



Study on Critical Lines Identification in Complex Power Grids

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Abstract. Some key lines in power system have important influence on the stability of a power grid operation. If the key lines can be identified, we can take some protection against these key lines to reduce the probability of blackouts. A comprehensive evaluation index which considers local information and global information is proposed for the identification of critical lines in power grids in this paper. From the point of view of network structure and synchronization dynamics, we compare the ranking results obtained by the comprehensive index method and the edge betweenness centrality method based on global information in IEEE39 and IEEE118. The simulation results show that the proposed index is better than edge betweenness centrality index on the recognition effect of the importance of edges.

Keywords: Power grid · Critical line identification
Synchronization dynamics · Complex network

1 Introduction

Many blackouts occurred in power grids at home and abroad, which have caused huge economic losses [1–7]. One of the largest blackouts in eight states in the eastern United States and parts of Canada paralyzed traffic and industry in August 2003 [6, 7]. The accident caused great shock in all countries. In order to prevent the occurrence of blackouts and maintain the stable operation of power grids, the researchers are aware that it is necessary to study the internal causes of blackouts. In fact, one component failure causes other elements to fail through lines, which is the cascading failure, thus find key lines and carry on supervision and protection to these key lines can improve the reliability of the power system and reduce the probability of blackout. With the rapid development of complex network theory, more and more researchers used complex network theory to identify key lines of power system and achieved a series of results [8–18]. This article quantified the impact of small world characteristics on cascading failure propagation based on complex network theory. The study found that high node betweenness centrality and edge betweenness centrality to fault propagation play a role in fueling [14]. From the point of view of network topology structure, the authors put forward the route betweenness centrality index using the number of the shortest distance between any two points through the line, which is used to measure edge importance. The simulation results show that the betweenness centrality index can find the key lines in power grids [15–17]. Considering the distribution of power flow

before and after disconnection of the line, a key line identification method was proposed. Using this identification method to identify the critical lines in a cascading failure process constructed by OPA blackout model. The experimental results show that the proposed identification method can identify the key lines in cascading failures, which can verify the effectiveness of the proposed method [9]. Key line identification methods based on cascading failure network diagram (CFG) were proposed respectively from node degree and different stages of cascading failures according to the propagation characteristics of cascading failures. Using the proposed key line identification methods to do simultaneous attack and timing attack experiment in IEEE118 node system. The simulation results show that the proposed critical line identification methods can not only reflect the vulnerability of power transmission lines, but also can reflect fault propagation relationship between lines, so the line identification methods have certain practical significance [18]. The above literatures all use single network feature index to identify key links in power grids.

The key line evaluation index is constructed from the network topology based on complex network theory in this paper. This index takes into account global information of edge betweenness centrality and local information of node degree, which overcomes the problem that a single index can not describe edge importance completely. In order to illustrate the effectiveness of the proposed index, the edge ranking result is compared with the results obtained by betweenness centrality method based on global information in IEEE39 and IEEE118 systems.

2 Network Model and Key Lines Identification Method

2.1 The Power Grid Model

The data of IEEE test system is the most recognized power grid data in the world at present, so power system researchers usually use IEEE standard test data for experimental simulation, such as IEEE14, IEEE30, IEEE39, IEEE57, IEEE118. Using graph theory to abstract topology structure of complex power system, it is convenient for us to do research on power grids. Generally, each component (generator, substation, user) is regarded as a node, and a power transmission line is regarded as an edge, so that a complex power grid is abstracted into a graph by node and edge. IEEE14 system has 5 generators, 9 loads, and 20 lines. There are 6 generators, 24 loads, and 41 lines in IEEE30 system. IEEE39 system consists of 39 nodes and 46 edges, including 10 generators and 29 loads. IEEE118 standard test system has 118 nodes and 179 edges, including 54 generators and 64 loads. The topology diagram of complex power networks is shown in Fig. 1. In this paper, we do the experiment simulation using unweighed and undirected networks.

2.2 The Power Grid Dynamic Model

The power grid dynamic model is the two order Kuramoto-like oscillator model [19–21], which is widely used in synchronous performance analysis and stability research. The model is described as follows:

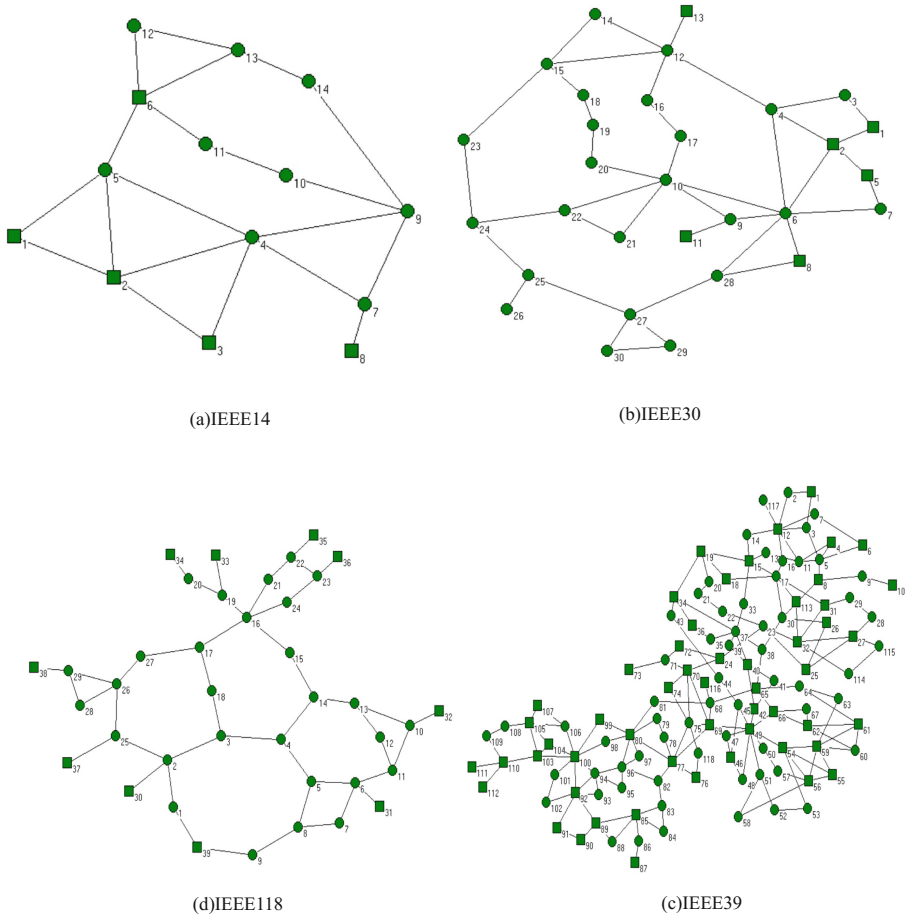


Fig. 1. Topology diagram of IEEE system: square nodes represent generator nodes, ellipse nodes represent load nodes

$$\ddot{\theta}_i = P_i - \alpha \dot{\theta}_i + K \sum_{j=1}^N a_{ij} \sin(\theta_j - \theta_i) \quad (1)$$

In order to facilitate numerical simulation, the above two order differential equation is rewritten into two first order differential equations:

$$\begin{cases} \dot{\theta}_i = \omega_i \\ \dot{\omega}_i = P_i - \alpha \omega_i + K \sum_{j=1}^N a_{ij} \sin(\theta_j - \theta_i) \end{cases} \quad i = 1, 2, \dots, N \quad (2)$$

Where θ_i is phase offset of node i , P_i is power of node i , α is loss parameter, K is coupling strength between nodes, $A = (a_{ij})$ is adjacency matrix of network connection which describes topological structure of network. If there is a link between node i and node j in unweighed and undirected networks, so $a_{ij} = a_{ji} = 1$, otherwise $a_{ij} = a_{ji} = 0$.

It can be seen from formula (2) that node phase and node frequency jointly determine motion state of node i . When a power network works normally, all nodes run at the frequency of 50 Hz or 60 Hz and the power grid is in synchronization state.

We use phase order parameter to measure synchronization ability of network [22, 23]. Order parameter is defined as follows:

$$r(t)e^{i\varphi(t)} = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j(t)} \tag{3}$$

Where $\theta_j(t)$ represents phase of oscillator j , $\varphi(t)$ represents a mean value of all oscillator phases in a network, N represents number of oscillators. It can be seen from formula (3) that order parameter $r(t)$ describes synchronization situation of all oscillators at a given moment. Steady order parameter r_∞ represents a mean value of order parameter of one system in a stationary state. The concrete formula of steady order parameter r_∞ is as follows:

$$r_\infty = \lim_{t_1 \rightarrow \infty} \frac{1}{t_2 - t_1} \int_{t_1}^{t_1 + t_2} r(t) dt \tag{4}$$

2.3 The Key Line Identification Method

Considering node degree and edge betweenness centrality, comprehensive evaluation index about importance of edges is put forward as follows:

$$NL_{ei-j} = \frac{(k_i + k_j)}{N * \langle k \rangle} * \frac{B_{ei-j}}{\max(B)} \tag{5}$$

Where k_i is degree of node i , k_j is degree of node j , N is node number, $\langle k \rangle$ is average degree of network, so $N * \langle k \rangle$ is sum of all node degree. $\frac{(k_i + k_j)}{N * \langle k \rangle}$ reflects proportion that local degree information which is composed of two nodes i and j connecting one edge occupies whole network node degree. Edge betweenness centrality B_{ei-j} is number of shortest paths between any two nodes passes edge e_{i-j} , which describes edge importance in the global information transmission. B is all edge betweenness centrality set. From the above analysis, it can be seen that comprehensive evaluation index considers not only local characteristics of nodes, but also global characteristics of edges.

According to the definition of each index, we calculate the index value of each edge in IEEE39 and IEEE118 standard test system, then we arrange edge in accordance with the index value from large to small order. Due to the limitation of length in this paper,

Tables 1 and 2 lists the edge numbering of top 30 of each index in IEEE39 system. The higher an edge sorting position is, the more important it is.

Table 1. Edge betweenness centrality sort table

Order number	Edge	Order number	Edge	Order number	Edge
1	e_{14-15}	11	e_{2-25}	21	e_{26-29}
2	e_{15-16}	12	e_{25-26}	22	e_{21-22}
3	e_{16-17}	13	e_{26-27}	23	e_{23-24}
4	e_{2-3}	14	e_{16-21}	24	e_{5-8}
5	e_{4-5}	15	e_{16-24}	25	e_{10-13}
6	e_{4-14}	16	e_{3-18}	26	e_{1-39}
7	e_{16-19}	17	e_{5-6}	27	e_{8-9}
8	e_{3-4}	18	e_{17-18}	28	e_{6-11}
9	e_{13-14}	19	e_{19-20}	29	e_{12-13}
10	e_{17-27}	20	e_{1-2}	30	e_{2-30}

Table 2. Comprehensive index sort table

Order number	Edge	Order number	Edge	Order number	Edge
1	e_{16-17}	11	e_{25-26}	21	e_{5-8}
2	e_{15-16}	12	e_{16-21}	22	e_{10-13}
3	e_{16-19}	13	e_{16-24}	23	e_{19-20}
4	e_{2-3}	14	e_{5-6}	24	e_{21-22}
5	e_{4-5}	15	e_{26-27}	25	e_{23-24}
6	e_{14-15}	16	e_{17-27}	26	e_{6-11}
7	e_{4-14}	17	e_{26-29}	27	e_{8-9}
8	e_{3-4}	18	e_{3-18}	28	e_{26-28}
9	e_{13-14}	19	e_{17-18}	29	e_{1-39}
10	e_{17-27}	20	e_{1-2}	30	e_{12-13}

3 Comparison of Line Identification Methods Based on Network Structure

We compare the sorting methods about edge importance from the perspective of network structure. Network efficiency $C(e_{i,j})$ is defined as follows [24]:

$$C(e_{i-j}) = N_{ei-j}^R / N \quad (6)$$

Where N_{ei-j}^R says node number in maximum connected subnet of network after the removal of the edge $e_{i,j}$, N says primitive network node number. N_{ei-j}^R / N represents the vulnerability of the network after the failure of the edge. The smaller the value of $C(e_{i,j})$ after removing edge $e_{i,j}$ is, the more important of edge $e_{i,j}$ to a network connectivity is

from formula (6). According to the index value, an edge is removed in order of large to small, and the relationship between the number of removing edges and network efficiency is studied in each power grid mode. The simulation results are shown in Fig. 2.

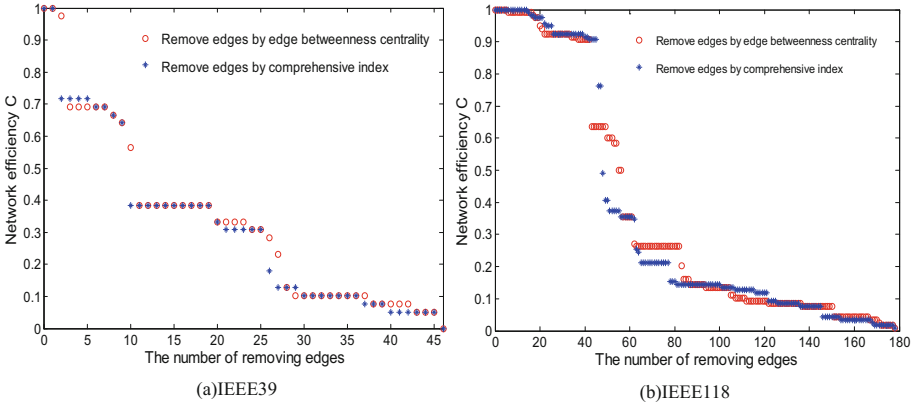


Fig. 2. Network efficiency under different ways to remove edges

In view of Fig. 2, it is found that removing edges number is inversely proportional to network efficiency value. The more edges are removed, the smaller network efficiency value is in each way, which indicates that connected network is divided into several scattered small connected region or isolated nodes and thus it reduces node number on the largest connected branch. If one edge is deleted, the number of nodes on the largest connected branch is reduced faster, then it can be explained that the deletion of the edge is more destructive to the network. In view of Fig. 2(a), when deleting the first 2 edges, network efficiency value is greatly reduced using comprehensive index method. At this time, network efficiency $C_{\text{comprehensive method}} < C_{\text{betweenness centrality method}}$ which shows that the destruction of a network is greater with the removal of edges by comprehensive method. According to the sorting result of Tables 1 and 2, it is found that the first 2 edges contain e_{15-16} and the other edge is different, which shows that e_{16-17} is more important than e_{14-15} . In addition, when removing the third edge of e_{16-17} by betweenness centrality method, we found $C_{\text{comprehensive method}} > C_{\text{betweenness centrality method}}$ which shows that the removal of edge e_{16-17} is more destructive to connectivity of a network. The above analysis illustrates that comprehensive index method is more reasonable in edge importance ranking. When removing the tenth edge by comprehensive index method, network efficiency $C_{\text{comprehensive method}}$ drops faster. According to edge importance ranking result using comprehensive index method, we found that this edge correspond to e_{2-25} while taking e_{2-25} in eleventh place in betweenness centrality method. The higher the sorting location is, the more important the link is. From this point of view, comprehensive index can find key edges of a network more effectively than betweenness centrality method. Combining Tables 1 and 2, we observe network efficiency of removing the twenty-first edge and the twenty-sixth edge in different ways, and we also find that the comprehensive method is more

effective in identifying important edges. Figure 2(b) can also obtain the above conclusions.

4 Comparison of Line Identification Methods Based on Dynamics

When an important line fails and is removed from a power grid in the operation of power system, synchronous capability of a power grid decreases, which affects the stable operation of a power grid. We usually use critical synchronous coupling strength k_c to measure synchronization capability of the power grid. In general, critical synchronization coupling strength k_c of a power grid will increase with the decrease of synchronization capability. We use two order Kuramoto-like model as the power grid dynamics model to study synchronous dynamic behavior of a power grid. This paper studies synchronization ability of power grids when deleting a link in different ways, and the simulation results are shown in Fig. 3.

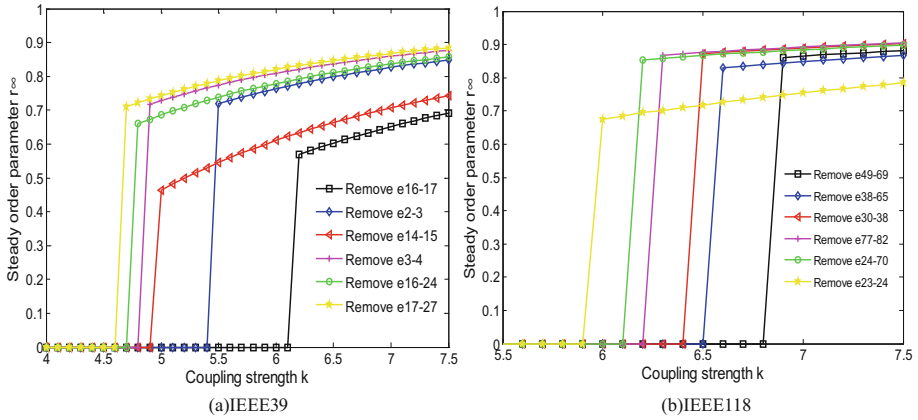


Fig. 3. Power grid synchronization capability when deleting a link

We use the edge importance index IM_{ei-j} to indicate importance of edge. The results of critical synchronization coupling strength k_c of a power grid when links are removed in Fig. 3(a) is as follows: $k_{c,remove\ e16-17} > k_{c,remove\ e2-3} > k_{c,remove\ e14-15} > k_{c,remove\ e3-4} > k_{c,remove\ e16-24} > k_{c,remove\ e17-27}$, and these links importance is sorted as follows: $IM_{e16-17} > IM_{e2-3} > IM_{e14-15} > IM_{e3-4} > IM_{e16-24} > IM_{e17-27}$. The ordering result of these edges importance by betweenness centrality method is as follows: $IM_{e14-15} > IM_{e16-17} > IM_{e2-3} > IM_{e3-4} > IM_{e17-27} > IM_{e16-24}$. These edges importance ranked by comprehensive method is as follows: $IM_{e16-17} > IM_{e2-3} > IM_{e14-15} > IM_{e3-4} > IM_{e16-24} > IM_{e17-27}$.

In addition, the ranking result of edge importance obtained by experimental simulation in Fig. 3(b) is as follows: $IM_{e49-69} > IM_{e38-65} > IM_{e30-38} > IM_{e77-82} > IM_{e24-70} > IM_{e23-24}$. The order of importance of these edges determined by betweenness

centrality is as follows: $IM_{e38-65} > IM_{e30-38} > IM_{e49-69} > IM_{e24-70} > IM_{e23-24} > IM_{e77-82}$. The order situation of importance of these links obtained by comprehensive method is as follows: $IM_{e49-69} > IM_{e38-65} > IM_{e30-38} > IM_{e77-82} > IM_{e24-70} > IM_{e23-24}$. By comparing simulation results and edge importance ranking results in different ways, it is found that the ranking results of edge importance obtained by comprehensive method and the experimental simulation are consistent. It shows that comprehensive method is more effective than betweenness centrality method in identifying the importance of links. Figure 3(b) can also get the same conclusion.

5 Conclusion

A comprehensive evaluation index is proposed in this paper, which considers local information of node degree and global information of edge betweenness centrality. Comparing the identification effect of comprehensive index method and betweenness centrality method on important edges from network structure and synchronization dynamics in IEEE39 and IEEE118 systems. The simulation results show that comprehensive method is more effective than betweenness centrality method to find key links in power grids.

Both edge betweenness centrality method and comprehensive index method all need to calculate betweenness, so the amount of calculation is large. From this point of view, these two methods are usually used for key line identification in small networks, which are not very suitable for large networks.

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