Chapter 2
Probabilistic Thinking and Young Children: Theory and Pedagogy

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Abstract Over the last decades, there has been a lot of interest in exploring young children's early probabilistic thinking, considering educational, cognitive and mathematical dimensions in children's learning and development. Today, probability is incorporated in many mathematical and statistical curricula and the ongoing research on children's probabilistic competencies has produced remarkable and educationally valuable conclusions. The aim of this chapter is to critically review key theoretical models of probabilistic thinking that cover the period of early childhood and to highlight a number of pedagogical implications while introducing probabilistic concepts in early childhood educational contexts. The traditional Piagetian claim that children during the preoperational period find it difficult to differentiate certainty and uncertainty seems to be replaced by findings that support children's capacity to engage with notions of probability. Recent research underlines how intuitions and experience, informal mathematical knowledge, probability literacy as well as curriculum development and task design play a significant role in shaping and enhancing preschoolers' probabilistic thinking, not only while they are young but with a lifelong perspective.

2.1 Setting the Scene: Probability, Literacy and Children

From early in life, children experience and interact with the world around them while making sense of the possible, random and impossible. They develop their understanding of the world through causal and statistical reasoning (Gopnik & Schulz, 2007), by making connections and using information and cues from around them, in order to predict and expect outcomes, when possible. Learning about the world requires learning about probabilistic relationships (Yurovsky, Boyer, Smith, & Yu, 2013) in framing what is likely and what is not. On many occasions, children through expe-

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rience and cognitive processing develop an understanding of probabilities as part of the development of their scientific, mathematical and social knowledge.

The nature of probability has three main approaches. The classical interpretation of probability is simply the fraction of the total number of possibilities in which the event occurs. Laplace (1814, 1951: 6–7) noted: '*The theory of chance consists in reducing all the events of the same kind to a certain number of cases equally possible, that is to say, to such as we may be equally undecided about in regard to their existence, and in determining the number of cases favorable to the event whose probability is sou*ght'. The second interpretation is the frequentist, where the possibilities of events may be assigned unequal weights and probabilities can be computed a posteriori. In this case, probability is based on the long-run behaviour of random outcomes (Konold, 1991). The third approach is subjective probability where probability tends to be 'a degree of belief', where biases, heuristics and intuitions interplay. '*Probability does not consist of mere technical information and procedures leading to solutions. Rather, it requires a way of thinking that is genuinely different from that required by most school mathematics. In learning probability, students must create new intuitions'* (Fischbein & Schnarch, 1997, p. 104). Thus, probability can become very complex and sophisticated and embraces a way of thinking in enabling us to cope with randomness, uncertainty and unpredictability through computational and subjective ways.

Gal (2005) highlights that many school curricula focus on the classical and/or frequentist views of probability instead of considering the big picture. He puts forward the notion of probability literacy, as '*the ability to access, use, interpret, and communicate probability-related information and ideas, in order to engage and effectively manage the demands of real-world roles and tasks involving uncertainty and risk*' (Gal, 2012, 4). His model suggests that five knowledge bases as well as supportive dispositions form probability literacy. Gal (2005) lists the five knowledge elements of probability literacy as the exploration of big ideas, like, variation, randomness, predictability/uncertainty; the estimation of probabilities; the use of language to communicate chance; the understanding of the context where probabilities are applied and the consideration of critical questions when dealing with probabilities. Nevertheless, according to Gal (2005) the dispositional elements are equally important building blocks of probabilistic literacy. These are: critical stance; beliefs and attitudes and personal sentiments regarding uncertainty and risk.

Similarly, Borovcnik (2016) defines probability literacy as '*the ability to use relevant concepts and methods in everyday context and problems*' (p. 1500). Probability literacy is essential in modern times, and recently, there has been growing interest in identifying ways and approaches in incorporating probabilities in education. Batanero et al. (2016) confirm the recognition of probability literacy by educational authorities globally, but also encourage more attention towards practical and pedagogical issues in implementing probability in curricula. The main aim of including probability in schools relates to its usefulness for daily life, its instrumental role in other disciplines and its important role in decision-making as a skill for competent and knowledgeable future citizens (Gal, 2005; Fisk, Bury, & Holden, 2006). Most studies focus on school-aged children and adolescents, but there is limited investigation capturing early experiences during preschool years. In this direction, the aim of this chapter is twofold: to present an overview of key theories and models on the development of probabilistic thinking, with an emphasis on early childhood, and to address pedagogical factors while implementing probabilities in early years. The first part identifies key characteristics and concepts that young children engage with at ages 3–7, based on theory and research, and the second part reviews important curriculum-related aspects in embedding probabilities in early years practice.

2.2 Children's Probabilistic Thinking: From Piaget and Fischbein to Contemporary Studies

The development of probabilistic thinking was traditionally and systematically explored by Piaget and Inhelder (1975). Within the Piagetian theory of cognitive development, it is recognised that the evolution of the idea of chance and probabilities is '*a kind of synthesis between operations and the fortuitous*' (p. 216). In the book 'The origin of the idea of chance in children' (1951), Piaget recognised that all occurrences in our daily lives are complex and proposed three developmental stages of the idea of chance. The first stage, which relates to early childhood, the preoperational period before the age of 7 or 8, is characterised by prelogical reasoning and limited cognitive capacity to understand irreversibility, deduction, random mixing and random distribution. Through a number of studies, Piaget concluded that children during this period do not distinguish the possible from the necessary and mark the development of the idea of chance during the second stage, when more advanced logical and arithmetical operations appear.

During this first stage, children base their judgements regarding random draws on phenomenism, passive induction and egocentrism. In a heads and tails game (with crosses and circles), where 10–20 counters were thrown at once, children were asked to predict the outcomes. Piaget would substitute the counters with fixed counters showing a cross on both sides aiming at recording children's reactions. Children in stage I would accept what they saw (phenomenism) as possible, whereas children in stage II would refuse to accept the coincidence of all counters showing a cross. Children also at stage I would try to justify this occurrence as part of their subjective and personal beliefs in that, for example, one can get crosses only if they toss the counters in a certain way (egocentrism). Also, young children would judge the outcome based on the immediately preceding facts (passive induction); if crosses came out previously, then crosses are likely to come out again.

For Piaget, children at the preoperational stage find it difficult to combine multiple interactions of phenomena as well as their irreversibility or independence. He tested this through another task, with the use of a rectangular box based on a sloping pivot and equal numbers of two sets of coloured balls. The two sets of balls are separated by a divider at one end of the box, and then children would be asked to predict after a number of tippings of the box the position and arrangement of the balls. Children in stage I would initially expect the balls after the first tipping to land in their original positions, and even if there was an element of mixture (one white ball ending up in the other coloured set of balls), they would seek for uniformities. This indicates that children at these ages have difficulty in understanding the idea of irreversibility and randomness.

Fischbein (1975) provided a more educational approach to probabilistic thinking in children and emphasised the role of intuitions in developing understandings of probabilistic notions. He focused on the importance of the intuitive endowment of the child and defined intuitions as '*forms of immediate cognition in which the justifying elements, if any, are implicit*' (p. 5). He underlined that when facing probabilistic events our behaviour requires specific intuitions, in the sense of predictions and responses 'guessed at' that are characterised by immediacy, globality, extrapolative capacity, structurality and self-evidentness. These intuitions are long-verified mechanisms, stabilised by social learning and personal experience. Fischbein (1975) provided different categories of intuitions and proposed that there should be separation between the concept of probability as an explicit, correct computation of odds and the intuition of probability, as a subjective, global evaluation of odds. He agreed that there is a developmental pattern in the emergence of probabilistic thinking in that through age and experience children develop more profound understandings. Therefore, he recommended the necessity to '*train, from early childhood, the complex intuitive base relevant to probabilistic thinking*' (p. 131).

Primary intuitions are cognitive acquisitions formed by experience before systematic instruction and are found in the preschool child. Secondary intuitions are formed by scientific instruction, mainly through school and formal education and transfer social experience to scientific truth. Another dichotomy in intuitions, proposed by Fischbein (1975), is between affirmatory and anticipatory intuitions. Affirmatory intuitions are based on the feeling of certitude of events; thus, anticipatory intuitions are global views on the solution to a situation which precede the problem-solving process. After several studies, Fischbein (1975) reached the conclusion that, in contradiction to Piaget, the intuition of chance is present before the age of 6–7. He argued that young children can indicate the capacity to evaluate chance and estimate the odds in a probabilistic manner, as they develop their primary and preoperational intuitions through daily experiences and subjective or perceptual considerations. This is confirmed in cases where the number of possibilities is small, where the nature of the problem is clear, where rewards for correct answers are present and where prior instruction in the concepts of chance and probability is enhanced. Thus, intuitions are key in the learning process and construction of probabilistic knowledge.

On a number of occasions, there has been a misinterpretation in that Piaget and Fischbein were contradictory. This is not the case; instead, they attempted to study and explore the development of probabilistic thinking in young children through different lenses. Piaget had a more developmental approach focusing on the role of intellectual ability, while Fischbein emphasised the role of intuitive thinking and pedagogy. More recently, and particularly over the last 20 years, there has been an ongoing interest in exploring the characteristics of young children's probabilistic thinking in formal and informal contexts of learning.

Jones et al. (1997) were pioneers in proposing a framework for assessing and nurturing children's thinking in probability from early ages. The aim of their framework was to provide aspects of children's probabilistic thinking in a comprehensive way that would support curriculum designers and pedagogues to implement probabilities in mathematical curricula and instruction. They examined four core constructs: sample space, probability of an event, probability comparisons and conditional probability.

Sample space: The sample space, Ω, is a key construct in understanding randomness (Nunes et al. 2014) and is about the set of all possible outcomes in a given situation. For example, the sample space (Ω) for rolling an ordinary dice would be $\Omega = (1, 2, 3, 4, 5, 6)$.
Probability of an event: The probability of an event A is the likelihood that event will occur P (A). With young children, this construct could be explored through guessing in a set task what is more or less likely to occur and reasoning why. For example, what is more likely to come up, if you turn over a card from a set of 6 hidden cards, where 5 are red and 1 is yellow?
Probability comparisons: Probability comparisons reflect the capacity to determine and justify: a. which probability situation is most likely to produce the desired outcome and b. whether two or more probability situations offer the same or a fair chance for the desired outcome. For example, if green wins, which of the following two spinners should I choose to increase the likelihood of winning: the one with ¾ in blue and ¼ in green or the one with ¼ blue and ¾ green?
Conditional probability: Conditional probability provides a way to reason about the outcome of an experiment, based on partial information or on additional information; the probability for an event A given B is denoted by $P(A \mid B)$. For example, in a box, there are 4 black and 2 white bears. If we shake the box and a bear is drawn, what colour is it likely to be? If this bear is not repositioned in the box and there is a second draw, what colour is this second bear likely to be?

Through observations and interviews with children at different ages, Jones et al. (1997; 1999) proposed 4 levels of probabilistic thinking. They defined as probabilistic thinking '*children's thinking in response to any probability situation*' (Jones et al., 1999, p. 488). By probability situation, or a situation involving uncertainty, they consider an activity or random experiment where more than one outcome is possible; thus, the actual outcome cannot be predetermined but only inferred. In detail, Level 1 is associated with subjective thinking; here, children make intuitive judgements based on their imagination or personal preferences, consistent with reasoning that is subjective or influenced by irrelevant aspects. Level 2 is transitional between subjective and naive quantitative thinking, where students often make inflexible attempts to quantify probabilities. Level 3 involves the use of informal quantitative thinking in that students use more generative strategies in listing the outcomes of two-stage experiments and in coordinating and quantifying thinking about sample space and probabilities. Finally, Level 4 incorporates numerical thinking and students demonstrate the use of valid numerical measures to describe the probabilities.

Table 2.1 Jones et al. (1997) framework for assessing probabilistic thinking—*Level 1: Subjective*

Concept	Type of response
Sample space	• lists an incomplete set of outcomes for one-stage experiments
Probability of an event	• predicts most/least likely event based on subjective judgements • distinguishes 'certain', 'impossible', possible events in a limited way
Probability comparisons	• compares the probability of an event in two different sample spaces, usually based on various subjective or numeric judgements
Conditional probability	• following a particular outcome, predicts consistently that it will occur next time, or alternatively that it will not occur again (overgeneralises)

Level 1 is age-related to early childhood (Table 2.1). In the same direction, Way (2003) recorded that around 5 years and 8 months, during the non-probabilistic thinking stage that she proposed, children show minimal understanding of randomness and are strongly reliant on visual comparisons. Children under 6 years may possess intuitive notions of probability, but these are unstable. Likewise, Nikiforidou and Pange (2010) found that 4-year-olds rely on visual comparisons and distinguish 'impossible' and 'possible' events in a limited way. Also, Sobel et al. (2009) found that children's probabilistic inferences develop into early elementary school, but preschoolers might also have some understanding of probability when reasoning about causal generalisation.

Examples from recent studies confirm that preschoolers have a sophisticated understanding of probability concepts. For example, Kushnir and Gopnik (2005) found in their study that children aged 4–6 apply the probabilistic element of the frequency of co-occurrence when developing causal relationships. Boyer (2007) used a computerised decision-making task to find that 5–6-year-olds select the more probable outcome by demonstrating intuitive sensitivity to probability. Girotto and Gonzalez (2008) found through three different studies that when preschoolers, around 5 years old, are faced with uncertain events, they are able to integrate a new piece of information in making inferences and as such indicate adaptation to posterior probability. Fisk et al. (2006) found that children aged 4–5 would commit the conjunction fallacy while participating in tasks involving choice between the more likely of two events, a single event and a joint event (conjunctive or disjunctive). Moreover, Girotto et al. (2016) found that in probabilistic choice tasks, 5-year-olds made optimal choices, whereas 3–4-year-olds based their responses on randomness and/or superficial heuristics. Such studies, as well as others, provide insights on preschoolers' probabilistic reasoning in diverse probabilistic situations. These, in turn, can inform practice and ways of fostering children's probabilistic literacy in educational contexts.

2.3 Probability in Early Childhood Educational Practice

2.3.1 Curriculum Design: Constructivism and Proposed Instructional Models

Early years education provides the foundation for fundamental conceptual understanding, knowledge and dispositions needed for further learning. In his idea of the *spiral curriculum*, Bruner (1960) addresses the role of probability and underlines: '*If the understanding of number, measure, and probability is judged crucial in the pursuit of science, then instruction in these subjects should begin as intellectually honestly and as early as possible in a manner consistent with the child's forms of thought. Let the topics be developed and redeveloped in later grades*' (pp. 53–54). He emphasises the necessity of introducing the equally important concepts of number, measure and probability as early as possible, in a way that relates to the child's cognition. These can then be revisited and reconstructed through time and progression.

Cobb (2007) agrees that mathematical learning is an interactive as well as a constructive process. It is a process where prior knowledge and experiences are used as the foundation for constructing and reorganising conceptual and theoretical ideas. In a similar direction, Sharma (2014) believes that probabilistic thinking can be developed slowly and systematically through carefully designed activities in appropriate learning environments. She favours the learning context that challenges students to explore and reflect on any discrepancies they observe and the one that facilitates evaluations and justifications in both verbal and representational modes.

Jones et al. (1999) took a socio-constructivist position in their study, supporting that probability knowledge can arise from students' attempts to solve problems, to build on and reorganise their informal knowledge, and to resolve conflicting points of view. Under this position, social processes are important mechanisms through which participants negotiate meaning and co-construct knowledge in collaborative learning environments (Cobb, 2007). The instructional sequence argued begins with the presentation of a meaningful task or problem and continues with an invitation to solve that problem in multiple ways, which leads to the sharing, justifying and discussing of those problem-solving strategies in small or large group discourses (Garfield & Ben-Zvi, 2009). This links to the predict–observe–explain (White & Gunstone, 1992) strategy that probes understanding, especially in science education. First, the students must predict the outcome of some event and reason about their prediction; then they must describe what they see happening; and finally, they must reconcile any conflict between prediction and observation.

Likewise, Sharma (2014) proposes a possible teaching sequence to explore probability, based on the example of a die rolling game:

1. Posing a task: introduce the task in a meaningful context
2. Making predictions: individually and next in pairs encourage students to discuss and record their predictions

3. Playing the game: encourage students in pairs/small groups to interact with the game and record the actual outcomes
4. Planning explorations: the whole class shares and discusses their ideas
5. Data collection and analysis: in groups, students collect and record data
6. Further exploration: representation of dice outcomes in various ways (i.e. tree diagrams, tables) (pp. 81–82).

Constructivist approaches to teaching and learning consider intuition and prior knowledge as a starting point for further learning (Gelman & Brenneman, 2004). Nikiforidou et al. (2013) found that the linkage between informal and formal probability learning in the preschool classroom can be enabled if the subject content and the cognitive capacity of children match. Young children not only know some mathematics before reaching formal schooling, but they are ready and eager to learn more of it (Greenes, 1999). Children encounter probabilistic judgements and relationships in their daily routines and develop an informal understanding of what is likely, possible, uncertain or random. It is these initial understandings and personal experiences that can be the stepping stone in instruction. As a matter of fact, children learn through physical and social interactions, before school, and formulate informal knowledge and understanding (Ginsburg, Lee, & Boyd, 2008). HodnikČadež and Škrbec (2011) propose that the probability contents in the preschool and early school period should be related to using everyday probability language, answering probability or likelihood questions about specific data, answering probability or likelihood questions about specific situations and collecting and reflecting on empirical data.

2.3.2 Aspects of Instruction and Task Design in Probability Learning

Manipulatives play a key role in children's mathematical understanding as they offer ways of connecting mathematical ideas to real-world experiences (McNeil & Jarvin, 2007). Manipulatives, both concrete and virtual, enable children to experience consciously and unconsciously mathematical thinking through their senses (Swan & Marshall, 2010); through exploration, manipulation, interaction and observation. Their design and how they are introduced in practice are key in children's meaning-making and reasoning. However, their presence only is not adequate for meaningful learning to occur. Instead, their effectiveness depends on how they are embedded in comprehensive, well-planned activities (Sarama & Clements, 2009), which gradually build on more advanced knowledge through play, exploration, repetition and stimulation (DeVries, Zan, Hildebrandt, Edmiaston, & Sales 2002). Falk et al. (2012) support that children's implicit probabilistic knowledge can be strengthened by devising hands-on educational measures and interactions through a playful way. Furthermore, HodnikČadež and Škrbec (2011) agree that concrete experiences and experimentation are key in teaching probabilistic concepts in preschool children.

An example of such experiences can originate from picture story books. Picture books have been found to can act as means for the construction of knowledge and higher understanding in mathematics instruction (Elia, van den Heuvel-Panhuizen, & Georgiou, 2010). They provide opportunities for meaningful connections between young children's prior knowledge and the content presented. Kinnear (2013) in her study on children's statistical reasoning used picture books as a task and data context, with children aged 5. She found that children responded to the uncertainty created by an unresolved problem in the story and by making predictions if the book generated personal interest either through the illustrations, the characters or the mystery presented.

In probability learning, manipulatives, concrete and technological, could be dice, spinners, cards, board games, tinker cubes, urns, boxes or bags composed by variant ratios and proportions of items, stories and scenarios, visual stimuli, props and tools, toys. Batanero et al. (2016) observe that as these physical devices can be acted upon, they are increasingly used in probability education aiming to induce chance events (e.g. by rolling, spinning, choosing) and the development of key probabilistic concepts. Some examples are presented in Table 2.2. However, as Pratt (2011) recommends, more research is needed in exploring the role of these artefacts in the development of new curricula and the linkage between probability and real-world phenomena. He mentions: '*…it is debatable whether there is much advantage in maintaining the current emphasis on coins, spinners, dice and balls drawn from a bag… now that games take place in real time on screens, probability has much more relevance as a tool for modelling computer-based action and for simulating real-world events and phenomena*' (Pratt, 2011, p. 892).

Falk et al. (2012) emphasise that the structure of the problem is a key learning factor when using probabilistic tasks. They found that young children from the age of 4 can be introduced to probability through playful ways. Furthermore, Schlottmann and Wilkening (2012) underline that task complexity, in relation to linguistic, memory and meta-cognitive demands, can define children's probabilistic thinking. Skoumpourdi et al. (2009) supported that the important factors in the nature and structure of the particular probabilistic task or problem situation for preschoolers are: a meaningful context, the manipulation of concrete materials, the facilitation of rich discussions, the reflective process and children's informal knowledge of probability. Thus, if there is a play element, materials, discourse and simplicity in the task, children can interact with probabilistic notions.

Another important pedagogical factor when introducing probability in early childhood relates to the significance of questions initiated by the teacher. According to Sharma (2014), the teacher plays an important role in posing questions that prompt students' thinking and reasoning. Through open-ended questions, students get the opportunity to deepen their perceptions and share them with others. Similarly, according to Friel et al. (2001, p. 130), the questions have to '*provoke students' understanding of the deep structure of the data presented*'. Sharma (2014) also recommends the use of some sentence beginners to help students write/express their responses.

Table 2.2 Examples of probabilistic tasks for preschoolers

Materials	Task	Key concept
 – a bag/box – different sums of items	After introducing children to the materials, we ask them to place the items in the bag and mix them up. Without seeing, we ask them ‘*If one item is selected* (either by themselves, or a puppet) *what do you think will come out*?’ Children can record their answers in 2 stages: prior to the draw (their predictions) and after the draw (actual outcome). Discussion can be facilitated in comparing and analysing the data. Variations to the distribution of the sample space are encouraged	– sample space – likelihood of events
 – discs with variations in the sample space	After introducing children to the materials, we ask them the following questions: ‘*If I want to bring orange, which spinner should I choose?*’, ‘*If I want to bring blue for the next 5 times, which spinner should I choose?*’ Again, children can record their predictions and actual outcomes for further discussion	– probability comparisons – sample space – likelihood of events

These could be, for example, ‘From the table it can be seen that… because…’. Way (2003) also noticed that teachers may build awareness of the relationship between the sample space and the likelihood of events through the repetition of games and the use of guiding questions.

Technology and its role in statistics and probability education is a growing field of interest (Batanero et al., 2016; Tishkovskaya & Lancaster, 2012). Chance et al. (2007) discuss how recent and ongoing developments in using technology in teaching statistics correlate with changes in course content, pedagogical methods and instructional formats. Batanero et al. (2016) agree that technology provides a big opportunity for probability education, and Borovcnik and Kapadia (2009) underline that probabilistic software offers more efficient, graphically oriented possibilities to supply experience with randomness. For example, Paparistodemou et al. (2008) examined the strategies through which children aged 5.5 and 8 years engaged in constructing a fair spatial lottery game. They found that the microworld enhanced children’s deterministic and stochastic thinking when exploring fairness and randomness. Haworth et al. (2010) found that in designing digital games, additional visual representations like decision trees that represent probabilistic reasoning support children's thinking processes. In another study, Paparistodemou et al. (2002) built a probability game to study young children's understanding of random mixture. Children as young as six could make sense of random mixtures in this game-like environment. Nowadays, the discussion has moved beyond technology itself, towards ways in which technological programs and tools can support the teaching and learning process of probability.

2.4 Conclusion

Probability can be approached not only through mathematical calculations but also through subjective intuitions. Children, from a really young age, through experience and experimentation construct knowledge and dispositions towards probabilistic concepts. They encounter situations where uncertainty and randomness apply. It was initially Piaget and Fischbein who explored the origins of probabilistic thinking in young children, and subsequent research has revealed ways through which children think and act within probabilistic contexts. Developmentally, children as young as four show engagement with notions of probabilistic thinking, and it is argued that the early childhood classroom can set foundations for probability literacy.

In this direction, it is proposed that there are some pedagogical implications to be considered when implementing probability in the preschool setting. These implications derive from both more generic approaches characterising early childhood pedagogy and more specific features applying to probabilistic thinking and reasoning in young children. Prior knowledge, intuitions, meaningful tasks in connection to children's personal worlds are important. Simple concepts of probability can be explored through discussion, group work, collaborative learning, concrete experiences and co-construction of knowledge. The concepts presented in this chapter (sample space, probability of an event, probability comparisons and conditional probability) are based on the framework proposed by Jones et al. (1997).

These concepts can be approached through instructional sequences, like predict–observe–explain. In these, children can engage with problem-solving situations underpinned by possibilities and probabilities. The learning experience can be enriched through a number of ways; the use of manipulatives (like dice, spinners, boxes), scenarios and story picture books (that encourage inferencing about specific data and situations), technological tools and software (like microworlds, digital games), discourse (like the use of open-ended questions), repetition and collaboration. However, further research is needed to investigate in more detail approaches through which probabilistic thinking can be fostered in a child-centred way. In particular, the role of manipulatives, physical and virtual, as transitional objects that enable doing and thinking—action and perception—needs more exploration. Other possible directions for future research could involve the role of the practitioner, the role of technology, the connection of probability to statistics and other fields.

To sum up, children's probabilistic competence is more profound than previously thought. The early childhood classroom can be the starting point of a spiral curriculum that introduces probability, aiming at probability literate future citizens.

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