Chapter 1
Theorising Links Between Context and Structure to Introduce Powerful Statistical Ideas in the Early Years

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Abstract Recent literature in the early years has emphasised the benefits of introducing children to powerful disciplinary ideas. Powerful ideas in statistics such as variability, aggregate, population, the need for data, data representation and statistical inquiry are generally introduced in the later years of schooling or university and therefore may be considered too difficult for young children. However, at an informal level, these ideas arise in contexts that are accessible to young children. The aim of this chapter is to theorise important relations between children's contextual experiences and key structures in statistics. It introduces the notion of *statistical context–structures,* which characterise aspects of contexts that can expose children to important statistical ideas. A classroom case study involving statistical inquiry by children in their first year of schooling (ages 4–5) is included to illustrate characteristics of age-appropriate links between contexts and structures in statistics. Over time, engaging children in significant activities that rely on statistical context–structures can provide children with multiple opportunities to experience statistics as a coherent and purposeful discipline and develop rich networks of informal statistical concepts well before ideas are formalised. For teachers and curriculum writers, statistical context–structures provide a framework to design statistical inquiries that directly address learning intentions and curricular goals.

1.1 Introduction

Researchers have long argued that powerful mathematical ideas are accessible to young children (e.g. Alexander, White, & Daugherty, 1997; English & Mulligan, 2013; Greer, Verschaffel, & Mukhopadhyay, 2007; Perry & Dockett, 2008). Yet many approaches to teaching young children undervalue their capacity—and therefore limit their opportunities—to access powerful statistical ideas. Content is often disconnected from purposeful activity, and learning sequences tend to focus on small

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increments building from simple to complex. Incremental approaches tend to isolate and disconnect statistical ideas from their rich contextual and structural relations with other key ideas, making them less coherent from the students' perspective (Bakker & Derry, 2011).

Addressing the gap between the conviction that children can benefit from access to powerful statistical ideas and the operationalisation of this conviction is critical. How does one design age-appropriate learning experiences with complex content? In this paper, I theorise how the context of a problem can be a powerful scaffold for children to engage informally with powerful statistical ideas. The paper introduces the theoretical notion of *statistical context–structures,* which characterise aspects of problem contexts that can expose children to key statistical ideas and structures (concepts with their related characteristics, representations and processes). Using statistical context–structures to create repeated opportunities for children to experience informal statistical ideas has the potential to strengthen their understanding of core concepts when they are developed later. Exposure to informal concepts across a variety of problem contexts highlights their relationships to other core concepts, develops coherence of how statistical ideas work together, assists students to recognise contexts in which the ideas are appropriate and potentially useful, and improves the sense of relevance of statistical ideas.

The aim of this paper is to illustrate how a teacher in an early years classroom (children aged 4–5 years) used a personal problem context to informally introduce, scaffold and develop informal yet powerful statistical content. Over the course of two lessons, she used an inquiry approach and a context familiar to students to leverage initial conceptions of variability, aggregate, population, a need for data and the value of representation to record, analyse and communicate ideas about data.

1.2 Literature Review and Theoretical Framework

Statistical concepts that are isolated become atomistic and impoverished (Bakker & Derry, 2011). To develop rich statistical understandings, students must see how statistical concepts and structures are related to one another, to practices and conventions, to their prior knowledges and experiences, and their utility for solving problems. The focus of this literature review is on understanding links between students' reasoning in problem contexts and their reasoning about key structures in the discipline (mathematics or statistics).

Literature on informal learning environments has begun to establish how reasoning in context can strengthen students' valuing of mathematics and relationships between concepts. There has long been acknowledgement of a gap between students' formal and informal knowledge and reasoning (Confrey & Kazak, 2006; Raman, 2002; Sadler, 2004). Much of this is the result of teaching formal concepts before students have developed understanding of both their usefulness for solving problems and their connections to students' prior knowledge and belief structures. Because "mathematical ideas are fundamentally rooted in action and situated in activity"

Table 1.1 Mapping of statistical context–structures in Makar (2014)

Context entity	Statistical structures	Statistical context–structure and reasoning
Height	data	The measure of how tall a person is can be collected and recorded as height (cm) data
Height of a child	Single data point	A child is associated with their height data
Heights of students in the class	Aggregate	Collectively, the heights of the children in the class can be considered as an entity to investigate
Heights of children in the class differed	Variability	Because all heights in the class were not the same, the children had to grapple with how to manage the variability of the height data
Organised heights clumped in the middle	Distribution shape	When children invented ways to record and organise the data, they noticed that most heights were in the middle and fewer heights were high or low in value; this feature was stable across both classes
Typical height	Average	To find the typical height, children invented a point estimate to capture the most common height (mode) and an interval estimate to capture where “most” heights clumped. They used these estimates to predict (with uncertainty) the typical heights of children in other classrooms
Height of very tall child	Outlier	One child was substantially taller than the others and they considered this student to have atypical height. They reasoned that it was unlikely to see this height in other classes

(continued)

Table 1.1 (continued)

Context entity	Statistical structures	Statistical context–structure and reasoning
The heights of children in another class were collected and compared to their class	Sampling variability	Their surprise that the data in the class next door were similar to but different than their own class data prompted discussions about what aspects of their data were likely or unlikely to be encountered in other classes (e.g. similar values but different frequencies of each height; similar but possibly not exactly the same typical height)
The typical height of the children in one class was used to predict the typical height of children in another class and across Australia	sample-population inference	One Vietnamese child argued that her mother was considered short in Australia, but was of typical height in Vietnam. This prompted students to clarify that their classroom was not representative of other countries and that data would need to be collected from a country to find the typical heights there

(Confrey & Kazak, 2006, p. 322), learning concepts first informally as they are situated in problems allows students to build experiences over time with rich mathematical structures. These experiences with informal ideas also develop students' sense of the utility of mathematical ideas before their formalisation. "People extract information about the world more often than they are aware and that this knowledge exists in tacit form, influencing thought and behaviour while itself remaining mostly concealed from conscious awareness" (Litman & Reber, 2005, p. 440). For example, social practices (including mathematical conventions) can become adopted without the learner being conscious of what is being learned. Boekaerts and Minnaert (1999) argue that the active, non-threatening and explorative nature of informal learning can assist to develop and sustain students' learning in line with social goals and expectations elicited by the context, since "most informal learning contexts are more powerful for developing criteria for success, progress, and satisfaction, which are in accordance with the students' own need structure" (p. 542). Boekaerts and Minnaert further contend that informal learning can heighten students' valuing and learning goals because they perceive learning to be natural and spontaneous.

The theoretical framework in this chapter develops the idea of *statistical context–structures*. Statistical structures maintain consistent patterns (invariances), despite statistics being a field of variability. Statistical context–structures are conceptualised as a mapping between a connected web of statistical structures (concepts with their related characteristics, representations and processes) and contextual entities that stand in for the statistical structures, with relationships between the contextual entities corresponding to the relationships between the statistical structures. Reasoning about the contextual entities is analogous to reasoning about the statistical structures.

For example, the typical height of children in a classroom is a contextual entity that would allow students to reason about the concept of central tendency without explicitly learning about the statistical mean. Students' reasoning about the mean as a representative measure of Year 3 students' heights is still possible even though they have not formally learned what a mean is or how to calculate it. A key benefit is that their reasoning can include the relationship of the mean to other statistical concepts. A study by Makar (2014), for example, highlighted how Year 3 children (aged 7–8) reasoned about variability, distribution (shape, spread, centre, outliers) and sample-population inference as they wrestled with how to find the "typical height" of the children in their classroom. In the process, they invented and critiqued iterations of data displays of increasing sophistication resulting in a graph similar to a histogram. In this example, the children encountered multiple statistical context–structures (Table 1.1). None of the statistical structures they encountered were formalised, but by repeatedly reasoning about the context, the students gained important experiences with informal versions of advanced statistical structures on which they could later map onto the formal ideas (McGowen & Tall, 2010), while formally addressing the content for their own year level.[1] The role of the teacher was critical here to scaffold student learning through engineering learning experiences and using questioning to guide students' ideas. For example, the heights of the children in the class differed (see column 1, Table 1.1). Children were not formally taught the statistical structure "variability" (e.g. the concept of variability with its related terminology, characteristics, representations, measures and relationships with other statistical structures such as "distribution"), as this would not be appropriate content for 7–8-year-olds. Even without formally learning the statistical structure "variability" (see column 2), the children were able to work with variability in the context of managing the differing heights of the children in their class (see column 3). When children had to predict the typical height of Year 3 students in the class next door, they had to grapple with the variability of the height data in their class. Reasoning about differing heights in that context was analogous (and more age-appropriate) to reasoning about variability. The characteristics, representations and processes related to variability were, to the children, the characteristics, representations and processes needed for making sense of the differing heights.

In contrast, the mean is often taught as a calculation of a set of numbers to work out the "average" of that set. Multiple studies have highlighted how this approach has created an impoverished conceptualisation of central tendency as students neither

[1] In the Year 3 curriculum in Australia (Australian Curriculum: Mathematics, 2016), students would be expected to be able to identity an issue/question and relevant data to collect (ACMSP068), carry out a simple data investigation (ACMSP069) and interpret and compare data displays (ACMSP070).

see the mean as a representative value of a data set nor link it to related ideas of distribution, sampling or inference (Bakker & Derry, 2011; Konold et al., 2002; Mokros & Russell, 1995; Watson, 2006). Bakker and Derry (2011) have argued that an atomistic approach to learning in statistics, where ideas are taught in isolation, has resulted in a lack of coherence in students' statistical thinking. They contend that this has been one of the key challenges in statistics education. However, within rich well-engineered contexts, there are multiple and diverse ways and opportunities to work informally with foundational relationships among statistical structures.

1.3 Methodology

This article is based on a case study of a classroom of young children in the first year of schooling (called Foundation or Prep in Australia). Case study is beneficial to generate insights through "the complexities and contradictions" (Flyvbjerg, 2006, p. 237) of narrative as a problem is played out in practice. It creates opportunities for the researcher to wrestle with a theoretical problem through issues that arise, including serendipitously, in empirical details of the case.

> As an account of practice, explained analytically, case study is a valuable methodology for the research of educational practice, particularly given the scope for the representation of complex practice with multiple and bundled trajectories. Thus, while on the one hand the case attempts to represent complex practice; the case study is the analytical explanation, constructed and crafted to recount, analyse and generate … new ways of understanding complex practices. (Miles, 2015, pp. 315–316)

The case reported in this article used a retrospective analysis of data collected from a larger study that aimed to understand teachers' experiences over time in teaching mathematics through inquiry (e.g. Makar, 2012). At the time the lessons were conducted, the teacher and researcher were interested more generally in how young children respond to and are guided in inquiry. The retrospective analysis of the two lessons captured in this article allowed the researcher to study these lessons anew to seek insight into the way that the teacher and students utilised the problem context of the inquiry to scaffold the children's thinking about statistical concepts, representations and processes. In order words, the retrospective analysis was used by the author to identify the use of statistical context–structures and how the teacher used them to guide students' statistical reasoning.

1.3.1 Participants and Lessons

The participants in the case study were in a prep class (about 20–25 children, aged 4–5 years old) in a suburb of a major city in Australia (prep is equivalent to kindergarten in most countries). The teacher was highly experienced in teaching with inquiry but this was her first time teaching this age of class (previously she taught

Year 3, ages 7–8 years). The data in this paper relied on classroom videos from two 40 min lessons taught on consecutive days at the end of the second month of the school year (in Australia, the school year runs from late January to mid-December). In the first lesson, the teacher introduced the question, "Do most students in Prep L have blue eyes?" and as a class the students sought a method to find out. Iterations of investigation and discussion were used to build on children's experiences and resulting ideas, scaffolded by the teacher. Children individually followed methods that made sense to them, observed their peers' work and discussed their ongoing progress with the teacher and/or as a class. In the second lesson, children continued their progress towards answering the inquiry question using iterative cycles of investigation work and whole class discussion. The lesson wrapped up by counting children with each eye colour.

1.3.2 Data Collection and Analysis

Video data are not objective, nor do they capture all of what is happening in a class (Roschelle, 2000). The choice of placement is deliberate and depends on the research aims. In this study, there were two key placements of the camera—stationary or roving. In either case, the choices that were made were based on seeking insights into students' ideas and the teacher's interaction with them. The camera was used in a stationary mode (on the tripod) if the focus was on the whole class, for example, during sessions when students were seated altogether on the carpet (e.g. when lessons were introduced or during sharing sessions). This allowed for the researcher to gain both general context for the timeline of events and also captured individual contributions by the teacher and students. In particular, this was a critical aspect of data collection to focus on the teacher's questions and how she guided the learning, as well as students' articulation of their thinking at a particular stage of the lesson. Together, this focus on the teacher and students' sharing allowed for the evolution of ideas to be traced to when they were first introduced. The camera was in roving mode (on or off the tripod) when students were working at their tables. During working sessions, the camera either followed the teacher as she interacted with students or it captured students working at one of the tables.

The data were analysed retrospectively using a video analysis process adapted from Powell, Francisco and Maher (2003). The process included seven stages: (1) intent viewing, (2) describing the video data, (3) identifying critical events, (4) transcribing, (5) annotating, (6) constructing a storyline and (7) composing narrative (p. 413). In the initial three stages, the videoed lessons were observed and a video log was created with timestamps, screen-captured images and short-running descriptions of what was happening. Critical events were marked in the video log as rich segments for potential analysis to help focus the observation. These first three stages provided an overall picture of the lesson to ensure that the data were fit for purpose to move to the fourth stage (transcription). The transcript was used to select and annotate excerpts and construct a preliminary (but disjointed) storyline. The author met with the teacher of the lesson to discuss the storyline, clarify the researcher's observations and focus the direction of the narrative. The resulting narrative was developed by iteratively reviewing, editing and elaborating the initial storyline including a second consultation with the teacher.

1.4 Results

The results section will use data from a prep class (ages 4–5) as they investigated the question, *Do Most Children in Prep L have Blue Eyes?* This question came from a comment made in the class by one of the children during an activity about their own eye colour. In setting up this question, the teacher used this problem context to informally introduce five key statistical ideas and structures: (1) acknowledging variability as an issue to resolve; (2) recognising that the individual and the aggregate are related, but not the same; (3) distinguishing what the population is for the investigation; (4) being aware of the need for data and evidence; and (5) valuing representations as ways to record, analyse and communicate results from data in solving problems. The data across the two lessons are presented chronologically in order to illustrate the development of students thinking over the lessons, although the entirety of the lessons is not presented. The critical role that the teacher played is highlighted to scaffold and progress reasoning using the statistical context–structures.

1.4.1 Informally Introducing Variability, Aggregate and Population

In introducing the inquiry question, the teacher Ms Louarn asked students to express their initial thoughts about whether most students in the class had blue eyes. Because this question is about a characteristic of the class as a whole, it is a question about the aggregate. Ms Louarn encouraged students to share their ideas and emphasised when students observed that there were different eye colours in the class (variability). At the same time, she nudged their anecdotal comments towards thinking about the aggregate.

Oliver:	Some people have green eyes too.
Ms Louarn:	They certainly do. So, do you think that more people in prep would have green eyes or blue eyes?
Oliver:	Green eyes.
Ms Louarn:	You think lots of people would have green eyes. What do you think, Kai? …
Kai:	The lessest have green eyes
Ms Louarn:	Less. Is that what you're saying? So you think fewer people in prep have green eyes than blue eyes. [Lesson 1; starting at video timestamp 1:04]

Oliver's response could either have been an observation, or perhaps a counter-example to the question. That is, his point that "Some people have green eyes too" may have been an answer to the investigation question (Do most students in Prep L have blue eyes) using anecdotal evidence. To encourage Oliver to think about the aggregate question, Ms Louarn incorporated his response into the investigation question to ask him again. His response, again green eyes, was acknowledged before she moved on to another response. The teacher emphasised two key points: first, that there was variability in the class in relation to eye colour (linking difference between individuals with the variability of the aggregate), and second, that there was a lack of consensus about which eye colour in the class was the most common (an aggregate question). This second point suggested a need for evidence (data), a point Ms Louarn would return to. The problem under investigation allowed for students to reason about variability because not all eye colours were the same. It also allowed them to reason about characteristics of the aggregate (whether the majority of the class had blue eyes) as opposed to individuals, giving them experience reasoning about the aggregate.

As students continued to share, the opportunity arose to clarify the population under investigation when students mentioned their parents' eye colours.

Ava:	I think most of the people in this class, they have brown eyes.
Ms Louarn:	Do you know anybody with brown eyes?
Ava:	… Um, my mum does, my dad doesn't.
Ms Louarn:	Are your mum and dad in prep?
Ava:	No.
Ms Louarn:	It's great to know mum and dad's eyes. Let's just think about children in prep at the moment. … Kai?
Kai:	My dad has green eyes.
Ms Louarn:	Yes, so sometimes our parents have different eyes from us, and obviously you have got brown eyes and you're saying your dad has got green eyes. We are just going to talk about people in prep at the moment. [1:56]

Ava introduced a third eye colour, brown, as a possible answer to the inquiry question. She also went further to bring in others she knew, like her mother, who had brown eyes. This allowed Ms Louarn to press further to informally clarify the population that was the target of their inquiry. The response from Kai suggested that this point was not yet acknowledged by the children. Note, however, that the variability of eye colour was a tacit assumption within the problem space.

By this stage, early in the lesson, the children had begun to experience several statistical context–structures through discussing the question, *Do most students in Prep L have blue eyes?* Four statistical structures that they encountered at an informal level (recognised by adults as data, variability, aggregate, population) were not experienced in isolation, and they were experienced by the children within the problem context (their personal context), as context–structures. That is, when children reasoned about "eyes", they were reasoning about "data". As context–structures, the statistical structures were considered in relation to one another (e.g. different colours of eyes created a challenge to consider a question about "eyes" as an aggregate; the aggregate in question did not include their parents, who were outside the population). Variability, aggregate and population were also considered in relation to the statistical idea that data are evidence, which is the focus in the next section.

1.4.2 Suggesting a Need for Data

Throughout the sharing session, the teacher guided the discussion within the familiarity of the context, while concurrently and informally emphasising statistical relationships. It would have been possible for her not to emphasise these aspects by exploring, for example, children's eye colours in relation to their parents or encouraging general sharing about people who children knew had various eye colours. Ms Louarn also could have curtailed the discussion above by asking the children to sort their eye colour drawings into categories or stacking them like a bar graph. However, the teacher instead used the investigation to begin to informally develop statistical ideas, the need for evidence and the important role that data play in answering a statistical question.

Following the discussion above, Ms Louarn moved to elicit from the children an approach to address the inquiry question. Some of the seeds of this investigation had already been sown: the lack of consensus about which eye colour was most common, discussions of evidence (individual anecdote and aggregate) and suggestion of the population of focus. Students shared their ideas as Ms Louarn recorded them. Most children focused on initially just looking at their peers' eyes. For example, Will said, "We, um, we could go and look at eyes. We should go and look in the eyes". After this idea was repeated by other children, the teacher confirmed with a show of hands that most in the class agreed that they would go around and look at everyone's eyes in the class.

At this point, the children had (with assistance) suggested that in order to find out whether most children in the class had blue eyes, they would need to look at the eyes of the children in their class. Although this may seem obvious to an adult, this was an initial and tentative link between the question and a suggestion that evidence was needed to check if this claim was true. At this age, they were not yet thinking about how just looking at everyone's eyes could help them to answer the inquiry question. They were yet to recognise a need for data: to record their observations as they looked at eyes or to analyse their recordings to determine an answer.

Ms Louarn:	Who's got a different idea?
Mila:	I will look at, um, um, everyone's colours eyes, and I will, um, um, make a picture.
Ms Louarn:	Ah! Mila has got an interesting thing, she says she is going to look at everybody's eyes and then she is going make a picture. What sort of picture you would make Mila?
Mila:	(unintelligible) then I'm gonna to paint all of the eyes and then, I am gonna, um, um, and then I'm gonna put them in my, and then I'm going to make my own shop, and then I am gonna make lots of different colours of friends!
Ms Louarn:	So, I think this what you said. That are going to find out what colour eyes everybody's got and you're going to draw a picture of their eyes. Is that what you said? (Mila smiles and nods) That's a really an interesting idea. I'd like to think about that. (To the class) Do you think that might help us remember, whose eyes that we've got?
Students:	yes
Ms Louarn:	That's a great idea. We go and look at everybody's eyes and then we draw a picture, so that we can remember the colour of everybody eyes. Thank you Mila, I like that idea. [7:57]

Mila's mention of a drawing gave Ms Louarn an opportunity to reframe her suggestion as a way of recording their observations, emphasising the benefit of recording as a way to remember and keep track of whose eyes were observed. Sienna built on Mila's idea and suggested using the drawings to find out what everyone's eye colours were (and they'd be done).

Ms Louarn:	Ah! So you are suggesting that if we look at the pictures of ourselves that we could find out from them what colour eyes people have got. That's a good idea too. And what you would do after that? So you would look at ourselves over there, and then what would you do?
Sienna:	Then you look if you're right and if they're right. And you can see that they are right. [10:33]

Using Sienna's mention of their drawings, Ms Louarn privileged Sienna's idea to emphasise the benefit of using representations (rather than just "looking" at eyes); she further elaborated to suggest to students that these recorded drawings would still require another step. Jack further built on Sienna's idea, suggesting how having the drawings would allow them to go further to count.

Ms Louarn:	Yes, Jack?
Jack:	Look at everybody's eyes, look at my eyes and see if umm, count how many eyes is blue or not.
Ms Louarn:	Well, Jack just said something very interesting. So he is going to look at eyes as well, but then, then we can count the eyes when we make a picture, that is good idea! [11:31]

Three tentative statistical ideas were initiated in the discussion, ideas to build on over the course of the lessons: (1) a need for data (e.g. Will: "look at eyes") to answer the inquiry question; (2) the benefit of recording (e.g. Mila: "make a picture") to remember; and (3) recording was not enough, there was a need to analyse the data (e.g. Jack: "count how many eyes are blue or not"). These three ideas, in context, maintained a coherence of experiencing data as a statistical structure, with its characteristics (as an observation), representation (recording for memory) and processes (data collection was not enough; analysis was needed to answer the question).

1.4.3 Recording and Analysing the Data

The teacher decided to let them begin even though their plan was only partially constructed. Several children walked around and observed their peers' eyes and reported to Ms Louarn. Her response was to emphasise a need to record.

Thanh:	I found 8 blue eyes.
Ms Louarn:	You found 8 blue eyes! How are you going to remember that next time?
Thanh:	Try and remember?
Ms Louarn:	You're going to try and remember. And so do you think if you found 8 blue eyes, do you think more people in prep have blue eyes? (Student shakes head no and then shrugs shoulders.) [15:26]

After a few minutes, most children were at least looking at eyes. For some, they saw this as collecting evidence, and for others they were likely mimicking their peers.

A few children drew pictures of children's eyes, their own and/or others', with eyes coloured (Fig. 1.1). For students who were colouring only eyes (and not other facial features), they appeared to have moved towards an image of the eyes as the relevant aspect of the context to record (as opposed to other facial features). This abstraction of the eyes suggested a move towards seeing the recording as data. Even if only one child did so purposefully, others often followed. The discussions then became important to connect these practices to their utility in solving the problem.

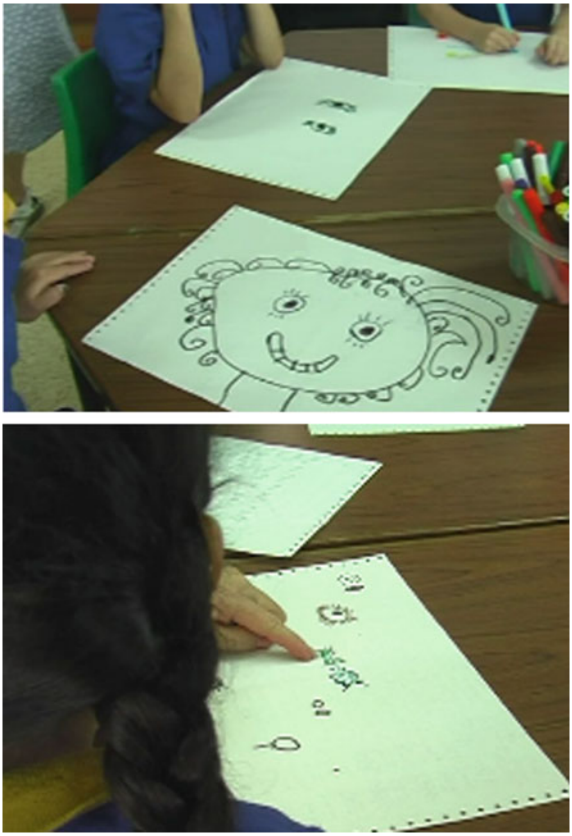

Fig. 1.1 Children's recordings of eye colours in the class

Ms Louarn temporarily stopped the class as they were working and reiterated the problem, "We're going to find if it is true that most people in prep have blue eyes". She suggested a number of options that students were undertaking to find out. After another 10 min, Ms Louarn asked students to bring their ideas back to the circle on the carpet, including drawings if they had them.

Ms Louarn:	[Children] did what they said they were going to do: Look at the eyes, some people said make a picture of the eyes, and some people said counting the eyes. So some people have done that. Would someone like to put their hand up and tell us what they found out about our question? What did you do Aisha?
Aisha:	Um I didn't get to do the Bec's hair. (She shows her drawing with two people's faces including hair, nose, eyes and mouth).
Ms Louarn:	… So you've got two people there. Are you going to draw a picture of everybody in the class and a picture of their eyes?
Aisha:	I don't know if I will be able to fit them on here.

Ms Louarn:	But is that your idea? (Aisha nods) I think that is a really clever idea. Aisha would draw a picture of everybody in the class and she would draw the colour of their eyes and that's a good way of making a picture isn't it? Tomorrow when we come back she will be able to remember it all. Thank you Aisha I think that is a really clever idea. You're right it might take a little while … but it's a great idea. [30:48]

Other students shared who had drawn the full face, hair and eyes of one or more people. Sienna had drawn eyes and numbers next to them (Fig. 1.2).

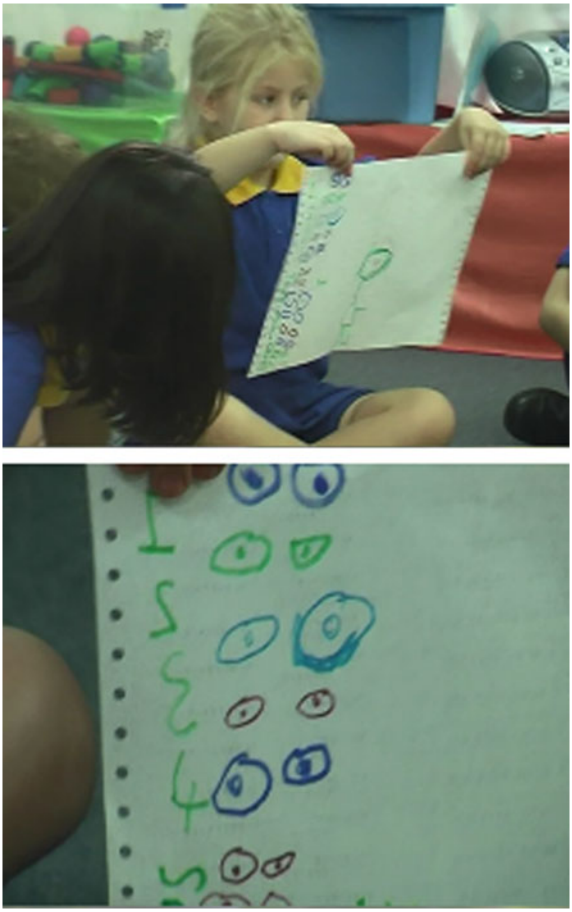

Fig. 1.2 Sienna's representation of eye colours

Ms Louarn:	What have you got there Sienna? Show everybody what you've done. And can you tell us all about that.
Sienna:	It's a list about people who have brown eyes and blue eyes and green eyes. Um most people do have the same colour eyes. I couldn't draw everyone's eyes.
Ms Louarn:	Why was that? Did you run out of time?
Sienna:	Yes.
Ms Louarn:	Is that what happened you ran out of time. (Sienna nods) So how many have you done so far? How many people have blue eyes?
Sienna:	(Sienna counts each individual blue eye and the teacher asks clarification if it is 12 eyes or 12 people. She counts again) … 1, 2, 3, 4, 5, 6 …
Ms Louarn:	So you got 6 people with blue eyes. … Whose eyes have you got there Sienna? (Sienna recalls the names.) Right, Sienna tomorrow that's going to be my first question so I want you to have a think between now and tomorrow, what can you do on your drawing—which is sensational by the way—to remember whose eyes they are? [35:01]

An emphasis throughout the lesson was on enculturating students into an expectation of representing and providing evidence of their investigation towards addressing the inquiry question, *Do most students in Prep L have blue eyes?* This consistent focus allowed students to enrich the connection between the problem context (responding to the inquiry question using their everyday knowledge) and relevant statistical structures (evidence which relied on data, representation, aggregate and analysis). For example, slowly through the lesson, more students adopted the practice of using eyes (rather than entire drawings) labelled with names to represent the students in the class. This strengthened the relationship between children's eyes (context) and structures (eyes as data, moving towards aggregate).

Sienna's acknowledgement showed emerging awareness that the drawings of eyes were contextual representations of data. This context–structure link allowed her to discuss "eyes" as "data". Ms Louarn recapped the ideas that had been presented and encouraged the other students to think about some of these ideas as they continued working towards addressing the inquiry question. The pattern continued the following day, periods of working interspersed with sharing; through iteration, most children adopted practices of drawing people or eyes recorded as data, as the teacher continually emphasised the benefits of observing, recording and counting to focus on the aggregate question.

1.5 Discussion

The focus of this paper was to examine the use of problem context as a proxy for working with statistical structures in a class of young children. It was not to provide evidence of individual success in understanding the links between the context and the statistics, but at an informal level provide children with a low-stakes opportunity to be exposed to and engage in reasoning with powerful statistical structures.

In the lessons presented, Australian children in Foundation Year (also called "Prep", which is similar to kindergarten, ages 4–5) sought to evaluate a peer's claim that most children in the class had blue eyes. The key structural elements of statistics that were informally introduced—variability, aggregate, population, data and representation—are critical as foundations for understanding any statistical concepts and practices. As these ideas were informally introduced, they became part of the problem space in subsequent discussions. The familiarity of the shared context of eye colour gave the children a way to reason about concepts—concretely and informally—through the context of the problem. The focus on data as evidence throughout the lessons allowed for discussion of informal versions of several statistical structures by allowing the context to stand in for those structures. This reasoning was similar to what would be done in later years using the more abstract statistical structures as part of that discussion. One mapping is given in Table 1.2 of the contextual elements that students experienced through the familiar context and the related statistical structure.

Although the statistical structures themselves and the links between the context and the related statistical structures were unknown to the children, their reasoning about the context (or emerging reasoning, or mimicking) paralleled more formal statistical reasoning that would be developed over time. For example, focusing on only considering the eye colours of the children in the classroom (rather than their parents) was explained in relation to the inquiry question about eye colour in their class. Their classmates were the population relevant to the inquiry, and the children's reasoning about their classmates' eye colours was analogous to reasoning more abstractly about a population. The statistical structures encountered by the children were not limited to this specific context. That is, although statistics is a field based on variability, the patterns and invariances within variability expose important statistical structures within the field. For example, relationships between data and population hold regardless of the context.

The context of the problem was content-rich and complex, allowing for multiple statistical concepts, relationships, tools and structures that had analogies in the context to be used not in isolation, but in relation to each other (holistically) and purposefully to solve a problem (cf. Bakker & Derry, 2011). Furthermore, it provided opportunity to enculturate the children in statistical practices. Ms Louarn, the teacher, played a key role in her questioning and privileging of focus ideas. She used the children's ideas to generate, build on and challenge their emerging strategies. Asking questions and critiquing ideas were also seen as valued practice (not the emphasis of this paper). The norms that were developed in the classroom allowed for productive interactions as children became accustomed to what was valued and normalised as part of the classroom culture. For example, in publicly sharing their ideas, children influenced peers to shift inefficient practices (e.g. drawing an entire person), provided ideas when others were stuck and generated opportunities for feedback (e.g. comparing one's own drawing with those shared in the circle).

Being aware of statistical context–structures is a valuable framework for teachers. By identifying elements of the problem context that stand in for statistical structures,

Table 1.2 Mapping statistical context–structures in the lessons

Context element	Related statistical structure	Links between contexts and context–structures
Eye colour	Data	It was necessary to observe children's eye colours to answer the inquiry question
Multiple eye colours	Variability	The variability of eye colour was the problem to be managed (otherwise no investigation would be needed)
Children in the class	Population	The eye colours of people outside of the class, like parents, were not relevant
Drawing of self	Single data point	Children drew themselves or a friend; these drawings represented a single data point
Drawings of eyes	Data representation	The need to record eye colour (and not hair colour) as evidence focused children on salient aspects to represent or ignore
Counting eyes	Data analysis	Counting provided a way to compare groups (blue- vs. brown-eyed children) to answer the inquiry question
Drawings of collections of eyes	Collection of data points	When students drew collections of eyes, their drawing represented collection of data points
Questions about the class	Focus on aggregate	The inquiry question required students to look beyond single or multiple individuals to consider collective qualities of the aggregate

teachers can become sensitised to problems that would likely engage in content aligned with the teacher's goals. This is often a challenge in inquiry when it can appear as though the content cannot be determined in advance. This will also assist the teacher in developing questions that will emphasise (through privileging and revoicing) or develop (through questioning) desired content out of children's ideas.

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