Chapter 9
Unpacking Implicit Disagreements Among Early Childhood Standards for Statistics and Probability

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Abstract Numerous recent curriculum documents around the world recommend that children begin to develop understanding of probability and statistics during early childhood and primary school. Although there is widespread agreement that such learning should occur, standards documents are not uniform in their specific recommendations. In particular, there are implicit disagreements about the roles of student-posed statistical questions, probability language, and variability in children's learning. Unpacking these implicit disagreements is in the interest of teachers, researchers, and curriculum developers because it can stimulate thought and debate about the proper emphasis for the concepts in standards documents. This chapter will help define the space for such thought and debate by summarizing how some key concepts are addressed differently in various early learning standards for probability and statistics. Defensible interpretations of the research literature are considered. Strategies teachers and curriculum developers can use to cope with situations in which standards documents conflict with desirable learning goals for children are also described. Boundary objects, which allow related communities of practice to operate jointly in absence of consensus, are discussed as a means for advancing teaching and research despite the existence of disagreement. Suggestions for working toward a greater degree of consensus across early childhood standards for statistics and probability are also offered.

9.1 Introduction

Statistics is relatively new to early childhood and primary curricula. The first recommendations to include statistics in school mathematics appeared in the first half of the twentieth century and focused on secondary school; it was not until the second half of the twentieth century that statistics appeared in curriculum recommendations for early childhood and primary level students as well (Jones & Tarr, 2010). The

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standards documents of the National Council of Teachers of Mathematics (NCTM, 1989, 2000) were groundbreaking in recommending specific statistical concepts for study during the early years of school. At the outset of the twenty-first century, it is now unusual for school mathematics standards documents *not* to include statistics in the course of study for the youngest students.

With the proliferation of standards documents that include statistics, implicit disagreements have arisen in the concepts recommended for study and their sequencing. These disagreements can be seen as one compares standards from various nations (Jones, Langrall, & Mooney, 2007) or even those within a single nation such as the USA (Dingman, Teuscher, Newton, & Kasmer, 2013). Such disagreements can be counterproductive because they may make research and development efforts in statistics curriculum and instruction more difficult. Research and curricula from one setting may have limited use in another if the learning standards governing each setting differ. Hence, disagreements among standards documents have some negative aspects.

Although disagreements are often seen in a negative light, they do not have to be. Discourse devoted to respectfully expressing and unpacking disagreements can lead to deeper examination of different positions even if consensus is not obtained (Matusov, 1996). Since standards documents are usually written by separate groups that do not always have direct contact with one another, discourse about curricular disagreements can be limited. In this chapter, I aim to create a space in which disagreements about curriculum standards for early childhood and primary statistics are made explicit and then respectfully analyzed. I also consider steps that can be taken to support early statistics education in the absence of consensus on curriculum standards.

9.2 Scope of the Chapter

This chapter is not an exhaustive treatment of all points of disagreement among all curriculum documents. Instead, it deals with three salient issues at the foundation of early statistics education: the posing of statistical questions, the development of probability language, and the study of variation. In order to illustrate the nature of disagreement in regard to each issue, I compare recommendations from several standards documents (Table 9.1), including: *Principles and Standards for School Mathematics* (PSSM, NCTM, 2000), *Guidelines for Assessment and Instruction in Statistics Education* (GAISE, Franklin et al., 2007), *Common Core State Standards for School Mathematics* (CCSSM, Common Core State Standards Initiative (CCSSI), 2010b), Turnonccmath.net bridging standards (Confrey et al., 2012), the New Zealand Curriculum (Ministry of Education, 2014), the Australian Curriculum (Australian Curriculum, Assessment, & Reporting Authority (ACARA), 2015), and the National Curriculum in England (Department for Education (DfE), 2013). Although this sample is not inclusive of all curriculum standards around the world, these documents collectively bring to light the important differences in approaches to early childhood and primary statistics. Several of these differences are summarized in Table 9.2 and subsequently explored in this chapter.

9.3 Statistical Questions

All of the curriculum standards documents shown in Tables 9.1 and 9.2 recommend the posing of statistical questions during the early years of school. However, they differ in regard to the types of questions students are to pose. Some documents prescribe specific types of questions to be posed, whereas others are more open. The documents also differ in their portrayal of the nature and purpose of statistical questioning.

Some standards documents espouse a relatively narrow perspective on posing statistical questions in the early grades. The CCSSM include just one explicit mention of student-posed statistical questions in the measurement and data strand for Grades

Table 9.1 Standards documents considered in this chapter and their sections for early statistics

	Document sections for early statistics education	Explanation of section content
PSSM (NCTM, 2000)	Grades Pre-K-2 data analysis and probability standard	Recommendations for children younger than 5 (Pre-K) through approximately age 8 (Grade 2)
GAISE (Franklin et al., 2007)	Level A	Beginning level in GAISE Pre-K-12 report; precise grade levels/ages not specified
CCSSM (CCSSI, 2010b)	Grades K-5 measurement and data strand	Recommendations for children approximately 5 years old (K) through approximately age 11 (Grade 5)
Turnonccmath.net bridging standards (Confrey et al., 2012)	Grades K-5 elementary data and modeling	Written to enhance the teaching of the CCSSM Grades K-5 Measurement and Data strand
New Zealand Curriculum (Ministry of Education, 2014)	Years 1–3 statistics strand of mathematics and statistics learning area	Learning objectives to be accomplished after 1, 2, and 3 years in school
Australian Curriculum (ACARA, 2015)	Foundation Year—year 2 statistics and probability strand of mathematics learning area	Proficiencies to be attained by children approximately 5–8 years of age
National Curriculum in England (DfE, 2013)	Key Stage 1—years 1–2 statistics portion (starting year 2) of mathematics program of study	Prescribed program of study for children approximately 5–7 years of age

Table 9.2 Summary of recommendations across standards documents

	Statistical questions	Probability language	Variability
PSSM (NCTM, 2000)	Teachers encourage children to ask questions about their experiences and develop ways to gather data (data analysis and probability standard)	Earliest emphasis on distinguishing among *likely, unlikely, more likely*, and *less likely* (Pre-K-2 data analysis and probability standard)	No explicit recommendations for the types of variability to be encountered during first years of school
GAISE (Franklin et al., 2007)	Children pose questions about contexts of interest with teachers' help (Level A)	Development of a continuum from *impossible* to *certain*, with *less likely, equally likely*, and *more likely* lying in between (Level A)	Children's early experiences should encompass measurement, natural, and induced variability (Level A)
CCSSM (CCSSI, 2010b)	Children ask questions about the total number of points in a data set, how many in each category, and how numbers in categories compare (Grade 1 measurement and data standard)	No explicit recommendations for development of probability language	Variability is not explicitly mentioned until Grade 6, where the focus is on quantifying it with formal statistical measures
Turnonccmath.net bridging standards (Confrey et al., 2012)	Children pose questions of interest to launch statistical investigations (Grade 1)	No explicit recommendations until Grade 7	In Grades 2 and 3, children should work with natural, experimental, and measurement-related variation
New Zealand Curriculum (Ministry of Education, 2014)	Children engage in statistical enquiry cycle, which includes statistical questions, with support in years 1–3	Children describe likelihoods with everyday language in Year 2, language such as *most likely* and *least likely* used in Year 3	No explicit recommendations for the types of variability to be encountered during first years of school
Australian Curriculum (ACARA, 2015)	Children pose simple questions of interest; emphasis on categorical variables through year 3	In Year 1, children use *will happen, won't happen*, and *might happen*; In Year 2, likely, *unlikely, certain*, and *impossible* are used	No explicit recommendations for the types of variability to be encountered during first years of school
National Curriculum in England (DfE, Department for Education 2013)	Children ask questions that require counting and comparing the numbers of objects in different categories (Key Stage 1, Years 1–2)	No explicit recommendations at Key Stages 1 and 2	No explicit recommendations for types of variability to be encountered during first years of school

K-5. It appears in Grade 1, where students are to, "Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another" (CCSSI, 2010b, p. 16). The National Curriculum in England takes a similar approach, stating that children should "ask and answer questions about totaling and comparing categorical data" (DfE, 2013, p. 16) during Key Stage 1.

Other standards documents put less limitations on the types of statistical questions young children are to pose. The Australian Curriculum, for example, is more restrictive only during the Foundation Year, stating that children should "answer yes/no questions to collect information and make simple inferences" (ACARA, 2015, p. 9). It prescribes work with categorical variables during Years 2 and 3, but it does not specify the types of questions children are to ask about the data after the Foundation Year. PSSM, GAISE, and the Turnonccmath.net bridging standards are even more open in their recommendations. These three documents emphasize the importance of having children choose questions of interest. Although children's questions may often involve categorical variables, there is no recommendation to limit their questions to those types of variables. GAISE does, however, emphasize the importance of teacher guidance in helping students select appropriate questions for investigation during level A.

In some cases, standards documents portray the posing of statistical questions as part of an iterative investigative cycle that includes activities such as gathering data, constructing representations, drawing inferences, and perhaps revising the original question. All of the recommendations in the GAISE document are situated within this type of cycle. The Turnonccmath.net bridging standards and New Zealand Curriculum situate questioning within a statistical inquiry cycle as well. These types of documents characterize question-posing as part of an overall process for conducting statistical investigations. Participating in this process brings the work students do in the classroom closer to what statisticians do during the course of professional practice.

Reviewing the range of recommendations for statistical question-posing, some readers might be inclined to criticize overly restrictive curriculum standards that do not prioritize students' interests and do not situate question-posing within an investigative cycle. Before making such judgments, however, it is important to consider what is known about children's tendencies in posing questions and teachers' abilities to support them. When prompted to pose questions about a given situation, children's initial questions may be too ambiguous to yield useful data; in such cases, they often need help to make the questions more precise (Russell, Schifter, & Bastable, 2002). It takes considerable skill for teachers to help students transform such questions into manageable ones. Research suggests that such skill may be elusive for teachers, as they can exhibit many of the same statistical difficulties as younger students (Groth, 2007) and may have trouble formulating interesting statistical questions of their own (Heaton & Mickelson, 2002). Given these circumstances, one might argue that it is appropriate to constrain the types of classroom questions to those which teachers and students are likely to be able to manage.

On the other hand, if one believes that standards documents should set forth aspirational goals rather than those that may be more immediately achievable, then setting ambitious goals for the range of questions to be asked and situating question-posing within a close approximation of statistical practice are advisable. In general, it is best to avoid the tendency to impose ceilings on young children's thinking, as they frequently show the ability to exceed adults' expectations (Moss, Bruce, & Bobis, 2016). Additionally, prompting students to pose enticing questions about a context of interest can encourage them to persevere through all phases of a statistical investigation (Konold & Higgins, 2003). As children pose questions, teachers need to be ready to help them form questions that can be addressed with data and avoid those that are too broad, inadequate, or produce too much data (English, 2014). Preparing teachers to support statistical question-posing in such a manner is a non-trivial task (Franklin et al., 2015), so standards for children's learning should not be developed without considering teacher preparation. Teachers must learn to focus children's attention on both the process of inquiry and the statistical content to be addressed to scaffold children's abilities to pose statistical questions (Fielding-Wells, this volume). Ideally, teacher preparation and standards documents for students would be developed and supported in tandem so that the written curriculum set forth in ambitious standards documents has a greater chance of becoming the enacted curriculum (Stein, Remillard, & Smith, 2007) in classrooms.

9.4 Probability Language

Standards documents can be separated into two broad groups in their recommendations for young children's development of probability language. One group makes no explicit recommendations for probability language development, and the other group does. In the latter group, there is not uniform agreement about how probability language should initially be developed. The lack of agreement across documents raises a number of issues to consider.

The CCSSM, the National Curriculum in England, and the Turonccmath.net bridging standards exemplify documents that contain no explicit recommendations for young children's development of probability vocabulary. The Turnonccmath.net bridging standards do address the development of probability vocabulary starting in Grade 7, but do not speak to the issue in the earlier grades. In the Turnonccmath.net bridging standard for Grade 7, students are to learn language to express certain events, impossible ones, and those whose probabilities lie in between. In the same grade level, they are to learn to quantify probabilities. The bridging standard is intended to help students prepare to succeed in their work with the CCSSM probability content standards, which also first appear in Grade 7.

PSSM and the New Zealand Curriculum take the approach of focusing on initially developing young children's use of everyday language to describe probabilities. In the Pre-K-2 portion of PSSM, this includes developing children's use of *likely* as a standalone word to describe the chance of an event and adding qualifiers to form everyday phrases such as *more likely* and *less likely*. Children are also to encounter *impossible* events and describe them as such. In PSSM, the use of everyday terms is to precede quantification of probabilities. PSSM recommend that calculations of exact probabilities occur in the later grades.

GAISE and the Australian curriculum also seek to leverage children's experiences with everyday language to describe probabilities, but take a slightly different approach. The earliest levels in both of these documents explicitly refer to everyday language that can be used to describe both ends of the probability scale (0 and 1) as well as terms in between. According to GAISE, at Level A, "Events should be seen as lying on a continuum from impossible to certain, with less likely, equally likely, and more likely lying in between" (Franklin et al., 2007, p. 33). Level A students are also to informally assign numerical probabilities to events corresponding to these terms. The Australian curriculum is also designed to help young children begin to think about both ends of the probability scale, but includes a scaffold not present in GAISE. During Year 1, Australian children are to use *will happen, won't happen*, and *might happen* to describe events. During Year 2, these terms *certain, impossible, likely*, and *unlikely* come to the forefront, presumably under the assumption that *will happen* is more understandable than *certain*, and *won't happen* more so than *impossible*. Terms associated with these key points on the probability scale continuum are included in other curriculum documents as well, but not necessarily at the earliest levels.

Examining the disparate recommendations for probability language development in light of existing literature is informative. One of the robust findings of research is that vocabulary learning tends to take place over multiple word encounters (Leung, 2005) and even several high-quality encounters with words do not ensure learning (McKeown, Beck, Omanson, & Pople, 1985). Everyday encounters with probability vocabulary are not necessarily of high quality. The word *certain*, for example, has a colloquial use of describing something that is very likely to occur (Certain, n.d., n.p.). This use in the everyday register differs from the more precise meaning in the mathematical register. One might conjecture that the more encounters one has with the word in the everyday register, the more difficult it becomes to incorporate the meaning from the mathematical register into existing cognitive structures. If this is, in fact, the case, then postponing explicit attention to probability vocabulary until the later grades is not advisable. Young students need multiple opportunities to distinguish colloquial meanings of probability words from mathematical ones.

The literature contains multiple examples of how students at times struggle to use probability vocabulary in a manner resonant with conventional mathematical discourse. In one study, Fischbein, Nello, and Marino (1991) found that some students used the word *possible* to describe events that were certain to occur, and that many used *rare* to describe impossible events. Nacarato and Grando (2014) reported students' use of *less probable* to describe events that cannot occur, and their use of *improbable* to describe events that occur frequently. In some studies, students have used the phrase *50–50 chance* to describe events they believe to be possible (Watson, 2005) or to describe outcomes that do not have the same probability of occurring (Tarr, 2002). Given well-documented examples of this nature, teachers may find themselves trying to counteract meanings students bring to school for probability terms rather than leveraging them as an intermediary step toward quantifying probabilities. Early attention to the mathematical meanings of vocabulary heard in everyday language could help counteract this problem, making it difficult to defend standards documents that include no explicit early attention to probability vocabulary development.

Even among standards documents that do include attention to early probability language development, there are issues to resolve. The most pressing of these appears to be deciding on the most appropriate way to scaffold children's learning of the probability scale continuum. Different sections of the continuum may require different amounts and types of scaffolding. For example, Fischbein et al. (Fischbein et al. 1991) found that students were more successful using *impossible* than *certain* in a mathematical manner. Such a finding seems to support the PSSM approach of drawing students' attention to *impossible* events in Grades Pre-K-2 and delaying formal work with *certain* until Grades 3–5. The Australian Curriculum presents an interesting alternative, however, having students first work with *will happen* and *won't happen* rather than *impossible* and *certain* immediately. Under the Australian progression, children may come to see *certain* as a synonym for *will happen* and *impossible* as a synonym for *won't happen* with teachers' guidance. The extent to which the progression achieves this goal, and in the process counteracts students' difficulties dealing with *certain*, is an empirical question awaiting investigation.

In general, the discrepancies in recommendations for supporting children's learning of probability vocabulary suggest an array of questions in need of systematic research attention, such as:

- To what extent does explicit attention to probability vocabulary in the early grades contribute to probability learning in later years?
- How does children's learning of words and phrases representing the parts of the probability continuum generally progress?
- How might informal outside-of-school encounters with probability vocabulary interfere with or support formal learning?
- Which approaches to introducing probability vocabulary, reflected in different curriculum documents, are the most effective?

As investigations of such questions take place, perhaps a greater degree of uniformity will be achieved across documents.

9.5 Variability

Standards documents can be grouped in two categories in regard to their treatment of statistical variability: those that explicitly identify variability as an object of study for young children and those where experiences with variability may be incidental to work with other standards. Attending to the manner in which variability is treated is a core consideration in teaching statistics. A primary reason statistics exists as a discipline is to study the variability we see in everyday life. Snee (1999) stated, "If there was no variation, there would be no need for statistics and statisticians" (p. 257). Likewise, Cobb and Moore (1997) argued that the need for statistics "arises from the omnipresence of variability" (p. 801). Variability is pervasive in data and distinguishes the study of statistics from the study of mathematics.

GAISE and the Turnonccmath.net bridging standards identify variability as a core object of study for young children and specify the types of variability they should encounter. GAISE recommends structuring curricula so that children encounter three types of variability at Level A: natural, measurement, and induced. Natural variability is encountered as children measure the same quantity across individuals, such as height, weight, or arm length, and observe that these measurements vary from one individual to the next. Measurement variability occurs when repeated measurements of the same thing vary due to characteristics of the measuring device or because of the system being measured. Induced variability occurs when intentional changes are made to a system to observe their effects. For example, planting one crop in a sunny area and another in a shady area may produce variability in yield from each one. The Turnonccmath.net bridging standards closely mirror GAISE in their treatment of variability, recommending the same three types of experiences starting in Grade 2. In both GAISE and the Turnonccmath.net bridging standards, priority is placed on having children work with the recommended types of variability but not necessarily learning the names for each type immediately.

GAISE and the Turnonccmath.net bridging standards are unusual in identifying specific types of variability students should encounter. The other standards documents shown in Table 9.1 do not explicitly identify types of variability to be studied by young children. For example, the word "variability" is not mentioned at all in the Grades K-5 CCSSM. Instead, the K-5 CCSSM data and measurement standards focus on constructing line plots, picture graphs, and bar graphs. After constructing these displays, students are to perform tasks such as finding the difference between the highest and lowest observation and determining how many more or less one category may contain than another. Although such activities may allow students to encounter different types of variability incidentally, the systematic treatment of different types of statistical variability is not prioritized.

Because variability is a core concept in statistics, it is worth considering how children whose curricula are not guided by documents such as GAISE might still have rich experiences with different types of variability. Although explicit mention of statistical variability might ultimately occur across more curriculum documents in the future, it is likely a long-term aspirational goal rather than something more readily attainable. Since standards documents are often criticized for containing too much content (Schmidt, McKnight, & Raizen, 1997), there is a natural tendency to resist adding to prescribed curricula. The content that ultimately does make its way into a standards document will also reflect the values and beliefs of the document writers. In some cases, standards document writers are driven by the desire to emphasize number and operation to a greater extent during the early years. This is sometimes done at the expense of de-emphasizing statistics. The first page of the CCSSM, for example, cites Ginsburg and Leinwand's (2009) argument that mathematics curriculum standards in higher achieving countries include less emphasis on data analysis in the early grades in favor of more attention to number, measurement, and geometry. This citation helps explain why statistics is de-emphasized in the early portions of the document, shifting much of the load to a compressed timeframe in the middle grades.

9.6 Boundary Objects

Given the lack of agreement about the content important for young children to study, creative ways to ensure rich variability experiences are needed. One theoretical construct that can be of assistance in this endeavor is the notion of *boundary object* (Star & Greismer, 1989). Boundary objects help different communities of practice operate collectively in the absence of consensus. In some cases, instructional plans can serve as boundary objects. For example, when teaching elementary students whose curriculum was driven by CCSSM, I created lessons that included the different types of variability identified in GAISE as they addressed the statistical graphing standards included in CCSSM (Groth, 2015). Careful attention to the contexts in which students produced graphs required in CCSSM helped ensure they would experience the different types of variability described in GAISE without adding requirements or extra time to the curriculum. Proponents of both CCSSM-like curricula and GAISE-like curricula can be satisfied with such a lesson sequence, even though their standards for early statistics differ substantially. As proponents of these different types of curricula discuss boundary objects like this lesson sequence, the prospects for greater consensus about early experiences in statistical variability may improve.

Boundary objects can also play roles in addressing the earlier-identified standards-related dilemmas for probability language and statistical questions. One possible approach to resolving these dilemmas is to look to content areas other than mathematics. For example, the teaching of probability vocabulary has natural connections to the language arts. Although it is not reasonable to expect language arts instruction to be guided by statistics standards, statistics educators can position themselves to collaborate on the design of lessons that meet elementary language arts standards such as, "Use precise language and domain-specific vocabulary to inform about or explain the topic" (CCSSI, 2010a, p. 20). Lessons that use words such as *certain* and *likely* as examples of domain-specific vocabulary could promote probability proficiency without adding extra language arts standards to the curriculum. In science, it is natural to pose questions that generate investigative cycles. Statistics educators can collaborate with science teachers to design classroom investigations that satisfy many existing science standards and simultaneously are motivated by rich statistical questions. Such collaborations with language arts and science teachers could ultimately provide avenues to expand the teaching of statistics and probability significantly beyond mathematics classes.

9.7 Conclusion

Disagreements about statistics standards for young children have a variety of sources. To conclude, I consider several sources of disagreement discussed in this chapter: beliefs about students' abilities, beliefs about teachers' abilities, robustness and influence of the research literature, and priorities for mathematics education in the early grades. In considering these sources, I also propose directions the field might take in order to provide high-quality statistics education for all young children even in a climate of disparate curricular recommendations.

Knowledge of students' and teachers' abilities should, to an extent, drive curriculum recommendations. For instance, recommendations to have students focus on categorical data when first posing statistical questions are reasonable from the standpoint that these may be among the most accessible types of questions for students in the early grades. However, including language only about categorical variables in a standard can have the effect of putting a ceiling on children's activities, even if the standard is meant only to be a minimum expectation. Because high-stakes assessments are often attached to standards, and there are many standards to address over the course of a school year, teachers tend to limit instruction to what is prescribed in the required standards (Breault, 2014). The highest priority for professional development then becomes learning to help students attain what is in the text of the standards, essentially putting a ceiling on teachers' growth as well.

To avoid putting ceilings on students' and teachers' growth, writers of standards documents can take a number of steps. One step would be to carefully phrase standards in a manner that identifies essential content but also encourages deeper study as opportunities arise. The PSSM document does so in its recommendations for children's posing of statistical questions, saying that such questions should arise from students' curiosity about the world around them. Of course, open recommendations of this nature put a greater burden on the teacher, who must skillfully handle unusual or unwieldy questions that students may pose. The presence of this greater burden suggests the desirability of forming standards for students and standards for teacher preparation in tandem, so that teachers might be better prepared to handle challenges that may come about as a result of more ambitious standards for students. At present, the two types of standards documents are usually written by separate groups and/or at different points in time; greater success might be realized by writing the two simultaneously.

Although more ambitious standards documents are desirable, simply having the goal of writing ambitious standards is not enough. The research community has considerable work to do to help guide the process. This is vividly illustrated by the current situation with standards for learning probability language. The disagreements in this area suggest a number of research questions in need of investigation, including: (i) To what extent does early instruction focused on probability language help improve students' probabilistic thinking and discourse throughout their years of school? (ii) How should children's experiences with probability language be sequenced? (iii) What kinds of scaffolding should teachers be prepared to furnish as children begin to distinguish between colloquial and technical meanings of probability vocabulary words and phrases? Answers to questions of this nature can help guide the formation of learning expectations for students and professional development goals for teachers. Along with conducting such studies, researchers need to be conscious of presenting their findings in venues and formats likely to be accessed and understood by writers of standards documents.

Of course, research studies will not resolve all disagreements because research is invariably interpreted in different ways by different individuals (Sierpinska & Kilpatrick, 1998). Some differences in interpretation and use of research stem from different priorities and beliefs about what is important in early childhood mathematics education. Overcoming such philosophical differences may ultimately prove to be the greatest challenge in providing quality early childhood experiences in regard to statistical questions, probability language, and variability. Achieving uniform consensus is not likely. However, boundary objects (Star & Greismer, 1989) allow groups with different beliefs to operate collectively even in absence of consensus. As noted earlier, one promising direction for the creation of boundary objects is designing lesson sequences and tasks that satisfy multiple standards documents simultaneously without over-burdening the curriculum. Designing, implementing, and analyzing such lessons can provide space for collective work among those holding different beliefs about the appropriate focus for the early study of statistics. Even if this collective work does not move individuals toward complete consensus, it can prompt deeper consideration of the beliefs and positions they hold (Matusov, 1996). As beliefs and positions are re-examined, a foundation is formed for well-constructed recommendations for children's and teachers' statistical learning.

References

Australian Curriculum, Assessment, and Reporting Authority. (2015). *Mathematics*: *Sequence of content F-6*. Retrieved from https://acaraweb.blob.core.windows.net/resources/Mathematics_-_Sequence_of_content.pdf.

Breault, R. (2014). Mining the potential in your state standards. *Kappa Delta Pi Record, 50*(1), 4–8.

Certain. (n.d.). In *Oxford's living dictionary*. Retrieved from https://en.oxforddictionaries.com/definition/certain.

Cobb, G., & Moore, D. (1997). Mathematics, statistics, and teaching. *The American Mathematical Monthly, 104*(9), 801–823.

Common Core State Standards Initiative. (2010a). *Common core state standards for English language arts*. Retrieved from http://www.corestandards.org.

Common Core State Standards Initiative. (2010b). *Common Core state standards for mathematics*. Retrieved from http://www.corestandards.org.

Confrey, J. Nguyen, K. H., Lee, K., Panorkou, N., Corley, A. K., & Maloney, A. P. (2012). *TurnOnCCMath.net: Learning trajectories for the K-8 Common Core State Math Standards*. Retrieved from https://www.turnonccmath.net.

Department for Education. (2013). *Mathematics programmes of study*: *Key stages 1 and 2*. Retrieved from https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/335158/PRIMARY_national_curriculum_-_Mathematics_220714.pdf.

Dingman, S., Teuscher, D., Newton, J. A., & Kasmer, L. (2013). Common mathematics standards in the United States: A comparison of K-8 and Common Core standards. *The Elementary School Journal, 113*(4), 541–564.

English, L. (2014). Statistics at play. *Teaching Children Mathematics, 21*(1), 36–44.

Fischbein, E., Nello, M. S., & Marino, M. S. (1991). Factors affecting probabilistic judgements in children and adolescents. *Educational Studies in Mathematics, 22*(6), 523–549.

Franklin, C., Bargagliotti, A. E., Case, C. A., Kader, G. D., Scheaffer, R. L., & Spangler, D. A. (2015). *Statistical education of teachers*. Alexandria, VA: American Statistical Association.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., et al. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report: A PreK-12 curriculum framework*. Alexandria, VA: American Statistical Association.

Ginsburg, A., & Leinwand, S. (with Decker, K.). (2009, December). *Informing grades 1–6 mathematics standards development*: *What can be learned from high-performing Hong Kong, Korea, and Singapore?* Washington, DC: American Institutes for Research. Retrieved from http://www.air.org/sites/default/files/downloads/report/MathStandards_0.pdf.

Groth, R. E. (2007). Toward a conceptualization of statistical knowledge for teaching. *Journal for Research in Mathematics Education, 38*(5), 427–437.

Groth, R. E. (2015). Royalty, racing, rolling pigs, and statistical variability. *Teaching Children Mathematics, 22*(4), 218–228.

Heaton, R. M., & Mickelson, W. T. (2002). The learning and teaching of statistical investigation in teaching and teacher education. *Journal of Mathematics Teacher Education, 5*(1), 35–59.

Jones, D., & Tarr, J. E. (2010). Recommendations for statistics and probability in school mathematics over the past century. In B. J. Reys & R. E. Reys (Eds.), *Mathematics curriculum issues, trends, and future directions: Seventy-second yearbook of the National Council of Teachers of Mathematics* (pp. 65–75). Reston, VA: National Council of Teachers of Mathematics.

Jones, G. A., Langrall, C. W., & Mooney, E. S. (2007). Research in probability: Responding to classroom realities. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 909–955). Charlotte, NC: Information Age & National Council of Teachers of Mathematics.

Konold, C., & Higgins, T. (2003). Reasoning about data. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 193–215). Reston, VA: National Council of Teachers of Mathematics.

Leung, C. (2005). Mathematical vocabulary: Fixers of knowledge or points of exploration? *Language and Education, 19*(2), 127–135.

Matusov, E. (1996). Intersubjectivity without agreement. *Mind, Culture, and Activity, 3*(1), 25–45.

McKeown, M., Beck, I., Omanson, R., & Pople, M. (1985). Some effects of the nature and frequency of vocabulary instruction on the knowledge and use of words. *Reading Research Quarterly, 20*(5), 522–535.

Ministry of Education. (2014). *The New Zealand Curriculum online: Mathematics and statistics*. Retrieved from http://nzcurriculum.tki.org.nz/The-New-Zealand-Curriculum/Mathematics-and-statistics/Achievement-objectives.

Moss, J., Bruce, C. D., & Bobis, J. (2016). Young children's access to powerful mathematics ideas: A review of current challenges and new developments in the early years. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (3rd ed., pp. 153–190). New York: Routledge.

Nacarato, A. M., & Grando, R. C. (2014). The role of language in building probabilistic thinking. *Statistics Education Research Journal*, *13*(2), 93–103. Retrieved from http://iase-web.org/documents/SERJ/SERJ13(2)_Nacarato.pdf.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Russell, S. J., Schifter, D., & Bastable, V. (2002). *Working with data casebook*. Parsippany, NJ: Dale Seymour Publications.

Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (1997). *A splintered vision*: *An investigation of U.S. science and mathematics education*. Boston, MA: Kluwer Academic Press.

Sierpinska, A., & Kilpatrick, J. (Eds.). (1998). *Mathematics education as a research domain: A search for identity*. Dordrecht, The Netherlands: Kluwer.

Snee, R. (1999). Discussion: Development and use of statistical thinking: A new era. *International Statistical Review, 67*(3), 255–258.

Star, S. L., & Greismer, J. R. (1989). Institutional ecology, 'translations', and boundary objects: Amateurs and professionals in Berkeley's Museum of Vertebrate Zoology, 1907–39. *Social Studies of Science, 19*(3), 387–420. https://doi.org/10.1177/030631289019003001.

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 319–369). Reston, VA: National Council of Teachers of Mathematics; Charlotte, NC: Information Age.

Tarr, J. E. (2002). The confounding effects of "50-50 chance" in making conditional probability judgments. *Focus on Learning Problems in Mathematics, 24*(4), 35–53.

Watson, J. M. (2005). The probabilistic reasoning of middle school students. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 145–168). New York: Springer.