Indian Astronomy and Mathematics in the Eleventh-Century Spain

According to Ibn al-Ādamī (c. 950 AD) as quoted by Qāḍī Ṣā'id al Andalūsī (d. 1070), Caliph al-Manṣūr of Baghdad (755–775 AD) ordered a Sanskrit astronomical work to be translated into Arabic.[1] This translation was made by al-Fazārī with the help of the Indian astronomer who had brought the said Sanskrit astronomical (i.e. *Siddhānta*) work to the Abbasid Court (in 771 or 773), and the Arabic version was called *Zij al-Sindhind* from which descended a long tradition within Islamic astronomy extending upto Spain for several centuries.[2]

Al-Fazārī also composed (c. 780) the *ZIj al-Sindhind al-Kabīr* (The Great Sindhind) which was based on the *Zij al-Sindhind*, and in which three Indian values, namely 3270, 3438 and 150, were used for the *Sinus Totus (i.e. trijyā)* or radius. A similar Arabic work called *Zīj maḥlūl fī al-Sindhind li daraja daraja* ("Astronomical Tables in the Sindhind Resolved for every Degree") was composed by Ya'qūb ibn Ṭāriq who had collaborated personally with the Indian astronomer who went to Baghdad in 771 or 773 (as mentioned above).[3]

Al-Khwārizmī who flourished under the region of Caliph al-Ma'mūn (813–833), made extensive use of the *Zīj al-Sindhind* and its derivative works in composing his *Zīj* (Astronomical Tables) which became famous through out the Islamic world upto Spain and in Europe through subsequent Latin translations. It is said for his astronomical work, al-Khwārizmī was fār more heavily indebted to Indian work than to other sources.[4] His *Astronomical Tables* were redacted by Masalama al-Majrīṭī who flourished in Spain and died there about 1007 (or later).[5] This was one of the channels through which Indian astronomy and mathematics penetrated Spain and the influence of Indian astronomy represented by the tradition of *Sindhind* continued there even after Ptolemy's *Almagest* (on Greek astronomy) came to be known.[6]

In the *Astronomical Tables* of al-Khwārizmī, the corrections for the planets and the reckoning of time are made with reference to the central place of the earth,

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called Arin, i.e. Ujjain which is the zero meridian of Hindu astronomy.[7] However, it was but natural for al-Majrīṭī to work out his adaptation of these Tables for the longitude of Córdoba (Spain) where he flourished.[8] Thus the crescent visibility table was computed on the basis of Indian visibility theory for the latitude of northern Spain.[9]

Al-Majrīṭī had several disciples who made his work known throughout the peninsula and through them exercised considerable influence on the work of later scientists.[10] For instance, one of the disciples was Ibn al-Samḥ (d. 1035) of Granada (Spain) who followed Indian astronomical techniques and about 1010 AD wrote a *zīj* (not extant) which is reported to be based on the methods of the famous *Sindhind*.[11]

Such was the impact of Indian scientific achievements in Spain that Qaḍī Sā'id (d. Toledo, Spain, 1070) included Indians among the nations which have cultivated the sciences in his *Ṭabaqāt al-Umam* ("Category of Nations") which he wrote there in 1062. He says:[12]

> The first nation (that has cultivated the sciences) is the people of India who form a nation vast in numbers, powerful, within great dominations. All former kings and past generations have acknowledged their wisdom and admitted their pre-eminence in various branches of learning....
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> The king of India was called King of Wisdom because of the concerns of the Indians for the sciences and their distinction in all branches of knowledge....
>
> Among all nations, during the course of centuries and throughout the passage of time, India was known as the mine of wisdom,... and the Indians were credited with excellent intellects, exalted ideas, universal maxims, rare inventions and wonderful inventions.
>
> .... they (the Indians) have studied arithmetic and geometry. They have also acquired copious and abundant knowledge of the movements of the stars, the secrets of the celestial sphere and all other kinds of mathematical sciences.

About the systems of astronomy followed in India Qāḍī Sā'id says:[13]

> Among the Indian systems of astronomy, there are three famous schools, i.e. those of Sindhind, Arjabhar and Arkand. Exact information has reached us only about the school of Sindhind (*Siddhānta*) which has been adopted by a group of Muslim scholars who have used it for the compilation of astronomical tables ....
>
> The followers of Sindhind state that the apogee *(awjāt)* and nodes *(Jawzahrāt)* of seven planets are all assembled at the head of Aries once in 4320,000,000 solar years, and they name this period as 'world-period' *[*cf. kalpa*]*. . . . As for the followers of Arjabhar [Āryabhaṭa], they are in agreement with the followers of Sindhind except with regard to the length of the 'world-period' [which is taken to be 4320,000 solar years in this system]. . . . As regards the followers of Arkand school, they differ from the former schools in respect of movements of planets and the 'world-period' but exact nature of this difference is not known to us.

Al-Zarqālī or Al-Zarqāll (Azarchiel of the Latins), the most celebrated astronomer of his time, lived in Toledo and Córdova (both in Spain) where he died in 1100 AD His name is associated with the famous *Toledo Tables* which enjoyed an enormous circulation. They were extraordinarily successful in the Latin world, and by the twelfth century they were used throughout Europe.[14]

The *Toledo Tables* have lot of material on Indian astronomy. It has a set of tables, of Indian origin, for computing oblique ascensions in terms of right ascensions.[15] Its table of Sines (with $R = 150'$) and the table of Solar declinations (with $e = 24°$, the Hindu value of the obliquity of the ecliptic) are identical with the corresponding tables in the Sanskrit *Khaṇḍa-khādyaka* (665 AD) of Brahmagupta.[16]

Like Indian, al-Zarqāli assumed circumference equal to 360° and diameter equal to $300'$, and defined *kardaga* as arc of 15° (cf. *gṛhārdha* or half sign which is used as a tabular interval by Brahmagupta).[17] He defined Sinus Rectus and Sinus Versus like the Indian *Kramajyā* and *Utkramajyā,* and gave the usual Indian methods for computing the tabular Sines.[18] Using the standard Hindu gnomon of 12 units and the Indian radius of 150 minutes, al-Zarqālī found the shadow of the sun from its altitude and vice versa. For π, he gave the approximation $\frac{22}{7}$, $\sqrt{10}$ and $\frac{62832}{20000}$, the last two of which are obtained from Indian sources.[19] The value $\sqrt{10}$ is found in old Jaina Canonical works, and $\frac{62832}{20000}$ is exactly the approximation given by Āryabhaṭa I (born 476 AD).[20]

In 1089, al-Zarqālī elaborated the *Almanac* of Ammonius in which the trigonometrical portion presents the mingling of the Indian material with that from other sources.[21]

Regarding the knowledge of Indian arithmetic in the eleventh-century Spain, we again quote the words of Qāḍī Sa'id who says:[22]

> In the domain of numerical sciences, we have their (i.e. of Indians) *hisāb al-ghubār* which was explained by *al-Khwārizmī*. It is a very compendious and quick system of calculation, easy to understand, simple to adopt, and remarkable in its composition, bearing testimony to the sharp intelligence, creative power and remarkable faculty of invention of the Indians.

Unfortunately, like his *Zīj*, the Arabic original of al-Khwārizmī's work on Indian arithmetic is lost, one of its suggested titles is *Kitāb Ḥisāb al-'Adad al-Hindi* ("Treatise on Calculation with Hindu Numerals").[23] However, its Latin version entitled *Algoritmi De Numero Indorum* is well known, and this played a very important role in introducing the Indian decimal place-value system of numerals and the corresponding computational methods in Europe.[24]

Similarly, the first serious Latin work on astronomy was a translation (via Arabic) of a redaction of *Sindhind* which was a translation of an Indian astronomical work in Sanskrit.[25]

We have already mentioned al-Majrīṭī's redaction (made in Spain) of al-Khwārizmī's *Zīj* which had strong influence of Sindhind.[26] Ibn al-Muthannā wrote a commentary (lost) on the original *Zīj* of al-Khwārizmī, but a Hebrew translation (of this lost commentary) by Ibn Ezra (born in Toledo, ca. 1090 and died at Calahorra, Spain, c. 1165) is extant. In some of the matters, e.g. table of ascensional difference which is missing in al-Majrīṭī's version, Ibn al-Muthannā's version shows further Indian influence by the use of Indian Sinus Totus of 150' and an interval of a *kardaga* of 15°.[27]

Ibn Ezra himself composed *Sefer ha-Mispar* ("Book of the Number") which describes the decimal system of numerals with zero showing a deep influence, although he often used the letters of the Hebrew alphabet as numeral-signs.[28] Just a few years before and after Ibn Ezra put the material, containing Indian techniques and parameters, into Hebrew, a host of other scholars, like Adelared of Bath (fl. 1116–1142), Plato of Tivoli (fl. 1132–1146) and Gerard of Cremona (d. Toledo, 1187), translated Graeco-Arabic and Indo-Arabic scientific literature into Latin. There were several centres where this translation work was done but Spain had major share in the activity. And it was through these Latin translations that astronomy and mathematics flowed wider into Europe causing a step towards renaissance.

References and Notes

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Ibn Ezra also knew Āryabhaṭa I's value of π (attributing it to "Indian sages") although there is some mistake or scribal error in quoting the value (see *Historia Mathematica,* I, 1974, 25).