

Multi-Objective Optimization and Cluster-Wise Regression Analysis to Establish Input–Output Relationships of a Process



Amit Kumar Das, Debasish Das and Dilip Kumar Pratihar

1 Introduction

We, human beings, have a natural tendency to know input–output relationships of a process. Goldberg (2002) claimed that a genetic algorithm (GA) is competent for yielding innovative solutions in a single-objective optimization (SOO) problem domain. An SOO is generally used for finding out a single optimal solution out of several possibilities. However, a multi-objective optimization (MOO) problem involves at least two conflicting objective functions and a Pareto-optimal front of solutions can be obtained for the same. This present chapter deals with an application of MOO.

For the last few decades, MOO had been applied for solving various research and industrial problems (Ahmadi et al. 2015, 2016; Aghbashlo et al. 2016; Jarraya et al. 2015; Khoshbin et al. 2016; Marinaki et al. 2015; Ahmadi and Mehrpooya 2015; Sadatsakkak et al. 2015). Making large data intelligent transportation system (Wang et al. 2016), fabrication and optimization of 3D structures in bone tissue engineering area (Asadi-Eydivand et al. 2016), portfolio optimization with functional constraints (Lwin et al. 2014), optimization of building design (Brownlee and Wright 2015), disaster relief operations (Zheng et al. 2015), etc. are some of the worth-mentioning examples, where MOO has been utilized successfully in the recent times. By using a multi-objective evolutionary algorithm (MOEA), a Pareto-front of solutions is obtained. These solutions are not generated randomly (Askar and Tiwari 2009) and

A. K. Das · D. Das · D. K. Pratihar (✉)
Indian Institute of Technology Kharagpur, Kharagpur 721302, India
e-mail: dkpra@mech.iitkgp.ac.in

A. K. Das
e-mail: amit.besus@gmail.com

D. Das
e-mail: dd4311@gmail.com

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they must satisfy mathematically Karush–Kuhn–Tucker (KKT) conditions (Miettinen et al. 1999). It is argued (Deb and Srinivasan 2008) that there is a high chance of finding some commonalities among these high-performing solutions of the Pareto-front. Nevertheless, it is also debated that this commonality may exist for either the whole Pareto-front or different subsets of the same. However, if the said commonalities are embedded in those optimal or near-optimal solution sets, then it is expected to obtain the design principles for that process after some numerical analysis of the solutions.

The said fact can have an immense effect in designing a product or process. If a designer does have this type of information a priori, then it will help him to do his job more efficiently. For example, let us take the case of metal cutting operation with the inputs, such as cutting speed, feed, and depth of cut, and take two conflicting outputs as minimizing the machining time and minimizing the surface roughness of the machined product. By using the MOEA, if it is found that one or more parameters are varying in a particular fashion within the Pareto-front or the objective functions are changing with the inputs in a certain manner, then this information can provide an extra advantage to the process designer to set the same in a more efficient way. Similar things are applicable to other processes as well.

This fact of obtaining various design principles has another significance for the manufacturing industries. Reduction of cost without compromising the safety and quality of the products has always been the primary concern of any industry. The scope of cost minimization through proper inventory and manpower management would be possible, if the facts discussed above are available. Moreover, similar prior information may assist the process designer to establish and stabilize the process of interest with ease.

To use an MOEA, input–output relationships are required to define the objective functions. In many instances, these equations are not obtained from the literature and we have to derive these using the statistical tools on the experimental data. Now, generation of a wide range of experimental data requires not only proper facilities, but also a sequence of several tedious steps, and it demands time, cost, and effort. Moreover, noticeable variations in output data of any experiments are likely to be observed for a given set of input parameters. This results in fuzziness and inaccuracy in the experimental dataset, as shown by Gil and Gil (1992). The inaccuracy of the data can be minimized through multiple repetitions of the experiments, thereby providing the upper and lower limits of variation in the developed dataset, for a given set of input parameters. However, it again becomes difficult due to time and resource constraints in most of the cases. Gil (1987) also argued that experimental data may suffer from the loss of information about the state or parameter space owing to the fuzziness in it. Therefore, the generated Pareto-optimal solutions using an MOEA may also be affected by the fuzziness and inaccuracy of the experimental data, and we may end up with largely inaccurate design principles for the analyzed process. Due to this issue, it is desirable to tackle inherent fuzziness of the experimental data to establish input–output relationships of the process more accurately.

2 Literature Survey

Evolutionary multi-objective optimization has been used to establish input–output relationships by several researchers. The existence of the resemblance among the Pareto-optimal front of solutions was highlighted by Deb (2003). He also suggested that these commonalities could be revealed through regression and manual plotting. Obayashi and Sasaki (2003) used the self-organizing map to view the higher dimensional objective space and design variable space on a lower dimensional map. They also used a clustering technique to make the clusters of decision parameters, which showed the role of the variables in improving the design and trade-offs. Taboada and Coit (2006) suggested applying k-means clustering technique on the Pareto-optimal solutions for ease of further analysis. Deb and Srinivasan (2008) used Benson’s method to obtain a modified Pareto-front from the initial one and after that, they used statistical regression analysis to get different design principles. This method had also been adopted and implemented by various researchers in their works (Deb et al. 2009; Deb and Jain 2003; Deb and Sindhya 2008). An analytical approach was suggested by Askar and Tiwari (2011) to get a Pareto-front for multi-objective optimization problem, and the obtained Pareto-optimal solutions were analyzed to derive several innovative commonality principles. Deb et al. (2014) proposed the method of automated innovization to decode several important relationships through the extensive use of an evolutionary algorithm. They used the word, innovization, which means the act of obtaining innovative solutions through optimization. In this case, they had not used regression tools to decipher the said principles. Later, a concept of higher level innovization was introduced by Bandaru et al. (2011) and Bandaru et al. (2015) in a generalized form. Also, a simulation-based innovization procedure was developed by Dudas et al. (2011). In this approach, they tried to evaluate the effect of variables on the performance of the process and they showed a method of getting in-depth knowledge about a process after these analyses.

Among all these stated approaches, no one adopted any method to model the inherent fuzziness of the Pareto-optimal solutions. As already discussed, if this work is not done, then there will be a high chance that we shall obtain imprecise design principle for the process of interest. Therefore, a method has been developed here to obtain different input–output relationships after modeling the fuzziness in the Pareto-optimal dataset. This method has been applied for an electron beam welding process on SS304 plates, and the obtained results clearly show the significance of this developed approach. The rest of the text has been arranged as follows:

The developed method has been described in Sect. 3, whereas Sect. 4 deals with the experimental data collection procedure. The results and discussion are provided in Sect. 5, and some concluding remarks are made in Sect. 6.

3 Developed Approach

In the proposed approach, an attempt has been made to establish the input–output relationships of a process through the extensive use of a multi-objective evolutionary algorithm (MOEA). The approach has been explained in the following steps:

Step 1: Develop initial Pareto-optimal front

In this step, an initial Pareto-front is obtained using an MOEA, where the input–output relationships are used to determine the fitness values of the objective functions. The obtained Pareto-front is subjected to some inherent fuzziness in it, which is going to be removed in further steps.

Several strategies for the MOEA are available in the literature, such as niched Pareto genetic algorithm (NPGA) (Horn et al. 1994), strength Pareto evolutionary algorithm (SPEA) (Zitzler and Thiele 1998), Pareto-archived evolution strategy (PAES) (Knowles and Corne 1999), non-dominated sorting genetic algorithm-II (NSGA-II) (Deb et al. 2002), multi-objective algorithm based on decomposition (MOEA/D) (Zhang and Li 2007), multiple populations for multiple objectives (MPMO) (Zhan et al. 2013), and others. In our study, NSGA-II has been used as the MOEA and its working principle is described in Fig. 1.

Step 2: Train a neuro-fuzzy system

Using the initial Pareto-front of solutions, a neuro-fuzzy system (NFS) is trained. In NFS, a fuzzy logic controller (FLC) is expressed as the form of a neural network. During the training, an evolutionary optimization technique is used to tune the NFS. The initial Pareto-optimal solutions are clustered using a clustering algorithm, and the number of rules of the NFS is kept as the same with that of a total number of clusters obtained. The data obtained through experiments are subjected to inaccuracy and fuzziness. This may be due to various reasons, such as experimental inaccuracy, error due to the unskilled operator, instrumental inaccuracy, and others. To take care of this inherent fuzziness, NFS is an efficient tool (Mitra and Pal 1996), which works based on the principles of fuzzy sets. In NFS, the advantages of both fuzzy logic controller and neural network are clubbed together to design and remove the uncertainty and imprecision of a set of data. The NFS has been used successfully to solve a variety of problems related to several fields of research (Takagi and Hayashi 1991; Takagi et al. 1992; Keller et al. 1992; Berenji and Khedkar 1992; Jang 1993; Ishibuchi et al. 1994). Here, an NFS with Mamdani approach (Mamdani and Assilian 1975) is used to model fuzziness in the initial Pareto-optimal solutions. The said NFS has mainly five layers, namely, input layer, fuzzification layer, And operation layer, fuzzy inference layer, and defuzzification layer. Gaussian type of membership functions, which has been used in input and output layers of the NFS, can be expressed using Eq. (1):

$$\mu_{Gaussian} = e^{-\left[\frac{(x-m)^2}{2\sigma^2}\right]}, \quad (1)$$

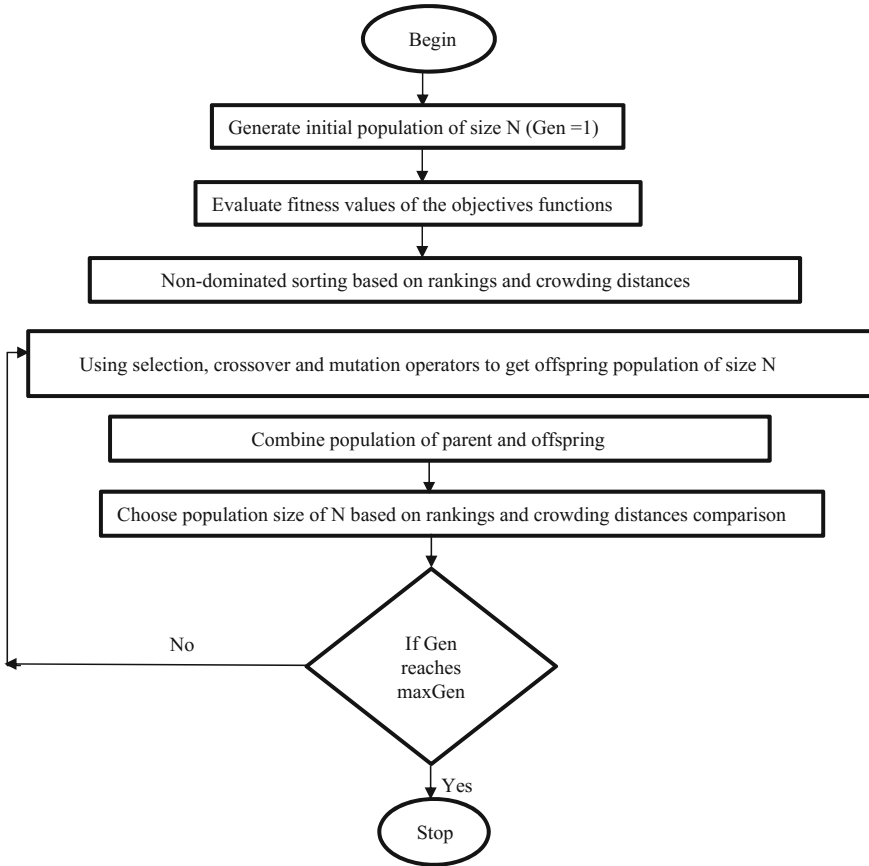


Fig. 1 Flowchart of NSGA-II algorithm

where σ and m are the standard deviation and mean of the Gaussian distribution, respectively. During the training, the NFS is evolved with the help of a genetic algorithm, where the Gaussian parameters (σ and m) are used as the design variables and the objective is to minimize the root-mean-square error (RMSE) value (here, error is the amount of deviation in prediction).

Step 3: Obtain a modified Pareto-front

In this step, the trained NFS is used in another MOEA to evaluate the fitness values of the objective functions. As NFS is a strong tool to take care the fuzziness of the used data, there is every possibility to get a better quality of Pareto-front compared to that of the initial one. In this way, the imprecision of the initial Pareto-optimal solutions is removed using the NFS and a modified and better Pareto-front in terms of both the objective functions' values is obtained.

Step 4: Clustering of the obtained modified Pareto-front

The modified Pareto-optimal dataset, obtained in Step 3, is clustered using a clustering algorithm. The purpose of carrying out this operation is to find that whether the design principles of the process are varying for the different clusters, or they are kept the same for the whole set of Pareto-front.

There are several techniques of clustering available in the literature. However, we have considered three popular clustering algorithms like fuzzy C-means clustering (FCM) (Bezdek 1973), entropy-based fuzzy clustering (EFC) (Yao et al. 2000), and density-based spatial clustering application with noise (DBSCAN) (Ester et al. 1996), in our study. In case of FCM, a data point may belong to several predefined numbers of clusters with different membership values. It is an iterative method, where the cluster centers and the membership values of the cluster members are going to be updated with the iterations. In every step of the algorithm, the focus is to reduce the dissimilarity measurement (which is evaluated in terms of Euclidean distance) of the cluster members. The members of the same cluster are likely to have the higher values of similarity, whereas the data points belonging to different clusters should be dissimilar in nature. The EFC algorithm works based on the entropy measures of the data points, which are calculated depending on the values of the distances (say Euclidean distances) among them. A data point is declared as a cluster center, if it is found with the minimum entropy value. The other points, which have the similarity measures with the cluster center greater than a user-defined value, will be put into the cluster. In case of DBSCAN, two user-specified parameters are used, such as neighborhood radius and minimum number of neighborhood points. The philosophy of this algorithm is that the density of the members lying within a cluster is higher than that of the outsider points. The details of these algorithms can be found out in (Pratihari 2007).

Step 5: Analysis of the modified Pareto-front

The obtained modified Pareto-optimal solutions are statistically analyzed cluster-wise to get several input–output relationships for the problem. Moreover, any relationship, which is common to all the clusters, has to be checked. The developed approach has been described through a flowchart, as shown in Fig. 2. Generally, the outputs of a natural process vary nonlinearly with the input parameters. Keeping this idea in mind, a nonlinear regression tool (using MINITAB 16.0 software (<http://www.minitab.com>)) has been used to determine various input–output relationships in power form.

4 Experimental Data Collection

To explain the proposed approach in more details, an engineering problem, namely, electron beam welding (EBW), has been selected and the developed approach has been implemented for the said process. The details of the experimental procedure, along with the setup information, have been provided in this section.

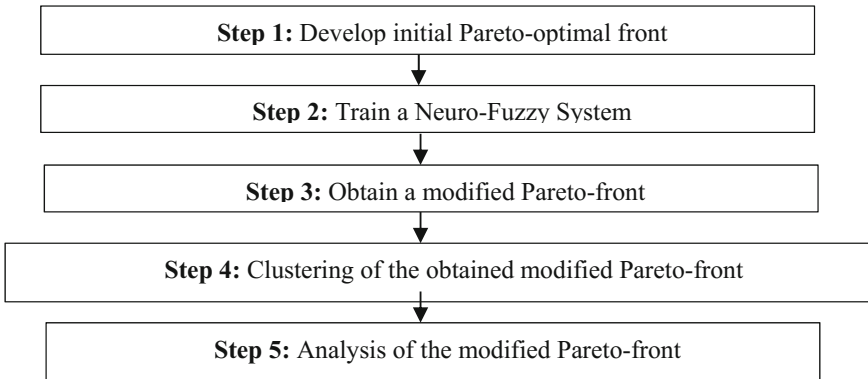


Fig. 2 Flowchart of the developed approach

4.1 Experimental Setup and Procedure

Bead-on-plate welding was carried out on EBW facility, developed by Bhabha Atomic Research Centre (BARC), Mumbai, at IIT Kharagpur (refer to Fig. 3). The machine has a maximum power rating of 12 kW. The beam was kept stationary, while the table containing the workpiece, fixture, and other arrangements traveled in the horizontal plane at a predefined welding speed. A vacuum was provided in the work chamber and gun chamber with the help of vacuum pumps before the initiation of the welding process. The stream of highly accelerated electrons was made incident on 20-mm-thick AISI 304 stainless steel workpiece in the vacuum environment. The chemical composition of the used material can be found out in (Das et al. 2016). The EBW experiments had been carried out following a multilevel full-factorial design. This study aims to investigate the effects of beam power and welding speed on the depth of penetration and bead width of the weld.

4.2 Data Collection

Two input parameters, namely, beam power (P in W) and welding speed (S in mm/min), were considered in this study. Considering four levels of the input parameters, the experiments had been carried out according to the multilevel full-factorial design with $2^4 = 16$ combinations of design variables. For each combination of input variables, welding was carried out three times in order to ensure repeatability.

These samples were sectioned, polished, etched, and observed under the microscope in order to obtain the desired measurements. The average values of the bead width (BW in mm) and depth of penetration (DP in mm) were calculated and used in this study. The details of the experimental data used for developing the model and testing the same are shown in Appendices 1 and 2, respectively.

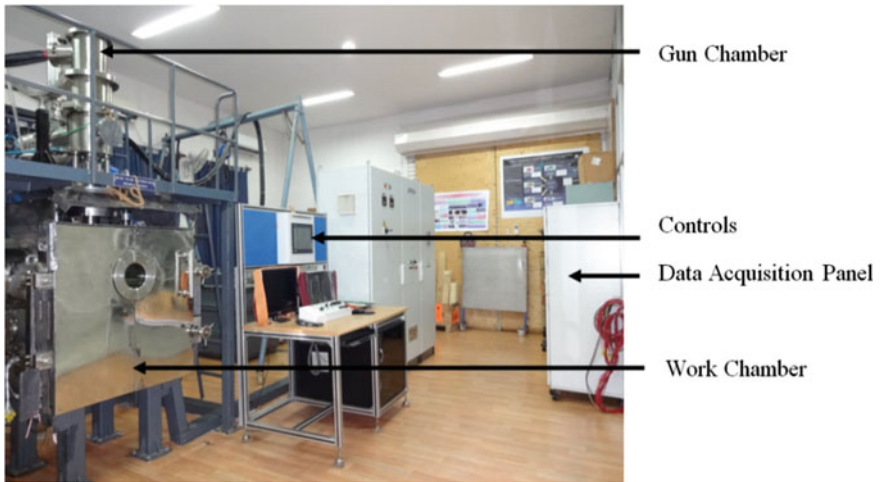


Fig. 3 Electron beam welding (EBW) setup, IIT Kharagpur, India

5 Results and Discussion

As like other natural processes, electron beam welding process has also nonlinear input–output relationships (Jha et al. 2014). These relationships were obtained from the experimental data using a nonlinear regression tool. After this, the developed approach was used to get the relationships among the responses and design variables, and other important information regarding the said process.

5.1 Obtaining Nonlinear Input–Output Relationships from the Experimental Data

Using the statistical software Minitab 16.0, a nonlinear regression analysis had been carried out to obtain the input–output relationships from the experimental data (refer to Appendix 1) collected within the upper and lower limits of the input variables, as provided in Table 1.

The following expression was obtained for depth of penetration:

Table 1 Input variables and their ranges

Sl. no	Input variables	Symbols	Minimum value	Maximum value
1	Beam power (W)	P	3200	5600
2	Welding speed (mm/min)	S	900	1800

$$DP = -2.31965 + 0.00443709 \times P - 0.00486308 \times S - 2.08008E - 07 \times P^2 + 2.67361E - 06 \times S^2 - 1.16208E - 06 \times P \times S. \quad (2)$$

The model was found to be capable of predicting accurate results because of a high value of regression coefficient of 0.97. A confidence level of 95% was considered. The other response, BW was also expressed in terms of the input parameters as follows:

$$BW = 18.0029 - 0.00554238 \times P + 0.000857167 \times S + 6.32813E - 07 \times P^2 - 5.69444E - 07 \times S^2 - 2.61667E - 07 \times P \times S. \quad (3)$$

The regression coefficient for this case was found to be 0.8.

5.2 Formulation of the Optimization Problem

The objective was to maximize the depth of penetration (DP), while keeping the bead width (BW) at a lower value. These said goals are contradictory to each other, as with the increase of DP, and output parameter BW increases. Therefore, this is an ideal problem for MOO and it can be expressed as follows:

$$\begin{aligned} & \text{Minimize} && 1/DP \\ & \text{Minimize} && BW \\ & \text{subject to} && \\ & && 3200 \leq P \leq 5600 \\ & && 900 \leq S \leq 1800 \end{aligned} \quad (4)$$

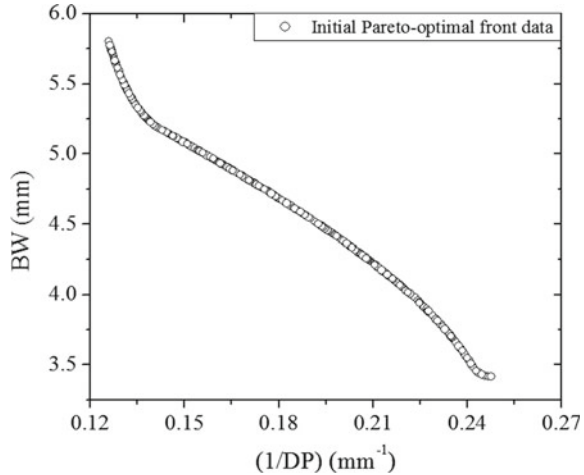
5.3 Obtaining Initial Pareto-Front

Using NSGA-II, an initial Pareto-front was obtained (refer to Fig. 4). In this case, Eqs. (2) and (3) were used to evaluate the numerical values for the outputs DP and BW, respectively. The user-defined parameters for the NSGA-II, such as probability of crossover (p_c), probability of mutation (p_m), population size (N), and maximum number of generations (G_{max}), were selected through a detailed parametric study. This study had been carried out by varying parameters one at a time and keeping the others fixed. The sequence for varying the parameters was taken as follows: p_c , p_m , N and G_{max} , as suggested in Pratihari (2007). The best parameters were chosen based on the maximum spread of the Pareto-front. The details of this parametric study have been provided in Table 2, where the selected parameters are written in bold.

Table 2 Results of the parametric study to select NSGA-II parameters

Exp no.	Fixed GA parameters' values	Varying GA parameter value	Spread of Pareto-optimal data set (X- and Y-coordinates represent 1/MRR and SR values, respectively)	Is the parameter selected?
1	$p_m = 0.06, N = 300$ and $G_{max} = 1000$	$p_c = 0.6$	(0.125994, 5.80199) to (0.247707, 3.41509)	No
2		$p_c = 0.7$	(0.125994, 5.80199) to (0.247708, 3.41509)	No
3		$p_c = 0.8$	(0.125994, 5.80199) to (0.247716, 3.41509)	Yes
4		$p_c = 0.9$	(0.125994, 5.80199) to (0.247704, 3.41509)	No
5		$p_c = 1.0$	(0.125994, 5.80199) to (0.247707, 3.41509)	No
6	$p_c = 0.8, N = 300$ and $G_{max} = 1000$	$p_m = 0.02$	(0.125994, 5.80199) to (0.247703, 3.41509)	No
7		$p_m = 0.04$	(0.125994, 5.80199) to (0.247715, 3.41509)	No
8		$p_m = 0.06$	(0.125994, 5.80199) to (0.247716, 3.41509)	Yes
9		$p_m = 0.08$	(0.125994, 5.80199) to (0.247711, 3.41509)	No
10		$p_m = 0.1$	(0.125994, 5.80199) to (0.247708, 3.41509)	No
11	$p_c = 0.8, p_m = 0.06$ and $G_{max} = 1000$	$N = 100$	(0.125994, 5.80199) to (0.247701, 3.41509)	No
12		$N = 200$	(0.125994, 5.80199) to (0.2477, 3.41509)	No
13		$N = 300$	(0.125994, 5.80199) to (0.247716, 3.41509)	Yes
14		$N = 400$	(0.125994, 5.80199) to (0.247709, 3.41509)	No
15		$N = 500$	(0.125994, 5.80199) to (0.247708, 3.41509)	No
16	$p_c = 0.8, p_m = 0.06$ and $N = 300$	$G_{max} = 1000$	(0.125994, 5.80199) to (0.247716, 3.41509)	Yes
17		$G_{max} = 2000$	(0.125994, 5.80199) to (0.247708, 3.41509)	No
18		$G_{max} = 3000$	(0.125994, 5.80199) to (0.247705, 3.41509)	No
19		$G_{max} = 4000$	(0.125994, 5.80199) to (0.247709, 3.41509)	No
20		$G_{max} = 5000$	(0.125994, 5.80199) to (0.247709, 3.41509)	No

Fig. 4 Obtained initial Pareto-front of solutions



In NSGA-II, different genetic operators, such as tournament selection, arithmetic crossover, and Gaussian mutation, were used. Now, the initial Pareto-front of optimal solutions was obtained utilizing the selected parameters, as shown in Fig. 4.

5.4 Training of an NFS

Using the initial Pareto-front dataset, a neuro-fuzzy system (NFS) had been trained. The training dataset had been clustered using a clustering technique, namely, fuzzy C-means clustering and in this case, the level of cluster fuzziness was considered as 2.0. To obtain the best results, the said data set was clustered into 16 different clusters. Therefore, the total number of rules for the NFS became equal to the number of clusters made. The structure of the developed NFS has been shown in Fig. 5.

In the used NFS, a supervised learning with a batch mode of training method had been applied. In the input and output layers, the Gaussian type of membership functions had been used. So, the total number of unknown parameters of this model was found to be equal to $(16 \times 4 \times 2 =) 128$ (as there are two inputs and two outputs each in the model, and each Gaussian function has two unknown parameters, that is, σ and m). For the training purpose, 300 input data points of the initial Pareto-front had been used and a root-mean-square error (RMSE) was calculated each time. Here, the error is nothing but the deviation in prediction. This NFS was evolved using a genetic algorithm, where the objective was set to minimize the RMSE value and the design variables were those 128 numbers of unknown parameters of the model. Different genetic operators, such as roulette wheel selection, linear crossover, and random mutation, had been utilized in the GA, and to obtain the best results, the selected GA parameters were as follows: crossover probability ($p_c = 0.9$), mutation probability ($p_m = 0.1$), and population size ($N = 60$).

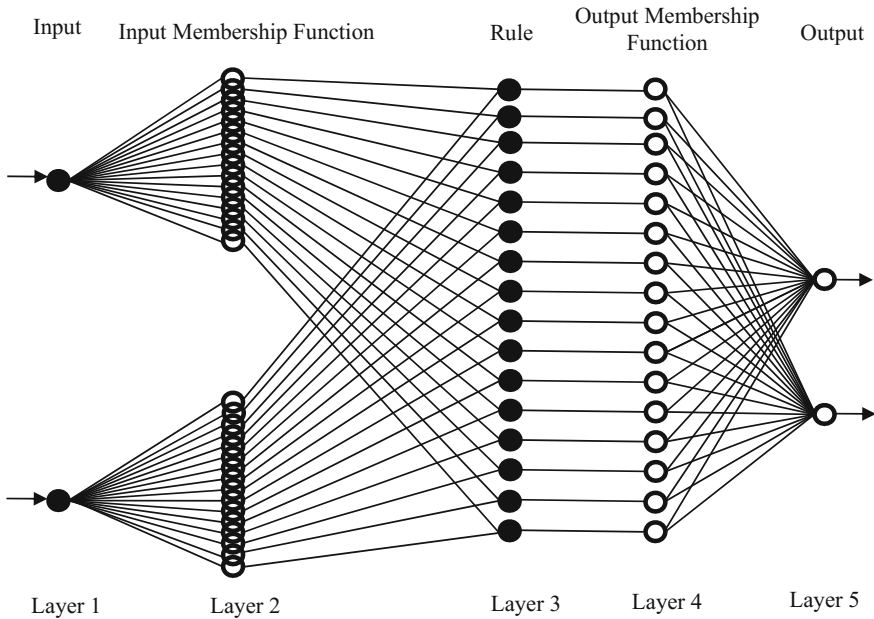


Fig. 5 Structure of the neuro-fuzzy system

5.5 Obtaining Modified Pareto-Front

In this step, the trained NFS had been used in an NSGA-II for determining the fitness values of the objectives. It is important to note that Eqs. (2) and (3) were used previously in the NSGA-II for obtaining the initial Pareto-front of solutions. The parameters for this NSGA-II were kept the same as in that of the previous case described in Sect. 5.3. We could get a modified Pareto-front, as shown in Fig. 6, and the quality of this Pareto-front had been improved in terms of the objective function values compared to that of the initial one.

5.6 Clustering of the Modified Pareto-Front Data Set

The obtained modified Pareto-optimal solutions were clustered using three different algorithms, namely, fuzzy C-means clustering (FCM), entropy-based fuzzy clustering (EFC), and density-based spatial clustering application with noise (DBSCAN). By using FCM algorithm with the level of cluster fuzziness kept equal to 1.25, two clusters were obtained, as shown in Fig. 7.

Fig. 6 Initial and modified Pareto-front of solutions

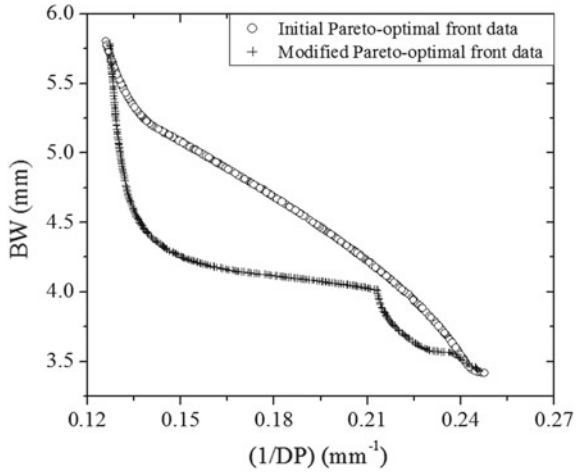
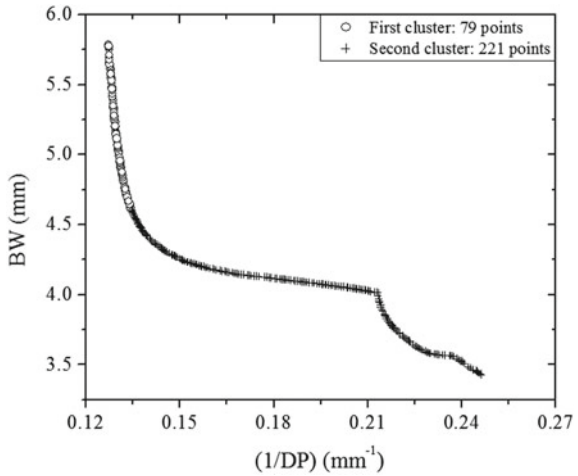


Fig. 7 Clustering of the modified Pareto-front using FCM



In case of EFC, two distinct clusters (refer to Fig. 8) were obtained with the user-defined parameters like the constant of similarity ($\alpha = 0.12$) and threshold value of similarity ($\beta = 0.9$).

In another case, where the clustering was done using DBSCAN algorithm, three distinct clusters were obtained, as shown in Fig. 9. In this algorithm, a point was considered to form a cluster, when a minimum of three other points were found to be present in a neighborhood radius of 0.032.

For the different clusters obtained using the said three clustering algorithms, the respective ranges of variation for the two outputs, such as DP and BW, are given in Table 3.

Fig. 8 Clustering of the modified Pareto-front using EFC

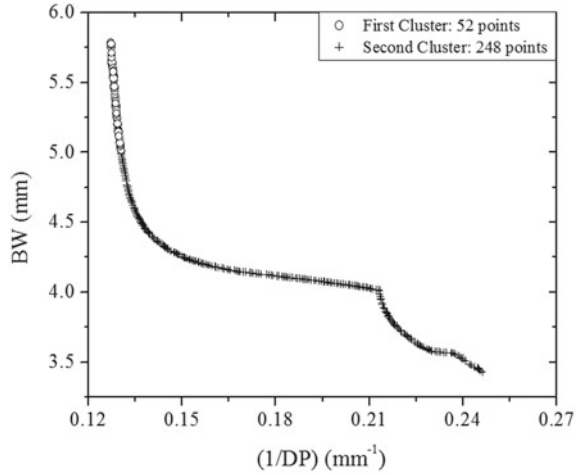
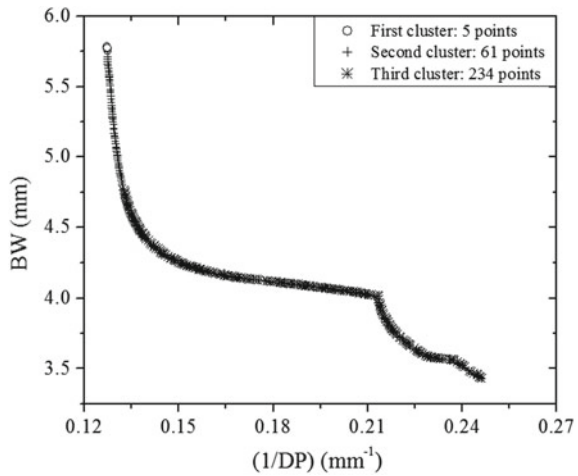


Fig. 9 Clustering of the modified Pareto-front using DBSCAN



The clustered solutions were analyzed using nonlinear regression analysis. For this purpose, a Gauss–Newton approach was used with 95% confidence level for all intervals and the convergence tolerance was assumed to be equal to 0.00001. In Table 4, cluster-wise obtained different input–output relationships are provided.

The two extreme points on the modified Pareto-front correspond to the maximum and minimum values of the two outputs of the EBW process. One of these points shows the highest values of DP and BW with the input parameters as follows: $P = 5600\text{ W}$, $S = 900\text{ mm/min}$. On the other hand, the other point provides the information regarding the lowest values of the outputs with an input variables setting as $P = 4750.98\text{ W}$, $S = 1800\text{ mm/min}$. It is observed (refer to Fig. 10) that for increasing the depth of penetration (DP), beam power (P) has to be increased and

Table 3 Ranges of DP and BW in different clusters

Cluster algorithm	Cluster number	Output parameters			
		DP (mm)		BW (mm)	
		Minimum	Maximum	Minimum	Maximum
FCM	1	7.4441	7.8581	4.6191	5.7872
	2	4.0632	7.4252	3.4281	4.6039
EFC	1	7.6593	7.8581	5.0077	5.7872
	2	4.0632	7.6535	3.4281	4.9897
DBSCAN	1	7.8502	7.8581	5.7519	5.7872
	2	7.5697	7.8472	4.7989	5.7124
	3	4.0632	7.5481	3.4281	4.7629

Table 4 Relationships among decision variables and responses for EBW problem

Clustering algorithm	Cluster number	Relationships among design variables and objectives
FCM	1	$DP = P^{0.443585} \times S^{-0.257345}$
	1	$BW = P^{1.13416} \times S^{-1.17341}$
	2	$DP = P^{0.790491} \times S^{-0.713536}$
	2	$BW = P^{0.40339} \times S^{-0.290404}$
EFC	1	$DP = P^{0.351876} \times S^{-0.14233}$
	1	$BW = P^{0.848204} \times S^{-0.814444}$
	2	$DP = P^{0.880063} \times S^{-0.818614}$
	2	$BW = P^{0.450339} \times S^{-0.345143}$
DBSCAN	1	$DP = P^{0.228065} \times S^{0.013738}$
	1	$BW = P^{0.228118} \times S^{-0.0315064}$
	2	$DP = P^{0.454036} \times S^{-0.26975}$
	2	$BW = P^{1.23061} \times S^{-1.29156}$
	3	$DP = P^{0.835263} \times S^{-0.765954}$
	3	$BW = P^{0.422925} \times S^{-0.313142}$

welding speed (S) should be at its lower value. In other situation, where a user requires a lower value of bead width (BW), the input parameter, P, has to be decreased and S is needed to be increased. Moreover, both DP and BW are found to be proportional to the heat input, which is a unified effect of beam power and welding speed on the weld geometries. This trend is in accordance with the literature (Das et al. 2017; Kar et al. 2015). Therefore, a user may be recommended to choose input parameters setting to avail the high depth of penetration (7.01 mm) and low bead width (4.36 mm) as follows: $P = 5548.8\text{ W}$, $S = 1096.46\text{ mm/min}$.

The obtained input–output relationships were used on some test data (refer to Appendix 2), and an average absolute percentage error (AAPE) was calculated for each of the cases of clustering techniques. These were compared to the results

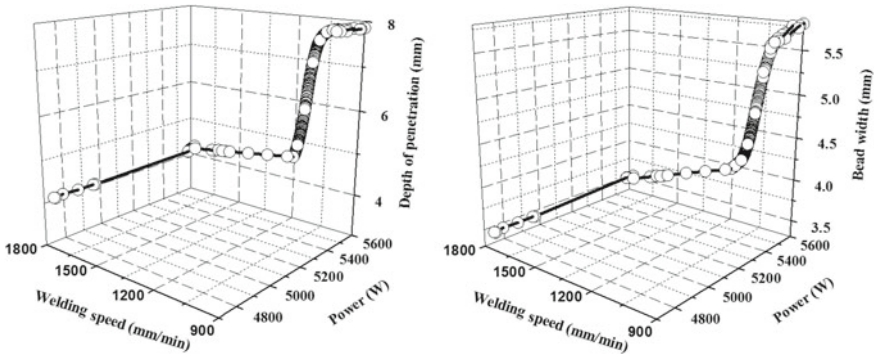


Fig. 10 Variations of DP and BW with the inputs, P and S, in the modified Pareto-front

Table 5 Comparison of results using the relationships obtained from developed approach and experimental data

Output	AAPE in case of FCM (%)	AAPE in case of EFC (%)	AAPE in case of DBSCAN (%)	AAPE in case of Eqs. from experimental data (%)
DP	2.9822	3.2329	2.9903	3.7288
BW	5.4764	5.4009	5.6698	6.3340

obtained using Eqs. (2) and (3). The comparison (refer to Table 5) clearly indicates the fact that the relationships derived by the developed approach could predict more accurately compared to the regression equations did. In addition, FCM algorithm could perform a slightly better compared to the other two algorithms of clustering.

Another interesting fact to observe here is that the range for the input variable power (P) had been squeezed from (3200, 5600 W) to (4750.98, 5600 W) in the modified Pareto-optimal dataset. This fact denotes that the effective range of the input parameter P for this process has been shortened and it is advisable to operate only in this squeezed range to get the best results. This information will surely help the designers to design and establish the process efficiently.

6 Conclusion

An approach was developed to obtain different input–output relationships of the EBW process by the extensive use of a multi-objective optimization technique and a neuro-fuzzy system. It is quite different from the approaches available, because it adopts a method to model the inherent fuzziness in the experimental data and at the end, it could generate more accurate input–output relationships for a process. The approach was applied for an EBW process, and the results obtained were superior

to that of the other methods in terms of precision and accuracy. This happens due to the fact that in the developed approach, an NFS, which is capable of handling uncertainty and inaccuracy of the data, is used and the imprecision in the data is removed to come up with the more accurate relationships.

The solutions of the modified Pareto-front were clustered and analyzed to obtain input–output relationships cluster-wise. A comparison has also been made among the relationships obtained in different cases of clustering algorithms based on the results of the test cases. Another interesting fact has been found out that for an input parameter, the effective range to get the best results has been squeezed. This prior information may increase the opportunities to reduce the operating cost and make the process more stable. Moreover, some physical aspects of the process are derived after analyzing the modified Pareto-optimal dataset, and the conclusions are seen to be inline with those made by other researchers for the process. Therefore, the developed approach can be applied to any process for obtaining input–output relationships and other important facts of the same.

Appendices

Appendix 1 collected experimental data

Sl. no.	Power (W)	Speed (mm/min)	Depth of penetration (mm)	Bead width (mm)
1	3200	1800	2.73	4.82
2	3200	1500	3.27	5.36
3	4000	1800	3.43	4.46
4	3200	1200	4	3.4
5	4000	1500	4.13	4.4
6	4800	1800	3.41	5.54
7	5600	1800	4.55	3.4
8	4800	1500	4.5	3.5
9	4000	1200	4.6	4.7
10	3200	900	3.9	6.92
11	5600	1500	4.8	5.1
12	4800	1200	5.29	5.31
13	4000	900	5.69	4.99
14	5600	1200	5.8	5.5
15	4800	900	7.15	5
16	5600	900	8.2	5.6

Appendix 2 Experimental data collected for testing the performance of the developed approach

Sl. no.	Power (W)	Speed (mm/min)	Depth of penetration (mm)	Bead width (mm)
1	4800	1650	4.37	3.38
2	4800	1325	5.03	3.87
3	5200	1650	4.39	3.86
4	5200	1325	5.27	4.51
5	5600	1000	7.91	5.28

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