Chapter 12
Social and Occupational Mobility

12.1 Introduction

The best way of quantifying human populations is by classifying their members on the basis of some personal attribute. One may classify families according to where they reside or workers by their occupations. Thus to study the dynamics of social processes, it is natural to start by looking at the movement of people between categories. Since such moves are largely unpredictable at the individual level, it is necessary for a model to describe mechanism of movement in probabilistic terms. The earliest paper in which social mobility was viewed as stochastic processes appears to be that of Prais (1955a, b). Since then, their has been grown up a large literature. A distinction has to be made between intergenerational mobility and intra-generational mobility. The former refers to changes of social class from one generation to another. Here the generation provides a natural discrete time unit. This phenomenon is usually called social mobility. Intra-generational mobility refers to changes of classes which take place during an individual's life span. This type of movement is called occupational or labor mobility since it is usually more directly concerned with occupations. Many deterministic and stochastic models have been developed to study social and occupational mobility situations in the different parts of the world. Several empirical studies of mobility have been published.

To study the movement of individuals over occupational categories, it is natural to start by looking at the movement of people between different categories and also at the process of recruitment of new entrants. Since such moves are largely unpredictable at the individual level, it is necessary to find a model to describe the mechanism of movement in probabilistic terms.

Study on occupational mobility is an important part of manpower planning. Such studies always help different organization, institutes, companies to properly build up their future plans regarding the number of new recruits and also help their staff members to properly plan their career. Different organization having the same setup may use the same model to study the promotion pattern. All such studies together are really helpful for proper manpower planning of the country.

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Studies related to the dynamics of social systems whose members move among a set of classes are of great importance for manpower planning. In manpower planning, the classes represent grades whose sizes are fixed by the budget or amount of work to be done at each level. Recruitment and promotion can only occur when vacancies arise through leaving or expansion. The stochastic element in such processes occurs due to loss mechanisms. Individuals leaving or moving create vacancies which generate sequence of internal moves. There may also be randomness in the method by which vacancies are filled. Development of such models has been done using replacement theory. Originally, such problems have arisen in connection with the renewal of human population through deaths and births. In recent years, the main application has been in the context of industrial replacement and reliability theory. The key random variables in all cases are the length of time an entity that remains active in a particular grade.

Let us start with mobility models and some related measures. There are several models and measures based on Markov chain. Prais (1955a, b) was probably the first author to apply Markov chain theory to social mobility. The society is characterized by the transition probability matrix P, and most of the measures proposed are based on this matrix. Some examples are listed in Matras (1960). In a completely immobile society, 'son' will have the same class as their father and P will be an identity matrix. Prais (1955a, b) defined a perfectly mobile society as one in which the 'son's' class is independent of his/her'father's.' For such a society the rows of P will be identical. A third situation can be identified as extreme movement in which every 'son' has a different class from his/her 'father.'

Bartholomew (1982) proposed an idea of social mobility based on the matrix P and the elements of π (vector giving the limiting distribution of the population among the classes).

A measure of generation dependence can be developed by considering the extent to which a son's class depends on that of his/her father's. A method of doing this is suggested by considering spectral representation of P in the form

$$P = \sum_{r=1}^{k} \theta_r A_r$$

The matrices $\{A_r\}$ constitute the spectral set. The coefficients $\{\theta_r\}$ are the eigenvalues of P and since P is stochastic $1 = \theta_1 \geq |\theta_2| \geq \cdots \geq |\theta_k|$

A measure proposed by both Shorrocks (1978) and Sommers and Conlisk (1979) is based on the second largest, in absolute values, of the θ_s. If it is denoted by θ_{max}, then the measure in $\mu_1(P) = \theta_{max}$.

Bartholomew (1982) proposed two other measures given by

$$\mu_2(P) = \frac{1}{k-1} \sum_{r=2}^{k} \theta_r$$

$$\mu_3(P) = |\theta_2\theta_3 \ldots \theta_k|^{\frac{1}{(k-1)}}$$

By regarding the distribution of the population at times t and $(t+1)$ as two multinomial populations, by using Bhattacharyya distance (1945–46) a measure of divergence has been suggested by Mukherjee and Basu (1979) as below

$$cos\Delta = \sum_i \sqrt{\pi_i^{(t)}\pi_i^{(t+1)}}$$

where $\Delta = 0$ if $P = I$

By considering measures of association as inverse measure of mobility, Mukherjee and Chattopadhay (1986) proposed a number of measures based on different coefficients of association. They also proposed another measure based on minimum discrimination information statistic $(m\ d\ i\ s)$ given by

$$J(1,2) = \sum_{j=1}^{k}(\pi_j^{(t)} - \pi_j^{(t+1)})log_e\left(\frac{\pi_j^{(t)}}{\pi_j^{(t+1)}}\right)$$

Occupational mobility refers to the movement of employees between jobs or job categories. For job changes over different organizations, the time interval between successive changes is likely to be random. As a result, for such situations simple Markov model does not provide a satisfactory representation. Attempts have been made to describe occupational mobility patterns in terms of semi-Markov processes (Ginsberg 1971, 1972). Bartholomew (1982) suggested one measure based on the transition probability matrix and the stochastic process $\{m(T)\}$ where $m(T)$ is the random number of decision points in the interval $(0, T)$. Mukherjee and Chattopadhay (1986) developed one measure based on renewal process. Starting with semi-Markov process, Mukherjee and Chattopadhyay proposed another measure in terms of the number of occupation changes during an interval of time. The same authors have also considered the situations where the job categories may be ordered in some sense.

Chattopadhyay and Gupta (2003) considered a discrete time Markov process where states of the system denote grades of the employees in an organization. The total size of the system is fixed. The recruitment needs are determined by the losses, together with any change in the size of the system. A specific version of the model with a fixed total size is due to Young and Almond (1961) who applied the model to the staff structure of a British University. The proposed model has been developed to study the career prospect on the basis of the staff categories and promotion pattern for non-teaching staff members of University of Calcutta.

Chattopadhyay and Khan (2004) has extensively studied the nature of job changes of staff members of University of Southern Queensland, Australia, on the basis of stochastic model. Khan and Chattopadhay (2003) have also derived corresponding prediction distribution on the basis of job offers received by the employees. Such type of works are very useful to investigate the manpower planning condition in different organization.

12.2 Model 1

The present model has been developed on the basis of the staff categories and promotion pattern for non-teaching staff members of University of Calcutta. Suppose that the members of the organization are divided into k strata (grades) in which there are r strata where direct appointments from outside are allowed (together with the promotion of existing staff members) and in the remaining $(k - r)$ strata posts no new appointment from outside is allowed. Only internal staff members are promoted to those positions. Let $n_i(t)$ denote the number of people in the first type of category i at time t $(i = 0, 1, 2, \ldots, r)$ and $z_j(t)$ denote the number of people in the second type of category j at time $t(j = r + 1, r + 2, r + 3, \ldots, k)$. The initial grade sizes are assumed to be given. Also let

$$N(t) = \sum_{i=1}^{r} n_i(t) + \sum_{j=r+1}^{k} z_j(t) \tag{12.2.1}$$

$$N(0) = \sum_{i=1}^{r} n_i(0) \tag{12.2.2}$$

where $N(0)$ is the total number of first type vacancies available. For $t > 0$, the grade sizes are random variables. Let us denote by $e(t)$ the number of new entrants in the system at time t and by p_{ij}, the probability of transition from grade i to grade j for an individual. These transition probabilities are assumed to be time homogeneous. The system from 1 to r be open system and grade (r + 1) to k be closed system. The allocation of new entrants in the system is specified by p_{oj} which gives the expected proportion of new entrants to the jth grade $(j = 0, 1, 2, \ldots, r)$. Here, we also assume that a person only moves to the next grade.
when $j \leq r - 1$,

$$E(n_j(t + 1)) = \sum_{i=1}^{r-1} p_{ij} E(n_i(t)) + e(t + 1)\ p_{oj}, \quad t = 1, 2, 3 \ldots \qquad j = 1, \ldots, r - 1 \tag{12.2.3}$$

when $j = r$,

the expected value of $n_j(t + 1)$ has been divided into two parts, one part due to changes by promotion and new appointment (n_{1_r}) and other part only due to promotion (n_{2_r}),

$$E(n_{1_r}(t + 1)) = p_{r-1r} E(n_{r-1}(t)) + e(t + 1) p_{0r} \quad t = 1, 2, 3 \ldots \tag{12.2.4}$$

$$E(n_{2_r}(t + 1)) = p_{rr} E(n_r(t)) \tag{12.2.5}$$

$$E(n_r(t+1)) = E(n_{1_r}(t+1)) + E(n_{2_r}(t+1)) \tag{12.2.6}$$

when $j \geq (r+1)$,

$$E(z_j(t+1)) = \sum_{i=r+1}^{k} p_{ij} E(z_i(t)) \quad t = 1, 2, 3 \ldots$$
$$j = r+1, \ldots, k \tag{12.2.7}$$

12.2.1 Some Perfect Situations

Let us define the following two perfect situations regarding promotion.

I. Perfect promotion situation

Under this situation, a particular individual has the equal chances of moving into two successive categories.

$$P^{k \times k} = \begin{pmatrix} 1/2 & 1/2 & 0 \ldots & 0 \\ 0 & 1/2 & 1/2 \ldots & 0 \\ \vdots & & & \\ 0 & \ldots & 1/2 & 1/2 \\ 0 & \ldots & \ldots & \ldots 1 \end{pmatrix}$$

II. No promotion situation

Under this situation, a particular individual has no chance of promotion.

$$P^{k \times k} = \begin{pmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & & & \\ 0 & 0 & \ldots & 1 \end{pmatrix}$$

12.2.2 Possible Measures of Career Pattern

The extent to which an individual changes his/her job from one category to another higher category can be measured by using different indices.

A measure can be defined as a continuous function $M(.)$ defined over the set of transition matrices $\mathcal{P}$ such that

$$0 \leq M(P) \leq 1 \quad \text{for all} \quad P \in \mathcal{P}.$$

For this, the function $M(.)$ requires no significant constraint on the set of potential measures since a change of origin and scale can always be found such that the transformed variables take values within the chosen interval. The function $M(.)$ is monotonic in nature because the probability of movements between grades are given by the off-diagonal elements of the transition matrix. The increasing off-diagonal elements indicates the higher level of mobility among the career pattern of individuals and hence

$$M(P) > M(P') \text{ when } P > P' \tag{12.2.8}$$

12.2.3 Measure of Career Pattern Based on Mahalanobis Distance

We introduce a new measure of occupational mobility in terms of distance of two populations. Here,we consider occupational situation at time t as one population and at time (t+1) as another. A common distance measure is *Mahalanobis distance*.
Let $\upsilon = (X_1(t), X_2(t), X_3(t), \ldots, X_k(t))'$

$$\upsilon \sim Multinomial(N(t), \pi_1(t), \pi_2(t), \ldots, \pi_k(t)), \sum_{i=1}^{k} \pi_i(t) = 1;$$

$$uY = (Y_1(t+1), Y_2(t+1), Y_3(t+1), \ldots, Y_k(t+1))'$$

$$uY \sim Multinomial(N(t+1), \pi_1(t+1), \pi_2(t+1), \pi_3(t+1), \ldots, \pi_k(t+1)), \sum_{i=1}^{k} \pi_i(t+1) = 1;$$

where,
$X_i(t)$ = Number of persons belonging in category i at time t;
$\pi_i(t)$ = Prob. of a person belonging in category i at time t;
N(t) = Total Number of persons in entire system at time t;
$Y_i(t+1)$ = Number of persons belonging in category i at time (t + 1);
$\pi_i(t+1)$ = Prob. of a person belonging in category i at time (t + 1);
$N(t+1)$ = Total Number of persons in entire system at time (t + 1);

$$E(uX) = N(t)u\pi(t)$$

$$E(uY) = N(t+1)u\pi(t+1)$$

$$uA_1 = \begin{pmatrix} \pi_1(t)(1-\pi_1(t)) & -\pi_1(t)\pi_2(t) & -\pi_1(t)\pi_3(t)\ldots\ldots & -\pi_1(t)\pi_{k-1}(t) \\ -\pi_1(t)\pi_2(t) & \pi_2(t)(1-\pi_2(t)) & -\pi_2(t)\pi_3(t)\ldots & -\pi_2(t)\pi_{k-1}(t) \\ \vdots & \vdots & \vdots & \vdots \\ -\pi_1(t)\pi_{k-1}(t) & \ldots & \ldots & -\pi_{k-1}(t)(1-\pi_{k-1}(t)) \end{pmatrix}^{(k-1)\times(k-1)}$$

Let, $t' = t + 1$;

$$uA_2 = \begin{pmatrix} \pi_1(t')(1-\pi_1(t')) & -\pi_1(t')\pi_2(t') & -\pi_1(t')\pi_3(t')\ldots & -\pi_1(t')\pi_{k-1}(t') \\ -\pi_1(t')\pi_2(t') & \pi_2(t')(1-\pi_2(t')) & -\pi_2(t')\pi_3(t')\ldots & -\pi_2(t')\pi_{k-1}(t') \\ \vdots & \vdots & \vdots & \vdots \\ -\pi_1(t')\pi_{k-1}(t') & \ldots & \ldots & \pi_{k-1}(t')(1-\pi_{k-1}(t')) \end{pmatrix}^{(k-1)\times(k-1)}$$

$$M - D = (u\pi(t) - u\pi(t+1))' uS^{-1}(u\pi(t) - u\pi(t+1)). \qquad (12.2.9)$$

where

$$uS^{(k-1)*\times(k-1)} = uA_1 + uA_2 \qquad (12.2.10)$$

Here actually we measure the shifting of the mean of population to study the mobility pattern.

12.2.4 Measure of Career Pattern Based on Entropy

Entropy as defined in a thermodynamical context arises naturally as additive quantity. Under this setup, probabilities are multiplicative. It can be shown that if the entropy S is a function of the probability P of a state, then S must be proportional to lnP. When we come to consider information as a function of probability, the same kind of relationship will apply.

Information is a statistical property of the set of possible messages, not of an individual message. If the probability of occurrences of symbol i in a system is p_i, Kendall (1973) observed the following requirements for a measure H of 'information' produced, which is continuous in the p_i. He then showed that only measure confirming to these requirements is

$$H_1 = -const \sum_{i=1}^{n} p_i ln(p_i) \qquad (12.2.11)$$

where p_i is the probability of a person belonging to the ith category.

Under the present setup, an appropriate measure on the absolute difference between the entropies of the classifications (distributions) corresponding to t and $t + 1$ is defined as,

$$E = |\sum_{i=1}^{k} \pi_i(t) ln(\pi_i(t)) - \sum_{i=1}^{k} \pi_i(t+1) ln(\pi_i(t+1))| \qquad (12.2.12)$$

$M - D$ and E can be estimated by replacing the $\pi(t)$ and $\pi(t+1)$ values by their corresponding MLE. $M - D$ and E are exactly equal to zero under no promotion situation. Under perfect promotion situation, the values may be obtained by using the relation $u\pi(t+1) = u\Pi' u\pi(t)$. The exact value will depend on the estimated values of $u\pi(t)$ and the number of categories. $M - D$ measure completely depends upon the data set and on the distribution of population. But E measure depends upon only data set. The value of $M - D$ for perfect promotion situation also depends upon the data set, but the value of E under the perfect promotion situation does not depend upon the data set and it is always equal to unity.

12.2.5 An Example

Consider the following real-life example on the non-teaching stuff of University of Calcutta officer class of the year 1990 and 2000. This was a six-grade hierarchical system. Here, r = 3, i.e., direct appointment be allowable upto third category. The estimated transition probability matrix from the flow data for the years 1990–91 and 2000–01 is as follows:

$$P = \begin{pmatrix} .7 & .3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & .5 & .5 & 0 & 0 \\ 0 & 0 & 0 & .834951 & .165049 & 0 \\ 0 & 0 & 0 & 0 & .457143 & .542857 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}^{(6\times 6)}$$

$$\hat{\pi}(t) = (0.134, 0.109, 0.369, 0.276, 0.093, 0.276, 0.093, 0.016)'$$

$$\hat{\pi}(t+1) = (.093, .1501, .184987, .41555, .088472, .067024)'$$

$$P_1 = \begin{pmatrix} .7 & .3 \\ 0 & 1 \end{pmatrix}^{2\times 2}$$

$$P_2 = \begin{pmatrix} .834951 & .165049 & 0 \\ .457143 & .542857 & \\ 0 & 0 & 1 \end{pmatrix}^{3\times 3}$$

$$p_{33} = .5$$

$$p_{34} = .5$$

$$p_{23} = 0$$

For the above example, the values of $M - D$ and E measures as well as the values of those measures under perfect promotion and no promotion are given in table:

	Observed	Perfect Promotion	No promotion
$\widehat{M - D}$	0.05	0.085	0
$\widehat{E}$	0.024201	0.0468460	0

Since the observed value of $M - D$ and E measures is near to the values under perfect promotion situation, it may be inferred that the chances of promotion are very high.

12.3 Model 2

The following measure was developed by Chattopadhyay and Khan (2004). Suppose that the service life of a person be comprised of k intervals of equal fixed width, t. The person gets at least one job offer within each such interval. The worth of an offer being determined by the associated salary (reward). The individual (assumed to be in service already) decides to leave the present job or not, at the end of each interval. One moves to a new job for the first time at the end of an interval in which the maximum of the remunerations associated with different job offers (within that interval) exceeds a fixed amount. This is the minimum wage at which the individual is willing to enter the job market for the first time. Subsequently, one changes the current job at the end of a particular interval only when the maximum of the wages associated with the offers received during that interval exceeds the wage of the current job. A change of job in this paper means that an individual may move from one occupation to another or within the same occupation. Let the individual gets N_i new job offers in the ith interval and let X_{ij} be the salary corresponding to the jth job offer in the ith interval, for $j = 1, 2, \ldots, n_i$, and $i = 1, 2, \ldots, k$. Note that to reflect the real-life situation, it is necessary to assume that n_i is strictly greater than zero since none can enter into the job market without a job offer. Both X_{ij} and N_i are assumed to be independently and identically distributed with pdf $g(x)$, $0 < x < \infty$, and pmf $h(y)$, $y = 1, 2, \ldots, \infty$, respectively. Define

$$Z_i = max(X_{i1}, X_{i2}, \ldots, X_{in_i}). \quad (12.3.13)$$

Here Z_i is the maximum wage of all job offers during the ith interval. Since Z_i is the largest order statistic, for a given n_i, the pdf of the conditional distribution of Z_i is

$$f(z_i|n_i) = n_i[G(x_{ij})]^{n_i-1}g(z_i)$$

where $G(\cdot)$ is the cdf of the distribution of X_{ij}. Hence, the distribution of Z_i is given by

$$f(z_i) = \sum_{n_i=1}^{\infty} n_i [G(z_i)]^{n_i - 1} g(z_i) h(n_i) \tag{12.3.14}$$

where $g(\cdot)$ and $h(\cdot)$ have the same specifications as before.

Let $F_{Z_i}(z)$ denote the corresponding cdf. Let z_0 be the minimum wage for which the individual accepts the first job offer at the i^{ih} interval. Then we can define

$$F_{Z_i}(z_0) = P[Z_i < z_0] \tag{12.3.15}$$

and its complement

$$\bar{F}_{Z_i}(z_0) = 1 - F_{Z_i}(z_0) = P[Z_i > z_0]. \tag{12.3.16}$$

Chattopadhyay and Khan defined a measure of occupational mobility as below. Define $N(k)$ = total number of job changes within the service life of the individual and $p_r^{(k)}$ = the probability of r job changes in the entire service life of the individual. Then

$$p_r^{(k)} = P[N(k) = r]. \tag{12.3.17}$$

A measure of occupational mobility using $p_r^{(k)}$ can be defined as

$$\mathrm{E}[N(k)] = \sum_{r=0}^{k} r p_r^{(k)} = [(k+1)\bar{F} - \bar{F}F^k - F(1 - F^k)]/2\bar{F}. \tag{12.3.18}$$

In the computation of $\mathrm{E}[N(k)]$, different binomial and geometric series are involved. After normalization with respect to k, the measure becomes $\mathrm{E}[N(k)/k]$.

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