Fuzzy Probability Approach in Seismic Hazard Analysis

Prasad Hiremath, Mohammad Muzzaffar Khan and G. Kalyan Kumar

Abstract Seismic hazard assessment like many other problems in seismology is a complicated problem, owing to the variety of parameters affecting the occurrence of the earthquake. Assessment of seismic hazard based on the theory of probability becomes hard particularly under highly uncertain conditions, i.e., where neither the statistical data nor the physical knowledge required for a purely probabilistic risk analysis is sufficient. Insufficiency of information will afflict the calculated risk probabilities with imprecision and will to the underestimation of the risk. This uncertainty, which is a result of vagueness and incompleteness of the data, should be considered in a rationale way. Essentials of seismic hazard analysis are identified and the concept of fuzzy logic is applied to seismic hazard analysis, generalizing the conventional probabilistic seismic hazard analysis (PSHA) to fuzzy probabilistic seismic hazard analysis (FPSHA). The β and λ values which are the input parameters for the PSHA are evaluated using earthquake data for the Warangal region. The uncertainty associated with these values will be too high for a site with moderate earthquake activity, and the previous earthquake data is scarce. Fuzzy logic will help in containing these uncertainties by its property of considering the possibility of a certain event occurring. This study presents an approach for seismic hazard analysis based on fuzzy set theory. Seismic activity is based on the seismic activity rate, λ, which is equal to the number of events with magnitudes equal to or greater than a defined magnitude level, say M_0, during a specified time period, T; the parameter b or β ($\beta = b \ln 10$). The variables β and λ are first converted into Gaussian fuzzy sets using α-cut method. The ranges of β and λ are chosen based on previous studies. The fuzzified variables are used in seismic hazard analysis. The outputs are defuzzified using the center of area method, and fuzzy hazard curve is

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developed for the study region. The output is compared with PSHA results. The horizontal PGA expected in Warangal on stiff ground, with a 10% probability of exceedance in 50 years (which corresponds to a return period of 475 years) is 0.0836 g, whereas, that with a 2% probability of exceedance in 50 years (return period = 2475 years) is 0.153 g.

Keywords Fuzzy logic · PSHA · Gaussian fuzzy sets · Peak ground acceleration

1   Introduction

The study of earthquakes dates back many centuries. Written records of earthquakes in China date back as far as 3000 years. Still, compared with millions of years where earthquakes have been occurring, humankind's experience with earthquake is very brief. Today, hundreds of millions of people throughout the world live with a significant risk to their lives and property from earthquakes. The health of many local, regional, and even national economies is also at risk from earthquakes. It is impossible to prevent earthquakes from occurring, but it is possible to mitigate their adverse effects: to reduce life losses, injuries, and damage. In order to realize this, the starting point is being able to estimate the adverse outcomes of the earthquake by means of risk assessment methods in a reliable and realistic way.

Seismic hazard such as ground shaking, liquefaction, landslides, fault movement, which are associated with an earthquake, may produce adverse effects on human activities. Seismic hazard analyses involve the quantitative estimation of ground shaking hazards at a particular site. Seismic hazards may be analyzed deterministically, as when a particular earthquake scenario is assumed, or probabilistically, in which uncertainties in earthquake size, location, and time of occurrence are explicitly considered.

The accuracy of PSHA depends on the accuracy with which uncertainty in earthquake size, location, recurrence, and effects can be characterized. Therefore, it is quite sensible to employ a concept which can model different forms of uncertainty present in natural hazards. A reasonable option is complementing probability theory with an extra dimension of uncertainty provided by fuzzy set theory. This concept has received the generic name of fuzzy probability after Zadeh. However, this generic term has been interpreted and mathematically formalized in various ways. One of the most attractive interpretations of fuzzy probability is where probability of a crisp event, due to sparsity of data sample, is expressed in terms of a fuzzy number. It will be shown that this concept is especially appropriate for situations for which our physical knowledge and available statistical data are insufficient to characterize their likelihood of occurrence with a probability distribution.

The term fuzzy logic was first coined for the proposal of fuzzy set theory by Zadeh (1965). Brown and Yao (1983) in their pioneering research on the application of fuzzy set theory in structural engineering have tried to make use of fuzzy

set theory for two of the structural engineering-related problems. Initially, a justification for the theory is provided, and then, simple fuzzy operations are developed and contrasted with those of conventional set theory. Frangopol et al. (1987) proposed a model which is able to reproduce both the randomness and the imprecision in conjunction with earthquake occurrences. Elham et al. (2015) in their study presented a new model for probabilistic seismic hazard assessment based on fuzzy set theory.

In the current study, fuzzy sets have been developed which can be used in the probabilistic seismic hazard assessment of Warangal, Telangana. For this, all the variables to be used in seismic hazard assessment must be defined in terms of fuzzy sets. Then, using α-cut method, the seismic hazard equations are evaluated. The outputs will be in fuzzy sets which will be defuzzified using the center method. At last, the expected peak ground acceleration values from the seismic sources are calculated. The results obtained are compared with the results of PSHA.

2   Methodology

In the present study, four input parameters, i.e., distance (R), magnitude (M), λ, and β, are defined as fuzzy sets by the discrete membership functions $\mu(R)$, $\mu(M)$, $\mu(\lambda)$, and $\mu(\beta)$, respectively. The input parameters, λ and β, for single seismic zone were taken from the previous study of the Warangal region by Deviprasad (2014). Due to the uncertainty of the four mentioned parameters, maximum and minimum values were determined by a Gaussian membership function. It may be noted that λ and β are parts of G–R relationship and R and M are uncertain inputs of an attenuation relationship. These uncertain parameters used in G–R relation and attenuation relationship are assessed using fuzzy logic approach. The results obtained with these fuzzified inputs will be defuzzified using centroid method. The defuzzified values are used for the seismic hazard assessment by Cornell–McGuire approach.

2.1   Gutenberg–Richter Frequency–Magnitude Recurrence Relationship

Gutenberg–Richter parameters 'a' and 'b', which are the basic input for the seismic hazard analysis, have been carefully evaluated through regression analyses using the established catalogue. Table 1 presents the completeness intervals and exceedance rates computed by the CUVI method (Mulargia and Tinti 1985) and Stepp's (1972) method.

The prime input values for the probabilistic seismic hazard analysis, β and λ, will be further processed with the fuzzy approach. To minimize the vagueness, a suitable range for β value and λ value was chosen. For the chosen ranges, using

Table 1 Seismicity parameters from CUVI and Stepp's method

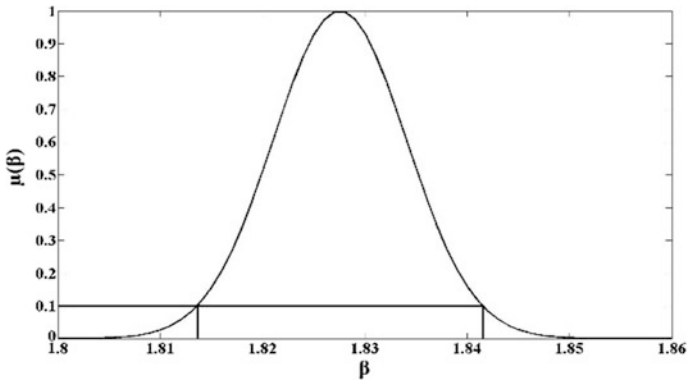| Single source zone	N	a	b	$M_{\min}$	$\lambda = 10^{(a-bM)}$	$\beta = 2.303b$
Stepp's method	248	3.43	0.80	3	10.69	1.85
CUVI method	248	3.33	0.78	3	9.75	1.80

Fig. 1 Gaussian membership curve for β values
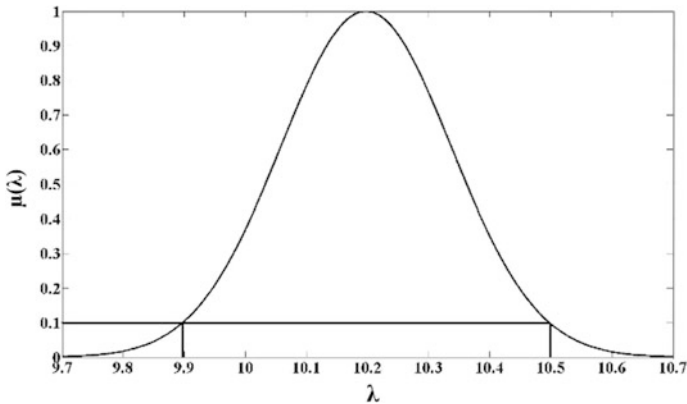
Fig. 2 Gaussian membership curve for λ values

their standard deviation and mean Gaussian membership curve was drawn as shown in Figs. 1 and 2.

As an example, in the first trial minimum value of β from the α-cut = 0.1 and minimum value of λ from the α-cut = 0.1 can be used as inputs. In the second trial, minimum value of β and maximum value of λ can be given as inputs. In that case, we will be having four possible combinations of β and λ values.

2.2 Fuzzy Attenuation Relation

Attenuation relationship given by Abrahamson and Silva has been used for cal-
culating PGA values. Fuzzy logic approach is used for different ranges of magni-
tude (M) and distance (R). Gaussian curves were developed for both M and R as it
was done in the case of G–R relations.

2.2.1 Fuzzification of the Magnitude

The magnitude range for a PSHA analysis that was done earlier was 3–6.5. The
interval chosen was 0.4375. For each interval, using standard deviation and mean,
Gaussian membership curves were drawn. α-cut of 0.1 was applied to get minimum
and maximum values of magnitude. One interesting observation that can be made
here is that the magnitude range is 6.2812–6.7187 which is greater than the highest
value chosen in the conventional PSHA. 6.2812 is the median value of the last but
one magnitude range. The fact that the magnitude value higher than the maximum
magnitude value for the region being covered under the fuzzy universe makes this
method more credible.

Gaussian membership curve for magnitude range 3–3.4375 is shown in Fig. 3.
The thick horizontal line at $\mu(M) = 0.1$ cutting the Gaussian membership curve at
two points is the α-cut at 0.1.

2.2.2 Fuzzification of Hypocentral Distance

The distance is also divided into ranges for the conventional PSHA. The least
distance is 11.46 km, and 350 km is the maximum distance. As it is done in the
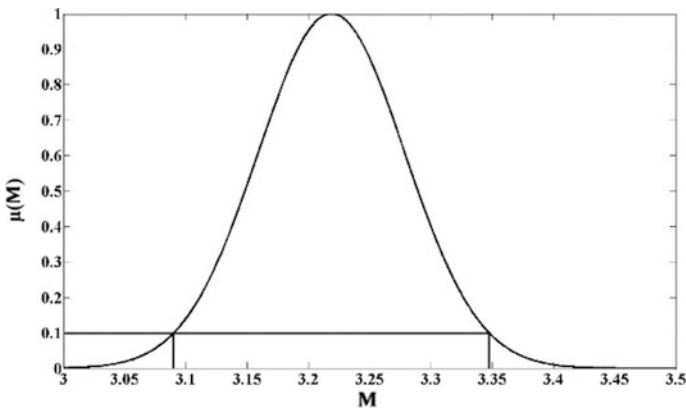

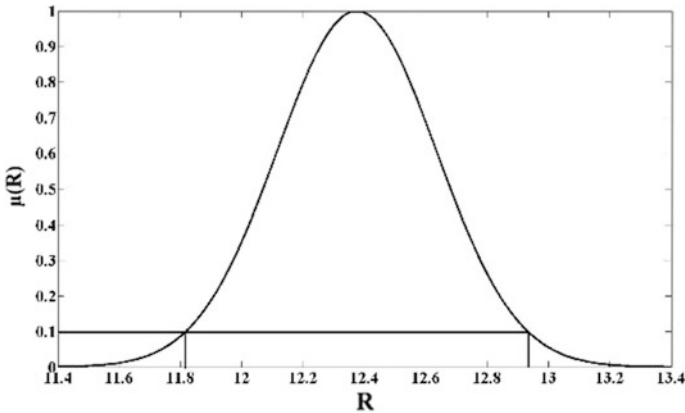**Fig. 3** Gaussian membership curve for magnitude range of 3–3.4375

Fig. 4 Gaussian membership curve for the distance range 11.46–13.29

earlier cases of fuzzification, here also for each distance range, a Gaussian membership curve is drawn and from α-cut method. Minimum and maximum values of distance were noted for further calculations. Gaussian membership curve drawn for the distance of range 11.46–13.29 km is shown in Fig. 4.

Distance and magnitude are to be given as inputs in the attenuation relationship of Abrahamson and Silva (1997). We are having two values for magnitude and two values for distance. Similar to the case of G–R relationship, here also four combinations of magnitude and distance are possible. Spectral acceleration values are calculated for all these four combinations of M and R. As there are nine ranges of magnitude chosen, for each of these nine ranges four values of spectral acceleration are obtained. Out of these four values, minimum and maximum values are chosen.

2.3 Probabilistic Seismic Hazard Analysis

In the present study, Cornell–McGuire approach is used to estimate the seismic hazard. The computations needed in the PSHA are performed using the program CRISIS 2007 (Ordaz et al. 2007).

In the analysis, we have four possible combinations of λ and β values and two sets of spectral acceleration values, i.e., minimum and maximum spectral acceleration values sets. It makes a total of eight possible combinations. Hence, eight trials have been run in CRISIS 2007 software. One of the best ways to represent the values obtained using fuzzy logic is to present the results in the form of ranges. From the values obtained from all the eight trials, minimum and maximum values of annual rate of exceedance are extracted and are shown in Fig. 5.

2.3.1 Seismic Hazard Curves

The effect of all the earthquakes at different locations, sizes, sources with different probability of exceedance is integrated into one curve. It shows the probability of different ground motions at a site, exceeding during a specified period of time in the form of a range.

The expected value of the PGA ranges corresponding to a return period of typical engineering interest, e.g., for 475 years (10% probability of exceedance in 50 years exposure time), and the range of peak horizontal acceleration can be read from Fig. 5 as 0.038–0.064 g. Different return periods can also be selected for reporting PGA values depending on the type and importance of the structure. All the hazard curves are developed for the rock sites. For any particular site, the local soil profile (thickness of the layers and type of soil) will transform the predicted ground motions.

2.3.2 Uniform Hazard Spectrum

Uniform hazard spectrum (UHS) is the spectral amplitudes of acceleration which can be evaluated at all the natural periods for a constant probability of exceedance at a site. Such a response spectrum is commonly known as the UHS. Fig. 6 shows the UHS corresponding to the upper bounds of 10% probability of exceedance in 50 years (475 years return period).

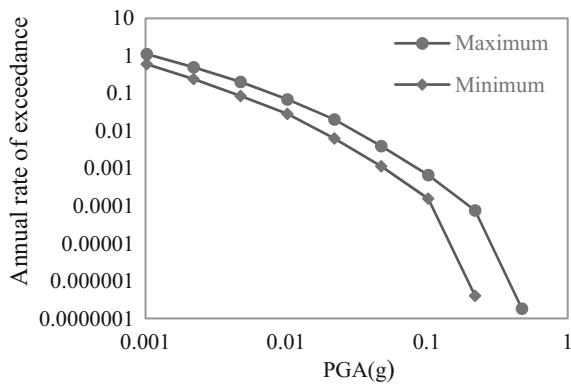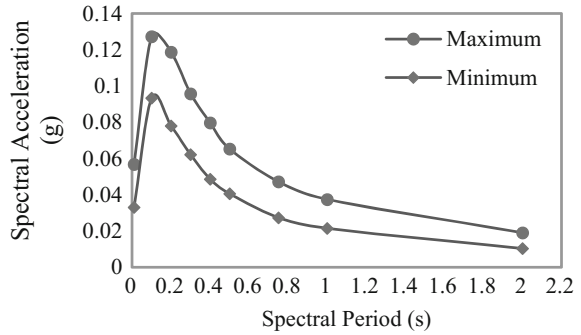**Fig. 5** Hazard curve for single seismic source zone

Fig. 6 UHS corresponding
to 10% probability of
exceedance in 50 years

3 Conclusion

In this study, a modular framework for seismic hazard assessment was presented. The quintessential strategy in developing the new framework is to improve and enhance each component of the conventional probabilistic seismic hazard analysis.

- The horizontal PGA expected in Warangal, with a 10% probability of exceedance in 50 years (which corresponds to a return period of 475 years), is varying from 0.038 to 0.064 g, whereas that with a 2% probability of exceedance in 50 years (return period = 2475 years) is in range of 0.08–0.113 g.
- The annual rates of exceedance and expected PGA calculated using PSHA method are observed to be lying in the ranges that are given by the FPSHA.
- The current study revealed that fuzzy sets could help to incorporate all the uncertainties in PSHA. However, a complete comparison could not be pulled off with previous studies, since not only the applied methods are different, as we have used fuzzy set theory, but also many of the input parameters and attenuation relationships are different.

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