Optimization of Constrained Engineering Design Problems Using Cohort Intelligence Method

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Abstract This paper proposes Cohort Intelligence (CI) method as an effective approach for the optimization of constrained engineering design problems. It employs a probability-based constraint handling approach in lieu of the commonly used repair methods, which exhibits the inherent robustness of the CI technique. The approach is validated by solving three design problems. The solutions to these problems are compared to those evaluated from Simple Constrained Particle Swarm Optimizer (SiC-PSO) and Co-evolutionary Particle Swarm Optimization based on Simulated Annealing (CPSOSA) (Cagnina et al., Informatica 32(3):319–326, [1]). The performance of Cohort Intelligence method is discussed with respect to best solution, standard deviation, computational time, and cost.

Keywords Cohort intelligence · Constrained optimization · Engineering design problems

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1 Introduction

Cohort Intelligence method proposed by Kulkarni et al. [2] is a bio-inspired optimization technique, in which a cohort refers to a group of candidates/individuals with the shared characteristic of achieving their individual goals [2]. It is based on the natural tendency of an individual to evolve its behavior by observing the behavior of other candidates of the cohort and emulating it. Every candidate follows a certain behavior and imbibes the associated qualities which may improve its own behavior. Roulette wheel approach based on the probability of the behavior being followed is used by the individual candidates to decide the candidate whose behavior is to be followed. It results in every candidate learning from the other and leads to the evolution of the overall behavior of the cohort. In order to arrive at the best solution, every candidate shrinks the sampling interval associated with every variable by using a reduction factor r, which along with the number of candidates is determined based on preliminary trials. The cohort behavior is considered to be saturated if significant improvement in the behavior is not seen or it becomes difficult to distinguish between the behaviors of all the candidates in the cohort. In this case, convergence is said to have occurred and the solution thus obtained is considered to be the optimum solution. The CI methodology was validated by solving four test problems such as Rosenbrock function, Sphere function, Ackley function, and Griewank function.

In [2], the CI methodology was validated as a robust, viable, and a competitive alternative to its contemporaries. However, the computational performance was governed by the reduction factor r and could be improved further to make it solve real-world problems which are generally constrained in nature. Hence for solving constrained optimization problems, probability collectives and a penalty function approach were introduced in [3]. It involves decomposing a complex system into subsystems in order to optimize them in a distributed and decentralized way. The approach produced competitive results, if not better compared to the GA techniques. It was concluded that the approach could be made more generalized and the diversification of sampling, rate of convergence, quality of results, etc., could be improved. Additionally, a local search technique can be incorporated for assisting in a neighborhood search to reach the global optimum [3].

Working on the following lines, CI method with probability-based constraint handing approach was proposed in [4]. Instead of the commonly used repair methods like penalty function method, in this approach a probability distribution is devised for every individual constraint. The lower and upper bounds of the distribution are chosen by finding the minimum and maximum values among all the constraints by substituting the lower and the upper boundaries of the variables in all the constraints. Based on the range in which the value of each of the constraint lies, the probability and the probability score are calculated. Roulette wheel approach based on the probability score is used to select the behavior to be followed, and similar to CI method range reduction is used to arrive at the best solution.

It was observed that a probability-based constraint handling approach was not only robust but also reduced the computational time as compared to Differential

Evolution (DE), Genetic Algorithm (GA), Evolutionary Strategy (ES), and Particle Swarm Optimization (PSO) which employ various repair methods [4]. In this paper, CI method with probability-based constraint handling approach was used to solve three engineering design problems, namely spring design, welded-beam design, and pressure vessel design.

2 Literature Review

Over the years, various optimization approaches have been applied to the selected engineering design problems. Among the commonly used techniques for these problems are GA-based co-evolution model and a feasibility-based tournament selection scheme, an effective Co-evolutionary PSO (CPSO) for constrained engineering design problems and a hybrid PSO (hPSO) with a feasibility-based rule for constrained optimization [5–7].

Cagnina et al. [1] developed a constrained version of PSO for solving engineering optimization problems. In this method, two particles are compared, and if both are feasible the one with better fitness function value is selected. If both are infeasible, the one with lower infeasibility is selected. The approach contains a constraint handling technique as well as a mechanism to update the velocity and position of the particles. The approach was tested by solving four problems, three of the problems have been adopted here (A01, A02, and A03). The best solution they found was compared to that obtained from different methods like Mezura and CPSO. The values obtained by this method are similar to those obtained in [8].

The hybrid CPSO exploits simulated annealing and penalty based method for handling constraints.

The approach was tested by solving three of the problems adopted here (A01, A02, and A03). The best solution they found was compared to that obtained from different methods like CPSO, structural optimization, self-adaptive constraint handling [5, 6, 9–11]. The solution attained in [8] was significantly better for all the problems (A01, A02, and A03) as compared to the other methods.

Cohort Intelligence is a relatively recent technique which has been used to solve engineering design optimization problems in this paper. The performance of CI method is compared to the techniques in hCPSO as they have attained better results as compared to the earlier methods [1, 8].

3 Problem Definition

A01: Tension/compression spring design optimization problem.

The spring design problem is taken from [1] in which a tension/compression spring is designed for minimum weight, subject to constraints of minimum deflection, shear stress, surge frequency, and limits on outside diameter and on design variables. There are three design variables: the wire diameter d (x_1), the mean coil diameter D (x_2), and the active coils P (x_3). The mathematical formulation of this problem is:

Minimize: $f(\overrightarrow{x}) = (x_3 + 2)x_2x_1^2$

Subject to:

$$g_1(\overrightarrow{x}) = 1 - \frac{x_2^3x_3}{7178x_1^4} \le 0$$

$$g_2(\overrightarrow{x}) = \frac{4x_2^2 - x_1x_2}{12{,}566(x_2x_1^3) - x_1^4} + \frac{1}{5108x_1^2} - 1 \le 0$$

$$g_3(\overrightarrow{x}) = 1 - \frac{140.45x_1}{x_2^2x_3} \le 0$$

$$g_4(\overrightarrow{x}) = \frac{x_2 + x_1}{1.5} - 1 \le 0$$

where $0.05 \le x_1 \le 2$, $0.25 \le x_2 \le 1.3$, and $2 \le x_3 \le 15$ (Table 1).

A02: Welded-beam design optimization problem.

The welded-beam design problem is referred to from [12] in which the welded beam is designed for minimum fabrication cost, subject to constraints of shear stress τ, bending stress in the beam σ, buckling load on the bar Pc, and end deflection on the beam δ. Four design variables x_1, x_2, x_3, and x_4 are considered for minimizing the fabrication cost. The mathematical formulation of this problem is:

Minimize: $f(\overrightarrow{x}) = 1.10471x_2x_1^2 + 0.04811x_3x_4(14 - x_2)$

Subject to:

$$g_1(\overrightarrow{x}) = \tau(\overrightarrow{x}) - 13{,}000 \le 0$$

$$g_2(\overrightarrow{x}) = \sigma(\overrightarrow{x}) - 30{,}000 \le 0$$

$$g_3(\overrightarrow{x}) = x_1 - x_4 \le 0$$

$$g_4(\overrightarrow{x}) = 1.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5 \le 0$$

$$g_5(\overrightarrow{x}) = 0.125 - x_1 \le 0$$

$$g_6(\overrightarrow{x}) = \delta(\overrightarrow{x}) - 0.25 \le 0$$

$$g_7(\overrightarrow{x}) = 6000 - Pc(\overrightarrow{x}) \le 0$$

where

$$\tau(\overrightarrow{x}) = \sqrt{(\tau')^2 + (2\tau\prime\,\tau'')\frac{x_2}{2R} + (\tau'')^2}$$

$$\tau\prime = \frac{6000}{\sqrt{2x_1x_2}}, \tau\prime\prime = \frac{MR}{J}$$

Table 1 Solution vector A01 (spring design)

Parameter	x_1	x_2	x_3	$g_1(\overrightarrow{x})$	$g_2(\overrightarrow{x})$	$g_3(\overrightarrow{x})$	$g_4(\overrightarrow{x})$	$f(\overrightarrow{x})$
Best solution	0.05157	0.35418	11.43864	−9.00443	−0.13467	−4.04831	−0.72948	0.01266

$$M = 6000\left(14 + \frac{x_2}{2}\right),\ R = \sqrt{\left(\frac{x_2^2}{4}\right) + \left(\frac{x_1 + x_3}{2}\right)^2},$$

$$J = 2\left\{x_1 x_2 \sqrt{2}\left[\left(\frac{x_2^2}{12}\right) + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}$$

$$\sigma\left(\overrightarrow{x}\right) = \frac{504{,}000}{x_4 x_3^3},\ \delta\left(\overrightarrow{x}\right) = \frac{65{,}856{,}000}{\left(30 \times 10^6\right) x_4 x_3^3}$$

$$Pc\left(\overrightarrow{x}\right) = \frac{4.013\left(30 \times 10^6\right)\sqrt{\frac{x_3 x_4^6}{36}}}{196}\left(1 - \frac{x_3\sqrt{\frac{30 \times 10^6}{4\left(12 \times 10^6\right)}}}{28}\right)$$

where $0.1 \le x_1 \le 2,\ 0.1 \le x_2 \le 10,\ 0.1 \le x_3 \le 10,\ 0.1 \le x_4 \le 2$ (Table 2).

A03: Pressure vessel design optimization problem.

The welded-beam design problem is referred to from [6, 12]. A cylindrical vessel is capped at both ends by hemispherical heads as shown in figure. Using rolled steel plate, the shell is made in two halves that are joined by two longitudinal welds to form a cylinder. The objective is to minimize the total cost, including the cost of the materials forming the welding. The design variables are: thickness x_1, thickness of the head x_2, the inner radius x_3, and the length of the cylindrical section of the vessel x_4. Consider a compressed air storage tank with a working pressure of 3000 psi and a minimum volume of 750 ft^3. The mathematical formulation of this problem is:

Minimize:

$$f\left(\overrightarrow{x}\right) = 0.6624 x_1 x_3 x_4 + 1.7781 x_2 x_3^2 + 3.1661 x_1^2 \mathrm{x}_4 + 19.84 x_1^2 x_3$$

Subject to:

$$g_1\left(\overrightarrow{x}\right) = -x_1 + 0.0193 x_3 \le 0$$
$$g_2\left(\overrightarrow{x}\right) = -x_2 + 0.00954 x_3 \le 0$$
$$g_3\left(\overrightarrow{x}\right) = -\pi x_3^2 x_4^2 - \tfrac{4}{3}\pi x_3^2 + 1{,}296{,}000 \le 0$$
$$g_4\left(\overrightarrow{x}\right) = x_4 - 240 \le 0$$

where $1 \times 0.0625 \le x_1 \le 99 \times 0.0625$, $1 \times 0.0625 \le x_2 \le 99 \times 0.0625$, $10 \le x_3 \le 200$, and $10 \le x_4 \le 200$.

As stated in [1], the variables x_1 and x_2 are discrete values which are integer multiples of 0.0625 in. Hence, the upper and lower bounds of the ranges of x_1 and x_2 are multiplied by 0.0625 as shown in Table 3.

Table 2 Solution vector for A02 (welded beam)

Parameter	x_1	x_2	x_3	x_4	$g_1(\vec{x})$	$g_2(\vec{x})$	$g_3(\vec{x})$	$g_4(\vec{x})$	$g_5(\vec{x})$	$g_6(\vec{x})$	$g_7(\vec{x})$	$f(\vec{x})$
Best solution	0.2057	3.4704	9.0366	0.2057	0.0449	0.0924	−1.8924E−12	−3.4329	−0.0807	−0.2355	0.0559	1.7248

Table 3 Solution vector for A03 (pressure vessel)

Parameter	x_1	x_2	x_3	x_4	$g_1(\overrightarrow{x})$	$g_2(\overrightarrow{x})$	$g_3(\overrightarrow{x})$	$g_4(\overrightarrow{x})$	$f(\overrightarrow{x})$
Best solution	0.81249	0.43750	42.09844	1.76636E+02	6.96000E−09	−0.03588	−0.03588	−63.36340	6059.71438

Table 4 Comparison of CI with existing algorithms

Problem		CPSOSA	SiC-PSO	CI
A01	Best	0.01266	0.01266	0.01266
	Mean	0.01267	0.0131	0.01266
	SD	3.68569E−6	4.1E−4	3.4552E−07
A02	Best	1.72485	1.72485	1.72484
	Mean	1.72485	2.0574	1.72484
	SD	1.70514E−05	0.2154	3.61161E−11
A03	Best	6059.7143	6059.714335	6059.71438
	Mean	6059.7143	6092.0498	6059.71438
	SD	2.26415E−06	12.1725	5.02269E−08

4 Results and Discussion

The CI algorithm for the stated engineering design optimization problems was coded in MATLAB 7.8.0 (R2009a), and simulations were run on a Windows platform using Intel Core i5 processor. The suitability of CI algorithm to solve constrained optimization problems was reaffirmed by applying it to engineering design problems. These well-known design problems have been solved by various optimization techniques and solutions from different methods that are available for comparison. Table 1 represents the comparisons between the solutions obtained from CI algorithm and that obtained from SiC-PSO and CPSOSA in terms of the best solution, mean, and standard deviation (SD).

It can be observed from Table 1 that the solution obtained from CI method in terms of the best solution is precisely comparable to those obtained by CPSOSA. Whereas the standard deviation (SD) obtained from CI algorithm is substantially better than those obtained from CPSOSA and SiC-PSO. The values of the mean obtained for CI algorithm are consistent with the best solution. The performance of Cohort Intelligence method was also measured in terms of the computational time, number of function evaluations (FE), and convergence. Parameters such as the number of candidates and reduction factor are also listed. The function evaluations (FE) required for CI algorithm are less as compared to 24,000 objective function evaluations performed by SiC-PSO, as stated in [1]. The time required for computation is also reasonable.

The solution convergence plots for the spring design function, welded-beam design function, and pressure vessel design function are presented in Figs. 1, 2, and 3, respectively, which exhibit the overall evolution of the cohort by improving individual behaviors as shown in Table 4. Initially, the behavior of the individual candidates can be distinguished. As the sampling interval reduces and the cohort evolves, we do not see significant improvement in the behavior of the candidates. At a particular point, it is difficult to distinguish between the behaviors of the candidates. The cohort is considered to be saturated, and convergence is said to have occurred.

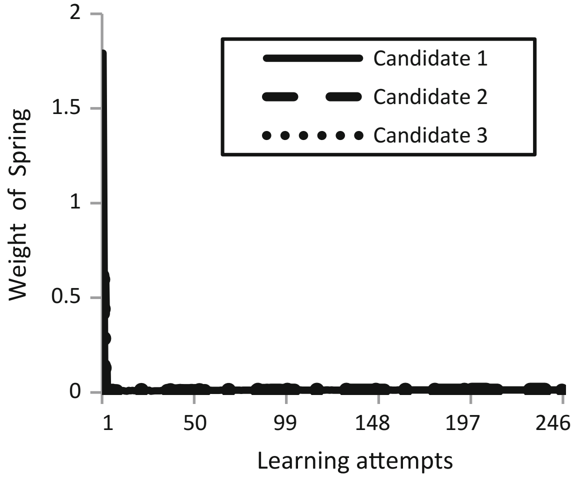

Fig. 1 Spring design function

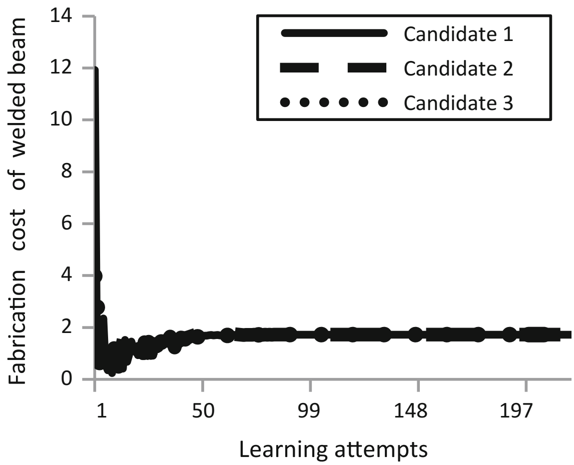

Fig. 2 Welded-beam design function

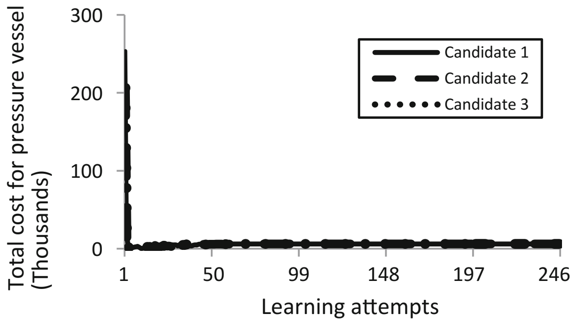

Fig. 3 Pressure vessel design function

Table 5 Performance of CI algorithm

Problem	Convergence	FE	Time (s)	Parameters (C, r)
A01	190	570	0.12	3, 0.95
A02	118	354	0.13	3, 0.95
A03	133	399	0.24	3, 0.95

5 Conclusion and Future Work

We have presented Cohort Intelligence method with probability-based constraint handling for constrained optimization problems. Cohort Intelligence approach has shown very good performance when applied to the given engineering design optimization problems as shown in Table 5. CI generates results which are comparable precisely to two other algorithms with respect to best solution and the individual variable values. Thus, the approach could be considered as a credible alternative to solving constrained optimization problems due to its robustness and high computational speed.

In the future, CI method could be applied for solving resource utilization-related problems by formulating them as a goal programming problems.

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