perturbation, the use of potentials to solve a dynamical problem, slow collapse times, the inability to handle unequal probabilities, and the dictatorial role of the flea perturbation, lead us to believe that the Landsman–Reuvers proposal is lacking in both physical grounding and theoretical promise. Finally, an overview is given of the challenges that were encountered in this attempt to solve the measurement problem from within quantum theory.

Keywords Measurement problem · Quantum measurement · Bohrification

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1 Introduction

In 2013, Landsman, and his student Reuvers, proposed a reformulation of the measurement problem of quantum mechanics [13]. Using the sensitivity of bound states with respect to small perturbations in the semi-classical limit, it was claimed that this reformulation may hold the possibility of resolving the measurement problem within the formalism of quantum theory. We shall refer to this idea and the (simple) mathematical model expressing this idea as the *flea model* or *flea approach*. Later [10], Landsman embedded his treatment of the measurement problem within a larger framework regarding, but not limited to, the relations between classical and quantum physics. Following the contribution of Landsman and Lindenhovius in this volume, we refer to this framework as *asymptotic Bohrification*.

Our goals are twofold:

1. To investigate the feasibility of the flea model for dynamical collapse.
2. To analyse to what extent asymptotic Bohrification captures the measurement problem.

Apart from this introduction, the final discussion and the appendices, the text is divided into two parts. Each part deals with one of the two goals. The introductory section only contains a single subsection, where a rather general formulation of the measurement problem is recalled. This subsection contains no new insights, but helps us to provide something to compare the measurement problem of the flea approach to.

In Sect. 2, we consider the reformulation of the measurement problem within asymptotic Bohrification. Section 2.1 introduces the measurement problem and highlights some of the underlying ideas. The asymptotic Bohrification formulation of the measurement problem requires a more liberal notion of outcome state for measurements. The lack of empirical grounding for such states is briefly treated in Sect. 2.2. Next, in Sect. 2.3, we consider the pressing problem of obtaining the correct Born probabilities for the outcomes in the flea model. The main problem here is that getting the Born probabilities correct may require us to abandon independence between the initial system state and the measurement apparatus. Finally, in Sect. 2.4 we attempt to relate the flea model to a conventional version of the measurement problem. The flea approach is incomplete in the sense that it is silent about setting up system-pointer correlations and gives no physical background on the flea as well as the wave function which is to be collapsed.

Subsequently, in Sect. 3 we consider the effectiveness of the collapse mechanism of the flea model. In Sect. 3.1 we consider the sizable obstables in generalising the collapse mechanism of the flea model to more outcomes, and unequal Born probabilities. In Sect. 3.2 we investigate the time scale of the collapse. There it turns out that the collapse of the wave function takes an unrealistically long time.

We conclude regretfully that as a model for dynamical collapse the flea model as it now stands is not feasible. Although the underlying idea looks good statically, there is no reason to believe that it would work dynamically. But even if the collapse were effective, it is not clear what this would say about measurements. The model is silent on the system-pointer correlation and the physics behind the flea and the potential, and seems far removed from existing models of quantum measurement. And even if a working model of quantum measurement, based on the flea model, could be found, the problem of replicating the Born probabilities may still require the addition of some sort of conspiracy theory or superdeterminism. If this is the case, then it is not clear what more the flea model has to offer, with respect to conventional formulations of the measurement problem.

1.1 The Measurement Problem

We briefly consider (a simple version of) the measurement problem as formulated in [2]. This formulation is of interest because it uses a sufficiently liberal notion of outcome, and because it does not rely on idealisations, making it relevant to the flea model. We ask to what extent there is still room for the flea model to circumvent the measurement problem posed here.

Consider a two-level system $\mathscr{S}$. Let $|+\rangle$ and $|-\rangle$ be an orthonormal basis of the Hilbert space $\mathscr{H}_{\mathscr{S}}$. The post-measurement states are written in the form $|A, \alpha\rangle$. The label A denotes a value of a certain macroscopic variable, which we call the pointer variable. With respect to the state $|A, \alpha\rangle$, the pointer is assigned the value A. The second index, α, refers to all other degrees of freedom we deem relevant. These might belong to the system $\mathscr{S}$, or be environmental, and may even include the whole universe (apart from the pointer variable).

Let V_A denote the set of all states $|A, \alpha\rangle$, for which we agree that it is sensible to say that the pointer variable has value A. Suppose that A and B are two macroscopically distinguishable values for the pointer variable. We shall not assume that the sets V_A and V_B are closed linear subspaces of the relevant Hilbert space, or, even stronger, that there are associated projections which are orthogonal. We instead consider a weaker relation between the elements of V_A and V_B. This weaker relation allows for the possibility of states in which the pointer has an outcome, but which display some form of tail with respect to the pointer variable. Recall that if $|\Phi\rangle$ and $|\Psi\rangle$ are orthogonal vectors, then $\||\Phi\rangle - |\psi\rangle\|^2 = 2$. We assume that elements of V_A and V_B are close to orthogonal in the sense that there exists a number $\eta \ll 1$ such that for all $|A, \alpha\rangle$ in V_A and all $|B, \beta\rangle$ in V_B we have,

$$\||A, \alpha\rangle - |B, \beta\rangle\| \geq \sqrt{2} - \eta. \tag{1}$$

Alternatively, this assumption follows from assuming that for macroscopically distinguishable pointer values A and B, there exists a positive number $\varepsilon \ll 1$, such that the transition probability satisfies

$$p(|A, \alpha\rangle \mid |B, \beta\rangle) = |\langle A, \alpha|B, \beta\rangle| \leq \varepsilon, \tag{2}$$

for all states in V_A paired with states in V_B. The initial pre-measurement state is assumed to be factorised

$$(a|+\rangle + b|-\rangle) \otimes |\Phi\rangle, \tag{3}$$

where $|\Phi\rangle$ is an appropriate initial state of the measurement apparatus and the environment. We do not need to further specify this state. The post-measurement state is not assumed to be factorised. Time evolution is assumed to be given by a unitary transformation, and is in particular linear. An initial state of the form $|+\rangle \otimes |\Phi\rangle$ evolves to a state $|P, \alpha\rangle$, with pointer value P. We assume that initial states of the form $|-\rangle \otimes |\Phi\rangle$ evolve into states $|M, \beta\rangle$ for which we attribute the value M to the pointer.

The measurement problem arises when the initial system state is taken to be

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right). \tag{4}$$

By linearity the measurement interaction yields the evolved state

$$|\psi\rangle \otimes |\Phi\rangle \mapsto U(t_0, t_f)|\psi\rangle \otimes |\Phi\rangle = \frac{1}{\sqrt{2}}\left(|P, \alpha\rangle + |M, \beta\rangle\right). \tag{5}$$

We assume that the pointer values M and P are macroscopically distinguishable. As a result $|P, \alpha\rangle \in V_P$ and $|M, \beta\rangle \in V_M$ are almost orthogonal in the sense of the inequality (1). But from (5) we deduce that with respect to the norm metric, the distance of the evolved state to both V_P and V_M must at least be large enough that the post-measurement state is forbidden to be in V_P, and forbidden to be in V_M. As a consequence, the post-measurement state is neither an element of V_P, nor an element of V_M, and hence we are unable to assign any outcome to the evolved state.

This formulation of the measurement problem does not seem to depend on any idealisation. In anticipation of later sections, the usage of a *pure* initial state may itself be seen as an idealisation, but this has no impact on the issue at hand. The use of the same unitary operator for different runs of the experiment (in particular, when considering different initial states $|\phi\rangle \otimes |\Phi\rangle$ of the measurement, where $\phi \in \{+, -, \psi\}$) may be viewed as an idealisation. In fact, for the flea model, we shall see that different runs of an experiment use slightly different hamiltonians. In a similar vein, van Wezel [21] adds a small non-hermitian term to the hamiltonian in order to create a dynamical collapse. Regardless of the origin of the variation, we assume that (possibly after tracing out many environmental degrees of freedom) the different unitary operators $U(t_0, t_f)$ are sufficiently close in operator norm with respect to each other. Under this assumption, the previous conclusion that the post-measurement state is neither in V_P, nor in V_M, remains valid.

As the flea model aims to solve the measurement problem from within quantum mechanics. the above formulation of the measurement problem seems to provide a serious obstacle for the flea model. Before considering the reformulation of the measurement problem for the flea mechanism, we consider whether there are still some loopholes left. There is at least one loophole, but it comes at a hefty price. We assumed the initial state to be factorised between system and the rest, as $|\phi\rangle \otimes |\Phi\rangle$. For different runs we assumed that $|\Phi\rangle$ was the same each time. Effectively, we were assuming that the system and apparatus states are independent. This independence is needed to derive the measurement problem. However, exploiting this independence loophole immediately puts us on thin ice, as it questions the very notion of measurement.

2 Bohrification and the Measurement Problem

The flea model provides a new formulation for the measurement problem, as part of a programme called asymptotic Bohrification, as well as a collapse mechanism for the wave function within this formulation. In this section we concentrate on the reformulation of the measurement problem, whereas the next section focuses on the collapse mechanism. In Sect. 2.1 we introduce asymptotic Bohrification, wherein lies the formulation of the measurement problem which the flea model aims to solve. This formulation of the measurement problem requires us to rely on an approximate version of outcome states, unfortunately without providing physical grounding of how to understand such states, as discussed in Sect. 2.2. However, such approximate outcome states are typical of approaches to the measurement problem which take the formalism of quantum mechanics seriously, as elaborated in Appendix 6 and should not (in itself) be seen as a weakness of the approach. Next, in Sect. 2.3 we briefly consider the pressing problem of replicating the Born rule in the flea model. Finally, in Sect. 2.4 we look at the connection between the asymptotic Bohrification formulation of the measurement problem, and more conventional formulations of this problem, such as in Sect. 1.1.

2.1 Asymptotic Bohrification

Consider the following incarnation of the measurement problem, adapted from [15]. According to Maudlin, the measurement problem is the incompatibility of the following three assumptions:

1. Quantum mechanical pure states are complete in the sense that they specify all physical properties of a system.
2. Time-evolution of the states is described by a linear unitary operator.
3. Measurements always (or at least usually) have single outcomes, i.e. at the end of the measurement, the measuring device indicates a definite physical state.

Indeed, in the previous section we saw this incompatibility formulated in a general setting, without any apparent reliance on idealisations. The only loophole mentioned in that section was the independence implicit in the form of the initial state, factorised between system and environment. Note that this assumption is not listed by Maudlin, in particular since denial of this assumption challenges the very notion of measurement.

The various approaches to the measurement problem differ in which of the above three assumptions are challenged. In hidden variable models the first assumption is denied. For dynamical collapse theories such as the GRW models, the second assumption is rejected. For the many-worlds interpretations the third assumption gets a new perspective. But how would the practitioner of the Copenhagen interpretation consider this problem? He (or she) would most likely frown at the first assumption. Properties are classical concepts and have no place in quantum theory. For now, let us ignore this issue. At this point, the Copenhagenist may claim that there is no measurement problem, in the sense of a contradiction. Indeed. assumptions (1) and (2) are about the quantum mechanical formalism whereas assumption (3) deals with the purely classical notions of outcomes and measurements. If we are to connect these assumptions, then we need to use classical approximations at some point of the description. It is exactly because of the need of these approximations that the irreducible probabilities of quantum theory arise. There is nothing mysterious about these probabilities, in the sense that in a purely quantum mechanical description probabilities need not arise. However, aside from putting us in the awkward position that a theory which supposedly generalises classical physics, also needs classical physics for its formulation, the Copenhagen move teaches us little, if anything, about measurements in quantum theory.

And so we ask: why would we *need* classical approximations in the first place? Consider Bohr's doctrine of classical concepts:

> However far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms. (?) The argument is simply that by the word experiment we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account of the experimental arrangements and of the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics.

Although this strikes us as common sense, there are different ways in which we can implement this philosophy with respect to the measurement problem. Note that if we accept the need to use the *language* of, and experience from, classical physics, this does not automatically entail that we need to replace approximations from the formalism of classical mechanics in studying the measurement problem. Just to be clear, we do not see wisdom in attempting to describe any realistic measurement apparatus (and environment) completely at the level of the standard model, and to use this as a model of measurement. However, we could endorse Bohr's doctrine of classical concepts, without making the jump to the rest of the Copenhagen interpretation. That is we could take the stance that the language of classical physics is essential to the measurement problem, but, unlike the Copenhagen interpretation not a priori assuming that the occurrence of an outcome cannot be explained by using a suitable quantum-mechanical model.

This brings us to the *Bohrification* approach to the measurement problem, proposed by Landsman. In this approach, Bohr's doctrine of classical concepts is given the following mathematical interpretation: study non-commutative C*-algebras, such as the algebra of all bounded operators on a Hilbert space, by means of commutative C*-algebras. There are two different ways in which this is done. The first approach, called *exact Bohrification* replaces a non-commutative C*-algebra by its partially ordered set of commutative C*-subalgebras, where the order is given by inclusion. Exact Bohrification is the central theme of the contribution of Landsman and Lindenhovius to this volume. Since this approach has not been applied to the measurement problem, we shall not consider it any further. In the second approach, called *asymptotic Bohrification*, the commutative and non-commutative C*-algebras, no longer related by an inclusion relation, are glued together in a bundle called a countinuous field of C*-algebras. Rather than discuss this approach in full generality, we first consider the motivating example of the flea approach to the measurement problem, as introduced by Landsman and his student Reuvers in [13], and later embedded by Landsman in the asymptotic Bohrification programme in [10]. The terminology which we adopt here was introduced by Landsman in [12].

Consider the following simplistic model used to reformulate the measurement problem. The relevant Hilbert space is $\mathscr{H} = L^2(\mathbb{R})$. The dynamics is generated by the hamiltonian

$$\hat{H}_\hbar = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{\lambda}{8}\left(x^2 - a^2\right)^2 =: \frac{\hat{p}^2}{2m} + V(x), \tag{6}$$

using a symmetric double well potential $V(x)$, with barrier height $V_\mathrm{b} = \lambda a^4/8$. The subscript $\hbar$ was added because we consider different values of $\hbar$, and are in particular interested in the limit $\hbar \to 0$. In this model, the value of $\hbar$ is used to represent a scale for macroscopicity, where smaller values of $\hbar$ correspond to more macroscopic situations. The reader who is uncomfortable about varying a constant of nature may instead consider the limit $\lambda \to \infty$ where the potential energy becomes steeper, or the limit $m \to \infty$.

The state for the model on which we initially focus is the (non-degenerate) ground state ψ_0 for this hamiltonian. The characteristic length of the problem, determining the scale on which the wavefunctions vary, is $l = (\hbar^2/mk)^{1/4}$. Here $k = \lambda a^2$ is the spring constant as determined by the quadratic approximation at the well minima $x = \pm a$. Consider the case with fixed λ, a but variable $\xi = \sqrt{\hbar^2/m}$. We are interested in the setting where $l \ll a$, i.e., large mass or small $\hbar$, where the lowest energy eigenstates are localised within the two wells, see Fig. 1.

The ground state ψ_0 is to represent the state of a pointer or some other macroscopic variable at some point of time during the measurement interaction. Supposedly, there is a system $\mathscr{S}$ which was initially in a Schrödinger cat state. Through its interaction with the measurement apparatus, this Schrödinger cat state was passed on to the pointer. The two wells in the potential correspond to two macroscopically distinct values that the pointer variable may take. The x-variable need not correspond to a physical distance; we only identify the two wells with two pointer values. The

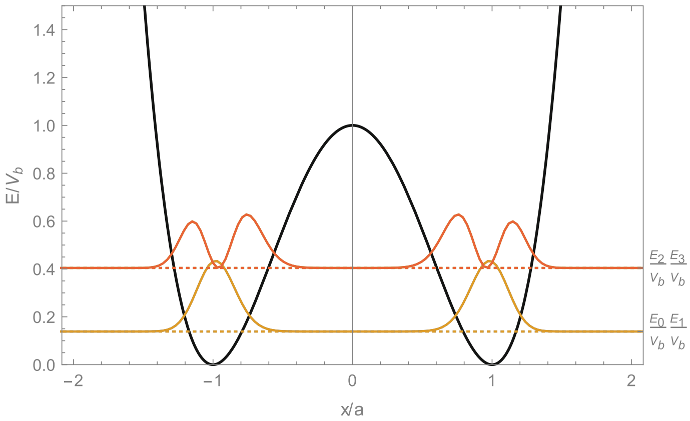


Fig. 1 The first four eigenfunctions as $|\psi(x)|^2$ for the double well potential with $l \ll a$. The first and second, and the third and fourth, wavefunctions overlap due to the symmetry

system $\mathscr{S}$ itself is not visible in this simple model. Neither does the model include an environment $\mathscr{E}$, which contains the uncontrollable degrees of freedom of the apparatus and any other degrees of freedom we may consider to be of interest. The environment plays an important conceptual role in the flea approach to the measurement problem, but the environmental degrees of freedom do not themselves appear in the model.

It may strike the reader as odd to use a bound state, the textbook example of a stable state, as the of the model to focus on. But keep in mind that this state of the model is not the initial state of the measurement. The initial apparatus state may very well have been metastable, as is typical for models of quantum measurement. In addition, we shall see that the flea approach uses a time-dependent hamiltonian, so the initial state does not remain a bound state. Regardless, we may still be sceptical whether the setting of a bound state sporting a macroscopic superposition actually occurs during *any* measurement. But we postpone further discussion with regard to the justification of the model until the end of this section.

At this point we can discuss the reformulation of the measurement problem. The ground state $\psi_0(x)$ has a $\mathbb{Z}_2$-symmetry, through reflection in the y-axis, a symmetry inherited from the invariance of $\hat{H}_\hbar$ under $x \mapsto -x$. Without any further additions or modifications (such as the flea) the wave function does not change over time and retains this symmetry, even when it becomes the post-measurement state. This happens for all non-zero values of $\hbar$. In the limit $\hbar \to 0$, the state - which we now think of as a post-measurement state - $\psi_0(x)$ converges, in a sense we will make precise mathematically in a moment, to the following classical mixed state

$$\rho_0^{(0)} = \frac{1}{2}\left(\rho_0^+ + \rho_0^-\right), \tag{7}$$

where the two phase space points

$$\rho_0^{\pm} = (p = 0, q = \pm a)$$

are the two classical ground states of the (classical) hamiltonian

$$h_0(p, q) = \frac{p^2}{2m} + \frac{\lambda}{8}\left(x^2 - a^2\right)^2. \tag{8}$$

From the point of view of asymptotic Bohrification, the measurement problem is that the quantum-mechanical post-measurement pure state converges to a classical mixed state. Or, alternatively stated, it is not the case that for sufficiently large mass m or small $\hbar$, the post-measurement state approximates a classical pure state.

So from this perspective the measurement problem can be seen as the incompatibility of the following three assumptions:

1. Measurements and their outcomes are notions from classical physics.
2. In many cases of interest, the transition from quantum physics to classical physics can be described by a limit such as $\hbar \to 0$ (or, to be briefly considered at the end of the following section, $N \to \infty$, where N is the number of degrees of freedom in the model).
3. Whenever such a limit is applicable, any physical effect in classical physics must be foreshadowed in quantum physics.

Let us consider the third assumption, which is vital. Even though the notion of outcome only has a meaning in the limiting classical theory, the classical limit itself is an idealisation and any phenomenon cannot be counted as genuinely physical if it only appears in this idealised limitting theory. The philosophy adopted in asymptotic Bohrification, telling us that outcomes should have approximate quantum-mechanical counterparts, is partly captured by *Earman's principle* [5]:

> While idealizations are useful and, perhaps, even essential to progress in physics, a sound principle of interpretation would seem to be that no effect can be counted as a genuine physical effect if it disappears when the idealizations are removed.

The rest is captured by *Butterfield's Principle* [3]:

> there is a weaker, yet still vivid, novel and robust behaviour that occurs before we get to the limit, i.e. for finite N. And it is this weaker behaviour which is physically real.

Thus the measurement problem, as it arises in the simple double well model, amounts to the problem that for small non-zero values of $\hbar$, the post-measurement state does not *approximate* one of the classical pure states $(p = 0, q = \pm a)$, which we identify as outcomes in this model. Mathematically, the term *approximate* has a precise meaning in asymptotic Bohrification, expressed in the language of continuous fields of C*-algebras. The definition can be found in Appendix 5.

It is crucial to understand the way in which the post-measurement state should approximate an outcome. By assumption, the very notion of outcome is a classical one. Yet, in following Butterfield's principle we should concentrate on quantum mechanics for small values of $\hbar$ (or in another suitable limit). It is the notion of convergence that tells us in which way the quantum mechanical states should be close to classical outcomes. Therefore, in addition to a precise mathematical formulation, we need a physical grounding for convergence of states. The following quote, taken from [11] on p. 9, should help in providing insight into the physical grounding of convergence of states, as it explains the way that the doctrine of classical concepts is understood in the operator algebraic setting of asymptotic Bohrification. The quantisation map $Q_\hbar$ used in this quote is defined in Appendix 5.

> The map $Q_\hbar$ is the quantization map at value $\hbar$ of Planck's constant; we feel it is the most precise formulation of Heisenberg's original *Umdeutung* of classical observables known to date. It has the same interpretation as the heuristic symbol $Q_\hbar$ used so far: the operator $Q_\hbar(f)$ is the quantum-mechanical observable whose classical counterpart is f.

Thus, Classical physics enters the measurement problem through the theory of quantization rather than through classical approximations, as used in the Copenhagen interpretation. The Umdeutung of the quotation is central to asymptotic Bohrification. Because of this, one might think that when a salesman, selling asymptotic Bohrification, is at the door, Heisenberg, rather than Bohr, is the one who knocks.

Although this gives some physical underpinning for the quantum mechanical approximate outcomes, we will argue in what follows that this understanding is incomplete, at least for solving the measurement problem.

To make the discussion more concrete, briefly consider the flea proposal [13], intended to alleviate the measurement problem. The time-independent hamiltonian of (6), which, from now on, we denote as $\hat{H}_{\hbar,0}$ is replaced by a time-dependent one $\hat{H}_\hbar(t) = \hat{H}_{\hbar,0} + f(t)\delta V$, where δV is a small perturbation of the potential, localised in one of the two wells, and $f(t)$ is some scalar function changing the hamiltonian from $\hat{H}_{\hbar,0}$ initially, to $\hat{H}_{\hbar,0} + \delta V$. The motivation for this move can be found in the work of Jona-Lasinio et al. [8] regarding the sensitivity of ground states with respect to small perturbations in the semi-classical setting. Provided that $\hbar$ is taken to be small enough, even for a minute perturbation, the ground state of $\hat{H}_{\hbar,0} + \delta V$ is highly concentrated in one of the two wells, as in Fig. 2.

More physically, there exists a scale for the size of the flea perturbation, dependent on $\xi = \sqrt{\hbar^2/m}$ and the shape of the flea, such that if the flea is larger than this scale it causes the groundstate to be almost completely localized in a single well, as shown in Fig. 3. This scale can be used to verify the static prediction of the flea model, namely that a small symmetry-breaking perturbation of the potential can cause the groundstate to be localized, by performing many experiments determining the groundstate of a double well for several sizes of a flea perturbation. Note that for larger masses m (smaller ξ), we need a smaller flea to localize the wavefunction!

This result of the (static) flea model thus begs to be applied to the (dynamical) measurement problem. The idea is that for a suitable dynamical introduction of the flea, the wave function, initially expressing a Schrödinger cat state, could evolve to the ground state with the perturbed potential resulting in a post-measurement state which is highly concentrated in one of the two wells. In Sect. 3 we explore the feasibility of such an approach. For the remainder of this section, however, we

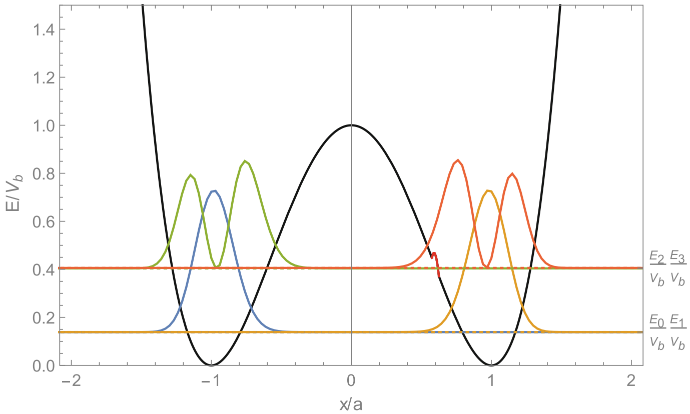


Fig. 2 The first four eigenfunctions as $|\psi(x)|^2$ for the double well potential with flea perturbation (marked red). From left to right, bottom to top, the maxima give the first (blue), second (yellow), third (green) and fourth (red) wavefunctions

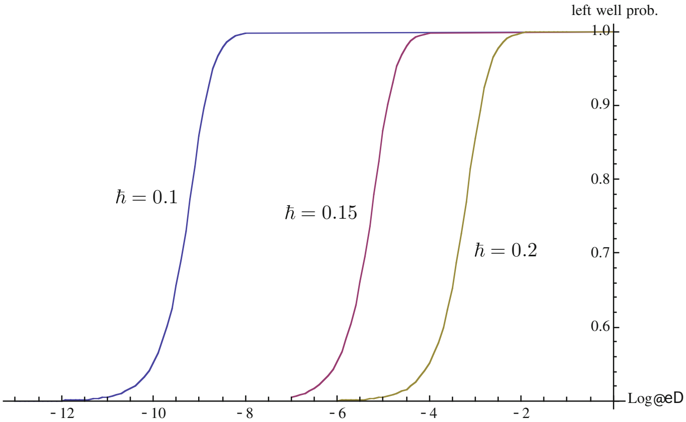


Fig. 3 Left-well probability of the groundstate of a double-well potential as a function of flea size (flea is scaled by shrinking parameter ε), for $m = 1$ and $\hbar = \xi \in \{0.1, 0.15, 0.2\}$ and a parabolic flea such as that shown in Fig. 2. The shrinking parameter ε is scaled logarithmically with base 10. This shows that there exist a scale for the size of the perturbation, dependent on $\xi = \sqrt{\hbar^2/m}$, above which the flea causes the groundstate to be almost completely located in a single well. As will be discussed in Sect. 3.1.2, however, it also shows that adding an asymmetry (interpreted as a flea) to the double well to allow for unequal initial probabilities relies critically on the size of the asymmetry

ask a different question. Starting from the assumption that the flea can (be made to) provide an effective collapse mechanism, how much closer would that bring us to solving the measurement problem?

2.2 Generalised Outcomes

Asymptotic Bohrification relies on a more generalized notion of outcome state, which have non-zero Born probabilities associated to macroscopically distinct pointer values. Even though these probabilites vanish in the semi-classical limit, it is the states *before* taking the limit which count as physically real, as made explicit by Butterfield's principle. This raises the question of how to understand such states as outcomes. In this subsection we argue that we lack a physical grounding for understanding post-measurement states of the flea model as outcomes. However, we do not consider this to be weakness of asymptotic Bohrification, but rather a challenge for any approach to the measurement problem, at least when the approach sticks within quantum mechanics.

Let us start with the second point: As argued in Appendix 6, in any physically realistic setting, the fundamental uncertainties of quantum mechanics entail that we cannot prepare a pure state for a system. Instead of an eigenstate such as $|+\rangle$ as in Sect. 1 of the measurement problem, it is more realistic to have an entangled state such as

$$\sqrt{1-\varepsilon^2}\,|+\rangle \otimes |\Phi\rangle + \varepsilon e^{i\phi}\,|-\rangle \otimes |\Psi\rangle,$$

with $\varepsilon > 0$.

The pragmatic physicist can of course safely ignore this and, for example, trade in the mathematically cumbersome initial states where the system is entangled with the environment, for a state such that the reduced system state is assumed pure. For a well designed preparation method, the difference between the eigenstate and the exact state is negligible as far as the statistics of outcomes of the subsequent measurements are concerned. The Born rule provides physical grounding for dismissing the small 'ε' terms in the state. However, when targeting the specific foundational issue of explaining the occurrence and statistics of outcomes, such a move may very well throw the baby out with the bath water.

As with the initial state in the previous example, the Born rule would also provide empirical grounding to understand the ε-states which arise as outcomes in the flea model. However, we should be cautious about a priori assuming the Born rule. In particular, as argued in the following subsection, this is because the Born rule dictates the statistics of the flea perturbations, thus challenging the independence assumption between system and pointer.

A different attempt at defining (approximate) outcomes can be found in the many worlds interpretation, or Everrettian quantum mechanics, where the ε-states obtain an even grander status than that of approximate outcomes, namely distinct branching worlds, and the Born rule can then allegedly be derived.[1] The discrete branching ontology of Everettian quantum mechanics is not exactly realised by decoherence, but only approximated and results in ubiquitous generation of ε-states. The problem

[1] The derivation of the Born rule has faced much criticism, such as its reliance on decoherence, which produces the ε states, through which the derivation becomes circular, as noted by various authors such as Zurek [22, 23], Baker [1], and Kent [9].

thus remains to bridge the gap between the ε-states and the exact branching ontology without smuggling in new concepts, such as the Born rule. Smallness of ε by itself does not provide any justification, especially if we are not clear on what these numbers mean. According to Zurek [22, 23], to pass from this effective description to the setting where classical decision theory can be used, we need to assume the Born rule. Proponents of Everettian quantum mechanics have, of course, been aware of such circularity objections. Wallace [20], for example, might dismiss the circularity objection by denying that the last step is needed. According to Wallace, the discrete branching structure of Everettian quantum mechanics is understood as a *robust* yet *emergent* feature of reality. But as is typical for discussions surrounding questions of emergence and reduction, it is hard to understand what this robustness exactly means. In a critique of such robustness, Dawid and Thébault [4] argue that the only robustness that holds explanatory power is *empirically grounded* robustness. In their words:

> The first crucial distinction that can be made is between a notion of robustness that is empirically grounded and one that is not. By this we mean some qualification such that whether a structure within the formalism of a theory is taken to be robust is dependent upon some interpretational connection between that structure and empirical phenomenology.

Similarly, the GRW dynamical collapse theories [2], although not within the framework of standard quantum theory, suffer from the same problem of epsilonics, in their case called "the tail problem". The collapsed states are approximately Gaussian and therefore the associated wavefunctions are non-zero throughout the whole space. From the point of view of standard quantum theory, this once again raises the question of how such states can be understood as outcomes. Ghirardi, Grassi and Benatti, reply that the tails do not constitute a problem, as these can be seemingly defined away in the matter density formulation of the theory [6]. The tails of a collapsed state correspond to low-density matter regions of the wavefunction, which are deemed not relevant because they are inaccessible. But what does it mean for a matter distribution to be accessible or inaccessible? If "inaccessible" means that any observer is unable to measure it, then we share the worries expressed in [14, 19] that we should not rely on observers before the tail problem is resolved. Thus the problem of connecting ε-states to physical reality without a priori assuming the Born rule again rears its head.

In short, ε-states, such as the generalised outcomes of asymptotic Bohrification are ubiquitous in the foundations of quantum mechanics, and the status of such states (whenever assuming the Born rule is out of the question) is a source of controversy. However, we leave this challenge of understanding outcome states for now, and move on to more pressing problems for asymptotic Bohrification.

2.3 Independence and Idealizations

If a flea-type perturbation is effective in collapsing the wave function, then the location of the flea determines to which well the state collapses. In order to be empirically consistent with quantum theory, the statistics of the flea locations should match the Born probabilities of the initial system state under consideration. Not much is said about the origin of the flea, other than it being of environmental nature. If the flea is thought to be some random perturbation, present simply because we do not want the physics of our model to hinge on mathematical idealizations, then indeed we expect it to be independent of the initial system state. But if that is the case, nothing short of a conspiracy theory would be needed to replicate the Born rule. This question is therefore; how could the flea be dependent, in the appropriate way, on the initial system state?

What makes this question particularly hard to answer, is that the flea model is not explicit on what role the system plays in the model. It may very well be that the problem of independence in replicating the Born rule simply amounts to the need to violate the independence between the system and the other degrees of freedom: as discussed at the end of Sect. 1.1, where this was stated as a loophole in avoiding the measurement problem. However, starting out with dependencies between pointer and system, it becomes unclear in what sense the flea model describes measurements. This is a pressing problem as it challenges the relevance of the flea model to the measurement problem.

In order to address the issue of independence, we need a physical underpinning for the flea. Related to the point of interpreting the flea, is the question of what the potentials represent. Recall that the flea proposal is based on the observation that two different potentials, one with a flea and the other without, behave quite differently when the parameter ξ is changed. This is, in itself, a completely static argument, void of any dynamical considerations. Note that the proposal relies heavily on potentials, which, according to quantum theory, are themselves always to be seen as an approximation of some dynamic quantum fields. For example, the possible energy of a photon emitted by an atom can be understood in terms of the energy levels of the electromagnetic potential governing the electrons. However, to understand the dynamics of the emission, the electrons must be coupled with an electromagnetic field. This is because the dynamics depends not only on the system's state but also on the state of the environment, in this case the electromagnetic field, whose interplay determines the speed of the changes in the system. This means that to connect the two different potentials, the flea must be dynamically added, and thus it must be treated as a dynamic quantum field.

In short, the flea model needs further work in making the system explicit and the providing the flea with a physical underpinning, before it can be considered as an actual model of quantum measurement. And even then it remains to be seen if the problem of independence can be resolved in such a way that the model has something to offer to the measurement problem without resorting to a version of superdeterminism.

2.4 Interpreting the Ground State

How is it possible that in the setting of quantum measurements, where entanglement plays such an important role, the flea model describes the dynamics of the pointer in terms of a single pure state?

The flea model assumes that, from the moment at which the flea is introduced, up to the end of the measurement, the degrees of freedom which are explicitly modelled are described by a pure state. It is tempting to think of the wave-function under investigation as representing the state of the pointer variable, but things cannot be that simple. Certainly, there is a relation between the pointer variable and the wave function since the two wells of the potential correspond to the two possible values of the pointer variable. However, if we think of the flea model as being obtained from a more complete measurement model by tracing out the system and environmental degrees of freedom, then we would expect entanglement between the system and the pointer variable to result in a non-pure state. This is crucial to the flea model since it relies on properties of bound states.

For the sake of concreteness, consider the time evolution for a von Neumann ideal measurement

$$\frac{1}{\sqrt{2}}\left(|m_1\rangle + |m_2\rangle\right) \otimes |r\rangle \mapsto \frac{1}{\sqrt{2}}\left(|m_1\rangle \otimes |E_1\rangle + |m_2\rangle \otimes |E_2\rangle\right). \tag{9}$$

Here $|r\rangle$ is the initial environmental state, and the environmental states $|E_i\rangle$ are close to orthogonal. When we trace out everything but the pointer, the entanglement causes the end result to be a non-pure state.

We could, as in Sect. 1.1, argue that different runs of the measurement correspond to different initial environmental states. However, if (still as in Sect. 1.1) after tracing out all environmental degrees of freedom, this would only lead to a small variation in the time-evolution operator, then this would have no impact on the above conclusion. The final state would still be close enough to (9) to yield a non-pure state. By the previous reasoning, the wave function of the flea model cannot represent the state of the pointer variable in a straightforward way. But then, what does it actually describe?

More pressingly: do asymptotic Bohrification and the flea model even deal with measurements? The flea model concentrates solely on collapsing a wave function, but much more is needed for any model of measurement. The post-measurement state not only needs to assign a value to the pointer variable, but also to the system's measured observable in such a way that these values are correlated. In this sense the flea model, concentrating only on the collapse of what we presume to be the pointer variable, is far from complete.

It should be noted that *we assumed* we should apply the flea perturbation solely to try to collapse the *pointer* to a definite outcome, which goes against the recommendations given to us by Landsman himself. In his view, the collapse should occur on the level of the combined pointer and observable.

Aside from the issues presented above, there is at least one other good reason for not wanting to identify the wave function with the pointer. To illustrate this, suppose we wish to include something akin to a flea perturbation in existing quantum measurement models. As an example, consider the model by Haake and Spehner [18], who use a double well for the pointer and an appropriate interaction to correlate the pointer's position with a system's observable (z-component of a spin-1/2 system). The obvious place in the Haake–Spehner model to apply the flea is to the potential of the pointer. However, since the original hamiltonian of the model commutes with the observable, and the model is only modified at the level of the pointer, the Born probabilities for the observable are unaffected by the introduction of the flea. Even if the flea is effective in causing a collapse of the pointer wave function, the correlation between pointer and observable disappears. As a side note, introducing the flea model to the Haake–Spehner model is not as straightforward as one might think from the previous remarks. For the model contains two separate potentials which are heavily-slanted double wells depending on the spin component, making it quite different from the flea model.

Thinking of entanglement and in keeping the observable/pointer correlation, one naturally thinks one should consider Landsman's proposal of the flea model as representing the combined observable and pointer. However, it is not clear to the authors what is meant by this statement, and, more concretely, how to apply this idea to any physical model. In the case where the collapse occurs on the level of the pointer, it is intuitively clear how to construct a model. Namely: x can, for example, describe the center-of-mass position of all particles in a gauge on the measurement device (pointer) that is subject to a electromagnetic potential due to its interaction with the particles in the measurement device and the wavefunction is the center-of-mass wavefunction of the particles in the gauge. The flea can be imagined to be a result of some change in the electromagnetic potential, although whether its cause should come from outside or inside the measurement device is unclear. But what if the collapsing wave function somehow represents the state of both pointer and observable? Where does the potential for this model come from, and how should it be interpreted? What would be the physical significance of x? What model can we build to give rise to a potential in x? It seems that the variable x is now much harder to interpret. Without knowing what x might stand for, it is hard to answer questions such as: why is it energetically unfavourable to have large x or $x = 0$ for the pointer+observable? The values of x are clear in the case of only a pointer (as in the Haake–Spehner model). But what does it mean in the case of a pointer and observable: does $x > 0$ correspond to spin up and $x < 0$ to spin down or only in the minima? What does $x = 0$ mean for the observable?

We end up in the situation where we do not have a physical interpretation for the potential, the wavefunction, or the flea, and where we are unable to connect with existing models of quantum measurement. Although this state of affairs is already troubling, it becomes more so after the next section where we conclude that the flea model is unable to perform its task of collapsing the wave function effectively.

To sum up the situation: How do you salvage a model if it is so incomplete at an interpretational level (i.e., what does the wave function mean, where does the flea come from, what does the potential represent) and at the same time so far removed from any other model of quantum measurement (it would need a model containing a bound state representing the combined observable and pointer state, and which is completely quantum mechanical)?

3 The Collapse Mechanism

In the previous section we looked at the measurement problem through the eyes of asymptotic Bohrification, without considering the effectiveness of the collapse mechanism. The collapse mechanism is the central topic of this section. In Sect. 3.1 we consider obstacles to generalising the collapse mechanism of the flea model to more outcomes, and unequal Born probabilities. In Sect. 3.2 we investigate the time scale of the collapse.

First, let us recall the setting of Sect. 2.1. The Hilbert space is $\mathscr{H} = L^2(\mathbb{R})$, and the initial state is the symmetric ground state of the hamiltonian in (6), i.e.,

$$\hat{H}_\hbar = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{\lambda}{8}\left(x^2 - a^2\right)^2 =: \frac{\hat{p}^2}{2m} + V(x), \tag{10}$$

The two wells of the potential represents two distinct values which the pointer variable of the measurement device can assume, and the symmetric ground state represents a Schrödinger cat like state, presumably transferred from the superposed state of some quantum two-level system. Thus the starting point of the flea model is supposed to take place near the end of a measurement interaction. The flea is a small (in sup-norm) asymmetric potential $W(x)$ which is localised near the bottom of one of the two wells of the symmetric double well potential $V(x)$. For a small value of $\hbar$, the ground state of the perturbed hamiltonian $\hat{H}_\hbar + W$ is well-nigh completely localised in a single well.

Ultimately, however, one of the main challenges of applying the flea model to the measurement problem is to find a time dependence for the flea, e.g. a function $t \mapsto f(t)$, in such a way that the initial symmetric wave function evolves to the localised ground state of the perturbed setting, as the hamiltonian evolves as $t \mapsto \hat{H}_\hbar + f(t)W$.

3.1 Generalisations

We consider two generalisations of the flea proposal; measurements with more than two possible outcomes, and system preparations with unequal associated Born probabilities.

We could have chosen to generalise in a different direction. For example, we could trade in the Hilbert space $L^2(\mathbb{R})$, the hamiltonian with a double well potential, and the semi-classical limit $\hbar \to 0$ for the spin chain Hilbert space $\mathscr{H}_N = \otimes^N \mathbb{C}^2$, the quantum Curie–Weisz hamiltonian,

$$H_N = -\sum_{-\frac{N}{2}}^{\frac{N}{2}-1} \sigma_i^z \sigma_{i+1}^z - B \sum_{i=1}^{N} \sigma_i^z$$

and the limit $N \to \infty$, where N is the number of sites on the lattice. This example was treated in the setting of asymptotic Bohrification in [10]. The only thing that we need to add is a flea, for example in the shape of a matrix $W = \text{diag}(0, 0, \ldots, 0, \varepsilon, 0, \ldots, 0)$, where ε is some small number and the matrix representation is relative to the basis of $\mathscr{H}_N$ generated by the eigenstates of the operators σ_i^z. However, for the sake of brevity we will not discuss such examples any further, at the risk of giving the impression that the idea of the flea model is limited to n-well potentials and the semi-classical limit ($\hbar \to 0$).

3.1.1 More Outcomes

The two wells of the potential correspond to the two values of the observable. How should we generalise the model to observables with n distinct values, where $n > 2$? One strategy is to consider the groundstate of an n-well potential with periodic boundaries. As an example we use $V(x) = V_b \cos^2(\pi x/2a)$ on $x \in [-na, na]$. This choice has the advantage that the energy eigenstates are known, namely they are the Mathieu functions, shown in Fig. 4.

A flea perturbation is added to the potential and the eigenfunctions are determined numerically. From Fig. 4 it is seen that the groundstate need not localise in a single well when only a single flea perturbation is added. In the figure, the flea perturbation is parabolic with finite support, although these details have no impact on the discussion. Since a single flea does not provide a fully localised ground state, one should consider adding multiple flea perturbations in the different wells. These multiple fleas must be chosen in such a way that the resulting potential no longer has any symmetry. If many fleas are added, and their locations, sizes and shapes are chosen largely at random, then this should be no problem.

A potential cause of problems for this generalisation is the increase in collapse times, especially when there are many wells, providing many barriers through which parts of the wave function have to tunnel. In Sect. 3.2 it will become clear that collapse times are already problematic for the double well setting. Therefore we shall not explore the increase of collapse times due to the added wells.

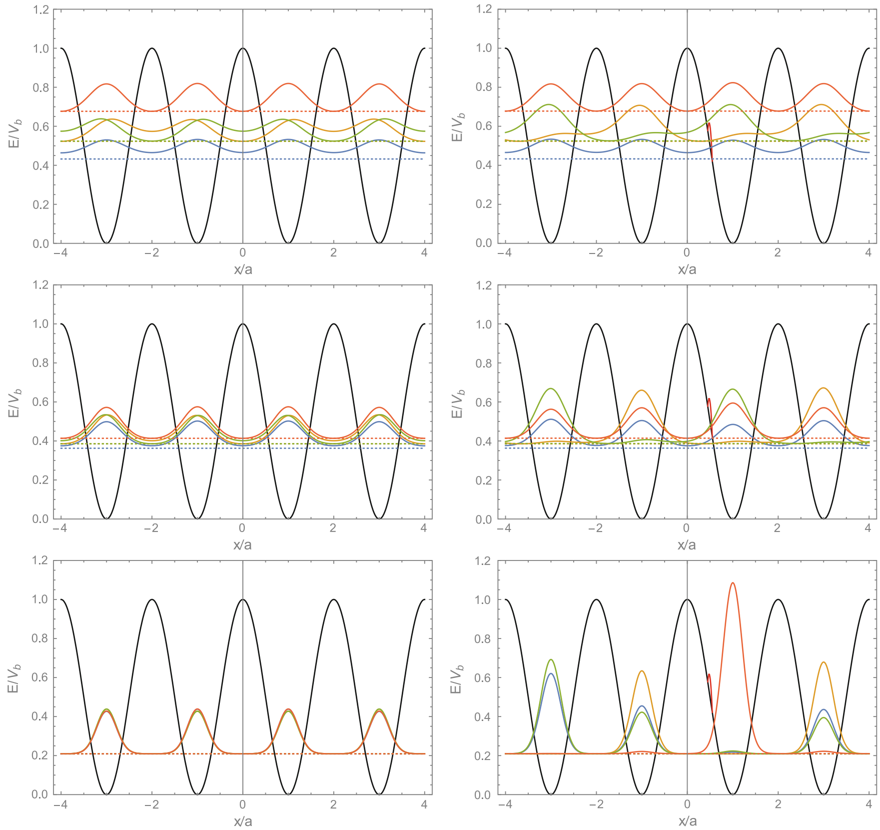


Fig. 4 The first four eigenfunctions as $|\psi(x)|^2$ of the periodic potential without (left) and with (right) a flea for $\xi = \{0.6, 0.4, 0.2\}$ from top to bottom, respectively. Clearly, without a flea the solutions are symmetric and will remain present in all wells as ξ becomes smaller. With the flea the symmetry is broken, and the groundstate (blue) will move out from the well with the Flea

3.1.2 Unequal Born Probabilities

How can we adapt the flea model to deal with other Born probabilities than the 50/50 case? At least two options are available. The first option is to keep using a symmetric potential, but to add additional wells. Suppose we consider a two level system, and that the Born probability assigned to one of the two values of the observable can be expressed as a rational number p/q, where we assume that 1 is the only common divisor of natural numbers p and q. We can consider q wells, p of which correspond to the value with the Born probability p/q, and the other $q - p$ correspond to the other value. Then we can proceed as before. This option will typically involve many wells, making it complicated and providing potential additional problems with regards to collapse times. In addition, this option looks far removed from the idea that the system state becomes correlated to the pointer variable. Since the connection between the system's measured observable and pointer is already so weak in the flea model, we consider this to be a serious drawback.

A second option, which does not share the previous drawbacks, is to replace the symmetric ground state of the symmetric double well potential by an asymmetric potential yielding a ground state such that the probabilities assigned to the wells are equal to the desired Born probabilities. However, since we are working in the semi-classical limit, the asymmetry of the potential should shrink with the value of $\hbar$. Otherwise, the asymmetry localises the wave function, just as the flea would.

In addition to the symmetric potential V there are three additional asymmetric terms added to the potential:

$$V(x) + W_0(x) + W_b(x) + W_f(t). \tag{11}$$

There is some noise W_0 which is too small (in sup-norm) to affect localisation, and which may be time dependent. We only add it to emphasise that the initial potential need not be completely symmetric in order to start out with a (sufficiently) symmetric ground state. Otherwise, we would need to worry whether the flea model itself conflicts with Earman's principle. Then there is a contribution W_b which ensures that the initial state of the flea model has the desired Born probabilities. Since it affects the Born probabilities, it is larger (in sup-norm) than the noise, but must be smaller than the flea in order to avoid localisation. The final contribution is the flea $W_f(t)$ which localises the wave function.

Consider again Fig. 3 in Sect. 2.1 where, for three different values of $\hbar$, we considered a parabolic flea W. We shrunk this flea as εW, where ε is a number ranging from 10^{-1} to 10^{-12}. As the flea shrinks we consider the probability assigned to the left well. The sensitivity of the ground state relative to small perturbations is seen as the curves shift to the right, when $\hbar$ decreases. Now suppose we are given a value of $\hbar$ and some perturbation W. If W is to be part of the noise, then there is an upper bound such that if we shrink W below this bound, then it will not affect the Born probabilities. Likewise, if W is intended as the flea, then there is a lower bound, and if W is, relative to the sup norm, larger than this boundary, it will allow localisation. If W is needed to set the initial Born probabilities, then there is only a single value of ε_0, such that $\varepsilon_0 W$ provides the right Born probabilities. Any deviation from this value affects the Born probabilities. Note that the variations around ε_0, such that the Born probabilities do not change significantly, decrease in order of magnitude as $\hbar$ decreases. More importantly, in all cases these asymmetries that we are fine-tuning are necessarily smaller than the flea perturbation. The flea is supposed to represent the result of an extremely small environmental fluctuation. Yet, we are fine-tuning even smaller fluctuations, just to get the right initial state.

In addition to the previous fine-tuning problem, the problem of independence also plays a role here. For recall that it is the location of the flea that determines in which well the final state is localised! The Born probabilities, which we have just modeled using a finely tuned asymmetry, do not influence how a flea is chosen. In [13], the flea perturbation was compared to a hung parliament, where a small political party acquired influence far exceeding its relative size. We feel that the flea is more like a dictator, determining what is going to happen, unfettered by any democratic constraint. The Born probabilities of the initial wave function, motivated by the correlation between observable and pointer variable, do not play any active role in the model.

We can only conclude that none of the options presented here provides a satisfactory extension of the flea model to arbitrary Born probabilities. But of how much interest can a solution to the measurement problem be, if it can only be applied to the special 50/50 case?

3.2 Problem of Collapse Times

Next, we consider the time scales of the collapse for the flea model. For a macroscopic device, modelled by a small value for $\hbar$, we find that a collapse takes an unrealistically long time, because we are considering a tunnelling problem with respect to a relatively large potential barrier.

Recall the work on Schrödinger operators by Jona-Lasinio et al. [8], which plays a key role in the flea approach. In the words of Landsman and Reuvers [13]:

> ...the ground state of a symmetric double-well Hamiltonian (which is paradigmatically of Schrodinger's Cat type) becomes exponentially sensitive to tiny perturbations of the potential as $\hbar \to 0$.

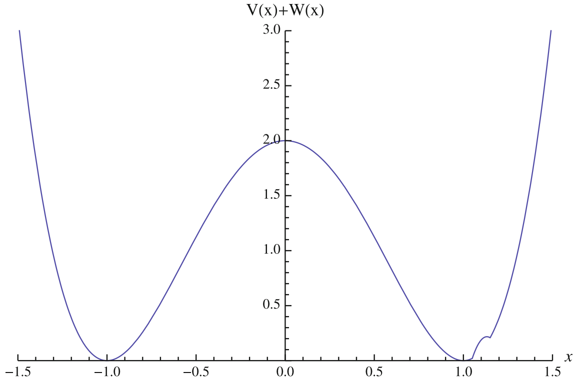
This phenomenon may have relevance for the quantum to classical transition. However, being a relation between bound states for slightly different potentials, it is a *static* phenomenon that has no obvious connection to the *dynamical* problem of the collapse of the wave function. In fact, as we argue below for the double well, when we decrease $\hbar$, we are effectively increasing the potential barrier between the wells, and the time needed for symmetric wave function to localise increases at least exponentially in the semi-classical limit, regardless of the way in which the flea is introduced. Since smaller values of $\hbar$ are supposed to represent a larger degree of macroscopicity, such an increase in collapse times undermines the flea model.

How should we choose the time dependence of the flea? Since there is presently no physics explaining the emergence of the flea we treat this to a large extent as a mathematical problem, rather than a physical one. The only physical restriction is that the collapse times should decrease as the scale $\hbar$ decreases, since this parameter is used to represent the degree of macroscopicity. Landsman and Reuvers [13] consider various ways of dynamically introducing a flea term $W(x)$ to the potential. The first option is to introduce it as a 'quench',

$$H(t) = H_0 + \theta(t)\varepsilon W, \tag{12}$$

where $\theta(t)$ is the Heaviside step function, and $\varepsilon > 0$ is a real number. Depending on the size of ε, introducing the flea either results in a wave function which oscillates between the two wells, or the original symmetric wave function barely changes at

Fig. 5 Potential with flea



all. For no value of ε however, do we obtain a wave function which is localised in a single well.

Other attempts, such as adding white noise or Poisson noise did not lead to a localisation either. In all these attempts both the ground state and the first excited state of the perturbed setting contribute significantly to the wave function. In order to avoid this, Landsman and Reuvers consider introducing the flea in the adiabatic limit, ensuring that we end up with the ground state of the perturbed setting. As it turns out, the combination of the adiabatic limit in quantum mechanics and the semi-classical limit poses a problem of too large collapse times.

For the sake of concreteness, consider the time-dependent Hamiltonian:

$$H(t) = H(0) + \sin\left(\frac{\pi t}{2T}\right) W, \quad \text{for } t \leq T \tag{13}$$

and $H(t) = H(0) + W$ for $t > T$. For the potential V of $H(0)$ we use the symmetric double well potential as in (6) and the flea W is a parabolic shape, as shown in Fig. 5.

Let $(\psi_n)_{n\in\mathbb{N}}$ be a (time-dependent) orthonormal basis of eigenfunctions of $H(t)$, where the eigenvalues $E_n(t)$ are ordered in increasing size. The wave function can be expressed as:

$$\Psi(t) = \sum_{i=1}^{\infty} c_n(t)\psi_n(t) \exp\left(\frac{i}{\hbar}\int_0^t ds\, E_n(s)\right), \tag{14}$$

where the x-dependence is suppressed in the notation. We start out in the ground state of $H(0)$, so $c_0(0) = 1$ and $c_n(0) = 0$ for all $n > 0$. The time dependence of $c_n(t)$ is given by

$$\dot{c}_n = c_n\langle\psi_n|\dot{\psi}_n\rangle - \sum_{m\neq n} c_m \frac{\langle\psi_n|\dot{H}|\psi_m\rangle}{E_m - E_n} \exp\left(-\frac{i}{\hbar}\theta_{mn}(t)\right), \tag{15}$$

$$\theta_{mn}(t) = \int_0^t ds(E_m(s) - E_n(s)). \tag{16}$$

In what follows we concentrate on $|\dot{c}_1(0)|$, for several reasons. As already noted, the first excited state is localised in the other well with respect to the ground state, therefore, $|\dot{c}_1(t)|$ is of interest. In addition, consider the energy splitting $\Delta(t) = |E_0(t) - E_1(t)|$, which rapidly decreases in the semi-classical limit. During the collapse, the splitting $\Delta(t)$ takes on its smallest value for the unperturbed setting $t = 0$. It is also at this time that we expect the overlap $|\langle\psi_1(t)|W|\psi_0(t)\rangle|$, appearing in the matrix element $|\langle\psi_1(t)|\dot{H}|\psi_0(t)\rangle|$, to be at its largest. In other words, if $|\dot{c}_1(0)|$ turns out to be sufficiently small, then this may help yield a final state which is close to the ground state of the perturbed potential. Thus, we ask for which order of magnitude of T, does the quantity

$$|\dot{c}_1(0)| = \frac{\pi}{2T\Delta(0)}|\langle\psi_1(0)|W|\psi_0(0)\rangle| \tag{17}$$

become sufficiently small. As in [13] on p. 8 as $\hbar \to 0$, $\Delta(0)$ decreases as

$$\Delta(0) \approx \frac{2\hbar a\sqrt{\lambda}}{\sqrt{e\pi}} e^{-\frac{d_V}{\hbar}}. \tag{18}$$

Unless $|\langle\psi_1(0)|W|\psi_0(0)\rangle|$ decreases with a rate of at least $e^{-d_V/\hbar}$ in the semi-classical limit, the time $T \to +\infty$ must increase exponentially fast in that same limit, in order to keep $T\Delta(0)$ finite. Here d_V denotes the WKB-factor.

In itself, the limit $T \to +\infty$ as $\hbar \to 0$ need not be problematic. As $\hbar$ decreases, we can shrink the flea W, and consequently shrink $|\langle\psi_1(0)|W|\psi_0(0)\rangle|$, whilst retaining a localised perturbed ground state. Consider the quantity:

$$\Gamma = \frac{|\langle\psi_1(0)|W|\psi_0(0)\rangle|}{\Delta(0)}, \tag{19}$$

which we use to quantify the rate at which T needs to increase. Figure 6 shows how $\mathrm{Log}(\Gamma)$ varies with $\hbar$ for the fixed parabolic flea of Fig. 5. Note the exponential growth, indicating that T needs to increase at a rate $x \mapsto e^{e^x}$. In the range of the graph, if we halve the value of $\hbar$, then Γ increases roughly 15 orders of magnitude.

How much can we shrink the flea $W(x) \mapsto 10^{-n}W(x)$ in order to compensate for this effect. For any $\hbar$ in the range of Fig. 6, if $n \geq 12$, the perturbed ground state is no longer localised in a single well. In other words, the collapse times rapidly increase as $\hbar \to 0$ even if we shrink the flea W as much as possible along the way. Why do we expect this behaviour to hold in general? Instead of $\hbar$, consider $\lambda := 1/\hbar$. Regardless of the details of the flea, we are considering situations where part of the wave function has to tunnel through a barrier of increasing height λV_0.

If the flea is introduced too fast, then the first excited state becomes occupied and we retain the superposition. If the flea is introduced slower, to ensure that we

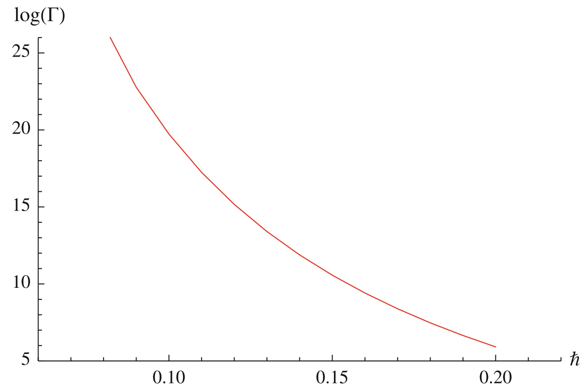


Fig. 6 Collapse times

remain in the ground state, then the collapse times increase dramatically as the macroscopicity scale parameter $\hbar$ decreases. The cause of the increase of collapse times is the rapid decrease of the initial splitting $\Delta(0)$. Yet, this same smallness of the splitting with regard to the semi-classical limit is important to the asymptotic Bohrification programme, as argued in [10] on p. 12:

> The essential point is that in our models, the energy difference $\Delta E_\bullet = E_\bullet^{(1)} - E_\bullet^{(0)}$, between $\Psi_\bullet^{(1)}$ and $\Psi_\bullet^{(0)}$ vanishes exponentially as $\Delta E_N \sim exp(-C \cdot N)$ for $N \to \infty$, or as $\Delta E_\hbar \sim exp(-C'/\hbar)$ for $\hbar \to 0$ respectively. This means that asymptotically any linear combination of $\Psi_\bullet^{(1)}$ and $\Psi_\bullet^{(0)}$ is almost an energy eigenstate.

The flea model hinges on the precept that first a macroscopic superposition for the pointer variable is formed, and subsequently this superposition is broken down dynamically. As argued in this subsection, this is an open invitation to problems about tunnelling times. However, conceptually it is suspicious as well. Already from decoherence we know that setting up a macroscopic superposition is highly non-trivial.

4 Conclusion

The most important physical results of the flea model is that in the static setting it gives a scale for the size of the flea, dependent on $\hbar^2/m$, above which the groundstate is almost fully located in one well, as was shown in Fig. 3, and that with increasing mass a smaller perturbation is needed. This static property of potentials could, in principle, be confirmed in experiments by determining the shape of the groundstate of a double well with varying sizes of an artificial flea perturbation. The drastic effect of such small perturbations on the form of the groundstate remains a remarkable feature of quantum theory.

Although the static picture looks appealing, when we consider the collapse using a time-dependent flea, we run into various problems:

- Only adiabatic introduction of the flea provides a collapse, but it comes with unrealistically long collapse times. For faster introduction of the flea, we find that the first excited state becomes relevant and no collapse is observed.
- We cannot generalise to system states with unequal associated Born probabilities.
- It is unclear how to scale to more than two outcomes.

When considering the statistics of outcomes in addition to individual runs, the following problem of independence is added:

- It is only the statistics of the flea distribution that determines the statistics of outcomes. Without any relation between the flea and the initial system state, there is no reason why the Born probabilities (apart from the special 50/50 case) should be replicated by the flea model.

In some sense the wave function, representing the system and pointer state, and the flea, which is assumed to be of environmental origin, need to be related. As the flea model, as so far formulated, is incomplete concerning the interpretation of the flea and the dynamics between system and pointer, it is hard to understand the exact ramifications of imposing relations between flea and system state. However, it would not be too surprising if, after further working out the flea model, it does turn out that replicating the Born probabilities simply means denying independence between the system state and the degrees of freedom of the measuring apparatus. If that is the case, we need to ask in what sense we are still considering a measurement.

With regard to interpretation of the flea model we are left with the following issues:

- The problem of potentials: The idea behind the flea proposal comes from comparing two separate potentials and thus has no bearing on dynamics. True dynamics only follows from a treatment in terms of two coupled quantum systems.
- Physical interpretational issues: The origin of both the potential and flea is unclear; which makes a realistic physical model almost impossible.
- Where is the system in the flea description, and how does it become correlated with the pointer? What kind of model would yield the double well ground state as an intermediate state for the combined system and pointer?

These are more than enough questions. But with the flea model being far removed from known models of quantum measurement, it is unclear how to construct an actual model which provides physical grounding for the potentials, the flea and/or the wave function. As a final point we mention that the generalised notion of outcome state in the flea approach still lacks an empirical grounding.

In short, the flea model as it now stands is not able to provide an effective collapse. Although the underlying idea looks good statically, there is no reason why it would work dynamically. Generalising the model to system states with unequal Born probabilities appears to provide a sizable obstacle as well. But even if the collapse were effective, it is not clear what this would say about measurements. The model is silent on the system-pointer correlation and the physics behind the flea and the potential. Thus, to the authors it is not clear that we are even dealing with measurements. And if a working model of quantum measurement, underlying the flea model, could be found, the problem of independence may still require the addition of some sort of conspiracy theory or superdeterminism. If this is the case, then it is not clear what more the flea model has to offer, about the measurement problem as formulated in Sect. 1.1.

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5 Appendix 1: Convergence of States

For concreteness, let $\hbar$ take values in the unit interval $[0, 1]$. To each strictly positive $\hbar > 0$, associate the non-commutative algebra $\mathfrak{A}_\hbar = \mathscr{K}(L^2(\mathbb{R}))$ of compact operators acting on the Hilbert space of square-integrable functions. To $\hbar = 0$ associate the commutative algebra $\mathfrak{A}_0 = C_0(\mathbb{R}^2)$ of continuous real-valued functions on the phase space $\mathbb{R}^2$, which vanish at infinity. Through their disjoint union $\mathfrak{A} = \coprod_{\hbar\in[0,1]}\mathfrak{A}_\hbar$ these algebras combine in a single algebra fibred over the unit interval, $\mathfrak{A} \to [0, 1]$. Dual to this bundle there is the bundle of state spaces $\mathscr{S} \to [0, 1]$ where $\mathscr{S} = \coprod_{\hbar\in[0,1]}\mathscr{S}_\hbar$, and $\mathscr{S}_\hbar$ denotes the state space of $\mathfrak{A}_\hbar$. For $\hbar > 0$ the states are density operators acting on $L^2(\mathbb{R})$, and for $\hbar = 0$ the states are probability measures on the phase space $\mathbb{R}^2$.

Next, we could consider the algebraic and topological aspects of these bundles. But since we are only concerned with the measurement problem, we refer the reader to [10] and proceed directly to the our main question; how is convergence of states defined in this scheme? More precise, when does a family of density operators $(\rho_\hbar)_{\hbar\in(0,1]}$ converge to a classical state $\mu_0 \in \mathscr{S}_0$? To define convergence, note that each density operator $\rho_\hbar$ defines a probability measure $\mu_\hbar$ on $\mathbb{R}^2$, through

$$\int_{\mathbb{R}^2} d\mu_\hbar f := Tr\left(\rho_\hbar Q_\hbar(f)\right), \quad \forall f \in C_0(\mathbb{R}^2) \tag{20}$$

where $Q_\hbar(f)$ is the compact operator acting on $L^2(\mathbb{R})$, called the 'Berezin quantisation' of f. The Berezin quantisation map $Q_\hbar$ is defined as

$$Q_\hbar(f) = \int_{\mathbb{R}^2} \frac{dpdq}{2\pi\hbar} f(p,q)|\Phi_\hbar^{(p,q)}\rangle\langle\Phi_\hbar^{(p,q)}|, \tag{21}$$

through the coherent states $\Phi_\hbar^{(p,q)} \in L^2(\mathbb{R})$;

$$\Phi_\hbar^{(p,q)}(x) = (\pi\hbar)^{-1/4} e^{-ipq/2\hbar} e^{ipx/\hbar} e^{-(x-q)^2/2\hbar}. \tag{22}$$

The states $\rho_\hbar$ converge to the classical (possibly mixed) state μ_0 iff the probability measures $\mu_\hbar$ converge weakly to μ_0 in the sense

$$\lim_{\hbar\to 0} \int_{\mathbb{R}^2} d\mu_\hbar f = \int_{\mathbb{R}^2} d\mu_0 f,$$

for each $f \in C_0(\mathbb{R}^2)$ with compact support.

For the double-well model, as $\hbar \to 0$, the ground state $\psi_0(x)$, or rather its associated density operator, converges to the classical mixed state (7), a convex combination of probability distributions with support in the two different wells. As emphasised in the main paper, this is a classical state which does not qualify as an outcome.

6 Appendix 2: Practical Necessities

The fundamental uncertainties of quantum theory prohibit the preparation of a pure state, let alone an eigenstate with respect to a fixed basis. For most purposes this is irrelevant since we can get close enough in terms of Born probabilities; but for the measurement problem, the distinction may matter. We illustrate the point using the simple example of preparing an initial state using a Stern–Gerlach experiment. The point will be that for any *fully* quantum mechanical treatment, the Born probabilities associated to both eigenvalues of the observable are always non-zero. If we started with an n-level system, the same would hold for all the eigenvalues of any observable.

Consider the Stern–Gerlach experiment where spin-$1/2$ particles are sent through an inhomogeneous magnetic field. The textbook view is that due to the spin-magnetic-field interaction the spin along the magnetic field gets correlated with the position of the particle. In this simple view of the device, the incoming particle is described by a wavepacket $\psi(\mathbf{x}, t)$ and then after some interaction time the position of the wavepacket is correlated to the spin in the z-direction

$$\begin{aligned} |\Psi(t=0)\rangle &= (\alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle) \int \psi(\mathbf{x}, 0)\,|\mathbf{x}\rangle\,\mathrm{d}\mathbf{x} \\ \to |\Psi(t)\rangle &= \alpha\,|\uparrow\rangle \int \psi_+(\mathbf{x}, t)\,|\mathbf{x}\rangle\,\mathrm{d}\mathbf{x} + \beta\,|\downarrow\rangle \int \psi_-(\mathbf{x}, t)\,|\mathbf{x}\rangle\,\mathrm{d}\mathbf{x}, \end{aligned}$$

where $\psi_\pm(\mathbf{x}, t)$ indicate the wavepackets as they leave the Stern–Gerlach apparatus. If the initial wavepacket is Gaussian, the wavepackets $\psi_\pm$ will also be approximately Gaussian but shifted upwards or downwards along the z-axis. For a derivation of the typical form of such wavepackets see [7, 16, 17].

For the Stern–Gerlach apparatus to serve its purpose, to distinguish spin states based on the position of the particle, the conditions of the experiment should be such that most particles will be detected at one of the two well-separated positions on a detector screen. Some of the dominant parameters, which determine the separation of the final wavepackets, are the initial wavepacket width, the strength and inhomogeneity of the magnetic field, and the time of flight during and after the interaction.

These parameters are varied by the experimenter when designing and testing the experiment until the overlap between the wavepackets $\int \psi_+^*(\mathbf{x}, t)\psi_-(\mathbf{x}, t)\mathrm{d}\mathbf{x}$ becomes extremely small such that for all practical purposes the wavepackets seem to be separated in space. However, according to quantum theory the overlap will in general always be non-zero. In other words, spin is not perfectly correlated with position on the detector screen.

If a small slit is made in the detector in the region we identify with "spin-up", the state immediately after the slit will be given by

$$\alpha \left|\uparrow\right\rangle \int \psi_+(\mathbf{x}, t) \left|\mathbf{x}\right\rangle \mathrm{d}\mathbf{x} + \beta \left|\downarrow\right\rangle \int \psi_-(\mathbf{x}, t) \left|\mathbf{x}\right\rangle \mathrm{d}\mathbf{x},$$

where now the integration is restricted to the small slit. If the experiment is well designed, one of the terms will be (exponentially) smaller than the other. In principle, we should also allow for an extremely small contribution where the particle tunnels through the detector screen.[2]

In practice, when the Stern–Gerlach apparatus is used as a preparation device, the smaller term will be discarded as the parameters of the setup were tuned specifically for reproducibility, i.e., it is tuned such that the smaller term is experimentally inaccessible to subsequent verification (using another device) due to the finite statistics and the resolution of any experiment. This leads to the erroneous conclusion that a pure state in spin-space can be obtained by application of the Stern–Gerlach apparatus. Theoretically, after the slit the following density matrix in the $\uparrow, \downarrow$-basis is obtained

$$\rho_s = \int \begin{pmatrix} |\alpha\psi_+|^2 & \alpha^*\psi_+^*\beta\psi_- \\ \alpha\psi_+\beta^*\psi_-^* & |\beta\psi_-|^2 \end{pmatrix} \mathrm{d}\mathbf{x}. \tag{23}$$

Experimentally, the factors in the density matrix can be tuned more-or-less continuously by the above-mentioned parameters, however, they will never be strictly equal to one or zero unless the exact initial spin-state was known, i.e., the exact value of α and β is known beforehand.

A further fundamental complication is that the magnetic field must have zero divergence, which implies that it cannot have a gradient in the field in only one direction [16]. Therefore, as the wavepacket has a finite width in space, each part of it couples to its local direction of the magnetic field; and these local directions are not precisely aligned with the single z-axis that is considered theoretically. Thus particles

[2] Similarly, in the two-slit experiment a photon is said to travel through both slits at once; however, in principle there is also a contribution that it tunnels through the screen itself, which is dependent on the thickness of the screen.

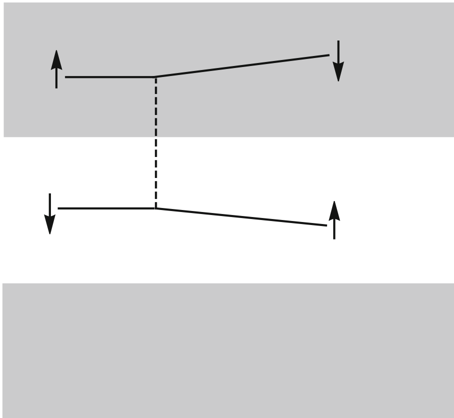

Fig. 7 A spin-flip process in the Stern–Gerlach experiment

with initially the same spin state along the quantization axis can, nevertheless, deviate according to that of the opposite spin.

Another important point is that the magnetic field and magnet were presumed to be classical. If the electromagnetic field is treated quantum mechanically as mediating interactions between the spin-1/2 test-particle and the particles in the Stern–Gerlach magnets, it would result in entanglement between the test-particle's position and those of the charge carriers in the coils of the Stern–Gerlach magnets. Namely, the charge carriers in the magnet would undergo a momentum increase, and thereby change their position, along the z-direction depending on the spin-component of the test-particle's wavefunction. After tracing out the states of the magnet, a density matrix is obtained similar to Eq. (23). Also, as spin exchange processes between the test-particle and the magnet's particles are always possible, there are again contributions which cause incorrect deflections to occur. Such processes are easy to visualize in the path-integral picture, which sums the amplitudes over all possible paths and interactions, see Fig. 7. Agreed, the suppression of spin-flips can be argued to be to be very strong under typical circumstances due to Pauli blocking of transitions to already occupied electronic states in the magnet: whereby we normally assume classical properties to the magnet.

Summarizing, the Stern–Gerlach experiment cannot be used as an ideal and reliable preparation device of spin states, even in principle, as there is no perfect one-to-one correspondence with position and spin. Note that the objections to this experiment creating pure states in spin are of a fundamental nature, namely they lie in the divergencelessness of the magnetic field, or the entanglement with the magnet with which it necessarily interacts, or the spatial extent of the wavefunctions.

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