

A Stochastic Feedback Queuing Model with Encouraged Arrivals and Retention of Impatient Customers



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Abstract Globalization has introduced never ending competition in business. This competition empowers customers and somewhat ensures quality at reduced cost. High competition along with uncertain customer behavior complicates the situation further for organizations. In order to stay ahead in the competition, organizations introduce various discounts and offers to attract customers. These discounts and offers encourage customers to visit the particular firm (online or offline). Encouraged arrivals result in heavy rush at times. Due to this, customers have to wait longer in queues before they can be serviced. Long waiting times results in customer impatience and a customer may decide to abandon the facility without completion of service, known as reneging. Reneging results in loss of goodwill and revenue both. Further, heavy rush and critical occupation of service counters may lead to unsatisfactory service and some customers may remain unsatisfied with the service. These customers (known as feedback customers) may rejoin the facility rather than leaving satisfactorily. Unsatisfactory service in these situations may cause harm to the brand image and business of the firm. If the performance of the system undergoing such pattern can be measured in advance with some probability, an effective management policy can be designed and implemented. A concrete platform for measuring performance of the system can be produced by developing a stochastic mathematical model. Hence, in this paper, a stochastic model addressing all practically valid and contemporary challenges mentioned above is developed by classical queuing theory model development approach. The model is solved for steady-state solution iteratively. Economic analysis of the model is also performed by introduction of cost model. The necessary measures of performance are derived, and numerical illustrations are presented. MATLAB is used for analysis as and when needed.

Keywords Stochastic models · Queuing theory · Globalization · Governance

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© Springer Nature Singapore Pte Ltd. 2019
A. K. Laha (ed.), *Advances in Analytics and Applications*, Springer Proceedings
in Business and Economics, https://doi.org/10.1007/978-981-13-1208-3_20

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1 Introduction

Contemporary business environment is highly challenging due to factors like uncertainty, globalization, and competition. Engaging new customers and retaining existing customers require a full proof strategy. The margin of error in given times is so low that a slight compromise with the strategy may result in loss to the business.

In order to engage new customers, firms often release offers and discounts. These discounts can be observed very frequently. Let it be online stores or offline stores, every firm comes up with discounts and offers every now and then. These offers and discounts attract customers to visit the stores or online web portals. These mobilized customers are termed as *encouraged arrivals* in this paper. At times customers also get attracted toward a firm once they see that lot of people are engaged in service with the particular firm already. Since a high volume of customers ensures a better quality, affordable price, or service, therefore, an arriving customer may get attracted toward a firm where there are customers already exist in large volumes. For example, in a healthcare facility, a large patient base ensures better doctors, better facilities, better treatment at affordable cost, or better service. This phenomenon was termed as *reverse balking* by Jain et al. (2014) which is contrary to classical queuing phenomenon named *balking* mentioned by Ancker and Gaffarian (1963a, b). Encouraged arrivals are different from reverse balking in the sense that reverse balking deals with probability calculated from existing system size and system capacity, while encouraged arrivals are directly related to the system size at given time or percentage increase in customers' arrival due to offers and discounts. The phenomenon of encouraged arrivals is contrary to discouraged arrivals discussed by Kumar and Sharma (2012).

Once the customers are encouraged, they result in higher volumes to the system. High volumes result in longer queues. These queues can be physical or digital. Service facility experiences increased load and smooth functioning becomes a tedious task. A dissatisfactory service at this point may result in customer impatience and a customer may decide to abandon the facility. This phenomenon is termed as customer *reneging* and discussed first by Haight (1957). Reneging is a loss to business, goodwill of the company, and revenue. A number of papers emerged on impatient customers since the inception of concept. Bareer (1957) studied phenomenon of balking and reneging in many ways. Natvig (1974) derived transient probabilities of queueing system with arrivals discouraged by queue length. Dooran (1981) further discussed the notion of discouraged arrivals in his work. Xiong and Altiook (2009) discussed impatience of customers. Kumar and Sharma (2013, 2014) studied queueing systems with retention and discouraged arrivals. Kumar and Som (2014) study single-server stochastic queueing model with reverse balking and reverse reneging. They mentioned that customer impatience will also decrease with increase in volume of customers. Kumar and Som (2015a, b) further present a queueing model with reverse balking, reneging, and retention. Kumar and Som (2015a, b) further added feedback customers to their previous model. Recently, Som (2016) presented a queueing system with reverse balking, reneging, retention of impatient customers, and feedback with

heterogeneous service. He discussed the case of healthcare facility going through mentioned contemporary challenges.

There is a need to have a strategy that can help in smooth functioning of the system for better result. Strategies driven by scientific methods in such cases result in better output. Owing to this practically valid aspect of business, we develop a single-server feedback stochastic queuing model addressing contemporary challenges in this paper. Cost-profit analysis of the model is discussed later.

2 Formulation of Stochastic Model

A Markovian single-server queuing model is formulated under the following assumptions:

- (i) The customers are arriving at the facility in accordance with Poisson process one by one with a mean arrival rate of $\lambda + n$, where “n” represents the number of customers exists in the system.
- (ii) Customers are provided service exponentially with parameter μ .
- (iii) An FCFS discipline is followed.
- (iv) Service is provided through single server and the capacity of system is finite (say, N).
- (v) The reneging times are independently and exponentially distributed with parameter ξ .
- (vi) Unsatisfied serviced customer may retire to the system with probability q and may decide to abandon the facility $p = (1 - q)$.
- (vii) A reneging customer may be retained with probability $q_1 = (1 - p_1)$.

In order to construct a stochastic model based on assumptions (i)–(viii), the following differential-difference equations are derived. These equations govern exhaustive stages the system can experience. Equation (1) governs initial stage of the system, Eq. (2) governs the functioning stage, and Eq. (3) governs the stage when system is full. The three differential-difference equations governing the system are given by

$$P'_0(t) = -\lambda P_0(t) + \mu p P_1(t) \tag{1}$$

$$P'_n(t) = \{\lambda + (n - 1)\}P_{n-1}(t) + \{(-\lambda + n) - \mu p - (n - 1)\xi p_1\}P_n(t) + (\mu p + n\xi p_1)P_{n+1}(t) \tag{2}$$

$$P'_N(t) = \{\lambda + (N - 1)\}P_{N-1}(t) - \{\mu p + (N - 1)\xi p_1\}P_N(t) \tag{3}$$

3 Steady-State Equations

In steady-state equation, the system of equations becomes

$$0 = -\lambda P_0 + \mu p P_1 \tag{4}$$

$$0 = \{\lambda + (n - 1)\}P_{n-1} + \{(-\lambda + n) - \mu p - (n - 1)\xi p_1\}P_n + (\mu p + n\xi p_1)P_{n+1} \tag{5}$$

$$0 = \{\lambda + (N - 1)\}P_{N-1} - \{\mu p + (N - 1)\xi p_1\}P_N \quad (6)$$

4 Steady-State Solution

On solving above equations iteratively, we get

$$P_n = \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i\xi p_1} P_0$$

As normality condition explains $\sum_{n=0}^N P_n = 1$

$$P_0 = \left\{ 1 + \sum_{n=1}^N \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i\xi p_1} \right\}^{-1}$$

And the probability that system is full is given by

$$P_N = \prod_{i=0}^{N-1} \frac{\lambda + i}{\mu p + i\xi p_1} P_0$$

5 Measures of Performance

5.1 Expected System Size (L_s)

$$L_s = \sum_{n=0}^N n P_n$$

$$L_s = \sum_{n=0}^N n \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i\xi p_1} P_0 \right\}$$

5.2 Expected Queue Length (L_q)

$$L_q = \sum_{n=0}^N (n-1)P_n$$

$$L_q = \sum_{n=0}^N (n-1) \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i \xi p_1} P_0 \right\}$$

5.3 Average Rate of Reneging Is Given by (R_r)

$$R_r = \sum_{n=1}^N (n-1) \xi p_1 P_n$$

$$R_r = \sum_{n=1}^N (n-1) \xi \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i \xi p_1} P_0 \right\}$$

5.4 Average Rate of Retention Is Given by (R_R)

$$R_R = \sum_{n=1}^N (n-1) \xi q_1 P_n$$

$$R_R = \sum_{n=1}^N (n-1) \xi \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i \xi q_1} P_0 \right\}$$

6 Numerical Illustration

Numerical validity of the model is tested by performing numerical analysis. The analysis is performed by varying parameters arbitrarily. Sensitivity of appropriate measures of performance is observed with respect to varying arbitrary values of parameters (Table 1).

We can observe that an increasing average rate of arrival leaves a positive impact on average system size; as a result queue length also increases, while increase in rate of renegeing means that high volume of customers put service under pressure and lot of customers fail to wait after a threshold value of time and renege thereafter, i.e.,

Table 1 Variation in L_s , L_q , and R_r and R_R with respect to λ . We take $N = 10, \mu = 3, \xi = 0.2, q = 0.2, q_1 = 0.3$

(λ)	(L_s)	(L_q)	(R_r)	(R_R)
2	9.511	8.5118	1.1917	0.5107
2.5	9.553	8.5528	1.1974	0.5132
3	9.585	8.5854	1.2020	0.5151
3.5	9.613	8.6127	1.2058	0.5168
4	9.636	8.6361	1.2090	0.5182
4.5	9.656	8.6564	1.2119	0.5194
5	9.674	8.6744	1.2144	0.5205
5.5	9.690	8.6904	1.2167	0.5214
6	9.705	8.7049	1.2187	0.5223
6.5	9.718	8.7179	1.2205	0.5231
7	9.730	8.7298	1.2222	0.5238
7.5	9.741	8.7407	1.2237	0.5244
8	9.751	8.7507	1.2251	0.5250
8.5	9.760	8.7599	1.2264	0.5256
9	9.768	8.7685	1.2276	0.5261
9.5	9.776	8.7764	1.2287	0.5266
10	9.784	8.7838	1.2297	0.5270

Source simulated data

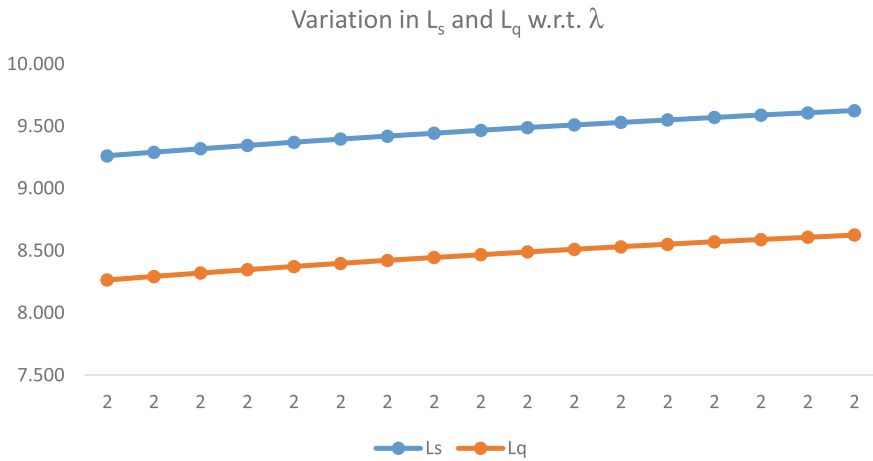


Fig. 1 Variation in system size and length of queue with respect to arrival rate

abandon the facility without completion of service. The following graphs explain the phenomenon (Fig. 1).

Similarly, the numerical results are obtained by varying service rate and rate of reneing. The following tables explain the same (Table 2).

Table 2 Variation in L_s, L_q, R_R and R_r with respect to μ . We take $N = 10, \lambda = 2, \xi = 0.2, q = 0.2, q_1 = 0.3$

(μ)	(L_s)	(L_q)	(R_r)	(R_R)
3	9.511	8.5118	1.1917	0.5107
3.1	9.492	8.4925	1.1890	0.5096
3.2	9.472	8.4722	1.1861	0.5083
3.3	9.450	8.4509	1.1831	0.5071
3.4	9.427	8.4283	1.1800	0.5057
3.5	9.403	8.4045	1.1766	0.5043
3.6	9.378	8.3792	1.1731	0.5028
3.7	9.351	8.3523	1.1693	0.5011
3.8	9.322	8.3237	1.1653	0.4994
3.9	9.291	8.2932	1.1611	0.4976
4	9.257	8.2606	1.1565	0.4956
4.1	9.222	8.2257	1.1516	0.4935
4.2	9.184	8.1882	1.1464	0.4913
4.3	9.142	8.1480	1.1407	0.4889
4.4	9.098	8.1048	1.1347	0.4863
4.5	9.050	8.0583	1.1282	0.4835

Source simulated data

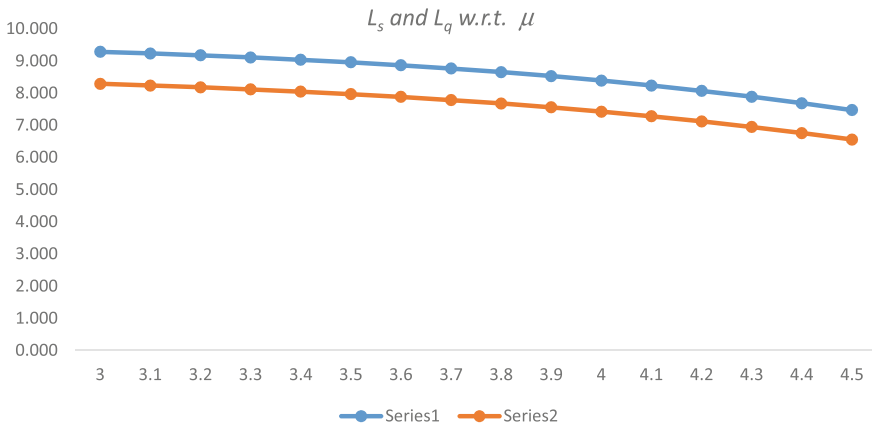


Fig. 2 System size and queue length with respect to service rate

We can observe as service rate increases, the expected system size decreases and so does expected length of queue, while decrease in rate of reneing, it means that decreasing volume of customers eases out the pressure on service and high number customers choose to wait rather than leaving the facility. The following graphs explain the phenomenon (Fig. 2).

From Table 3, it can be observed that feedback results in increasing expected system size and queue length. The rate of reneing goes high as more and more

Table 3 Variation in L_s, L_q, R_R and R_r with respect to q . We take $N = 10, \lambda = 2, \xi = 0.1, \mu = 3, q_1 = 0.3$

(q)	(L_s)	(L_q)	(R_r)	(R_R)
0	9.148	8.1539	1.1415	0.4892
0.05	9.232	8.2359	1.1530	0.4942
0.1	9.304	8.3065	1.1629	0.4984
0.15	9.366	8.3679	1.1715	0.5021
0.2	9.421	8.4217	1.1790	0.5053
0.25	9.469	8.4693	1.1857	0.5082
0.3	9.511	8.5118	1.1917	0.5107
0.35	9.550	8.5503	1.1970	0.5130
0.4	9.585	8.5855	1.2020	0.5151
0.45	9.618	8.6178	1.2065	0.5171
0.5	9.648	8.6478	1.2107	0.5189
0.55	9.676	8.6759	1.2146	0.5206
0.6	9.702	8.7023	1.2183	0.5221
0.65	9.727	8.7272	1.2218	0.5236
0.7	9.751	8.7508	1.2251	0.5250
0.75	9.773	8.7732	1.2283	0.5264

Source simulated data

customers in the system cause high level of delays in service and cause high level of impatience.

From Table 4, it can be observed that retention leaves a positive impact on system size. Though the queue length increases, retaining impatient customers is an important phenomenon that leads to increased revenue at the end.

7 Economic Analysis of the System

This section discusses the cost–profit analysis of the model. An algorithm is written for newly formulated functions of total expected revenue, total expected cost, and total expected profit for obtaining numerical outputs.

Total expected cost (TEC) of the model is given by

$$TEC = C_s\mu + C_hL_s + C_rR_r + C_L\lambda P_N + C_fqL_S + C_RR_R$$

$$TEC = C_s\mu + C_h \sum_{n=0}^N n \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i\xi p_1} P_0 \right\} + C_r \sum_{n=1}^N (n - 1)\xi p_1 \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i\xi p_1} P_0 \right\}$$

Table 4 Variation in L_s, L_q, R_R and R_r with respect to q_1 . We take $N = 10, \lambda = 2, \xi = 0.1, \mu = 3, q = 0.2$

Probability of retention (q_1)	Expected system size (L_s)	Expected queue length (L_q)	Average rate of reneing (R_r)	Average rate of retention (R_R)
0	9.263	8.2645	1.6529	0.0000
0.05	9.291	8.2929	1.5756	0.0829
0.1	9.319	8.3203	1.4977	0.1664
0.15	9.345	8.3469	1.4190	0.2504
0.2	9.371	8.3726	1.3396	0.3349
0.25	9.396	8.3975	1.2596	0.4199
0.3	9.421	8.4217	1.1790	0.5053
0.35	9.444	8.4450	1.0979	0.5912
0.4	9.467	8.4677	1.0161	0.6774
0.45	9.489	8.4896	0.9339	0.7641
0.5	9.510	8.5109	0.8511	0.8511
0.55	9.531	8.5315	0.7678	0.9385
0.6	9.551	8.5515	0.6841	1.0262
0.65	9.570	8.5709	0.6000	1.1142
0.7	9.589	8.5897	0.5154	1.2026
0.75	9.608	8.6079	0.4304	1.2912

Source simulated data

$$\begin{aligned}
 &+ C_L \lambda \prod_{i=0}^{N-1} \frac{\lambda + i}{\mu p + i \xi p_1} P_0 + C_f \times q \sum_{n=0}^N n \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i \xi p_1} P_0 \right\} \\
 &+ C_R \sum_{n=1}^N (n-1) \xi q_1 \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i \xi p_1} P_0 \right\}
 \end{aligned}$$

Total expected revenue (TER) of the model is given by

$$TER = R \times \mu \times (1 - P_0) + R_f \times q \times L_s$$

Total expected profit (TEP) of the model is given by

$$\begin{aligned}
 TEP = &R \times \mu \times (1 - P_0) + R_f \times q \times L_s - C_s \mu + C_h \sum_{n=0}^N n \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i \xi p_1} P_0 \right\} \\
 &+ C_r \sum_{n=1}^N (n-1) \xi p_1 \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i \xi p_1} P_0 \right\} + C_L \lambda \prod_{i=0}^{N-1} \frac{\lambda + i}{\mu p + i \xi p_1} P_0 + C_f \\
 &\times q \sum_{n=0}^N n \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i \xi p_1} P_0 \right\} + C_R \sum_{n=1}^N (n-1) \xi q_1 \left\{ \prod_{i=0}^{n-1} \frac{\lambda + i}{\mu p + i \xi p_1} P_0 \right\}
 \end{aligned}$$

Table 5 Variation in *TEC*, *TER* and *TEP* with respect to μ . We take $N = 10, \lambda = 2, \xi = 0.1, C_s = 10, C_l = 15, C_h = 2, C_f = 50, C_r = 2, R = 500$

(μ)	(TEC)	(TER)	(TEP)
3	83.995	1589.6643	1505.6697
3.1	84.883	1638.4003	1553.5177
3.2	85.760	1686.8742	1601.1145
3.3	86.624	1735.0321	1648.4078
3.4	87.475	1782.8114	1695.3363
3.5	88.310	1830.1395	1741.8294
3.6	89.128	1876.9333	1787.8058
3.7	89.925	1923.0979	1833.1726
3.8	90.701	1968.5263	1877.8251
3.9	91.453	2013.0986	1921.6456
4	92.178	2056.6818	1964.5036
4.1	92.874	2099.1296	2006.2553
4.2	93.539	2140.2829	2046.7443
4.3	94.169	2179.9708	2085.8021
4.4	94.762	2218.0114	2123.2494
4.5	95.316	2254.2144	2158.8984

Source simulated data

where

- C_s Cost of service,
- C_h Holding cost of a customer,
- C_r Cost of reneging,
- C_L Cost of a lost customer,
- C_f Cost of feedback,
- R Revenue earned from each customer,
- R_f Revenue earned from each feedback customer, and
- C_R Cost of retention of a customer.

The cost model formulated above is translated into MATLAB, and numerical results are obtained for varying rate of arrival and service (Table 5).

Though the cost increases with increase in the rate of service, as service cost increases but due to reduced rate of reneging, the revenue goes high and firms profit keeps on increasing with an improving rate of service (Table 6).

The table shows an increase in TEP with increase in average arrival rate which is obvious as increasing rate of arrival results in higher number of customers to the system and firm enjoys more revenue from each customer.

Figure 3 shows total expected profit is higher with improving service and fixed arrival (—). In comparison to the case when arrivals increase, the customers are serviced at a constant rate. This shows an improving service that ensures better profits and a firm shall focus more on providing better service rather than bringing more customers to the system.

Table 6 Variation in *TEC*, *TER* and *TEP* with respect to μ . We take $N = 10, \mu = 3, \xi = 0.1, C_S = 10, C_l = 15, C_h = 2, C_r = 2, R = 500$

(λ)	(TEC)	(TER)	(TEP)
2	83.995	1589.6643	1505.6697
2.5	91.747	1592.2844	1500.5369
3	99.422	1593.5561	1494.1341
3.5	107.054	1594.3050	1487.2510
4	114.660	1594.8157	1480.1558
4.5	122.248	1595.2010	1472.9531
5	129.823	1595.5115	1465.6884
5.5	137.389	1595.7726	1458.3841
6	144.946	1595.9983	1451.0521
6.5	152.498	1596.1970	1443.6994
7	160.044	1596.3744	1436.3306
7.5	167.586	1596.5342	1428.9486
8	175.124	1596.6793	1421.5556
8.5	182.659	1596.8119	1414.1533
9	190.191	1596.9337	1406.7430
9.5	197.720	1597.0461	1399.3258

Source simulated data

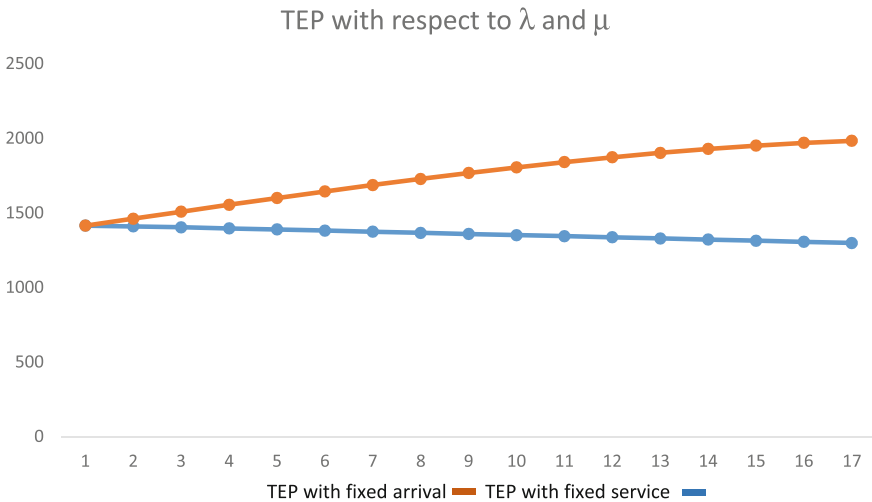


Fig. 3 TEP with respect to fixed and improving service rate

8 Conclusion and Future Scope

The results of the paper are of immense value for any firm encountering the phenomenon of encouraged customers and load on service. By knowing the measures of performance in advance, the overall performance of the system can be measured

and an effective strategy can be planned for smooth functioning. By adopting and implementing this model, the economic analysis of the facility can also be measured and bird's eye view on financial aspect of the business can also be observed.

Further optimization of service rate and system size can be achieved while the system can be studied in transient state. The system can also be studied for heterogeneous service. A multi-server model can also be explored.

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