

# Analytical Seismic Fragility Analysis of Existing Building Frame in Northeast India



S. Mukherjee, S. Ghosh, S. Ghosh and S. Chakraborty

**Abstract** The analytical seismic fragility analysis (SFA) of a typical mid-rise steel building frame considered to be located in the Guwahati city of Northeast India is presented in the performance-based earthquake engineering (PBEE) framework. The approach starts with a detailed probabilistic seismic hazard analysis (PSHA) of the Guwahati city to obtain hazard curve parameters. Subsequently, nonlinear time history analyses (NLTHA) are carried out to obtain displacement demand model parameters for the example frame. For dynamic analysis, a representative ground motion bin is prepared judiciously. As the recorded accelerograms in the region is limited, the bin is formed with recorded as well as artificially and synthetically generated data. The structural capacities at various limit states are obtained from random pushover analysis (RPA). With the estimated seismic hazard, demand and capacity parameters, fragility curves are generated for different performance levels. The annual probability of failure is finally estimated based on the seismic hazard and the fragility curves.

**Keywords** Northeast India · Probabilistic seismic hazard analysis  
Steel frame building · Nonlinear dynamic and pushover analysis  
Seismic fragility analysis

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## 1 Introduction

The Northeast (NE) India, comprising of the States of Arunachal Pradesh, Assam, Manipur, Meghalaya, Mizoram, Nagaland, Sikkim and Tripura, is one of the seismically most active regions of the world. The region has experienced 18 large earthquakes ( $M \geq 7$ ) during the last hundred years including the great earthquakes of Shillong (1897,  $M = 8.7$ ) and Assam–Tibet border (1950,  $M = 8.7$ ). The region marks the boundary between Indian and Eurasian plates. The complex tectonic and geological set up of the region cause plate movements in different directions. Due to the highly oblique continental convergence of the northward-moving Indian plate at the rate of  $20 + 3$  mm/year, earthquakes of magnitudes 8 and above have occurred in the past and may recur. But the big concern is that with the present state of the art knowledge it is not possible to predict when, where and how it will occur. Moreover, there has been a phenomenal increase in the population density and infrastructural development programmes in the region. These factors are further increasing the vulnerability of human population and structures to earthquakes. Thus, it becomes essential to assess the status of seismicity in the NE region realistically for an urgent and sustained mitigation measure.

A significant amount of research conducted on the subject and the subsequent advancements in the understanding and knowledge of earthquake-resistant structural design provided a better recognition of the need for Seismic vulnerability assessment (SVA) and retrofit of older existing buildings. The recent development of performance-based earthquake engineering (PBEE) in the context of probabilistic seismic response analysis of structures is believed to open a new way for SVA of structures considering parameter uncertainty. In fact, seismic fragility analysis (SFA) has emerged as an integrated platform for SVA in a probabilistic framework to quantify seismic risk during the life-cycle of a structure.

Though, the development in the field of SVA at international level is exhaustive [1, 2]; same is not the case for NE region. The studies on seismic hazard analysis of the NE region are notable [3, 4]. However, the studies on the seismic safety of existing structures are very scarce and limited to simplified evaluation in deterministic framework [5, 6]. Thus, realistic SFA of existing structures of the NE region for an urgent and sustained mitigation measure has become important.

The present study deals with analytical SFA of a typical mid-rise moment resisting steel building frame located in the Guwahati city of NE India. The approach starts with a detailed Probabilistic Seismic Hazard Analysis (PSHA) considering the seismicity around 300 km radius around the city to estimate the seismic hazard parameters corresponding to the desired hazard level. Subsequently, the nonlinear time history analyses (NLTHA) are carried out to obtain the displacement demand model and the demand parameters are calculated accordingly for the example frame. For NLTHA, a representative ground motion bin corresponding to the specified hazard level for the location of the structures is necessary. As the recorded accelerograms in the NE region is very scarce, the present study is limited to use of eight numbers of recorded accelerograms. And, to supplement the limited

recorded input, the accelerogram data are further generated artificially and synthetically so that statistically meaningful study can be performed for seismic demand analysis. Eight numbers of accelerograms are generated artificially and matched with IS code (IS 1893: 2000) specified pseudo-acceleration response spectrum for seismic Zone V and for rock and hard soil site corresponding to 5% damping. Another eight numbers are synthetically generated by stochastic point source modelling, identifying the most vulnerable source for the specific hazard level for the city. The structural capacities at various limit states are obtained from the random pushover analysis (RPA) of the considered frame. Finally, with the knowledge of the seismic hazard parameters, demand and capacity parameters, the analytical seismic fragility expression is obtained in the context of PBEE and fragility curves are generated corresponding to different structural limit states namely the immediate occupancy (IO), life safety (LS) and collapse prevention (CP). Annual probability of failure of the example frame is also estimated based on the seismic fragility and hazard data obtained by PSHA corresponding to specified hazard level.

## 2 Fundamental of Seismic Risk Analysis

The fundamental of seismic risk analysis procedure is to calculate the probability of exceeding a structural limit state. It is basically a time-dependent structural reliability analysis problem in which the limit state of interest is the difference between seismic demand and capacity. It can be mathematically expressed as

$$Z(\mathbf{X}_C, \mathbf{X}_D, t) = C(\mathbf{X}_C, t) - D(\mathbf{X}_D, t) \quad (1)$$

In the above,  $\mathbf{X}_C, \mathbf{X}_D$  are the variables governing the capacity and demand and  $t$  is the time parameter. The computation of probability that the limit state function is negative represents the seismic risk of the structure, i.e.

$$P_f = \int_{Z < 0} f_Z(\mathbf{X}) dZ \quad (2)$$

where  $\mathbf{X}$  is an 'n' dimensional vector having variables involving seismic demand and capacity,  $f_Z(\mathbf{X})$  is the joint probability density function (pdf) of the involved random variables. The exact computation of the above is often computationally demanding. In fact, the joint pdf of  $f_Z(\mathbf{X})$  is hardly available in closed form. Various approximations are usually adopted to obtain the probability of exceeding different limit states of damage for a response parameter under a specific seismic intensity measure. This is customarily termed as SFA. The SFA by PBEE in the probabilistic framework can be performed by two approaches: (i) Analytical fragility based on probabilistic seismic demand and capacity model and

(ii) Simulation-based fragility based on nonlinear PBEE using random field theory and statistical simulation. Considering the focus of the present study, the analytical SFA is presented here.

### 3 Analytical Seismic Risk Analysis

The problem of seismic risk evaluation for a structure at a site is to obtain the failure limit state probability,  $P_{LS} = P[D \geq C]$ , i.e. the exceedance of the structural demand value ( $D$ ) to its capacity ( $C$ ). In order to determine  $P_{LS}$ , the equation can be decomposed into parts with respect to an interface variable using the concept of total probability theorem [7]. Considering the spectral acceleration ( $S_a$ ) as the interface variable, the decomposition can be obtained as

$$\begin{aligned} P_{LS} &= P[D \geq C] = \sum_x P[D \geq C | S_a = x] \cdot P[S_a = x] \\ &= \int_x F_R(x) \cdot |d\lambda_{S_a}(x)| \end{aligned} \quad (3)$$

The problem of calculating the limit state probability in Eq. (3) involves solution of two problems. The first term in the integral represents the conditional limit state probability for a given level of ground motion intensity which is the seismic fragility,  $F_R(x)$ . The second problem is to estimate the spectral acceleration hazard for the location, which can be estimated from PSHA. For this, a detailed PSHA is carried out in the present study, considering the seismicity around the site from historical earthquake data. Evaluations of these are briefly discussed in the following subsections.

#### 3.1 Spectral Acceleration Hazard, $P[S_a \geq x]$

The spectral acceleration hazard at a site is based on evaluating the pdf of a random parameter  $Z$ , representing the strong ground motion parameter at a site due to all the earthquakes expected to occur during a specified exposure period in the region around the site. The peak spectral velocity,  $PSV(T)$  for a time period  $T$  is considered as the ground motion parameter here for PSHA. If,  $q(z|M_j R_i)$  denotes the probability that a given  $PSV(T)$  of amplitude ‘ $z$ ’ at a site will be exceeded during an event of magnitude  $M_j$  occurring at a source element  $R_i$ , the annual average occurrence rate of earthquakes  $\lambda(z)$  can be obtained as:

$$\lambda(z) = \sum_{i=1}^I \sum_{j=1}^J q(z|M_jR_i)n(M_jR_i) \tag{4}$$

The quantity  $n(M_jR_i)$  in the above equation is the annual rate of occurrence of the  $(M_j, R_i)$  event. The estimated value of the PSV(T) amplitude for a particular  $M_j$  and  $R_i$  combination will be  $\log[\text{PSV}'(T)] + \varepsilon(T)$ ,  $\varepsilon(T)$  is the error residual term obtained from the attenuation model [4]. The probability that the estimated value of PSV'(T) will be greater than a specific value 'z' can be obtained as,  $\varepsilon(T) > \log[z] - \log[\text{PSV}'(T)]$ . Based on the normal distribution assumption of the residuals  $\varepsilon(T)$ , the cumulative distribution function (CDF) can be obtained as

$$p(\varepsilon(T)) = \frac{1}{\sigma(T)\sqrt{2\pi}} \int_{-\infty}^{\varepsilon(T)} e^{-\frac{1}{2}\left(\frac{x-\mu(T)}{\sigma(T)}\right)^2} dx \tag{5}$$

Thus, the term  $q(z|M_jR_i)$  can be obtained as  $1 - p(\varepsilon(T))$ , where  $p(\varepsilon(T)) = \Phi(\varepsilon(T))$ . Assuming that  $\lambda(z)$  is the average occurrence rate of a Poisson process, the probability of exceeding z during an exposure period of Y years can be obtained as

$$P[\text{PSV}(T)] = 1 - \exp\{-\lambda(z).Y\} \tag{6}$$

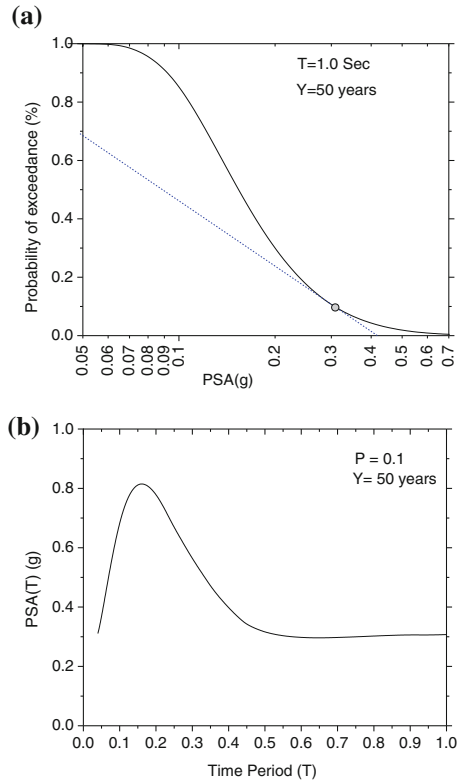
The CDF is obtained for a specified exposure period (50 years herein) using Eq. (4) in Eq. (6) with the knowledge of  $n(M_jR_i)$  around the site and an attenuation model. The details of those calculations may be seen in [4].

The steel frame taken up for the case study (detailed in Sect. 5) is considered to be located in the Guwahati city. Based on the procedure outlined above, the PSV hazard is obtained and converted to PSA hazard by multiplying its ordinates with  $2\pi/T$ . Figure 1a shows the peak spectral acceleration (PSA) hazard curve (in terms of 'g') for horizontal motions, for  $T = 1.0$  s,  $Y = 50$  years. Figure 1b shows the uniform hazard spectra (UHS) obtained for 10% probability of exceedance in 50 years. The hazard curve is approximated in the region of interest by a power-law relationship [7]:

$$\begin{aligned} \lambda_{\text{IM}}(x) &= P[\text{IM} \geq x] \\ &= 1 - \exp\left[-(x/u)^{-k}\right] \approx (x/u)^{-k} \approx k_0x^{-k} \end{aligned} \tag{7}$$

where  $u$  and  $k$  are the scale and shape parameter, respectively, and  $k_0 = u^k$ . Drawing a tangent line to the hazard curve (Fig. 1a) at the point of interest (i.e. points corresponding to  $p = 0.1$ ,  $Y = 50$  years), the slope and intercept of the tangent are obtained which provides the values of the parameters  $k_0$  and  $k$ . These are estimated as 0.0004 and  $-2.72$ , respectively. Thus, the power-law model of PSA hazard for

**Fig. 1 a** Hazard curve for  $T = 1.0$  s and exposure period of 50 years. **b** Uniform hazard spectra for 10% probability of exceedance in 50 years



Guwahati city is obtained as  $\lambda_{IM}(x) = 0.0004x^{-2.72}$  for  $T = 1.0$  s,  $p = 0.1$  and  $Y = 50$  years (i.e. 475 years of return period).

### 3.2 Damage Fragility $P[D \geq C | S_a = x]$

The next problem is to calculate the conditional failure probability of seismic demand reaching or exceeding random structural capacity given a specific value of seismic intensity, i.e.  $F_R(x) = P[D \geq C | S_a = x]$ . Assuming structural capacity is log-normal with uncorrelated  $D$  and  $C$ , following the fundamentals of first order reliability analysis method, the damage fragility (probability of failure) can be obtained as

$$F_R(x) = \Phi \left[ -\frac{\ln m_C - \ln m_D}{\sqrt{\beta_C^2 + \beta_{D|S_a}^2}} \right] = \Phi \left[ \frac{\ln m_{D|S_a} - \ln m_C}{\sqrt{\beta_{D|S_a}^2 + \beta_C^2}} \right] \quad (8)$$

Here  $F_R(x)$  is the fragility that the median demand will reach or exceed the median capacity for a specified value of  $S_a = x$ ,  $m_C$  is the median capacity and  $\beta_C$  is its log-normal standard deviation (SD). For a given level of intensity  $S_a$  there will be variability in displacement based demand due to randomness in seismic phenomenon. A functional relationship is conveniently introduced between the spectral acceleration and the median,  $m_D$  of the demand variable. In general, the conditional median of  $D$  for  $S_a = x$  can be expressed as

$$m_{D|S_a}(x) = g(x) \tag{9}$$

Assuming that the demand parameters are log-normally distributed, the above conditional probabilistic model can be represented as

$$D = m_{D|S_a}(x) \cdot \varepsilon \tag{10}$$

In which  $\varepsilon$  is a log-normal random variable with median equal to unity and conditional logarithmic SD  $\sigma_{\ln \varepsilon} = \beta_{D|S_a}$ . A power-law relationship can be readily obtained between the median demand and spectral acceleration as:

$$m_{D|S_a}(x) = a \cdot x^b \tag{11}$$

where, ‘ $a$ ’ and ‘ $b$ ’ are the regression parameters and can be obtained by probabilistic seismic demand analysis methods, e.g. cloud analysis (CA), strip method or incremental dynamic analysis method. In the present study, the CA method is used to obtain the median relationship which is further detailed in the case study section.

Now, substituting Eq. (11) in Eq. (8) yields

$$\begin{aligned} F_R(x) &= \Phi \left[ \frac{\ln(ax^b) - \ln m_C}{\sqrt{\beta_{D|S_a}^2 + \beta_C^2}} \right] \\ &= \Phi \left[ \frac{\ln x - \ln(m_C/a)^{1/b}}{\frac{1}{b} \sqrt{\beta_{D|S_a}^2 + \beta_C^2}} \right] = \Phi \left[ \frac{\ln(x/m_R)}{\beta_R} \right] \end{aligned} \tag{12}$$

where,  $m_R$  and  $\beta_R$  are the median and dispersion of the damage fragility defined as  $m_R = (m_C/a)^{1/b}$  and  $\beta_R = \frac{1}{b} \sqrt{\beta_{D|S_a}^2 + \beta_C^2}$ . The log-normal SD of capacity,  $\beta_C = \sqrt{\beta_{CR}^2 + \beta_{CU}^2}$ , where  $\beta_{CR}$  represents the effect of structural uncertainty propagation obtained through RPA of the considered structure for different performance level. While  $\beta_{CU}$  arises from the assumption of structural modelling. Due to the non-availability of specific information on epistemic uncertainty ( $\beta_{CU}$ ), it is considered to be 0.2 in the present study [8].

### 3.3 Analytical Seismic Risk

Once the conditional failure probability  $F_R(x)$  is obtained from Eq. (12), the analytical expression can be readily obtained to estimate the seismic risk. By substituting Eqs. (7) and (12) in Eq. (3) and carrying out the integral, the failure limit state probability,  $P_{LS}$  can be obtained as

$$\begin{aligned}
 P_{LS} &= P[D \geq C] = \sum_x P[D \geq C | S_a = x] \cdot P[S_a \geq x] \\
 &= \int_x F_R(x) \cdot d\lambda_{S_a}(x) = \lambda_{IM}(m_R) \exp\left(\frac{1}{2} k^2 \beta_R^2\right) \\
 &= \lambda_{IM}(m_R) \exp\left[\frac{1}{2} \frac{k^2}{b^2} (\beta_{D|S_a}^2 + \beta_C^2)\right]
 \end{aligned} \tag{13}$$

The above displacement based explicit format to obtain annual limit state frequency can be directly used by substituting the values of  $m_R$  and  $\beta_R$  in Eq. (13) and the use of the hazard curve relation as describe by Eq. (7).

## 4 Selection of Ground Motion Bin

The analytical SFA in the framework of PBEE as described in the previous section largely hinges on proper evaluation of structural demand parameters through nonlinear response history analysis. The most acceptable form for NLTHA of SFA of structures corresponds to the use of recorded accelerograms. However, due to the limited availability of recorded accelerograms specific to the hazard level for the focused region, the choice of natural ground motions here is limited to eight numbers. Thus, eight numbers of accelerograms are generated artificially and another eight numbers are synthetically generated identifying the most vulnerable  $M_j$  and  $R_i$  combination for the specific hazard level of the location under consideration. These are briefly discussed in the following.

The eight natural records are selected from the past earthquake data in the region which covers a surface wave magnitude range from 6.0 to 8.0 and epicentral distance within 300 km for rock site. For the disaggregation study, a target hazard level is identified and contribution of each source is calculated by finding out the probability of exceedance of the target hazard level for each of the sources. Due to limited resources of recorded accelerograms in the region, the accelerogram records are also selected from Northern Himalayan earthquakes corresponding to similar subsoil sites (available in the COSMOS virtual data centre). The earthquakes with an epicentral range within 10 km are avoided due to the possibility of directivity pulse effect.



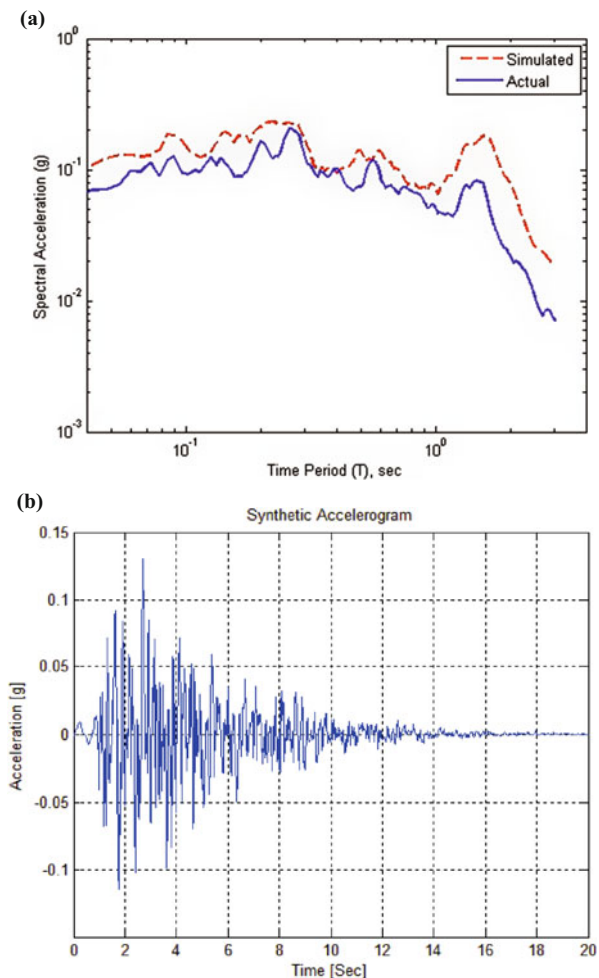
The artificial accelerograms are generated compatible with the acceleration response spectra for rock and hard soil for 5% damping [9]. The power spectral density (PSD) function is obtained following Kaul [10] and the accelerograms are generated accordingly following the methodology proposed by Gasparini and Vanmarcke [11]. Defining a vector of amplitudes and simulating different arrays of phase angles, the stationary stochastic process is obtained with the same general aspect but with different characteristics. These amplitudes are calculated using the PSD and the random phase angles are generated in the interval of  $0-2\pi$ . Further, to simulate the transient nature of the earthquakes, the steady-state motions are multiplied by a deterministic envelope function [12].

The stochastic ground motion model as proposed by Boore [13] is used for generation of synthetic acceleration time histories. Following this, eight accelerograms are generated for different magnitudes between 6.0 and 8.0 and epicentral range within 300 km. Figure 2a compares actual and one typical simulated acceleration spectra for Loharghat Station (closest to the Guwahati City) and Fig. 2b is the associated synthetic accelerogram generated for  $M_w = 6.0$  and  $R = 150$  km and focal depth = 15 km.

## 5 Case Study: A Six-Storey Steel Frame

A two-bay steel moment frame as shown in Fig. 3 considered to be located in the Guwahati city is undertaken for the numerical elucidation of the analytical SFA and seismic risk evaluation procedure. The structural analysis is performed by using SAP 2000 software. The grade of steel is Fe250 having expected yield stress of 275 MPa. The beams and columns are modelled with lumped plasticity at their ends. The nonlinear hinges are assigned at the beam and column ends. The beams are modelled with moment hinges (M3) whereas the columns are modelled with axial-moment (P-M3) hinges. Auto hinges are assigned at beam and column ends according to tables of FEMA 356 [14]. The NLTHA is carried out by Hilber–Hughes–Taylor integration scheme. The mass and stiffness proportional damping, i.e. Rayleigh's damping is considered as 2% for the first two modes.

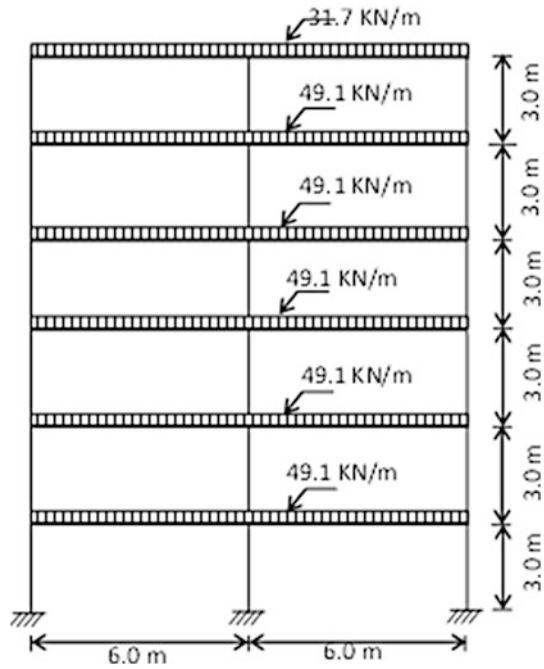
The NLTHA of the frame is carried out for the 24 numbers of ground motion inputs and the maximum interstory drift (ISD) values are obtained for each of ground motion input representing the structural demands  $D$ . The power-law relationship conveniently constructed between the median maximum ISD and spectral acceleration values and the regression parameters 'a' and 'b', obtained by the cloud analysis. For the considered building frame the power-law relationship between the median demand and spectral acceleration is obtained as:  $m_D(x) = 1.713 \cdot x^{0.828}$ ,  $\beta_{D|S_a} = 0.263$ .



**Fig. 2** **a** Acceleration spectra for actual and simulated accelerogram for Loharghat Station. **b** Generated accelerogram from the modified Fourier spectrum

The RPA is performed to obtain the probabilistic seismic capacity of the frame. For the present study, four structural parameters are considered to be random as detailed in Table 1. With the assumed distribution types of the parameters, random samples are generated using Latin Hypercube Sampling (LHS) with reduced correlation. With the generated 100 sets of random parameters and the maximum ISD values are calculated corresponding to each limit state. The three structural limit states or performance levels, i.e. IO, LS and CP as per FEMA 356 are considered for seismic risk evaluation. The median capacity values ( $m_C$ ) and the associated SD ( $\beta_C$ ) are obtained accordingly and are depicted in Table 2.

**Fig. 3** Elevation of the two-bay steel frame



**Table 1** The properties of the random parameters

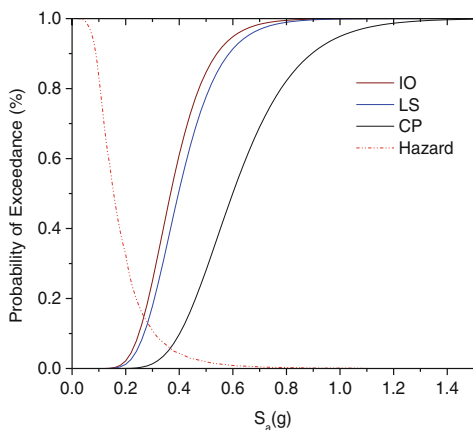
Uncertainty source	pdf	COV
Dead load	Normal	0.07
Live load	Normal	0.10
Steel yield strength	Log-normal	0.1
Steel elastic modulus	Log-normal	0.02

**Table 2** The median capacity and associated SD

Parameter	IO	LS	CP
$m_C$ (%)	0.65	0.7	1.04
$\beta_C$	0.06	0.07	0.10

With these parameters, the fragility curves corresponding to all the three limit states are obtained for the considered frame and shown in Fig. 4. With the knowledge of the fragility and hazard curve the annual failure probability, corresponding to the spectral acceleration hazard level of 10% probability of exceedance in 50 years, is calculated from Eq. (13) and are shown in Table 3 for different performance limit states.

**Fig. 4** The seismic fragility curves corresponding to different limit states



**Table 3** The annual probability of failure for different performance limit states

Limit state	$P_{LS}$
IO	0.0071
LS	0.0058
CP	0.0020

## 6 Discussion and Conclusions

The analytical SFA of steel frame located in the Guwahati city of the NE India is performed in the framework of PBEE and the associated seismic risks are estimated for different limit states. From the fragility curves, it is observed that the structure is associated with high probability of failure for IO limit state corresponding to the spectral acceleration hazard level (i.e. 0.3g) at fundamental period of the structure. However for LS and CP limit state, the performance of the structure is rather satisfactory. The hazard value corresponding to 10% probability of exceedance in 50 years as obtained from the PSHA at the fundamental period is close to the hazard value as specified by the Indian code, i.e. 0.36g. Hence the spectral acceleration hazard level considered for the present analysis may be regarded as the representative of the IS code specified hazard level. Based on the observations it may be opined that a steel moment frame building located in NE India with the moderate fundamental period when designed according to the IS code guidelines is most likely to perform beyond its elastic range, i.e. beyond IO level when subjected to the IS code specified hazard level. This clearly indicates the need for a more detailed study on nonlinear seismic performance evaluation of structures located in the NE India.

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