Solving LP Models for Multi-objective Matrix Games with I-Fuzzy Goals

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Abstract The principal objective of this work is to obtain the optimal strategies for a multi-objective two-person zero-sum matrix game with intuitionistic fuzzy goals (MOMGIFG). In this problem, the fuzziness in aspiration levels of both players are characterized by intuitionistic fuzzy sets. The developed linear models are solved in maxmin–minmax way using linear membership function (*mf*) and non-membership function (*nmf*). A numerical example is incorporated to demonstrate the proposed solution procedure.

Keywords Matrix games · Intuitionistic fuzzy goals · Optimal strategies Intuitionistic fuzzy sets

1 Introduction

Multi-objective game theory optimizes those multi-objective problems that involve two or more than two decision makers. In fact, real game problems cannot be characterized precisely because of fuzzy information about their elements. Various studies about the zero-sum matrix game models with two players have been done so far, e.g., [6–8, 10, 16] and references therein, where fuzziness in payoffs and goals are characterized by fuzzy sets. But, a situation in which an element feels a hesitation to belong or not belong to a subset of universe cannot be represented by fuzzy sets. Intuitionistic fuzzy sets (I-fuzzy sets) [4] can give a suitable description of such kind vague information. Firstly, Atanassov [5] used I-fuzzy set in game models. Thereafter, many researchers studied single- and multi-objective two-person zero-sum matrix game in I-fuzzy environment [1, 2, 11–13, 17, 18] and references therein.

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The focus of this paper is introducing a solution approach for MOMGIFG. The notion for the proposed technique is inspired from max–min principle of classical game theory.

The outline of this research work is as follows: Sect. 2 introduces some preliminaries which are relevant to this work such as I-fuzzy set, maxmin–minmax solution and decision-making principle in I-fuzzy environment. In Sect. 3, a single-objective game model in matrix form with I-fuzzy goals is reviewed under some assumptions. A solution procedure for MOMGIFG with a set of assumptions is proposed in Sect. 4. In Sect. 5, an example is given to demonstrate the effectiveness of present work.

2 Preliminaries

Present section concerns some necessary definitions and one principle which are used throughout this paper.

Definition 1 (*I-fuzzy Set*) An I-fuzzy set $\widetilde{T}$ on space S is defined by two functions, μ_+ and μ_-, such that $\mu_+(s) \in [0, 1]$ represents the grade of membership of s in $\widetilde{T}$ and $\mu_-(s) \in [0, 1]$ represents the grade of non-membership of s in $\widetilde{T}$ with condition $0 \leq \mu_+(s) + \mu_-(s) \leq 1$. The expression $\mu^h(s) = 1 - \mu_+(s) - \mu_-(s)$ is called degree of hesitancy of s in $\widetilde{T}$. An I-fuzzy set $\widetilde{T}$ is denoted by

$$\widetilde{T} = \{\langle s,\ \ \mu_+(s),\ \ \mu_-(s)\rangle \mid s \in S\}.$$

In this paper, the goals for each player are viewed as I-fuzzy sets. The meaning of the value of $\mu_+(s)$ for an I-fuzzy goal is the grade of satisfaction of I-fuzzy goal for an expected payoff, whereas the value of $\mu_-(s)$ represents the degree of dissatisfaction of I-fuzzy goal. Recently, some I-fuzzy and fuzzy programming in term of goal programming have been found in [9, 14, 15].

A MOMGIFG is described by multi-payoff matrices $M^1, M^2, \ldots, M^r$. In this problem, Player I and II are denoted by P_1 and P_2, respectively. Suppose that I-fuzzy goal for kth payoff for P_1 and P_2 is denoted by $\tilde{g}^k_{P_1}$ and $\tilde{g}^k_{P_2}$, respectively. It is supposed that the r objectives of P_1 are also the objectives for P_2.

Definition 2 The maxmin–minmax value w. r. t. the grade of satisfaction of an aggregated I-fuzzy goal to P_1 is

$$\max_{p \in U^m} \min_{q \in U^n} \min_{k} \quad \{\mu_{\tilde{g}^k_{P_1}+}(p^T M^k q)\} \tag{1}$$

$$\min_{p \in U^m} \max_{q \in U^n} \max_{k} \quad \{\mu_{\tilde{g}^k_{P_1}-}(p^T M^k q)\} \tag{2}$$

where U^m/U^n is mixed strategy space to P_1/P_2. Such a strategy p^* is known as the maxmin–minmax solution of matrix game with aggregated I-fuzzy goal for P_1.

Similarly, the maxmin–minmax value w. r. t. the grade of satisfaction of an aggregated I-fuzzy goal to P_2 is

$$\max_{q \in U^n} \min_{p \in U^m} \min_{k} \{\mu_{\tilde{g}^k_{P_2}+}(p^T M^k q)\} \tag{3}$$

$$\min_{q \in U^n} \max_{p \in U^m} \max_{k} \{\mu_{\tilde{g}^k_{P_2}-}(p^T M^k q)\}. \tag{4}$$

Such a strategy q^* is known as the maxmin–minmax solution of matrix game with aggregated I-fuzzy goal for P_2.

Definition 3 (*Angelov's Decision-Making Principle*) Suppose that there are m goals $A_1, A_2, \ldots, A_m$ and n constraints $B_1, B_2, \ldots, B_n$ in a domain of alternatives Ω. All these goals ($A_i's$) and constraints ($B_j's$) are I-fuzzy sets on Ω. Angelov [3] proposed that an I-fuzzy decision which is evaluated by a suitable aggregation of the I-fuzzy sets $A_i (i = 1, 2, \ldots, m)$ and $B_j (j = 1, 2, \ldots, n)$. He used fuzzy intersection and fuzzy union as aggregation operators. Therefore, an I-fuzzy decision D which is an I-fuzzy set, defined by $\mu_{D+} : \Omega \to [0, 1]$ given by $\mu_{D+}(\omega) = \min_{i,j} \left(\mu_{A_i+}(\omega),\ \mu_{B_j+}(\omega)\right)$ and $\mu_{D-} : \Omega \to [0, 1]$ given by $\mu_{D-}(\omega) = \max_{i,j} \left(\mu_{A_i-}(\omega),\ \mu_{B_j-}(\omega)\right)$.

The optimal decision can be obtained as $\max_{\omega} \mu_{D+}(\omega)$ and $\min_{\omega} \mu_{D-}(\omega)$.

According to this principle, the crisp version of above I-fuzzy optimization problem in linear programming (LP) form can be formulated as follows:

$$\max \quad (\alpha_+ - \alpha_-)$$

s.t.,

$$\begin{aligned}
\mu_{A_i+}(\omega) &\geq \alpha_+, \\
\mu_{A_i-}(\omega) &\leq \alpha_-, \quad (i = 1, 2, \ldots, m), \\
\mu_{B_j+}(\omega) &\geq \alpha_+, \\
\mu_{B_j-}(\omega) &\leq \alpha_-, \quad (j = 1, 2, \ldots, n), \\
\alpha_+ + \alpha_- &\leq 1, \\
\alpha_+ \geq \alpha_-, \ \alpha_- &\geq 0, \ \omega \geq 0.
\end{aligned} \tag{5}$$

Here, the optimal solution of model (5) is denoted by $(\omega^*, {\alpha_+}^*, {\alpha_-}^*)$.

3 Single-Objective Matrix Game with I-Fuzzy Goal (SOMGIFG)

Present section demonstrates in what way a SOMGIFG can be solved through a pair of linear programming problem (LPP).

Let $M = [m_{ij}]_{m\times n}$ denote a payoff matrix of real constants for P_1. Since game is zero-sum, so $-M = [-m_{ij}]_{m\times n}$ is payoff matrix for P_2. Here, U^m/U^n represents a set of mixed strategies for P_1/P_2. The sets U^m and U^n are defined as:

$$U^m = \{p = (p_1, p_2, \ldots, p_m)^T | \sum_{i=1 \text{ to } m} p_i = 1, p_i \geq 0\},$$

and

$$U^n = \{q = (q_1, q_2, \ldots, q_n)^T | \sum_{j=1 \text{ to } n} q_j = 1, q_j \geq 0\}.$$

In this work, the goals of P_1 and P_2 are characterized by I-fuzzy sets. Suppose that $\bar{v}_a$ is the aspiration level for P_1 with tolerance error p_a and $\bar{v}_r$ is the rejection level for P_1 with tolerance error p_r. For P_2, let $\underline{v}_a$ be aspiration level with tolerance error q_a and $\underline{v}_r$ be rejection level with tolerance error q_r.

To solve two-person zero-sum SOMGIFG, the following conditions are assumed as:

- (H_1) The I-fuzzy goals of both players P_1 and P_2 are represented by linear *mf* and *nmf*;
- (H_2) For P_1, $\bar{v}_r - p_r \leq \bar{v}_a - p_a$ & $\bar{v}_r \leq \bar{v}_a$;
- (H_3) For P_2, $\underline{v}_a + q_a \leq \underline{v}_r + q_r$ & $\underline{v}_a \leq \underline{v}_r$.

Using (H_1)–(H_2), the solution for optimization problem of P_1 will be produced as:

Theorem 1 [11] *The maxmin–minmax solution for P_1 is equivalent to the solution of a LPP which is described as*

$$\begin{aligned}
&\max \quad (\lambda_+ - \lambda_-) \\
&\text{s.t.,} \\
&\sum_{i=1 \text{ to } m} m_{ij} p_i + p_a - \bar{v}_a \geq p_a \lambda_+, \\
&\sum_{i=1 \text{ to } m} m_{ij} p_i - \bar{v}_r \geq -p_r \lambda_-, \quad (j = 1, 2, \ldots, n), \\
&\sum_{i=1 \text{ to } m} p_i = 1, 0 \leq \lambda_+, \lambda_- \leq 1, \\
&\lambda_+ + \lambda_- \leq 1, \lambda_+ \geq \lambda_-, p \geq 0.
\end{aligned} \tag{6}$$

Theorem 2 [11] *The maxmin–minmax solution for P_2 with assumptions (H_1) and (H_3) is equivalent to the solution of a LPP which is described as:*

$$
\begin{aligned}
&\max \quad (\eta_+ - \eta_-)\\
&\text{s.t.,}\\
&\sum_{j=1 \text{ to } n} m_{ij}q_j - \underline{v}_a - q_a \le -q_a\eta_+,\\
&\sum_{j=1 \text{ to } n} m_{ij}q_j - \underline{v}_r \le -q_r\eta_-, \quad (i = 1, 2, \ldots, m),\\
&\sum_{j=1 \text{ to } n} q_j = 1, 0 \le \eta_+, \eta_- \le 1,\\
&\eta_+ + \eta_- \le 1, \eta_+ \ge \eta_-, q \ge 0.
\end{aligned} \tag{7}
$$

4 Solution Procedure to MOMGIFG

In a multi-objective matrix game, each player has more than one objective and each objective is represented by a payoff matrix. Suppose that both players (P_1 and P_2) have same r objectives.

For this matrix game problem, following conditions are assumed as:

(H_4) The payoff values in each payoff matrix are real numbers;
(H_5) The fuzziness in aspiration level of each objective is represented by an I-fuzzy set; and
(H_6) *mf* and *nmf* for each I-fuzzy goal are linear.

Now, a methodology is proposed to obtain the models in LP form for strategic problem to P_1 and P_2, respectively, as follows:
Optimization problem for P_1
Suppose that *mf* and *nmf* of the I-fuzzy goal for kth objective of P_1 are denoted by $\mu_{\tilde{g}^k_{P_1}+}(p^T M^k q)$ and $\mu_{\tilde{g}^k_{P_1}-}(p^T M^k q)$, respectively. Using ($H_4$)–($H_6$), $\mu_{\tilde{g}^k_{P_1}+}(p^T M^k q)$ can be represented as

$$
\mu_{\tilde{g}^k_{P_1}+}(p^T M^k q) = \begin{cases} 0 & , \ p^T M^k q < \bar{v}^k_a - p^k_a,\\ 1 - \frac{\bar{v}^k_a - p^T M^k q}{p^k_a} & , \ \bar{v}^k_a - p^k_a \le p^T M^k q < \bar{v}^k_a,\\ 1 & , \ \bar{v}^k_a \le p^T M^k q, \end{cases} \tag{8}
$$

and *nmf* $\mu_{\tilde{g}^k_{P_1}-}(p^T M^k q)$ is

$$
\mu_{\tilde{g}^k_{P_1}-}(p^T M^k q) = \begin{cases} 1 & , \ p^T M^k q < \bar{v}^k_r - p^k_r,\\ 1 - \frac{p^T M^k q - (\bar{v}^k_r - p^k_r)}{p^k_r} & , \ \bar{v}^k_r - p^k_r \le p^T M^k q < \bar{v}^k_r,\\ 0 & , \ \bar{v}^k_r \le p^T M^k q, \end{cases} \tag{9}
$$

with conditions $\bar{v}^k_r - p^k_r \le \bar{v}^k_a - p^k_a$ and $\bar{v}^k_r \le \bar{v}^k_a$.

Using [3], *mf* and *nmf* for aggregated I-fuzzy goal to P_1 can be formed in respective order as:

$$\min_{k} \; \{\mu_{\tilde{g}^k_{P_1}+}(p^T M^k q)\} \tag{10}$$

and,

$$\max_{k} \; \{\mu_{\tilde{g}^k_{P_1}-}(p^T M^k q)\} \tag{11}$$

Assuming that

(H_7) The calculating *mf* in (10) and *nmf* in (11) are linear.

The maxmin–minmax value in terms of degree of acceptance of an aggregated I-fuzzy goal to P_1 is

$$\max_{p \in U^m} \min_{q \in U^n} \min_{k} \; \{\mu_{\tilde{g}^k_{P_1}+}(p^T M^k q)\},$$

$$\min_{p \in U^m} \max_{q \in U^n} \max_{k} \; \{\mu_{\tilde{g}^k_{P_1}-}(p^T M^k q)\}.$$

Theorem 3 *The maxmin–minmax solution for P_1 with assumption (H_7) is equivalent to the following LP model*

$$\begin{aligned}
&\max \quad (\lambda_+ - \lambda_-)\\
&\text{s.t.,}\\
&\sum_{i=1 \text{ to } m} m^k_{ij} p_i + p^k_a - \bar{v}^k_a \;\geq\; p^k_a \lambda_+,\\
&\sum_{i=1 \text{ to } m} m^k_{ij} p_i - \bar{v}^k_r \;\geq\; -p^k_r \lambda_-, \quad (j = 1, 2, \ldots, n),\\
&\sum_{i=1 \text{ to } m} p_i \;=\; 1, 0 \leq \lambda_+, \lambda_- \;\leq\; 1,\\
&\lambda_+ + \lambda_- \;\leq\; 1, \lambda_+ \geq \lambda_-, p \geq 0,
\end{aligned} \tag{12}$$

where $k = 1, 2, \ldots, r$.

Proof The maxmin–minmax problem for P_1 is

$$\max_{p \in U^m} \min_{q \in U^n} \min_{k} \; \{\mu_{\tilde{g}^k_{P_1}+}(p^T M^k q)\},$$

$$\min_{p \in U^m} \max_{q \in U^n} \max_{k} \; \{\mu_{\tilde{g}^k_{P_1}-}(p^T M^k q)\}.$$

For *mf*

$$\begin{aligned}
&\max_{p \in U^m} \min_{q \in U^n} \min_{k} \left(1 - \frac{\bar{v}_a^k - p^T M^k q}{p_a^k}\right) \\
&= \frac{1}{p_a^k} \max_{p \in U^m} \min_{q \in U^n} \min_{k} \left(\sum_{i=1 \text{ to } m} \sum_{j=1 \text{ to } n} m_{ij}^k p_i q_j + c^k\right) \\
&= \frac{1}{p_a^k} \max_{p \in U^m} \min_{k} \min_{q \in U^n} \sum_{j=1 \text{ to } n} \left(\sum_{i=1 \text{ to } m} m_{ij}^k p_i + c^k\right) q_j \\
&= \frac{1}{p_a^k} \max_{p \in U^m} \min_{k} \min_{j \in J} \left(\sum_{i=1 \text{ to } m} m_{ij}^k p_i + c^k\right).
\end{aligned}$$

Let $\min_{j \in J} \left(\sum_{i=1 \text{ to } m} m_{ij}^k p_i + c^k\right) = \lambda_{k+}$ and further let $\min_{k} \lambda_{k+} = \lambda_{+}$. In similar way, for *nmf*, letting $\max_{k} \lambda_{k-} = \lambda_{-}$. The maxmin–minmax problem for P_1 reduces to LP model (12).

Optimization problem for P_2

Let *mf* and *nmf* of an I-fuzzy goal for k^{th} objective of P_2 be denoted by $\mu_{\tilde{g}_{P_2}^k+}(p^T M^k q)$ and $\mu_{\tilde{g}_{P_2}^k-}(p^T M^k q)$, respectively. Using (H_4)–(H_6), $\mu_{\tilde{g}_{P_2}^k+}(p^T M^k q)$ can be represented as

$$\mu_{\tilde{g}_{P_2}^k+}(p^T M^k q) = \begin{cases} 1 & , \ p^T M^k q < \underline{v}_a^k, \\ 1 - \frac{p^T M^k q - \underline{v}_a^k}{q_a^k} & , \ \underline{v}_a^k \le p^T M^k q < \underline{v}_a^k + q_a^k, \\ 0 & , \ \underline{v}_a^k + q_a^k \le p^T M^k q, \end{cases} \tag{13}$$

and $\mu_{\tilde{g}_{P_2}^k-}(p^T M^k q)$ is

$$\mu_{\tilde{g}_{P_2}^k-}(p^T M^k q) = \begin{cases} 0 & , \ p^T M^k q < \underline{v}_r^k, \\ \frac{p^T M^k q - \underline{v}_r^k}{q_r^k} & , \ \underline{v}_r^k \le p^T M^k q < \underline{v}_r^k + q_r^k, \\ 1 & , \ \underline{v}_r^k + q_r^k \le p^T M^k q, \end{cases} \tag{14}$$

with conditions $\underline{v}_a^k + q_a^k \le \underline{v}_r^k + q_r^k$ and $\underline{v}_a^k \le \underline{v}_r^k$ for $k = 1, 2, \ldots, r$.

Using [3], *mf* and *nmf* for aggregated I-fuzzy goal can be calculated in respective order as

$$\min_{k} \ \{\mu_{\tilde{g}_{P_2}^k+}(p^T M^k q)\} \tag{15}$$

and,

$$\max_{k} \ \{\mu_{\tilde{g}^k_{P_2}-}(p^T M^k q)\} \tag{16}$$

In similar to problem of P_1, assuming that

(H_8) The calculating *mf* in (15) and *nmf* in (16) are linear.

The maxmin–minmax value in terms of the degree of acceptance of an aggregated I-fuzzy goal to P_2 is

$$\max_{p \in U^m} \min_{q \in U^n} \min_{k} \ \{\mu_{\tilde{g}^k_{P_2}+}(p^T M^k q)\},$$

$$\min_{p \in U^m} \max_{q \in U^n} \max_{k} \ \{\mu_{\tilde{g}^k_{P_2}-}(p^T M^k q)\}.$$

Theorem 4 *The maxmin–minmax solution for P_2 with assumption (H_8) is equivalent to the following LP model*

$$\begin{aligned}
&\max \quad (\eta_+ - \eta_-)\\
&\text{s.t.,}\\
&\sum_{j=1 \text{ to } n} m^k_{ij} q_j - \underline{v}^k_a - q^k_a \le -q^k_a \eta_+,\\
&\sum_{j=1 \text{ to } n} m^k_{ij} q_j - \underline{v}^k_r \le -q^k_r \eta_-, \quad (i = 1, 2, \ldots, m),\\
&\sum_{j=1 \text{ to } n} q_j = 1, 0 \le \eta_+, \eta_- \le 1,\\
&\eta_+ + \eta_- \le 1, \eta_+ \ge \eta_-, q \ge 0,
\end{aligned} \tag{17}$$

where $k = 1, 2, \ldots, r$.

Proof Proof is similar to Theorem 3.

5 Example

This section consists of an example of MOMGIFG which shows the validity of the proposed work.

The payoff matrices M^1, M^2 are separately indicated as:

$$M^1 = \begin{pmatrix} 4 & 2 & -1 \\ -2 & 0 & 1 \end{pmatrix}, M^2 = \begin{pmatrix} 10 & 24 & 9 \\ 7 & 15 & 11 \end{pmatrix}$$

Here, we assume that
$\bar{v}_a^1 = 3,\ p_a^1 = 4,\ \bar{v}_r^1 = 2,\ p_r^1 = 6$ and $\bar{v}_a^2 = 10,\ p_a^2 = 5,\ \bar{v}_r^2 = 7,\ p_r^2 = 4.$

Now, model (12) becomes,

$$\max \quad (\lambda_+ - \lambda_-)$$

s.t.,

$$
\begin{aligned}
4p_1 - 2p_2 + 1 \geq 4\lambda_+ \ &,\ 2p_1 + 1 \geq 4\lambda_+,\\
-p_1 + p_2 \geq 4\lambda_+ \ &,\ 10p_1 + 7p_2 - 5 \geq 5\lambda_+,\\
24p_1 + 15p_2 - 5 \geq 5\lambda_+ \ &,\ 9p_1 + 11p_2 - 5 \geq 5\lambda_+,\\
4p_1 - 2p_2 - 2 \geq -6\lambda_- \ &,\ 2p_1 - 2 \geq -6\lambda_-,\\
-p_1 + p_2 - 2 \geq -6\lambda_- \ &,\ 10p_1 + 7p_2 - 7 \geq -4\lambda_-,\\
24p_1 + 15p_2 - 7 \geq -4\lambda_- \ &,\ 9p_1 + 11p_2 - 7 \geq -4\lambda_-,\\
p_1 + p_2 = 1 \ &,\ \lambda_+ + \lambda_- \ \leq \ 1,\\
p_1, p_2 \geq 0 \ &,\ \lambda_+ \geq \lambda_-, \lambda_- \geq 0.
\end{aligned}
\tag{18}
$$

The optimal solution for P_1 is obtained as;
$\left(p^* = (0.3750,\ 0.6250)^T,\ \lambda_+{}^* = 0.3125,\ \lambda_-{}^* = 0.2917\right)$.

For P_2, we take $\underline{v}_a^1 = -2,\ q_a^1 = 5,\ \underline{v}_r^1 = 0,\ q_r^1 = 4$ and $\underline{v}_a^2 = 7,\ q_a^2 = 4,\ \underline{v}_r^2 = 10,\ q_r^2 = 5.$

Model (17) is reduced as follows,

$$\max \quad (\eta_+ - \eta_-)$$

s.t.,

$$
\begin{aligned}
4q_1 + 2q_2 - q_3 - 3 \leq -5\eta_+ \ &,\ -2q_1 + q_3 - 3 \leq -5\eta_+,\\
10q_1 + 24q_2 + 9q_3 - 11 \leq -4\eta_+ \ &,\ 7q_1 + 15q_2 + 11q_3 - 11 \leq -4\eta_+,\\
4q_1 + 2q_2 - q_3 \leq -4\eta_- \ &,\ -2q_1 + q_3 \leq -4\eta_-,\\
10q_1 + 24q_2 + 9q_3 - 10 \leq -5\eta_- \ &,\ 7q_1 + 15q_2 + 11q_3 - 10 \leq -5\eta_-,\\
q_1 + q_2 + q_3 = 1 \ &,\ \eta_+ + \eta_- \ \leq \ 1,\\
q_1, q_2, q_3 \geq 0 \ &,\ \eta_+ \geq \eta_-, \eta_- \geq 0.
\end{aligned}
\tag{19}
$$

The optimal solution for P_2 is obtained as;
$\left(q^* = (0.25,\ 0,\ 0.75)^T,\ \eta_+{}^* = 0.25,\ \eta_-{}^* = 0.0625\right)$.

These results are calculated by TORA software.

6 Conclusions

A solution procedure is introduced for MOMGIFG in this paper. This work shows that the strategic problems for both players are equivalent to two LPP. An example is given to show the existence of this theory. The author intends to study a case in which assumption (H_4) is violated, i.e., entries of payoff matrices having fuzziness in future.

References

1. Aggarwal, A., Dubey, D., Chandra, S., Mehra, A.: Application of Atanassor's I-fuzzy set theory to matrix games with fuzzy goals and fuzzy payoffs. Fuzzy Info. Eng. **4**, 401–414 (2012)
2. Aggarwal, A., Mehra, A., Chandra, S.: Application of linear programming with I-fuzzy sets to matrix games with I-fuzzy goals. Fuzzy Opt. Decis. Mak. **11**, 465–480 (2012)
3. Angelov, P.P.: Optimization in an intuitionistic fuzzy environment. Fuzzy Sets Syst. **86**, 299–306 (1997)
4. Atanassov, K.T.: Intuitionistic fuzzy sets. Fuzzy Sets Syst. **50**, 87–96 (1986)
5. Atanassov, K.T.: Ideas for intuitionistic fuzzy equations, inequalities and optimization. Notes Intuitionistic Fuzzy Sets **1**(1), 17–24 (1995)
6. Bector, C.R., Chandra, S., Vijay, V.: Matrix games with fuzzy goals and fuzzy linear programming duality. Fuzzy Opt. Decis. Mak. **3**, 255–269 (2004)
7. Campos, L.: Fuzzy linear programming models to solve fuzzy matrix games. Fuzzy Sets Syst. **32**, 275–289 (1989)
8. Kumar, S.: Max-min solution approach for multi-objective matrix game with fuzzy goals. Yugoslav J. Oper. Res. **26**(1), 51–60 (2016)
9. Kumar, S.: The relationship between intuitionistic fuzzy programming and goal programming. In: Proceedings of 6th International Conference on Soft Computing for Problem Solving, School of Mathematics, pp. 220-229. Thapar University, Patiaila, Punjab (India), 3–24 Dec 2017
10. Li, D.F.: A fast approach to compute fuzzy values of matrix games with payoffs of triangular fuzzy numbers. Eur. J. Oper. Res. **223**, 421–429 (2012)
11. Nan, J.X., Li, D.F.: Linear programming approach to matrix games with intuitionstic fuzzy goals. Int. J. Comput. Intell. syst. **6**(1), 186–197 (2013)
12. Nan, J.X., Li, D.F., Zhang, M.J.: A lexicographic method for matrix games with payoffs of triangular intuitionistic fuzzy numbers. Int. J. Comput. Intell. Syst. **3**(3), 280–289 (2010)
13. Pandey, D., Kumar, S.: Modified approach to multi-objective matrix game with vague payoffs. J. Int. Acad. Phy. Sci. **14**(2), 149–157 (2010)
14. Pandey, D., Kumar, S.: Fuzzy multi-objective fractional goal programming using tolerance. Int. J. Math. Sci. Eng. Appl. **5**(1), 175–187 (2011)
15. Pandey, D., Kumar, S.: Fuzzy optimization of primal-dual pair using piecewise linear membership functions. Yugoslav J. Oper. Res. **22**(2), 97–106 (2012)
16. Sakawa, M., Nishizaki, I.: Max-min solutions for fuzzy multiobjective matrix games. Fuzzy Sets Syst. **67**, 53–69 (1994)
17. Seikh, M.R., Nayak, P.K., Pal, M.: Application of intuitionistic fuzzy mathematical programming with exponential membership and quadratic non-membership functions in matrix games. Ann. Fuzzy Math. Infor. **9**(2), 183–195 (2015)
18. Seikh, M.R., Nayak, P.K., Pal, M.: Matrix games with intuitionistic fuzzy pay-offs. J. Inf. Opt. Sci. **36**, 159–181 (2015)