Chapter 6
Flood Coincidence Risk Analysis Using Multivariate Copula Functions

6.1 Introduction

Disastrous floods can be caused by unusual combinations of hydrometeorological factors and river basin conditions. Topography, land cover, and temporal and spatial distribution of rainfall play a dominant role in the generation of floods, which can be reflected in the contributions that major tributaries make to the mainstream flow. The coincidence of flood flows of mainstream and its tributaries may determine the peak flow. Therefore, the risk of flooding due to the combination of flood flows from different rivers is important for hydraulic design. The combination risk arises when large floods occur simultaneously in the mainstream as well as in its tributaries, and this risk is characterized regarding flood magnitude and occurrence date. Traditional methods focus only on the flood magnitudes, and a more realistic approach is therefore needed.

The traditional approach to the risk assessment entails determining the probability that a pre-selected value of the flood characteristic will be exceeded or equivalently determining the return period (Prohaska et al. 2008). This approach is based on univariate frequency analysis or regional frequency analysis. However, this approach does not consider the correlation of flows from different regions. The risk of combining floods involves at least two sites in the mainstream and its tributaries or two tributaries. This suggests that a multivariate hydrological analysis, which considers the dependence between flood variables, is needed.

Prohaska et al. (2008) used a two-dimensional probability distribution to evaluate the coincidence of floods on two adjacent streams, on the assumption that floods followed log-normal distribution. The use of the log-normal distribution for representing the frequency distribution of peak flow is not supported by hydrologic practices in many countries. For example, the Pearson three (P-Ш) distribution is assumed for frequency analysis of flood peaks in China (MWR 1993), log-Pearson

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three in the U.S. (IACWD 1982) and the generalized logistic (GL) distribution in the UK (Robson and Reed 1999). Further, Prohaska's study is limited to only two variables. It is usual that there is more than one tributary of the mainstream.

For these reasons, a new multivariate model, based on the copula function, is applied in this study. Most of these studies involve bivariate copulas (Kao and Govindaraju 2010; De Michele and Salvadori 2003; Favre et al. 2004; Shiau et al. 2006; Dupuis 2007; Zhang and Singh 2006, 2007b). Trivariate copula functions also have been used. Grimaldi and Serinaldi (2006) applied the Archimedean copula to model the trivariate joint distribution of floods. Serinaldi and Grimaldi (2007) described an inference procedure to carry out a trivariate frequency analysis via asymmetric Archimedean copulas. Zhang and Singh (2007a, c) applied the Archimedean copulas to trivariate frequency analysis of floods as well as rainfall events. Kao and Govindaraju (2008) applied the Plackett copulas to trivariate statistical analysis of extreme rainfall events (e.g., Song and Singh 2010b); Song and Singh (2010b) modeled the joint probability distribution of drought duration, severity and inter-arrival time using a trivariate Plackett copula. Applications of four-dimensional copula functions in hydrological fields have also been reported recently. De Michele et al. (2007) introduced a method for constructing multivariate distributions, given 2-copulas for each bivariate marginal law and applied the method to provide a four-dimensional characterization of sea state statistics. Serinaldi et al. (2009) used a four-dimensional student copula to analyze drought probabilistic characteristics. Since more variables are involved, four-dimensional copulas will be used in this study.

The content of this chapter is to apply a multivariate copula to analyze the coincidence flood risk of rivers. The upper Yangtze and Colorado River are selected as case studies. Daily flow data from four sites at the upper Yangtze and Colorado River is chosen. Four-dimensional copula functions are applied to construct the joint distribution of flood occurrence dates and magnitudes. The von Mises distribution is used to describe the flood occurrence dates, while the Pearson type three (P-Ш) and log Pearson type three distributions are selected as the marginal distribution of annual maximum flood peaks. The coincidence probabilities of flood magnitudes and occurrence dates are analyzed. The conditional probabilities for the Three Gorges Reservoir (TGR) are calculated.

6.2 Methodology

In this section, copula functions are selected to construct the joint distribution. The detailed information of copula theory can be found in Chap. 2. The von Mises distribution is selected as a marginal distribution function for flood occurrence dates, and the characteristic and expression of the von Mises distribution are described in Chap. 3.

The Flood Estimation Handbook (Reed 1999) states the flood risk assessment is to estimate the risk of a flood occurrence. The Environment Agency's Strategy for Flood Risk Management 2003/4-2007/8 (EA 2003) states that one task of flood risk estimation is to estimate the chance of a probability of a certain flood event. A methodology is presented herein for the estimation of a kind of special flood event, namely the coincidence of flood flows in the main river and its tributary. The term coincidence is used to denote the simultaneous occurrence of floods at two (or more) rivers. The degree of coincidence is measured by the probability of flood events. The theoretical background draws from the practical application of a multivariate probability distribution function, or its conditional probabilities (Prohaska et al. 2008). As flood events are characterized by flood occurrence dates and magnitudes, both of the two factors should be considered. This study considered the quantitative characteristics of simultaneous floods on the main river and its tributaries, and the flood dates of simultaneous floods.

First, flood magnitude is selected as a reference variable for analysis (Favre et al. 2004). The P-III and log P-III distributions are selected as marginal distribution functions for flood magnitude. The copula function is used to establish the joint distribution. The exceedance probability of coinciding flood volumes considered in flow profiles is defined as:

$$P_{Q_n}^T = P(Q_1 > q_1^T, Q_i > q_i^T, \ldots, Q_n > q_n^T) \tag{6.1}$$

where $P_{Q_n}^T$ is the exceedance probability of coinciding flood magnitudes; i is the ith gauge station; n is the number of variables and can be equal to two, three, and four in this study; $Q_1 \ldots Q_i \ldots Q_n$ are flow magnitudes; $q_1^T \ldots q_i^T \ldots q_n^T$ mean the design flood volume for the return period T.

Second, flood date is selected as a reference variable for analysis. In this study, if annual maximum floods occur within dt days, the floods were defined as contemporary temporal occurrences. The coincidence probability of flood dates at two or more considered inflow profiles is defined as:

$$P_n^t = P_t(t_k < T_i \le t_{k+1}, t_k - dt_{ij} < T_j \le t_{k+1} + dt_{ij}, \ldots, t_k - dt_{in} < T_n \le t_{k+1} + dt_{in}) \tag{6.2}$$

where i, j represent any river in the data set, and gauge station j is located downstream of the catchment; T_i means the random variable of flood occurrence dates, and dt is the time interval and equals one day in this study. The flood travel time between the two sites also should be considered. Equation 6.2 can compute the probabilities of simultaneous floods for two, three rivers in the basin. To calculate P_n^t, the marginal distribution for T_i is needed to build first. The von Mises distribution was selected as a marginal distribution function for flood occurrence dates. The detailed information for deriving the distribution of flood occurrence dates is given in Chaps. 3 and 4. Then, the joint distribution is built for evaluating the coincidence probability of flood dates. The detailed information for establishing copulas is given in Chap. 2.

Third, both the flood magnitudes and flood dates are selected as reference variables for risk analysis. Assuming that the flood occurrence dates are independent of flood magnitudes and flood peaks occur simultaneously at two or more rivers in the same basin, the flood coincidence probabilities of rivers for given flood magnitudes were estimated as

$$P_n^T = \sum_{t=1}^{N} P_n^t \cdot P(Q_1 > q_1^T, Q_i > q_i^T, \ldots, Q_n > q_n^T) \tag{6.3}$$

6.3 Data

The upper Yangtze River, which is the longest river in China and third longest in the world, is selected as a case study. The Three Gorges Project (TGP) is located on the Yangtze River. Floods in the middle and lower reaches of the Yangtze River mainly stem from the upper region of Yichang site, which is also the control site for TGP. Usually, the flood volume of upper Yichang site is about 50% of the total flow volume of the Yangtze River, about 90% of the Jingjiang River reach, which is regarded as the most key area for flood prevention. Hence, studying flood characteristics in upper Yangtze River is an important task for flood prevention.

The upper Yangtze River comprises a complex of tributaries, principally Yalong River, Min River, Jialing River on the left bank, and Wu River on the right bank. A schematic of the regional main tributary rivers and gauging stations is shown in Fig. 6.1. Some basic features of the available data are given in Table 6.1. Yalong

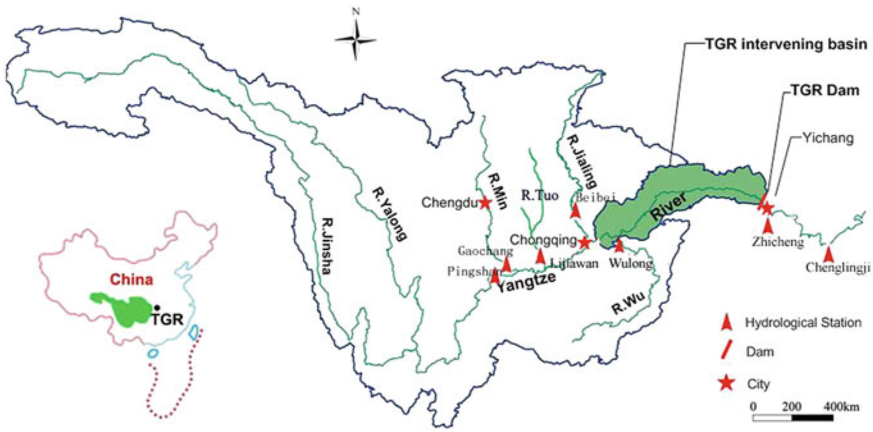


Fig. 6.1 Locations of regional tributary rivers and gaging stations

Table 6.1 Major tributaries to the upstream Yangtze River

Major tributary	Catchment area (km^2) >80,000 km^2	Record of length	Yangtze River	Major tributary	Catchment area (km^2) >80,000 km^2	Record of length
				Yalong River	144,200	1951–2007
	485,099	1951–2007	Jinsha River			
Min River	135,400	1951–2007				
Jialing River	157,900	1951–2007				
				Wu River	87,920	1951–2007
	1,005,501	1951–2007	Yichang (TGR)		1,005,501	1951–2007

River joins Jinsha River which is also recognized as part of the Yangtze River. Therefore, the Jinsha River, instead of the Yalong River, is used in this study. Relative frequencies of annual maximum floods in these rivers are calculated, as graphed in Fig. 6.1. It is shown that large floods always occur in July and August, except in the Wu River, in which the highest RF occurs in the middle of June. Therefore, it is more likely that floods in Jinsha River, Min River, and Jialing River occur simultaneously. Therefore, Jinsha, Min and Jialing rivers are selected in this study. Yichang site at the location of TGP is an important site on the Yangtze River and is also selected.

The Colorado River, in the Southwestern United States and northwestern Mexico, approximately 1450 miles (2330 km) long (Munro 1992), is also selected as a case study. The Colorado River above Lees Ferry is defined as upper Colorado River basin with about 17,800 square miles. The Colorado River originates in the mountains of central Colorado and flows about 230 miles southwest into Utah. There are some tributaries in the upper Colorado River basin, principally Green River, Gunnison River and San Juan River. A schematic of the regional main tributary rivers and gauging stations is shown in Table 6.2. The Green River, located in the western United States, is the chief tributary of the Colorado River. The watershed of the river, the Green River basin, covers parts of Wyoming, Utah, and Colorado. It is only slightly smaller than that of the Colorado when the two rivers merge. The average yearly mean flow of the river at Green River, Utah, is 173.3 m^3/s (6121 cubic feet) (Enright et al. 2008). The Gunnison River is a significant tributary of the Colorado River, 264 km (164 miles) long, in the southwest state of Colorado (U.S. Geological Survey 2011). It is the fifth largest tributary of

Table 6.2 Major tributaries to the upper Colorado River

Major tributary	Catchment area (km^2)	Record of length	Colorado River	Major tributary	Catchment area (km^2)	Record of length
	20,800	1933–2011	Above Cameo		20,800	1933–2011
Green River	44,850	1894–2011		Gunnison River	20,533	1896–2011
				San Juan River	12,000	1914–2011
	111,800	1921–2011	Lees Ferry		111,800	1921–2011

the Colorado River, with a mean flow of 122 m^3/s (4320 ft^3/s). The San Juan River is a tributary of the Colorado River in the southwestern United States, about 616 km (383 miles) long, the mean flow of which is about 62.4 m^3/s (2205 cubic feet per second) at its mouth (U.S. Geological Survey 2011). Comparing with the other two major tributaries, the mean flow of San Juan River is relatively smaller. Therefore, only Green and Gunnison River are considered in this study. As Lees Ferry is the division site between upper and lower Colorado River, this site is considered. The site near Grand Junction (named upper Cor. hereafter) is selected for analyzing the flow above Cameo of Colorado River. Therefore, four sites in Colorado River basin are considered in this study.

Pairwise dependence structures of the four stations in the two river basins are estimated. Empirical estimates of bivariate Kendall's τ of flood magnitudes and occurrence dates for all the pairs of interest here are given in Tables 6.3 and 6.4. The correlation coefficient between the Beibei and Yichang stations in upper

Table 6.3 Values of Kendall's τ of flood magnitudes and occurrence dates for all pairs of the four stations in the upper Colorado River

Stations	Above Cameo	Green River	Gunnison River	Lees Ferry
Above Cameo	1.00	0.68	0.66	0.49
Green River	0.37	1.00	0.58	0.42
Gunnison River	0.19	0.32	1.00	0.50
Lees Ferry	0.19	0.19	0.13	1.00

Note Upper triangular matrix is Kendall's τ of flood magnitude, and the lower triangular matrix is Kendall's τ of flood dates. The meaning is the same hereafter

Table 6.4 Values of Kendall's τ of flood magnitudes and occurrence dates for all pairs of the four stations in the upper Yangtze River

Stations	Pingsha	Gaochang	Beibei	Yichang
Pingsha	1.00	0.11	−0.08	0.28
Gaochang	0.07	1.00	0.03	0.21
Beibei	0.08	0.08	1.00	0.32
Yichang	0.19	0.18	0.34	1.00

Yangtze River is negative, but it is very small and close to 0. This means that the association between the two variables can be negligible and the Gumbel copula is therefore used.

6.4 Application

6.4.1 Estimation of Marginal Distributions

In order to show the validity of the mixed von Mises distribution, other distributions, such as Gumbel, normal, and Pearson III distributions, are selected as possible marginal distributions for the upper Yangtze River. Parameters of the mixed von Mises distribution are estimated by the maximum likelihood method. Parameters of other distributions are estimated by the L-moment method. Then these distributions are fitted to the data and compared with the mixed von Mises distribution. The best-fitted distributions are selected using the root mean square error (RMSE) values shown in Table 6.5 (Zhang and Singh 2007b). It is found that the mixed von Mises distribution has the smallest RMSE values for the flood dates

Table 6.5 RMSE Values of different probability distributions of flood occurrence dates in the upper Yangtze River (%)

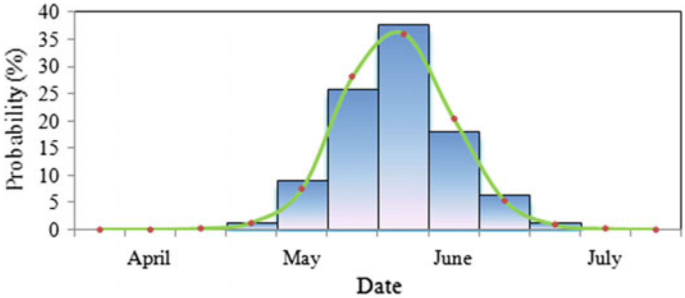
Distribution	Pingshan	Gaochang	Beibei	Yichang
Mixed von Mises	1.898	1.413	1.725	2.067
Generalized logistic (GLO)	4.028	3.291	3.646	7.148
Generalized Pareto (GP)	3.688	3.911	2.574	3.465
Pearson type 3 (P-III)	3.184	2.773	2.725	5.591
Generalized extreme-value (GEV)	2.927	2.688	2.654	5.988
Gamma distribution	4.539	2.806	4.450	6.280
Normal distribution	3.560	2.887	3.021	7.393
Gumbel distribution	6.641	4.525	4.255	5.985
Wakeby distribution	2.796	2.566	1.848	3.465
Kappa distribution	2.734	2.664	2.102	2.195
Exponential distribution	10.432	8.735	8.426	6.340

Note The bold characters mean the minimum value of each column at all four stations in upper Yangtze River. The values of estimated parameters of the von Mises distribution of both river basins are listed in Table 6.6. The Kolmogorov-Smirnov (KS) test is selected as the goodness-of-fit test to evaluate the validity of the assumption that the flood occurrence dates followed the mixed von Mises distribution. Results shown in Table 6.6 indicate that this assumption cannot be rejected at the 5% significance level. The frequency histograms of the flood occurrence dates fitted by the mixed von Mises distribution for AM sample series in upper Colorado River are shown in Fig. 6.2a–d. The marginal distribution curves of flood occurrence dates in upper Yangtze River are shown in Fig. 6.3, in which the line represents the theoretical distribution, and the crosses the empirical frequencies of observations. Figures 6.2 and 6.3 indicate that all the theoretical distributions fitted the observed data reasonably well.

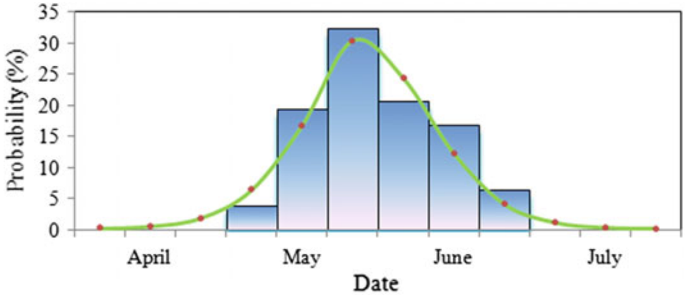
The values of estimated parameters of the P-III and log P-III distributions are given in Table 6.6. A chi-square goodness-of-fit test is performed to test the assumption, H_0, that the flood magnitude followed the P-III or LP-III distribution. It is shown that P-III or LP-III distribution is valid for flood magnitudes at four sites

Table 6.6 Parameters and hypothesis test results of margin distributions

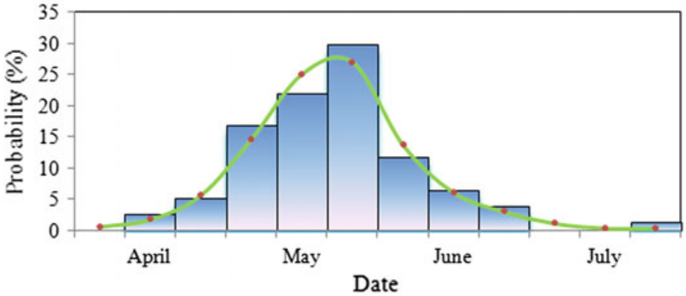
Stations	Mixed von Mises distribution				P-III distribution			
	u_i	K_i	p_i	K-S	α	β	δ	$\chi 2$
Pingshan	4.33 5.19 3.46	67.99 8.89 3.70	0.17 0.44 0.39	0.042 (0.176)	10.41	0.0008	3279.29	0.36 (3.84)
Gaochang	4.29 3.65 2.88	3.04 300.00 7.90	0.60 0.16 0.24	0.039 (0.176)	4.726	0.0005	6660.33	0.33 (3.84)
Beibei	5.44 2.96 5.29	0.00 7.20 4.25	0.21 0.46 0.33	0.035 (0.176)	330.6	0.002	−110580	0.57 (5.99)
Yichang	2.99 4.20 5.35	13.48 6.63 11.21	0.50 0.28 0.23	0.045 (0.176)	156.25	0.0016	−49825	0.92 (3.84)
Above Cameo	3.32	3.69	1.0	0.064 (0.155)	4.919	0.0896	3.783	0.13 (3.84)
Green River	3.02	2.38	1.00	0.057 (0.155)	3.493	0.1050	3.966	0.003 (3.84)
Gunnison River	2.57 4.29	2.25 4.97	0.96 0.04	0.031 (0.155)	30.451	0.0503	2.418	0.54 (3.84)
Lees ferry	2.29 3.52 5.84	198.29 1.00 260.56	0.09 0.74 0.17	0.144 (0.155)	10.058	0.0952	3.584	0.79 (3.84)



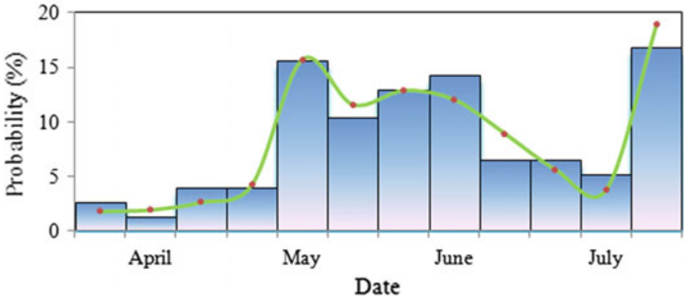
(a) Upper Cor.



(b) Green River



(c) Gunnison River



(d) Lees ferry

Fig. 6.2 Frequency histograms of flood occurrence dates fitted by the mixed von Mises distribution for the four stations in upper Colorado River

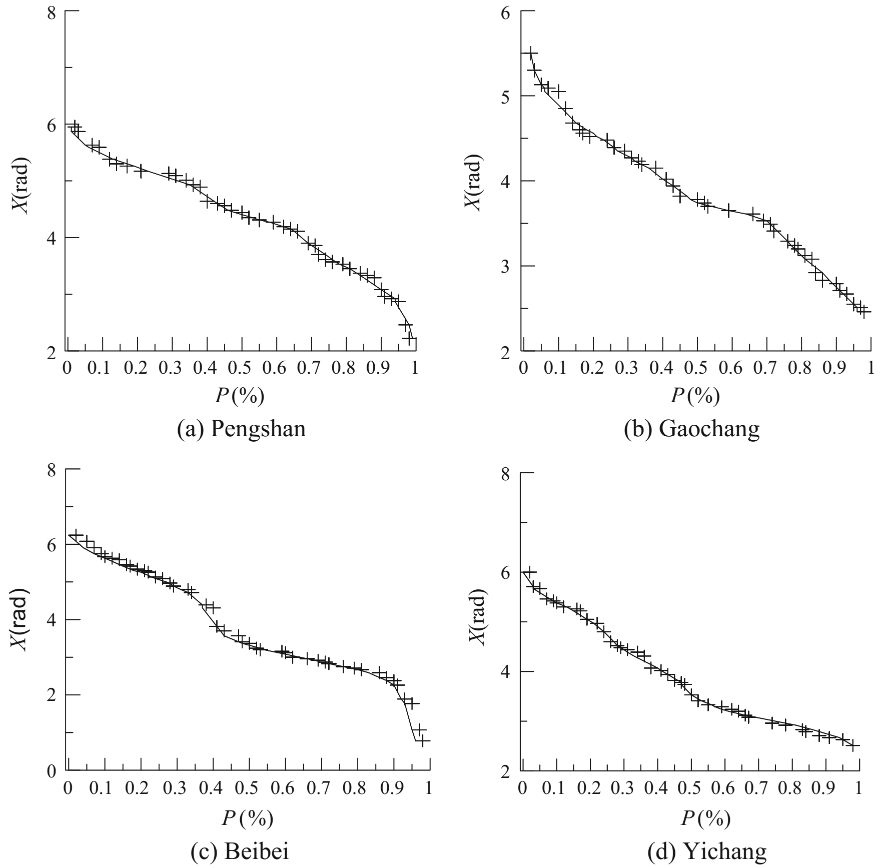


Fig. 6.3 Frequency curves of flood occurrence dates based on AM samples

studied with a critical value 0.05. The marginal distribution frequency curves of flood magnitudes in the Upper Yangtze River are shown in Fig. 6.4. It is seen that graphically the P-III distribution fit the empirical distribution.

6.4.2 *Estimation of Joint Distributions*

A four-variate symmetric Gumbel, asymmetric Gumbel, and X-Gumbel copulas are used for modelling the dependence amongst the four stations. The formulas of these copulas are given in Chap. 2.

A pseudo-likelihood technique involving the ranks of the data is used for estimating parameters of the four-variate symmetric Gumbel and asymmetric Gumbel copulas. For the Yangtze River, the value of estimated parameter of symmetric

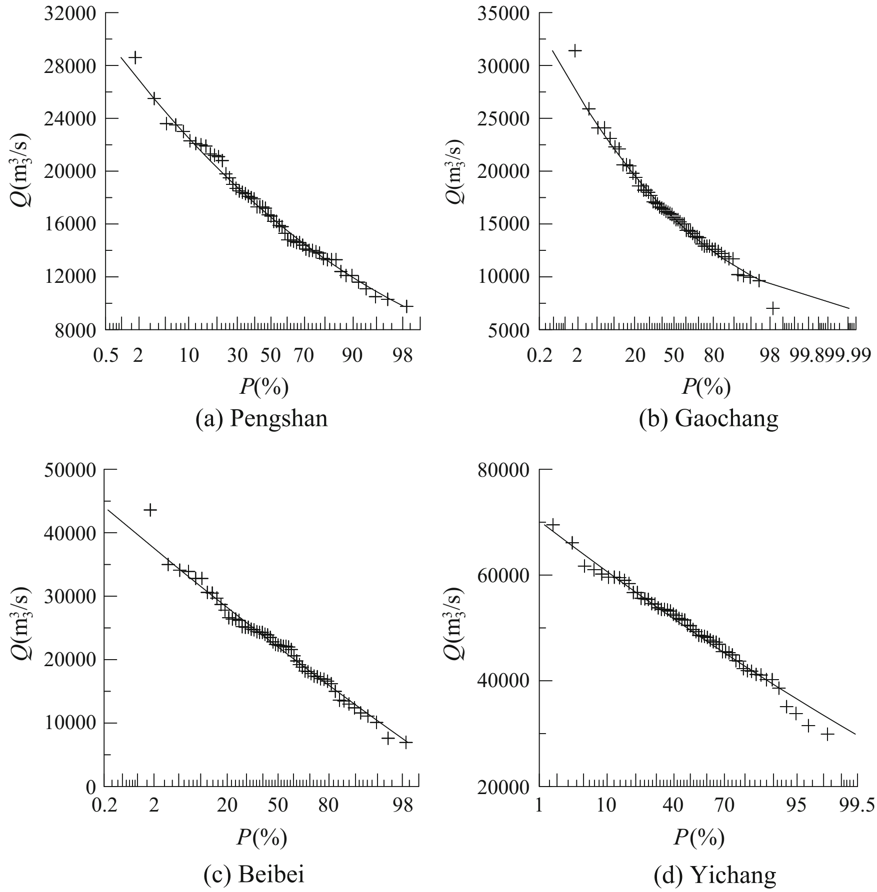


Fig. 6.4 Frequency curves of flood magnitudes based on AM samples

Gumbel is $\hat{\theta} = 1.14$ for flood magnitudes, and $\hat{\theta} = 1.20$ for flood occurrence dates. Estimates of parameters of the asymmetric Gumbel are $\hat{\theta}_1 = 1.06$, $\hat{\theta}_2 = 1.16$, and $\hat{\theta}_3 = 1.46$ for flood magnitudes; and $\hat{\theta}_1 = 1.18$, $\hat{\theta}_2 = 1.20$, and $\hat{\theta}_3 = 1.38$ for the flood occurrence dates. For the Colorado River, the value of estimated parameter of symmetric Gumbel is $\hat{\theta} = 1.99$ for flood magnitudes, and $\hat{\theta} = 1.22$ for flood occurrence dates. Estimates of parameters of the asymmetric Gumbel are $\hat{\theta}_1 = 1.82$, $\hat{\theta}_2 = 2.35$, and $\hat{\theta}_3 = 2.72$ for flood magnitudes; and $\hat{\theta}_1 = 1.13$, $\hat{\theta}_2 = 1.30$, and $\hat{\theta}_3 = 1.54$ for the flood occurrence dates. Pickand's dependence function, which was recommended by Salvadori and De Michele (2010), is used for estimating parameters of the X-Gumbel copula. The values of parameters of X-Gumbel for flood magnitudes and flood dates in Upper Colorado River are given in Table 6.7.

Table 6.7 Parameters of X-Gumbel joint distributions

Rivers	Parameters	a_1	a_2	a_3	a_4	x	s
Upper Yangtze River	Magnitude	0.039	1.0	0.92	0.63	2.99	1.46
	Dates	0.999	0.132	0.137	0.576	1.09	2.19
Upper Colorado River	Magnitudes	0.707	0.773	0.725	0.268	3.267	2.763
	dates	0.221	0.396	1.000	0.193	1.181	3.372

The empirical and fitted Pickands' function for all the pairs of stations and three copula models of flood magnitudes in the two basins are plotted in Figs. 6.5 and 6.6, respectively. The symmetric Gumbel dependence functions are the same in all the plots. The asymmetric Gumbel dependence functions are different corresponding to different pairs. The asymmetric Gumbel copula fits better than the symmetric one. The X-Gumbel provides a better fit than the other two models. The empirical joint probabilities of flood occurrence dates and flood peak magnitudes are plotted against theoretical probabilities, as shown in Fig. 6.7, in which the theoretical joint probabilities, F, of the real occurrence combinations of x and y are estimated. Figure 6.7 shows that no significant difference between empirical and theoretical joint probabilities can be detected.

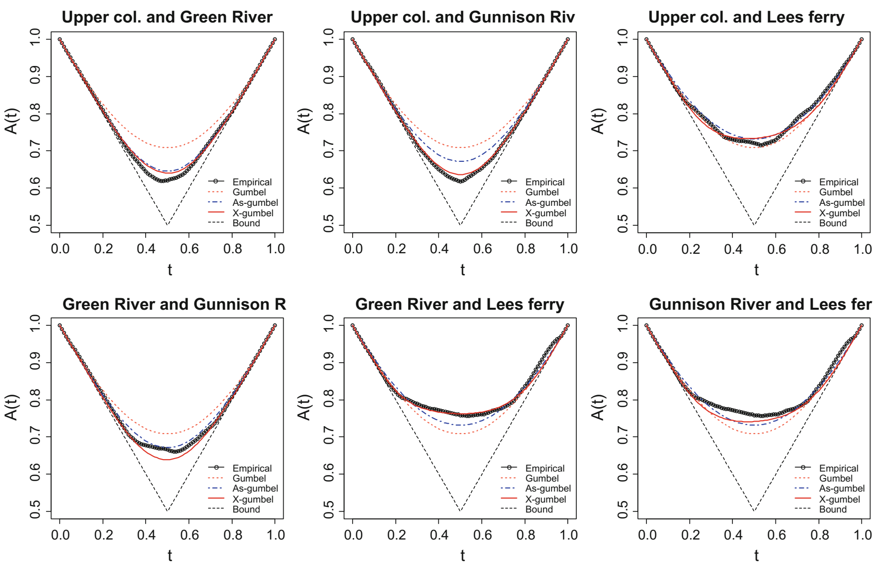


Fig. 6.5 Plots of empirical and fitted Pickand's dependence functions of flood magnitude for all pairs of stations and the three models

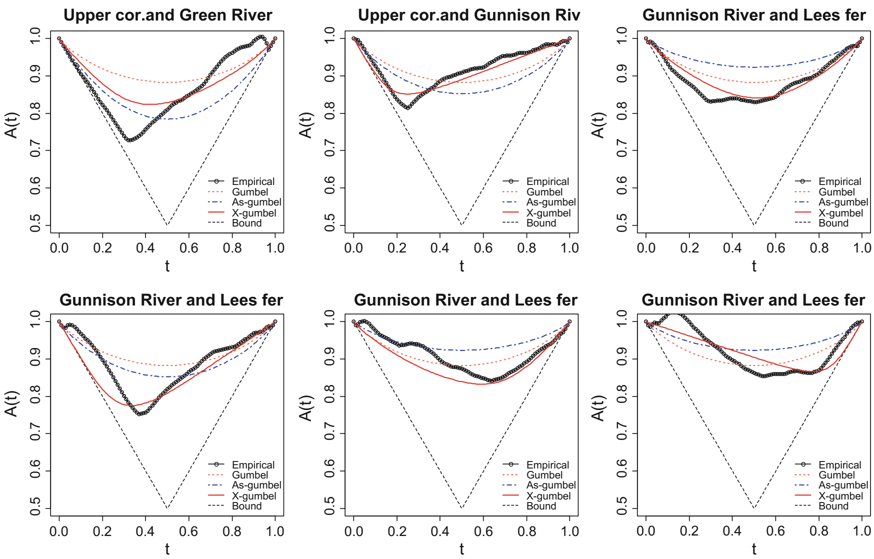


Fig. 6.6 Plots of empirical and fitted Pickand's dependence functions of flood occurrence dates for all pairs of stations and the three models

6.4.3 *Analysis of Flood Coincidence Risk*

6.4.3.1 Coincidence Probabilities Analysis

According to the analysis above, the X-Gumbel copula is used for the flood coincidence risk analysis hereafter. The exceedance probabilities of coinciding *T*-year flood volumes at two and three considered inflow profiles are calculated as shown in Tables 6.8 and 6.9. The average exceedance probabilities of 100, 50, 10, 5, and 2-year for the four sites are 0.0075, 0.015, 0.0763, 0.1561 and 0.4196, respectively.

The coincidence probabilities of flood dates in two, three and four rivers, P_2^t, P_3^t and P_4^t, are evaluated as shown in Fig. 6.8a–e, respectively. For the Jinsha and Min Rivers, the higher coincidence probabilities occur in late July and middle August. According to the observed data, there are seven times that the flood occurred simultaneously in the two rivers, five of which is within this period. For the Jinsha and Jialing Rivers, the curve demonstrates the multi-modal characteristic, and the higher coincidence probabilities occur in the middle July and early September, which indicates that the flood control water level of the Three Gorges Reservoir (TGR) should not be raised too high and certain flood control storage is needed for TGR. For the Jialing and Min tributaries, the highest probability occurs in July. Six of eight flood events that occurring simultaneously in the two rivers, are within this

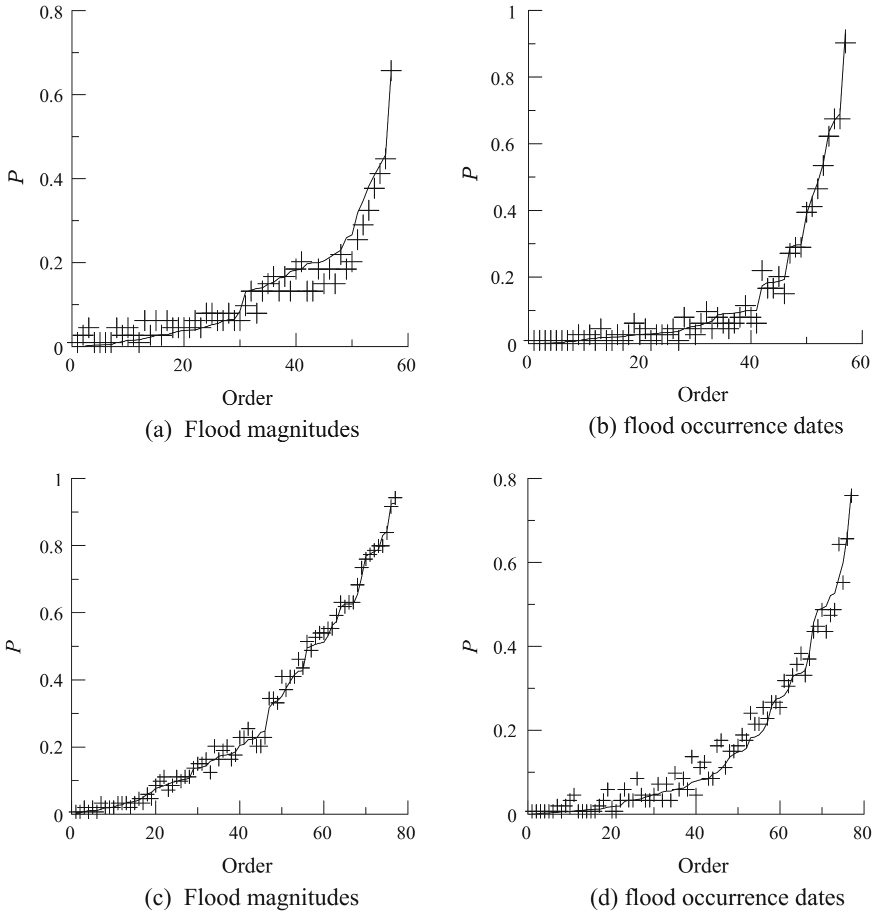


Fig. 6.7 Joint distribution and empirical probabilities of observed combinations based on **a** and **b** are for flood magnitudes, and flood occurrence dates in upper Yangtze River; **c** and **d** are for flood magnitudes and flood occurrence dates in upper Colorado River

period. For the three rivers in the upper Yangtze River, July has the highest coincidence probabilities. For the four stations, the higher probabilities occur in July. It is indicated that in May and June, the coincidence probabilities are very small, which means the low coincidence risk. Therefore, it is possible to raise the flood control water level of TGR in the two months. All the analysis mentioned above demonstrates that the calculated results are in accordance with historical data.

The coincidence probabilities of *T*-year design flood for two and three tributaries are calculated based on Eq. 6.3, and results are listed in Tables 6.10 and 6.11. Results are reasonable from the point of view that the coincidence probabilities

Table 6.8 The exceedance probability of coinciding *T*-year flood volumes at two considered inflow profiles

Tributaries	T	100	50	10	5	2
Upper Col. and Green Rivers	100	0.00746	0.00921	0.00997	0.00999	0.01000
	50	0.00929	0.01495	0.01976	0.01995	0.019994
	10	0.00998	0.01982	0.07607	0.09390	0.099553
	5	0.00999	0.01997	0.09458	0.15562	0.196065
	2	0.01000	0.02000	0.09969	0.19677	0.418765
Upper Col. and Gunnison Rivers	100	0.00751	0.00927	0.00997	0.00999	0.01000
	50	0.00929	0.01505	0.01979	0.01996	0.02000
	10	0.00998	0.01981	0.07654	0.09441	0.09962
	5	0.00999	0.01996	0.09458	0.15647	0.19651
	2	0.01000	0.02000	0.09966	0.19669	0.42025
Green and Gunnison Rivers	100	0.00749	0.00930	0.00998	0.00999	0.01000
	50	0.00925	0.01502	0.01982	0.01997	0.02000
	10	0.00997	0.01978	0.07639	0.09467	0.09969
	5	0.00999	0.01995	0.09420	0.15621	0.19682
	2	0.01000	0.01999	0.09959	0.19632	0.41979

increase when the return period T is decreasing. The average coincidence probabilities of 100 and 10-year design flood in two tributaries are 0.000143 and 0.001467, respectively. The coincidence probabilities of 1000 and 500-year design flood in three tributaries are 3.63×10^{-6} and 3.71×10^{-5}. From Tables 6.10 and 6.11, the coincidence probabilities of any other return period can be obtained directly or by interpolation.

6.4.3.2 Conditional Probabilities Analysis

The flood control standard of TGR is 1000 years. To analyze the effect of the upper tributaries on TGR, the conditional probabilities are calculated. The conditional probabilities of the occurrence of the T-year flood at the TGR, given the occurrence of flood in the upper tributaries can be defined as:

$$\begin{aligned} &P(Q_n > q_n^T | Q_1 > q_1^T, \ldots, Q_{n-1} > q_{n-1}^T) \\ &= P(Q_1 > q_1^T, \ldots, Q_n > q_n^T) / P(Q_1 > q_1^T, \ldots, Q_{n-1} > q_{n-1}^T) \end{aligned} \tag{6.4}$$

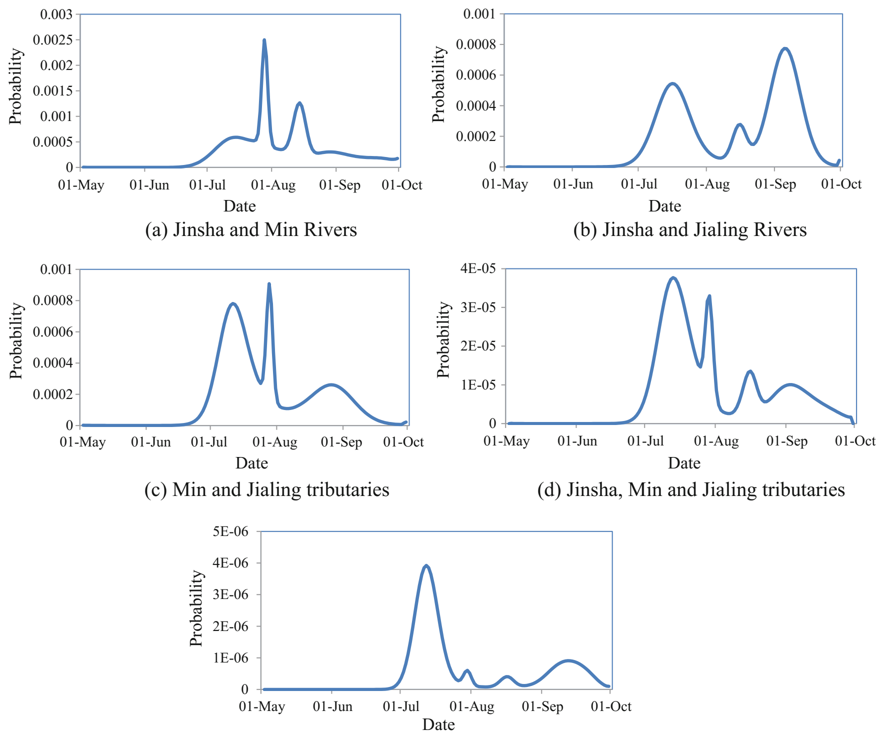
where n is the number of random variables and is from two to four; $Q_1, \ldots, \ldots Q_n$ are flow magnitudes in any of the two rivers; $q_1^T, \ldots, q_n^T$ mean the T-year design flood. For the case n equal to 2, the conditional probabilities of T-year design flood for the Yangtze River at TGR, given the flood volume in one of the upper tributary by

Table 6.9 The exceedance probability of coinciding T-year flood volumes at three considered inflow profiles

Upper Col.	Gunnison / Green	100	50	10	5	2
100	100	0.00671	0.00744	0.00751	0.00751	0.00751
	50	0.00744	0.00899	0.00929	0.00929	0.00929
	10	0.00749	0.00925	0.00996	0.00997	0.00998
	5	0.00749	0.00925	0.00997	0.00999	0.00999
	2	0.00749	0.00925	0.00997	0.00999	0.01000
50	100	0.00741	0.00896	0.00927	0.00927	0.00927
	50	0.00904	0.01346	0.01505	0.01505	0.01505
	10	0.00930	0.01502	0.01971	0.01980	0.01981
	5	0.00930	0.01502	0.01978	0.01994	0.01996
	2	0.00930	0.01502	0.01978	0.01995	0.01999
10	100	0.00746	0.00921	0.00996	0.00997	0.00997
	50	0.00929	0.01495	0.01969	0.01979	0.01979
	10	0.00997	0.01975	0.06874	0.07602	0.07654
	5	0.00998	0.01982	0.07605	0.09207	0.09456
	2	0.00998	0.01982	0.07639	0.09419	0.09944
5	100	0.00746	0.00921	0.00997	0.00999	0.00999
	50	0.00929	0.01495	0.01976	0.01993	0.01996
	10	0.00998	0.01982	0.07573	0.09180	0.09439
	5	0.00999	0.01995	0.09247	0.14128	0.15632
	2	0.00999	0.01997	0.09466	0.15613	0.19480
2	100	0.00746	0.00921	0.00997	0.00999	0.01000
	50	0.00929	0.01495	0.01976	0.01995	0.01999
	10	0.00998	0.01982	0.07607	0.09389	0.09939
	5	0.00999	0.01997	0.09457	0.15553	0.19452
	2	0.01000	0.02000	0.09954	0.19528	0.38732

specifying $Q_1 > q_1^T$, is obtained by Eq. 6.4. In a similar manner, the conditional probabilities of Yichang Station given the flood volume of two or three upper rivers are obtained. The calculated conditional probabilities are listed in the Table 6.12.

Table 6.12 shows that for a fixed return period in the upper rivers, the conditional probabilities show an increasing trend when the return period of TGR decreases. For example, given the occurrence of 1000-year design flood in Jinsha River, the conditional probabilities of 1000 and 10-year design flood in TGR are 0.35 and 0.71, respectively. The conditional probabilities of TGR given T-year design floods in three rivers are greater than those given T-year design floods in two rivers, and the conditional probabilities of TGR given T-year design floods in two rivers is greater than those only given T-year flood in one river. From these points of view, results of the calculation are reasonable. It is shown in Table 6.12, the



(a) Jinsha and Min Rivers

(b) Jinsha and Jialing Rivers

(c) Min and Jialing tributaries

(d) Jinsha, Min and Jialing tributaries

(e) Jinsha, Min, Jialing, and Yangtze (Yichang Station) Rivers

Fig. 6.8 The coincidence probabilities of flood dates on each day in the upper Yangtze River and its tributaries

Table 6.10 Coincidence probabilities considering flood magnitudes and occurrence dates in two of the tributaries in the upper Yangtze River

Rivers	*T*	100	50	10	5	2
Upper Col. and Green Rivers	100	0.00023	0.00029	0.00031	0.00031	0.00031
	50	0.00029	0.00047	0.00062	0.00062	0.00062
	10	0.00031	0.00062	0.00237	0.00293	0.00311
	5	0.00031	0.00062	0.00295	0.00486	0.00612
	2	0.00031	0.00062	0.00311	0.00614	0.01307
Upper Col. and Gunnison Rivers	100	0.00006	0.00007	0.00007	0.00007	0.00007
	50	0.00007	0.00011	0.00015	0.00015	0.00015
	10	0.00007	0.00015	0.00057	0.00070	0.00074
	5	0.00007	0.00015	0.00070	0.00116	0.00144
	2	0.00007	0.00015	0.00074	0.00144	0.00287
Green and Gunnison Rivers	100	0.00014	0.00018	0.00019	0.00019	0.00019
	50	0.00018	0.00029	0.00038	0.00038	0.00038
	10	0.00019	0.00038	0.00146	0.00181	0.00191
	5	0.00019	0.00038	0.00180	0.00299	0.00377
	2	0.00019	0.00038	0.00191	0.00376	0.00804

Table 6.11 Coincidence probabilities considering flood magnitudes and occurrence dates in three tributaries of the upper Yangtze River

Jinsha	Jialing Min	100	50	10	5	2
100	100	3.63E-06	4.02E-06	4.06E-06	4.06E-06	4.06E-06
	50	4.02E-06	4.86E-06	5.02E-06	5.02E-06	5.02E-06
	10	4.05E-06	5E-06	5.38E-06	5.39E-06	5.39E-06
	5	4.05E-06	5E-06	5.39E-06	5.4E-06	5.4E-06
	2	4.05E-06	5E-06	5.39E-06	5.4E-06	5.4E-06
50	100	4.00E-06	4.84E-06	5.01E-06	5.01E-06	5.01E-06
	50	4.88E-06	7.27E-06	8.13E-06	8.13E-06	8.13E-06
	10	5.03E-06	8.11E-06	1.06E-05	1.07E-05	1.07E-05
	5	5.03E-06	8.11E-06	1.07E-05	1.08E-05	1.08E-05
	2	5.03E-06	8.11E-06	1.07E-05	1.08E-05	1.08E-05
10	100	4.03E-06	4.98E-06	5.38E-06	5.39E-06	5.39E-06
	50	5.02E-06	8.08E-06	1.06E-05	1.07E-05	1.07E-05
	10	5.39E-06	1.07E-05	3.71E-05	4.11E-05	4.13E-05
	5	5.39E-06	1.07E-05	4.11E-05	4.97E-05	5.11E-05
	2	5.39E-06	1.07E-05	4.13E-05	5.09E-05	5.37E-05
5	100	4.03E-06	4.98E-06	5.38E-06	5.40E-06	5.40E-06
	50	5.02E-06	8.08E-06	1.07E-05	1.08E-05	1.08E-05
	10	5.39E-06	1.07E-05	4.09E-05	4.96E-05	5.10E-05
	5	5.40E-06	1.08E-05	5.00E-05	7.63E-05	8.45E-05
	2	5.40E-06	1.08E-05	5.11E-05	8.44E-05	1.05E-04
2	100	4.03E-06	4.98E-06	5.38E-06	5.40E-06	5.40E-06
	50	5.02E-06	8.08E-06	1.07E-05	1.08E-05	1.08E-05
	10	5.39E-06	1.07E-05	4.11E-05	5.07E-05	5.37E-05
	5	5.40E-06	1.08E-05	5.11E-05	8.40E-05	1.05E-04
	2	5.40E-06	1.08E-05	5.38E-05	1.06E-04	2.09E-04

Jialing River has the most significant impact on the flow of TGR. The coefficient of correlation between Jialing River (at Beibei Station) and Yangtze River (at Yichang Station) is 0.318, the largest value in Table 6.4, which shows the close relationship between the two rivers. It is demonstrated that from Table 6.13, the higher conditional probabilities of TGR are generally obtained when the flows of Jinsha and Jialing Rivers are known. Table 6.14 gives the conditional probabilities of TGR when *T*-year design flood in the upper three rivers are known. It can be seen that when the three rivers upper have a 1000-year flood, the conditional probabilities of TGR is 1.0.

Table 6.12 Conditional probabilities $P(Q_Y > q_Y^T | Q_1 > q_1^T)$ of TGR, under the condition of the flood occurring in one of upper Yangtze River

Yichang	Return Period	1000	500	100	50	10
Jinsha River	1000	0.35	0.47	0.71	0.78	0.89
	500	0.24	0.35	0.62	0.71	0.86
	100	0.07	0.12	0.36	0.48	0.74
	50	0.04	0.07	0.24	0.36	0.66
	10	0.01	0.02	0.07	0.13	0.41
Min River	1000	0.27	0.37	0.58	0.65	0.79
	500	0.18	0.27	0.50	0.58	0.75
	100	0.06	0.10	0.28	0.38	0.62
	50	0.03	0.06	0.19	0.28	0.55
	10	0.01	0.02	0.06	0.11	0.34
Jialing River	1000	0.39	0.53	0.77	0.83	0.93
	500	0.26	0.39	0.68	0.77	0.90
	100	0.08	0.14	0.40	0.54	0.79
	50	0.04	0.08	0.27	0.40	0.72
	10	0.01	0.02	0.08	0.14	0.44

Table 6.13 Conditional probabilities $P(Q_Y > q_Y^T | Q_1 > q_1^T, Q_2 > q_2^T)$ of TGR, under the condition of the flood occurring in two of upper Yangtze River

Yichang	Return period	1000	500	100	50	10
Jinsha and Jialing River	1000	0.99	1.00	1.00	1.00	1.00
	500	0.97	0.98	1.00	1.00	1.00
	100	0.72	0.81	0.93	0.98	1.00
	50	0.45	0.62	0.80	0.88	1.00
	10	0.05	0.10	0.31	0.40	0.76
Jinsha and Min River	1000	0.73	0.96	1.00	1.00	1.00
	500	0.42	0.73	1.00	1.00	1.00
	100	0.09	0.18	0.73	0.96	1.00
	50	0.04	0.09	0.42	0.73	1.00
	10	0.01	0.02	0.09	0.18	0.75
Jialing and Min River	1000	0.90	0.95	1.00	1.00	1.00
	500	0.81	0.89	0.99	1.00	1.00
	100	0.50	0.63	0.88	0.95	1.00
	50	0.32	0.47	0.75	0.86	0.99
	10	0.05	0.10	0.32	0.43	0.80

Table 6.14 Conditional probabilities $P(Q_Y > q_Y^T | Q_1 > q_1^T, Q_2 > q_2^T, Q_3 > q_3^T)$ of TGR, under the condition of the flood occurring in three of upper Yangtze River

Return period	1000	500	100	50	10
1000	1.00	1.00	1.00	1.00	1.00
500	0.98	0.99	1.00	1.00	1.00
100	0.76	0.86	0.97	1.00	1.00
50	0.50	0.68	0.86	0.94	1.00
10	0.06	0.12	0.38	0.48	0.87

6.5 Conclusions

The flood combination risk, which reflects the probability of coincidence of multi-dimensional flood peaks, is important for reservoir operation and flood management. The copula function is used to establish the joint distribution of flood magnitudes and flood occurrence dates. The coincidence probabilities of flood magnitudes and dates are calculated. The conditional probabilities of TGR for different return periods are analyzed. The main conclusions of this study are summarized as follows:

(1) Symmetric Gumble, asymmetric Gumble and X-Gumble copula function are used. The X-Gumble copula provides the best fit. Therefore, the X-Gumble copula is used for the combination risk analysis in this chapter.
(2) By analyzing the coincidence probabilities of flood magnitudes and flood dates, this Chapter contributes to better practical knowledge in the area of engineering hydrology, particularly about the assessment of flood events and the performance of comprehensive flood-risk analyses. According to the analysis results, it is possible to raise the flood control water level of TGR in May and June. To the contrary, in September, the flood control water level of the TGR should not be raised too high, and certain flood control storage is needed for TGR. The flow in Jialing River has the most significant impact on the inflow in TGR. If the three upper rivers have a 1000-year design flood, the TGR also experiences a 1000-year flood. The coincidence probabilities or conditional probabilities of any other return period can be obtained directly from Tables 6.8, 6.9, 6.10, 6.11, 6.12, 6.13 and 6.14 or by interpolation.

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