

# Asymptotic Symbol Error Rate Analysis of Weibull/Shadowed Composite Fading Channel



Puspraj Singh Chauhan and Sanjay Kumar Soni

**Abstract** In this work, we derive the asymptotic expressions of the average symbol error probability (SEP) of a wireless system over the Weibull-lognormal fading channel. First, we evaluate an approximation of the multipath distribution at the origin then the composite distribution is obtained by averaging the approximate multipath probability density function (PDF) with respect to shadowing. The result is further extended to include maximal ratio combining (MRC), equal gain combining (EGC), and selection combining (SC) PDF at the origin. The derived expressions of the composite PDF are further utilized to evaluate the average SEP for both coherent and non-coherent modulation schemes. The derived expressions have been corroborated with Monte-Carlo simulations.

**Keywords** Probability distribution function · Diversity · Symbol error probability

## 1 Introduction

A composite model is a class of mathematical model which includes both multipath and shadowing phenomena simultaneously and hence is a more realistic model. Among the available class of composite fading models, Weibull-lognormal (WLN) draws its significance from the fact that the Weibull distribution is known to characterize the multipath effects of an indoor and outdoor channel, based on its excellent matching with the measurements conducted in related environments [1–4]. The shadowing effect of the channel is best captured by the lognormal (LN) distribution [5]. Moreover, the LN distribution is shown to characterize a number of wireless applications such as an outdoor scenario, fading phenomenon in an indoor environment, radio channels affected by body worn devices, ultra wideband indoor channels, and

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weak-to-moderate turbulence channels found in free-space optical communications channels [5–7].

In the performance analysis of a wireless system, the closed-form solution facilitates better interpretation of system behavior. Yet, sometimes the complexities of the expression defies the basic purpose of the system optimization [8]. This motivates us to go for an asymptotic analysis of system performance. In the literature, various work related to asymptotic behavior of a system has been carried out [9–11]. For example, in [9], asymptotic bit error rate (BER) analysis has been presented for maximal-ratio combining with transmit antenna selection in flat Nakagami-m fading channels. In [10], simplified expressions of the BER for the  $\eta - \mu/\text{Gamma}$  composite fading channel in a high-power regime are derived. The asymptotic BER expressions for the  $\alpha - \eta - \mu$  fading channel have been derived for both coherent and non-coherent modulation schemes [11]. To date, the asymptotic analysis over W-LN fading channel with diversity reception has not been reported in the open literature. Recently, authors of a current paper have reported asymptotic closed-form expressions of the average symbol error probability (SEP) with maximal ratio combining (MRC) diversity [12]. The common approach adopted to derive the asymptotic solutions of the average SEP over the composite fading channel is to first derive the composite distribution by averaging the multipath with respect to shadowing, approximate the distribution at the origin as suggested in [8], then deduct the average SEP. Generally, composite distribution following the previous concept may lead to a result having a summation term, and thus the solution may not be tractable as far as the derivation of the probability density function (PDF) of the MRC, equal gain combining (EGC), and selection combining (SC) output is concerned, and usually does not lead to the closed-form solution.

In this chapter, we obtain the asymptotic expressions for the average SEP with all three diversity schemes such as MRC, EGC, and SC. While deriving the asymptotic solutions we have followed the following approach. First, we evaluate an approximation of the multipath distribution at the origin then the composite distribution is obtained by averaging the approximate multipath PDF with respect to shadowing. The result is further extended to include MRC, EGC, and SC PDFs at the origin. These expression have been used to evaluate the closed-form solutions of the average SEP. Furthermore, we have compared the performance of MRC, EGC, and SC in the context of error probability over the composite fading channel.

## 2 System Model

The Weibull envelope “X” has the PDF given as follows [13]:

$$f_X(x) = \frac{cA}{\Omega^{c/2}} x^{c-1} \exp \left[ -A \left( \frac{x^2}{\Omega} \right)^{c/2} \right] \quad (1)$$

where  $\Omega$  is the average fading power  $\Omega = E[X^{\frac{c}{2}}]$ ,  $A = [\Gamma(1 + \frac{2}{c})]^{c/2}$  and  $\Gamma(\cdot)$  is the Gamma function. Here  $c$  is the multipath parameter and the channel condition im-

proves as  $c \rightarrow \infty$ . As a special case, when  $c = 1, 2$  the Weibull distribution reduces to the exponential and Rayleigh distributions, respectively. An LN random variable (RV) “Z” has the PDF [5]:

$$f_Z(z) = \frac{1}{z\sigma\sqrt{2\pi}} \exp \left[ - \left( \frac{\ln z - \mu}{\sqrt{2}\sigma} \right)^2 \right] : z > 0 \quad (2)$$

where  $\sigma$  and  $\mu$  are the mean and standard deviation of  $\ln(Z)$ . The expected value of Z is  $E[Z] = \Gamma = Z_{avg} = \exp(\mu + \sigma^2/2)$ . As such, and by using Taylor’s series, the  $f_X(x)$  given in (1) can be rewritten as:

$$f_X(x) = \frac{cA}{\Omega^{\frac{c}{2}}} x^{c-1} + \mathcal{O} \quad (3)$$

where  $\mathcal{O}$  stands for higher order terms. First, substitute (3) and (2) into the definition of the composite distribution [14, Eq. (3)], then setting  $t = (\ln(z) - \mu)/\sqrt{2}\sigma$ , employing the identity [15, Eq. (3.323.2<sup>10</sup>)], and finally following the conversion  $\gamma = x^2\rho$ ,  $\bar{\gamma} = \Omega\rho$  and  $f_Y(\gamma) = f_X(\sqrt{\gamma/\rho})/2\sqrt{\gamma\rho}$ , where  $\rho = \frac{E_s}{N_0}$ ,  $E_s$  is the energy per symbol and  $N_0$  is the one-sided power spectral density of the additive white Gaussian noise (AWGN) [13], the signal-to-noise ratio (SNR) distribution of the composite distribution around origin can be given as:

$$f_Y(\gamma) \approx \frac{cAe^{-\frac{\mu c}{2}} e^{\frac{\sigma^2 c^2}{8}}}{2\rho^{\frac{c}{2}}} \gamma^{\frac{c}{2}-1} \quad (4)$$

The simplified PDF of (4) does not contain any summation term, thus enabling us to derive the PDF of the diversity combiner output in a convenient way, which is presented next.

## 2.1 Maximal Ratio Combining Probability Density Function at the Origin

For MRC with  $L$  independent and identically distributed (i.i.d.) diversity branches, the instantaneous SNR of the combiner output is given by:

$$\gamma_{mrc} = \sum_{j=1}^L \gamma_j \quad (5)$$

where  $\gamma_j$  is the instantaneous SNR of the  $j$ th branch. Since the  $L$  WLN RVs are i.i.d., the moment-generating function (MGF) of  $\gamma_{mrc}$  is expressed as  $M_{\gamma_{mrc}}(s) = \prod_{j=1}^L M_{\gamma_j}(s)$ , where  $M_{\gamma_j}(s)$  is the  $j$ th branch MGF and is deduced by taking the Laplace transform

of (4) with the aid of [15, Eq. (3.381.4)]. Thus, assuming the average SNR of each branch to be same, i.e.,  $\rho_1 = \rho_2 = \dots = \rho_L = \rho$ , the MGF of  $\gamma_{mrc}$  can readily be shown as:

$$M_{\gamma_{mrc}}(s) \approx \left( \frac{cAe^{-\frac{\mu c}{2}} e^{\frac{\sigma^2 c^2}{8}} \Gamma(\frac{c}{2})}{2(s\rho)^{\frac{c}{2}}} \right)^L \quad (6)$$

The PDF of the RV  $Y_{mrc}$  is deduced by performing the inverse Laplace transform of (6) with the aid of [15, Eq. (3.381.4)], yields [12]:

$$f_{Y_{mrc}} \approx \frac{(\vartheta)^L (\Gamma(\frac{c}{2}))^L}{\rho^{\frac{Lc}{2}} \Gamma(\frac{Lc}{2})} \gamma^{\frac{Lc}{2}-1} \quad (7)$$

where  $\vartheta = \frac{cAe^{-\frac{\mu c}{2}} e^{\frac{\sigma^2 c^2}{8}}}{2}$ .

## 2.2 Equal Gain Combining Probability Density Function at the Origin

For L i.i.d. diversity branches, the instantaneous SNR of the EGC output is given as:

$$\gamma_{egc} = \left( \frac{1}{\sqrt{L}} \sum_{j=1}^L \sqrt{\gamma_j} \right)^2 \quad (8)$$

The above equation can be further be expressed by taking the square-root of both sides as:

$$x_{egc} = \sum_{j=1}^L \frac{x_j}{\sqrt{L}} \quad (9)$$

In a similar context to MRC, the MGF for EGC is expressed as  $M_{x_{egc}} = \prod_{j=1}^L M_{x_j}$  ( $s/\sqrt{L}$ ). Now, following a similar approach to MRC, and with the aid of [13], the SNR distribution around the origin is deduced as:

$$f_{Y_{egc}}(\gamma) \approx \frac{\alpha^L (\Gamma(c))^L (\sqrt{L})^{Lc}}{2\Gamma(Lc)\rho^{\frac{Lc}{2}}} \gamma^{\frac{Lc}{2}-1} \quad (10)$$

where  $\alpha = Ace^{-\frac{\mu c}{2}} e^{\frac{\sigma^2 c^2}{8}}$ .

### 2.3 Selection Combining Probability Density Function at the Origin

The simplest approach for combining the signals from the channel branches is the SC method. From the practical point of view, this algorithm has the easiest implementation. In this, the output or branch is picked which has the highest SNR which can be defined mathematically as  $Y_{sc} = \max(Y_j), j = 1, 2, \dots, L$ . The PDF of the output SNR is defined as [16]:

$$f_{Y_{sc}} = L(F_Y(\gamma))^{L-1}f_Y(\gamma) \quad (11)$$

where  $F_Y(\gamma)$  is the cumulative distribution function (CDF). The CDF can be obtained by substituting (4) in the definition  $F_Y(\gamma_{th}) = P(Y < \gamma_{th})$  [16] and after some straight forward mathematical simplification:

$$F_Y(\gamma_{th}) = \frac{\vartheta \gamma_{th}^{\frac{c}{2}}}{\frac{c}{2} \rho^{\frac{c}{2}}} \quad (12)$$

Further, substituting (4) and (12) into (11) results in the closed-form expression of the SC distribution:

$$f_{Y_{sc}}(\gamma) \approx \frac{L\vartheta^L}{\left(\frac{c}{2}\right)^{L-1} \rho^{\frac{Lc}{2}}} \gamma^{\frac{Lc}{2}-1} \quad (13)$$

## 3 Average Symbol Error Probability Analysis

In this section, we analyse the performance of the composite fading channel over average SEP for both coherent and non-coherent modulation schemes. The general expression of the average SEP over a fading channel is obtained by taking an ensemble average of the instantaneous error probability over the fading distribution. The general expression of the average SEP over a fading channel is given by [16]:

$$\bar{P}_e = \int_0^{\infty} P_e(\gamma) f_Y(\gamma) d\gamma \quad (14)$$

where  $P_e(\gamma)$  is the instantaneous symbol error rate (SER) of the modulation technique.

### 3.1 Coherent Average Symbol Error Probability

The generalized probability of error for coherent modulation schemes is given by [11, Eq. (17)]:

$$P_e(\gamma) = A_p \operatorname{erfc}(\sqrt{B_p \gamma}) \quad (15)$$

where constants  $A_p$  and  $B_p$ , for different modulation techniques, are given in [11, Table I] for various constellation size.  $\operatorname{erfc}(\cdot)$  is the complementary error function and is defined as  $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2) dt$ .

### 3.1.1 Average Symbol Error Probability for Maximal Ratio Combining

By substituting (7) and (15) in (14), letting  $t = \sqrt{B_p \gamma}$ , and using [17, Eq. (2.8.2.1)], the asymptotic average SEP can be obtained as:

$$\bar{P}_{e,coh}^{mrc,asy} \approx \frac{2A_p \vartheta^L (\Gamma(\frac{c}{2}))^L \Gamma(\frac{Lc+1}{2})}{cL \sqrt{\pi} (B_p \rho)^{\frac{cL}{2}} \Gamma(\frac{Lc}{2})} \quad (16)$$

The result of the asymptotic average SEP can also be expressed in terms of coding gain ( $G_c$ ) and diversity gain ( $G_d$ ), i.e.,  $P_e^{asym} \approx (G_c \bar{\gamma})^{-G_d}$  [8, eq. (1)] as:

$$G_d = \frac{Lc}{2} \quad G_c = \left( \frac{2A_p \vartheta^L (\Gamma(\frac{c}{2}))^L \Gamma(\frac{Lc+1}{2})}{cL \sqrt{\pi} B_p^{\frac{cL}{2}} \Gamma(\frac{Lc}{2})} \right)^{-\frac{2}{cL}} \quad (17)$$

### 3.1.2 Average Symbol Error Probability for Equal Gain Combining

By substituting (10) and (15) in (14), and following a similar procedure as defined above, it follows immediately that:

$$\bar{P}_{e,coh}^{egc,asy} \approx \frac{A_p \alpha^L (\Gamma(c))^L (\sqrt{L})^{Lc} \Gamma(\frac{Lc+1}{2})}{\sqrt{\pi} \Gamma(Lc+1) (\rho B_p)^{\frac{cL}{2}}} \quad (18)$$

Diversity and coding gain are expressed as:

$$G_d = \frac{Lc}{2} \quad G_c = \left( \frac{A_p \alpha^L (\Gamma(c))^L (\sqrt{L})^{Lc} \Gamma(\frac{Lc+1}{2})}{\Gamma(Lc+1) \sqrt{\pi} B_p^{\frac{cL}{2}}} \right)^{-\frac{2}{cL}} \quad (19)$$

### 3.1.3 Average Symbol Error Probability for Selection Combining

By substituting (13) and (15) in (14), and following a similar procedure as defined in Sect. 3.1.1, it follows immediately that:

$$\bar{P}_{e,coh}^{sc,asy} \approx \frac{2A_p \vartheta^L \Gamma(\frac{Lc+1}{2})}{c \sqrt{\pi} (\frac{c}{2})^{L-1} (\rho B_p)^{\frac{Lc}{2}}} \quad (20)$$

The values of diversity and coding gain are expressed as:

$$G_d = \frac{Lc}{2} \quad G_c = \left( \frac{2A_p \vartheta^L \Gamma(\frac{Lc+1}{2})}{c \sqrt{\pi} (\frac{c}{2})^{L-1} B_p^{\frac{Lc}{2}}} \right)^{-\frac{2}{cL}} \quad (21)$$

### 3.2 Non-coherent Average Symbol Error Probability

The instantaneous SEP for different non-coherent modulation schemes is given by [12, Eq. (18)]:

$$P_e(\gamma) = A_n \exp(-B_n \gamma) \quad (22)$$

where the parameters  $A_n$  and  $B_n$  are defined in [12, Table 2].

#### 3.2.1 Average Symbol Error Probability Maximal Ratio Combining

The asymptotic average SEP is derived by substituting (7) and (22) in (14), which with the aid of [15, Eq. (3.381.4)], yields:

$$\bar{P}_{e,non}^{mrc,asy} \approx \frac{A_n (\vartheta)^L (\Gamma(\frac{c}{2}))^L}{(B_n \rho)^{\frac{Lc}{2}}} \quad (23)$$

The diversity and coding gain are expressed as:

$$G_d = \frac{Lc}{2} \quad G_c = \left( \frac{A_n (\vartheta)^L (\Gamma(\frac{c}{2}))^L}{B_n^{\frac{Lc}{2}}} \right)^{-\frac{2}{cL}} \quad (24)$$

#### 3.2.2 Average Symbol Error Probability Equal Gain Combining

The closed-form asymptotic solution to average SEP is derived by substituting (10) and (22) in (14), and repeating similar steps to those defined above:

$$\bar{P}_{e,non}^{egc,asy} \approx \frac{A_n \alpha^L (\Gamma(c))^L (\sqrt{L})^{Lc} \Gamma(\frac{Lc}{2})}{2 \Gamma(Lc) (\rho B_n)^{\frac{Lc}{2}}} \quad (25)$$

Diversity and coding gain are expressed as:

$$G_d = \frac{Lc}{2} \quad G_c = \left( \frac{A_n \alpha^L (\Gamma(c))^L (\sqrt{L})^{Lc} \Gamma(\frac{Lc}{2})}{2\Gamma(Lc) B_n^{\frac{cL}{2}}} \right)^{-\frac{2}{cL}} \quad (26)$$

### 3.2.3 Average Symbol Error Probability Selection Combining

The closed-form asymptotic solution is evaluated by substituting (13) and (22) in (14), and repeating similar steps to those defined in Sect. 3.2.1:

$$\bar{P}_{e,non}^{sc,asy} \approx \frac{L A_n \vartheta^L \Gamma(\frac{Lc}{2})}{(\frac{c}{2})^{L-1} (\rho B_n)^{\frac{Lc}{2}}} \quad (27)$$

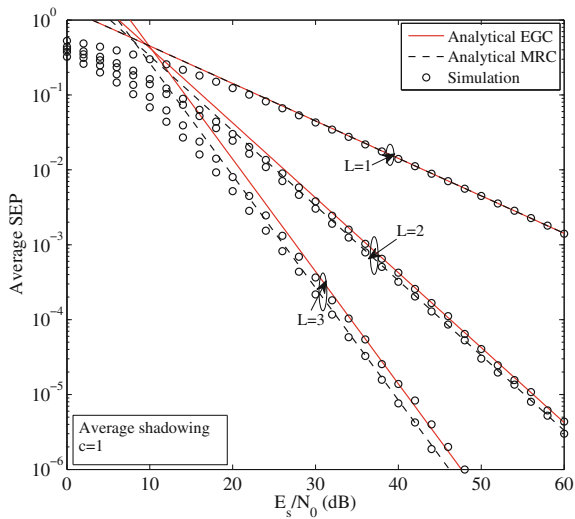
The values of diversity and coding gain are expressed as:

$$G_d = \frac{Lc}{2} \quad G_c = \left( \frac{L A_n \vartheta^L \Gamma(\frac{Lc}{2})}{(\frac{c}{2})^{L-1} B_n^{\frac{cL}{2}}} \right)^{-\frac{2}{cL}} \quad (28)$$

## 4 Numerical Analysis

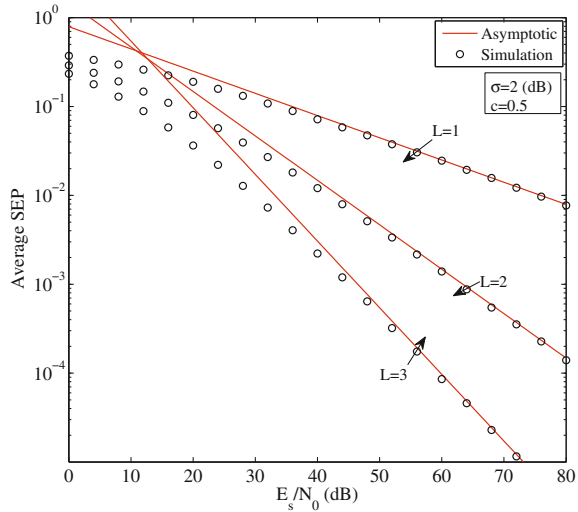
In this section, the asymptotic behavior of the average SEP for the WLN fading channel has been presented graphically. The Monte-Carlo simulations are also included in all the figures to validate the accuracy of the derived expressions.

**Fig. 1** Average SEP for MPAM with the MRC and EGC diversity scheme and constellation size  $M = 4$

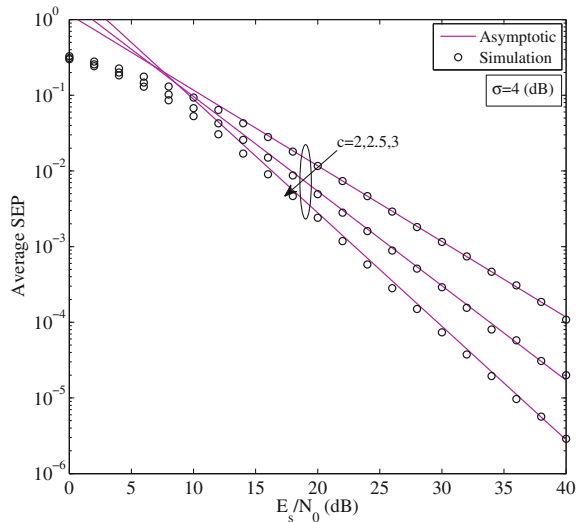




**Fig. 2** Average SEP for BPSK with the SC diversity scheme



**Fig. 3** Average SEP for non-coherent DBPSK with  $c = 2, 2.5, 3$



In Fig. 1, asymptotic plots of the average SEP for coherent M-ary pulse amplitude modulation (MPAM), with MRC (16) and EGC (18) side by side, are presented against  $E_s/N_0$ . The parameters under consideration are  $c = 1$ , infrequent light shadowing [18, 19], constellation size  $M = 4$ , and diversity order  $L = 1, 2, 3$ . It is clear from the figure that the asymptotic plot converges at high SNR and coincides with Monte-Carlo simulations. It is also observed from the plot that MRC is superior to EGC for all the diversity schemes, and the separation increases with increase in diversity order. The average SEP for coherent binary phase shift keying (BPSK) versus  $E_s/N_0$  is given in Fig. 2 with the SC diversity scheme. The Monte-Carlo simulations

are also included and shown to coincide with the closed-form solution at high SNR. In Fig. 3, the plot illustrates the non-coherent differential BPSK (DBPSK) (23) scheme versus  $E_s/N_0$ . As expected, it is revealed from the figure that increasing parameter  $c$  means that system performance improves.

## 5 Conclusion

The closed-form expressions of diversity PDF at the origin for the composite W-LN fading channel have been presented. The derived results were then extended to evaluate the asymptotic expressions of the average SEP for both coherent and non-coherent modulation schemes. It was shown that the asymptotic plot merges with Monte-Carlo simulations at high SNRs, verifying the accuracy of the derived expressions.

## References

1. Sagias NC, Karagiannidis GK, Bithas PS, Mathiopoulos PT (2005) On the correlated Weibull fading model and its applications. In: Ieee transactions on vehicular technology conference, pp 2149–2153
2. Cheng J, Tellambura C, Beaulieu NC (2004) Performance of digital linear modulations on weibull slow-fading channels. *IEEE Trans Commun* 52(8):1265–1268
3. Ibdah Y, Ding Y (2015) Mobile-to-mobile channel measurements at 1.85 GHz in suburban environments. *IEEE Trans Commun* 63(2):466–475
4. Bessate A, Bouanani FEL (2016) A very tight approximate results of MRC receivers over independent Weibull fading channels. *Phys Commun* 21:30–40
5. Khandelwal V (2014) Karmeshu: a new approximation for average symbol error probability over log-normal channels. *IEEE Wirel Commun Lett* 3(1):58–61
6. Navidpour SM, Uysal M, Kavehrad M (2007) BER performance of free-space optical transmission with spatial diversity. *IEEE Trans Wirel Commun* 6(8):2813–2819
7. Héliot F, Xiaoli C, Hoshyar R, Tafazolli R (2009) A tight closed-form approximation of the log-normal fading channel capacity. *IEEE Trans Wirel Commun* 8(6):2842–2847
8. Wang Z, Giannakis GB (2003) A Simple and General Parametrization Quantifying Performance in Fading Channels. *IEEE Trans Commun* 51(8):1389–1398
9. Chen Z, Chi Z, Li Y, Vucetic B (2009) Error performance of maximal-ratio combining with transmit antenna selection in flat Nakagami- $m$  fading channels. *IEEE Trans Wirel Commun* 8(1):424–431
10. Zhang H, Matthaiou M, Tan Z, Wang H (2012) Performance Analysis of digital communication systems over composite  $\eta\mu/\Gamma$  fading channels. *IEEE Trans Veh Technol* 61(7):3114–3124
11. Badarneh OS, Aloqlah MS (2016) Performance analysis of digital communication systems over  $\alpha - \eta - \mu$  fading channels. *IEEE Trans Veh Technol* 65(10):7972–7982
12. Chauhan PS, Tiwari D, Soni SK (2017) New analytical expressions for the performance metrics of wireless communication system over weibull/lognormal composite fading. *Int J Electron Commun (AEU)* 82:397–405
13. Simon MK, Alouini M (2005) Digital communication over fading channels, (2nd ed.), New York, Wiley

14. Shanker PM (2004) Error rates in generalized shadowed fading channels. *Wirel Person Commun* 28:233–238
15. Gradshteyn IS, Ryzhik IM (2007) *Table of integrals, series, and products*. (7th ed.), Academic Press, California
16. Rana V, Chauhan PS, Soni SK, Bhatt M (2017) A new closed-form of ASEP and channel capacity with MRC and selection combining over Inverse Gaussian shadowing. *Int J Electron Commun (AEU)* 74:107–115
17. Prudnikov AP, Brychkov YA, Marichev OI (1986) *Integrals and Series Volume 2: Special Functions*, 1st edn. Gordon and Breach Science Publishers
18. Loo C (1985) A statistical model for a land mobile satellite link. *IEEE Trans Veh Technol* 34:122–127
19. Loo C (1990) Digital transmission through a land mobile satellite channel. *IEEE Trans Commun* 38:693–697