Reliability Analysis of Slopes in Soils with Strain-Softening Behaviour

S. Metya, S. Dey, G. Bhattacharya and R. Chowdhury

Abstract This article presents a systematic approach for the reliability analysis of slopes in strain-softening soils based on the first order reliability method (FORM). The performance function is based on the Spencer method modified to take strain-softening into account in terms of the average residual factor R_F over a potential slip surface estimated based on a simple progressive failure model available in the literature. The shear strength parameters, peak and residual, are assumed as normally distributed random variables and the reliability analysis is performed on the probabilistic critical slip surface. For the residual factor R_F, bounded by 0 and 1, a generalized beta distribution has been assumed. Results obtained from an illustrative example indicate that a significant reduction (25%) occurs in the value of the minimum reliability index when R_F is considered as a random variable compared to when R_F is considered as a deterministic parameter. A FORM based sensitivity study also reveals that, amongst the five random variables, residual factor has the most dominating influence on the estimated reliability index and thus justifies its inclusion as one of the random variables.

Keywords Slope reliability · Peak and residual strengths · Residual factor Probability distribution

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1 Introduction

The conventional limit equilibrium method of slope analysis is based on the assumption of ideal plastic behaviour of soils. Such an assumption implies simultaneous failure of a slope. In contrast, slopes in strain-softening soils are associated with progressive failure. It is well known that in such soils the processes of progressive failure are often associated with a decrease in the values of shear strength parameters (Skempton 1964, 1985). Skempton proposed a definition of residual factor at a point in a soil mass as the extent to which shear strength has decreased from its peak value to its residual value. He also proposed a definition of the average residual factor over a slip surface as the proportion of slip surface length over which shear strength has reduced to a residual value. The average residual factor over a slip surface, denoted by R_F, therefore, deserves to be included as a parameter in the stability analysis of slopes in strain-softening soils associated with progressive failure. In the past, the residual factor, averaged over a potential slip surface, has been included, either directly or indirectly, as a deterministic variable even in probabilistic studies. However, because of uncertainties associated with the residual factor, it is very important to consider it as a random variable in slope reliability analyses within a probabilistic framework of slopes in strain-softening soils.

In this paper, a systematic approach for reliability analysis of a 2-D simple slope in a strain-softening soil considering the average residual factor over a potential slip surface as a random variable, has been developed on the basis of a limit equilibrium model, specifically, the Spencer's circular method. For the purpose of reliability analysis, the first order reliability method (FORM), widely accepted as the most versatile among the methods of reliability analysis based on analytical approximations (Haldar and Mahadevan 2000), has been made use of in this study. An assumption has been made regarding a suitable probability distribution of the average residual factor R_F.

2 Adopted Methodology

2.1 *Slope Stability Analysis—Deterministic and Probabilistic*

Deterministic slope stability analyses based on the limit equilibrium approach consists of two joint tasks, namely, computation of factor of safety of a given or trial slip surface, and then the search for the critical slip surface having the minimum factor of safety FS_{min} (called the deterministic critical slip surface) using an optimization technique. The Spencer method of slices (Spencer 1967), regarded as one of the rigorous methods, is used for the calculation of factor of safety and the sequential quadratic programming (SQP), rated as a powerful optimization technique (Hong and Roh 2008), is adopted in the MATLAB platform with its optimization toolbox.

Similar to the deterministic analysis, the probabilistic slope stability analysis can be viewed as the problem of locating the slip surface corresponding to the lowest value of reliability index $\beta_{\min}$ (or the highest value of probability of failure, p_{F}) (called the probabilistic critical slip surface of the slope). As already mentioned, for the determination of reliability index, the first order reliability method (FORM), has been adopted in this study. The detailed description of the computational procedure for the deterministic and probabilistic slope stability analysis can found elsewhere (Metya and Bhattacharya 2014) and the same computer programs have been made use of in this study. As a part of that study, these programs were validated with reference to two benchmark slope problems.

2.2 Modified Expression for Factor of Safety Based on Spencer Method Including Residual Factor

Based on the original definition by Skempton (1964, 1985), the average residual factor over a slip surface mass, denoted as R_{F}, can be expressed as

$$R_{\mathrm{F}} = \frac{s_{\mathrm{p}} - s}{s_{\mathrm{p}} - s_{\mathrm{r}}}, \quad (1)$$

where, s_{p}, s_{r}, and s denote the average values of the peak shear strength, the residual shear strength, and the current shear strength respectively, over a potential slip surface.

For the analysis of slopes in strain-softening soils, using the above definition of residual factor, the shear strength parameters have been redefined as follows:

$$c'_{rf} = R_{\mathrm{F}} c'_{\mathrm{r}} + (1 - R_{\mathrm{F}}) c'_{\mathrm{p}} \quad (2)$$

$$\tan \phi'_{rf} = R_{\mathrm{F}} \tan \phi'_{\mathrm{r}} + (1 - R_{\mathrm{F}}) \tan \phi'_{\mathrm{p}} \quad (3)$$

The expression for the factor of safety, FS, associated with a curved slip surface of circular shape for a simple slope, based on the Spencer method (Spencer 1967), has been modified for a strain-softening soil, by replacing c′ with c'_{rf} given by Eq. (2), tan ϕ' with tan ϕ'_{rf} given by Eq. (3).

2.3 Method of Estimation of Statistical Properties of the Residual Factor Using an LEM Based Progressive Failure Model (Chowdhury et al. 2010)

A simple model for progressive failure of slopes in strain-softening soils under the framework of the limit equilibrium methods of slices (LEM) has been given by Chowdhury et al. (2010). If the soil is assumed to be perfectly brittle strain-softening, the shear strength parameters of the overstressed slices will reduce to residual values, whereas the remaining segments will still be at the peak shear strength. Based on this model involving an iterative process of distribution excess shear stress from the overstressed slices, one can identify those segments of a potential slip surface for which the shear strength parameters have decreased from the peak to the residual values. Then the mean of the residual factor can be estimated as $R_F = L_r/L$ in which L is the total length of a slip surface of which the length L_r is at the residual shear strength, the remaining length ($L-L_r$) being still at the peak shear strength.

2.4 Probability Distribution for the Residual Factor

As regards the probability distribution of residual factor, a choice may be made between the assumption of a normal distribution and that of a generalized beta distribution. However, assuming normal distribution, errors will arise as the mean values approach the end points 0 and 1. Moreover, use of normal distribution excludes consideration of skewed distributions. A generalized beta distribution with the end points of 0 and 1 seems more appropriate. Once the mean of R_F is estimated based on the progressive failure model, as described above, the specific beta distribution to be used in a given situation depends on a reasonable assumption regarding the value of the COV of R_F. After making such an assumption, the beta-distribution parameters q and r corresponding to the specific shape of the beta distribution to be considered in the analysis are then calculated using Eqs. (4) and (5) (Harr 1977).

$$E[x] = a + \frac{q}{q+r}(b-a) \tag{4}$$

and,

$$V[x] = \frac{qr(b-a)^2}{(q+r)^2(q+r-1)} \tag{5}$$

where, $E[x]$ and $V[x]$ are the expected value and variance of residual factor as a beta distributed random variable with $a = 0$ and $b = 1$.

3 Illustrative Example

The application of the proposed procedure for reliability analysis of finite slopes in strain-softened soils is elucidated with the help of an example of a simple slope in a strain-softening soil selected from the literature (Chowdhury et al. 2010).

3.1 Slope Description and Input Data

A homogeneous slope is considered with height 25 m, inclination 22°, and unit weight of soil 18.8 kN/m^3. The statistical properties of the peak and the residual strength parameters, considered as random variables with normally distribution, are as given in Table 1. The pore pressure ratio r_u is taken as zero.

3.2 Studies Conducted

With the help of the illustrative example described above, studies have been conducted for purposes of numerical demonstration of the application of the proposed simplified procedure for 2-D reliability analysis of finite slopes in strain-softening soils. Of special interest is the analysis of the most likely scenario of strain-softening occurring in part of a slip surface (referred to as Case C) in which case the average residual factor has a value between 0 and 1. For such a scenario, it is necessary to quantify the extent to which strain-softening would advance in a given slope situation. As discussed in Sect. 2.3, for this purpose the simplified LEM-based progressive failure model proposed in Chowdhury et al. (2010) has been used. As against this general case, there are two extreme scenarios, namely, (i) the entire slip surface is at peak strength (R_F = 0) (Case A), and (ii) the entire slip surface is at residual strength (R_F = 1) (Case B). A comparison of results for the three cases will indicate the error introduced as a result of analysing a Case C scenario as either Case A (R_F = 0) or Case B (R_F = 1).

Further, to bring out the impact of including R_F as one of the random variables, reliability analyses have been performed first considering the residual factor as a deterministic parameter, and then, as a random variable. Moreover, in order to study

Table 1 Statistical properties of the strength parameters

Description of the parameter		Mean	Coefficient of variation
Peak strength parameters	c_p'	30.0 kPa	0.20
	tan ϕ_p'	tan(20)	0.10
Residual strength parameters	c_r'	10.0 kPa	0.20
	tan ϕ_r'	tan(12)	0.10

the relative influence of residual factor as a random variable on the reliability index, a sensitivity analysis based on the FORM has also been included.

3.3 Reliability Analyses and Corresponding Results

For slope reliability analysis, the probabilistic critical slip surface together with the value of the associated minimum reliability index $\beta_{\min}$ has been determined. In order to explore the effect of considering R_F as a random variable, analysis has been performed first (i) considering R_F as a deterministic (Study 1), and then (ii) considering R_F as a random variable (Study 2).

3.3.1 Study 1: Considering R_F as Deterministic

Considering the residual factor as deterministic (i.e. $R_F = 0$ for Case A and $R_F = 1$ for Case B), the probabilistic critical slip surfaces have been determined based on the computational procedure outlined in Sect. 2.1 using the FORM in conjunction with the SQP technique, and are shown in Fig. 1. The minimum reliability index ($\beta_{\min}$) associated with these two probabilistic critical slip the surfaces (for Case A and Case B) are obtained as 4.244 ($p_F = 1.10 \times 10^{-5}$) and −2.059 ($p_F = 9.80 \times 10^{-1}$) respectively.

Besides the two extreme scenarios presented above, the third or the most likely scenario (Case C) is considered next. For this case, unlike the above two cases, during the search for the probabilistic critical slip surface, the value of R_F is estimated for each trial slip surface based on the Chowdhury et al. (2010) simplified model. Following the same general procedure as used above, the probabilistic critical slip surface has been determined for the Case C as also shown in Fig. 1 in

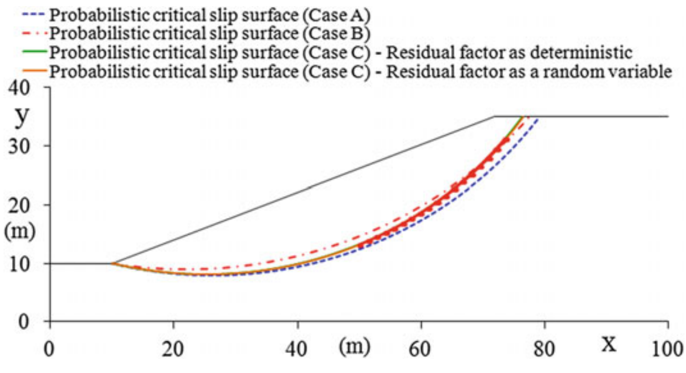

Fig. 1 Probabilistic critical surfaces for case A, case B and case C

which the strain-softened portions (failed slices) of the slip surfaces are highlighted in red bullet points. It is observed that all the three probabilistic critical slip surfaces are rather close to one another except in their upper segments. For Case C, the associated value of β_{min} is obtained as 3.555 (p_{F} = 1.89 × 10^{-4}) while the associated value of R_{F} is 0.395. It is thus seen that the β_{min} value for Case C is in between the two extreme cases i.e., β_{min} value for Case A and Case B, which is expected. A comparison of the β_{min} values for the three cases further reveals that assuming peak shear strength for the entire slip surface overestimates the reliability index by nearly 19%, while assuming residual shear strength for the entire slip surface underestimates the reliability index by nearly 158%.

3.3.2 Study 2: Considering R_{F} as a Random Variable

In this study, the residual factor R_{F} is considered as a statistically independent random variable. During the search for the probabilistic critical slip surface, for each trial slip surface, its mean value is estimated using the Chowdhury et al. (2010) model, while its COV is assumed to be 0.3. The obtained probabilistic critical slip surface is shown again in Fig. 1. The value of the associated β_{min} is obtained as 2.856 (p_{F} = 2.1 × 10^{-3}) while the associated value of R_{F} is 0.396.

3.3.3 Comparison Between Results from Study 1 and Study 2

It is seen that the β_{min} for Case C considering the residual factor R_{F} as a random variable is significantly lower than that assuming R_{F} as deterministic and percentage difference is nearly 25%. It is, however, observed from Fig. 1 that these two slip surfaces are rather close to each other.

3.3.4 FORM-Based Sensitivity Analysis

A FORM-based sensitivity analysis comparing the values of direction cosines for various random variables for the probabilistic critical slip surface is shown in Fig. 2. It is observed that the residual factor R_{F} has the largest influence on the reliability index among the five random variables considered in this study and this observation clearly indicates the consideration of the residual factor as one of the random variables in the reliability analysis of slopes in strain-softening soils.

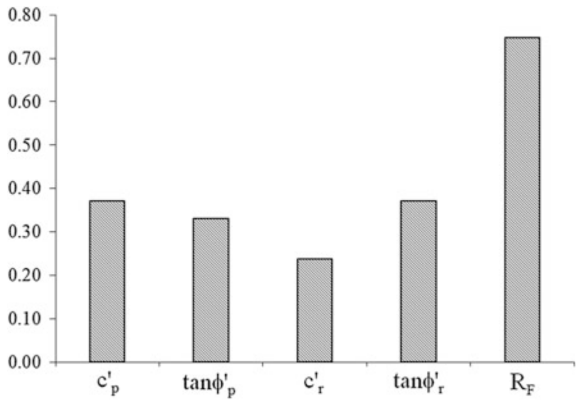

Fig. 2 Results of sensitivity analysis based on the FORM method

4 Conclusions

Based on the studies undertaken in this paper, the following concluding remarks can be made:

1. In this study, three possible scenarios have been studied, namely, the two extreme cases, Case A, when the entire slip surface is at peak strength ($R_F = 0$), and, Case B, when the entire slip surface is at residual strength ($R_F = 1$), and the most likely one, Case C, when strain softening has taken place over part of the slip surface ($0 < R_F < 1$). For Case C, reliability analyses have yielded a β_{min} value of 3.555 for deterministic R_F and 2.856 for random R_F. When R_F is deterministic, Case A overestimates the reliability index by nearly 19%, while Case B underestimates the reliability index by nearly 158%. When R_F is random these differences are even higher (49 and 172% respectively).
2. The effect of including R_F as one of the random variables (with an assumed COV of 0.3) in reliability analysis of a slope in strain-softening soil is substantial; the value of reliability index reduces by a margin of 25% compared to when R_F is considered deterministic. It, however, needs to investigate the effect of the assumption regarding a suitable value of COV for R_F.
3. FORM-based sensitivity studies confirm that, amongst the five random variables, residual factor has the most dominating influence on the estimated reliability index and thus justifies its inclusion as one of the random variables.

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